MEAM 520

Forward Kinematics and DH Parameters

Katherine J. Kuchenbecker, Ph.D.

General Robotics, Automation, Sensing, and Perception Lab (GRASP)

MEAM Department, SEAS, University of Pennsylvania

GRASP LABORATORY

Lecture 6: September 17, 2013

Homework 2:
Rotation Matrices and Homogeneous Transformations

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

September 10, 2013

This paper-based assignment is due on Tuesday, September 17, by midnight (11:59:59 p.m.) You should aim to turn it in during class that day. If you don't finish until later in the day, you can turn it in to Professor Kuchenbecker's office, Towne 224, in the bin or under the door. Late submissions will be accepted until Thursday, September 19, by noon (11:59:59 a.m.), but they will be penalized by 10% for each partial or full day late, up to 20%. After the late deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down should be your own work, not copied from any other individual or a solution manual. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. If you get stuck, post a question on Piazza or go to office hours!

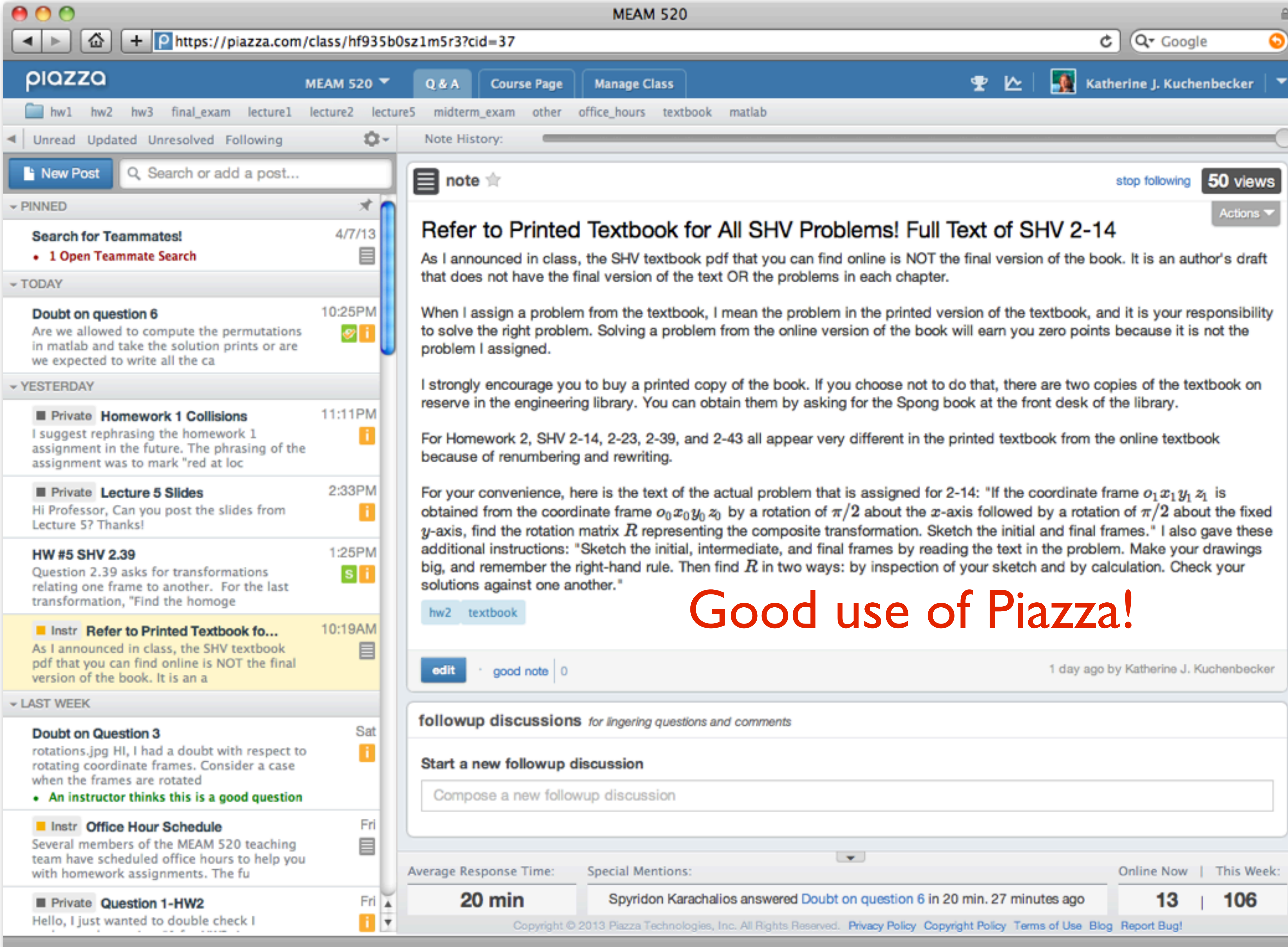
These problems are from the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions when provided. Write in pencil, show your work clearly, box your answers, and staple together all pages of your assignment. This assignment is worth a total of 20 points.

1. SHV 2-6, page 65 – Verifying Three Properties of $R_{z,\theta}$ *(2 points)*
2. SHV 2-10, page 66 – Sequence of Rotations *(2 points)*
   Please specify each element of each matrix in symbolic form and show the order in which the matrices should be multiplied; as stated in the problem, you do not need to perform the matrix multiplication.
3. SHV 2-14, page 67 – Rotating a Coordinate Frame *(4 points)*
   Sketch the initial, intermediate, and final frames by reading the text in the problem. Make your drawings big, and remember the right-hand rule. Then find R in two ways: by inspection of your sketch and by calculation. Check your solutions against one another.
4. SHV 2-23, page 68 – Axis/Angle Representation *(4 points)*
   Be careful with the sketch, and remember the right-hand rule.
5. SHV 2-39, page 70 – Homogeneous Transformations *(4 points)*
   Treat frame $o_2x_2y_2z_2$ as being located at the center of the cube's bottom surface (as drawn in Figure 2.14), not at the center of the cube (as stated in the problem). Be careful with notation; you are looking for H_1^0, H_2^0, H_3^0, and H_3^2.
6. SHV 2-43, page 71-72 – Commutativity of Homogeneous Transformations *(4 points)*

Due today
by midnight.

Late deadline is
Thursday at noon.

Turn in to bin at front
of class or outside
Dr. K's office (224 TB)


