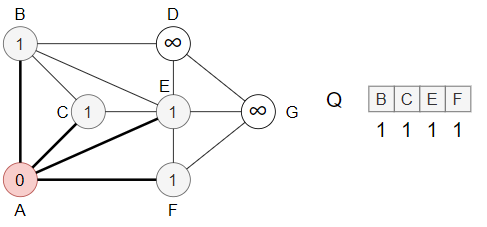
Problem 1.

Diagram

Description automatically generateda)



(a) (b)

Schematic

Description automatically generated with low confidenceA picture containing diagram

Description automatically generated

(c) (d)

Diagram, schematic

Description automatically generatedA picture containing text, clock

Description automatically generated

(e) (f)

A picture containing text, clock

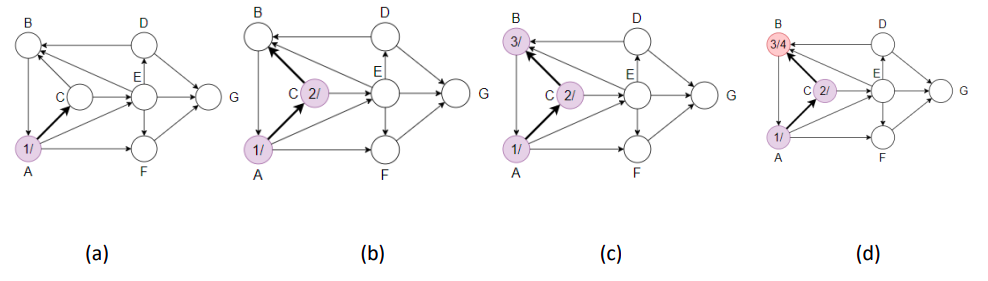
Description automatically generated

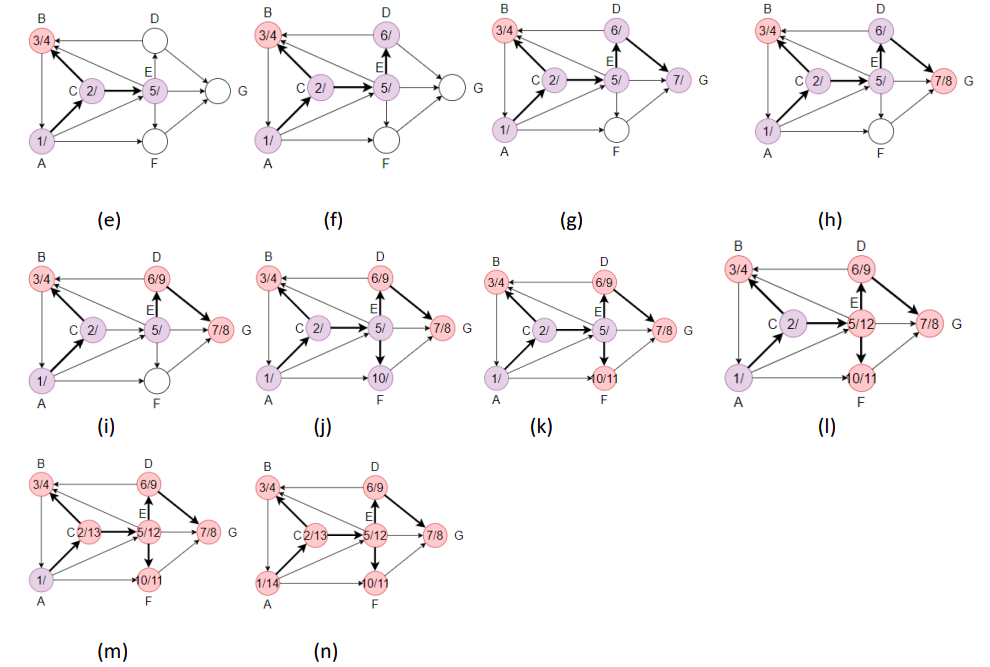
A picture containing text, clock

Description automatically generated

(g) (h)

b)

