

MEAM 520

Velocity Kinematics and Jacobians

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GRASP LABORATORY

Lecture 13: October 15, 2013



MEAM 520

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"Charismatic Machines" Seminar by Heather Knight at 10:45am on Tuesday 10/15 in Wu and Chen MEAM FALL SEMINAR

"Charismatic Machines"

Tuesday, October 15
Levine Hall, Wu and Chen Auditorium
10:45 a.m.
Coffee will be served starting at 10:30 a.m. in Levine Lobby

Heather Knight
PhD Candidate
Robotics Institute
Carnegie Mellon

Abstract:
You have probably heard about The Terminator's supposed "friend or foe" algorithm. This presentation will be about how humans make similar determinations about machines. Is a robot an agent or a device? Intelligent or a dolt? Charismatic or dull? If we are designing machines, we want to influence these first impressions, because that enables or limits the effectiveness and acceptance of these machines in human environments.

My current research investigates machine body language, namely motion. Generally humans interpret machine actions by modeling them back on archetypes of human or animal behaviors, neurologically and psychologically. Some motion communications require knowledge of the physiology of an agent, e.g., waving hello or shaking one's head. I am particularly interested in modeling motion characteristics that are form invariant, communicating task-relevant robot states, identifying features that non-anthropomorphic robots can use to interact with us successfully & appropriately. At the MEAM Seminar, assisted my pint-sized robot comedian Data, I share early results of how these principles can be applied to mobile robots or even flying machines. Turns out even utilitarian mobile robots can rub people the wrong way, just think of how angry people get at inconsiderately moving cars on the way to work. It might also be handy to quickly distinguish between a drone waging war, versus bringing you a cupcake.

Bio:
Heather Knight is a PhD candidate at Carnegie Mellon and founder of Marilyn Monrobot, which features comedy performances by Data the Robot, the annual Robot Film Festival and a one-off Cyborg Cabaret. Her current research involves human-robot interaction, non-verbal machine communications and non-anthropomorphic social robots. She was named to the 2011 Forbes List for 30 under 30 in Science. Her work also includes: robotics and instrumentation at NASA's Jet Propulsion Laboratory, interactive installations with SyyN Labs (including the award winning "This too shall pass" Rube Goldberg Machine music video with OK GO), field applications and sensor design at Aldebaran Robotics, and she is an alumnus from the Personal Robots Group at the MIT Media Lab.

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Please enter your comments by this Friday, October 18. Your responses are anonymous, so you should feel comfortable being honest.

We appreciate your taking the time to complete this evaluation; your feedback will help us improve the class and our teaching for everyone's benefit.

other

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followup discussions for lingering questions and comments

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Project I

PUMA Dance

MEAM 520 | Class Profile | Piazza

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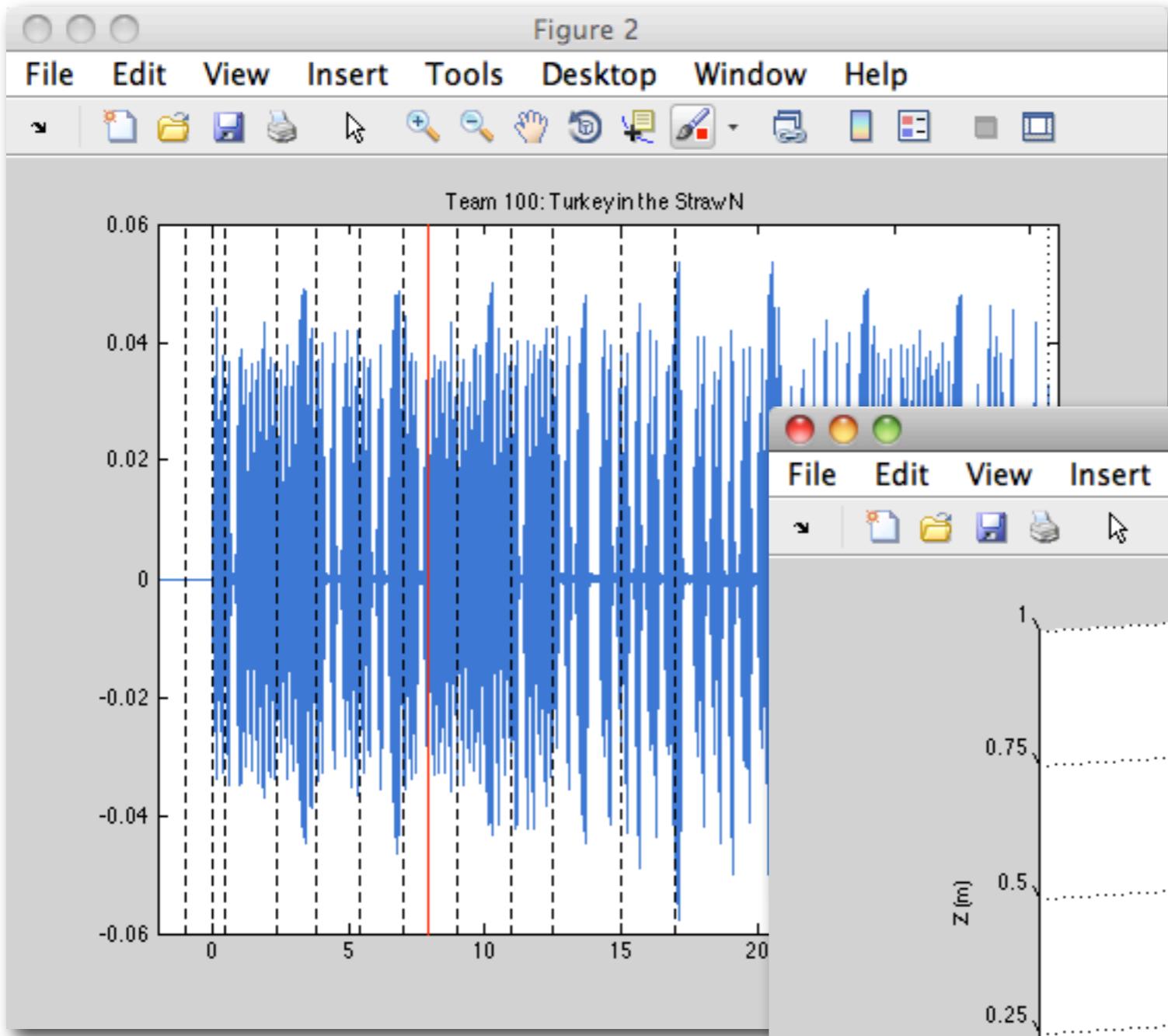
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Due by 11:59 p.m. on Sunday, October 20

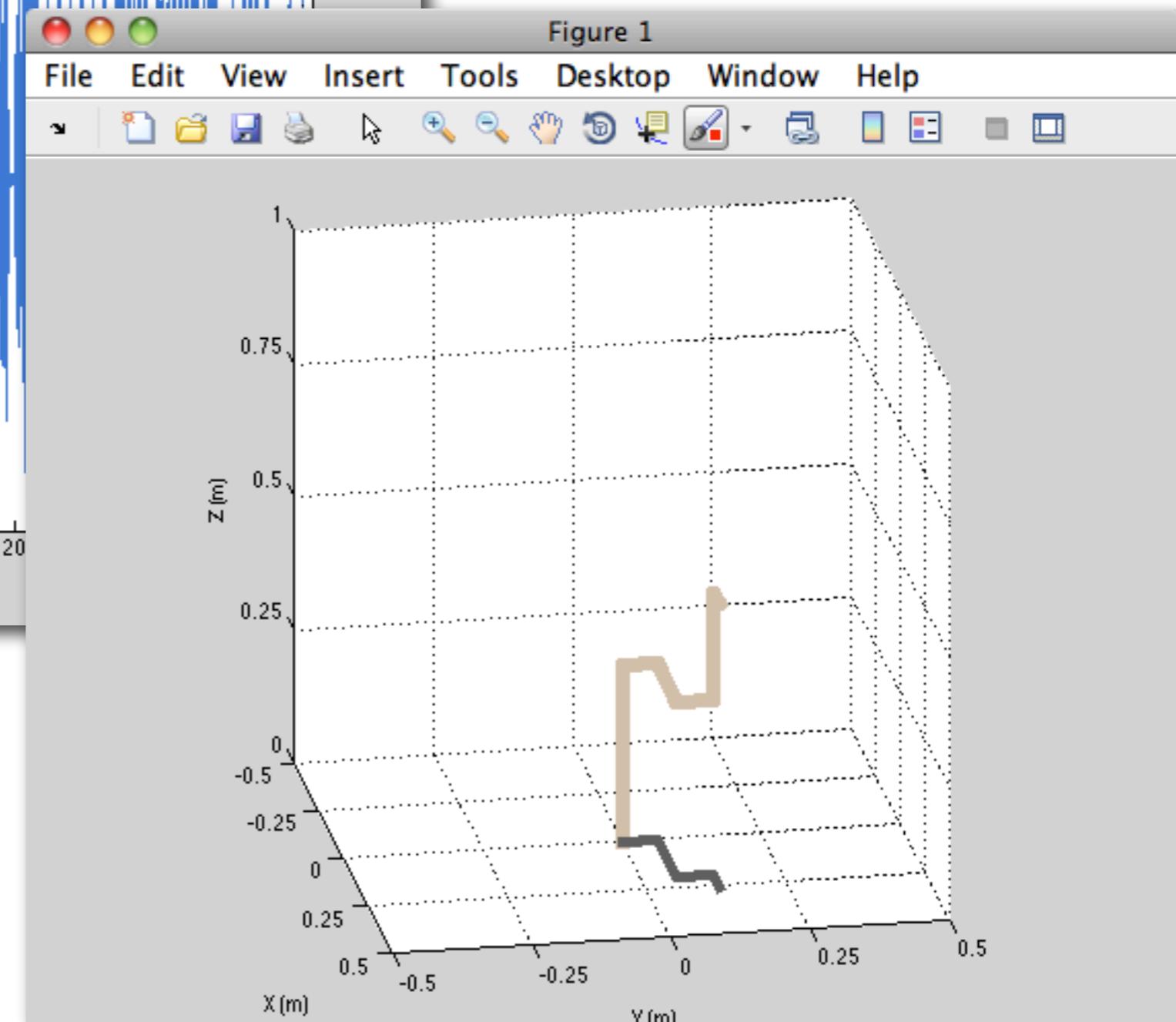
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Figure 2



