Upper Bounds For the Correlated Bayesian Information Filtering Problem

2015 Applied Probability Conference

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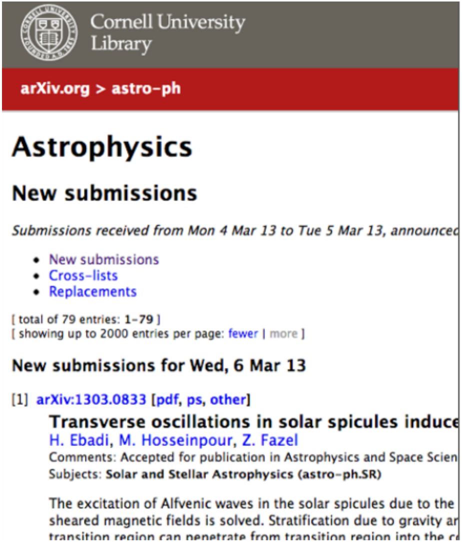
July 6th, 2015

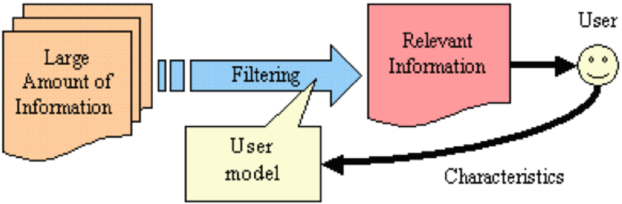
Information Filtering



- There are lots of new papers (roughly 80 new papers/day in astrophysics.
- Problem 1: Browsing this many papers is a lot of work for researchers.
- Problem 2: Researchers still miss important developments.

Information Filtering

- Information filtering systems automatically distinguish relevant from irrelevant items (emails, news articles, intelligence information) in large information streams.



Exploration vs. Exploitation

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- Exploration vs. exploitation tradeoff.

Traditional Approach

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    - Exploration and exploitation periodically.
- We present a Bayesian sequential decision-making formulation of this problem.

Our Results

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- An implementable policy, called Decompose Then Decide (DTD).
- Prove our upper bound is asymptotically tight.

Bayesian Model

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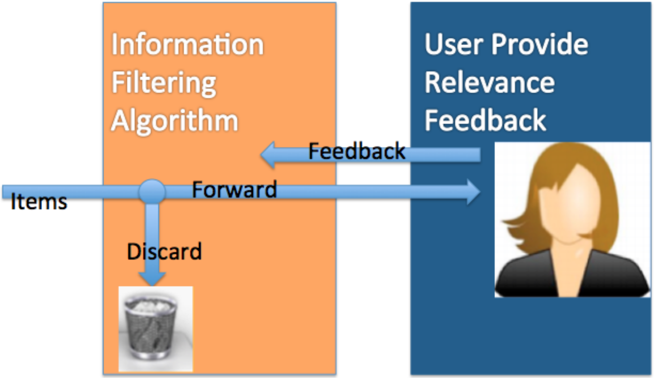
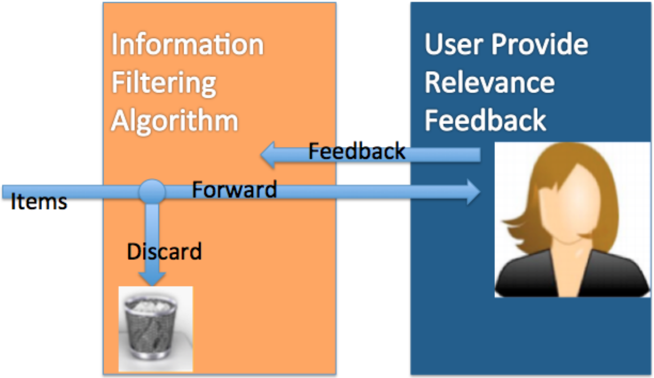


