

Given the lengths the links:

To the elbow: $l_1=1$ unit

To the end-effector: $l_2=1$ unit

Task 1: Calculating positions

The elbow is of length l_1 and at an angle q_1

Position of the elbow joint from the base(0,0):

$$x_e = l_1 \cos(q_1)$$

$$y_e = l_1 \sin(q_1)$$

The end-effector is at a distance of l_2 from elbow joint and at an angle q_2 relative to the elbow joint. Hence it is at a total angle of q_1+q_2

Position of the end-effector from the base (0,0):

$$x = x_e + l_2 \cos(q_1+q_2)$$

$$y = y_e + l_2 \sin(q_1+q_2)$$

Taking three configurations:

1. Straight Arm Configuration
2. Bent Elbow Configuration
3. Folded Arm Configuration

1. Straight Arm Configuration:

Since the arm is straight, this implies that the relative angle between the elbow joint and end-effector will be 0, i.e $q_2=0$

q_1 can be anything, as it is basically it's orientation in space.

Taking $q_1=0$:

$$x_e = l_1 = 1 \text{ unit}$$

$$y_e = 0$$

$$x = x_e + l_2 = 1 + 1 = 2 \text{ units}$$

$$y = y_e + 0 = 0 + 0 = 0$$

2. Bent Elbow Configuration:

Since the arm is bent, then, q_2 can be anything except 0 (as it will make the arm straight) and $\pm \pi$ (as it will fold it).

Again q_1 can be anything as it is just it's orientation in space.

Taking $q_1=q_2=\pi/4$:

$$x_e = l_1 \cos(\pi/4) = 1/\sqrt{2}$$

$$y_e = l_1 \sin(\pi/4) = 1/\sqrt{2}$$

$$x = x_e + l_2 \cos(\pi/2) = 1/\sqrt{2}$$

$$y = y_e + l_2 \sin(\pi/2) = 1/\sqrt{2} + 1$$

3. Folded Arm Configuration:

Since the arm is folded, then it implies that q_2 will be either π or $-\pi$.

Again q_1 can be anything.

Taking $q_1 = \pi/4$ and $q_2 = -\pi$

$$x_e = l_1 \cos(\pi/4) = 1/\sqrt{2}$$

$$y_e = l_1 \sin(\pi/4) = 1/\sqrt{2}$$

$$x = x_e + l_2 \cos(-3\pi/4) = 0$$
$$y = y_e + l_2 \sin(-3\pi/4) = 0$$

Task 3: Explanations

q_1 is the elbow joint angle, and increasing this angle will lift the arm, whereas decreasing the angle will lower the arm, in short, it will change the orientation of the workspace.

q_2 is the relative angle made between the two links, so, increasing/decreasing this angle will bend the arm more and when it is equal to 180 deg, it will fold the arm.

The workspace that the arm can reach to, is the annulus having radius lying between $|l_1 - l_2|$ (where it is a folded arm) and $l_1 + l_2$ (where it is a straight arm). Given, $l_1 = l_2 = 1$, the arm can reach to any point lying between radius 0 and 2.