MEAM 520
https://piazza.com/class/hf935b0sz1m5r3?cid=37
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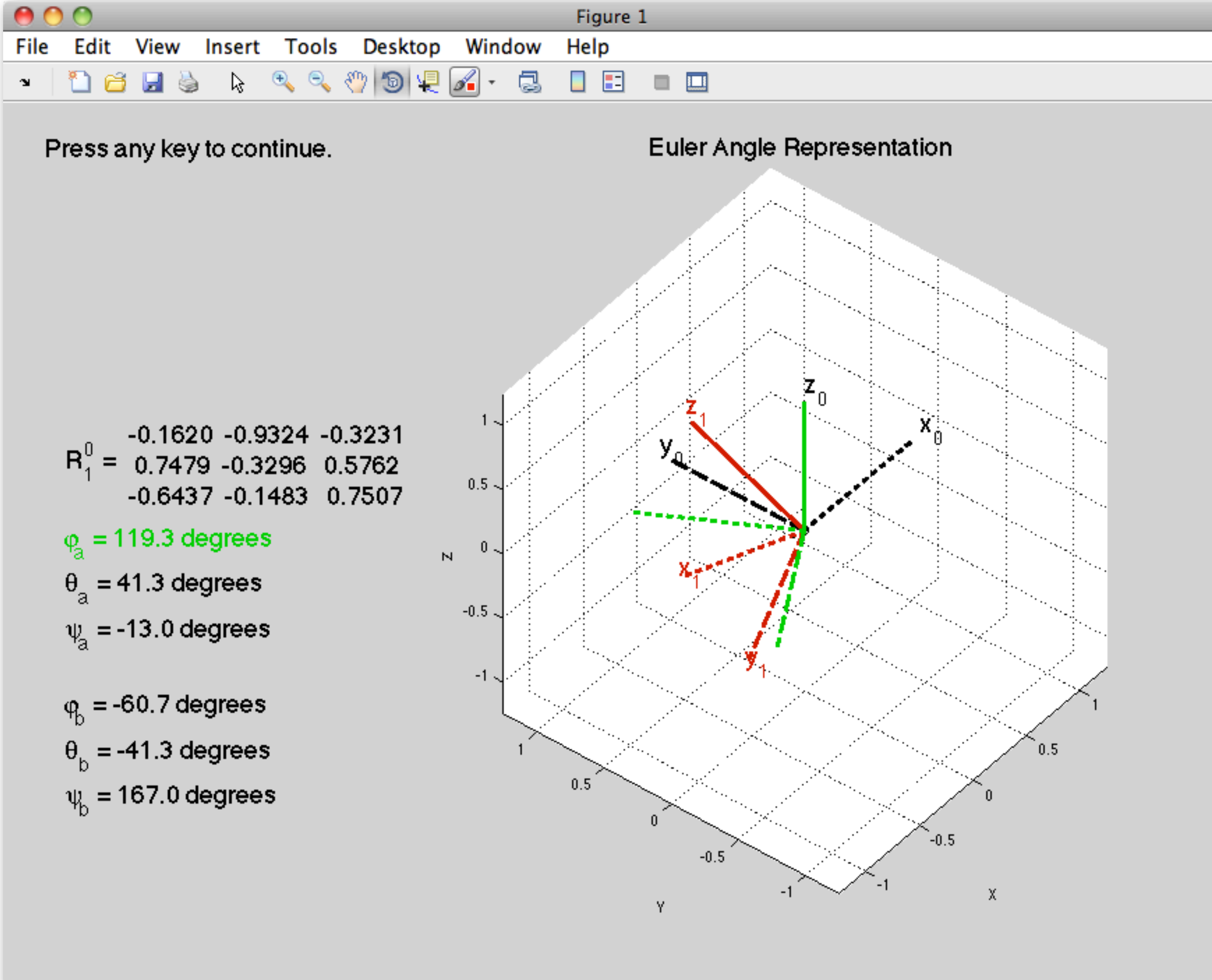
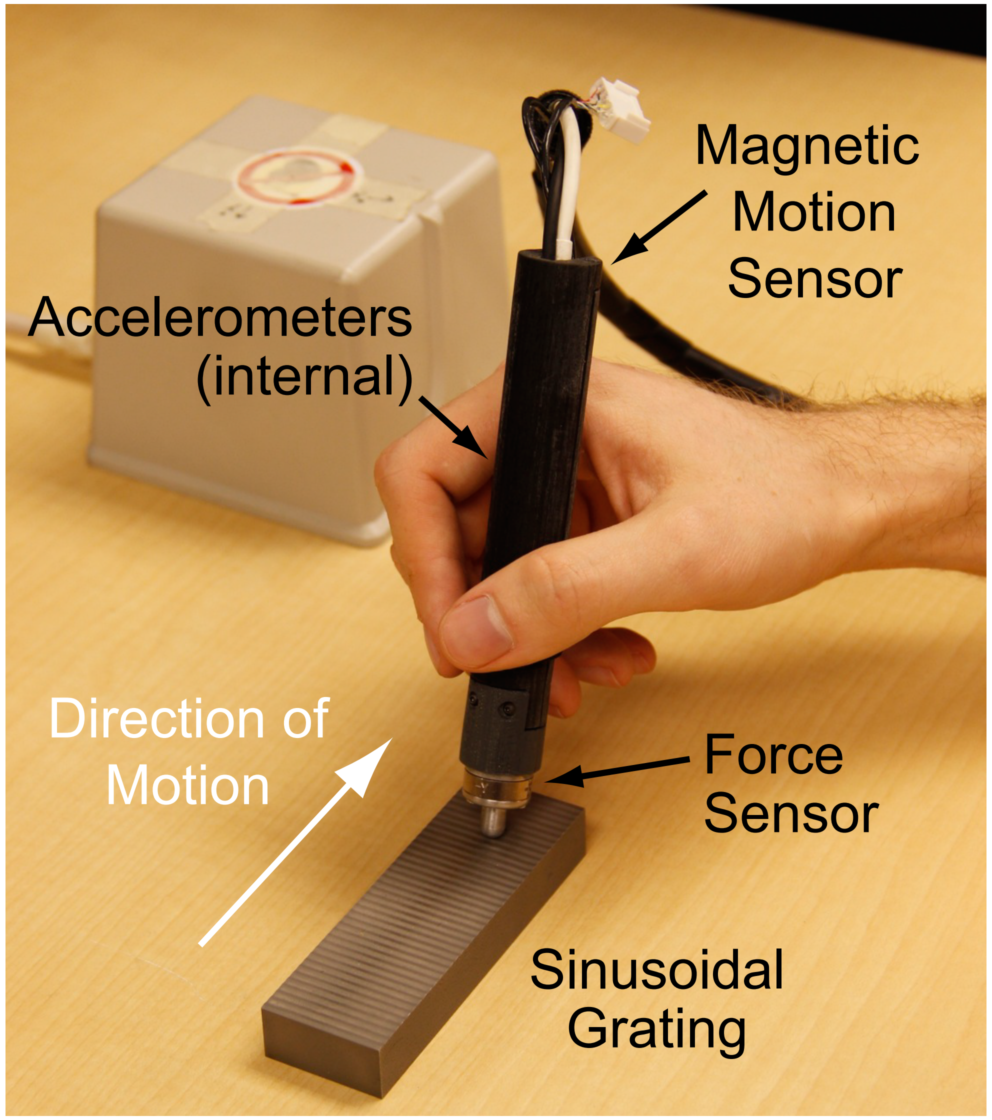
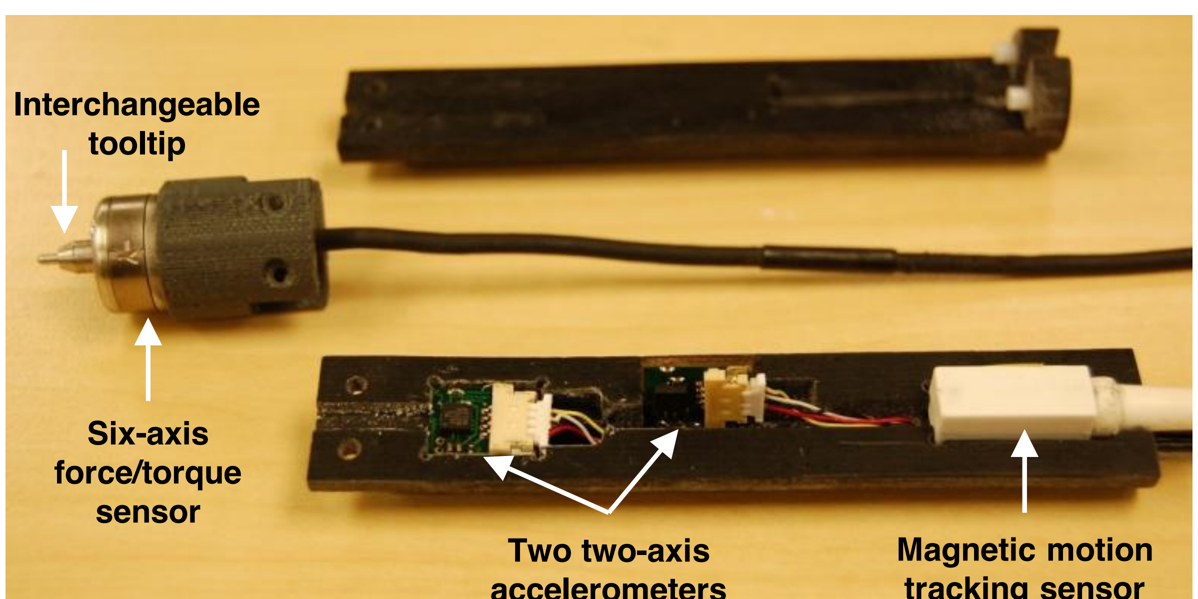
Katherine J. Kuchenbecker
hw1 hw2 hw3 final_exam lecture1 lecture2 lecture5 midterm_exam other office_hours textbook matlab
Unread Updated Unresolved Following
Note History:
New Post
Search or add a post...
PINNED
Search for Teammates! 4/7/13
1 Open Teammate Search
TODAY
Doubt on question 6 10:25PM
Are we allowed to compute the permutations in matlab and take the solution prints or are we expected to write all the ca
YESTERDAY
Private Homework 1 Collisions 11:11PM
I suggest rephrasing the homework 1 assignment in the future. The phrasing of the assignment was to mark "red at loc
Private Lecture 5 Slides 2:33PM
Hi Professor, Can you post the slides from Lecture 5? Thanks!
HW #5 SHV 2.39 1:25PM
Question 2.39 asks for transformations relating one frame to another. For the last transformation, "Find the homoge
Instr Refer to Printed Textbook fo... 10:19AM
As I announced in class, the SHV textbook pdf that you can find online is NOT the final version of the book. It is an a
LAST WEEK
Doubt on Question 3 Sat
rotations.jpg HI, I had a doubt with respect to rotating coordinate frames. Consider a case when the frames are rotated
An instructor thinks this is a good question
Instr Office Hour Schedule Fri
Several members of the MEAM 520 teaching team have scheduled office hours to help you with homework assignments. The fu
Private Question 1-HW2 Fri
Hello, I just wanted to double check I
note
stop following
50 views
Actions
Refer to Printed Textbook for All SHV Problems! Full Text of SHV 2-14
As I announced in class, the SHV textbook pdf that you can find online is NOT the final version of the book. It is an author's draft that does not have the final version of the text OR the problems in each chapter.
When I assign a problem from the textbook, I mean the problem in the printed version of the textbook, and it is your responsibility to solve the right problem. Solving a problem from the online version of the book will earn you zero points because it is not the problem I assigned.
I strongly encourage you to buy a printed copy of the book. If you choose not to do that, there are two copies of the textbook on reserve in the engineering library. You can obtain them by asking for the Spong book at the front desk of the library.
For Homework 2, SHV 2-14, 2-23, 2-39, and 2-43 all appear very different in the printed textbook from the online textbook because of renumbering and rewriting.
For your convenience, here is the text of the actual problem that is assigned for 2-14: "If the coordinate frame $o_1x_1y_1z_1$ is obtained from the coordinate frame $o_0x_0y_0z_0$ by a rotation of $\pi/2$ about the x-axis followed by a rotation of $\pi/2$ about the fixed y-axis, find the rotation matrix R representing the composite transformation. Sketch the initial and final frames." I also gave these additional instructions: "Sketch the initial, intermediate, and final frames by reading the text in the problem. Make your drawings big, and remember the right-hand rule. Then find R in two ways: by inspection of your sketch and by calculation. Check your solutions against one another."
hw2 textbook
Good use of Piazza!
edit
good note 0
1 day ago by Katherine J. Kuchenbecker
followup discussions for lingering questions and comments
Start a new followup discussion
Compose a new followup discussion
Average Response Time:
20 min
Special Mentions:
Spyridon Karachalios answered Doubt on question 6 in 20 min. 27 minutes ago
Online Now | This Week:
13 | 106
Copyright © 2013 Piazza Technologies, Inc. All Rights Reserved. Privacy Policy Copyright Policy Terms of Use Blog Report Bug!

Who tried the visualize_R.m script?



Rigid Motions and Homogeneous Transformations





I want to calculate the position of the tip of the handheld tool in the fixed frame based on magnetic motion tracking data.

$$\mathbf{v}_p^0 = \mathbf{R}_1^0 \mathbf{v}_p^1 + \mathbf{d}_1^0$$

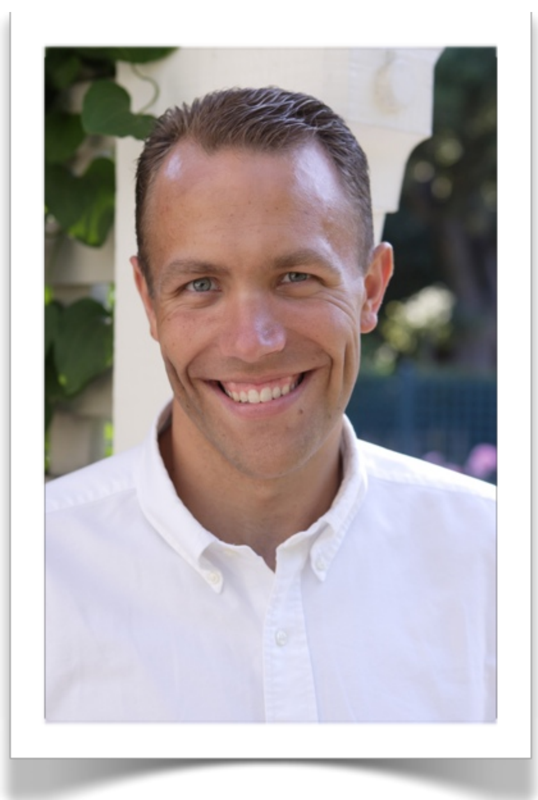
$$\begin{array}{cc} & \begin{array}{cccc} & \mathbf{R}_1^0 & & \mathbf{d}_1^0 \end{array} \\ \mathbf{H} = & \begin{bmatrix} n_x & s_x & a_x & d_x \\ n_y & s_y & a_y & d_y \\ n_z & s_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \mathbf{v}_p^1 \\ \mathbf{P} = & \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \end{array}$$

