Homework: Lambda Calculus

Learning Objectives:

1. Understand evaluation order
2. Understand church encoding
3. Learn to perform β-reduction

Instructions:

- Total points: 47 pt
- Early deadline: Mar 31 (Wed) at 11:59 PM; Regular deadline: Apr 2 (Fri) at 11:59 PM (you can continue working on the homework till TA starts to grade the homework).
- Submit one pdf file on Canvas under Assignments, Homework 6 submission. You are encouraged to use latex. But we will accept a scanned copy as well.

Questions:

1. (9 pt) [β-reduction] Perform β-reduction for the following λ expressions.
   (a) (3 pt) $(((\lambda(x)(\lambda(y)(x\ y)))((\lambda(a)\ a)\ a))((\lambda(b)\ b)\ b))$
   (b) (3 pt) $(((\lambda(x)(\lambda(y)(y)))((\lambda(z)\ z)\ a))\ b)$
   (c) (3 pt) $(((\lambda(x)(x\ x))(\lambda(y)\ y))\ x)$
2. (4 pt) [Evaluation order] The goal of this problem is to help you understand the evaluation order of lambda calculus.
   In the following, show the steps of β-reduction for the lambda expression using two types of evaluation orders
   $$((\lambda(x)\ p)((\lambda(y)(y\ y))(\lambda(z)(z\ z))))$$
3. (7 pt) [Church Encoding] Encode the following logic Boolean operations using the encoding of *true*, *false*, *ite*, *not* and *or* provided in the lecture.
   (a) (3 pt) *and a b*
   (b) (4 pt) $a \rightarrow b$
4. (16 pt) [Church Encoding and understanding the semantics of lambda expressions] Using *zero*, *one* and *two* as well as *succ*, *true* and *false* provided in the lecture, answer the following two questions:
   (a) (4 pt) What is the result of $((\lambda(z)((one\ f)\ z))\ (succ\ zero))$?

(b) Suppose we define *unknown*: $(\lambda(x)(\lambda(y)(\lambda(z)\ z)))$ and *g*: $(\lambda(n)((n\ unknown)\ true))$, what is the result of:

i. (4 pt) $(g\ zero)$
ii. (3 pt) $(g\ one)$
iii. (2 pt) $(g\ two)$
iv. (3 pt) What mathematical/logical operation is computed by g?

5. (11 pt) [Church Encoding, understanding the semantics of lambda expressions] Given:

data: $(\lambda(x)\ (\lambda(y)\ (\lambda(z)\ ((z\ x)\ y))))$
op1: $(\lambda(p)\ (p\ (\lambda(x)\ (\lambda(y)\ x))))$
op2: $(\lambda(p)\ (p\ (\lambda(x)\ (\lambda(y)\ y))))$
true: $(\lambda(x)\ (\lambda(y)\ x))$
false: $(\lambda(x)\ (\lambda(y)\ y))$

(a) (4 pt) What is the result of $(op1\ ((data\ a)\ b))$?

(b) (4 pt) What is the result of $(op2\ ((data\ a)\ b))$?

(c) (3 pt) What computation do *op*1 and *op*2 perform?