Task 2: Python script and images, shown below:

```
In [1]: import numpy as np
import matplotlib.pyplot as plt

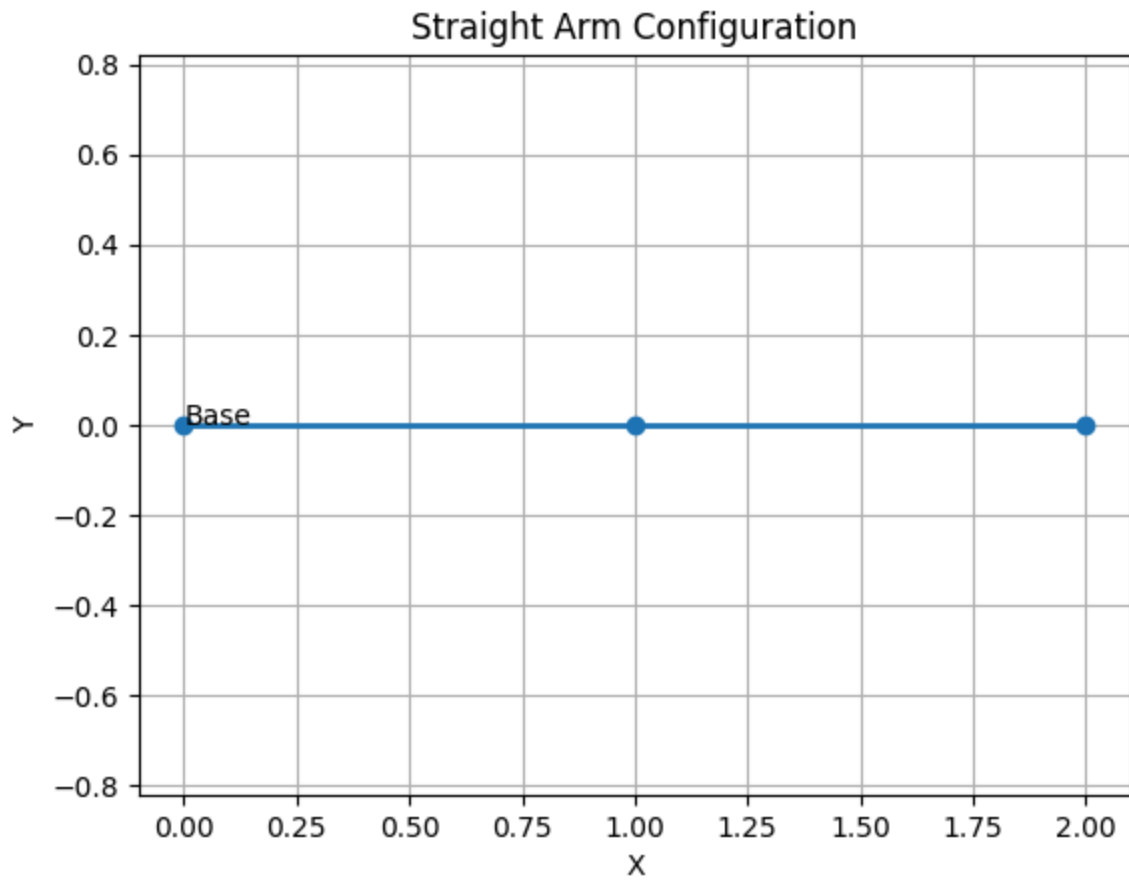
# Given Link Lengths
l1 = 1.0
l2 = 1.0
def forward_kinematics(q1, q2):
    # Elbow position
    xe = l1 * np.cos(q1)
    ye = l1 * np.sin(q1)

    # End-effector position
    x = xe + l2 * np.cos(q1 + q2)
    y = ye + l2 * np.sin(q1 + q2)

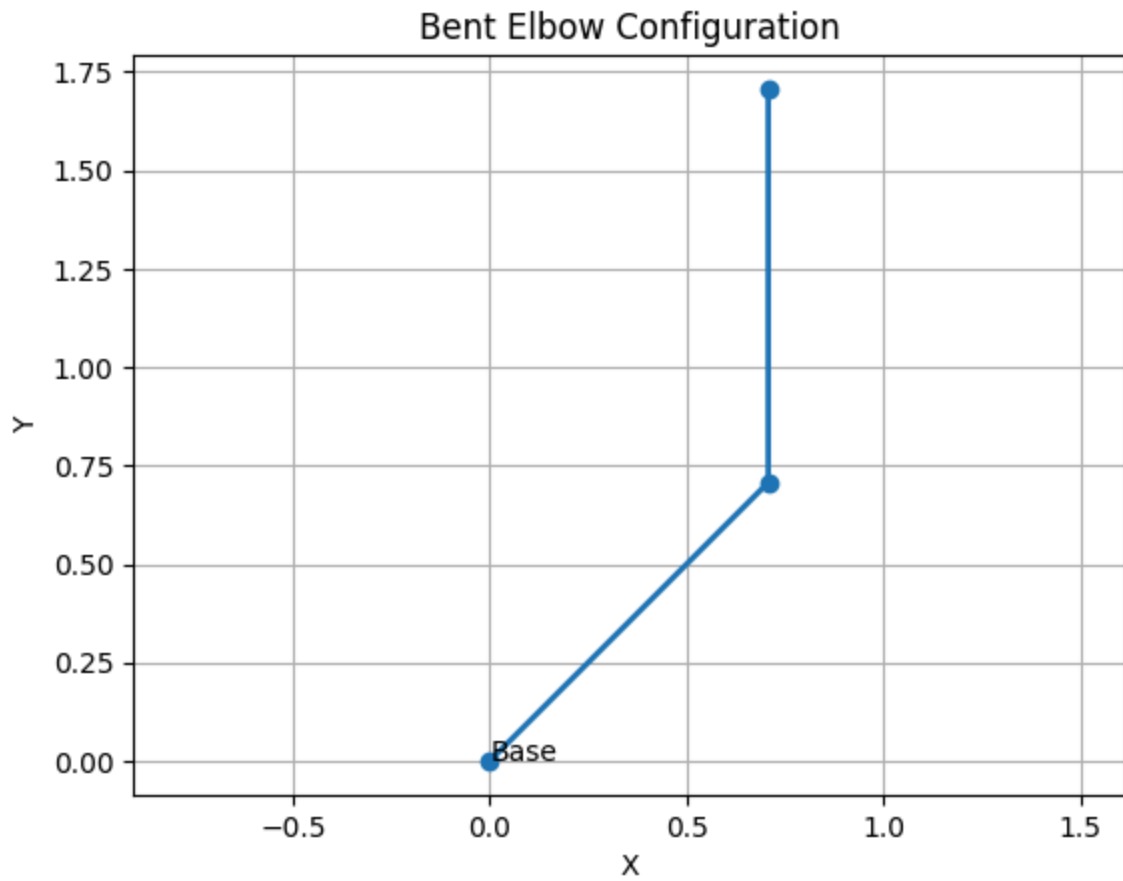
    return (xe, ye), (x, y)
def plot_arm(q1, q2, title):
    elbow, ee = forward_kinematics(q1, q2)
    x_points = [0, elbow[0], ee[0]]
    y_points = [0, elbow[1], ee[1]]

    plt.figure()
    plt.plot(x_points, y_points, '-o', linewidth=2)
    plt.text(0, 0, "Base")
    plt.axis('equal')
    plt.grid(True)
    plt.xlabel("X")
    plt.ylabel("Y")
    plt.title(title)
    plt.show()
```

```
In [2]: q1 = 0
q2 = 0
plot_arm(q1, q2, "Straight Arm Configuration")
```



```
In [3]: q1 = np.pi / 4  
q2 = np.pi / 4  
plot_arm(q1, q2, "Bent Elbow Configuration")
```



```
In [4]: q1 = np.pi / 4  
q2 = -np.pi  
plot_arm(q1, q2, "Folded Arm Configuration")
```

