MEAM 520

Inverse Position Kinematics

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General Robotics, Automation, Sensing, and Perception Lab (GRASP)

MEAM Department, SEAS, University of Pennsylvania

GRASP LABORATORY

Lecture 21: November 14, 2013

Name ______________________________

Resubmission of Midterm Exam

MEAM 520, Introduction to Robotics
University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

November 8, 2013

If you doubt the correctness of any of the answers you submitted on the in-class midterm, you may rework those problems to try to earn back a proportion of the points you may have lost. You must decide whether to submit new answers for each of the five problems on the exam; if you do any lettered sub-question in a problem, you should resubmit all of them, as they are interrelated. Your final score on each problem will be a weighted average of your in-class score and your resubmission score: $s_{\text{final}} = (1-p)\, s_{\text{in-class}} + p\, s_{\text{resubmission}}$. We anticipate that this proportion p will be about 30%, but it may be raised or lowered to match overall class performance. If you do not submit new answers for a certain problem, your in-class score will be your final score. Completing this exam resubmission is completely optional.

You must do this exam resubmission independently, without talking about it with anyone else. You may use a calculator, your notes, the textbook, the Internet, and any other reference materials you find useful. However, you must not take assistance from any individual (including electronic correspondence of any kind), and you must not give assistance to other students in the class. If you accidentally discuss part of the exam with someone, do not fill out that part of the exam resubmisson. Any suspected violations of Penn's Code of Academic Integrity will be reported to the Office of Student Conduct for investigation.

This resubmission is due **by the start of class (noon) on Tuesday, November 12**. Because we will be discussing the exam in class that day, late resubmissions cannot be accepted. If you need clarification on any question, please post a private note on Piazza. When you work out each problem, please show all steps and box your answer.

	Points	Score
Problem 1	20	
Problem 2	15	
Problem 3	10	
Problem 4	15	
Problem 5	40	
Total	100	

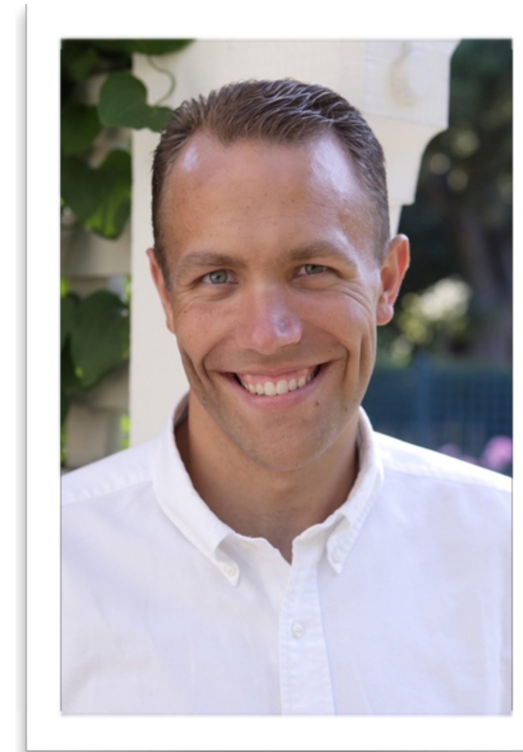
I agree to abide by the University of Pennsylvania Code of Academic Integrity during this exam resubmission. I pledge that all work is my own and has been completed without the use of unauthorized aid or materials.

Signature ______________________________ Date ______________

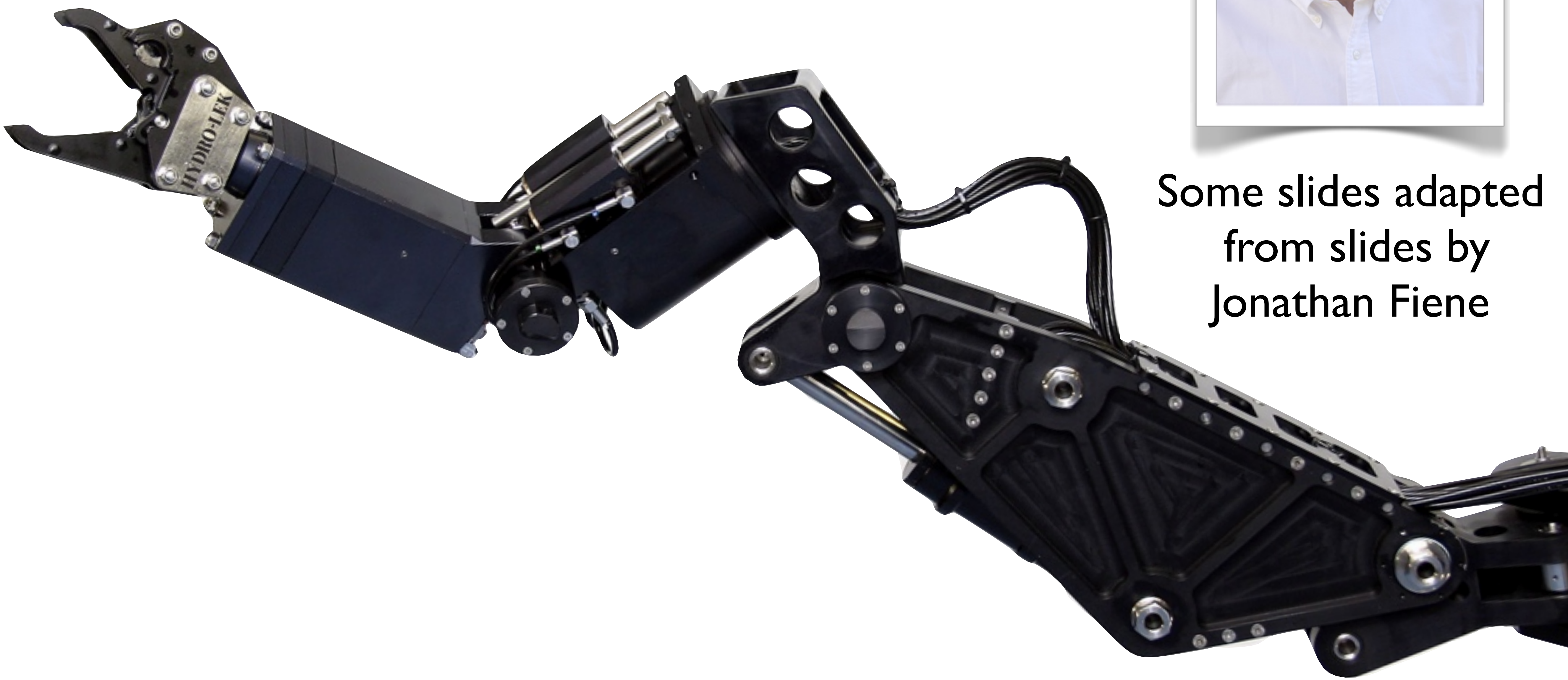
1

Please turn in your resubmission of the midterm exam.

Inverse Kinematics



Some slides adapted from slides by Jonathan Fiene



SHV 3.3

Inverse Kinematics

$$\text{given } \mathbf{H} = \begin{bmatrix} \mathbf{R} & o \\ 0 & 1 \end{bmatrix} \text{ and a certain manipulator with } n \text{ joints}$$

$$\text{find } q_1, \ldots, q_n \text{ such that } \mathbf{T}_n^0(q_1, \ldots, q_n) = \mathbf{H}$$

given $\mathbf{H} = \begin{bmatrix} \mathbf{R} & o \\ 0 & 1 \end{bmatrix}$ and a certain manipulator with n joints

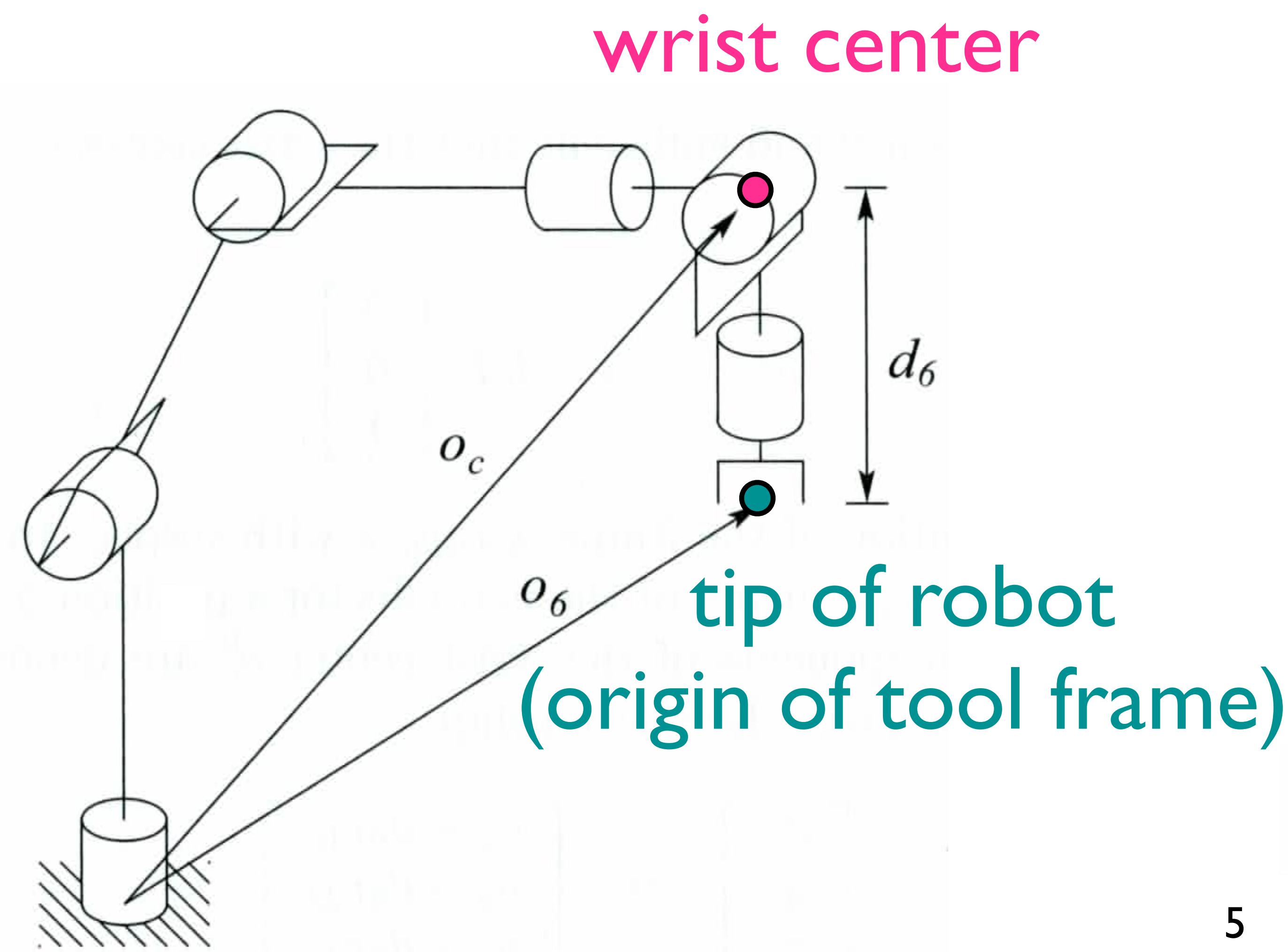
find $q_1, \ldots, q_n$ such that $\mathbf{T}_n^0(q_1, \ldots, q_n) = \mathbf{H}$ ugly!

The inverse kinematics problem may or may not have a solution.

Even if one exists, the solution may or may not be unique.

Trick: Exploit the kinematic structure of the manipulator.

If the robot has a spherical wrist, use **Kinematic Decoupling**.



$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

position

$$R = R_3^0 R_6^3$$

$$R_6^3 = (R_3^0)^{-1} R = (R_3^0)^T R$$

orientation

We want closed-form solutions (explicit equations):

$$q_k = f_k(h_{11}, \ldots, h_{34}), \quad k = 1, \ldots, n$$

What's the alternative?

