MEAM 520

Velocity Kinematics and Jacobians

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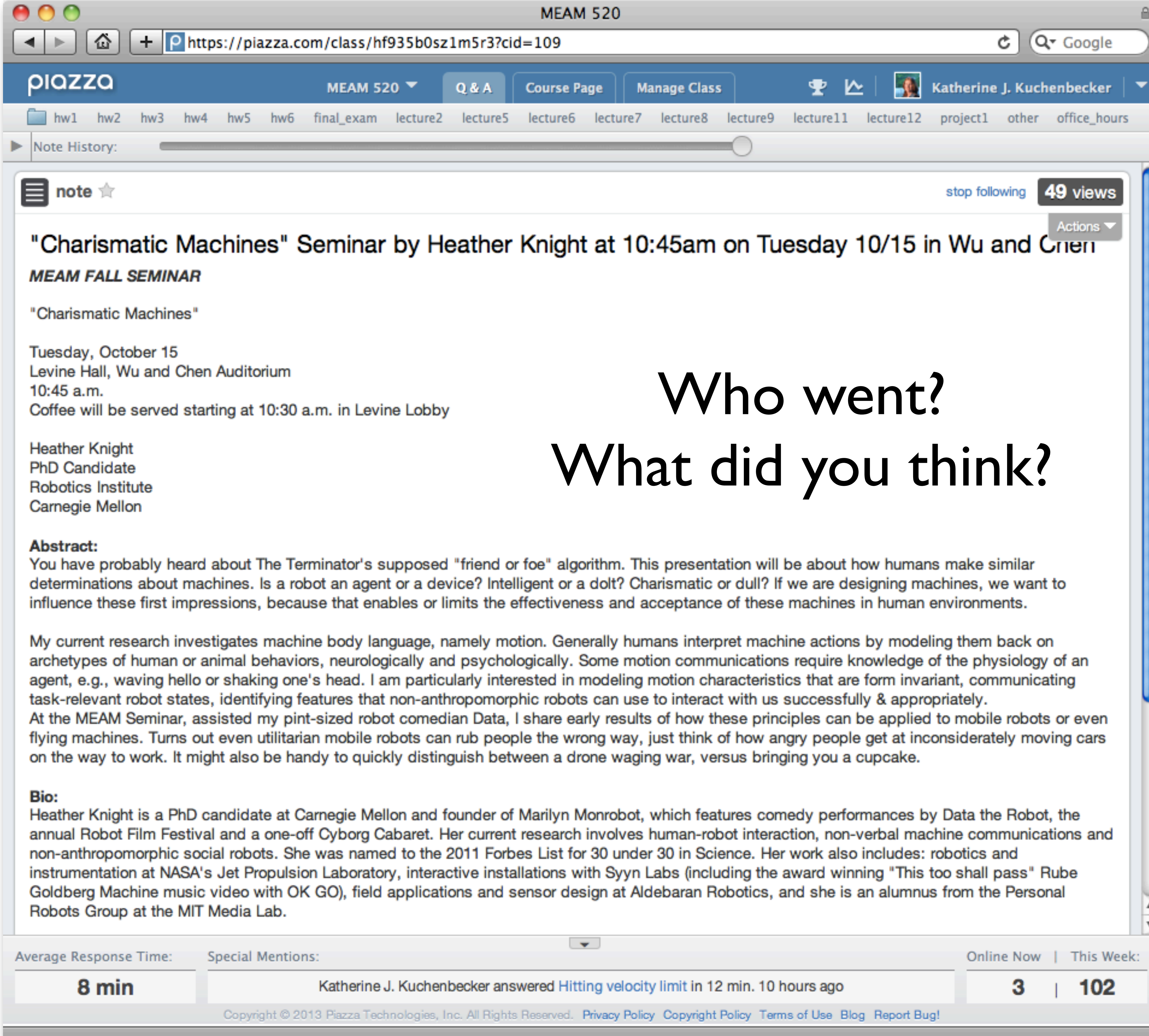
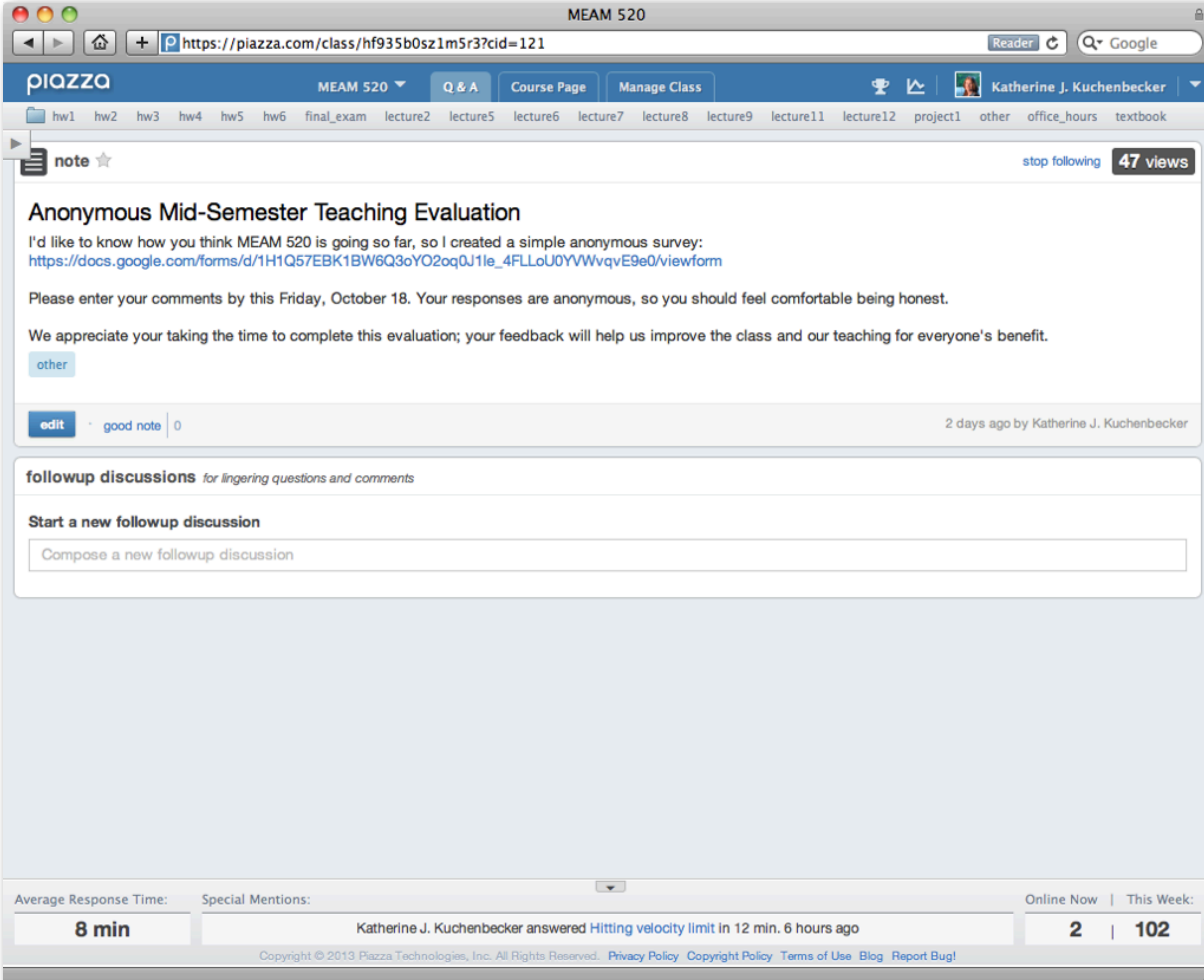
GRASP LABORATORY

Lecture 13: October 15, 2013



Who went?

What did you think?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Anonymous Mid-Semester Teaching Evaluation
I'd like to know how you think MEAM 520 is going so far, so I created a simple anonymous survey:
https://docs.google.com/forms/d/1H1Q57EBK1BW6Q3oYO2oq0J1le_4FLLoU0YVWvqvE9e0/viewform
Please enter your comments by this Friday, October 18. Your responses are anonymous, so you should feel comfortable being honest.
We appreciate your taking the time to complete this evaluation; your feedback will help us improve the class and our teaching for everyone's benefit.
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Project 1

PUMA Dance

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https://piazza.com/upenn/fall2013/meam520/resources

Lecture Slides
(10 out of 18 resources displayed)

