Homework Assignment 1

MCS/CS 401 Computer Algorithms I **due on Wednesday, 06/22/2022 at 11:59 PM**

Read Chapters 2,3,4 thoroughly and solve the following problems:

Problem 1. *Define the sequence a, such that* $a_1 = 3$, $a_2 = 5$. *For each* $n \geq 3$, $a_n = a_{n-1} + 2a_{n-2} - 2$. *Use strong induction to prove that for each* $n \in \mathbb{N}$, $a_n = 2^n + 1$.

Problem 2. *Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrence*

$$T(n) = \begin{cases} 2 & n = 2 \\ 2T(\frac{n}{2}) + n & n = 2^k,\ for\ k > 1 \end{cases}$$

is $T(n) = n\ lg(n)$. *Note that* $lg(n) = \log_2(n)$.

Problem 3. *Use a recursion tree to determine a good asymptotic upper bound on the recurrence* $T(n) = 3T(\frac{n}{2}) + n$. *Use the substitution method to prove your answer.*

Problem 4. *Let f(n) and g(n) be asymptotically positive functions.* ***Prove or disprove*** *each of the following conjectures.*

1. *f(n)* $= O(g(n))$ *implies g(n)* $= O(f(n))$
2. *f(n)+g(n)* $= \Theta(min(f(n), g(n)))$
3. *f(n)* $= \Theta(f(\frac{n}{2}))$
4. *f(n)* $= O(g(n))$ *implies g(n)* $= \Omega(f(n))$
5. *f(n)* $= O((f(n))^2)$

Problem 5. *Using the definitions for* O *and* Ω, ***prove formally*** *that*
a) $n^3 + 15n + 2$ *is* $O(n^4)$
b) $2n^3 + 25n$ *is* $\Omega(n^2)$
by identifying the constants n_0 *and* c *explicitly.*

Assignment Guidelines and Plagiarism Warning

This assignment will consist of 5 Problems and it is due on **Wednesday, 06/22/2022 at 11:59 PM**!

Your solution of this assignment must consist of a single, continuous PDF file, which you will upload to Blackboard-Gradescope on or before the above specified deadline.

This assignment must be solved **individually**. Under no circumstances are you allowed to copy or to collaborate with anyone else. **All submitted files will be automatically checked for plagiarism**. Regardless of who copied from whom, all caught in the act of plagiarism will be penalized, as specified in the course syllabus.

In particular, using internet resources of any kind is **not** allowed. Internet sites are routinely checked for similarity to your submission for content. Changing order or variable names will not prevent plagiarism detection. In addition, do not post any content of this assignment to any internet sites or make it public in any other form. **The content of this assignment is not in the public domain**!

You are free, however, to use our course resources, such as lecture notes and our text book, during the solving of this assignment. If you have questions about this assignment come to my online office hours, or those of the Teaching Assistants, using the usual Blackboard link.