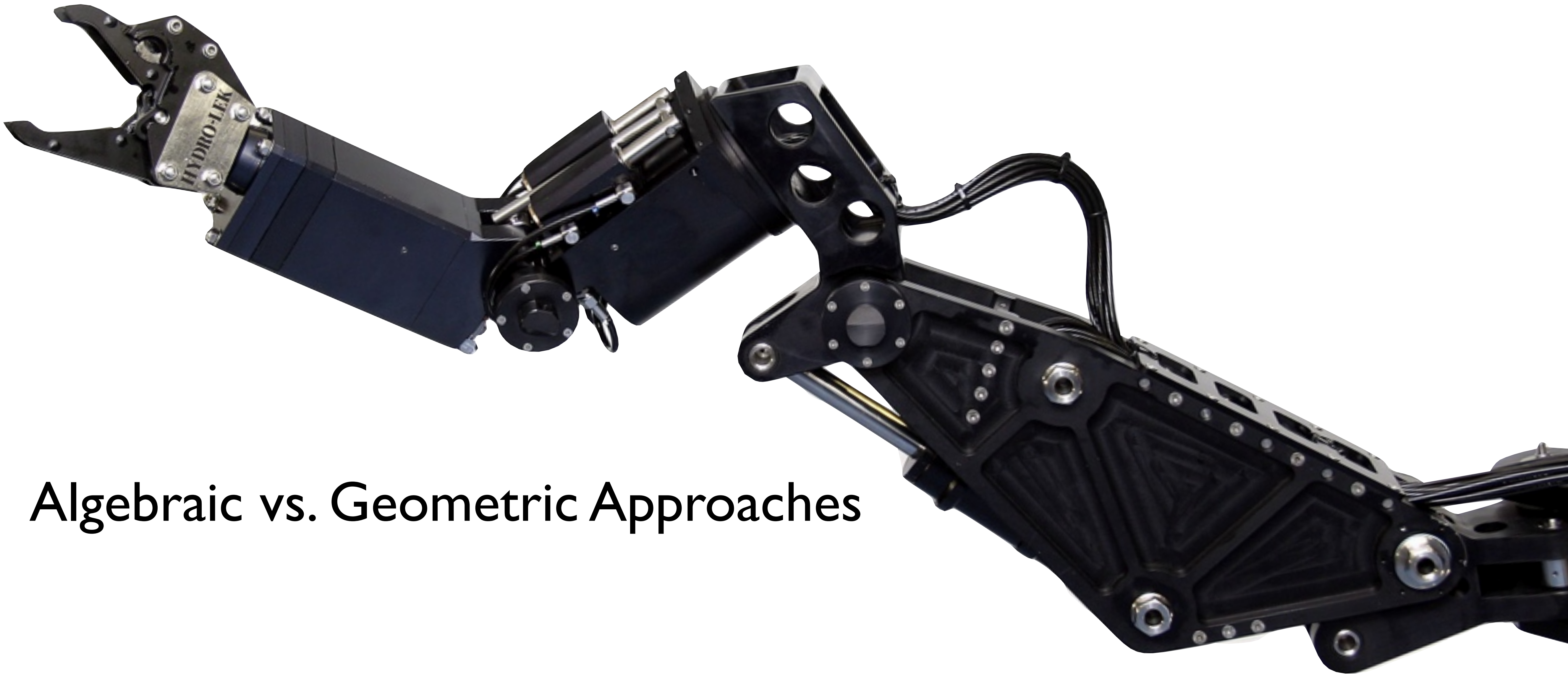
Numerical solution, where an algorithm iteratively looks for a solution to the system of equations.

Why do we want closed-form solutions?

1. IK must often be solved in real time, e.g., every 20 ms. Closed-form solutions evaluate much faster than numerical solutions and are guaranteed to yield an answer.

2. There are generally multiple solutions to IK. Having closed-form solutions lets you develop rules for selecting the best solution from the options.

Inverse Position Kinematics



Algebraic vs. Geometric Approaches

SHV 3.3

Inverse Position Kinematics: Algebraic Decomposition

given the forward transform matrix for a manipulator

$$\mathbf{T}_n^0 = \begin{bmatrix} \left[\mathbf{R}_n^0(\mathbf{q})\right]_{3\times 3} & \left[\mathbf{d}_n^0(\mathbf{q})\right]_{3\times 1} \\ [\mathbf{0}]_{1\times 3} & 1 \end{bmatrix}$$

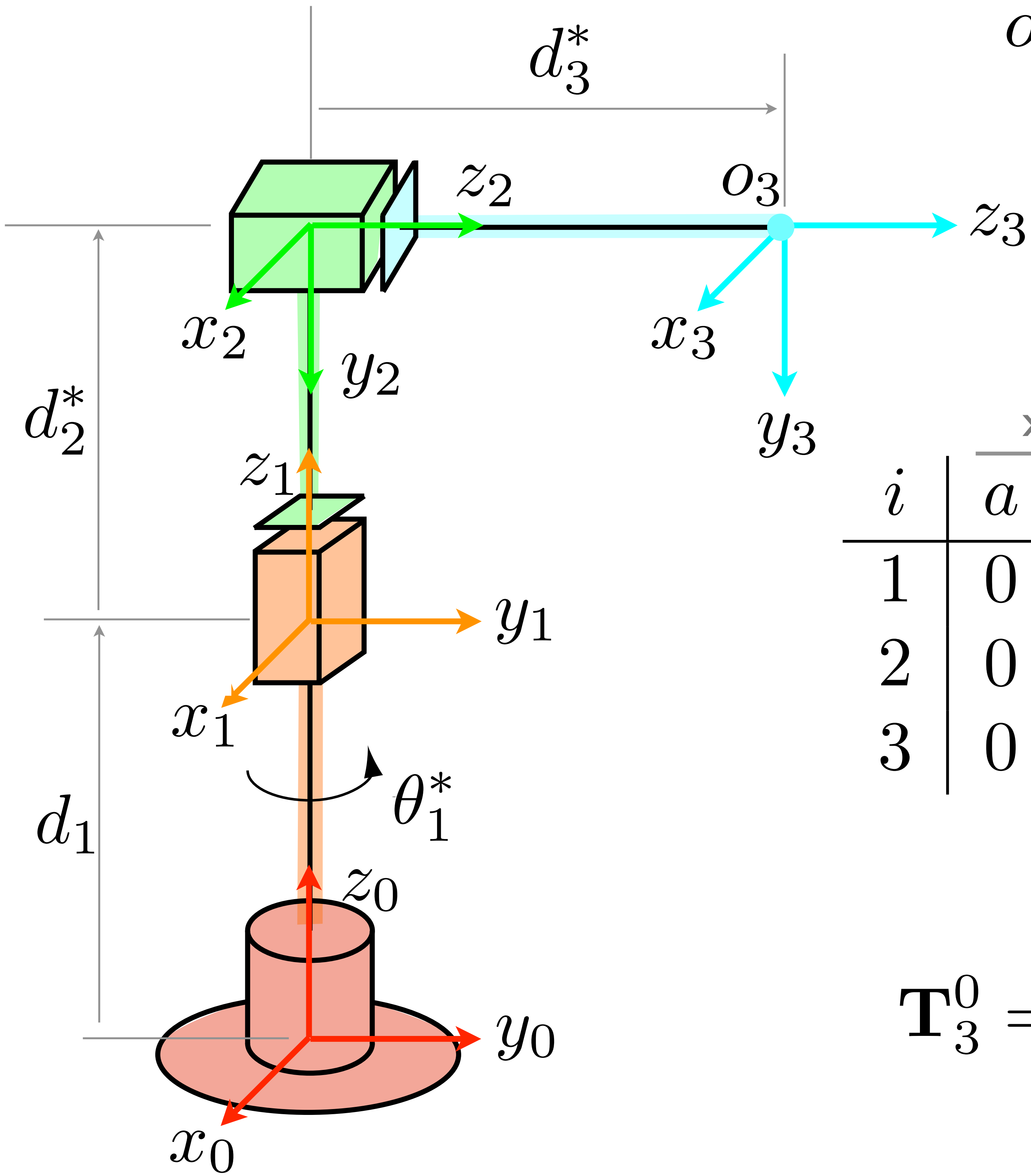
solve the system of 3 equations from the displacement vector

$$\begin{aligned} d_x &= \left[\mathbf{d}_n^0(\mathbf{q})\right]_1 \\ d_y &= \left[\mathbf{d}_n^0(\mathbf{q})\right]_2 \\ d_z &= \left[\mathbf{d}_n^0(\mathbf{q})\right]_3 \end{aligned}$$

to find the joint variables in terms of the end-effector position

$$\mathbf{q} = \begin{bmatrix} q_1(d_x, d_y, d_z) \\ q_2(d_x, d_y, d_z) \\ \vdots \\ q_n(d_x, d_y, d_z) \end{bmatrix}$$

The RPP Cylindrical Robot - Algebraic Approach



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ignoring joints past RPP

	x-step		z-step	
i	a	α	d	θ
1	0	$0°$	d_1	θ_1^*
2	0	$-90°$	d_2^*	$0°$
3	0	$0°$	d_3^*	$0°$

$$\theta_1^* = ?$$

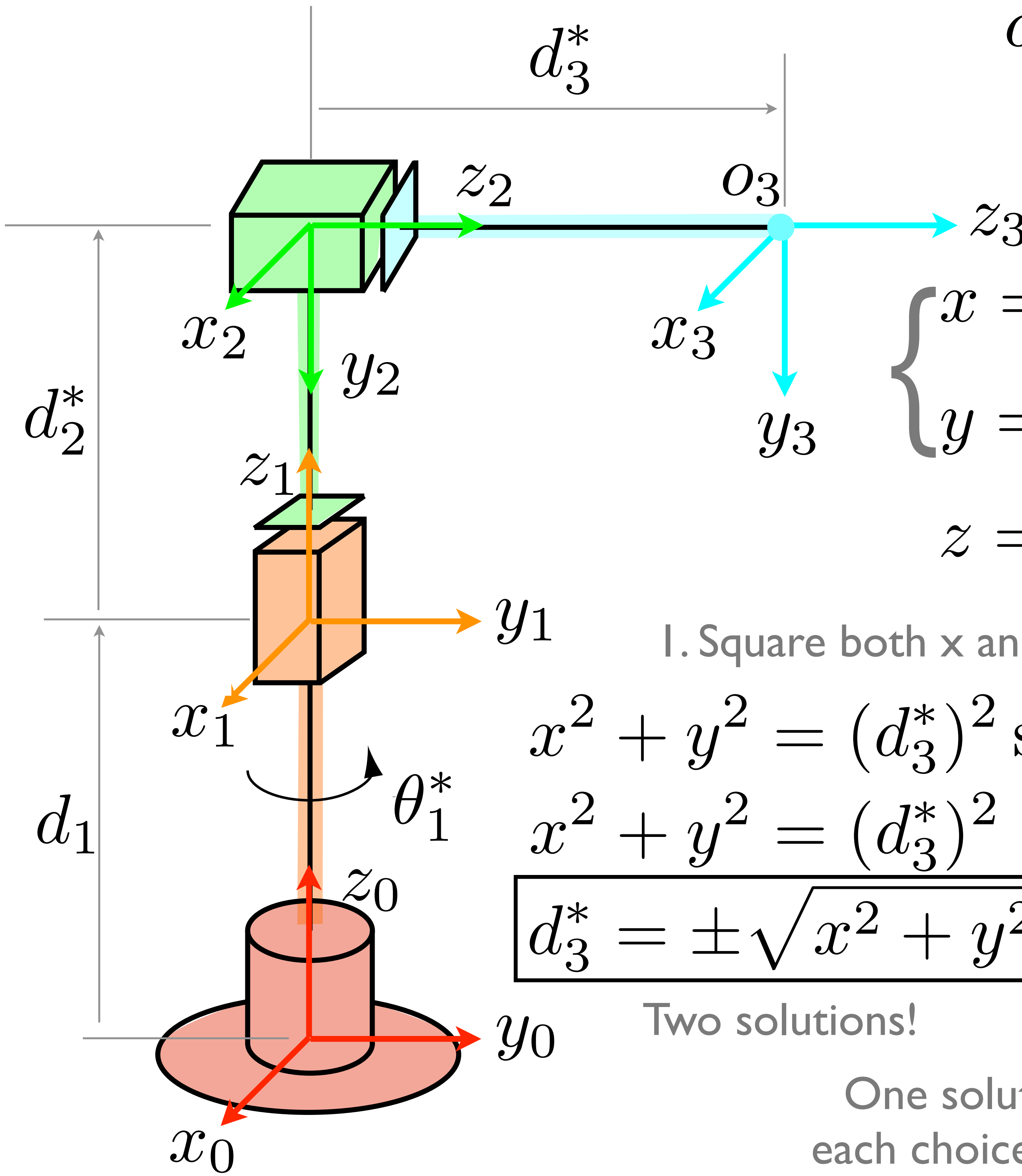
$$d_2^* = ?$$

$$d_3^* = ?$$

$$\mathbf{T}_3^0 = \begin{bmatrix} c_1^* & 0 & -s_1^* & -d_3^* s_1^* \\ s_1^* & 0 & c_1^* & d_3^* c_1^* \\ 0 & -1 & 0 & d_1 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The RPP Cylindrical Robot - Algebraic Approach

This slide was corrected to show both solutions for theta1.



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ignoring joints past RPP

$$\begin{cases} x = -d_3^* \sin(\theta_1^*) \\ y = d_3^* \cos(\theta_1^*) \end{cases}$$

$$z = d_1 + d_2^* \longrightarrow \boxed{d_2^* = z - d_1}$$

1. Square both x and y equations and add them.

$$x^2 + y^2 = (d_3^*)^2 \sin^2(\theta_1^*) + (d_3^*)^2 \cos^2(\theta_1^*)$$

$$x^2 + y^2 = (d_3^*)^2$$

$$\boxed{d_3^* = \pm\sqrt{x^2 + y^2}}$$

Two solutions!