$$\mathbf{v}_p^0 = \mathbf{H}_1^0 \, \mathbf{v}_p^1$$

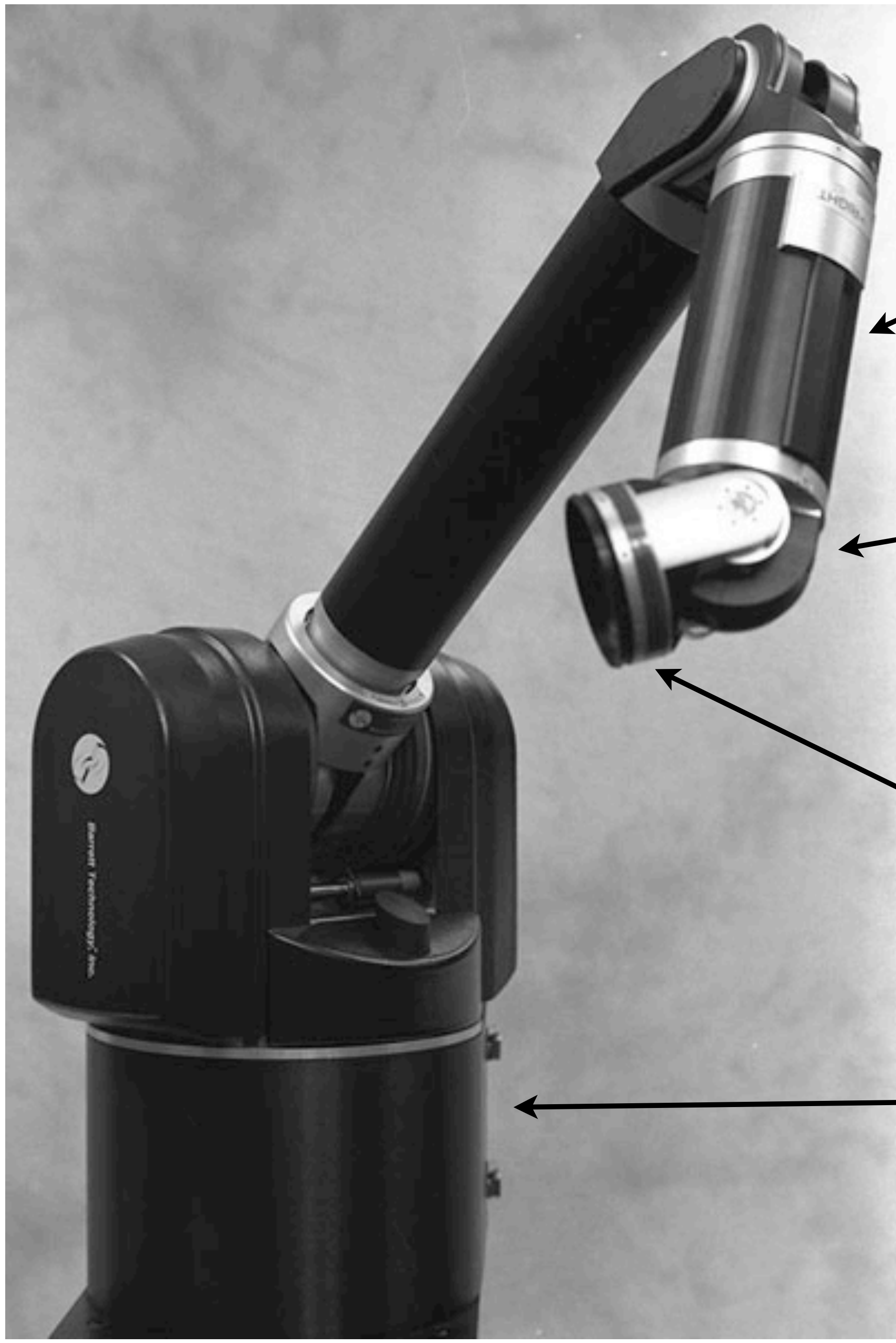
Manipulator Terminology Review

SHV 1.1-1.3, 3.1



Adapted from slides
created by Jonathan Fiene

General Terminology



Link : rigid body, 6 degrees of freedom

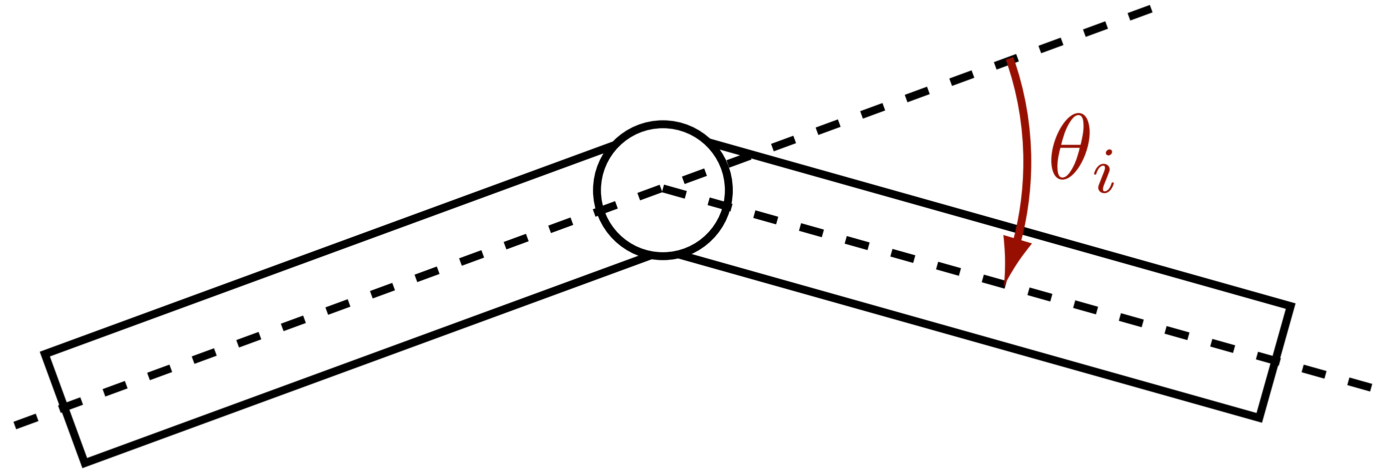
Joint : connection between two links

End-effector : interacts with the environment

