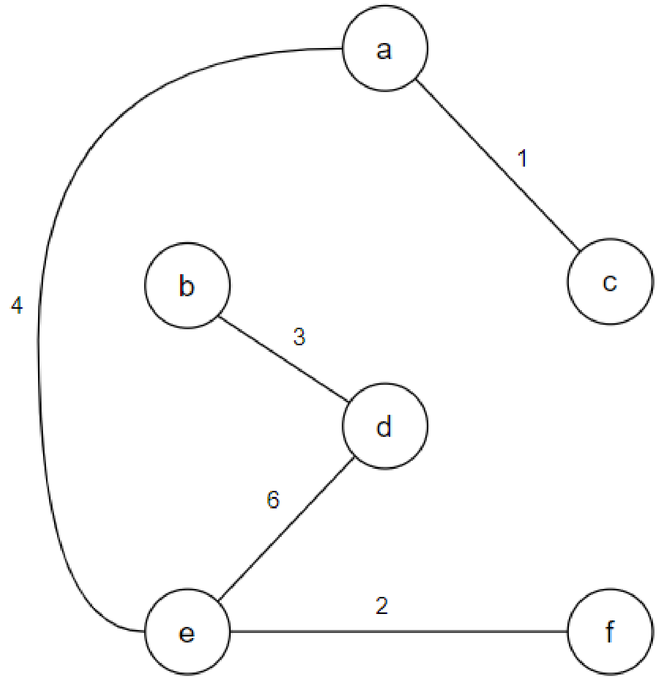
1.

a.



b.

	1	2	3	4	5
Kruskal's	(a, c)	(e, f)	(b, d)	(a, e)	(d, e)
Prim's	(d, b)	(a, c)	(e, d)	(a, e)	(e, f)

c.

Suppose there are two distinct MSTs A=(V, E1) and B=(V, E2).

Since A and B are distinct, the sets E1 - E2 and E2 - E1 are not empty.

let e1 be the one with least weight; this choice is unique because the edge weights are all distinct. Without loss of generality, assume e1 is in A. (e1 $\in$ E1 - E2)

As B is an MST, {e1} $\cup$ B must contain a cycle C.

As a tree, A contains no cycles, therefore C must have an edge e2 that is not in A.

If e2 = e1 then e2 $\in$ E1 (because e1 $\in$ E1 - E2)
If e2 $\neq$ e1 then e2 $\in$ E2

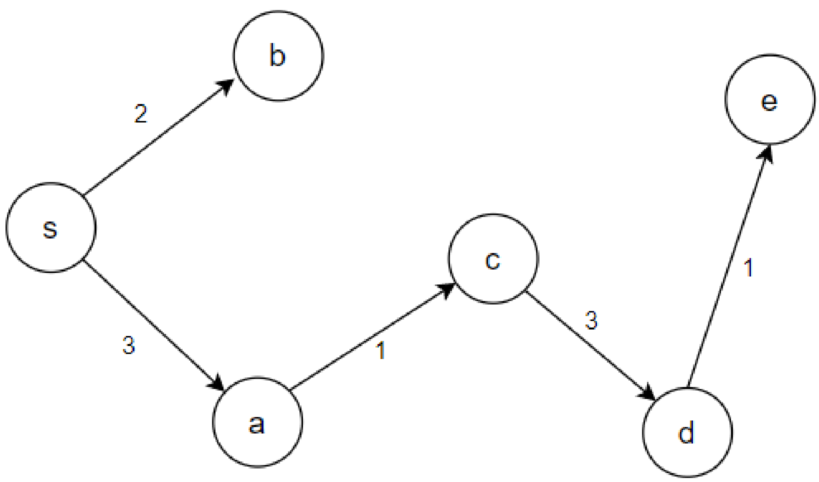
So, e2 is not in any MST. This contradicts the assumption that B is an MST.

2.

a.

iter	d[ ]						Selected node
	s	a	b	c	d	e	
0	0	3	2	6			s
1	0	3	2	4	10		a
2	0	3	2	4	10	11	b
3	0	3	2	4	7	11	c
4	0	3	2	4	7	8	d
5	0	3	2	4	7	8	e
6	0	3	2	4	7	8	

b.



c.

Suppose there is Vz≠Vk, the shortest path distance from s->k-closest is (Vz, k-closest).

So distance of Vz equals to distance of Vk because the distance from s→ v is unique and k-closest → v also is unique.

It contradicts “the shortest path distances in G from a source s ∈ V to each vertex v ∈ V are unique.”

So Vz = Vk, eth shortest path from the source vertex s ∈ V to a k-closest vertex x ∈ V consists only of vertices in Vk.

3.

Let M1 be the TM that can be solved in polynomial time.

Construct a TM M2 that can be solved the reverse complement of L in polynomial time:

with input M1:

Run M2 on w

if M2 accepts

then

reject

if M2 rejects

then

accept

M1 decides the reverse complement of L. Because M1 runs in polynomial time, M2 also runs in polynomial time.

4.

To prove the String transformation is in NP, there is an algorithm that solves in polynomial time.

Assume that the string transformation can be solved in O(|x|^k)
Language L belongs to NP only if there exists a polynomial-time algorithm A and constant k.

There exists a certificate y with O(|x|^k) runtime such that an inversion operation and deletion operation.

So, each language L in NP, there is an algorithm A that can verify each string x which is converted into y is L in polynomial time in k steps, given a certificate y with length

O(|x|^k).

The algorithm is try all possible S $\in$ {0,1}* with |y| = O(|x|^k).

So this problem is in NP

5.

I do not know how to solve this problem