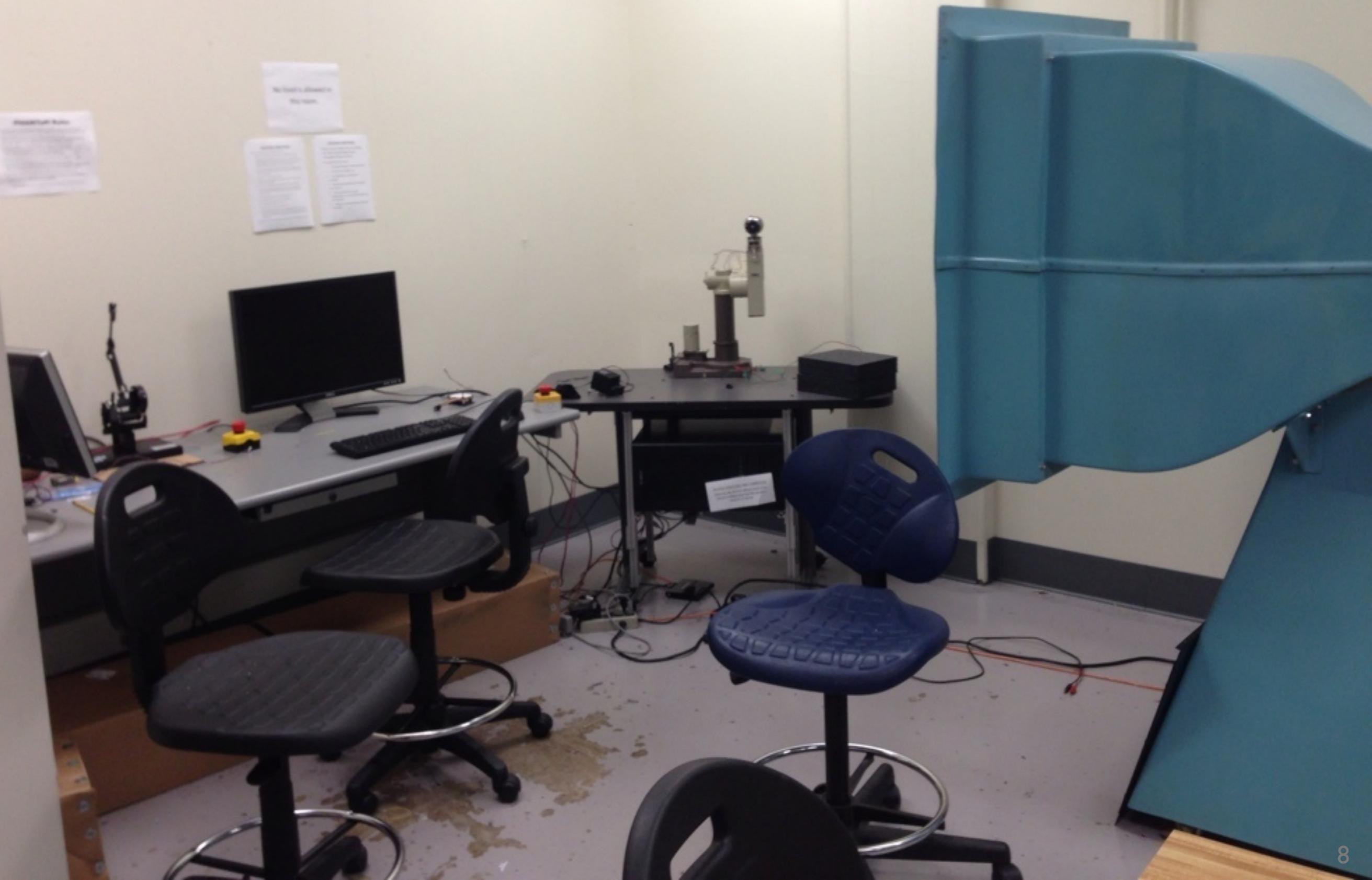
One team already submitted!

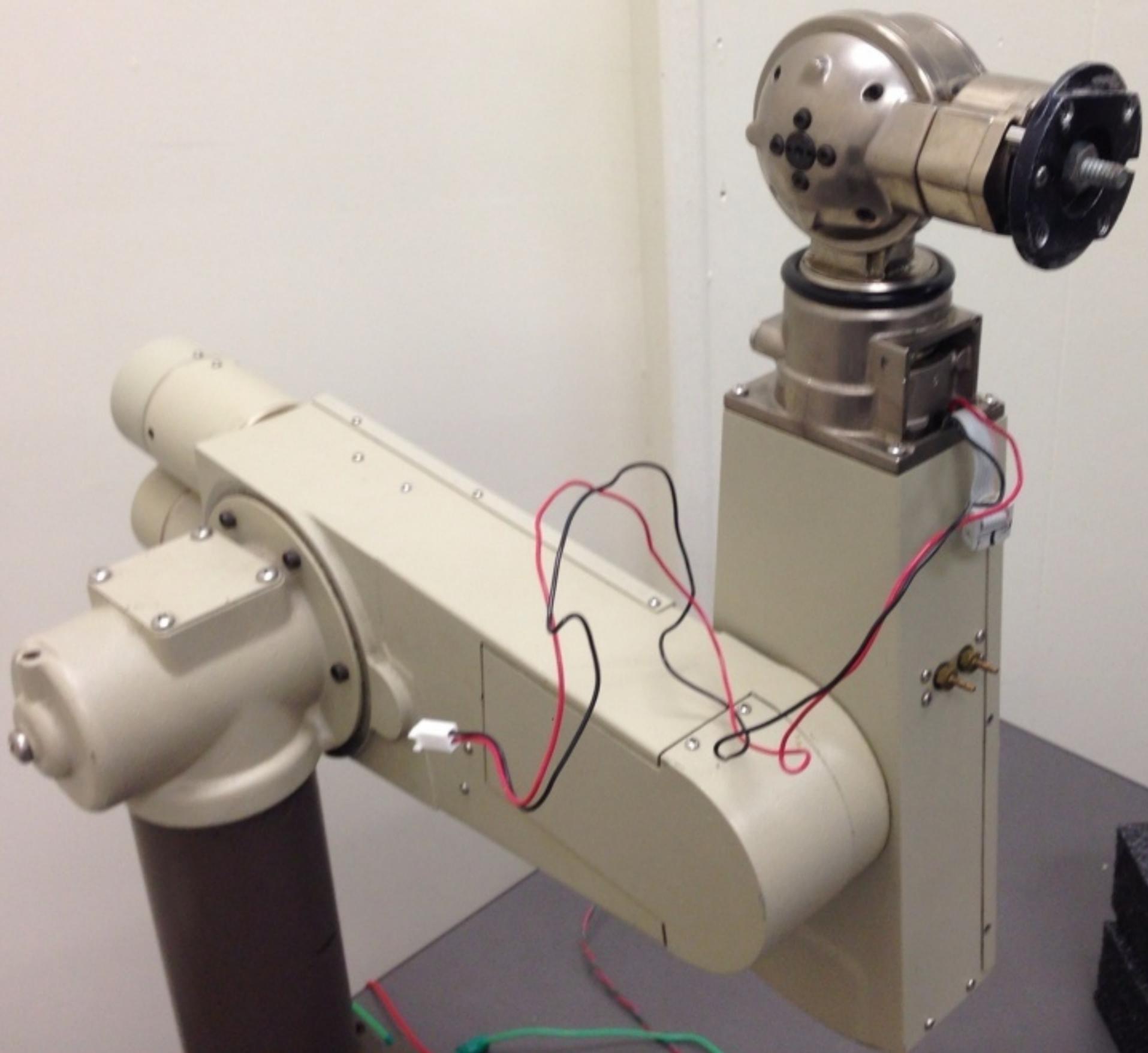


How is it going for your team?

What questions do you have
about Project I ?