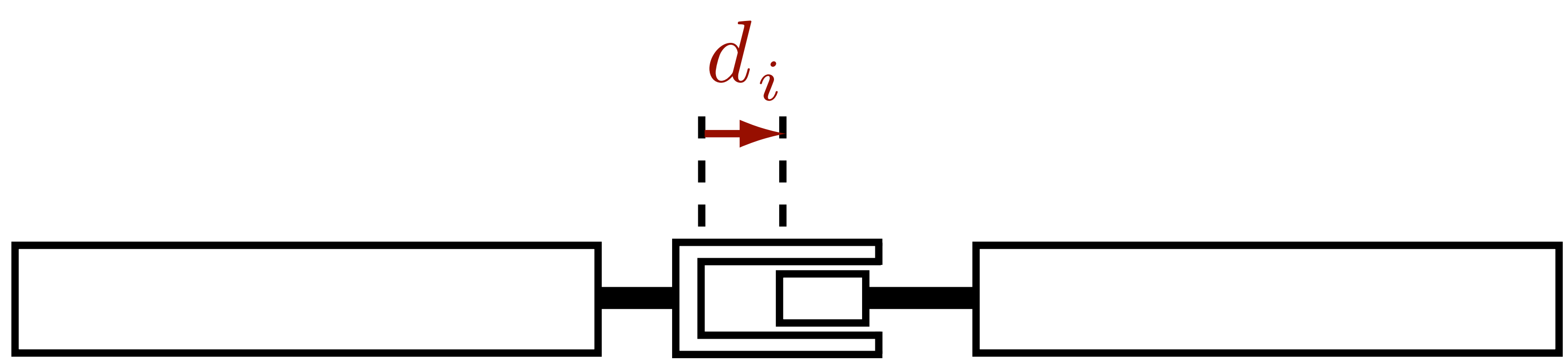
Base : connected to ground

Joint Descriptions

Revolute : angular displacement between adjacent links

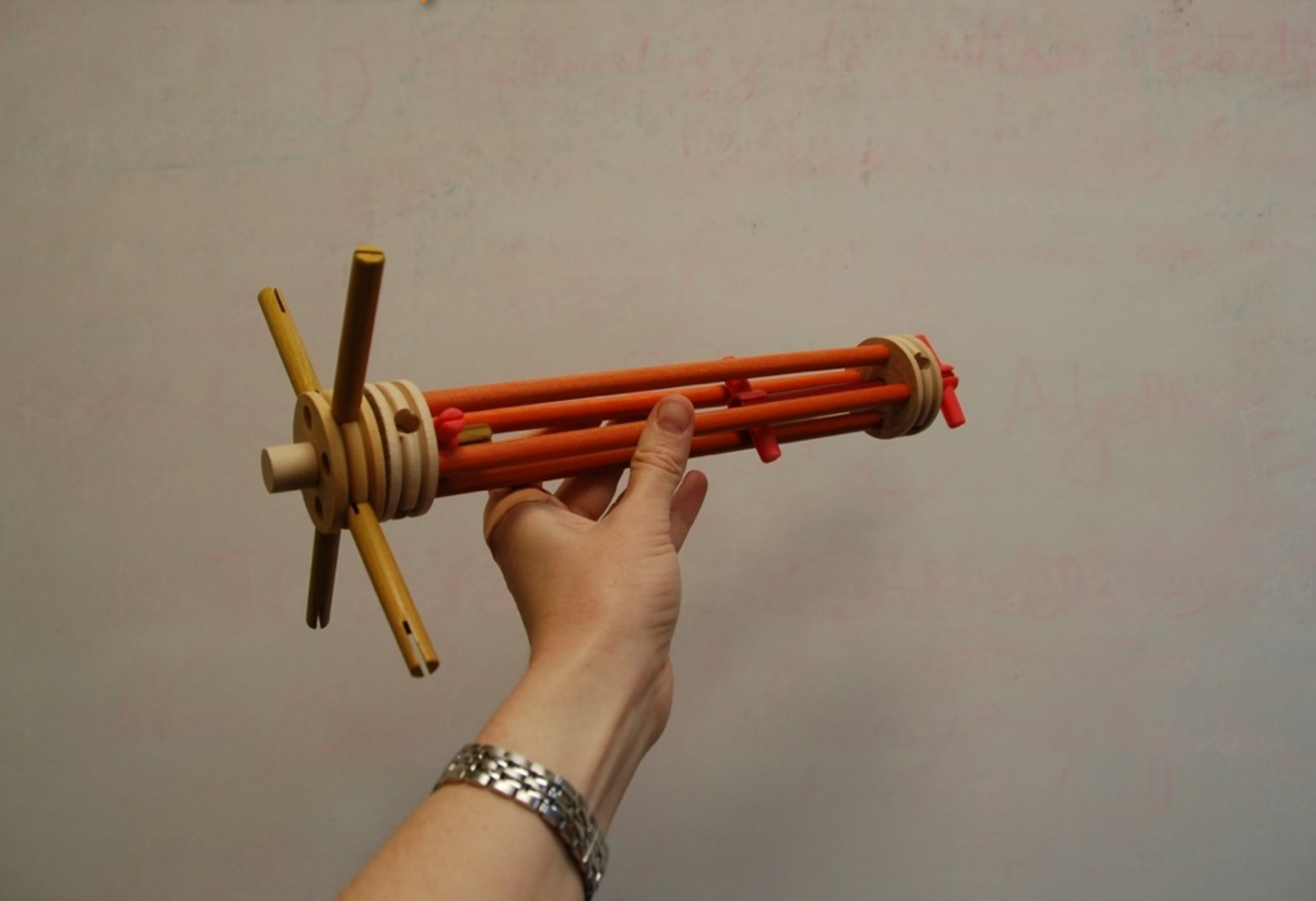


Prismatic : linear displacement between adjacent links

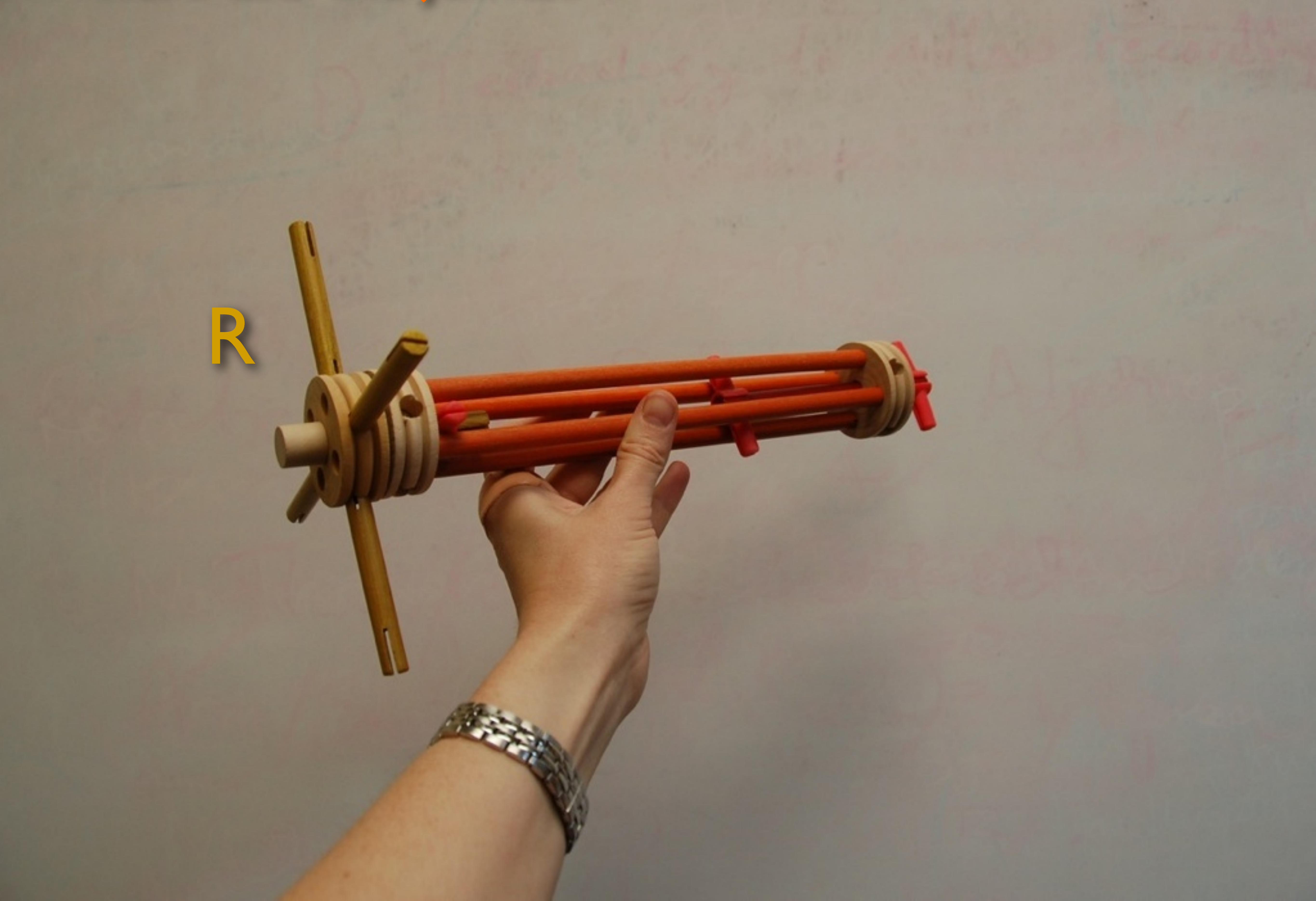


We use z-axes to denote joint axes.

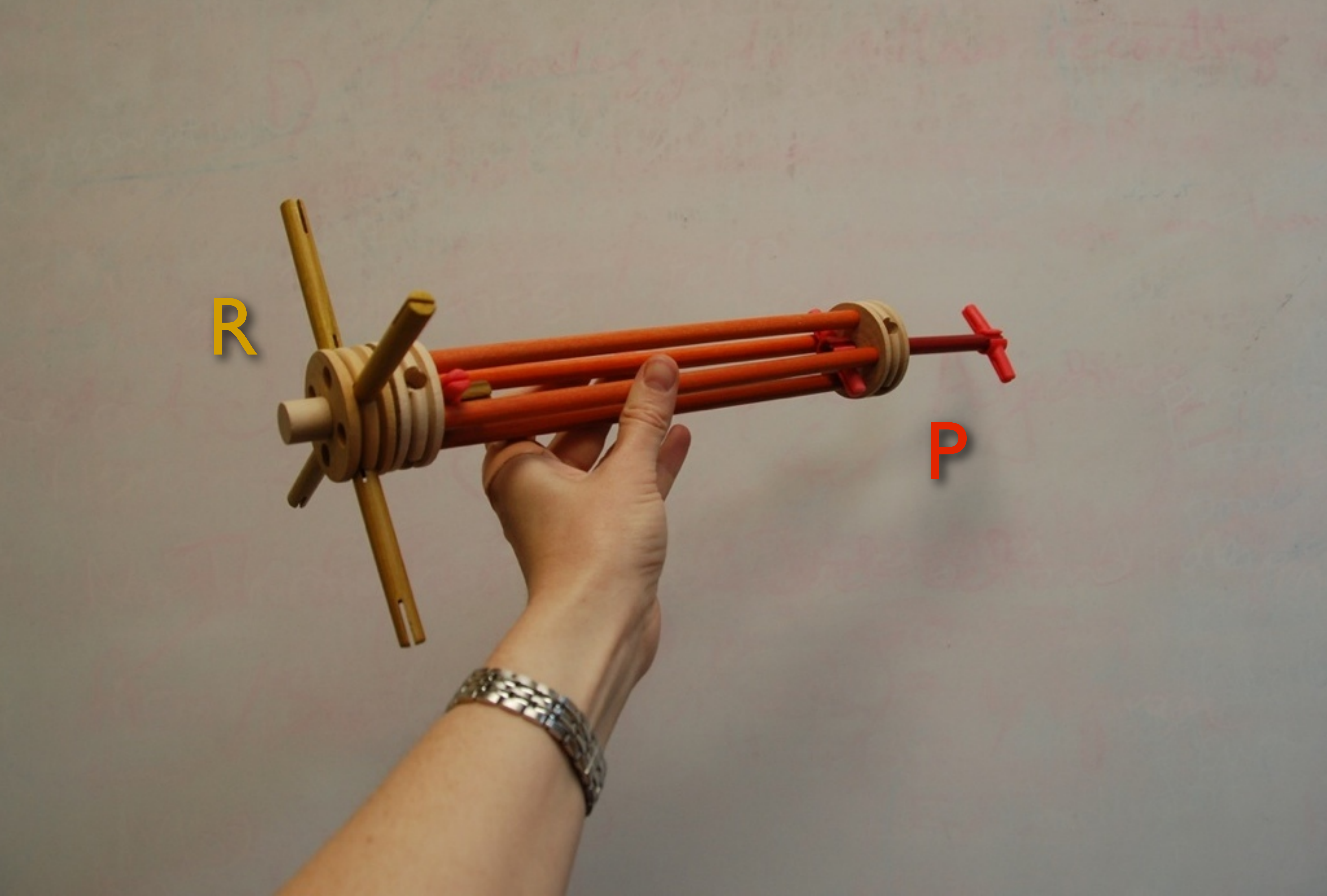
Where are the joints?



