

1.

a. $f(n) = 5n^2 + 6n^{(3/2)}$

$g(n) = n^2$

We need to prove

$f(n) \leq Cn^2$ for $n \geq n_0$, then:

$5 + 6n^{(-1/2)} \leq C$

\Rightarrow choose $C = 11, n_0 = 1$

$f(n) / g(n) = (5n^2 + 6n^{(3/2)}) / n^2 \leq 11$

true

b. $f(n) = 2n + 1$

$g(n) = n$

We need to prove

$f(n) \leq Cn$ for $n \geq n_0$, then

$2 + 1/n \leq C$

\Rightarrow Choose $n_0=1, C=3$

$f(n) / g(n) = (2n + 1) / n \leq 3$

true

c. $f(n) = n^2 - 5n$

$g(n) = n$

There are n_0 and C satisfy:

$f(n) / g(n) \leq C$

$\Rightarrow n - 5 \leq C$

for some large n , there's no constant C satisfy.

so it's not $O(n)$

true

d. $f(n) = (n-3)^2$

$g(n) = n$

Assume the statement is true, then there are n_0 and C satisfy:

$f(n) \leq Cn$ for $n \geq n_0$, then

$n - 6 + 9/n \leq C$

for some large n , there's no constant C satisfy.

so it's not $O(n)$

the statement is false

2.

a. $T(n) = 3T(n/3) + 1$

prove that $T(n) \leq cn$

Assume a bound hold for $n/3$

$T(n) \leq 3cn/3 \leq cn$ for some large n

$c > 1$, we done

b.

$T(n) = T(n-1) + 2n$

Guess $T(n) = n^2$

there exists a c and n_0 such that

$T(n) \geq cn$ for $n > n_0$ and $c > 0$
if $n=1$ then $T(1) = 1 > c \lg(1)$ or all $c > 0$

$T(n) = T(n-1) + 2n \geq c(n-1)$
 $\Rightarrow T(n) = c\Omega(n)$

3.

a.

$a=7, b=3, f(n) = n^2$

$n^{\log_3(7)} < n^2$

\Rightarrow case 2

$\Rightarrow T(n)$ is $\Theta(n^2)$

b. $a=3, b=3, f(n) = n^{1/2}$

$n^{\log_3(3)} > n^{1/2}$

\Rightarrow case 3

$T(n)$ is $\Theta(n^{\log_3(3)}) = \Theta(n)$

5.

a.

$cursum = array[0]$

for $i=0$ to $n-1$:

$cursum = \max(array[i], cursum + array[i+1])$

$maxsum = \max(maxsum, cursum)$

print $maxsum$

time complexity: $O(n)$