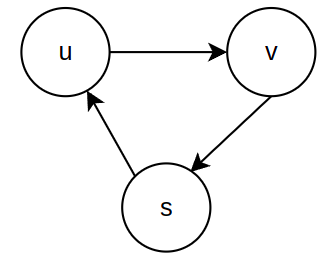




c)

|  |  |  |
| --- | --- | --- |
| Vertex | Discovery time | Finish time |
| A | 1 | 13 |
| B | 2 | 14 |
| C | 3 | 8 |
| D | 4 | 7 |
| E | 5 | 6 |
| F | 9 | 10 |
| G | 11 | 12 |

Problem 2.



In Grapth G, consider the Adjacent list below. Performing DFS starting at s, we have:

|  | d | f |  |
| --- | --- | --- | --- |
| s | 1 | 4 |  |
| v | 2 | 3 |  |
| u | 5 | 6 |  |

So v.d < u.f, but v is not a descendant of u in the depth-first forest.

Problem 3.

Suppose there is a MST called T that does not contain the minimum weight edge e from u->v. Since T is a connected tree, therefore adding the edge e (u, v) to T will form a cycle. Following this cycle, there must be at least one other edge in the cycle crossing the edge e. By removing this edge, and leaving (u, v), we have a tree with a lower total weight than the original T. But T was assumed to be an MST, this is a contradiction. So, minimum weight edge e form u->v must be part of every MSTs of graph G.

Problem 4.

Assume exist the graph has two distinct MST T1 and T2 . Let (u,v) be an edge in T1 but not in T2. Removing edge (u,v) will split the tree T into two components. Call T1[u] and T1[v] are those 2 components. Let (x,y) be the unique light edge crossing the cut (T1[u],T1[v]).

If (x,y) different from (u,v) then w(x,y) < w(u,v) and the spanning tree T1 − {(u,v)} ∪ {(x,y)} has a better cost than T1. This is a contradiction.

Assume that (x,y) = (u,v), the unique light edge (u,v) crossing the cut (T1[u],T1[v]) doesn’t belong to T2 . Consider the path p from u to v in T2, path p starts in T1[u] and ends in T1[v], therefore there have to be an edge e on it crossing the cut (T1[u], T1[v]). Because edge (u,v) is the unique light edge crossing this cut => w(u,v) < w(e). If we add edge (u,v) to T2 then will form a cycle composed of (u,v) and p. Removing edges from the cycle we get again a spanning tree. Hence T2 ∪ {(u,v)} − {e} is a spanning tree and the cost is better than T2 , a contradiction.

Counterexample for the converse., consider a graph with 3 a,b,c vertexes and weights

w(b,c) = w(b,a) = 3 and w(a,c) = 4 .

The MST is a – b – c . But the cut (b, (a,c)) doesn’t have a unique light edge crossing the cut

Problem 5.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Vertex  Steps | A | B | C | D | E | F | G |
|  | 0 | (∞,─) | (∞,─) | (∞,─) | (∞,─) | (∞,─) | (∞,─) |
| 1 | ─ | (∞,─) | **(2,A)** | (∞,─) | (3,A) | (4,A) | (∞,─) |
| 2 | ─ | (8,C) | ─ | (∞,─) | **(3,A)** | (4,A) | (∞,─) |
| 3 | ─ | (8,C) | ─ | (10,E) | ─ | **(4,A)** | (4,E) |
| 4 | ─ | (8,C) | ─ | (10,E) | ─ | ─ | **(4,E)** |
| 5 | ─ | **(8,C)** | ─ | (10,E) | ─ | ─ | ─ |
| 6 | ─ | ─ | ─ | **(10,E)** | ─ | ─ | ─ |
| 7 | ─ | ─ | ─ | ─ | ─ | ─ | ─ |

Shortest paths between vertex A and all others

A -> A = 0

A -> C -> B = 8

A -> C = 2

A -> E -> D = 10

A -> E = 3

A -> F = 4

A -> E -> G = 4