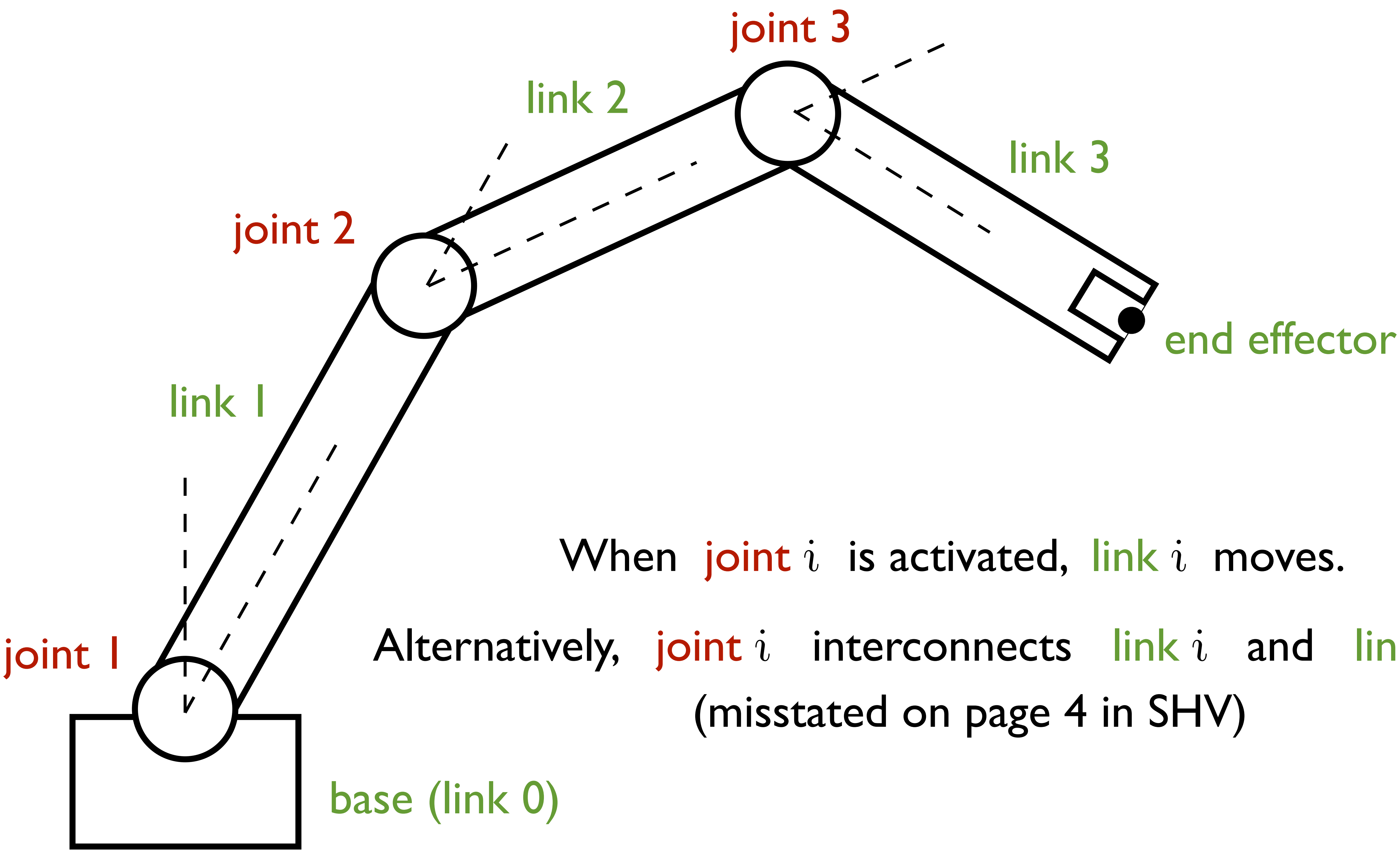
Where are the joints?



Where are the joints?



A **kinematic chain** is a system of rigid bodies connected by joints



When joint i is activated, link i moves.

Alternatively, joint i interconnects link i and link i-1

(misstated on page 4 in SHV)

In a **serial** kinematic chain, each intermediate link is connected to two others

Errata: Robot Modeling and Control

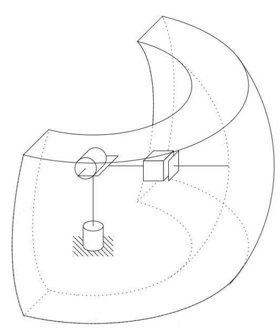
Mark W. Spong, Seth Hutchinson, and M. Vidyasagar

Compiled by MEAM 520 teaching team, expanding on the errata from Seth Hutchinson.

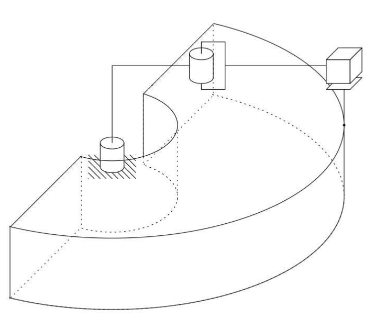
Chapter 1

Page 4 Near the bottom of the page, change "if the joint is the interconnection of links i and $i+1$" to "if the joint is the interconnection of links i and $i-1$".

Page 18 In Figure 1.17(a) the workspace for the spherical robot is drawn too low. The spherical surface should be centered on the intersection between the two revolute joints, as shown in the image below, which was taken from an early draft of the book:



Page 18 In Figure 1.17(b) the workspace for the SCARA robot is too far away from the base. The tip of the robot should be at the far arc of the workspace. The two end arcs should also be tangent to the outer arc and should have a radius equal to the length of the robot's forearm. Generally this is a poor illustration of the workspace of a SCARA robot. Here is a version that has the workspace in the right location, but it erroneously shows straight instead of curved edges:

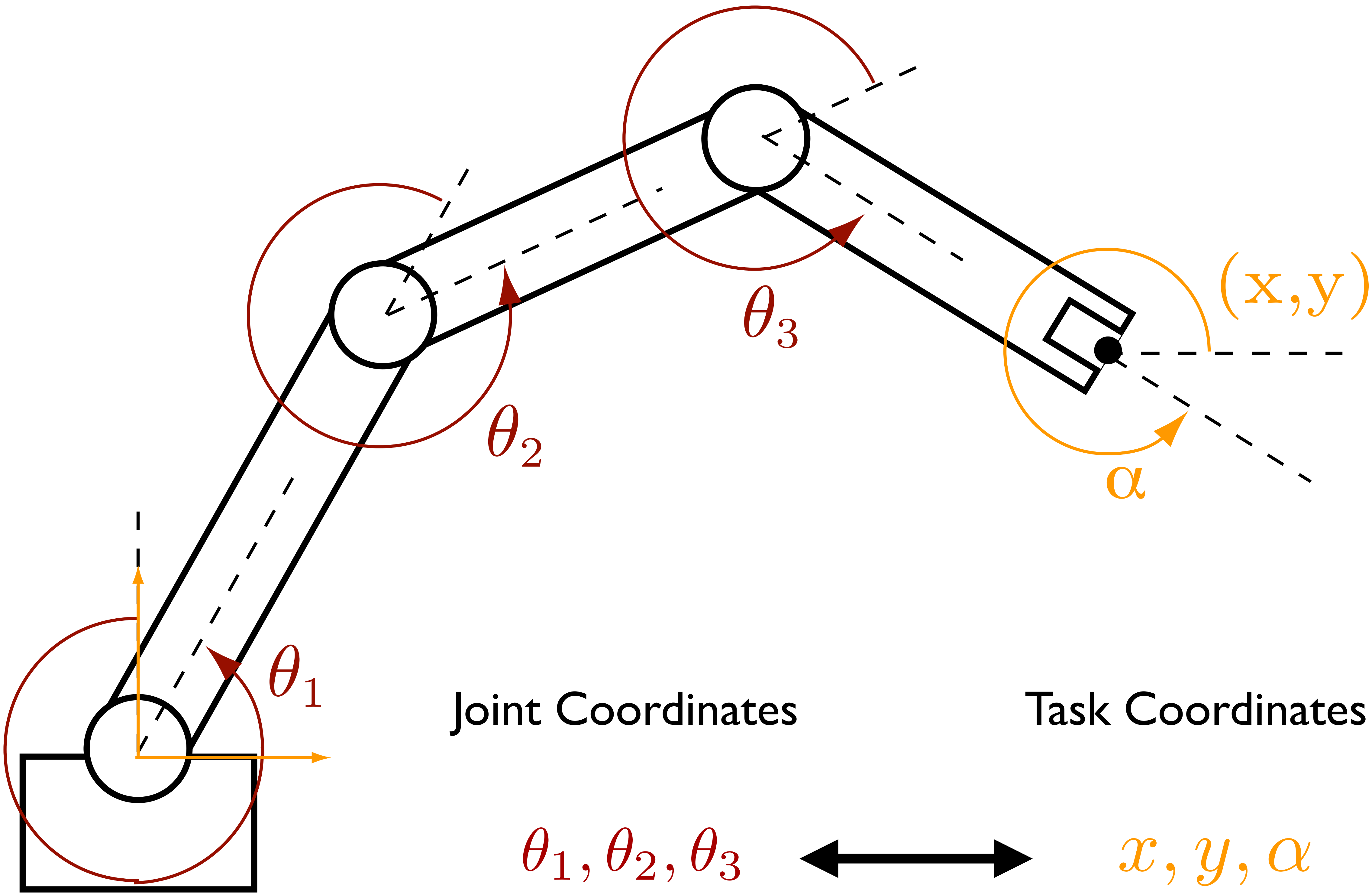


Who has looked at the errata?

Correct the known errors in your textbook.

Report any new errors that you find!