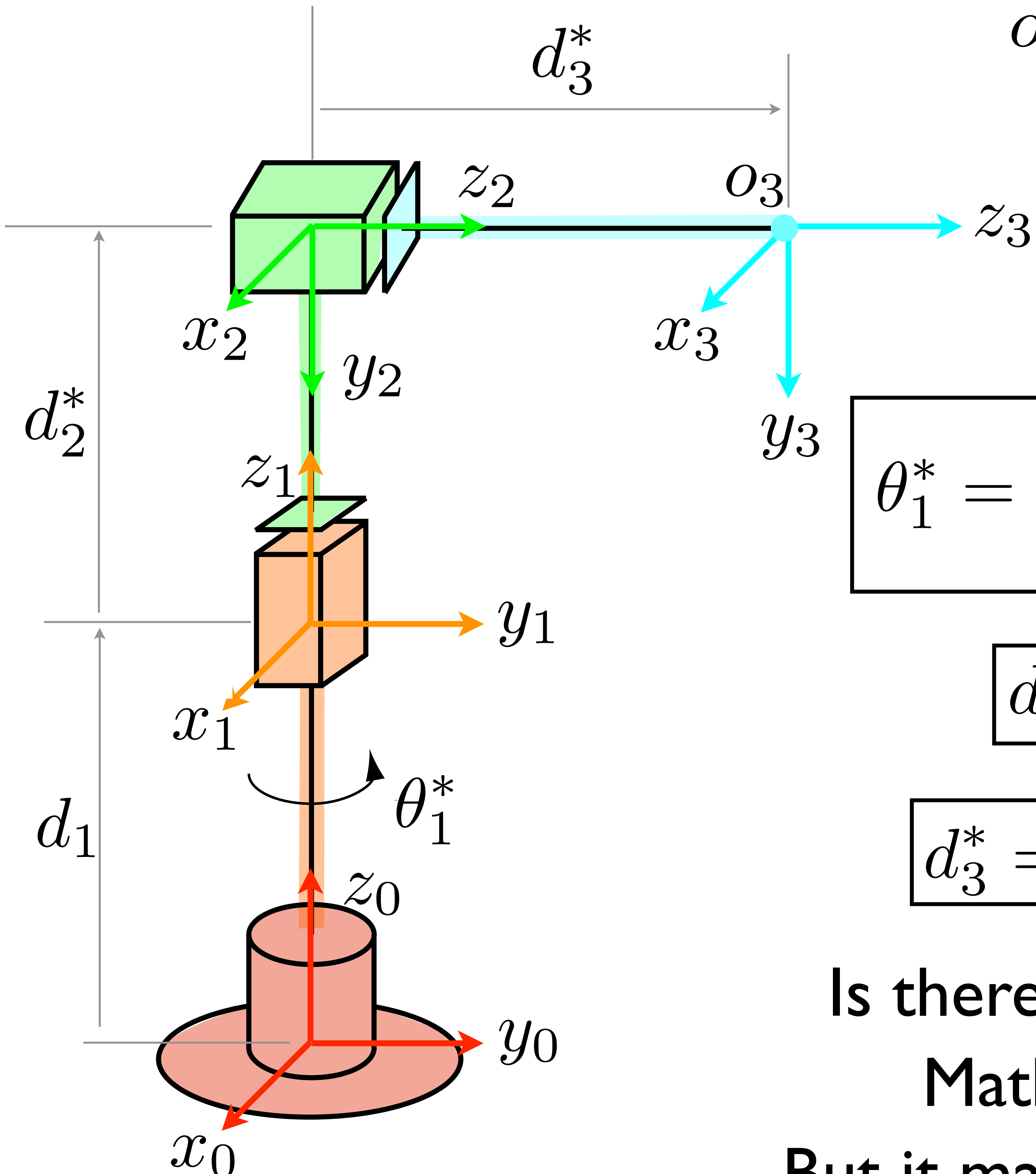
2. Solve for sin and cos of theta1 and take atan2

$$\sin(\theta_1^*) = \frac{-x}{d_3^*} \quad \cos(\theta_1^*) = \frac{y}{d_3^*}$$

One solution for each choice of d3*

$$\boxed{\theta_1^* = \operatorname{atan2}\left(\frac{-x/d_3^*}{y/d_3^*}\right)}$$

The RPP Cylindrical Robot - Algebraic Approach



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ignoring joints past RPP

$$\boxed{\theta_1^* = \operatorname{atan2}\left(\frac{-x/d_3^*}{y/d_3^*}\right)}$$

$$\boxed{d_2^* = z - d_1}$$

$$\boxed{d_3^* = \pm\sqrt{x^2 + y^2}}$$

Is there always a solution?

Mathematically, yes.

But it may violate joint limits.

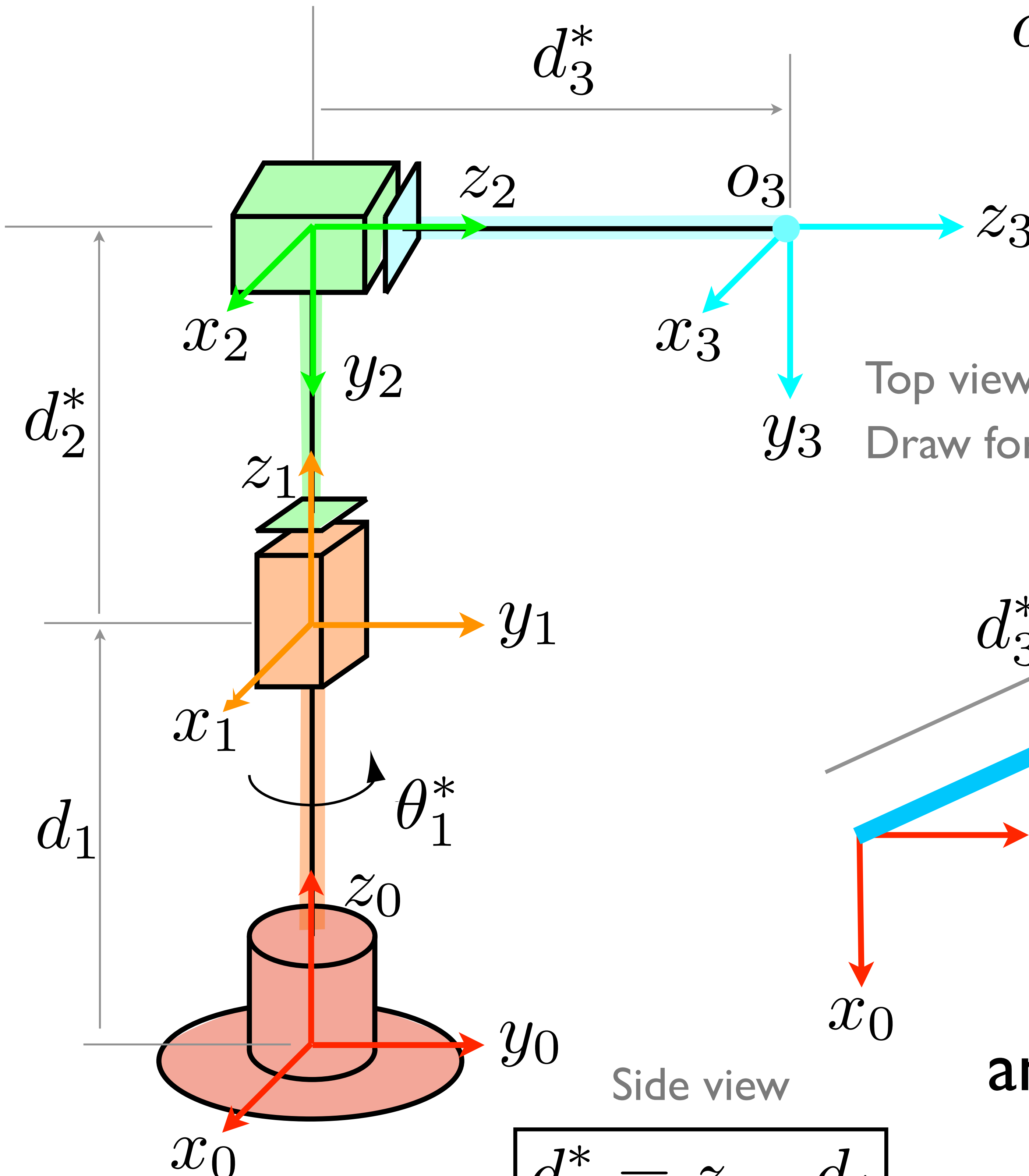
Geometric Analysis

For most simple manipulators, it is easier to use geometry to solve for closed-form solutions to the inverse kinematics

solve for each joint variable q_i
by projecting the manipulator onto the x_{i-1}, y_{i-1} plane

closed-form inverse kinematic solutions are not always possible, and if it is solvable, there are often multiple solutions

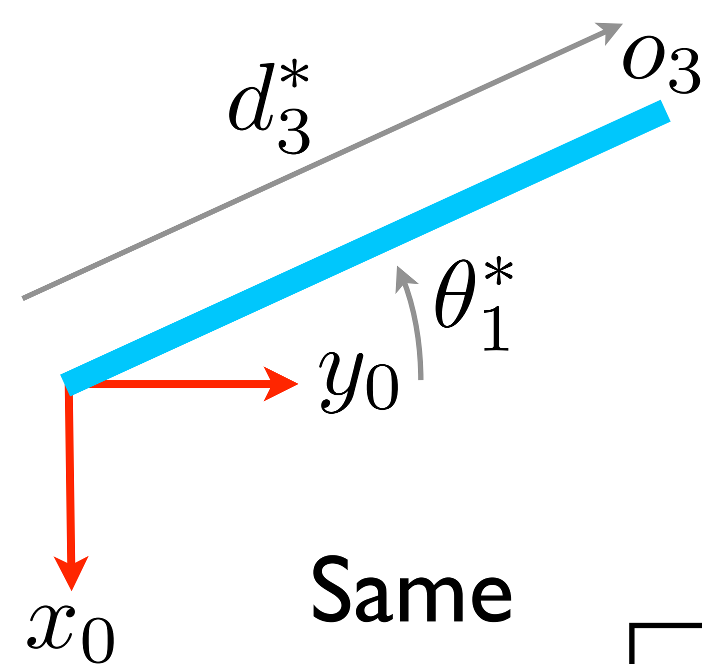
The RPP Cylindrical Robot - Geometric Approach



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ignoring joints past RPP

Top view (looking down along z0).
Draw for a small positive angle theta1.



$$\theta_1^* = ?$$

$$d_2^* = ?$$

$$d_3^* = ?$$

$$\boxed{d_3^* = \pm\sqrt{x^2 + y^2}}$$

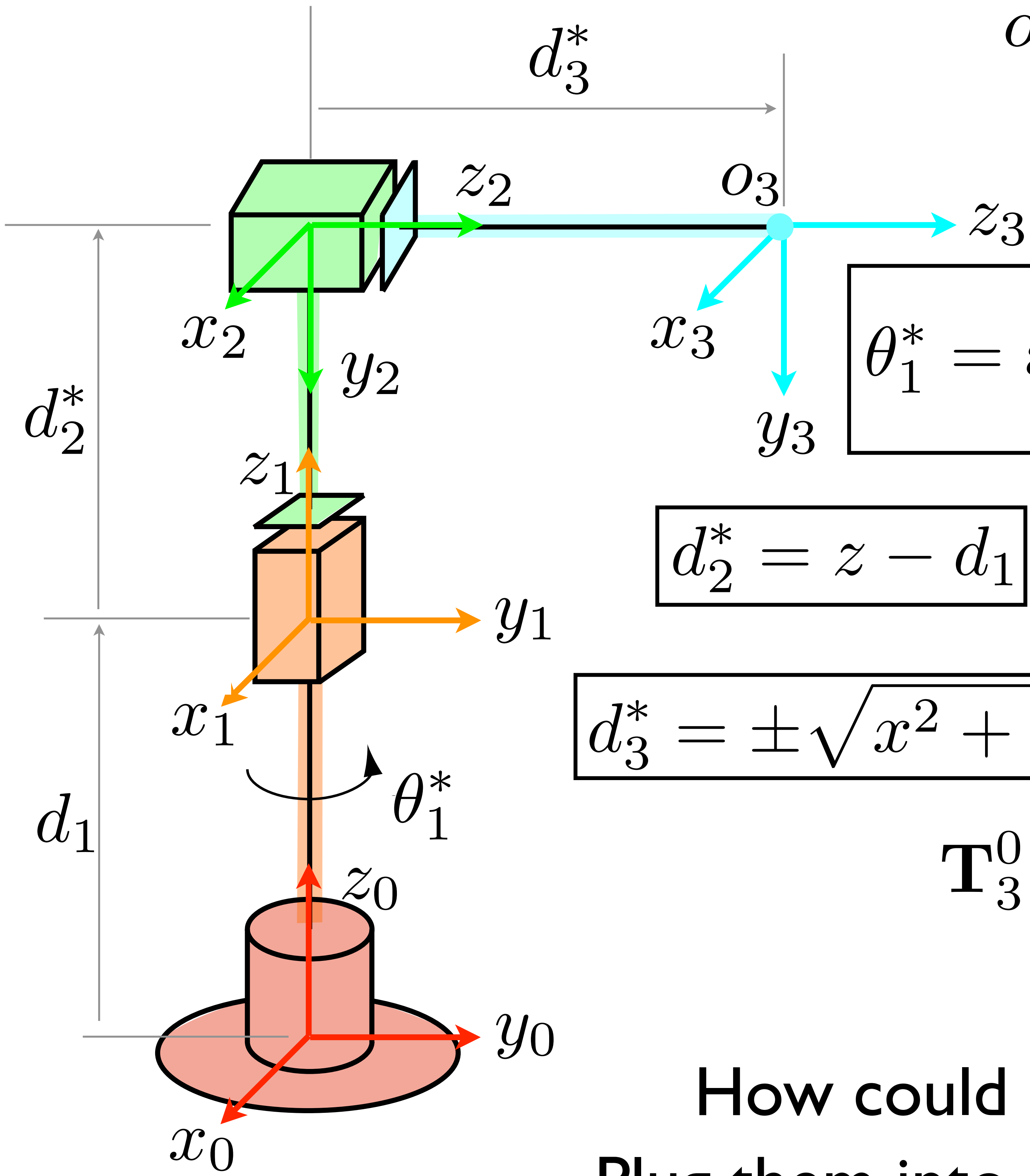
Side view

$$\boxed{d_2^* = z - d_1}$$

Same answers!

