Name ____

Midterm Exam

MEAM 520, Introduction to Robotics
University of Pennsylvania
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November 8, 2012

You must take this exam independently, without assistance from anyone else. You may bring in a calculator and two 8.5"×11" sheets of notes for reference. Aside from these two pages of notes, you may not consult any outside references, such as the textbook or the Internet. Any suspected violations of Penn's Code of Academic Integrity will be reported to the Office of Student Conduct for investigation.

This exam consists of several problems. We recommend you look at all of the problems before starting to work. If you need clarification on any question, please ask a member of the teaching team. When you work out each problem, please show all steps and $\boxed{\text{box your answer}}$. On problems involving actual numbers, please keep your solution symbolic for as long as possible; this will make your work easier to follow and easier to grade. The exam is worth a total of 100 points, and partial credit will be awarded for the correct approach even when you do not arrive at the correct answer.

	Points	Score
Problem 1	20	
Problem 2	20	
Problem 3	15	
Problem 4	20	
Problem 5	25	
Total	100	

I agree to abide by the University of Pennsylvania Code of Academic Integrity during this exam. I pledge that all work is my own and has been completed without the use of unauthorized aid or materials.

Signature ____

Date ____

Problem 1: Short Answer *(20 points)*

a. Why is it important to indicate the **coordinate frame** in which a vector is expressed, e.g., p^0? *(2 points)*

b. Compared to other methods such as Euler Angles, explain one **advantage** of using a 3×3 **rotation matrix** for representing the orientation of a three-dimensional object. *(2 points)*

c. Compared to other methods such as Euler Angles, explain one **disadvantage** of using a 3×3 **rotation matrix** for representing the orientation of a three-dimensional object. *(2 points)*

d. Serial robotic manipulators are composed of revolute and prismatic joints. Explain one advantage of using a **revolute** joint instead of a prismatic joint in a robot. *(2 points)*

e. Explain one advantage of using a **prismatic** joint instead of a revolute joint. *(2 points)*

f. Describing a rigid-body transformation in three dimensions generally requires six numbers. Why then are only four DH parameters (a, α, d, θ) needed to describe link i's pose relative to link $i-1$ in a serial manipulator? *(4 points)*

g. In the DH convention, the homogeneous transformation representing one step in a serial linkage chain can be represented as a product of four basic transformations, as follows:

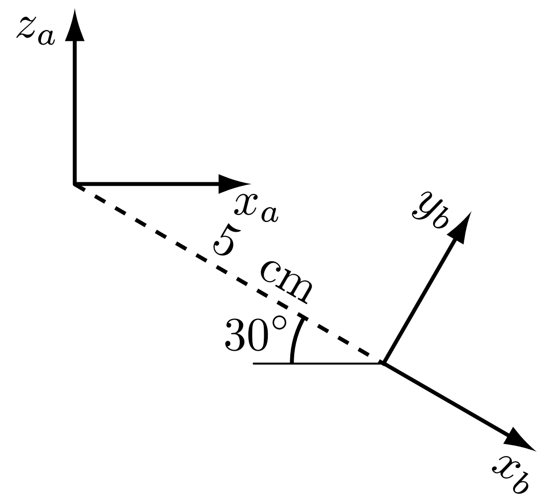
$$A_i = \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i}$$

Here, Rot indicates a rotation around the subscripted axis by the noted angle, and Trans indicates a translation along the specified axis by the noted distance.

Which pairs of the four matrices on the right-hand side commute? Explain why these pairs commute. Find all permutations of these four matrices that yield the same homogeneous transformation matrix, A_i. You do not need to calculate the product. *(6 points)*

Problem 2: Homogeneous Transformations ***(20 points)***

The diagram below depicts two coordinate frames in three dimensions. Note that all of the unit vectors shown and the dashed line segment labeled with a length are in the same plane.



a. What is H_b^a, the homogeneous transformation representing the position and orientation of frame b in frame a? *(10 points)*