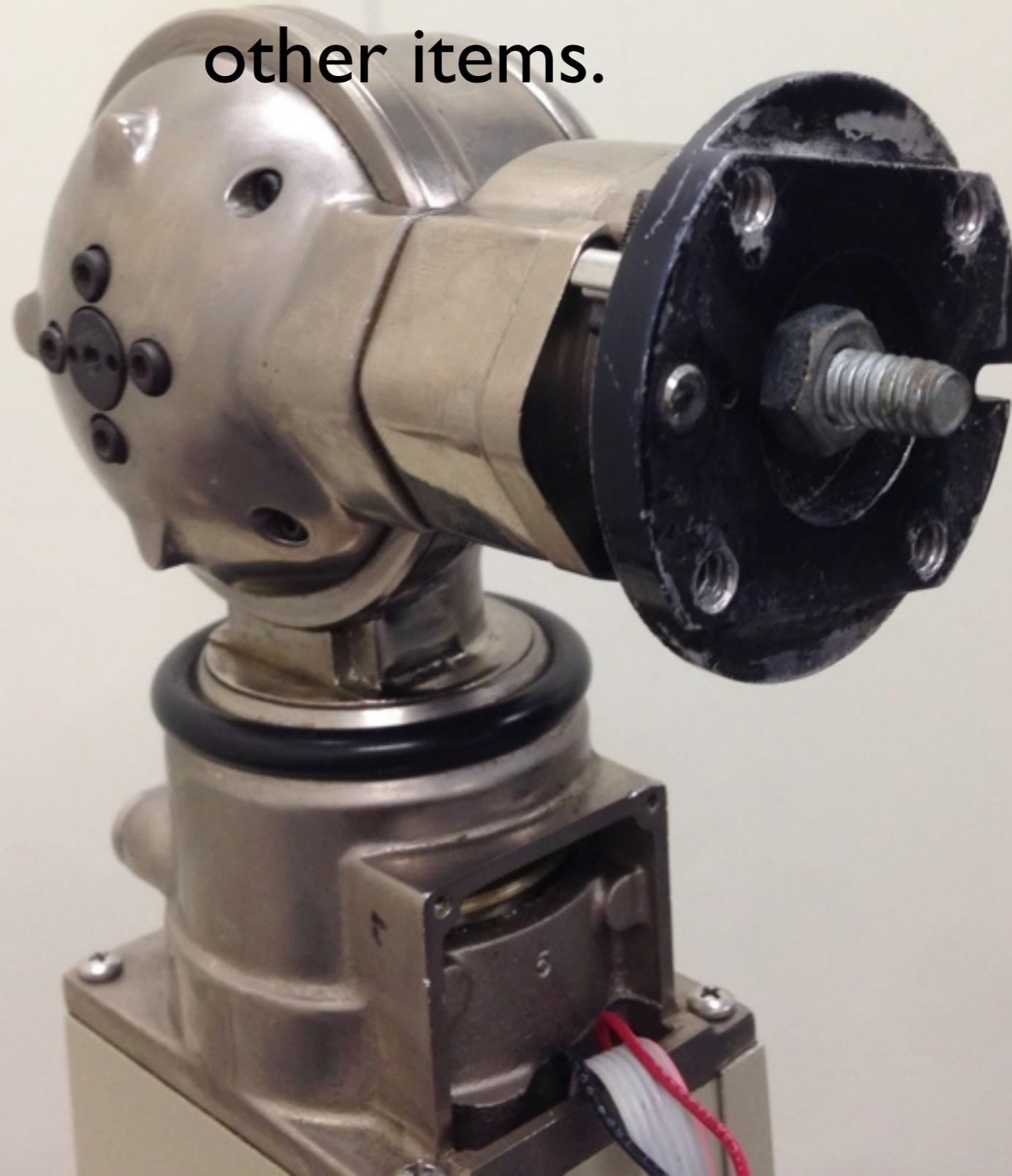
We are getting the robot ready.



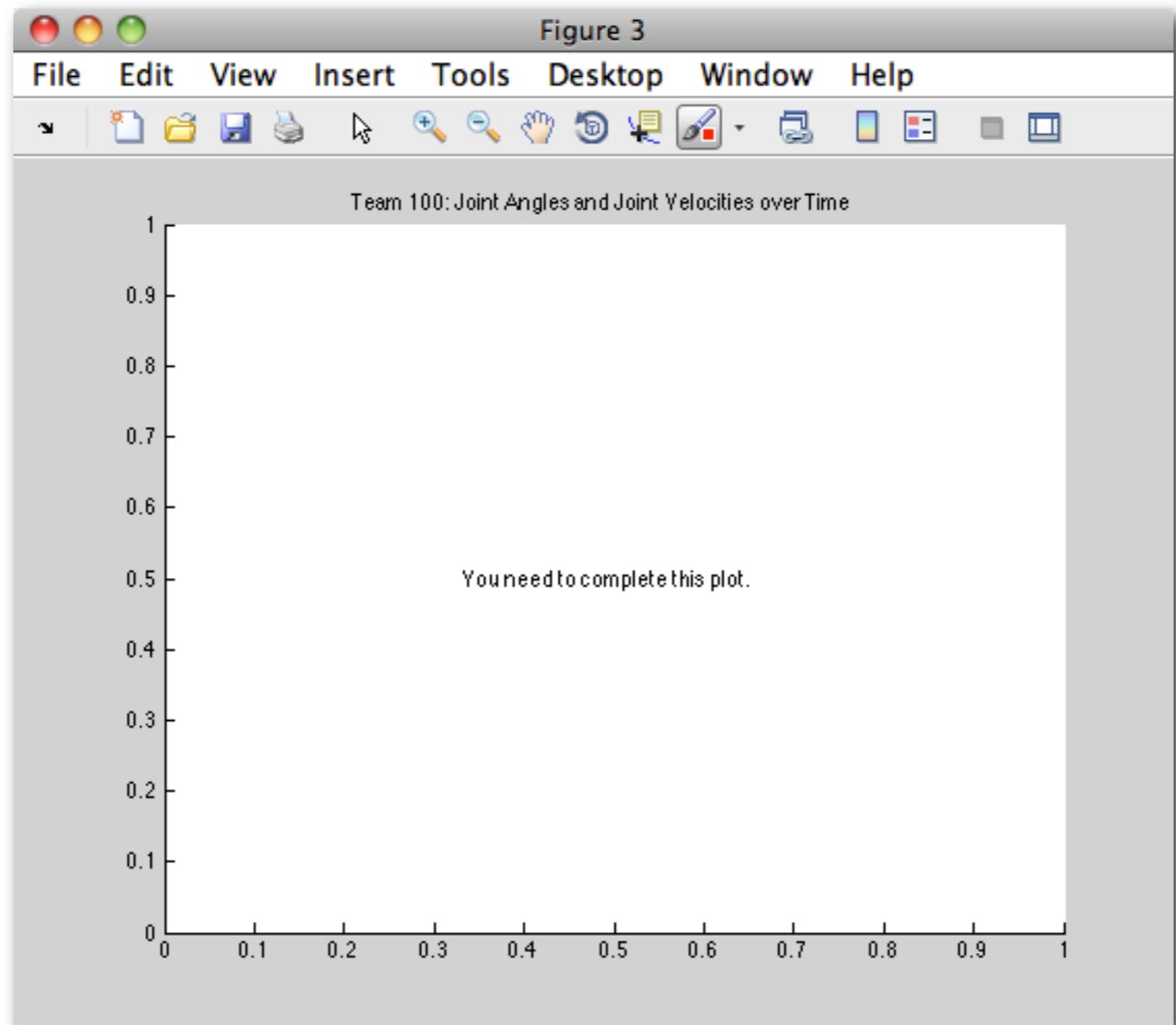


You may choose to attach something to the robot's end effector for the dance. We'll post attachment specs.

You must avoid collisions between the attached object and other items.

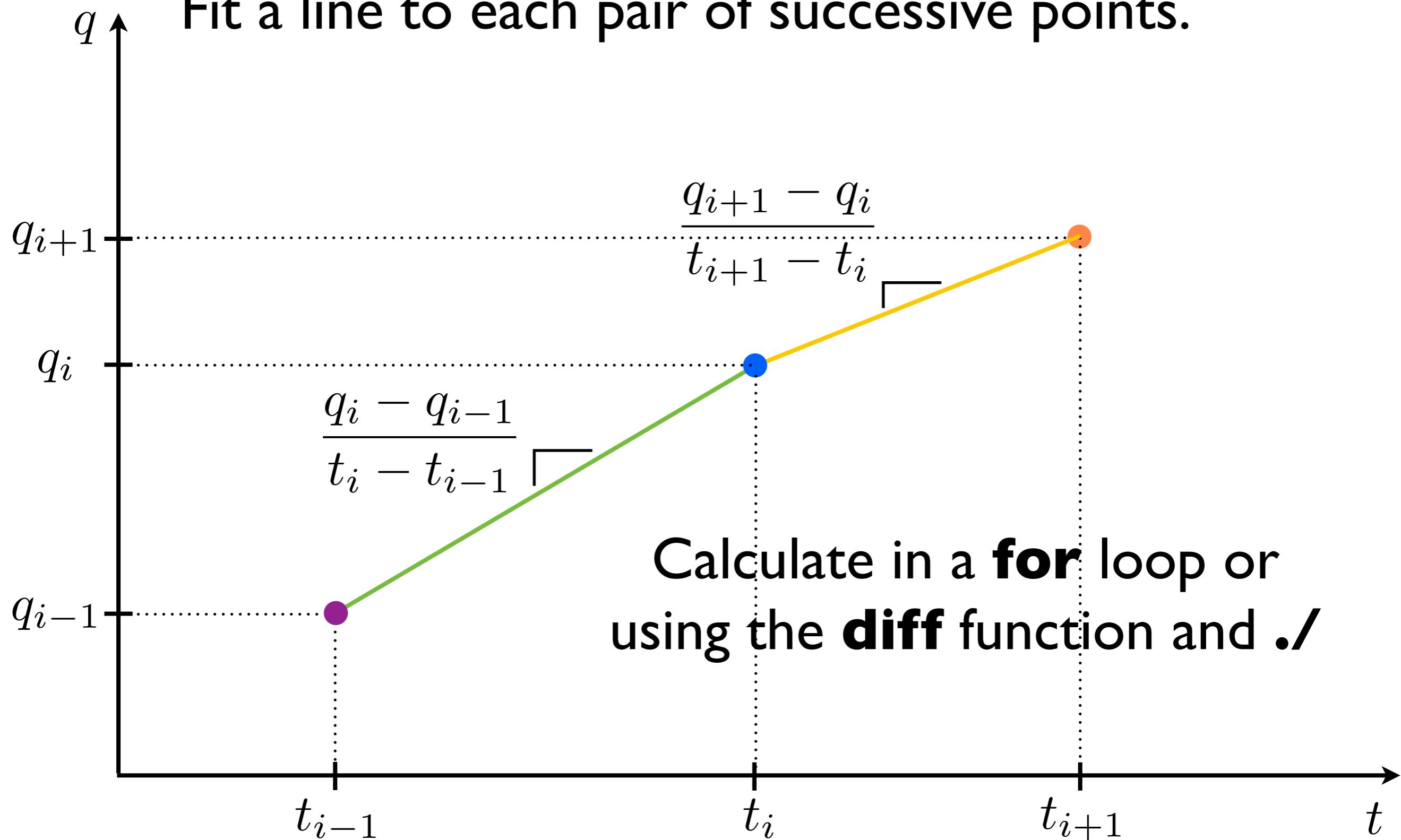


Need to plot joint angles and joint velocities over time. How to do that?



