This exam starts at 10 AM and lasts until 12:00 PM, and it consists of 10 problems. **Please upload a PDF version of the exam solution to Gradescope by 12:10 PM** The upload link will expire at this time and a submission will no longer be possible. Do not email me your solutions. They will not be accepted. If you have any questions, I will be in our regular online classroom for the duration of the exam.

You must solve these problems on your own! You may not collaborate with anyone during the solving of the exam. However, you are allowed to use the course book, lecture notes and a simple calculator.

Do not share this exam with anyone and do not post this exam to any online sites. Such an action is considered academic dishonesty and it will be treated as such.

[**Problem 1**] Formally prove or disprove the following claims

a) $\log_3(n^3)$ is $O(n)$, for $n \geq 1$ (10 Points)

b) 3^n is $\Omega(n^k)$ for any real constant $k \geq 1$ (10 Points)

[**Problem 2**] Using substitution method only, prove that

a) $T(n) = 6T(n/7) + 4n$ is $O(n)$ (10 Points)

b) $T(n) = T(n-1) + 16n$ is $O(n^2)$ (10 Points)

Any other method will receive zero points.

[**Problem 3**] Formally formally **prove** or **disprove** the following claims, using any method

a) $T(n) = 4T(n/2) + 1$ is $\Theta(n^2)$ (10 Points)

b) $T(n) = 3T(n/5) + 3$ is $\Theta(n^{\log_5(3)})$ (10 Points)

[**Problem 4**] Assuming that $n \geq 1$, formally **prove** or **disprove** the following claims

a) $n^2 - 6n + 3$ is $O(n)$ (10 Points)

b) 5^n is $O(4^n)$ (10 Points)

c) $5n(log(n))^3$ is $O(n^{4/3})$ (10 Points)

[**Problem 5**] Consider a 1-dimensional integer array A, with random entries, and a decision problem $P = \{A\ is\ an\ integer\ array\ with\ n\ elements\ :\ integer\ x \in A.\}$ Prove that problem P is in NP. (10 Points)

[**Problem 6**] Consider an undirected graph $G = (V, E)$, in which each edge has the *identical* weight. Consider a minimum spanning tree of the graph G, called $T = (V, E')$. For each question below, draw a picture and then formally show that

a) there is a unique path between u and v in T for all $u, v \in V$. (10 Points)

b) tree T is not unique. (10 Points)

You must provide a formal proof AND an illustrative picture and to receive points!

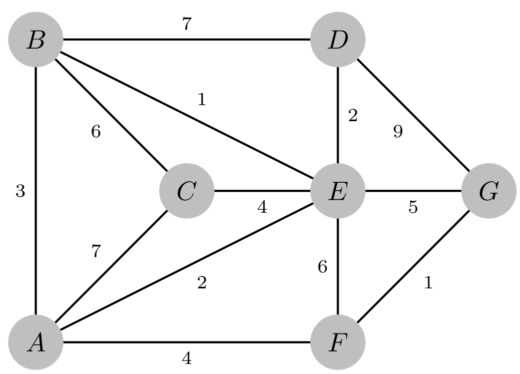
[**Problem 7**] Consider a 3CNF Boolean formula $\phi(x_1, x_2, ..., x_k)$, containing n clauses, and a certificate C for $\phi(x_1, x_2, ..., x_k)$. Describe *formally* an algorithm which can verify certificate C in polynomial time. Hint: Recall that a certificate is a variable assignment that makes ϕ evaluate to 1. (20 Points)

[Problem 8] Let F denote an algorithm with two input arguments A and B. Argument A represents an *unsorted* 2-dimensional integer array with n elements (for example, A = [[5,2],[8,1],[3,6], ...]), and B represents a *sorted*, standard, 1-dimensional integer array with m elements.

```
F(A, B)
  k = 0
  n = len(A)
  for i from 1 to n
      x = A[i][0]+A[i][1]
      if x is not contained in B
         k = k+x
  return k
```

What is the fastest running time for this algorithm, and give its upper bound (i.e $O()$). That is, give the upper bound for the running time of this algorithm for the most efficient implementation of this algorithm and properly justify your answer. Hint: Not all implementations lead to the fastest running time. (20 Points)

[Problem 9] Starting at vertex A, run Prim's algorithm on the following graph. You do not need to draw all steps but you must display the final picture of the generated minimum spanning tree and the order of inclusion of edges into the tree, using standard notation (u,v) to represent an edge between hypothetical vertices u and v, in order to receive points. (20 points)



[Problem 10] Run the Dijkstra's algorithm on the following directed graph and find all shortest paths between vertex A and all others. (20 points)