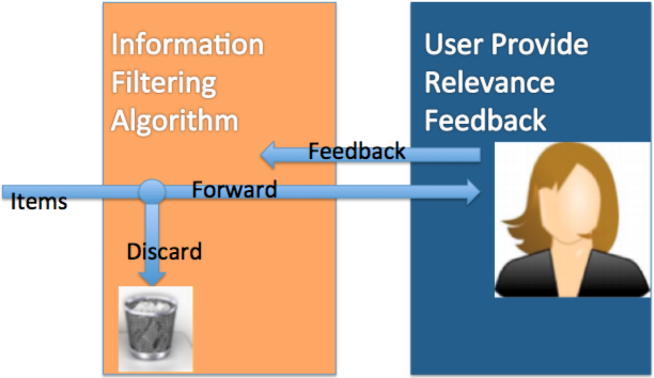
Bayesian Model

- We consider information filtering for a single user.
- The n^{th} arriving item is described by a k-dimensional feature vector $X_n = \{x_{1,n}, \cdots, x_{k,n}\}$. We assume that $x_{i,n} \geq 0$ for all i and n and $\sum_{i=1}^{k} x_{i,n} = 1$.



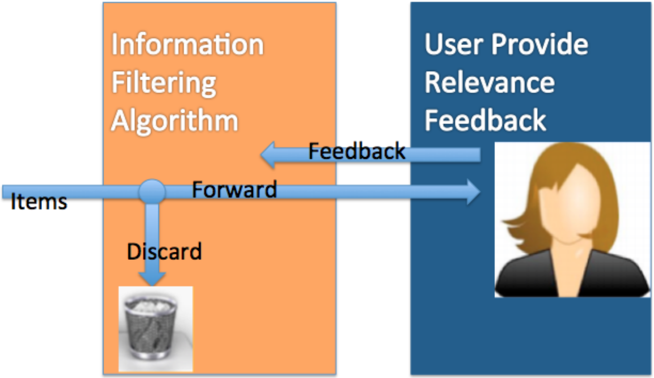
Bayesian Model

- Let $U_n \in \{0,1\}$ represent this decision for the n^{th} item, where 1 means to forward and 0 means not to forward.



Bayesian Model

- Let $U_n \in \{0,1\}$ represent this decision for the n^{th} item, where 1 means to forward and 0 means not to forward.
- Each time the system forwards, it pays a constant cost c and receives the item's relevance Y_n as a reward.



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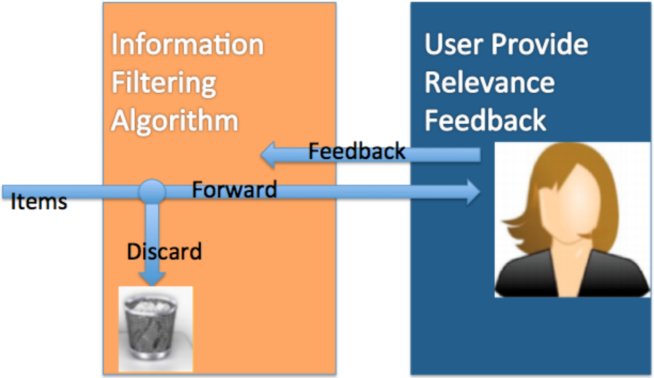
- Let $\theta = (\theta_1, \cdots, \theta_k)$ denote the single user's latent preference vector for the k different features.



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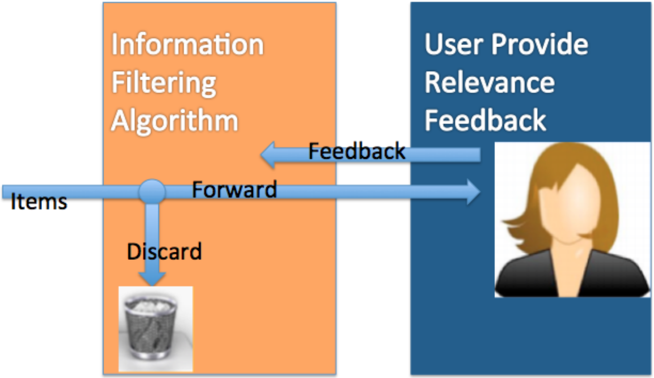
- Let $\theta = (\theta_1, \cdots, \theta_k)$ denote the single user's latent preference vector for the k different features.
- We assume $\theta \sim N(\mu_0, \Sigma_0)$.