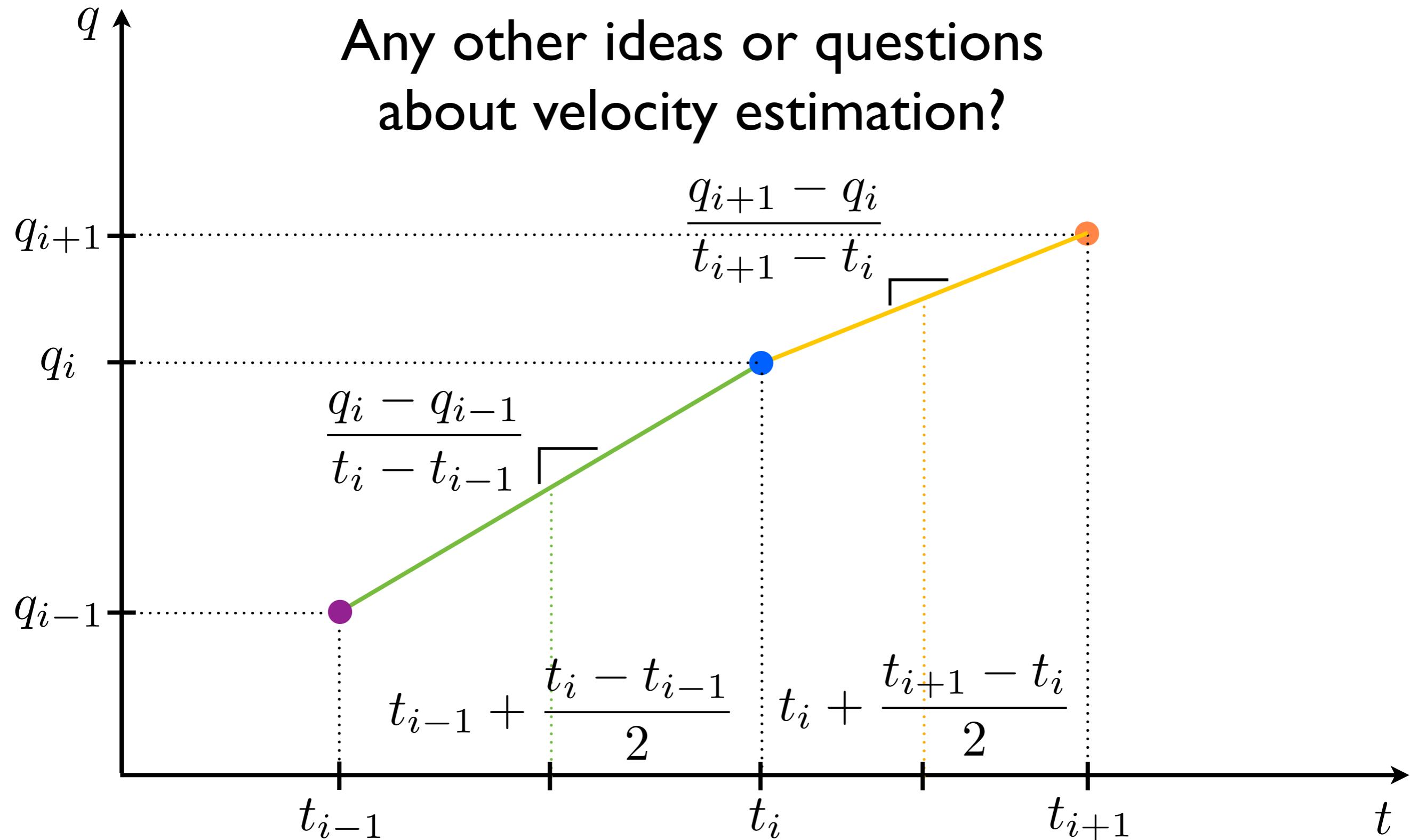
You have a list of times and associated joint angles.
How to estimate joint velocity?

Fit a line to each pair of successive points.



How many velocity estimates do you get? $n - 1$

What time should you associate with each velocity?

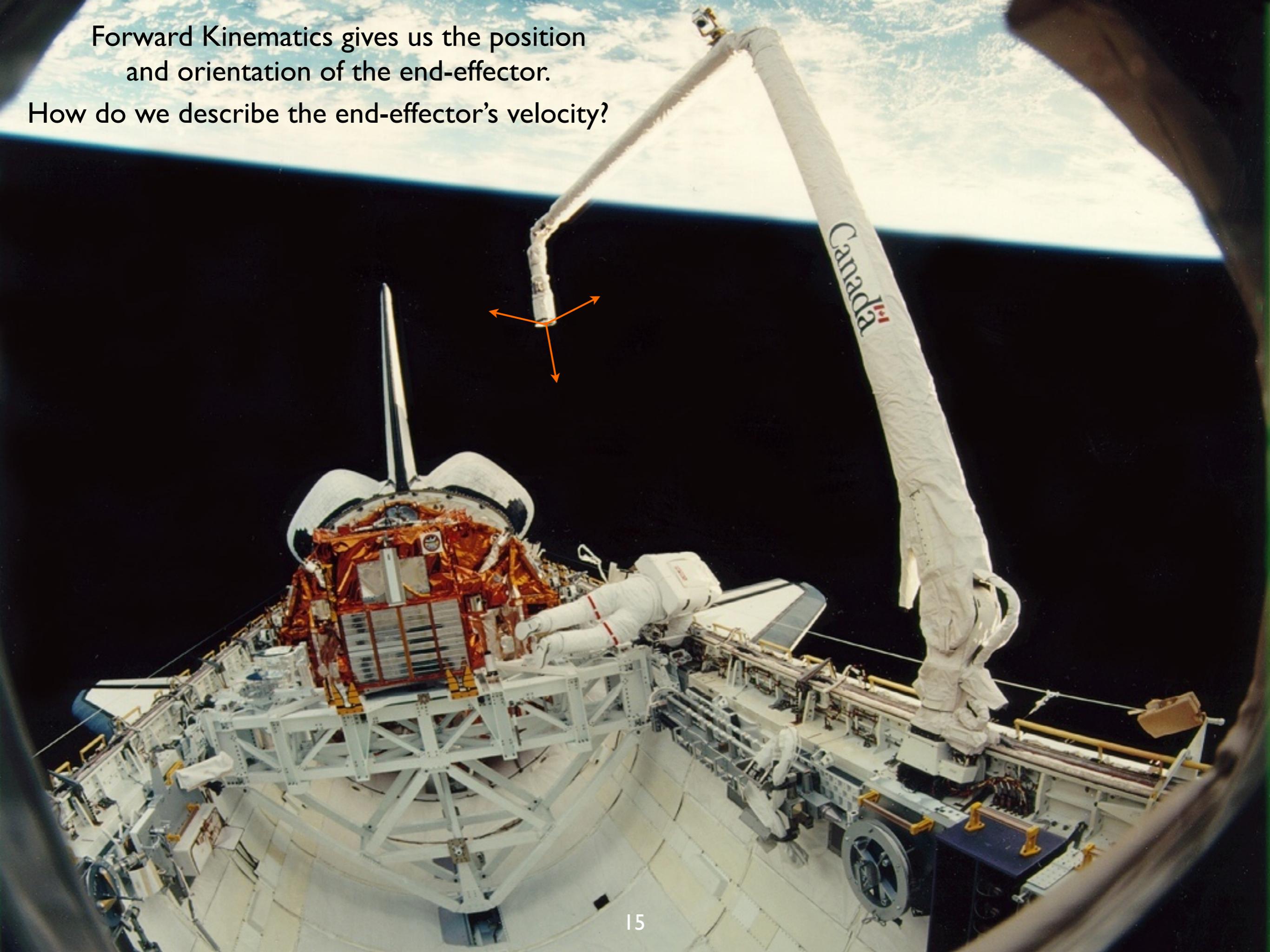


Velocity Kinematics

SHV Chapter 4

Forward Kinematics gives us the position
and orientation of the end-effector.

How do we describe the end-effector's velocity?

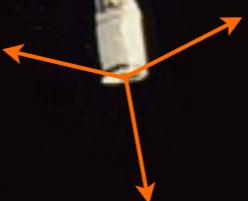




Consider the case of fixed-axis rotation.
All points on the body move in circles around the axis.
Linear velocity thus depends on the point's position.
Angular velocity is a property of the body.