$$\boxed{\theta_1^* = \text{atan2}\left(\frac{-x/d_3^*}{y/d_3^*}\right)}$$

The RPP Cylindrical Robot - Algebraic Approach



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ignoring joints past RPP

$$\boxed{\theta_1^* = \operatorname{atan2}\left(\frac{-x/d_3^*}{y/d_3^*}\right)}$$

$$\boxed{d_2^* = z - d_1}$$

$$\boxed{d_3^* = \pm\sqrt{x^2 + y^2}}$$

$$\mathbf{T}_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

How could I check my answers?

Plug them into the forward kinematics!

Questions ?

Name ________________________________

Midterm Exam

MEAM 520, Introduction to Robotics
University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

November 8, 2012

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	Points	Score
Problem 1	20	
Problem 2	20	
Problem 3	15	
Problem 4	20	
Problem 5	25	
Total	100	

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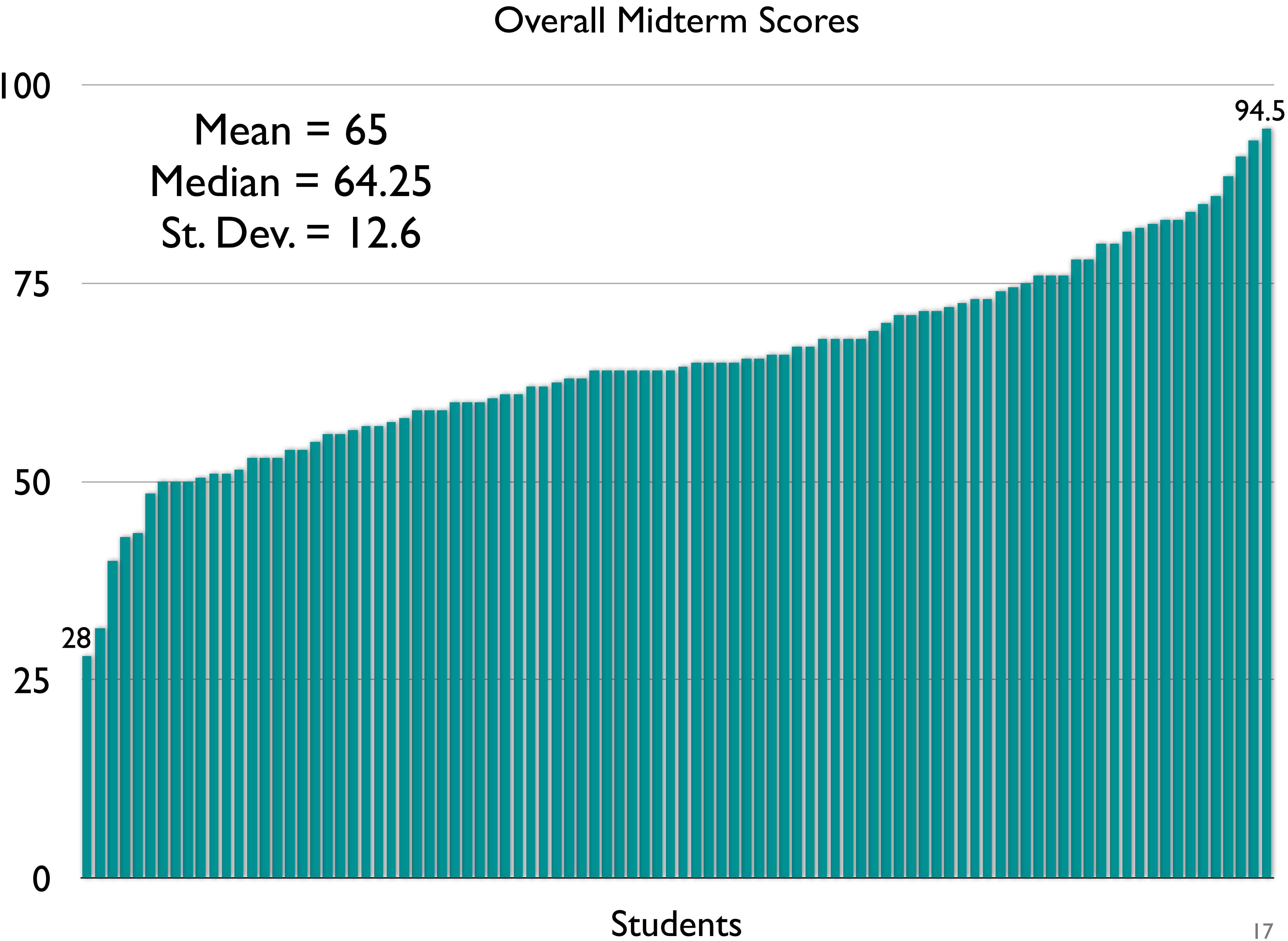
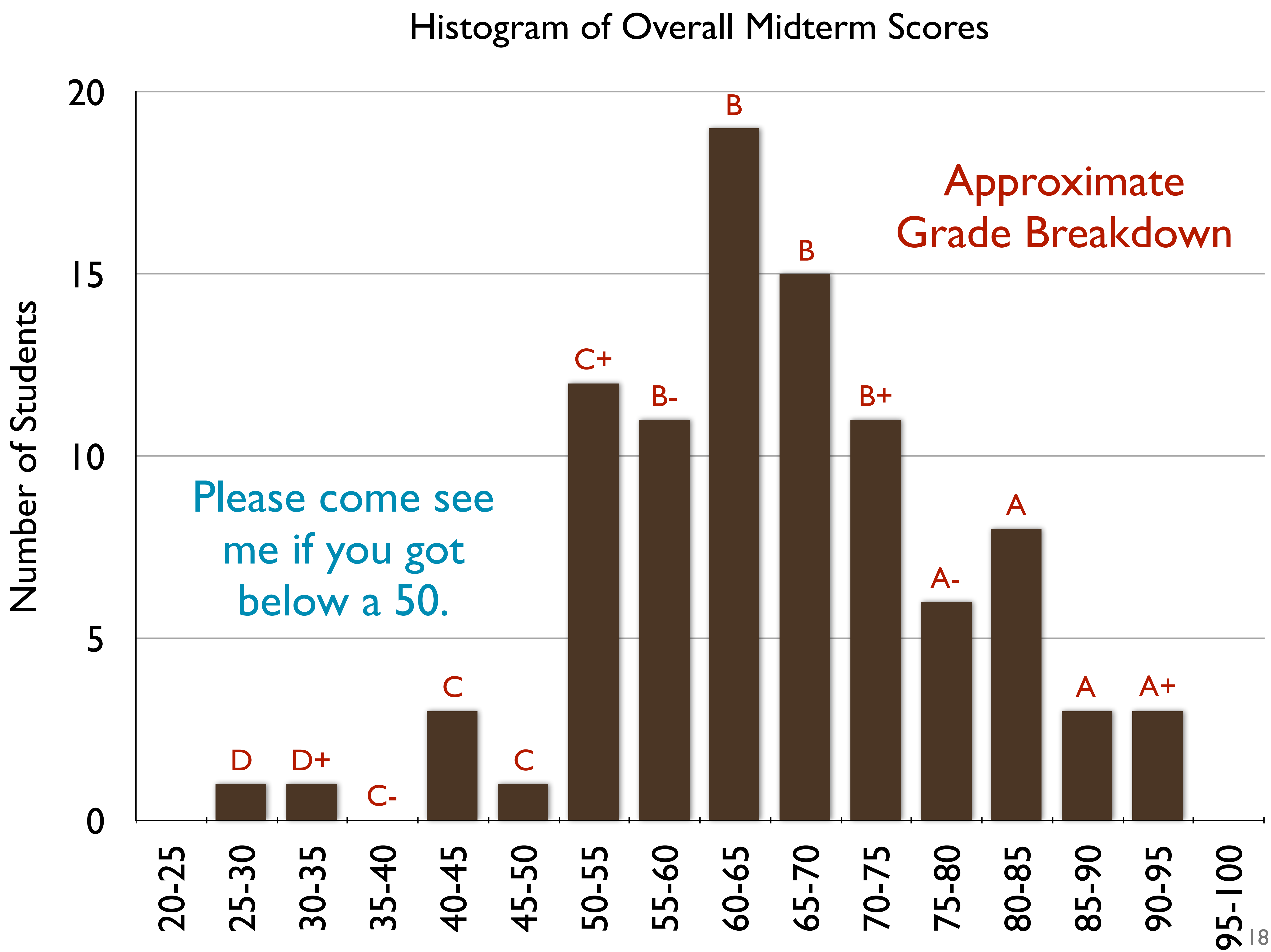
Signature ________________________________

Date ________________________________

1


Overall Midterm Scores
100
75
50
25
0
Mean = 65
Median = 64.25
St. Dev. = 12.6
94.5
28
Students



Histogram of Overall Midterm Scores
Number of Students
0
5
10
15
20
20-25
25-30
30-35
35-40
40-45
45-50
50-55
55-60
60-65
65-70
70-75
75-80
80-85
85-90
90-95
95-100
D
D+
C-
C
C
C+
B-
B
B
B+
A-
A
A
A+
Approximate Grade Breakdown
Please come see me if you got below a 50.

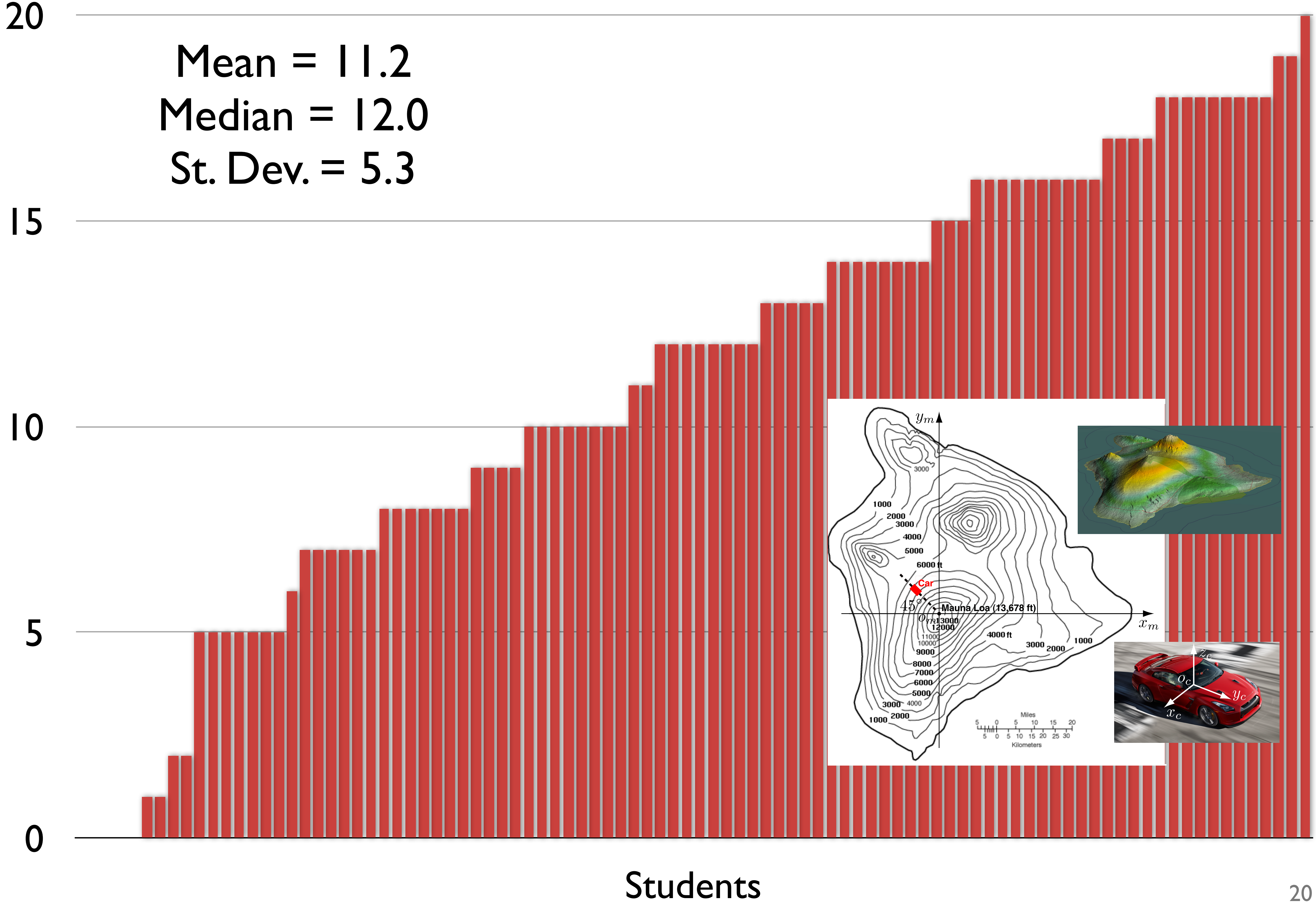
Handing back exams
in alphabetical order by last name