Lecture Slides	Lecture Date	Actions
robotics12project1.pdf	Oct 8, 2013	Edit · Post a note · Delete
robotics11trajectories-code.zip	Oct 3, 2013	Edit · Post a note · Delete
robotics11trajectories-handout.pdf	Oct 3, 2013	Edit · Post a note · Delete
robotics11trajectories.pdf	Oct 3, 2013	Edit · Post a note · Delete
robotics10trajectories-code.zip	Oct 1, 2013	Edit · Post a note · Delete
robotics10trajectories.pdf	Oct 1, 2013	Edit · Post a note · Delete
robotics09dh-handout.pdf	Sep 26, 2013	Edit · Post a note · Delete
robotics09dh.pdf	Sep 26, 2013	Edit · Post a note · Delete
robotics08dh-handout.pdf	Sep 24, 2013	Edit · Post a note · Delete
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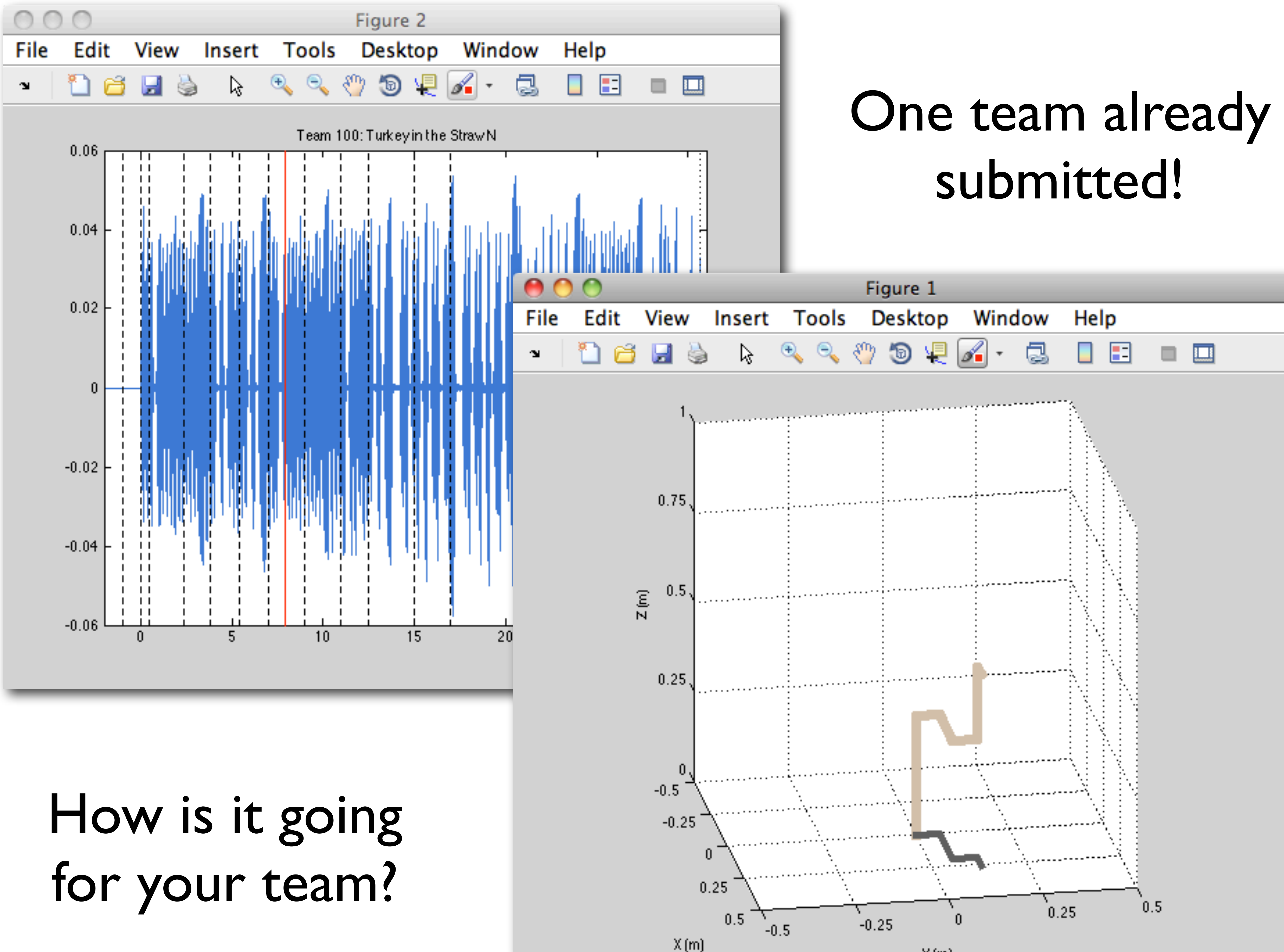
Due by 11:59 p.m. on Sunday, October 20

Homework
(10 out of 12 resources displayed)

Homework	Due Date	Actions
pumadancesim_v2.zip	Oct 20, 2013	Edit · Post a note · Delete

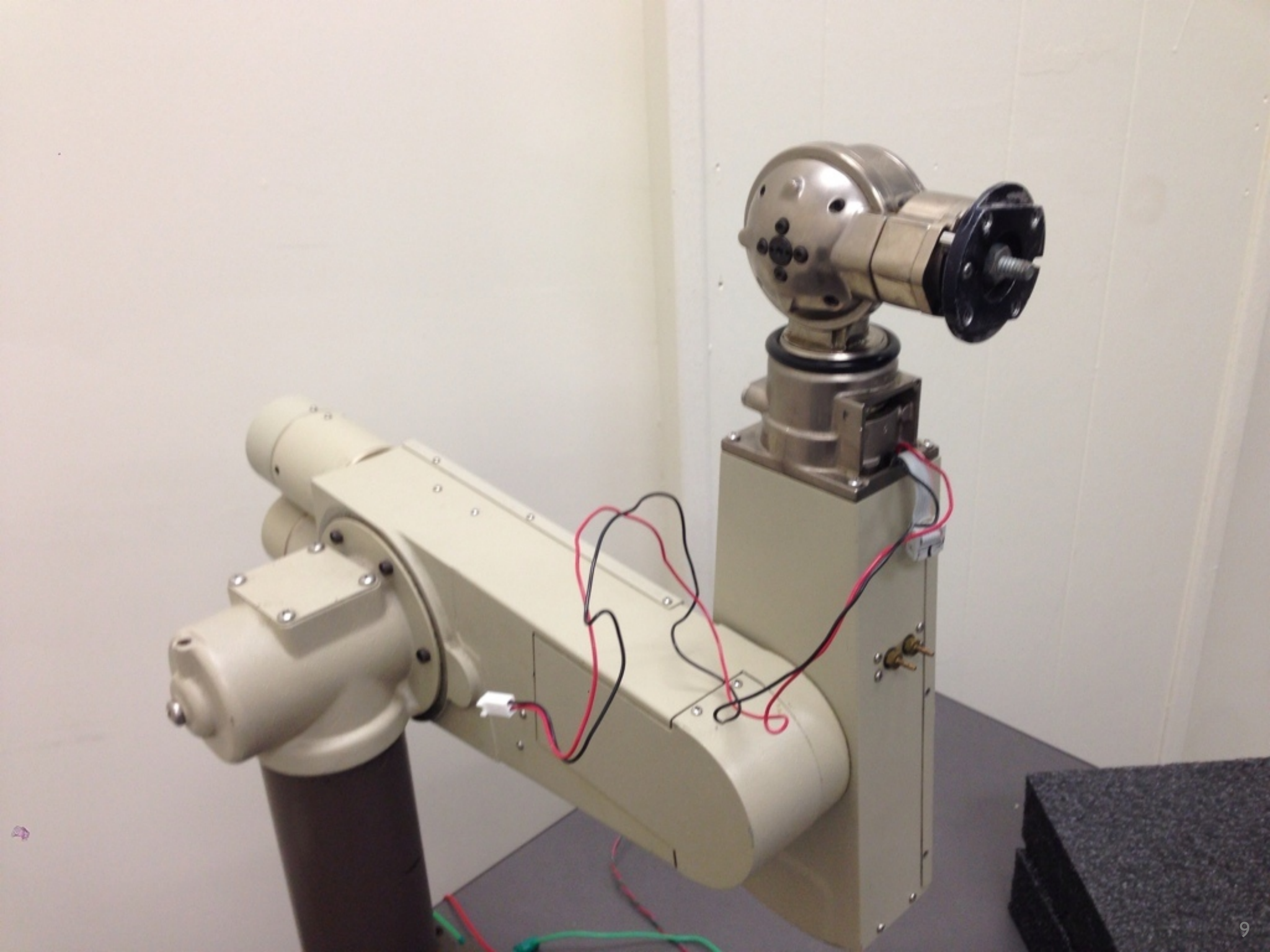


One team already submitted!

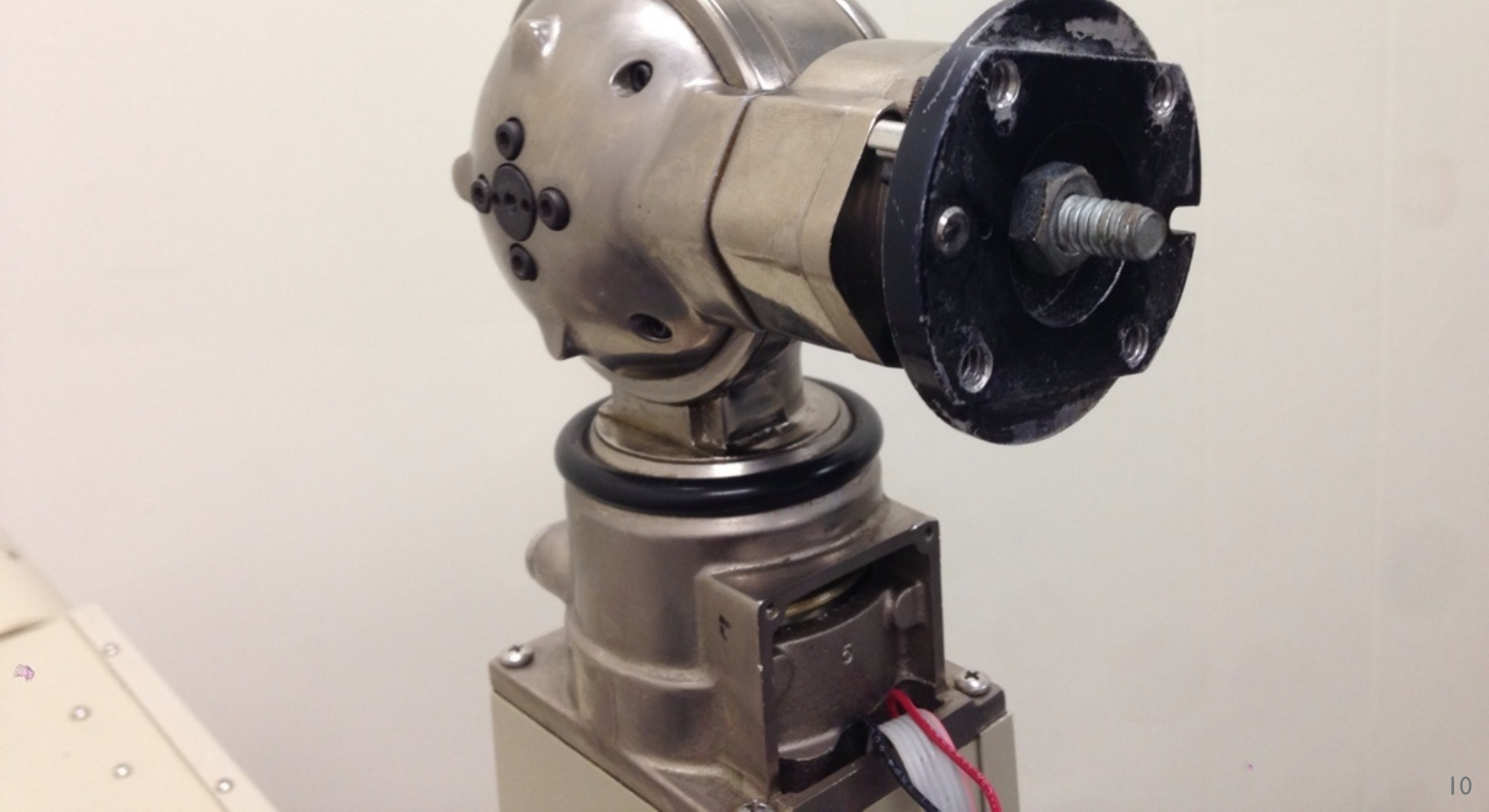
How is it going for your team?