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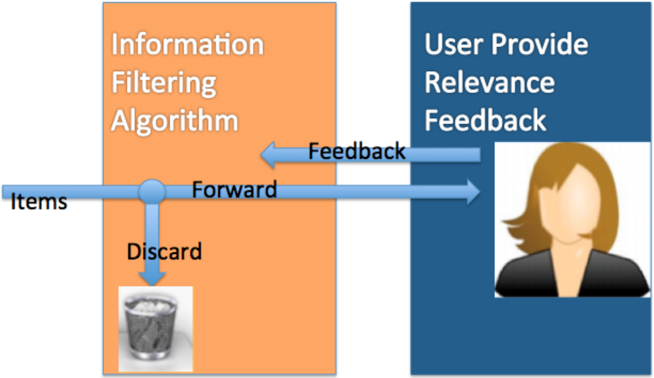
- We assume $\theta \sim N(\mu_0, \Sigma_0)$.
- This relevance is modeled as $Y_n = \theta \cdot X_n + \epsilon_n$, where $\epsilon_n \sim N(0, \lambda^2)$ (Similar in spirit to Bayesian linear bandit).



Our Goal

- The decision of whether or not to forward the n^{th} item can only depend on our current X_n as well as the previous information $H_{n-1} = (U_m, X_m, U_m Y_m : m \leq n-1)$.

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- Mximize the following:

$$\sup_{\pi \in \Pi} E^{\pi}\left[\sum_{n=1}^{N} U_n(Y_n - c)\right] \tag{1}$$

where E^{π} denotes the expected reward using policy π.

1. Motivation
2. Problem Formulation
3. Main Theoretical Results
4. Numerical Experiments
5. Conclusion

Upper Bounds

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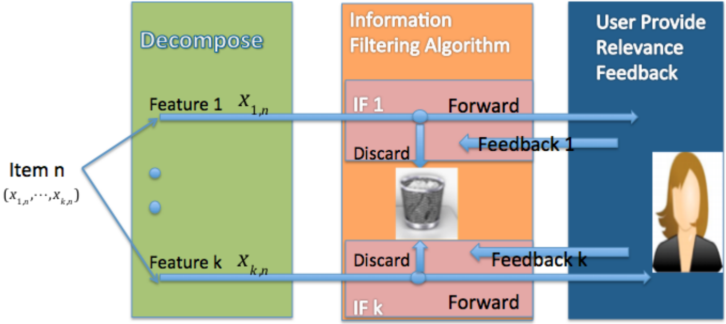
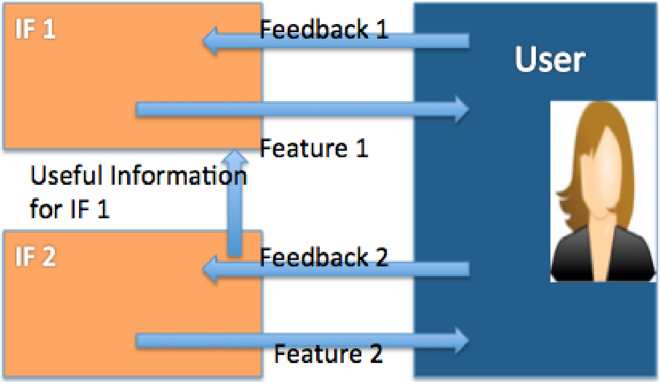
- Our idea is:



Why we need information relaxation?



The maximum information Feedback2 can contain is θ_2, which corresponds to the user's latent preference vector for the second feature.

Upper Bounds

- Define $Y_{i,n} = \theta_i + \epsilon_n^i$ and $\epsilon_n^i \sim N(0, \lambda^2)$ for $i = 1, 2, \cdots, k$. We may think of $Y_{i,n}$ as the reward that we would have seen if X_n were equal to e_i.

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- Rewrite $Y_n = \sum_{i=1}^k x_i\theta_i + \epsilon_n = \sum_{i=1}^k x_i(\theta_i + \epsilon_n^i) = \sum_{i=1}^k x_i Y_{i,n}$.