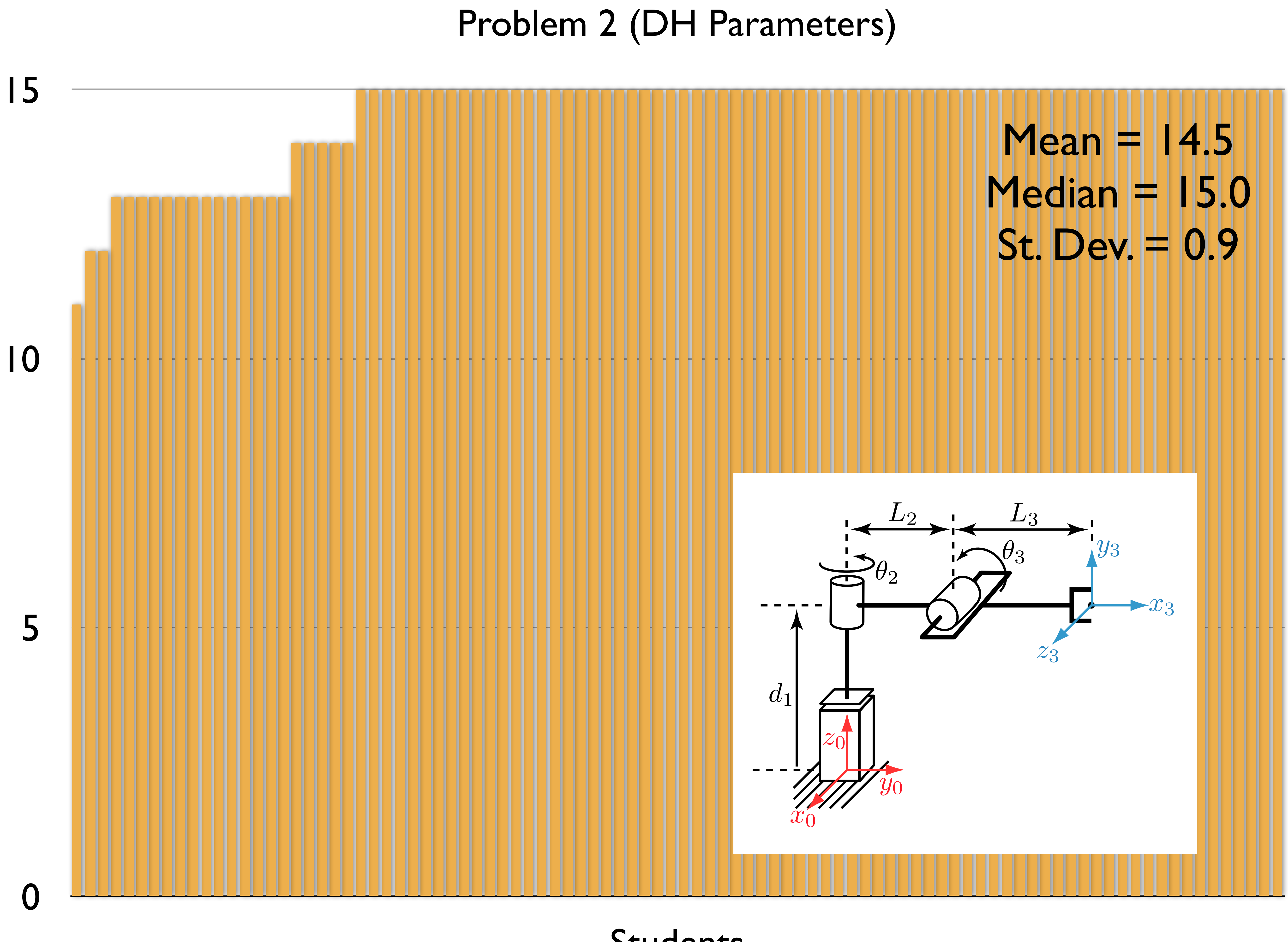
Naomi has last names A – K

Dr. K had last names L – Z

Graded homework assignments
are also attached to some exams.

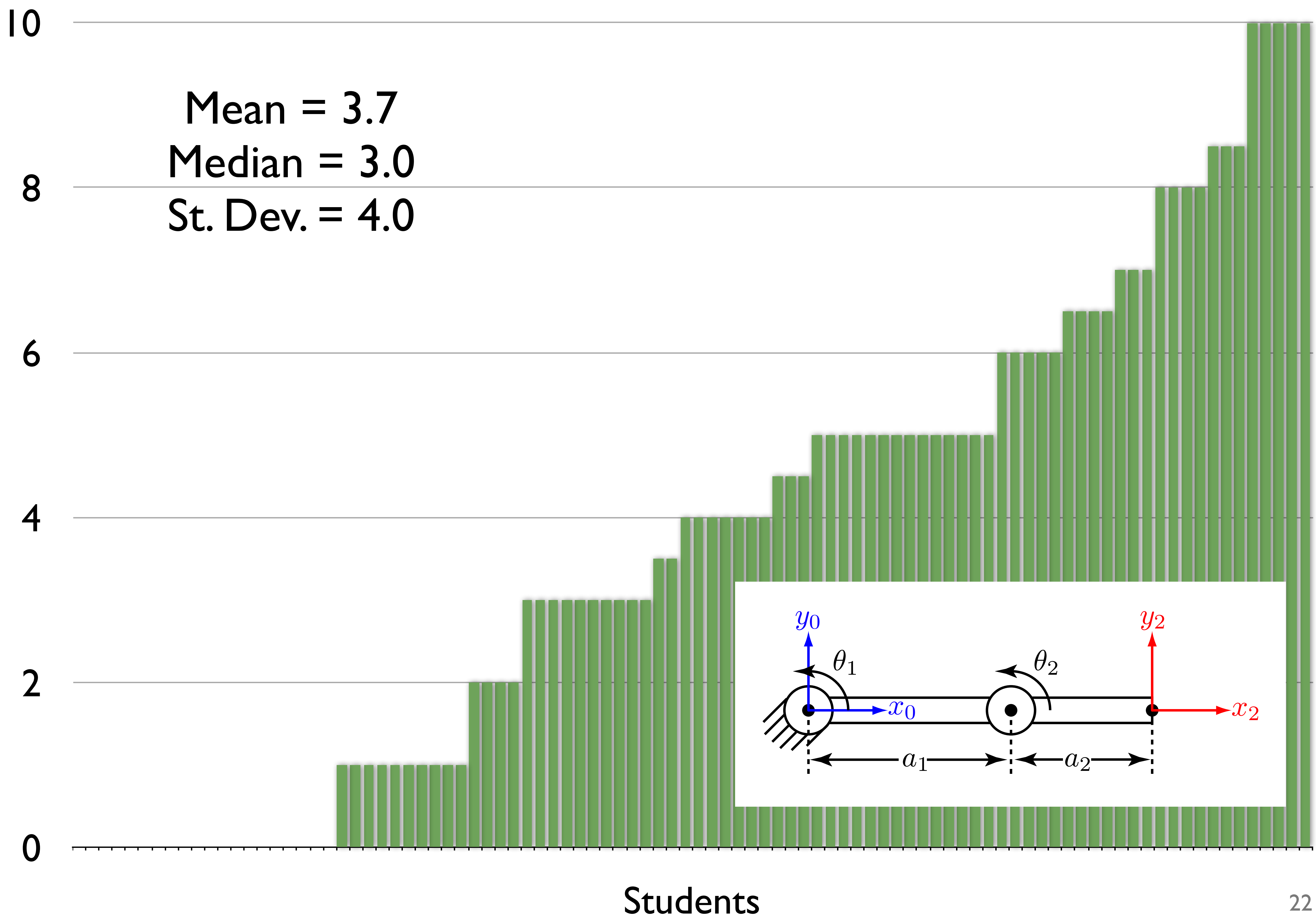
Problem 1 (Homogeneous Transformations)



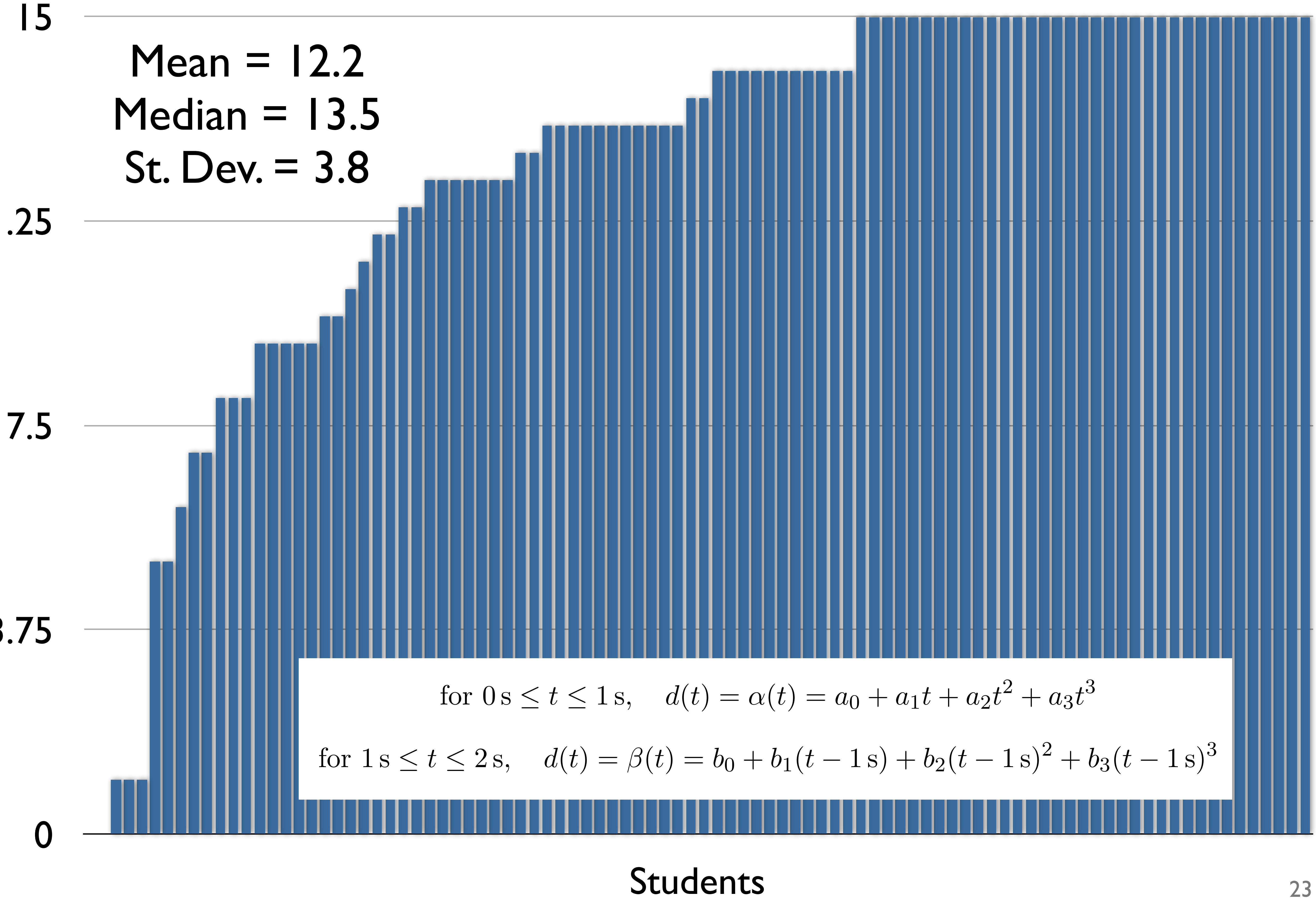


Problem 2 (DH Parameters)
15
10
5
0
Mean = 14.5
Median = 15.0
St. Dev. = 0.9
L_2
L_3
θ_2
θ_3
y_3
x_3
z_3
d_1
z_0
y_0
x_0
Students