What questions do you have about Project 1 ❓

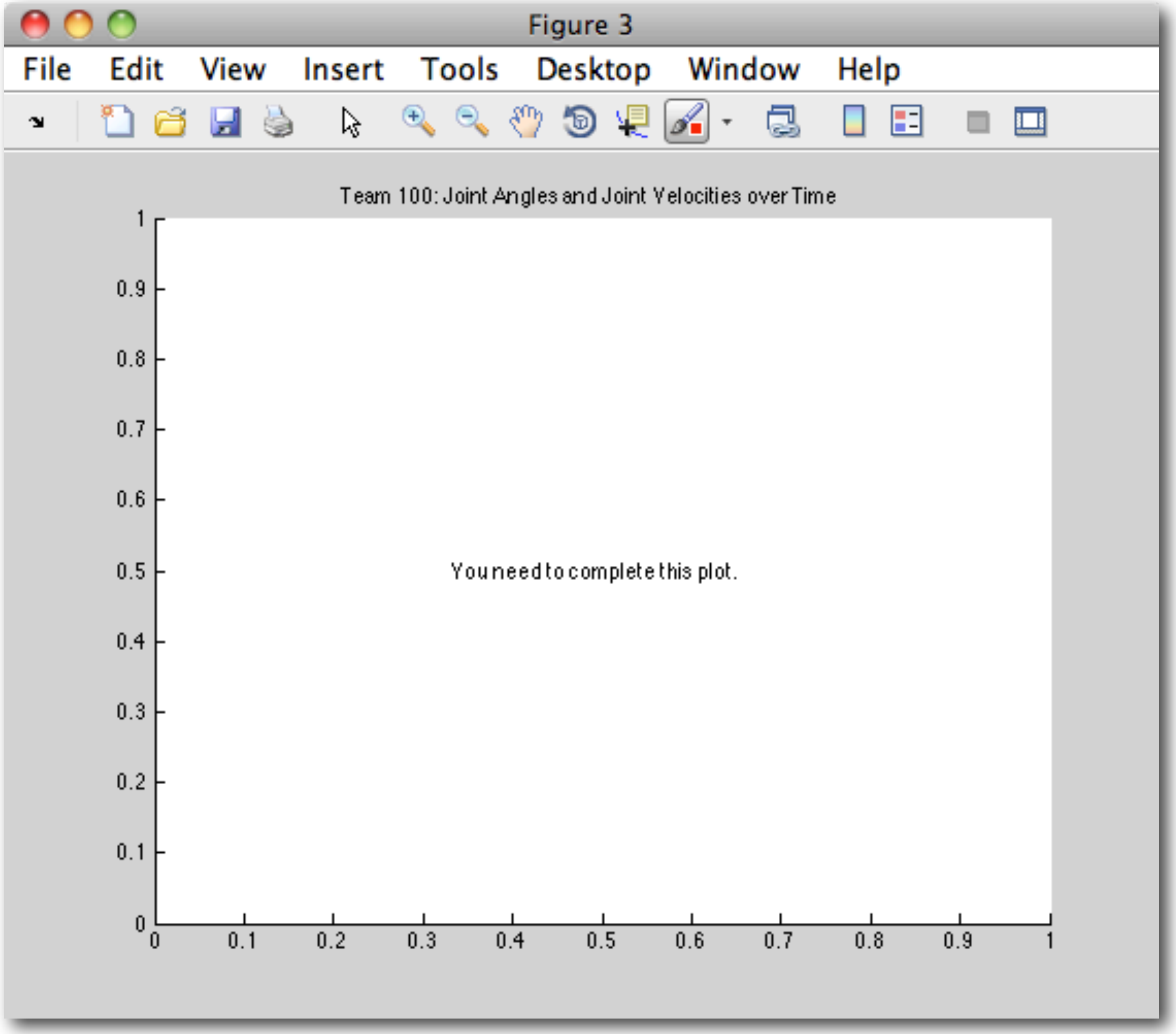
We are getting the robot ready.



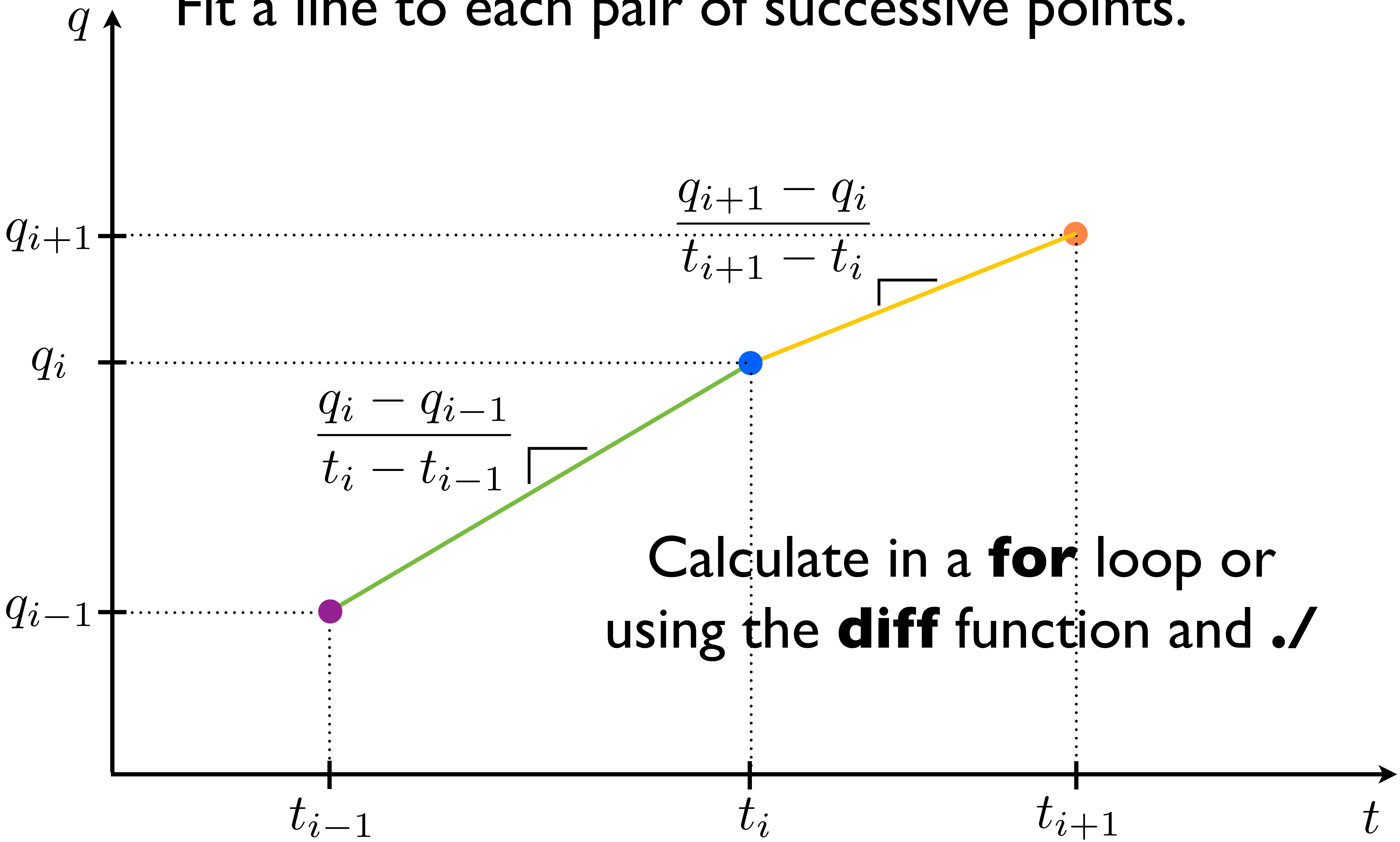
You may choose to attach something to the robot's end effector for the dance. We'll post attachment specs.
You must avoid collisions between the attached object and other items.



Need to plot joint angles and joint velocities over time. How to do that?



You have a list of times and associated joint angles.
How to estimate joint velocity?
Fit a line to each pair of successive points.