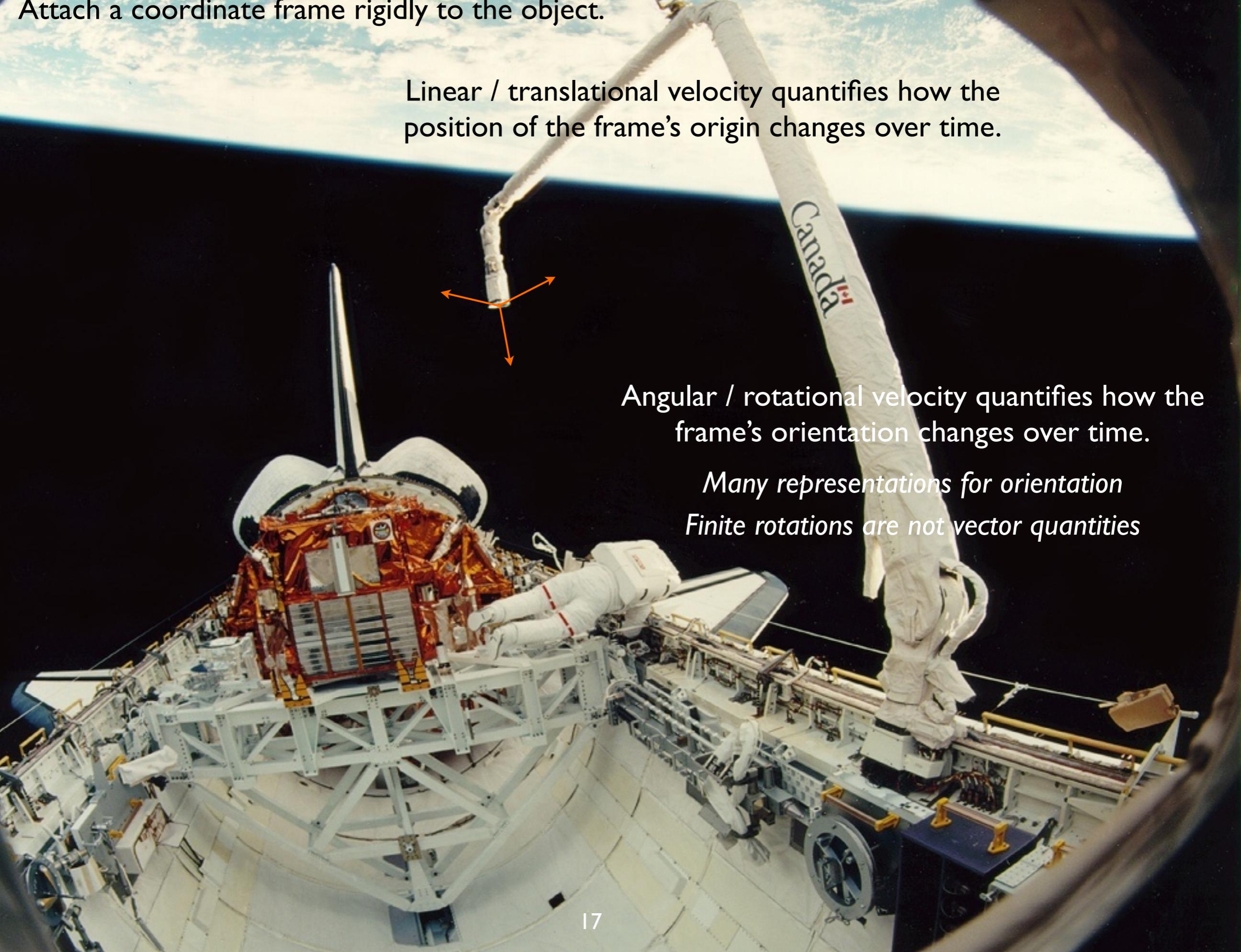
Attach a coordinate frame rigidly to the object.

Linear / translational velocity quantifies how the position of the frame's origin changes over time.



Angular / rotational velocity quantifies how the frame's orientation changes over time.

*Many representations for orientation
Finite rotations are not vector quantities*



What is the time derivative of a rotation matrix?

$$R = R(\theta) \in SO(3)$$

e.g., $R(\theta) = R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$

$$\dot{R} = \frac{dR}{dt} = ?$$

$$\dot{R} = \frac{dR}{dt} = \frac{dR}{d\theta} \frac{d\theta}{dt}$$

?

What is the derivative of a rotation matrix?

$$\frac{dR}{d\theta} = ?$$

$$R R^T = I$$

$$\frac{d}{d\theta} (R R^T) = \frac{d}{d\theta} (I)$$

product rule

$$\frac{dR}{d\theta} R^T + R \frac{dR^T}{d\theta} = 0$$

Interesting equation! Sum of two matrices equals **zero**.

$$\text{define } S = \frac{dR}{d\theta} R^T \qquad S^T = \left(\frac{dR}{d\theta} R^T \right)^T = R \frac{dR^T}{d\theta}$$
$$S + S^T = 0$$

Even more interesting! Sum of a matrix and its transpose equals **zero**.
What does that mean about the matrix S?

Skew-Symmetric Matrices

$$S + S^T = 0$$

$$S = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

What do you know about the elements of S?

Zeros along the diagonal.

Negative values across the diagonal.

$$S = \begin{bmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{bmatrix}$$

Skew-Symmetric Matrices

$$S + S^T = 0 \quad S = \begin{bmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{bmatrix}$$

Define the operator S

$$\vec{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \quad S(\vec{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

The operator S is linear

$$S(\alpha\vec{a} + \beta\vec{b}) = \alpha S(\vec{a}) + \beta S(\vec{b})$$

But what does S do?

$$S(\vec{a}) \vec{p} = ?$$

Skew-Symmetric Matrices

$$S(\vec{a}) \vec{p} = ? = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

$$= \begin{bmatrix} -a_z p_y + a_y p_z \\ a_z p_x - a_x p_z \\ -a_y p_x + a_x p_y \end{bmatrix}$$

$$= \begin{bmatrix} a_y p_z - a_z p_y \\ a_z p_x - a_x p_z \\ a_x p_y - a_y p_x \end{bmatrix}$$

$$S(\vec{a}) \vec{p} = \vec{a} \times \vec{p}$$

Skew-symmetric matrices are a compact way to represent a cross-product between vectors.



$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{v} = S(\vec{\omega})\vec{r}$$

What is the derivative of a rotation matrix?

$$\frac{dR}{d\theta} = ?$$

define $S = \frac{dR}{d\theta} R^T$ $S + S^T = 0$



This matrix is skew-symmetric.

It also contains the quantity we are seeking.

Multiply both sides on the right by R .

$$S R = \frac{dR}{d\theta} R^T R \quad R^T R = I$$

$$\boxed{\frac{dR}{d\theta} = S R}$$

Computing the derivative of a rotation matrix R is equivalent to multiplying that matrix R by a skew-symmetric matrix S .

$$S = \frac{dR}{d\theta} R^T$$

$$\frac{dR}{d\theta} = S R$$

Example

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \quad \dot{R}_{x,\theta} = ?$$

$$\dot{R}_{x,\theta} = \frac{dR_{x,\theta}}{dt} = \frac{dR_{x,\theta}}{d\theta} \frac{d\theta}{dt} = S R_{x,\theta} \frac{d\theta}{dt}$$

$$S = ? = \frac{dR_{x,\theta}}{d\theta} R_{x,\theta}^T$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin \theta & -\cos \theta \\ 0 & \cos \theta & -\sin \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = S(\hat{i}) \quad S(\vec{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$S = \frac{dR}{d\theta} R^T$$

$$\frac{dR}{d\theta} = S R$$

Example

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \quad \dot{R}_{x,\theta} = ?$$

$$\dot{R}_{x,\theta} = \frac{dR_{x,\theta}}{dt} = \frac{dR_{x,\theta}}{d\theta} \frac{d\theta}{dt} = S R_{x,\theta} \frac{d\theta}{dt}$$

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = S(\hat{i})$$

The skew-symmetric matrix S defines the axis about which rotation is occurring.