Joint and Task Coordinates



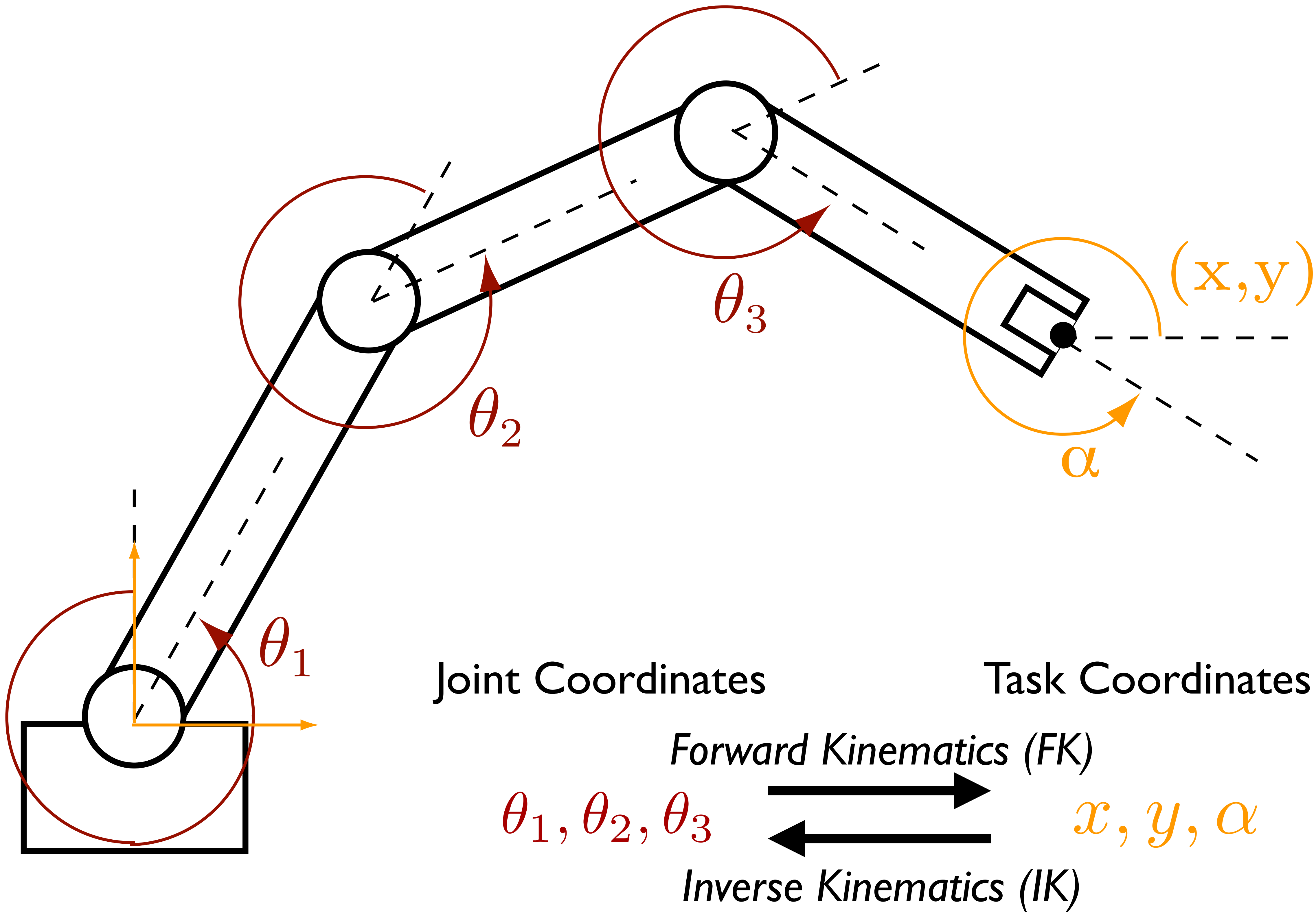
Joint Coordinates

Task Coordinates

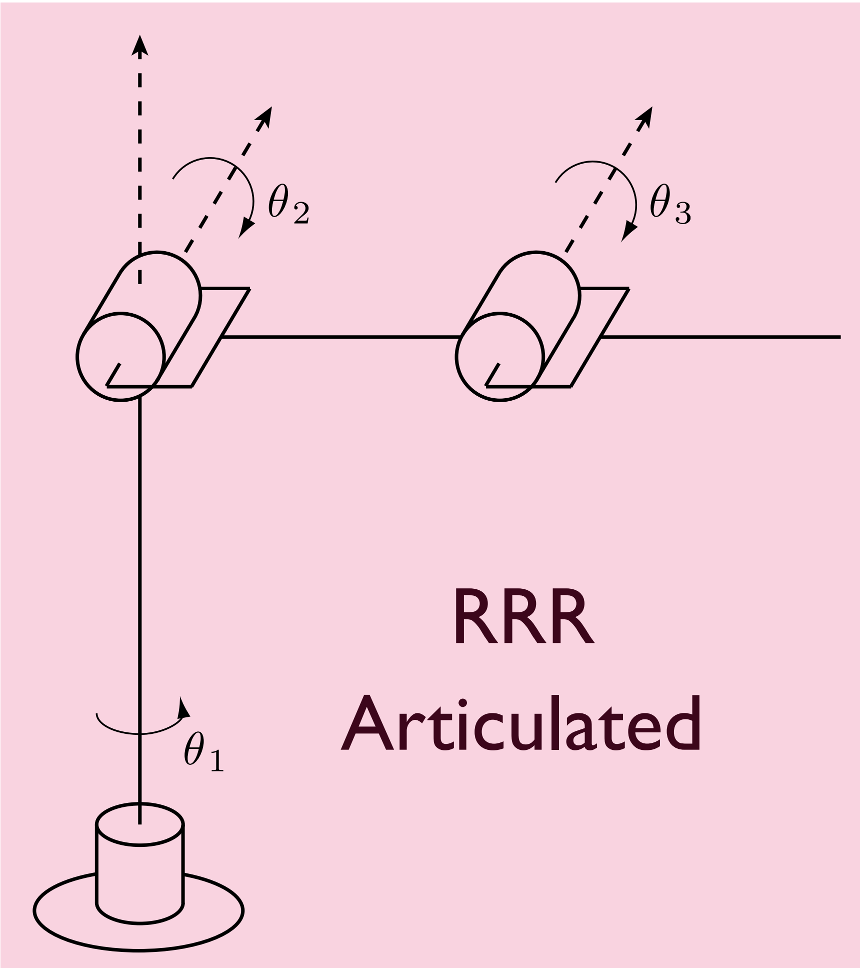
$\theta_1, \theta_2, \theta_3 \longleftrightarrow x, y, \alpha$