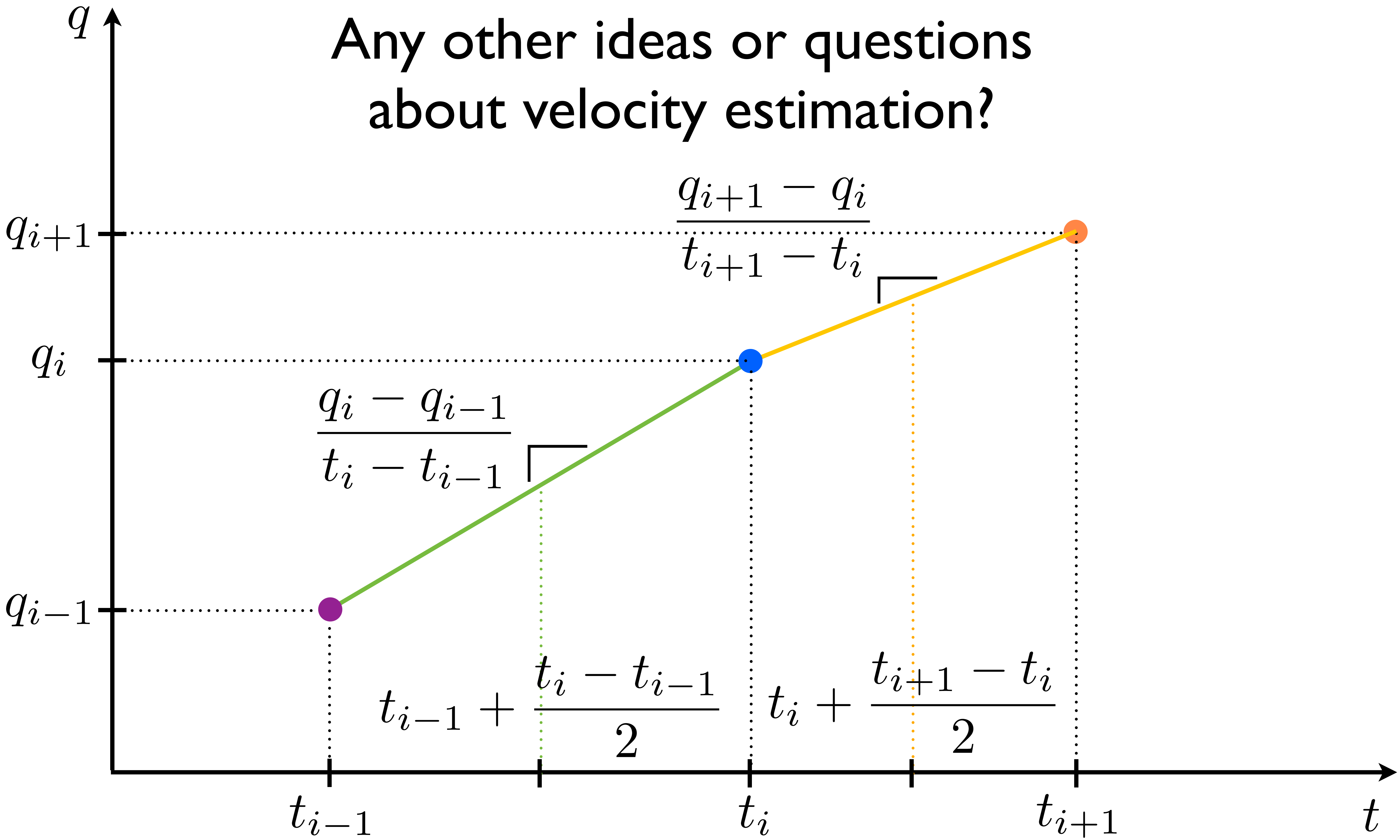


Calculate in a **for** loop or using the **diff** function and **./**

How many velocity estimates do you get? $n-1$

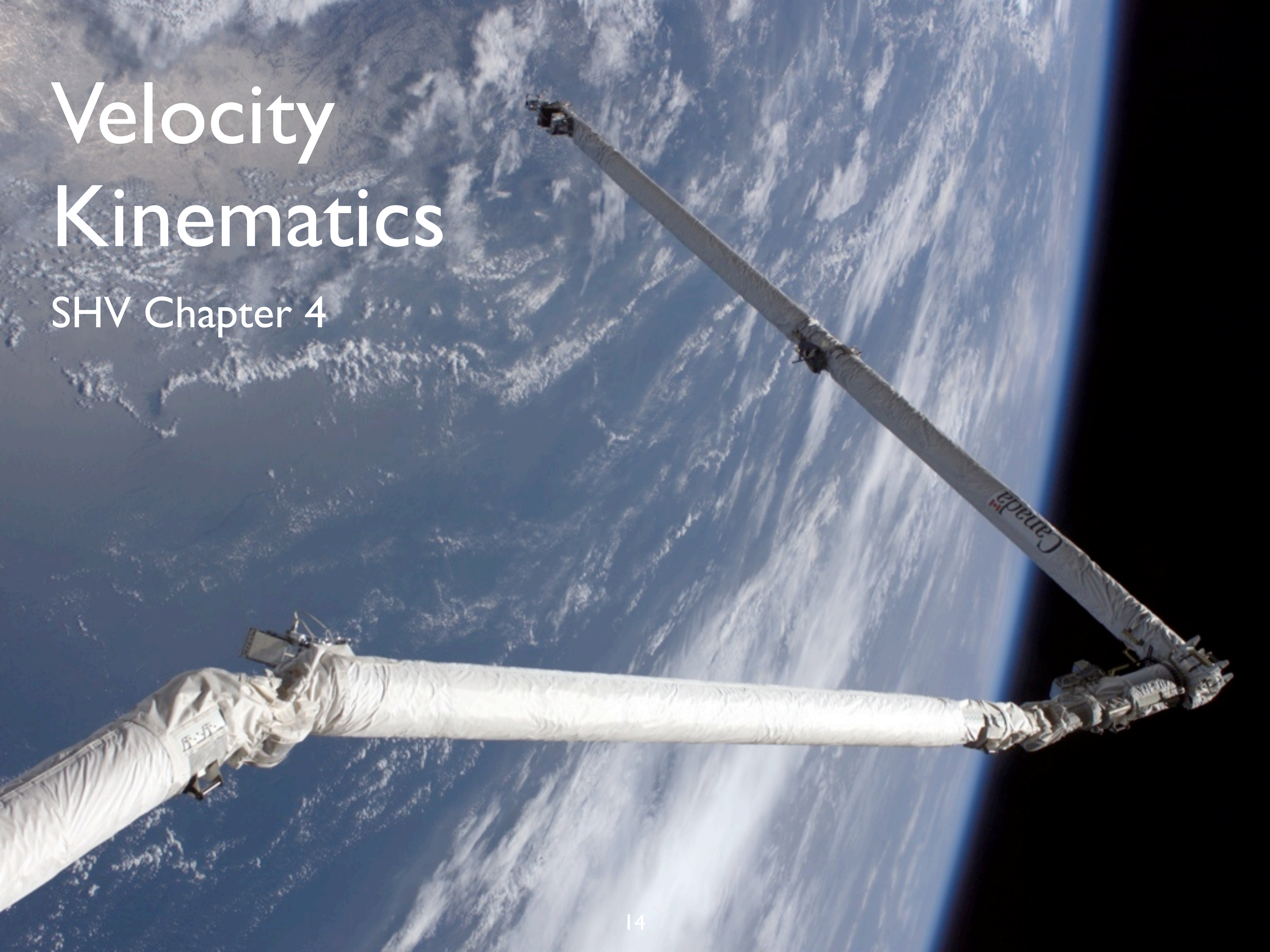
What time should you associate with each velocity?

Any other ideas or questions about velocity estimation?



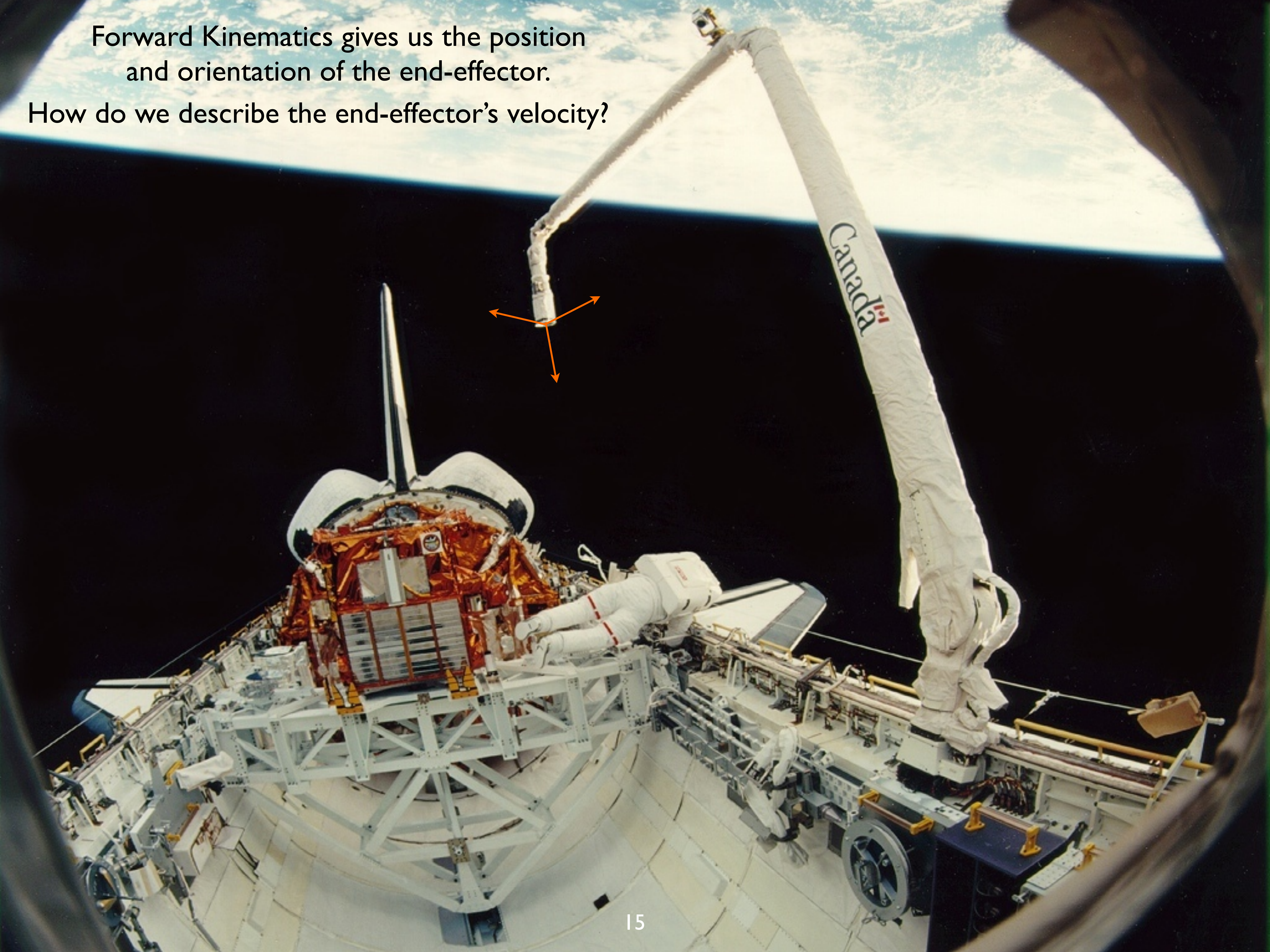
Velocity Kinematics

SHV Chapter 4



Forward Kinematics gives us the position and orientation of the end-effector.

How do we describe the end-effector's velocity?



Consider the case of fixed-axis rotation.
All points on the body move in circles around the axis.
Linear velocity thus depends on the point's position.
Angular velocity is a property of the body.