Problem 3 (More DH Parameters)



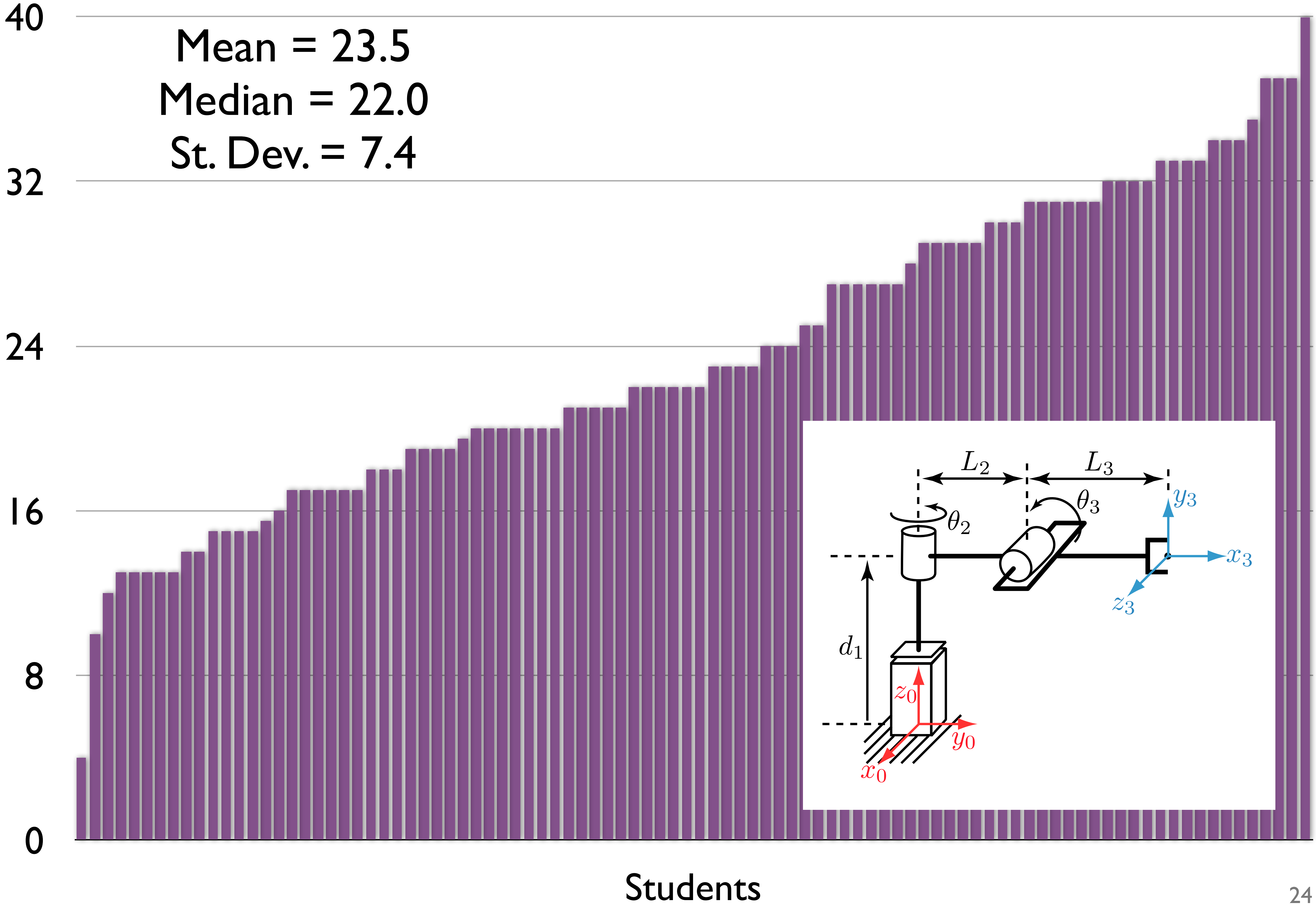
Problem 4 (Trajectory)



$$\text{for } 0\,\text{s} \le t \le 1\,\text{s}, \quad d(t) = \alpha(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\text{for } 1\,\text{s} \le t \le 2\,\text{s}, \quad d(t) = \beta(t) = b_0 + b_1(t - 1\,\text{s}) + b_2(t - 1\,\text{s})^2 + b_3(t - 1\,\text{s})^3$$

Problem 5 (Jacobians and Singularities)



Name Solutions by KJK

Midterm Exam

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University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

November 5, 2013

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Signature ____________________

Date ____________________

1

Solutions will soon be on Piazza.

Look over your exam and compare with the solution.

Check the grades posted on Canvas as well.

If you think we made a mistake in grading your test, write out an explanation on a separate piece of paper; do not write on or modify your exam at all.

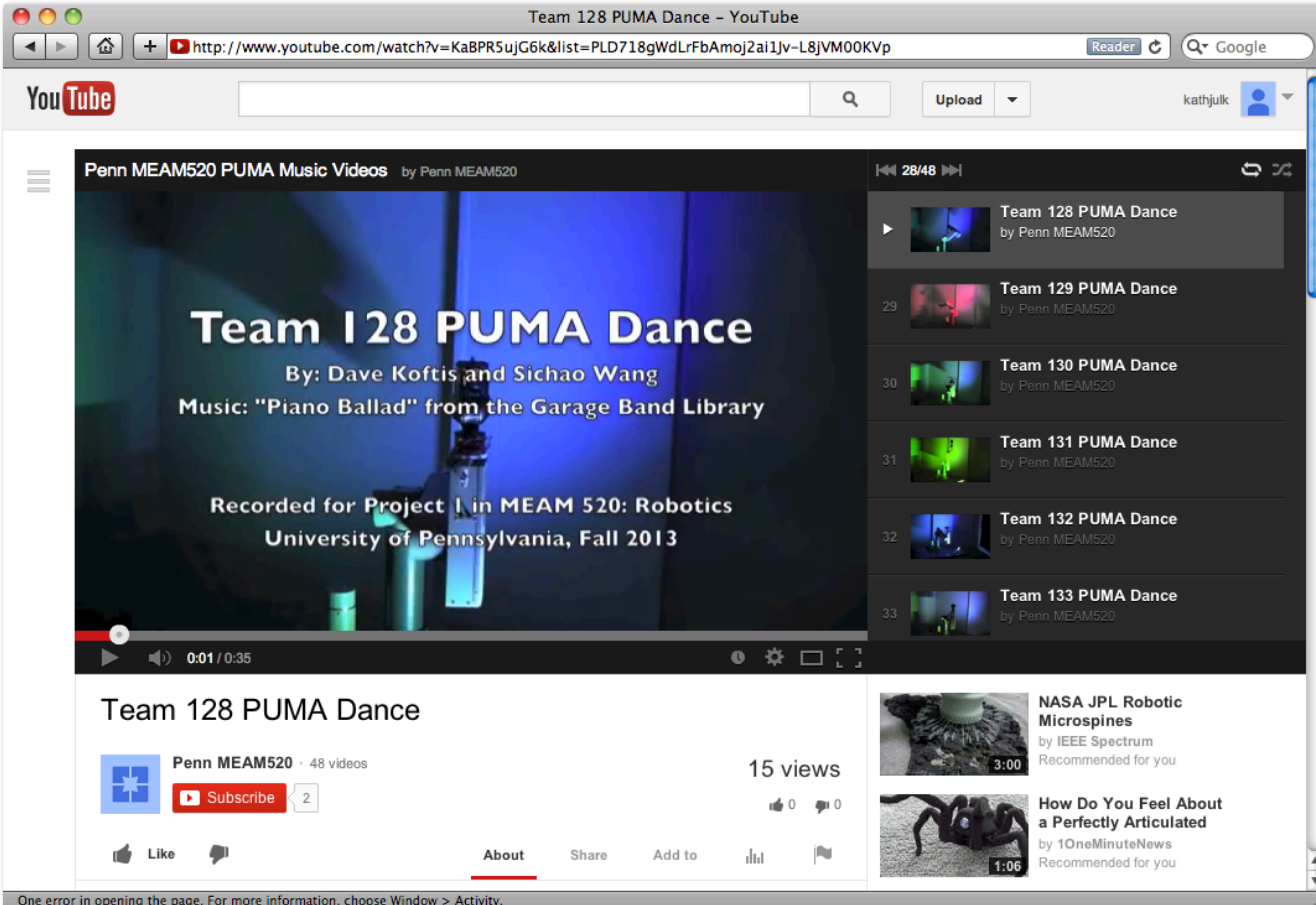
Attach your written inquiry to your test and give it to Naomi.

We will correct any grading mistakes.

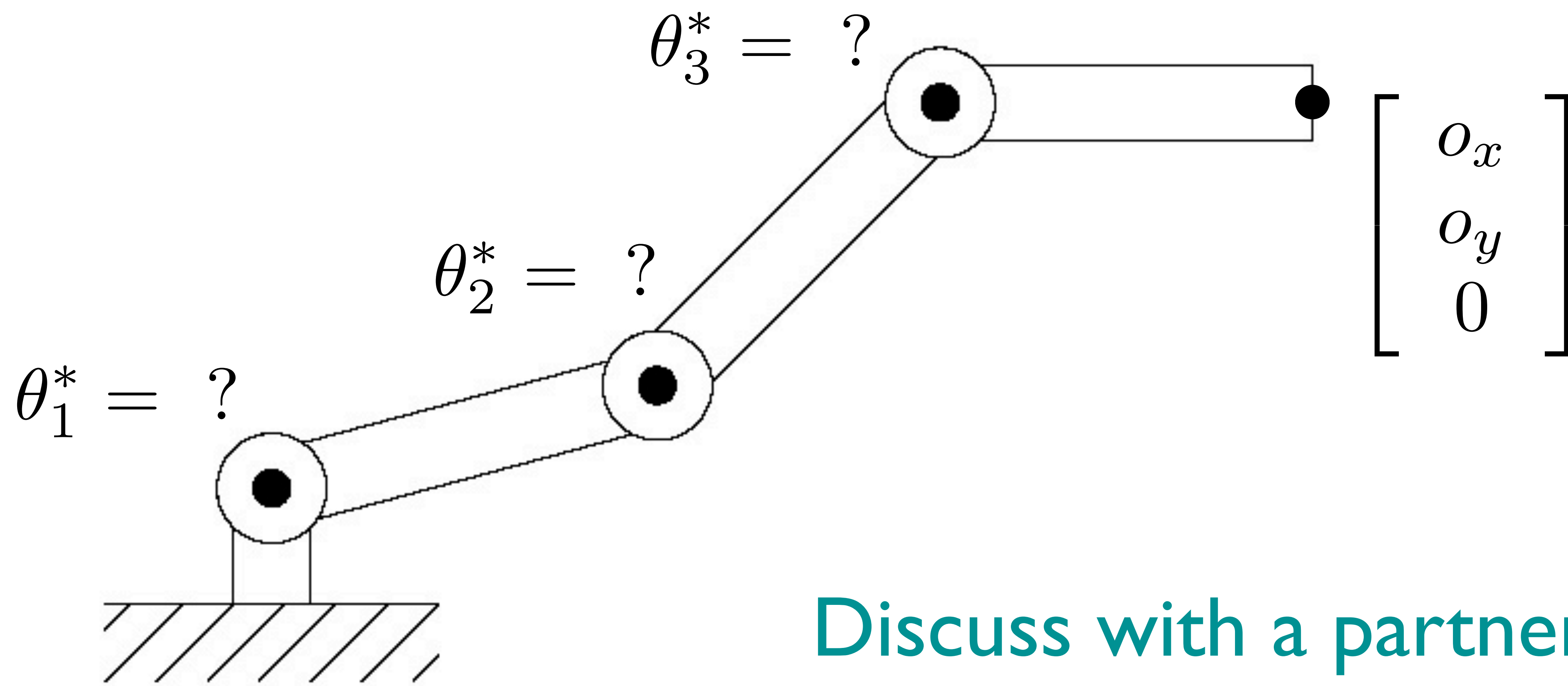
We will be grading the resubmissions this weekend.

What questions do you have?

Let's watch Teams 128–133.