$$\dot{R}_{x,\theta} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\dot{\theta} \sin \theta & -\dot{\theta} \cos \theta \\ 0 & \dot{\theta} \cos \theta & -\dot{\theta} \sin \theta \end{bmatrix}$$

$$\dot{R}_{x,\theta} = S(\hat{i}) R_{x,\theta} \dot{\theta}$$

$$\dot{R}_{x,\theta} = S(\ddot{\theta} \hat{i}) R_{x,\theta}$$

$$\vec{\omega} = \dot{\theta} \hat{i}$$

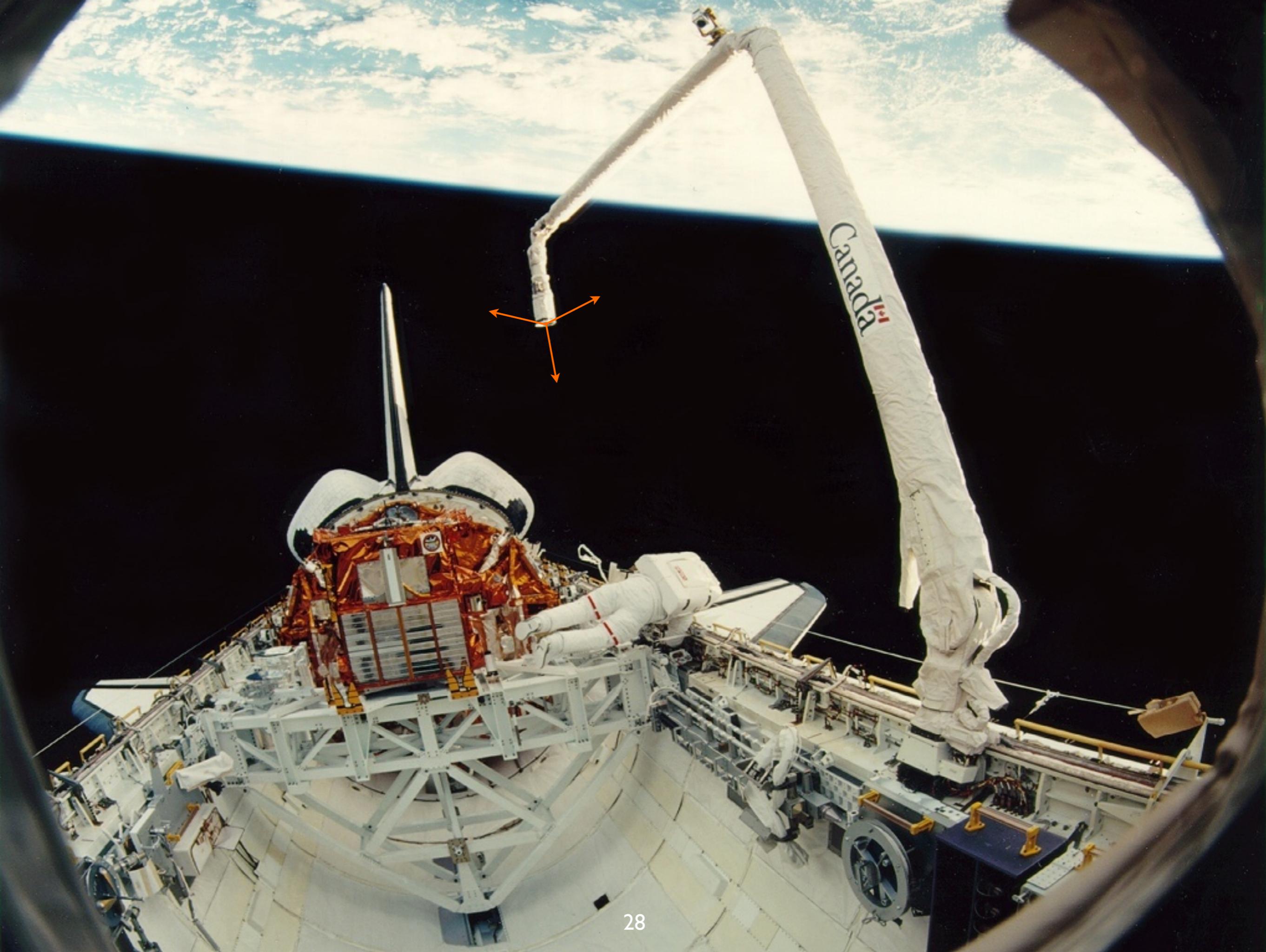
$$\dot{R}_{x,\theta} = S(\vec{\omega}) R_{x,\theta}$$

In general, you simply form S from the angular velocity vector and don't need to differentiate the matrix.

What is this useful for?

Understanding how angular velocities combine on a robotic manipulator.

Calculating the linear velocity of the end-effector of a robot.



Understanding how angular velocities combine on a robotic manipulator

See SHV 4.4: Addition of Angular Velocities

$$R_2^0(t) = R_1^0(t)R_2^1(t)$$

$$\omega_2^0 = \omega_{0,1}^0 + R_1^0\omega_{1,2}^1$$

The angular velocity of frame 2 relative to frame 0
is equal to the angular velocity of frame 1 relative to frame 0, expressed in frame 0,
plus the angular velocity of frame 2 relative to frame 1, expressed in frame 0

You can add angular velocity vectors!

Calculating the linear velocity of the end-effector of a robot

See SHV 4.5: Linear Velocity of a Point Attached to a Moving Frame

$$p^0 = R_1^0(t)p^1 + o_1^0(t)$$

$$\dot{p}^0 = \dot{R}_1^0 p^1 + \dot{o}_1^0$$

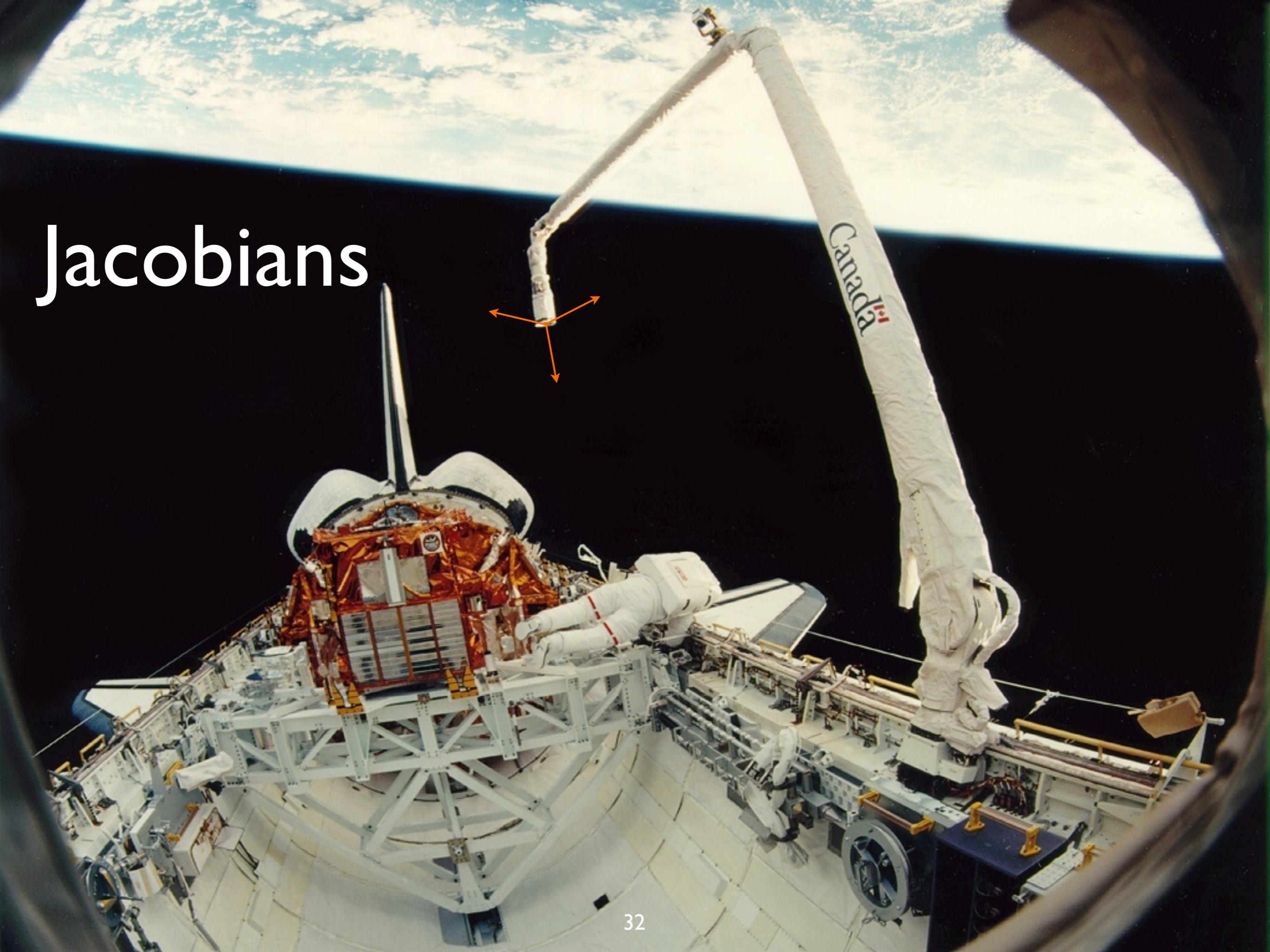
$$\dot{p}^0 = S(\omega^0)R_1^0 p^1 + \dot{o}_1^0$$

$$\dot{p}^0 = \omega^0 \times p^0 + \dot{o}_1^0$$

You can calculate the linear velocity of the end-effector from the angular velocity of its frame, its position relative to the rotational axis, and the linear velocity of its frame's origin.

Questions ?