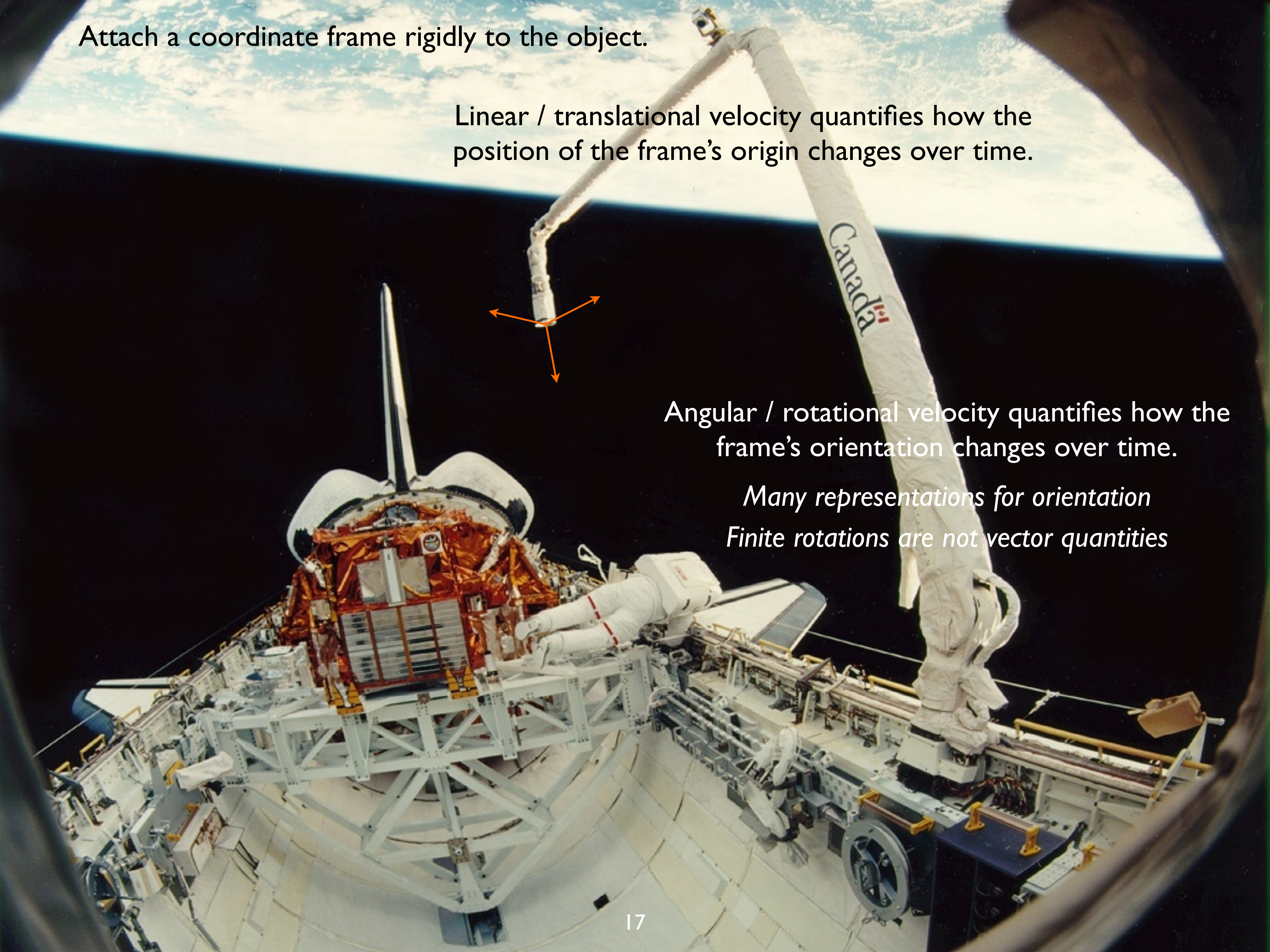
Attach a coordinate frame rigidly to the object.

Linear / translational velocity quantifies how the position of the frame's origin changes over time.

Angular / rotational velocity quantifies how the frame's orientation changes over time.

Many representations for orientation

Finite rotations are not vector quantities



What is the time derivative of a rotation matrix?

$$R = R(\theta) \in SO(3)$$

$$\text{e.g., } R(\theta) = R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\dot{R} = \frac{dR}{dt} = ?$$

$$\dot{R} = \frac{dR}{dt} = \frac{dR}{d\theta}\frac{d\theta}{dt}$$

?

What is the derivative of a rotation matrix?

$$\frac{dR}{d\theta} = ?$$

$$R\,R^T = I$$

$$\frac{d}{d\theta}\left(R\,R^T\right) = \frac{d}{d\theta}(I)$$

product rule

$$\frac{dR}{d\theta}R^T + R\frac{dR^T}{d\theta} = 0$$

Interesting equation! Sum of two matrices equals **zero**.

$$\text{define } S = \frac{dR}{d\theta}R^T \qquad S^T = \left(\frac{dR}{d\theta}R^T\right)^T = R\frac{dR^T}{d\theta}$$

$$S + S^T = 0$$

Even more interesting! Sum of a matrix and its transpose equals **zero**.
What does that mean about the matrix S?

Skew-Symmetric Matrices

$$S + S^T = 0$$

$$S = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

What do you know about the elements of S?

Zeros along the diagonal.
Negative values across the diagonal.

$$S = \begin{bmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{bmatrix}$$

Skew-Symmetric Matrices

$$S + S^T = 0 \qquad S = \begin{bmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{bmatrix}$$

Define the operator S

$$\vec{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \qquad S(\vec{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

The operator S is linear

$$S(\alpha\vec{a} + \beta\vec{b}) = \alpha S(\vec{a}) + \beta S(\vec{b})$$

But what does S do?

$$S(\vec{a})\,\vec{p} = ?$$

Skew-Symmetric Matrices

$$S(\vec{a})\,\vec{p} = ? = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

$$= \begin{bmatrix} -a_z p_y + a_y p_z \\ a_z p_x - a_x p_z \\ -a_y p_x + a_x p_y \end{bmatrix}$$

$$= \begin{bmatrix} a_y p_z - a_z p_y \\ a_z p_x - a_x p_z \\ a_x p_y - a_y p_x \end{bmatrix}$$

$$S(\vec{a})\vec{p} = \vec{a} \times \vec{p}$$

Skew-symmetric matrices are a compact way to represent a cross-product between vectors.


$\vec{v} = ?$
$\vec{r}$
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$\vec{v} = \vec{\omega} \times \vec{r}$
$\vec{\omega}$
$\vec{v} = S(\vec{\omega})\vec{r}$

What is the derivative of a rotation matrix?

$$\frac{dR}{d\theta} = ? \qquad \text{define } S = \frac{dR}{d\theta} R^T \qquad S + S^T = 0$$

This matrix is skew-symmetric.

It also contains the quantity we are seeking.

Multiply both sides on the right by R.

$$S\,R = \frac{dR}{d\theta} R^T R \qquad R^T R = I$$

$$\boxed{\frac{dR}{d\theta} = S\,R}$$

Computing the derivative of a rotation matrix R is equivalent to multiplying that matrix R by a skew-symmetric matrix S.

$$\boxed{\begin{array}{c} S = \dfrac{dR}{d\theta} R^T \\ \dfrac{dR}{d\theta} = S\,R \end{array}}$$

Example

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad \dot{R}_{x,\theta} = ?$$

$$\dot{R}_{x,\theta} = \frac{dR_{x,\theta}}{dt} = \frac{dR_{x,\theta}}{d\theta}\frac{d\theta}{dt} = S R_{x,\theta}\frac{d\theta}{dt}$$

$$S = ? = \frac{dR_{x,\theta}}{d\theta} R_{x,\theta}^T$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\theta & -\cos\theta \\ 0 & \cos\theta & -\sin\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = S(\hat{i}) \qquad S(\vec{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\boxed{\begin{gathered} S = \frac{dR}{d\theta}R^T \\ \frac{dR}{d\theta} = S\,R \end{gathered}}$$

Example

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad \dot{R}_{x,\theta} = ?$$

$$\dot{R}_{x,\theta} = \frac{dR_{x,\theta}}{dt} = \frac{dR_{x,\theta}}{d\theta}\frac{d\theta}{dt} = SR_{x,\theta}\frac{d\theta}{dt}$$

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = S(\hat{i})$$

The skew-symmetric matrix S defines the axis about which rotation is occurring.

$$\dot{R}_{x,\theta} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta \\ 0 & \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta \end{bmatrix}$$