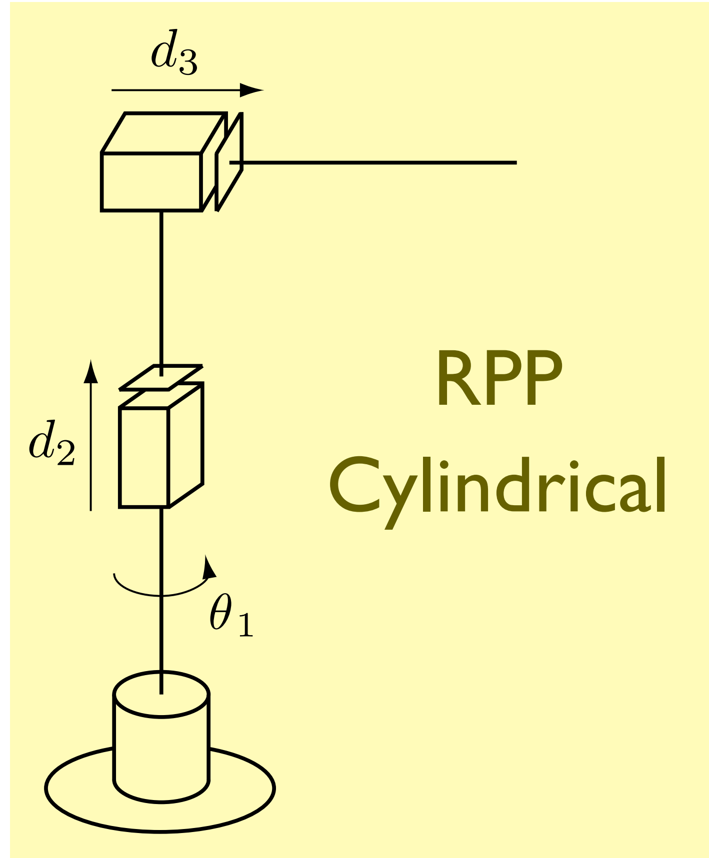
Kinematics

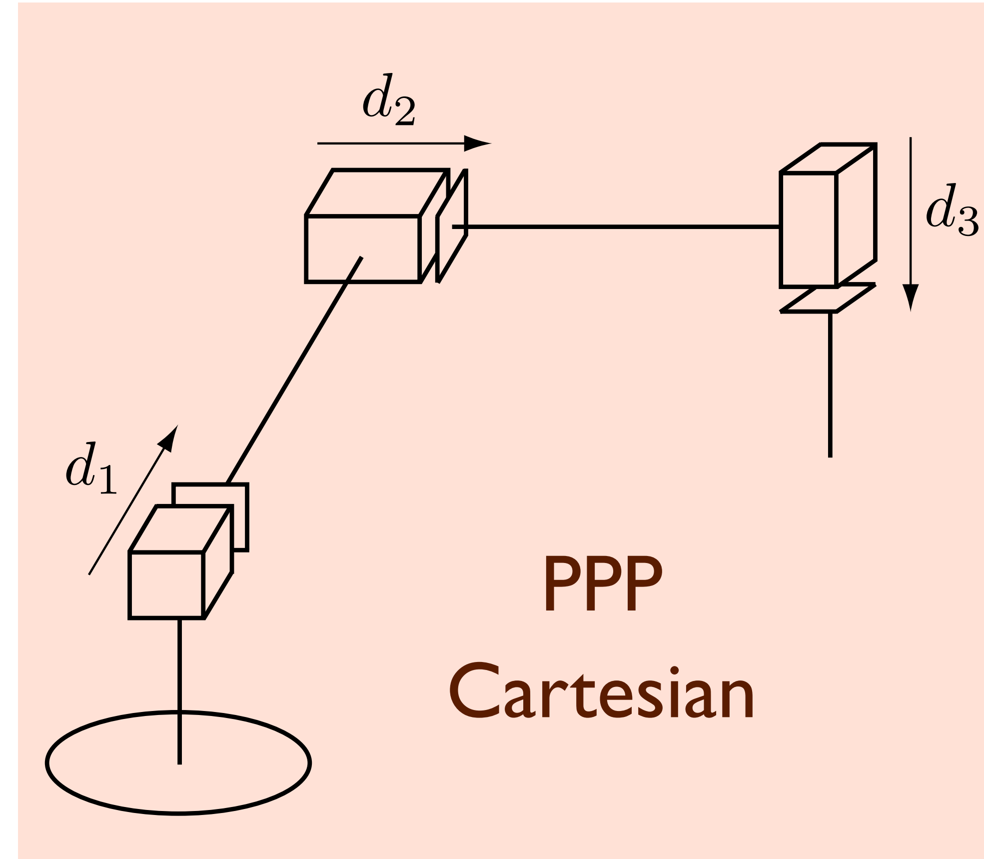
Joint and Task Coordinates

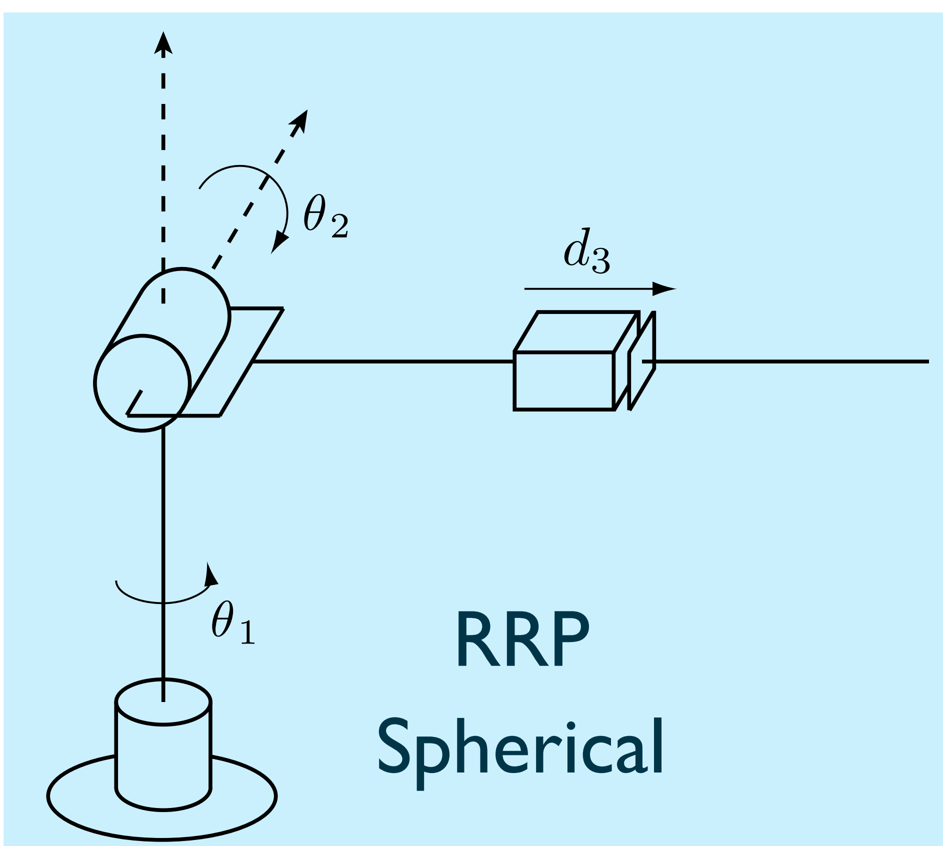


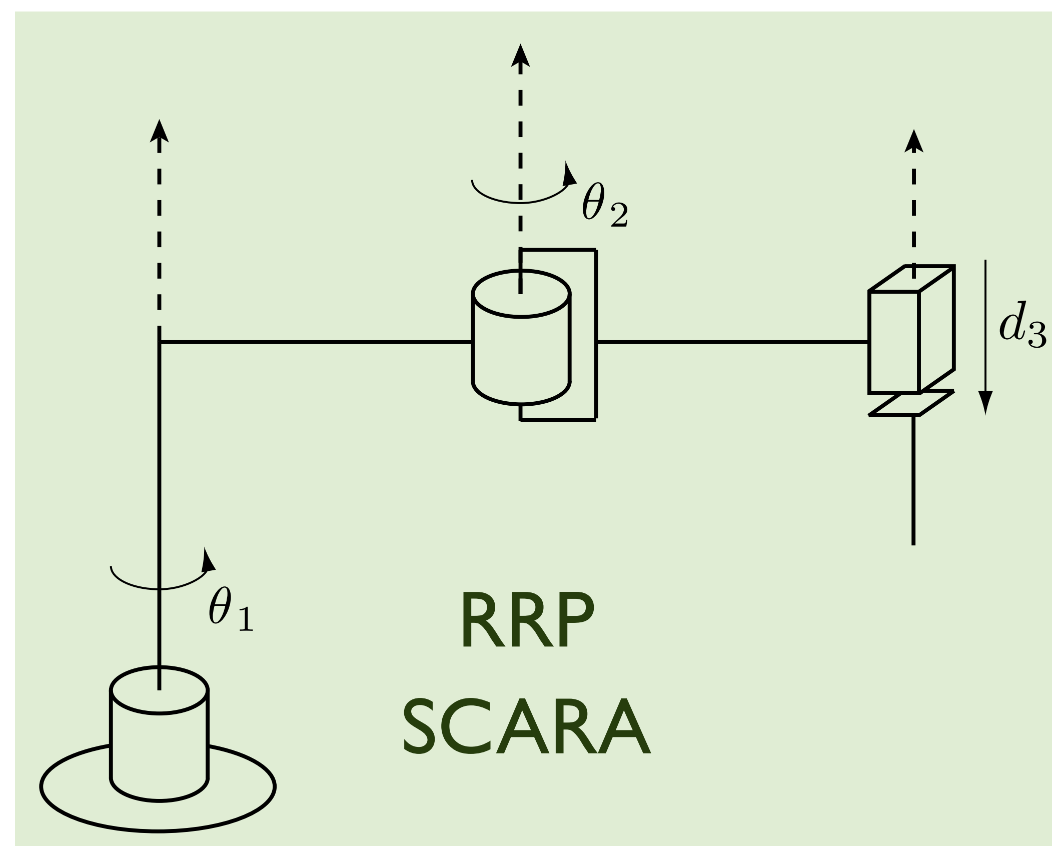
Common Manipulator Designs



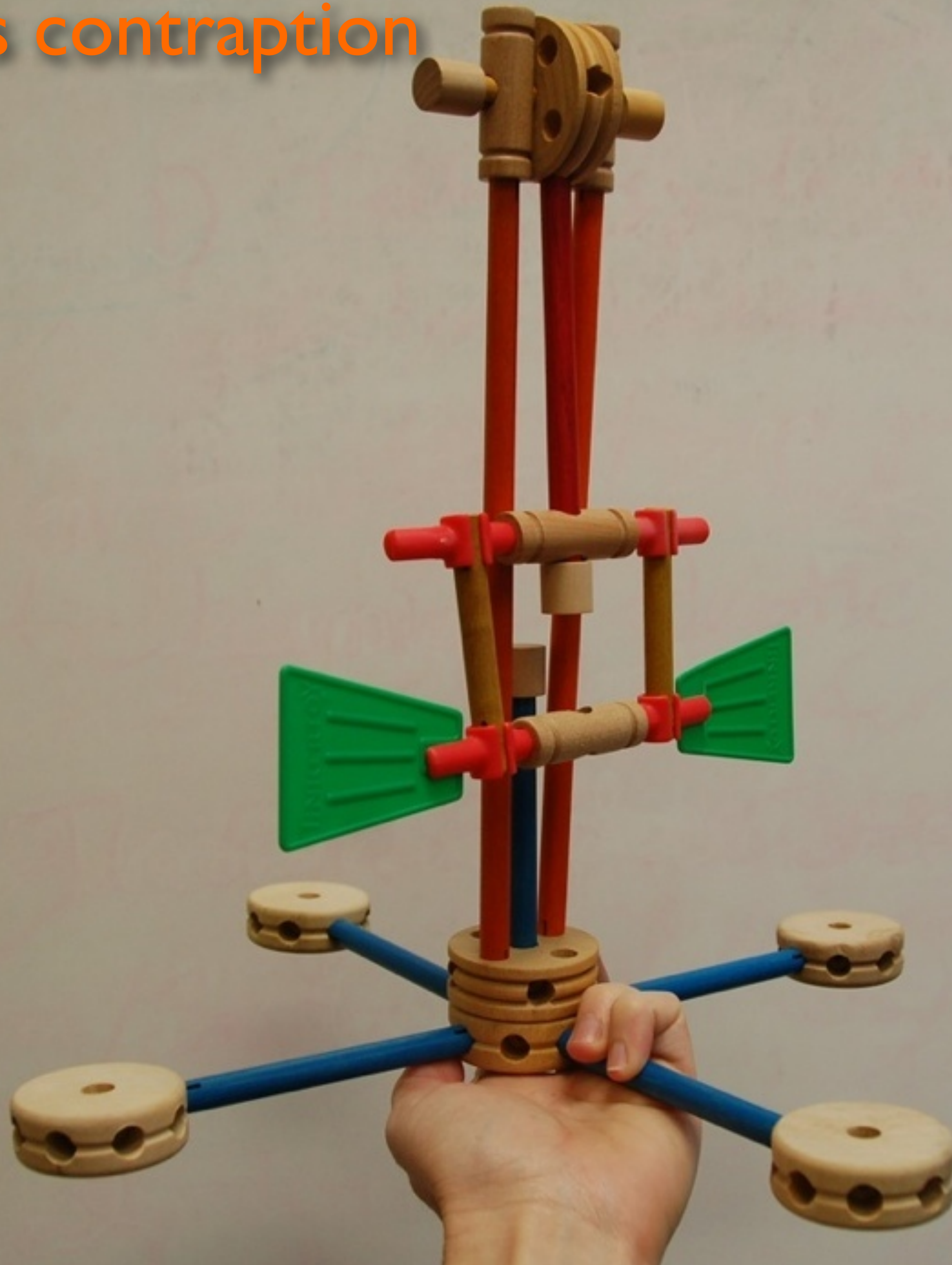




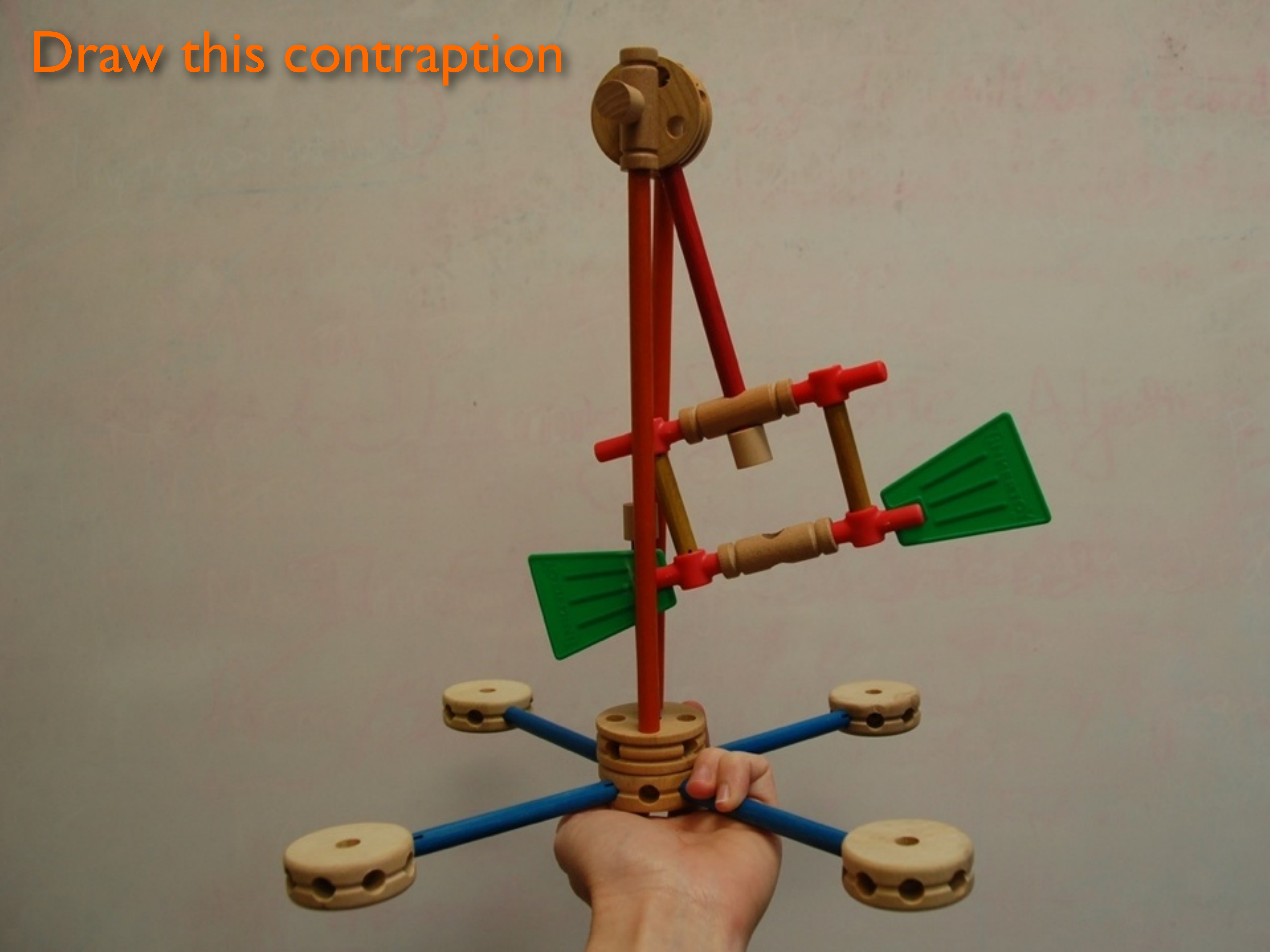




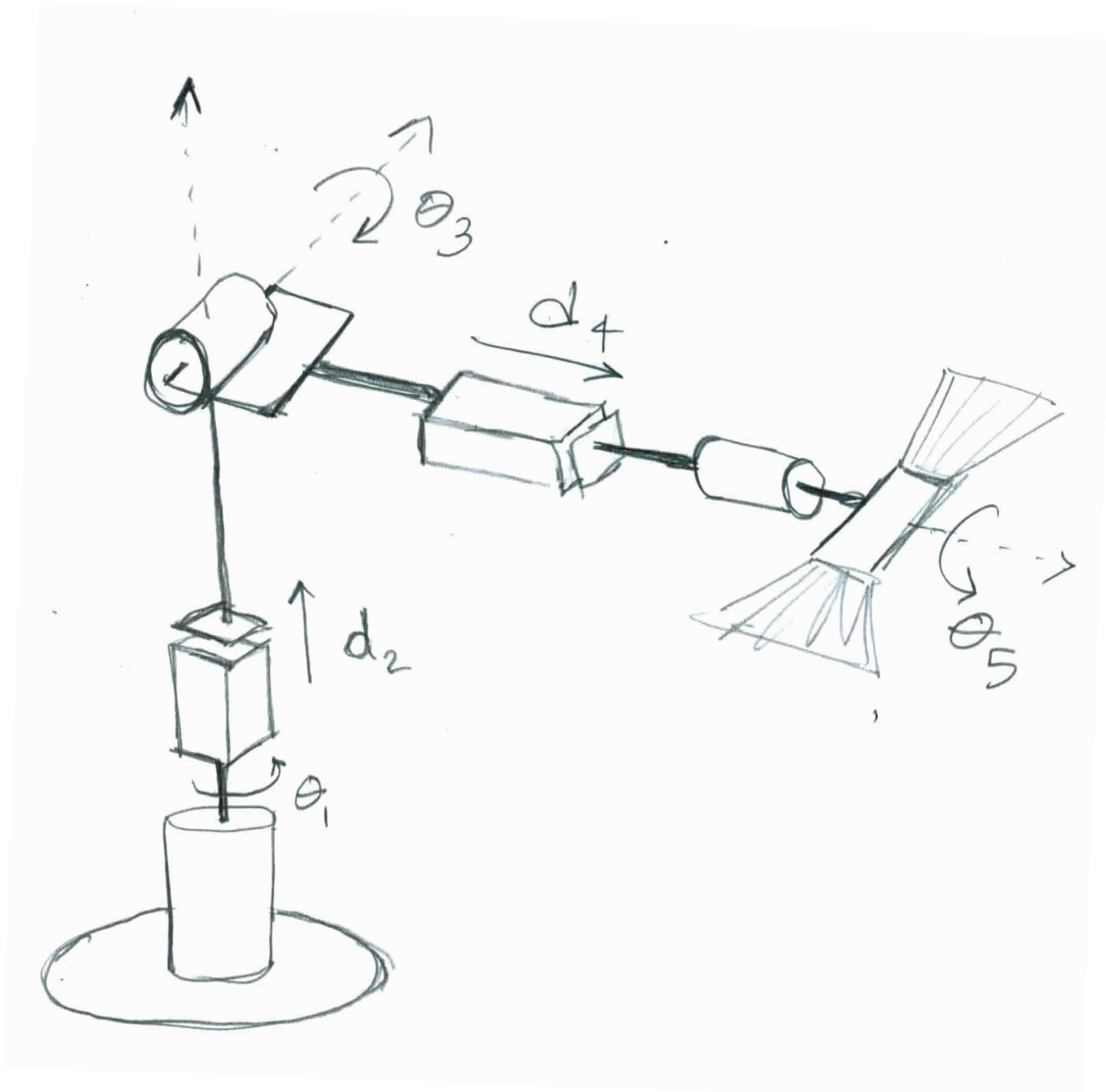
