CS 401-Computer Algorithms I (Prof. Asudeh)
Homework 2
Deadline: 3/2/22

February 23, 2022

1 Problem 1 (20 pts)

For every statement below, decide if the statement is true or false. If it is true, give a short explanation. If it is false, give a counter example. Assume that the graph $G = (V, E)$ is undirected and connected. Do not assume that all the edge weights are unique unless stated.

- (a) If every edge weight in G is unique, then G has an unique MST.
- (b) Suppose a graph G has a lightest edge e. Then e is present in all MSTs of G.

2 Problem 3 (20 pts)

KT (Chapter 4, Exercise 9): One of the basic motivations behind the Minimum Spanning Tree Problem is the goal of designing a spanning network for a set of nodes with minimum total cost. Here we explore another type of objective: designing a spanning network for which the most expensive edge is as cheap as possible. Specifically, let $G = (V, E)$ be a connected graph with n vertices, m edges, and positive edge costs that you may assume are all distinct. Let $T = (V, E)$ be a spanning tree of G; we define the bottleneck edge of T to be the edge of T with the greatest cost. A spanning tree T of G is a minimum-bottleneck spanning tree if there is no spanning tree T of G with a cheaper bottleneck edge.

- (a) Is every minimum-bottleneck tree of G a minimum spanning tree of G? Prove or give a counterexample.
- (b) Is every minimum spanning tree of G a minimum-bottleneck tree of G? Prove or give a counterexample.

3 Problem 3 (10 pts)

Show how to find the maximum spanning tree of a graph, that is, the spanning tree of largest total weight.

4 Problem 4 (30 pts)

Recall that everything is represented by binary codes in computer. For example, the ASCII code uses a fixed number of bits (one byte) to represent a character. For instance, the binary code for the letter ‘A’ is ‘01000001’. Suppose you are given a large text file, containing some characters. You can assign a variable number of bits to each character, as far as you can decode a binary file based on your coding. For example, assigning the code ‘0’ to ‘A’, ‘1’ to ‘B’ and ‘01’ to ‘C’ is not valid. Because then the sequence ‘0010’ can be interpreted as ‘AABA’ and ‘ACA’. However, the assignment ‘0’ for ‘A’, ‘10’ for ‘B’, and ‘11’ for ‘C’ is valid (assuming that there is no other character). Note that using this coding the sequence '0010' is uniquely decoded as 'AAB'. Following this coding, the text 'BBABCCA' is transformed into '101001011110' (requiring 12 bits). Given a large text, as a combination of n characters (such as 'a', 'b', 'c', ...), our goal is to find a valid coding that minimizes the number of bits to store the text (Huffman code).

- (i) Design an efficient algorithm for finding the optimal coding.
- (ii) Prove the algorithm is correct and finds the optimal solution.
- (iii) Given a Huffman coding and a binary file, design an efficient algorithm to decode the binary.

5 Problem 5 (20 pts)

In computer networking, broadcasting is to send a message to all nodes of the networks. A computer network can be modeled as a weighted graph where every edge (a,b,w) is shows a link from node a to node b with the (message-passing) cost w. It is easy to see that minimum spanning tree (MST) provides the broadcasting strategy with minimum cost (send messages across the edges of MST). The weights of the edges in the computer networks, however are dynamic. I.e., the weights of edges change arbitrarily. Assume that changes in the edge weights are provided in a streaming manner. An update request is in the form of update(a,b,w). That is, to change the weight of edge (a,b) to w.update(a,b,w):
...
updateMST(a,b,w) ...

- Design an efficient algorithm, updateMST(a,b,w), for updating the MST in this setting.