$$\dot{R}_{x,\theta} = S(\hat{i})R_{x,\theta}\dot{\theta}$$

$$\dot{R}_{x,\theta} = S(\dot{\theta}\hat{i})R_{x,\theta}$$

$$\vec{\omega} = \dot{\theta}\hat{i}$$

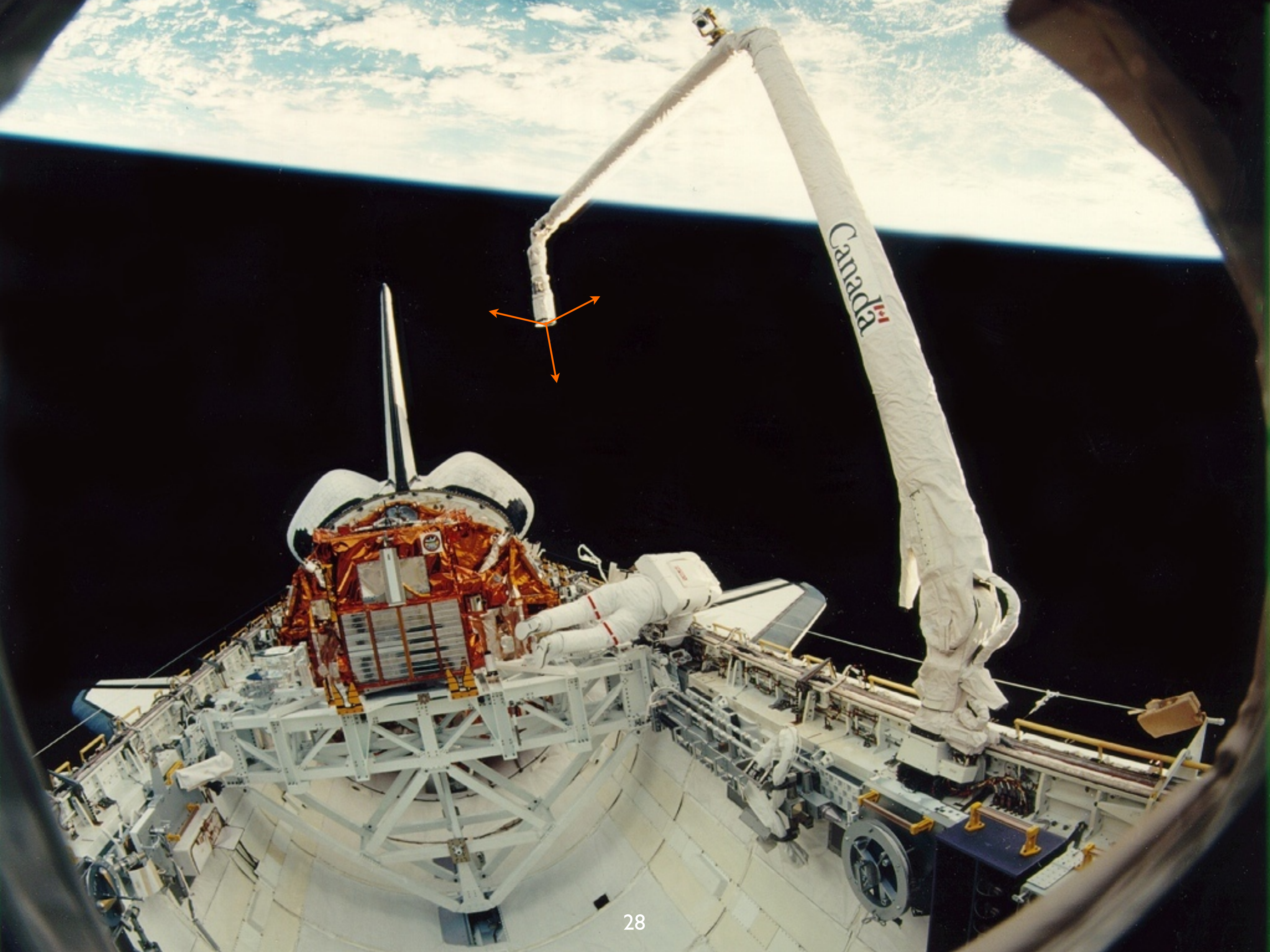
$$\dot{R}_{x,\theta} = S(\vec{\omega})R_{x,\theta}$$

In general, you simply form S from the angular velocity vector and don't need to differentiate the matrix.

What is this useful for?

Understanding how angular velocities combine on a robotic manipulator.

Calculating the linear velocity of the end-effector of a robot.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Understanding how angular velocities combine on a robotic manipulator

See SHV 4.4: Addition of Angular Velocities

$$R_2^0(t) = R_1^0(t)R_2^1(t)$$

$$\omega_2^0 = \omega_{0,1}^0 + R_1^0\omega_{1,2}^1$$

The angular velocity of frame 2 relative to frame 0
is equal to the angular velocity of frame 1 relative to frame 0, expressed in frame 0,
plus the angular velocity of frame 2 relative to frame 1, expressed in frame 0

You can add angular velocity vectors!

Calculating the linear velocity of the end-effector of a robot

See SHV 4.5: Linear Velocity of a Point Attached to a Moving Frame

$$p^0 = R_1^0(t)p^1 + o_1^0(t)$$

$$\dot{p}^0 = \dot{R}_1^0 p^1 + \dot{o}_1^0$$

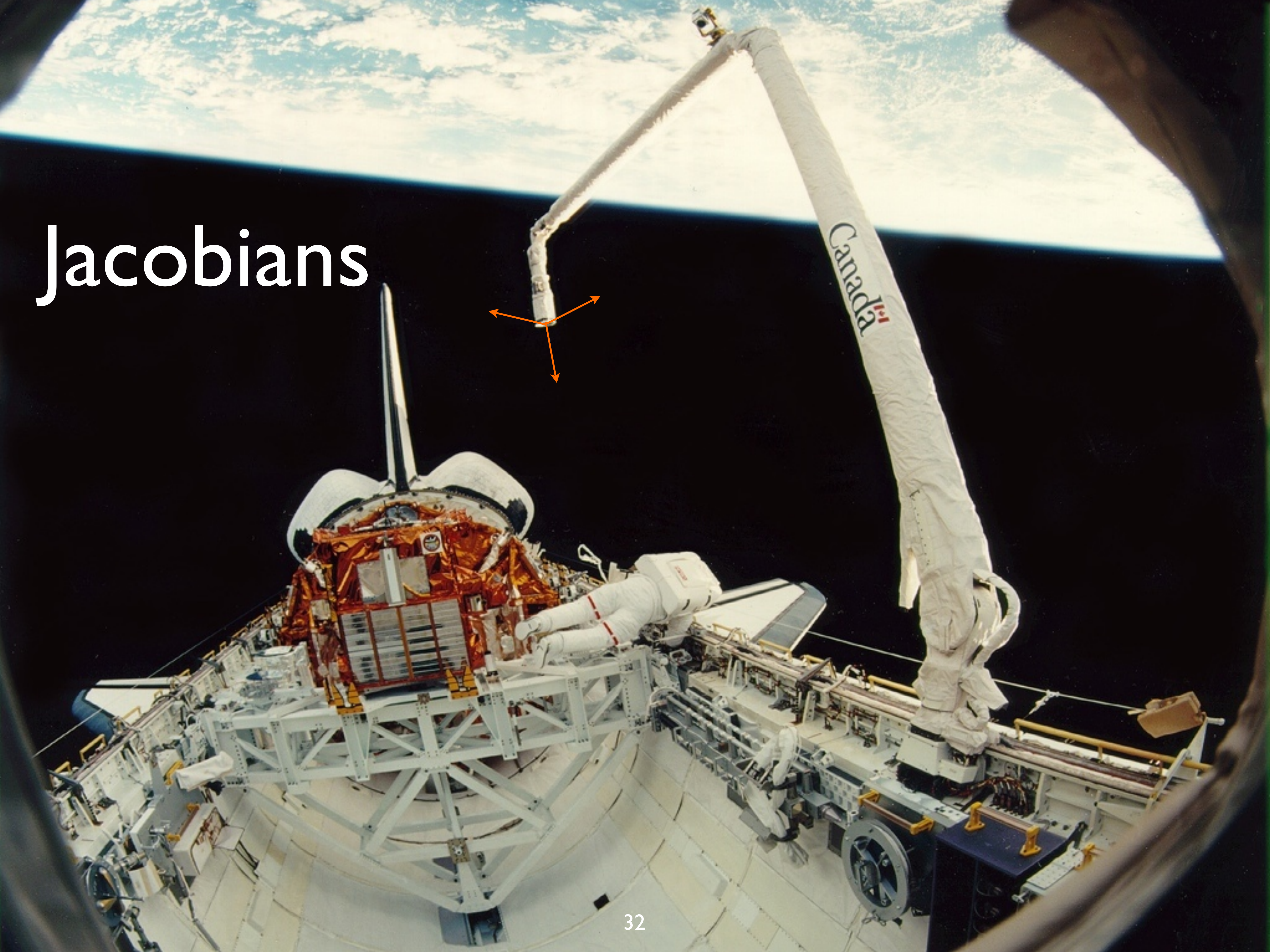
$$\dot{p}^0 = S(\omega^0)R_1^0 p^1 + \dot{o}_1^0$$

$$\dot{p}^0 = \omega^0 \times p^0 + \dot{o}_1^0$$

You can calculate the linear velocity of the end-effector from the angular velocity of its frame, its position relative to the rotational axis, and the linear velocity of its frame's origin.

Questions ?

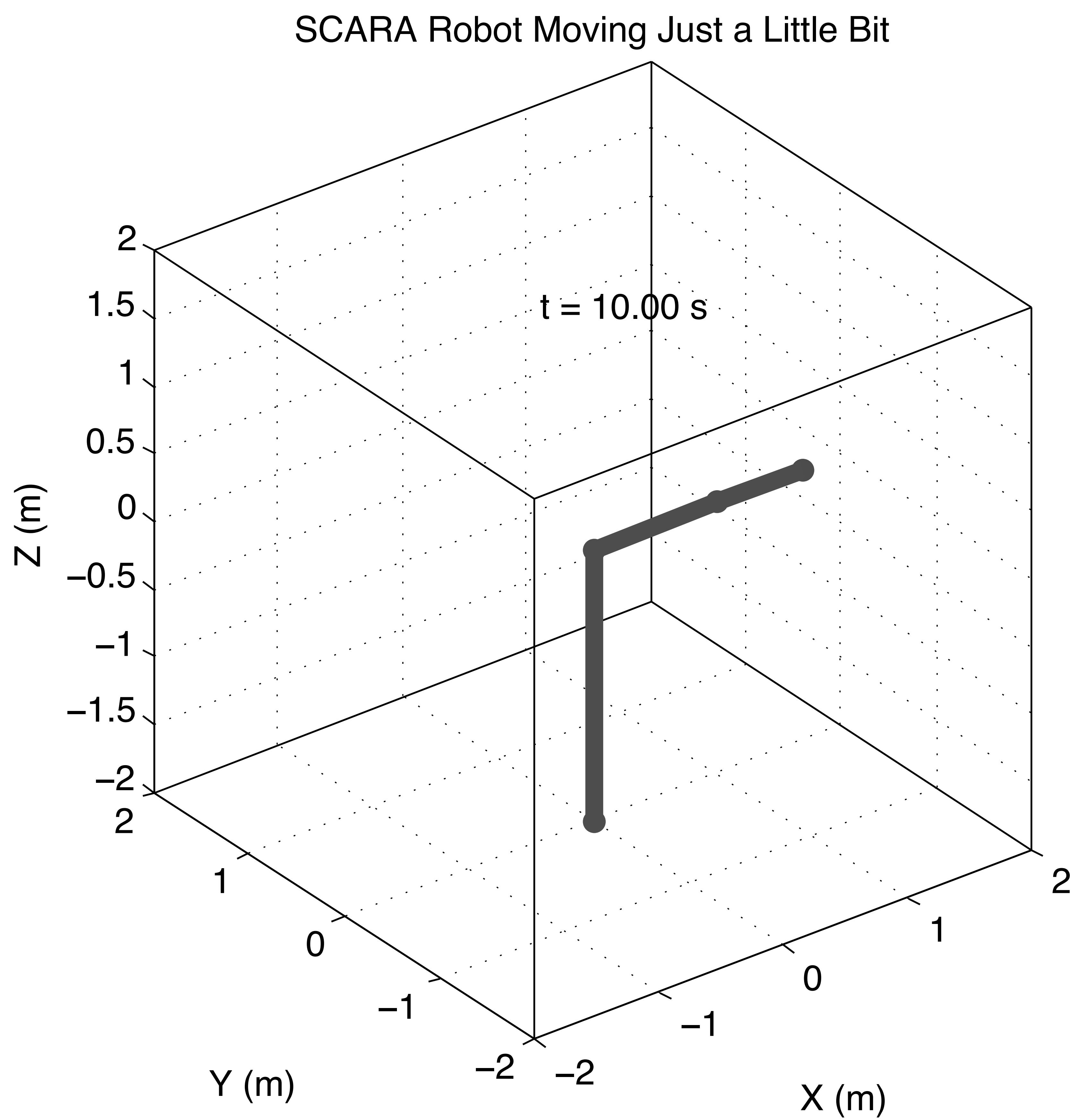
Jacobians



How do the velocities of the joints affect
the linear and angular velocity of the end-effector?