b. What is H_a^b? *(10 points)*

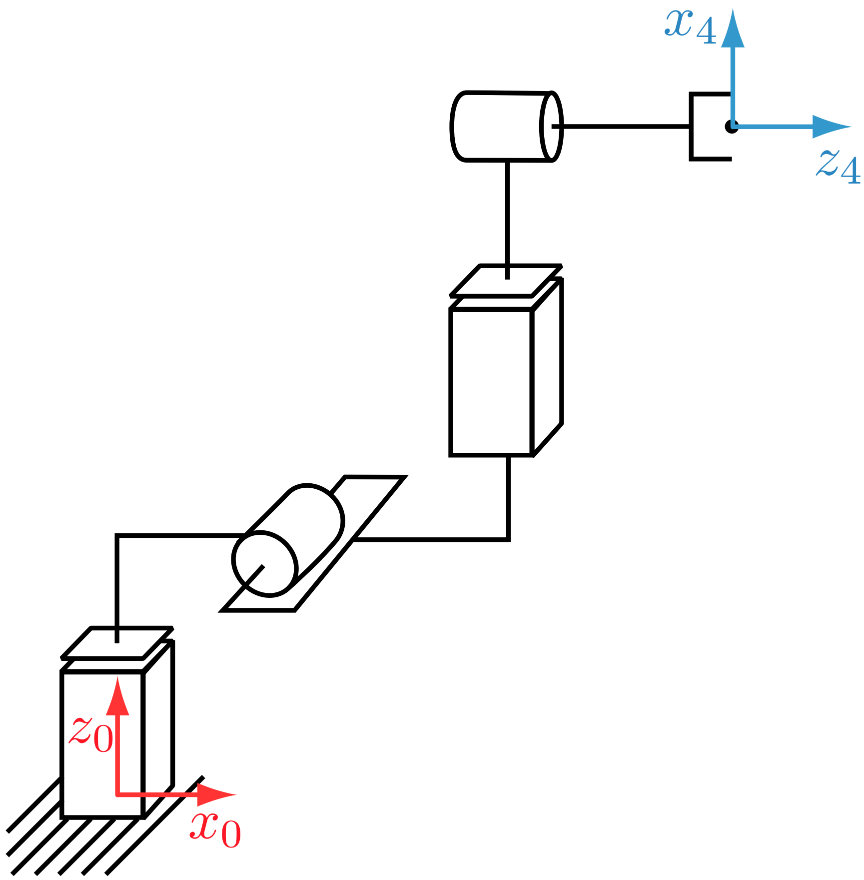
Problem 3: Calculating a Trajectory *(15 points)*

We seek a trajectory $\theta(t)$ that will move a revolute robot joint from rest (not moving) at position α at time $t = 0$ s to be at position β in 2 seconds with a final velocity of 1 rad/s. Assume $\alpha < \beta$.

a. **Roughly sketch the trajectory** as a function of time, labeling the constraints. *(3 points)*

b. Compute a **cubic polynomial** that satisfies these constraints. Be sure to provide the correct units for all coefficients in the equation. *(12 points)*

Problem 4: Forward Kinematics ***(20 points)***

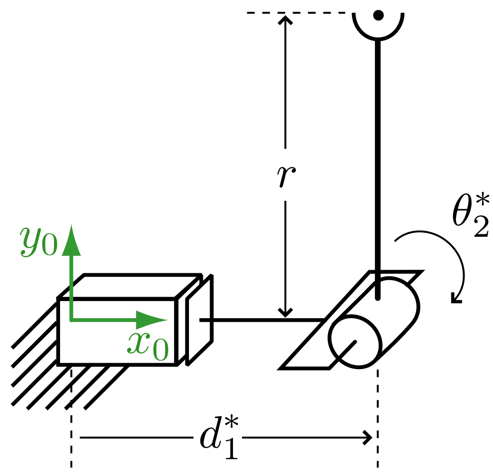


a. Draw **frames 1 through 3** on the above diagram, following the DH convention. *(4 points)*

b. The diagram shows both prismatic joints extended to a positive displacement, while both revolute joints are shown at zero. Fill in the table of **DH parameters** below. Use a superscript star to indicate joint variables, e.g., d_1^*. On the figure, label any DH parameters that you introduce and also mark the positive direction for all joint variables. *(16 points)*

i	a	α	d	θ
1				
2				
3				
4				

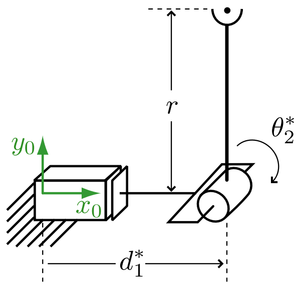
Problem 5: Inverse Position Kinematics and Jacobian ***(25 points)***



The diagram above shows a PR manipulator. The prismatic joint is drawn at a positive displacement, and the revolute joint is drawn at zero rotation. Positive joint directions are marked with arrows. The center of the gripper (black dot) has the following position with respect to frame 0:

$$p^0 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} d_1^* + r \sin \theta_2^* \\ r \cos \theta_2^* \end{bmatrix}$$

a. Assume this robot has no joint limits and will not collide with itself in any configuration. Given a desired position of the end effector $[x \quad y]^T$, find **all possible solutions** to this robot's planar inverse position kinematics. In addition to providing equations for d_1^* and θ_2^*, please state **how many solutions there are** to the inverse kinematics problem, and explain how the number of solutions depends on the desired position. *(10 points)*



$$p^0 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} d_1^* + r\sin\theta_2^* \\ r\cos\theta_2^* \end{bmatrix}$$

b. Calculate the **linear velocity Jacobian** J_v for this robot. *(7 points)*

c. Use your answers from above to derive the **singular configurations** of the arm, if any. **Sketch** the manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration. *(8 points)*