Jacobians

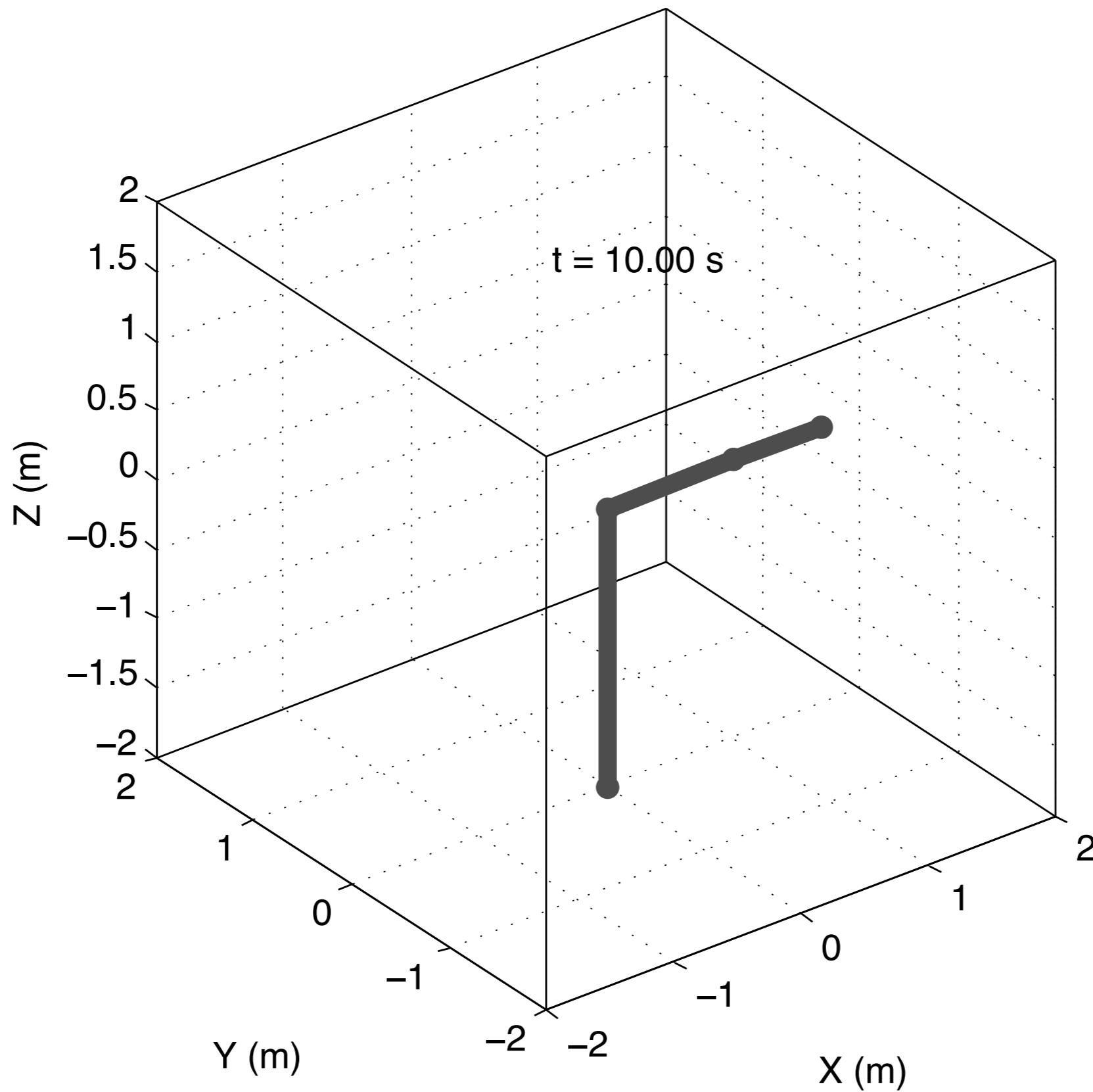


How do the velocities of the joints affect the linear and angular velocity of the end-effector?

These quantities are related by the **Jacobian**, a matrix that generalizes the notion of an ordinary derivative of a scalar function.

Jacobians are useful for planning and executing smooth trajectories, determining singular configurations, executing coordinated anthropomorphic motion, deriving dynamic equations of motion, and transforming forces and torques from the end-effector to the manipulator joints.

SCARA Robot Moving Just a Little Bit



explore how **changes** in joint values affect the end-effector movement

could have **N joints**, but only **six** end-effector velocity terms (xyzpts)

The **Jacobian** matrix lets us calculate how joint velocities translate into end-effector velocities (depends on configuration)

look at it in two parts - position and orientation

$$v_n^0 = J_v \dot{q}$$

$$\omega_n^0 = J_\omega \dot{q}$$

How do we calculate the position Jacobian?

for

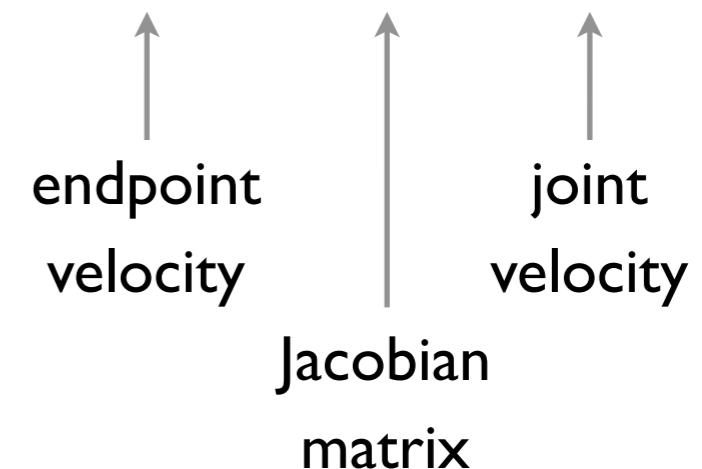
$$x(t) = f(q_1(t), q_2(t), \dots, q_n(t))$$

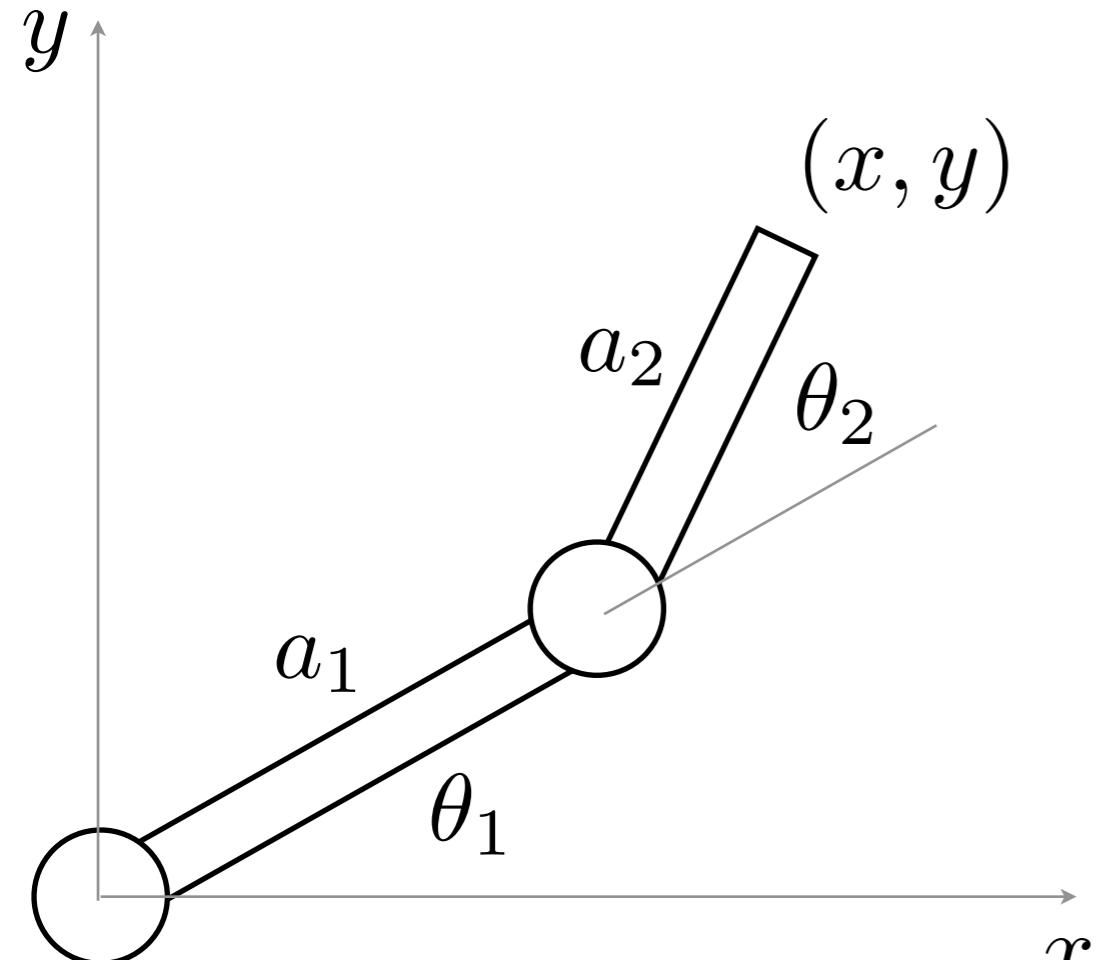
the time derivative can be found using

$$\frac{dx}{dt} = \sum_{i=1}^n \frac{\delta x}{\delta q_i} \frac{dq_i}{dt}$$

For an n-dimensional joint space and a cartesian workspace, the position Jacobian is a 3xn matrix composed of the partial derivatives of the end-effector position with respect to each joint variable.

$$\mathbf{J}_p = \begin{bmatrix} \frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \cdots & \frac{\delta x}{\delta q_n} \\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \cdots & \frac{\delta y}{\delta q_n} \\ \frac{\delta z}{\delta q_1} & \frac{\delta z}{\delta q_2} & \cdots & \frac{\delta z}{\delta q_n} \end{bmatrix}$$

$\dot{\mathbf{p}} = \mathbf{J}_p(q) \dot{\mathbf{q}}$

 endpoint velocity joint velocity
 Jacobian matrix



$$\mathbf{J}_p = \begin{bmatrix} \frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \cdots & \frac{\delta x}{\delta q_n} \\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \cdots & \frac{\delta y}{\delta q_n} \\ \frac{\delta z}{\delta q_1} & \frac{\delta z}{\delta q_2} & \cdots & \frac{\delta z}{\delta q_n} \end{bmatrix}$$