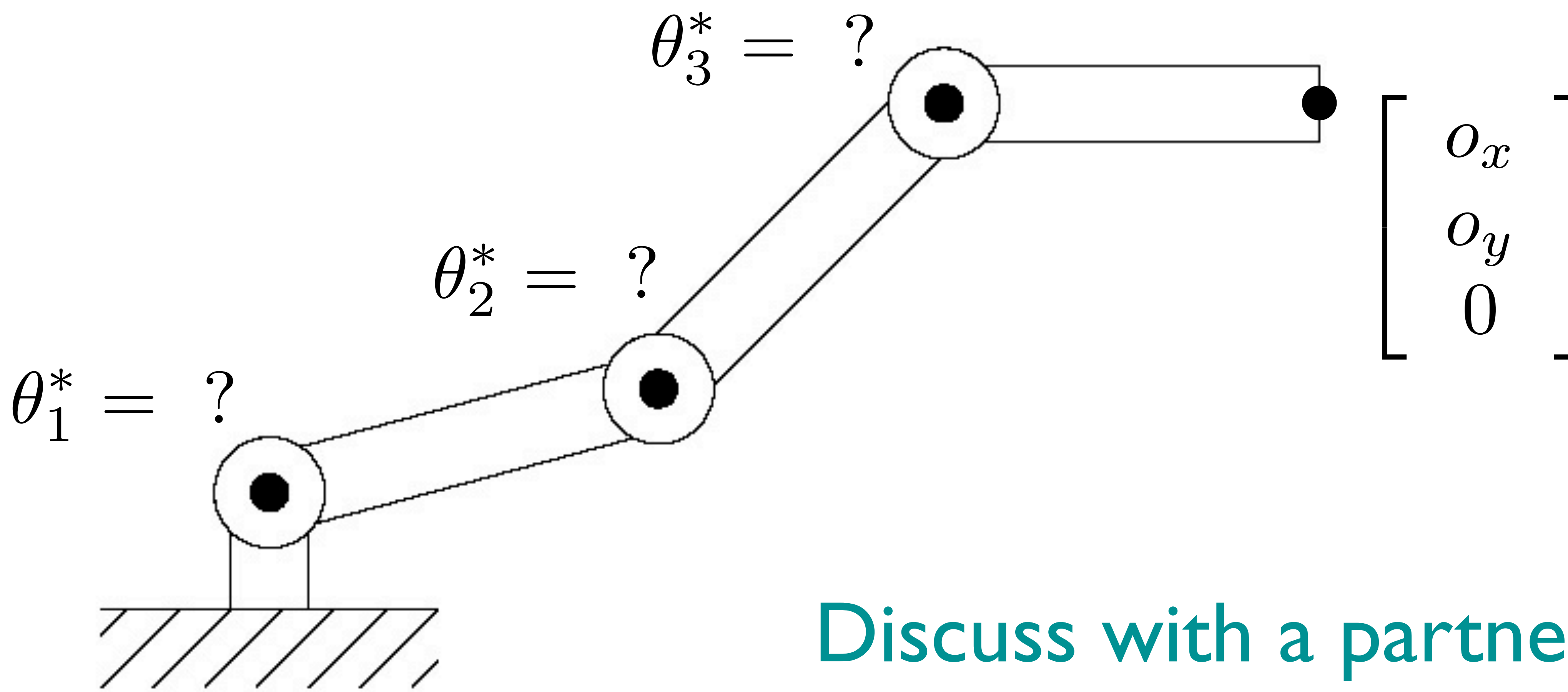
Another Example – Planar RRR Robot



Discuss with a partner.

Given a desired position of the end effector, how many solutions are there to the inverse kinematics for this robot?

Another Example – Planar RRR Robot

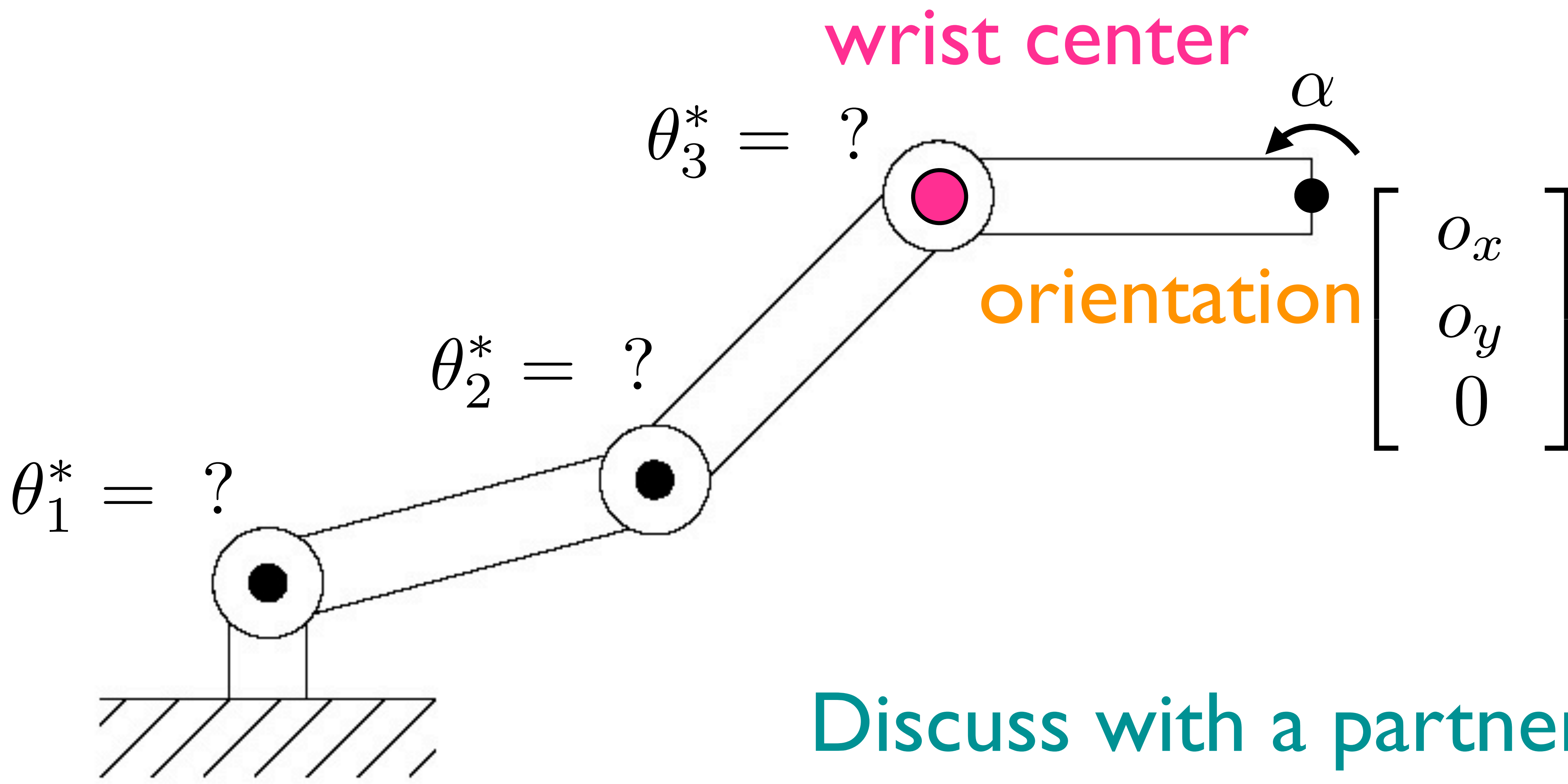


Discuss with a partner.

Given a desired position of the end effector, how many solutions are there to the inverse kinematics for this robot?

Infinitely many solutions if the target position is in the workspace
1 solution if the target position is on the workspace boundary
0 solutions if the target position is outside the workspace

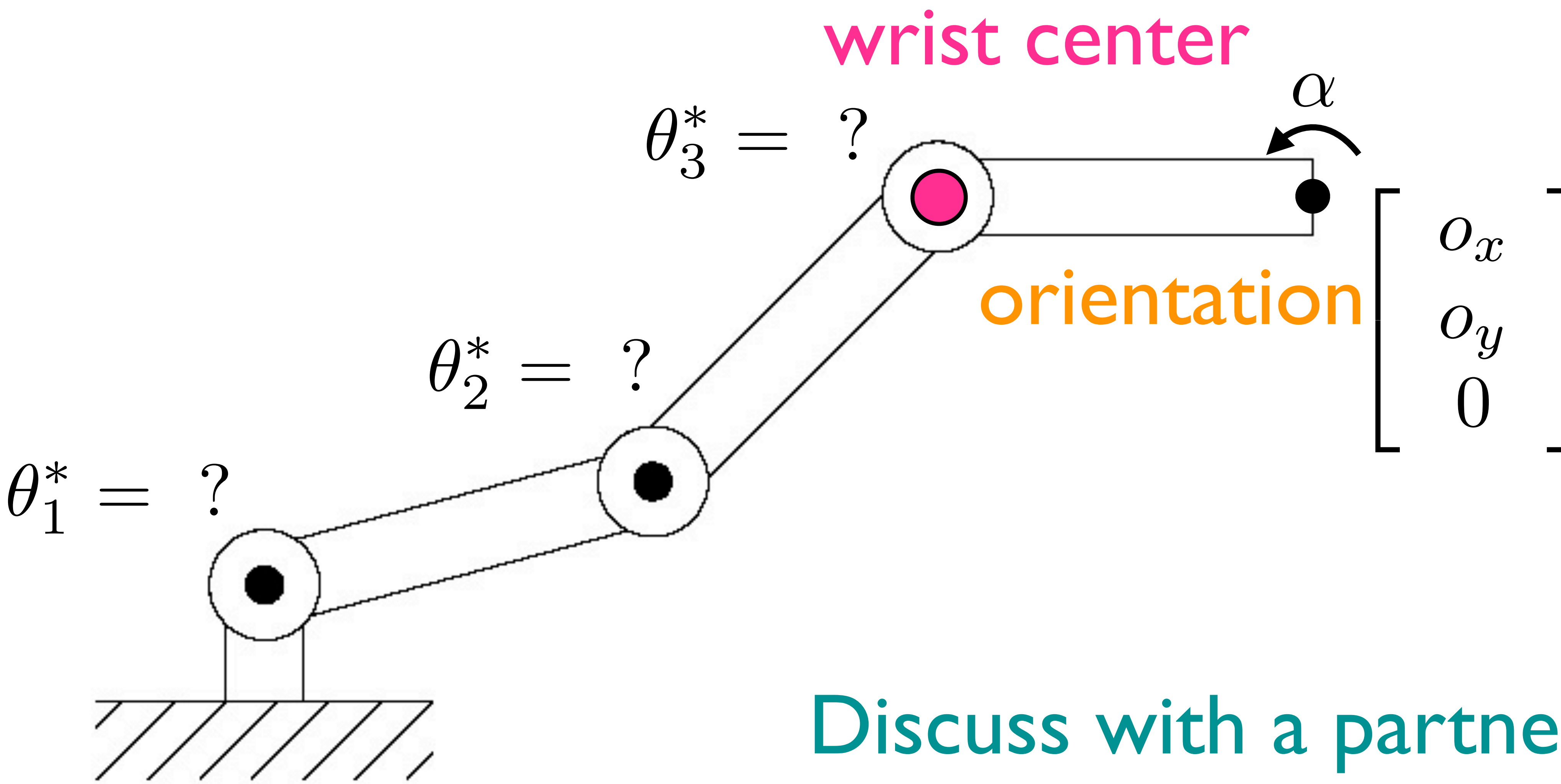
Another Example – Planar RRR Robot



Discuss with a partner.

If the orientation of the end effector is also specified, how many inverse kinematics solutions are there?

Another Example – Planar RRR Robot



Discuss with a partner.

If the orientation of the end effector is also specified, how many inverse kinematics solutions are there?