Upper Bounds

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- For each feature j, we now introduce a new set of policies Π_j and the decision in Π_j may depend on $\theta \cdot e_i$ for $\forall i \neq j$.

Upper Bounds

Theorem 1

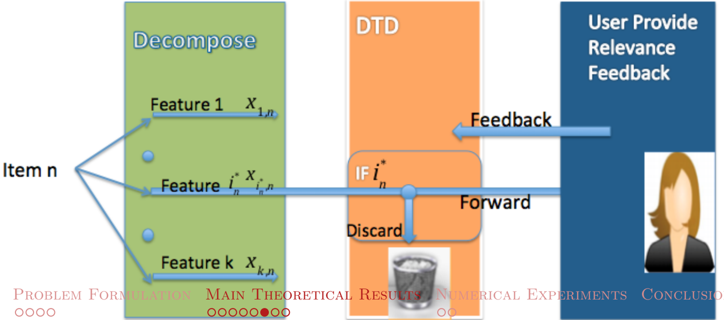
$$\sup_{\pi \in \Pi} E^{\pi}\left[\sum_{n=1}^{N} U_n(Y_n - c)\right] \leq \sum_{j=1}^{k} \sup_{\pi'' \in \Pi_j} E^{\pi''}\left[\sum_{n=1}^{N} U_{j,n}(x_{j,n} Y_{j,n} - x_{j,n} c)\right].$$

Decompose Then Decide (DTD)

DTD

Let π_i^* denote the optimal policy for the i^{th} single-feature subproblem. Then, DTD is defined as follows:

1. Find $i_n^* \in \arg\max\{X_n e_i, i = 1, \cdots, k\}$.
2. Set $U_n = \pi_{i_n^*}^*(\mu_{i_n^*}, \sigma_{i_n^*})$.
3. Update μ, Σ respectively.



Asymptotic optimality

Let $(\mathbb{P}^l : l = 1, 2, \cdots)$ be a sequence of probability distribution on R^k with $\mathbb{P}^l \to \mathbb{P}_*$ in distribution, where $\mathbb{P}_*$ has support on the unit vectors.

- Let $U(\mathbb{P})$ denote the upper bound from Theorem 1 in which $X_n \sim \mathbb{P}$.
- Let $V(\mathbb{P})$ denote the value of an optimal policy (i.e., the value function), and $V^{DTD}(\mathbb{P})$ as the value of the DTD policy, for this same problem instance.

Asymptotic optimality

Theorem 2

Suppose that Σ_0 is a diagonal matrix. Then

$$\lim_{l\to\infty} (U(\mathbb{P}^l) - V(\mathbb{P}^l)) = 0,$$

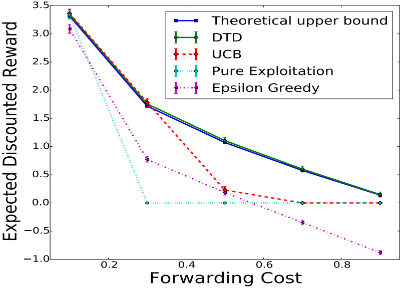
$$\lim_{l\to\infty} V(\mathbb{P}^l) - V^{DTD}(\mathbb{P}^l) = 0,$$

$$\lim_{l\to\infty} V(\mathbb{P}^l) = U(\mathbb{P}_*) = V(\mathbb{P}_*).$$

Numerical Experiments

- Simulated Data.
- Yelp Academic Dataset.

Numerical Experiments



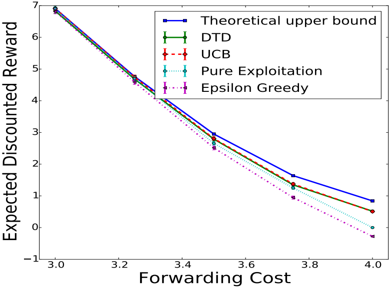


Figure : The left plot compares performance on simulated data and the right plot compares performance on the Yelp academic dataset.

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Conclusion

- An instance-specific computational upper bound.
- An implementable policy (DTD).
- Prove asymptotically optimality.

Question?

Thanks for your time!