Tag all questions on Gradescope after submitting the PDF file!

Read Chapters 22, 23 and 24 thoroughly and solve the following problems.

**Problem 1.** Consider the three graphs below. Graph in Figure 1 is an undirected graph. Graph in Figure 2 is a directed graph. Graph in Figure 3 is a directed acyclic graph.

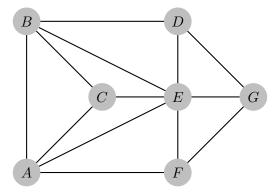


Figure 1:

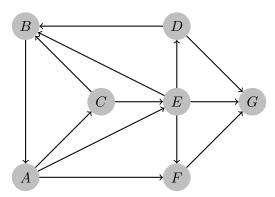


Figure 2:

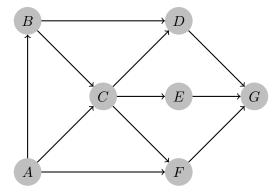


Figure 3:

Using the vertex A as a starting point and, otherwise, using the alphabetical order to select subsequent vertices where applicable:

- a) Run the breadth-first-search algorithm on the graph in Figure 1, showing the same steps as in Figure 22.3 (Lecture 14, page 3) as your answer. (2 Points)
- b) Run the depth-first-search algorithm on the graph in Figure 2, showing the same steps as in Figure 22.4 (Lecture 14, page 6) as your answer. (2 Points)
- c) Run the topological sort algorithm on the graph in Figure 3, showing the same table as in Lecture 14, page 8 as your answer (1 Point).

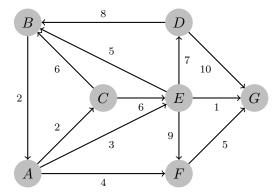
Use the vertex A as a starting point and, otherwise, use the alphabetical order to select subsequent vertices where applicable.

**Problem 2.** Give a counterexample to the conjecture that if a directed graph G contains a path from u to v, then any depth-first search must result in  $v.d \le u.f$ .

**Problem 3.** Let (u, v) be a minimum-weight edge in a connected graph G. Show that (u, v) belongs to some minimum spanning tree of G.

**Problem 4.** Show that a graph has a unique minimum spanning tree if, for every cut of the graph, there is a unique light edge crossing the cut. Show that the converse is not true by giving a counterexample.

**Problem 5.** Run the Dijkstra's algorithm on the following graph and find all shortest paths between vertex A and all others.



## Assignment Guidelines and Plagiarism Warning

This assignment will consist of 5 Problems and it is due on **due on Wednesday**, **07/27/2022** at **11:59 PM**!

Your solution of this assignment must consist of a single, continuous PDF file, which you will upload to Blackboard on or before the above specified deadline.

This assignment must be solved **individually**. Under no circumstances are you allowed to copy or to collaborate with anyone else. **All submitted files will be automatically checked for plagiarism**. Regardless of who copied from whom, all caught in the act of plagiarism will be penalized, as specified in the course syllabus.

In particular, using internet resources of any kind is **not** allowed. Internet sites are routinely checked for similarity to your submission for content. Changing order or variable names will not prevent plagiarism detection. In addition, do not post any content of this assignment to any internet sites or make it public in any other form. **The content of this assignment is not in the public domain!** 

You are free, however, to use our course resources, such as lecture notes and our text book, during the solving of this assignment. If you have questions about this assignment come to my online office hours, or those of the Teaching Assistants, using the usual Blackboard link.