2 solutions if the wrist center is inside the 2-link workspace
1 solution if the wrist center is on the 2-link workspace boundary
0 solutions if the wrist position is outside the 2-link workspace
Infinitely many solutions if the wrist center is the origin

Questions ❓

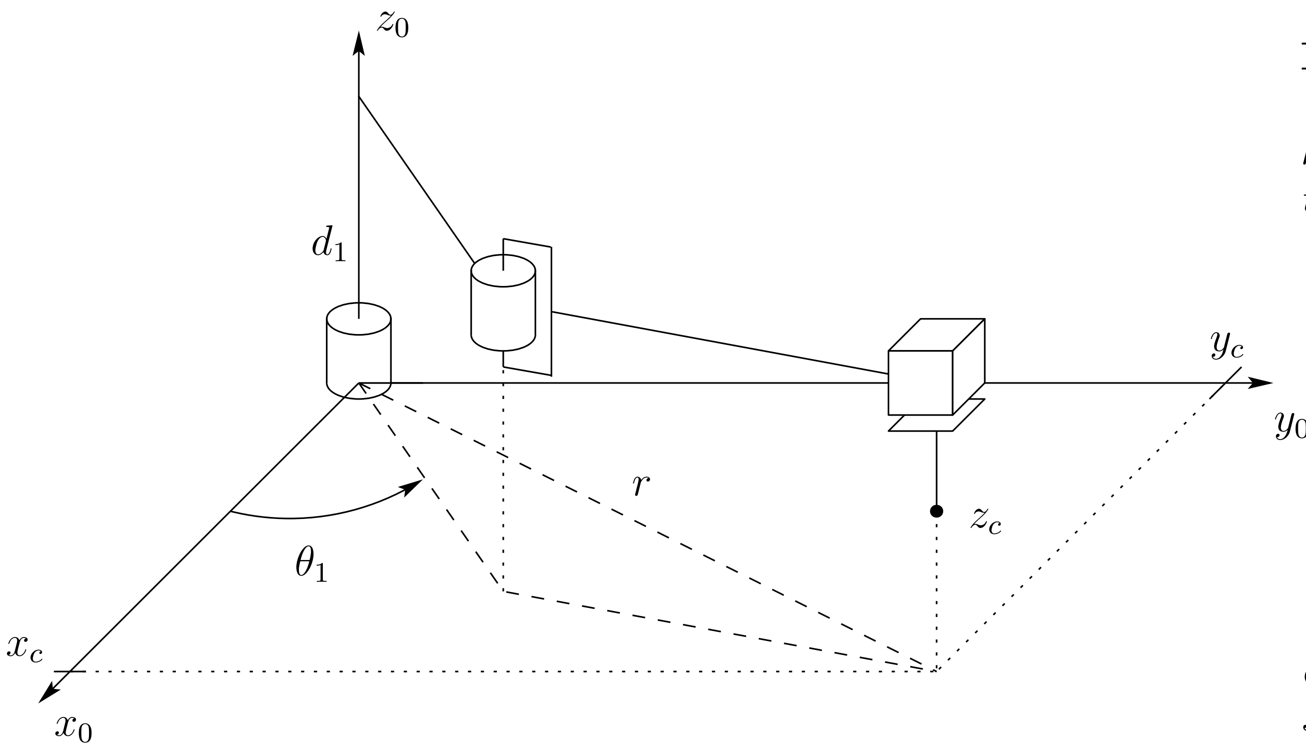
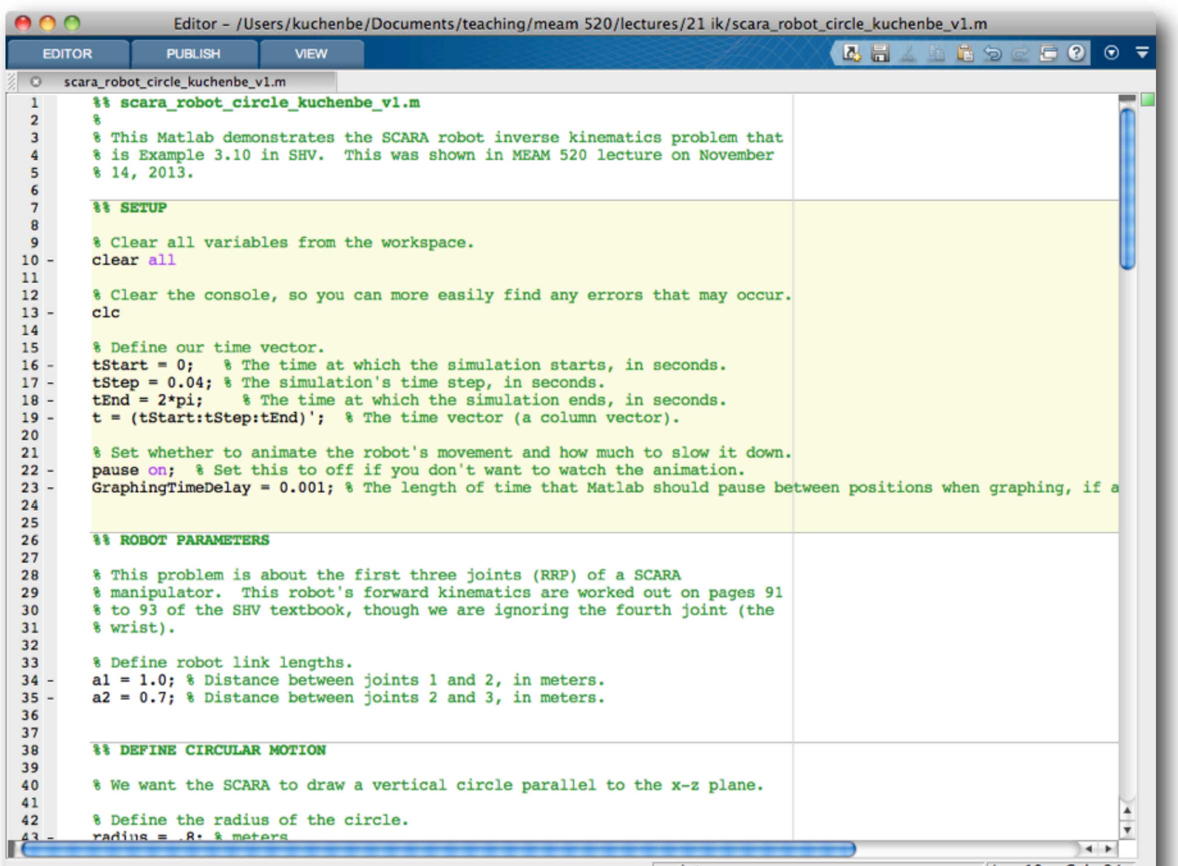


Fig. 3.22 SCARA manipulator




```
%% scara_robot_circle_kuchenbe_v1.m
%
% This Matlab demonstrates the SCARA robot inverse kinematics problem that
% is Example 3.10 in SHV.  This was shown in MEAM 520 lecture on November
% 14, 2013.

%% SETUP

% Clear all variables from the workspace.
clear all

% Clear the console, so you can more easily find any errors that may occur.
clc

% Define our time vector.
tStart = 0;   % The time at which the simulation starts, in seconds.
tStep = 0.04; % The simulation's time step, in seconds.
tEnd = 2*pi;    % The time at which the simulation ends, in seconds.
t = (tStart:tStep:tEnd)';  % The time vector (a column vector).

% Set whether to animate the robot's movement and how much to slow it down.
pause on;  % Set this to off if you don't want to watch the animation.
GraphingTimeDelay = 0.001; % The length of time that Matlab should pause between positions when graphing, if a


%% ROBOT PARAMETERS

% This problem is about the first three joints (RRP) of a SCARA
% manipulator.  This robot's forward kinematics are worked out on pages 91
% to 93 of the SHV textbook, though we are ignoring the fourth joint (the
% wrist).

% Define robot link lengths.
a1 = 1.0; % Distance between joints 1 and 2, in meters.
a2 = 0.7; % Distance between joints 2 and 3, in meters.


%% DEFINE CIRCULAR MOTION

% We want the SCARA to draw a vertical circle parallel to the x-z plane.

% Define the radius of the circle.
radius = .8; % meters
```

MATLAB code:
scara_robot_circle_kuchenbe_v1.m

Example 3.10 SCARA Manipulator

As another example, we consider the SCARA manipulator whose forward kinematics is defined by T_4^0 from (3.30). The inverse kinematics solution is then given as the set of solutions of the equation

$$T_4^0 = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & s_{12}c_4 - c_{12}s_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -c_{12}c_4 - s_{12}s_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.81}$$

We first note that, since the SCARA has only four degrees-of-freedom, not every possible H from $SE(3)$ allows a solution of (3.81). In fact we can easily see that there is no solution of (3.81) unless R is of the form

$$R = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ s_\alpha & -c_\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{3.82}$$

and if this is the case, the sum $\theta_1 + \theta_2 - \theta_4$ is determined by

$$\theta_1 + \theta_2 - \theta_4 = \alpha = \operatorname{atan2}(r_{11}, r_{12}) \tag{3.83}$$

Projecting the manipulator configuration onto the $x_0 - y_0$ plane immediately yields the situation of Figure 3.22. We see from this that

$$\boxed{\theta_2 = \operatorname{atan2}\left(c_2, \pm\sqrt{1 - c_2}\right)} \tag{3.84}$$

where

$$c_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1a_2} \tag{3.85}$$

$$\boxed{\theta_1 = \operatorname{atan2}(o_x, o_y) - \operatorname{atan2}(a_1 + a_2c_2, a_2s_2)} \tag{3.86}$$

We may then determine θ_4 from (3.83) as

$$\begin{aligned} \theta_4 &= \theta_1 + \theta_2 - \alpha \\ &= \theta_1 + \theta_2 - \operatorname{atan2}(r_{11}, r_{12}) \end{aligned} \tag{3.87}$$

Finally d_3 is given as

$$\boxed{d_3 = o_z + d_4} \quad d_4 = 0 \tag{3.88}$$

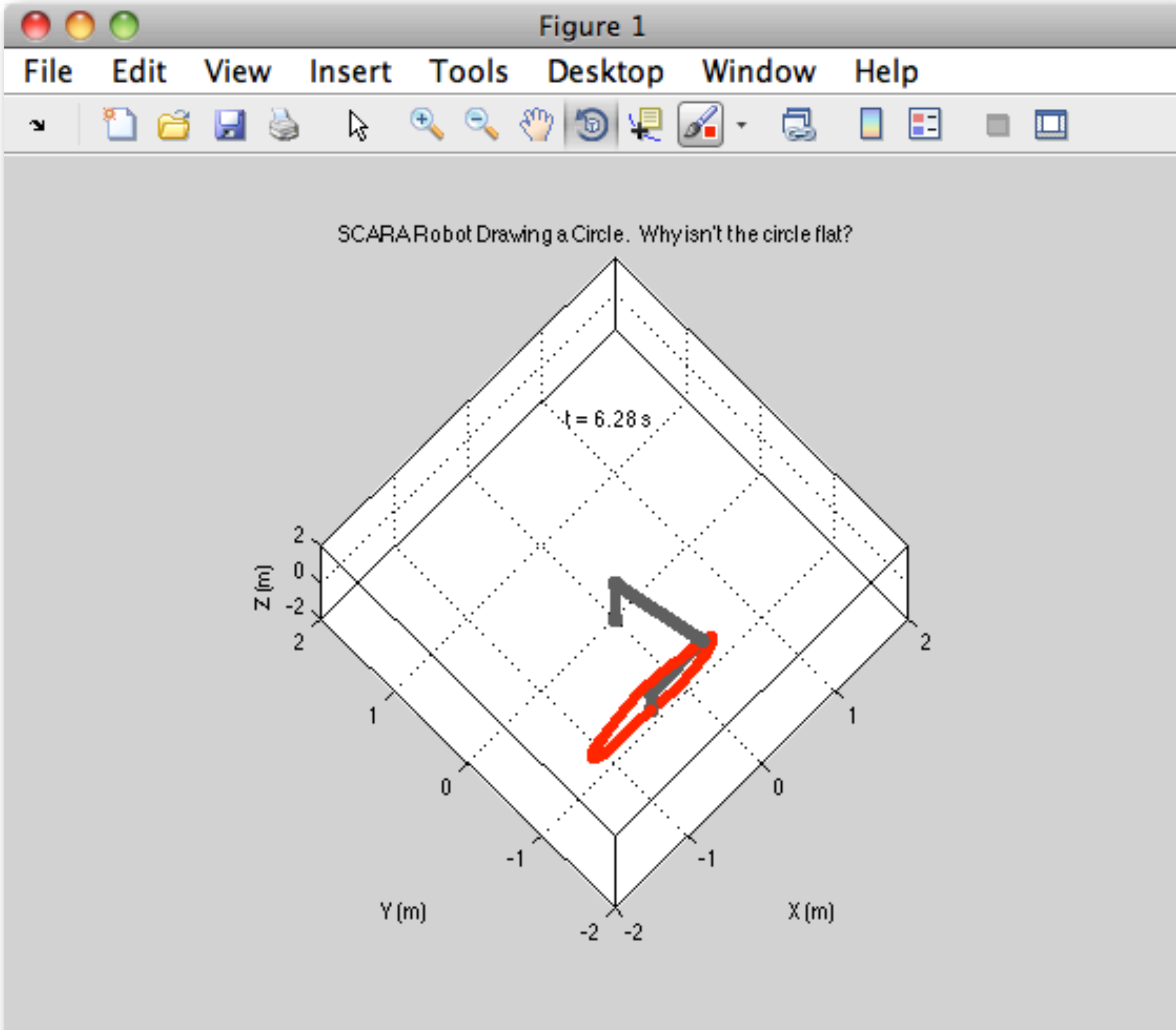


Figure 1
File Edit View Insert Tools Desktop Window Help
SCARA Robot Drawing a Circle. Why isn't the circle flat?
t = 6.28 s
Z (m)
Y (m)
X (m)
2
0
-2
2
1
0
-1
-2
-2
-1
0
1
2

What's wrong?