MODULE *total_order*

EXTENDS *skeen*

- Total Order: There exists a total order $<$ on all messages that are multicast in an execution trace such that, if process p delivers message m, then for all messages $m' < m$ such that p is one of addresses of message m', p delivers m' before m.
- Total Order can be formalized as the following formula

$$
\begin{array}{l}
GlobalTotalOrdering \triangleq \\
\quad \exists\ \text{ordering} \in [McastID \to 1 \,..\, M] : \\
\qquad \land \forall\, id1,\, id2 \in McastID :\ ordering[id1] = ordering[id2] \Rightarrow id1 = id2 \\
\qquad \land \forall\, p \in Proc : \forall\, id1,\, id2 \in McastID : \\
\qquad\quad (\ \land globalTS[p][id1] \neq TimestampNull \\
\qquad\quad\ \ \land globalTS[p][id2] \neq TimestampNull \\
\qquad\quad\ \ \land ordering[id1] < ordering[id2]\ ) \\
\qquad\quad\ \ \Rightarrow Less(globalTS[p][id1],\, globalTS[p][id2])
\end{array}
$$

- However, *APALACHE* cannot verify *GlobalTotalOrdering* because the initialization of ordering and its corresponding quantifiers.

The conjunction of *ConsistentGlobalTS* and *AsymmetricOrdering* implies Total Order

$$
\begin{array}{l}
ConsistentGlobalTS \triangleq \\
\quad \land \forall\, id \in McastID : \forall\, p,\, q \in Proc : \\
\qquad (\ \land globalTS[p][id] \neq TimestampNull \\
\qquad\ \ \land globalTS[q][id] \neq TimestampNull) \\
\qquad\quad \Rightarrow globalTS[p][id] = globalTS[q][id] \\
\quad \land \forall\, id1,\, id2 \in McastID : \forall\, p \in Proc : \\
\qquad (\ \land id1 \neq id2 \\
\qquad\ \ \land globalTS[p][id1] \neq TimestampNull \\
\qquad\ \ \land globalTS[p][id2] \neq TimestampNull) \\
\qquad\quad \Rightarrow globalTS[p][id1] \neq globalTS[p][id2]
\end{array}
$$

All addressees of message id must agree on its global timestamp.

Every message has a unique global timestamp.

$$
\begin{array}{l}
AsymmetricOrdering \triangleq \\
\quad \forall\, id1,\, id2 \in McastID : \forall\, p,\, q \in Proc : \\
\qquad (\ \land globalTS[p][id1] \neq TimestampNull \\
\qquad\ \ \land globalTS[p][id2] \neq TimestampNull \\
\qquad\ \ \land globalTS[q][id1] \neq TimestampNull \\
\qquad\ \ \land globalTS[q][id2] \neq TimestampNull \\
\qquad\ \ \land id1 \neq id2) \\
\qquad\quad \Rightarrow \neg(Less(globalTS[p][id1],\, globalTS[p][id2]) \land Less(globalTS[q][id2],\, globalTS[q][id1]))
\end{array}
$$

The global timestamps of messages preserve asymmetric order.

* Modification History
* Last modified *Thu Mar* 25 22:17:11 *CET* 2021 by *tran*
* Created *Thu Mar* 25 18:57:07 *CET* 2021 by *tran*