1.
a.
$f(n) = \log_3(n^3) = 3\log_3(n)$
$g(n) = n$
We need to prove
$f(n) <= Cg(n)$ for $n>=n_0$. Then:
$3\log_3(n) / n <= C$
=> choose $n_o = 1$ and $C = 3$
$3\log_3(n) / n <= 3$ with $n >= 1$
True

b.
$f(n) = 3^n$
$g(n) = n^k$ with $k >= 1$
Assume $3^n >= Cn^k$ then:
$3^n / n^k >= C$
The left side has no limit with any constant k when n to infinity.
False

3.
a. T (n) = 4T (n/2) + 1
According to Master Theorem, a=4, b=2, f(n)=1
n^$(\log_2(4) - \varepsilon) = n$^$0$
=> case 1
$\varepsilon = 2$
$T(n) = \Theta(n^2)$
True

b. T (n) = 3T (n/5) + 3
According to Master Theorem, a=3, b=5, f(n)=3
n^$(\log_5(3) - \varepsilon) = n$^$0$
=> case 1
$\varepsilon = \log_5(3)$
$T(n) = \Theta(n$^$(\log_5(3)))$
True

4.
a.
$f(n) = n^2 - 6n + 3$
$g(n) = n$
We need to prove
$f(n) <= Cg(n)$ for $n>=n_0$. Then:
$n – 6 + 3/n <= C$
no exist constant C with all $n >= 1$
False

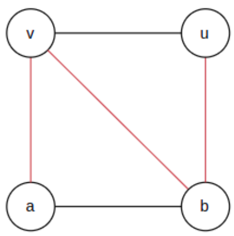
b.
$f(n) = 5^n$
$g(n) = 4^n$

We need to prove
$f(n) <= Cg(n)$ for $n>=n_0$. Then:
$(5/4)^n <= C$
=> The left side has no limit when n to infinity => no exist constant C with all n >= 1
False

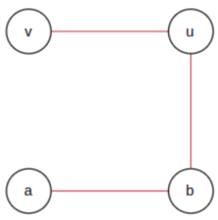
c.
$f(n) = 5n(log(n))^3$
$g(n) = n^{(4/3)}$
We need to prove
$f(n) <= Cg(n)$ for $n>=n_0$. Then:
$5n(log(n))^3 / n^{(4/3)} <= C$
<=> $5 log(n)^3 / n^{(1/3)} <= C$
<=> $5 log(n)^3 / (n^{(1/9)})^3 <= C$
compare n and $2^{(n^{1/9})}$, the left side doesn't increase speed as the right side
So the statement is true

5.
In order to check "A is an integer array with n elements" statement, we can iterate all elements in A to check whether all elements are integer and exist integer x in A elements.
It can be run in O(n) with n is size of A. It runs in polynomial time, therefore this P is in NP.

6.
a. Since T is MST, so there is a unique path (shortest path) from u → v with u,v in T and u,v ∈ V



In the image above, The MST is a-v-b-u, there is only path from u->v
b. Since the edges have the same weight, so the MST T is not unique, we can form another MST T'
In the image above we can form a MST like this:

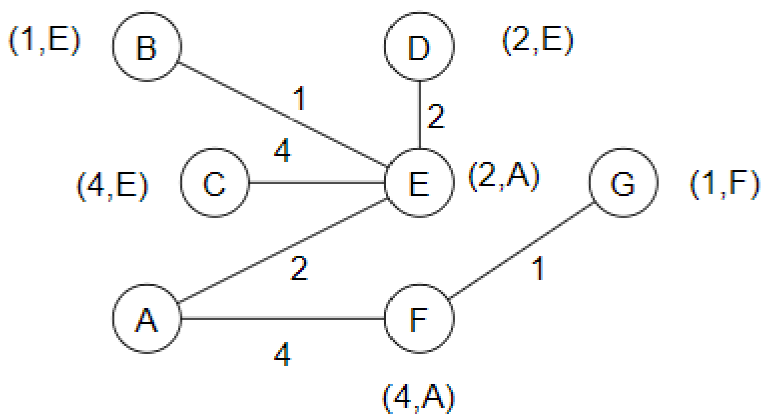


7.
We simply calculate n clauses with certificate C, then calculate the formula the formed from clauses.
Loop calculate 1 clause: k
Loop calculate n clauses: n
O(nk), run in polynomial time.

8.
This fasted algorithm running on O(n) if x is always the first element if B.
The efficient time complexity is O(nlogm) in case x is not in B, then we can find it by binary search tree since B is a sorted array, so to find x is in B it takes O(log(m)) time

2.

9.



10.
Shortest paths between vertex A and all others
A -> A = 0
A-> E -> B = 3
A -> C = 7
A -> E -> D = 4
A-> E = 2
A -> F = 4
A -> F -> G =5

Vexter / Step	A	B	C	D	E	F	G
0	0	(∞,-)	(∞,-)	(∞,-)	(∞,-)	(∞,-)	(∞,-)
1	-	(∞,-)	(7,A)	(∞,-)	**(2,A)**	(4,A)	(∞,-)
2	-	**(3,E)**	(7,A)	(4,E)	-	(4,A)	(7,E)
3	-	-	(7,A)	**(4,E)**	-	(4,A)	(7,E)
4	-	-	(7,A)	-	-	**(4,A)**	(7,E)
5	-	-	(7,A)	-	-	-	**(5,F)**
6	-	-	**(7,A)**	-	-	-	-
7	-	-	-	-	-	-	-