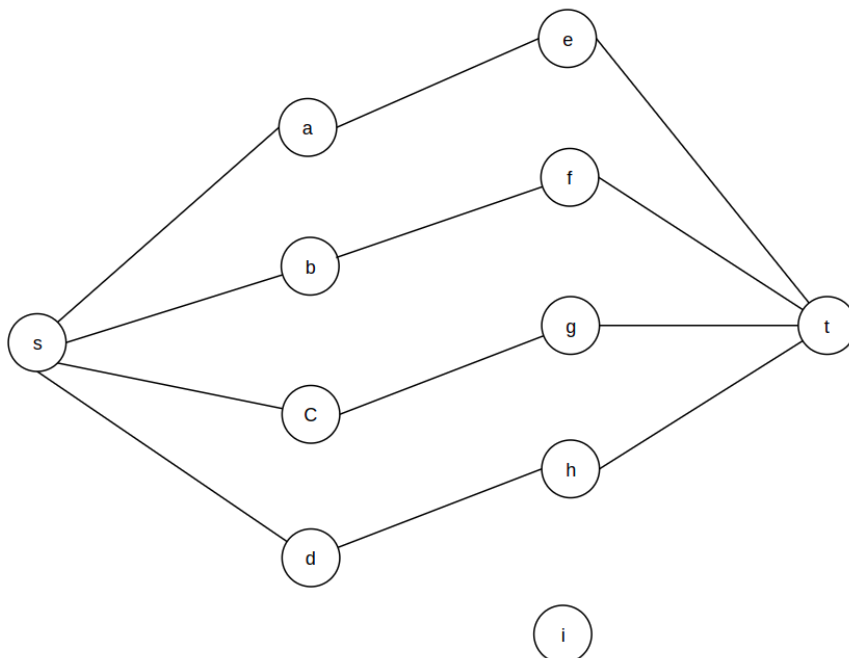
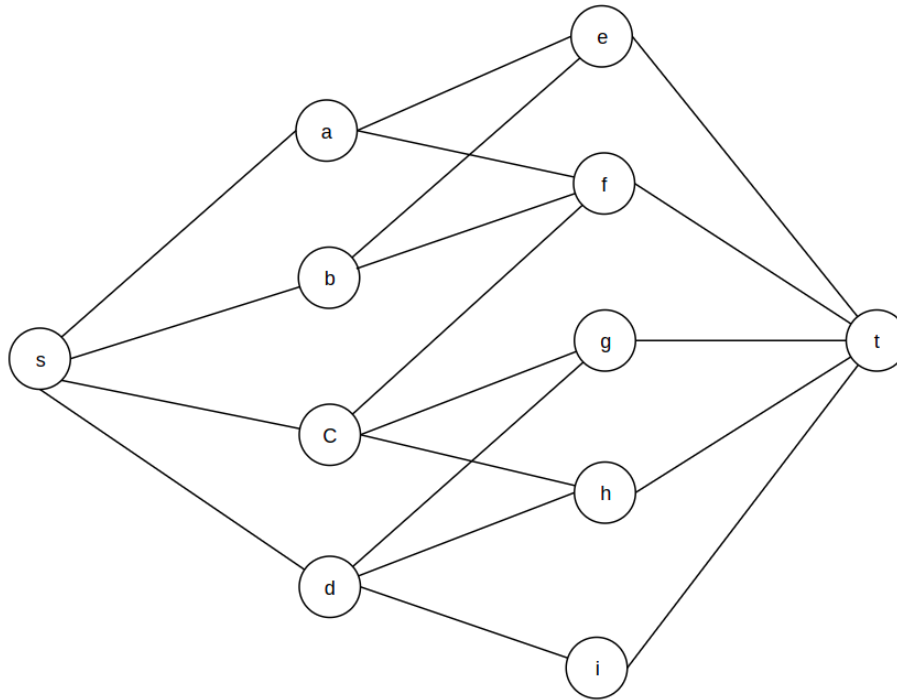


1.  
 flow:  $s \rightarrow v_1$ ,  $v_2 \rightarrow v_1$ ,  $v_4 \rightarrow v_3$ ,  $v_4 \rightarrow t$   
 $v_3 \rightarrow v_2$   
 capacity:  $11+1+4+7-4=19$

2.



3.

There is no edges between any two vertices of  $L, R$  in  $G'$  either because  $G$  is a bipartite graph.

In  $G'$ , The augmenting path (follow FORD-FULKERSON algorithm) has starting edge  $s$  and any one vertex in  $L$ . Similarly, the augmenting path has a last edge that connects any one vertex in  $R$  and the sink  $t$ . The remaining edges are the edges of  $G$  from  $L$  to  $R$ .

Because a bipartite graph may have a path of maximum length of  $2 \cdot \min(|L|, |R|) - 1$ , the remaining edges in the augmenting path are  $2 \cdot \min(|L|, |R|) - 1$ .

Therefore, the total length of an augment path =  $2 \cdot \min(|L|, |R|) - 1 + 2 = 2 \cdot \min(|L|, |R|) + 1$

=> The upper bound on the length of an augmenting path is  $2 \cdot \min(|L|, |R|) + 1$

4.

I think we can reduce 3sat from clique and reduce clique from independent-set

Reduce clique from Independent-set:

It takes polynomial time to verify a set is a clique or not,

Just iterate all vertexes in set to check no vertex has an out-degree.

$O(n^2)$  with  $n$  is number of vertices. So it belongs to NP. (1)

Clique: For Graph  $G=(V,E)$  and integer  $k$

Independent-set:  $G'=(V,E')$  and integer  $k'$

We have same vertices, Construct a graph  $G$  and  $G'$ .

If there is an independent set size  $k$  in  $G'$ , it implies no

2 vertices share an edge in  $G'$  which implies all vertices share an edge with all others in  $G$  forming a clique.

That is exist a clique size  $k$  in  $G$ .

If there is a clique size  $k$  in  $G$ , it implies all vertices share an edge with all others in  $G$ . which implies no 2 vertices share an edge in  $G'$  forming a independent-set.

That is exist a clique size  $k$  in  $G'$

So, clique can be reduced to independent-set. it belongs to NP-hard.(2)

(1) (2) -> it's NP-complete

Reduce 3sat from clique:

Construct graph  $G$  of  $k$  clusters with a maximum of 3 nodes in each cluster.

Each node in cluster is label with a literal.

5.

For checking the result of 3SAT, it needs to iterate all literals and formulas then compute the result true/false. It will take  $O(mn^8)$  to compute, it's in polynomial time, so it is in NP.