These quantities are related by the **Jacobian**,
a matrix that generalizes the notion of an ordinary
derivative of a scalar function.

Jacobians are useful for planning and executing smooth trajectories, determining singular configurations, executing coordinated anthropomorphic motion, deriving dynamic equations of motion, and transforming forces and torques from the end-effector to the manipulator joints.



SCARA Robot Moving Just a Little Bit
t = 10.00 s
Z (m)
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
Y (m)
2
1
0
−1
−2
X (m)
−2
−1
0
1
2

explore how **changes** in joint values affect the end-effector movement

could have **N joints**, but only **six** end-effector velocity terms (xyzpts)

The **Jacobian** matrix lets us calculate how joint velocities translate into end-effector velocities (depends on configuration)

look at it in two parts - position and orientation

$$v_n^0 = J_v \dot{q} \qquad \omega_n^0 = J_\omega \dot{q}$$

How do we calculate the position Jacobian?

for

$$x(t) = f(q_1(t), q_2(t), \ldots, q_n(t))$$

the time derivative can be found using

$$\frac{dx}{dt} = \sum_{i=1}^{n} \frac{\delta x}{\delta q_i} \frac{dq_i}{dt}$$

For an n-dimensional joint space and a cartesian workspace, the position Jacobian is a 3xn matrix composed of the partial derivatives of the end-effector position with respect to each joint variable.

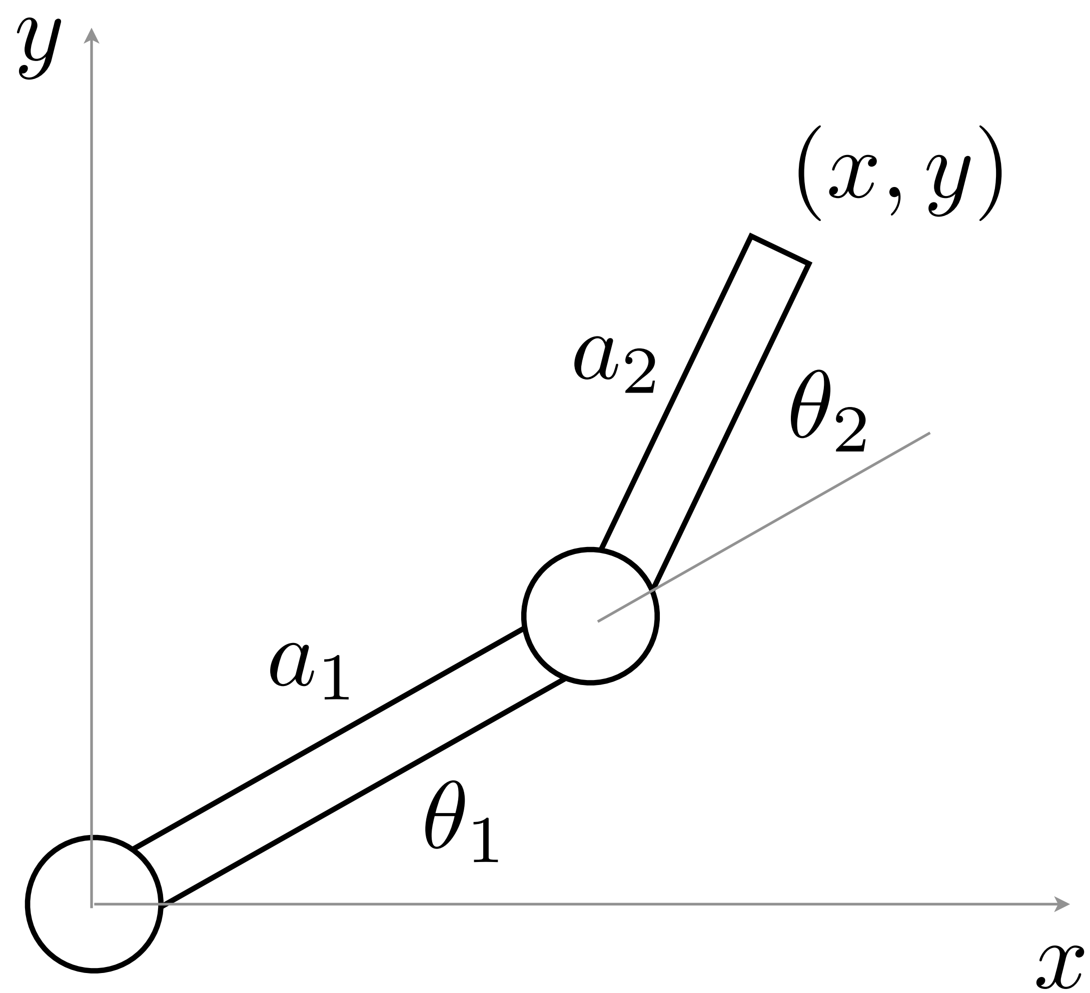
$$\mathbf{J}_p = \begin{bmatrix} \frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \cdots & \frac{\delta x}{\delta q_n} \\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \cdots & \frac{\delta y}{\delta q_n} \\ \frac{\delta z}{\delta q_1} & \frac{\delta z}{\delta q_2} & \cdots & \frac{\delta z}{\delta q_n} \end{bmatrix}$$

$$\dot{\mathbf{p}} = \mathbf{J}_p(q)\ \dot{\mathbf{q}}$$

endpoint velocity

Jacobian matrix

joint velocity



$$\mathbf{J}_p = \begin{bmatrix} \frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \cdots & \frac{\delta x}{\delta q_n} \\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \cdots & \frac{\delta y}{\delta q_n} \\ \frac{\delta z}{\delta q_1} & \frac{\delta z}{\delta q_2} & \cdots & \frac{\delta z}{\delta q_n} \end{bmatrix}$$

From the forward kinematics, we can extract the position vector from the last column of the transform matrix:

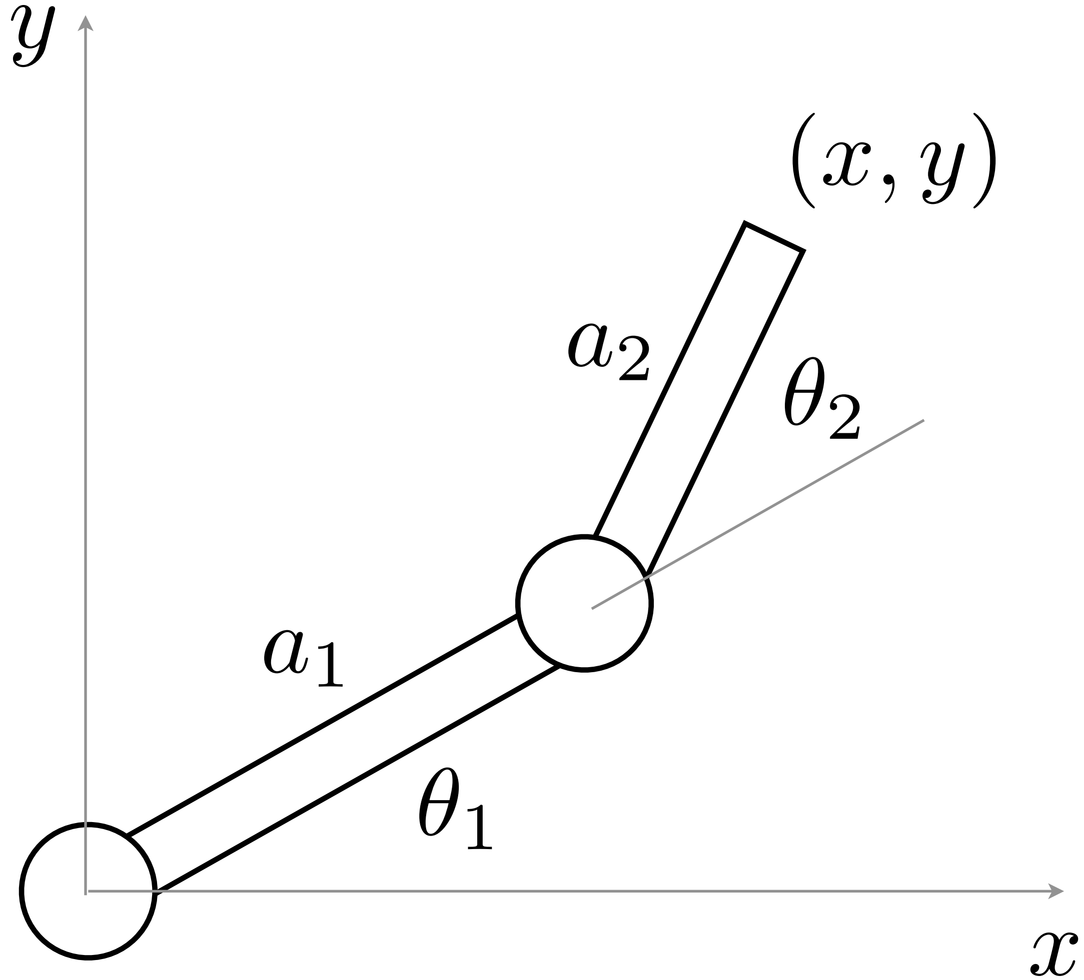
$$\mathbf{d}_2^0 = \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

