An End-to-end Framework for Energy-efficient Cascaded Dual-shop Collaborative Scheduling with Mating Operation

A. MILP Model of ECDCSP-M

Objectives (1) demonstrate two optimization objectives, minimizing the C_{max} and TEC, repectively. Constraint (2) ensures that each factory includes a dummy job. Constraint (3) guarantees that each job must be assigned to only one factory in Phase 1. Constraints (4) and (5) ensure that if the job is not assigned to a particular factory, there is no precursor and successor of the job in the factory. Constraint (6)-(8) guarantee that the sequence of jobs assigned to any factory cannot form a subring. Constraint (9) implies that the completion time of J_j is non-negative. Constraint (10) implies that each job can only undergo the current operation after the completion of its preceding operation. Constraint (11) represents the relationship between the completion times of two adjacent jobs on the same machine. Constraint (12) stipulates the completion time of the initial job in each factory. Constraint (13) guarantees that in Phase 2, each operation can only be assigned to one machine among multiple parallel machines for processing. Constraint (14) ensures that each stage includes a dummy job in Phase 2. Constraints (15) and (16) ensure that if the job is not assigned to a particular machine in Phase 2, there is no precursor and successor of the job on the machine. Constraint (17)-(19) guarantee that the sequence of jobs assigned to any machine cannot form a subring. Constraint (20) implies that the completion time of J_j is non-negative in Phase 2. Constraint (21) implies that all jobs immediately enter the transportation phase after completing processing in Phase 1. Constraint (22) ensures that the operation of a job may not start until the previous stage has completed. Constraints (23)-(27) require that the mating operations of the main order and the corresponding sub order can only start processing after the completion of their respective predecessor operations. Constraint (28) represents the relationship between the completion times of two adjacent operations on any machine. The EC of the machine in Phase 1 on processing mode, setup mode, and idle mode are defined by constraints (29)-(31), respectively. In Phase 2, constraints (32)-(34) define the EC in these modes. Constraint (35) represents the EC during the transportation phase. Equations (36) and (37) define the C_{max} and TEC, respectively.

B. State Representation

Each node and arc in the graph structure has its own features, which are normalized at the beginning of the training. First, the description for each feature is as follows:

- The features of the factory node F_f in Phase 1 are represented as vector $\nu_f^t = \{|\mathcal{N}^t(F_f)|, Lr_f^t\}$, where $|\mathcal{N}^t(F_f)|$ and $FL_f^t = \frac{\sum_{i=1}^{m}\sum_{j=0}^{n}\sum_{j'=1}^{n} x_{j,f}\cdot x_{j',f}\cdot(p_{j,i}^1+st_{j,j',i}^1)}{t}$ represent the number of neighboring job nodes and the factory load rate of F_f at time t, respectively.
- The features of the machine node $M_{k,l}^p$ in Phase 2 are represented as vector $\varpi_{k,l}^t = \{I_{k,l}^t, AT_{k,l}^t, |\mathcal{N}^t(M_{k,l}^p)|, ML_{k,l}^t\}$, where $I_{k,l}^t$ is a binary variable that takes value 1 if $M_{k,l}^p$ is idle at time t; and 0 otherwise. $AT_{k,l}^t$ represents the available time of $M_{k,l}^p$. $|\mathcal{N}^t(M_{k,l}^p)|$ indicates the number of neighbor operation nodes of $M_{k,l}^p$ at time t. $ML_{k,l}^t = \frac{\sum_{j=0}^{n}\sum_{j'=1}^{n} w_{j,j',l,k}\cdot(p_{j,l}^2+st_{j,j',l,k}^2)}{t}$ represents the machine load rate of $M_{k,l}^p$ at time t.
- The features of the operation node $O_{j,k}$ are represented as vector $\mu_{j,k}^t = \{I_{j,k}^t, MO_{j,k}, |\mathcal{N}^t(O_{j,k})|, pt_{j,k}, ST_{j,k}^t, CT_j, m_j, ec_{j,k}\}$, where $I_{j,k}^t$ is a binary variable that takes value 1 if $O_{j,k}$ is scheduled at time t; and 0 otherwise. $MO_{j,k}$ is also a binary variable that takes value 1 if $O_{j,k}$ is a mating operation; and 0 otherwise. $|\mathcal{N}^t(O_{j,k})|$ is the number of neighbor machine nodes of $O_{j,k}$ at time t. $pt_{j,k}$ represents processing time of $O_{j,k}$. In Phase 1, $pt_{j,k}$ equals $p_{j,i}^1$, but in Phase 2, due to the presence of unrelated parallel machines, $pt_{j,k}$ is uncertain. If $O_{j,k}$ is scheduled on $M_{k,l}^p$ at time t, $pt_{j,k}$ equals $p_{j,k}^2/v_{k,l}$. Otherwise, $pt_{j,k}$ is the average value $\bar{p}_{j,k} = \frac{\sum_{l=1}^{r_k} p_{j,k}^2/v_{k,l}}{r_k}$. $ST_{j,k}^t$ is the start time of $O_{j,k}$. If $O_{j,k}$ is scheduled at time t, $ST_{j,k}^t$ equals the actual value $ST_{j,k}$. Otherwise, $ST_{j,k}^t$ is an estimate computed with only precedence constraints. If its immediate predecessor $O_{j,k-1}$ is scheduled, then $ST_{j,k}^t = ST_{j,k-1} + pt_{j,k}$, otherwise, $ST_{j,k}^t = ST_{j,k-1}^t + \bar{p}_{j,k-1}$. CT_j represents the completion time of the J_j corresponding to $O_{j,k}$, $CT_j = ST_{j,s}^t + pt_{j,s}$. m_j denotes the number of operations left. $ec_{j,k}$ is processing EC of $O_{j,k}$. If $O_{j,k}$ is scheduled at time t, $ec_{j,k} = pt_{j,k} \times \beta_{k,l}^2$. Otherwise, $ec_{j,k}$ is the average value $\bar{e}_{j,k} = \frac{\sum_{l=1}^{r_k} pt_{j,k}\times\beta_{k,l}^2}{r_k}$.
- The disjunctive arcs $\mathcal{K}$ connects J_j corresponding to the operation $O_{j,k}$ with the compatible factory node F_f in Phase 1. Thus, the features jointly determined by the operation node and the factory node characterizes the disjunctive arc, embedding information between these two types of nodes. Let the representation vector of the disjunctive arc $K_{j,f} \in \mathcal{K}$ be $\lambda_{j,f} = \{tp_{j,f}\}$, where $tp_{j,f}$ denotes the transportation time of the J_j

from $M_{f,m}$ to Phase 2.

- The disjunctive arcs $\mathcal{H}$ connects the operation $O_{j,k}$ with the compatible machine $M_{k,l}^{p}$ in Phase 2. Similarly, the features jointly determined by the operation node and the machine node characterizes the disjunctive arc. Let the representation vector of the disjunctive arc $H_{j,k,l} \in \mathcal{H}$ be $\xi_{j,k,l} = \{pt_{j,k,l}, ec_{j,k,l}\}$, where $pt_{j,k,l} = \frac{p_{j,k}^{2}}{v_{k,l}}$ and $ec_{j,k,l} = \frac{p_{j,k}^{2}}{v_{k,l}} \times \beta_{k,l}^{2}$ denote the processing time and EC of $O_{j,k}$ on $M_{k,l}^{p}$, respectively.