From the forward kinematics, we can extract the position vector from the last column of the transform matrix:

$$\mathbf{d}_2^0 = \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

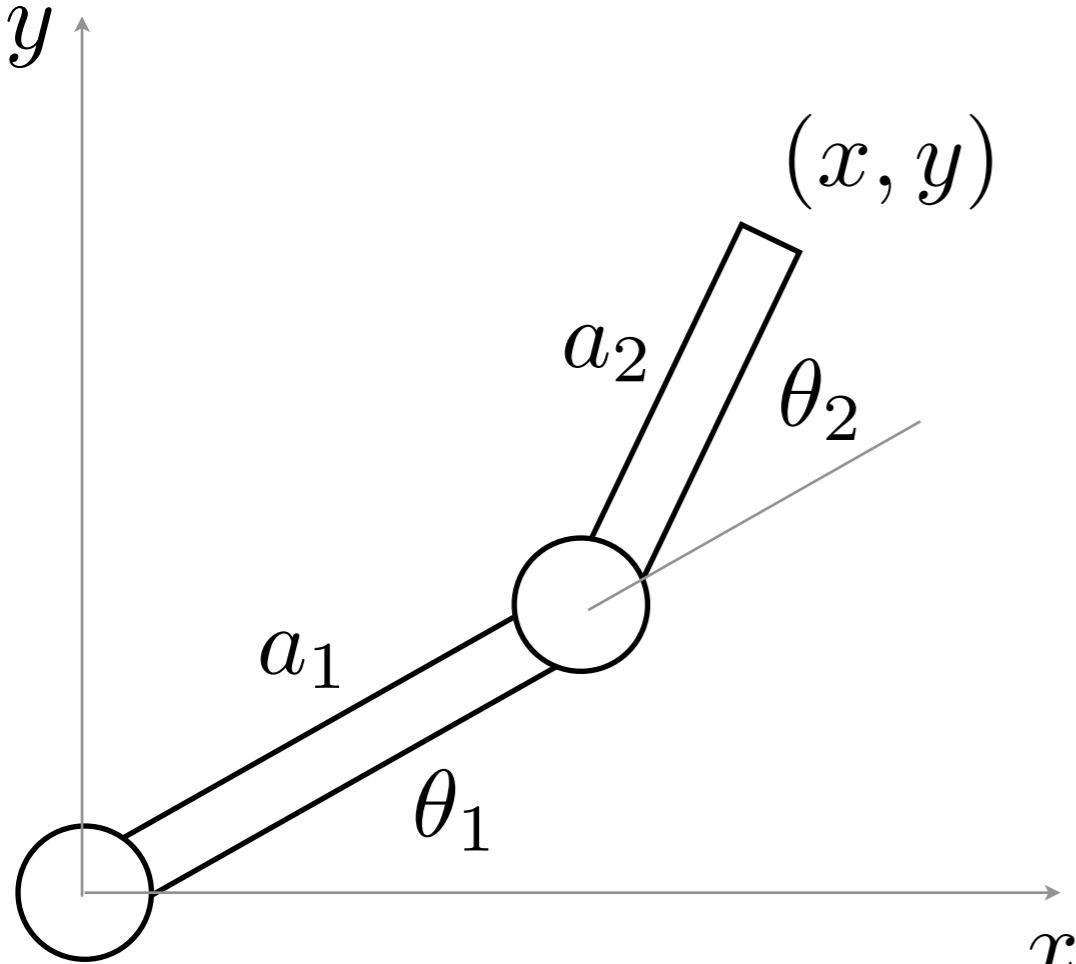
Taking the partial derivative with respect to each joint variable produces the Jacobian:

$$\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix}$$

which relates instantaneous joint velocities to endpoint velocities

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

The Position Jacobian : Planar RR



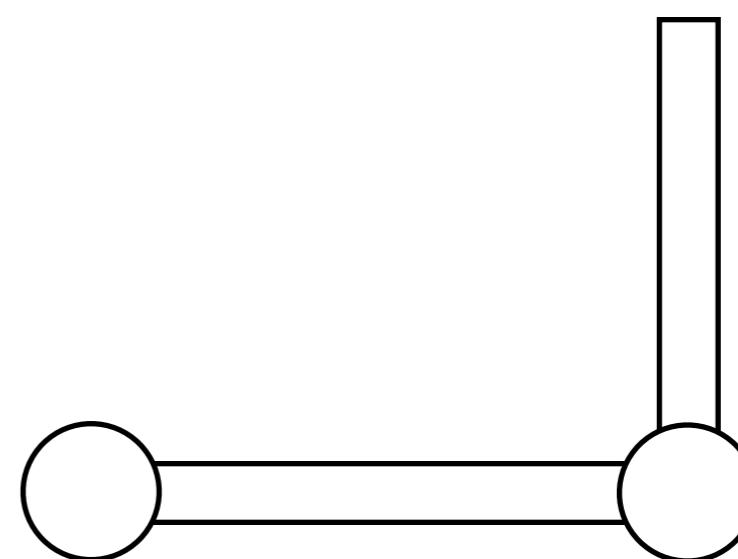
$$\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix}$$

$$\theta_1 = 0, \quad \theta_2 = \pi/2$$

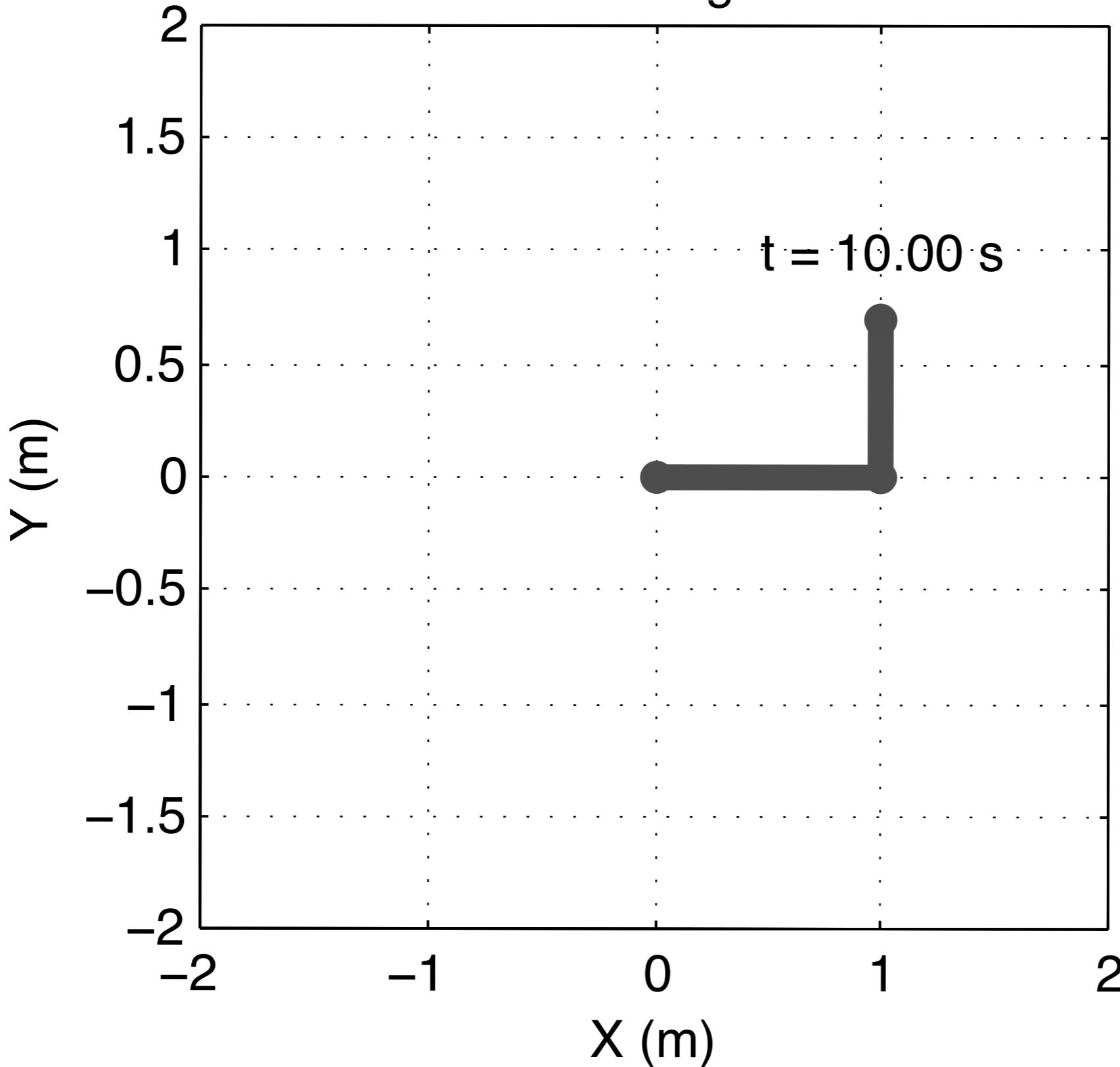
$$\dot{x} = -a_2 \dot{\theta}_1 - a_2 \dot{\theta}_2$$

$$\dot{y} = a_1 \dot{\theta}_1$$

$$\dot{z} = 0$$



SCARA Robot Moving Just a Little Bit



Proposed Midterm Dates

Thursday, October 31, in class

or

Tuesday, November 5, in class

Preferences?

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Please enter your comments by this Friday, October 18. Your responses are anonymous, so you should feel comfortable being honest.

We appreciate your taking the time to complete this evaluation; your feedback will help us improve the class and our teaching for everyone's benefit.

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