Taking the partial derivative with respect to each joint variable produces the Jacobian:

$$\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix}$$

which relates instantaneous joint velocities to endpoint velocities

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$



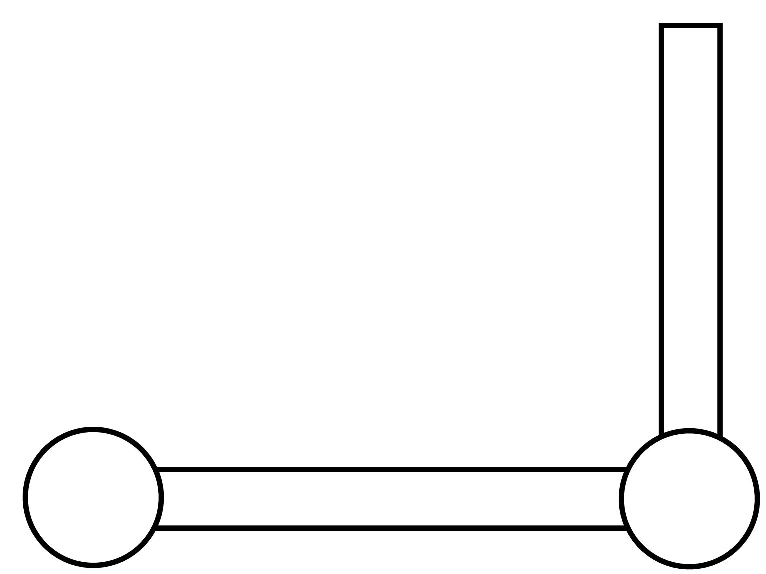
$$\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$
$$\begin{matrix} 0 & \qquad 0 \end{matrix}$$

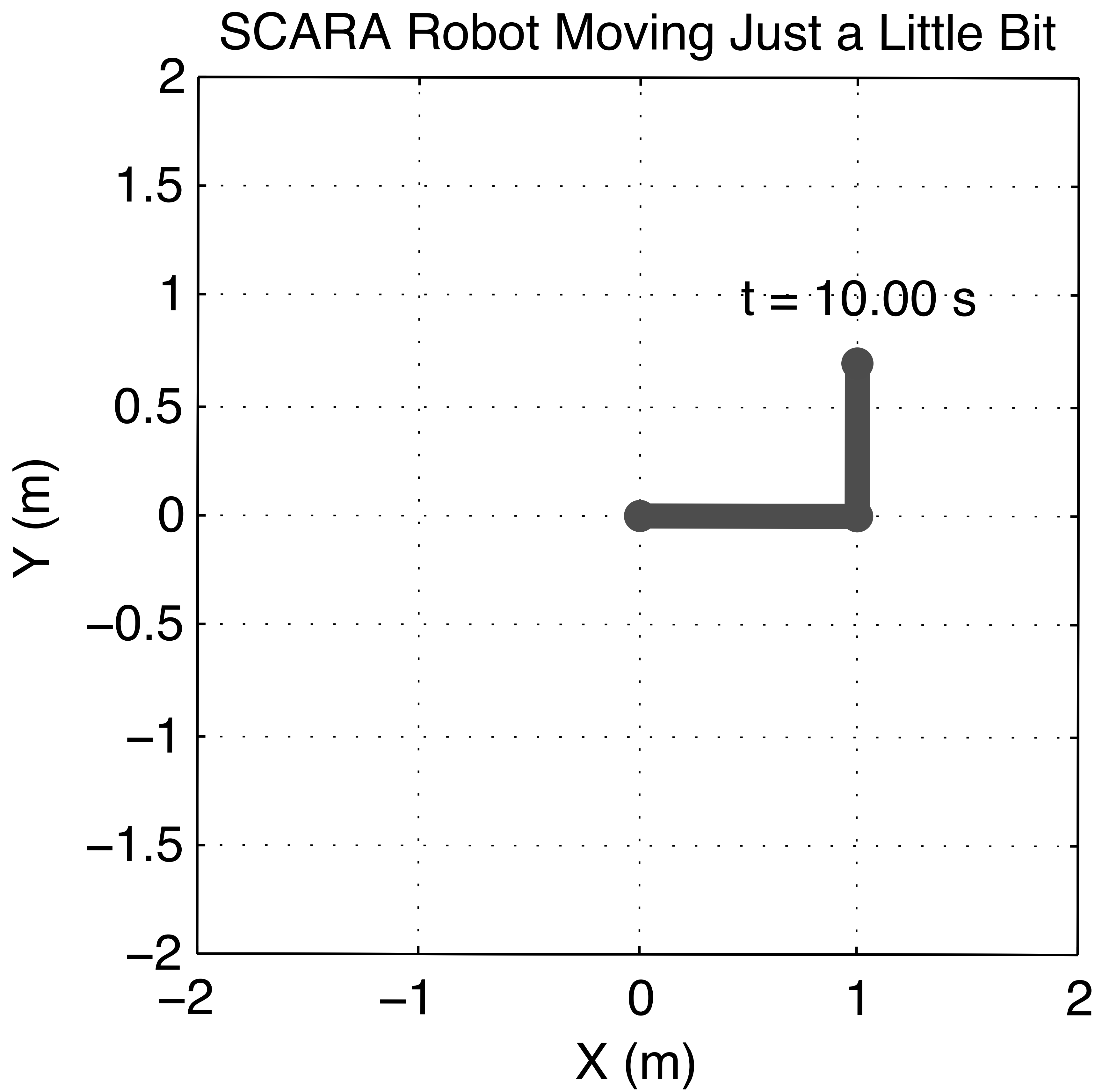
$$\theta_1 = 0, \ \theta_2 = \pi/2$$

$$\dot{x} = -a_2\dot{\theta}_1 - a_2\dot{\theta}_2$$

$$\dot{y} = a_1\dot{\theta}_1$$

$$\dot{z} = 0$$

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t = 10.00 s
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
Y (m)
−2
−1
0
1
2
X (m)

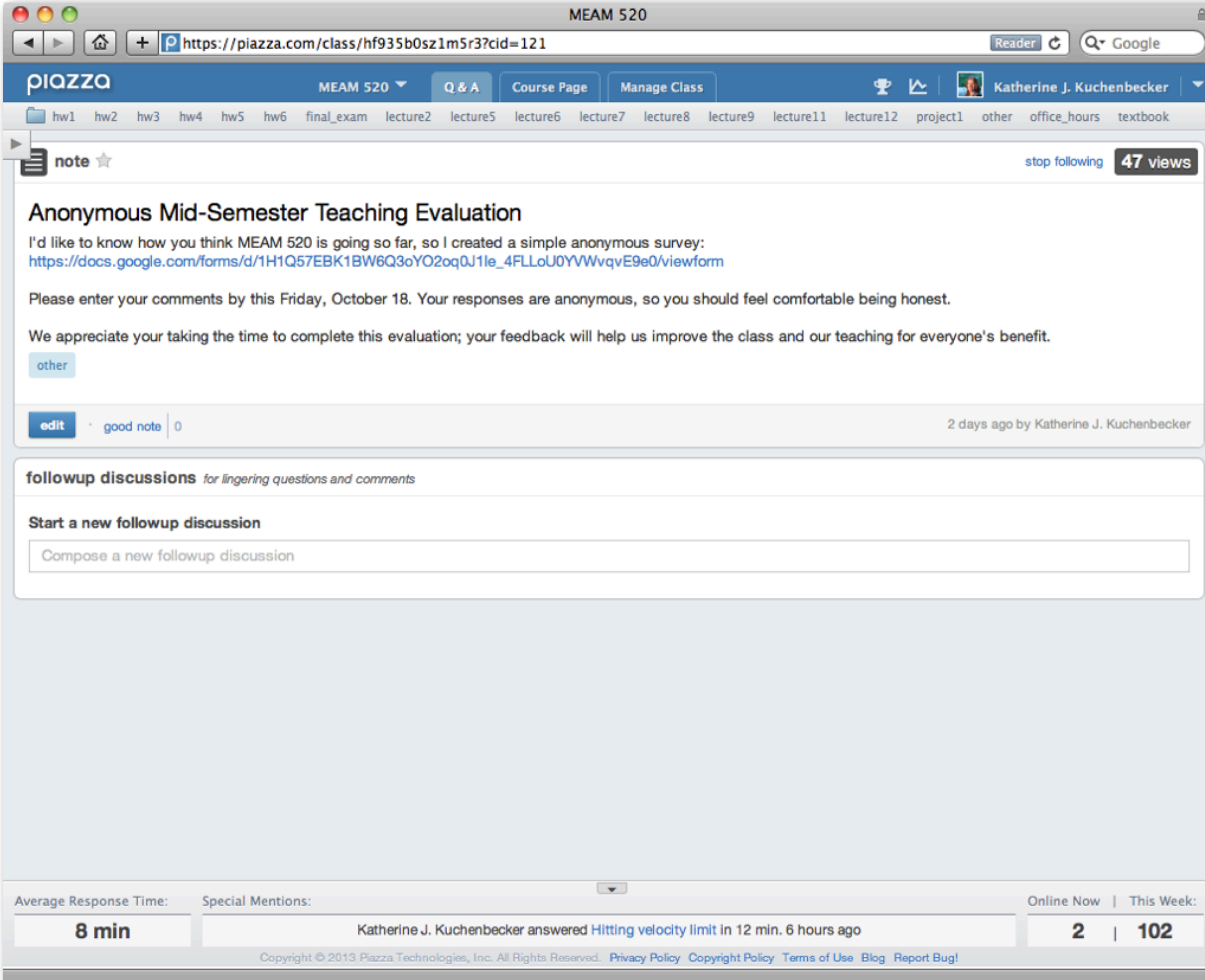
Proposed Midterm Dates

Thursday, October 31, in class

or

Tuesday, November 5, in class

Preferences?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note

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Anonymous Mid-Semester Teaching Evaluation

I'd like to know how you think MEAM 520 is going so far, so I created a simple anonymous survey:
https://docs.google.com/forms/d/1H1Q57EBK1BW6Q3oYO2oq0J1le_4FLLoU0YVWvqvE9e0/viewform

Please enter your comments by this Friday, October 18. Your responses are anonymous, so you should feel comfortable being honest.

We appreciate your taking the time to complete this evaluation; your feedback will help us improve the class and our teaching for everyone's benefit.

other

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2 days ago by Katherine J. Kuchenbecker

followup discussions *for lingering questions and comments*

Start a new followup discussion

Compose a new followup discussion

Average Response Time: 8 min

Special Mentions: Katherine J. Kuchenbecker answered Hitting velocity limit in 12 min. 6 hours ago

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