Draw this contraption



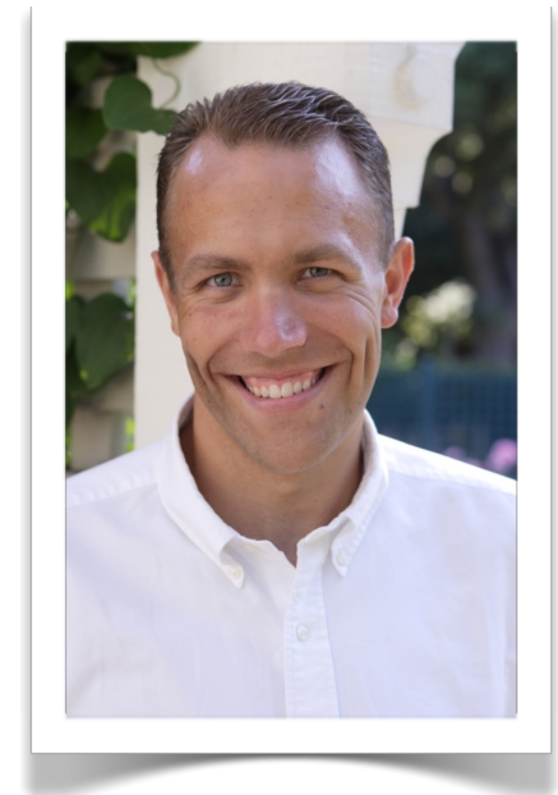
Draw this contraption



RPRPR



Questions ?

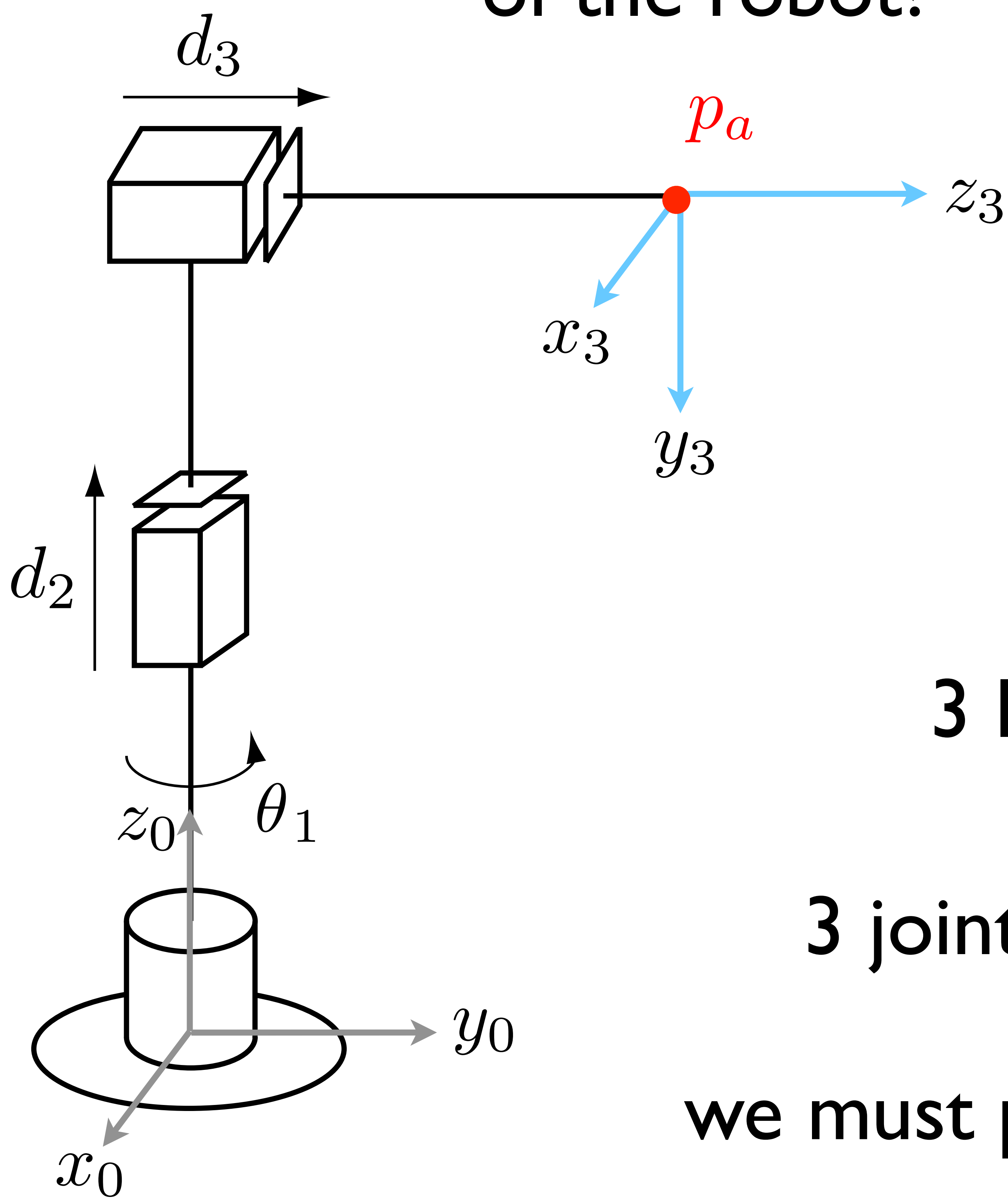


Adapted from slides created by Jonathan Fiene

Forward Kinematics

Where is the tip of the robot?

Given (q_1, q_2, q_3), where is the tip of the robot?



3 links plus ground

3 joints

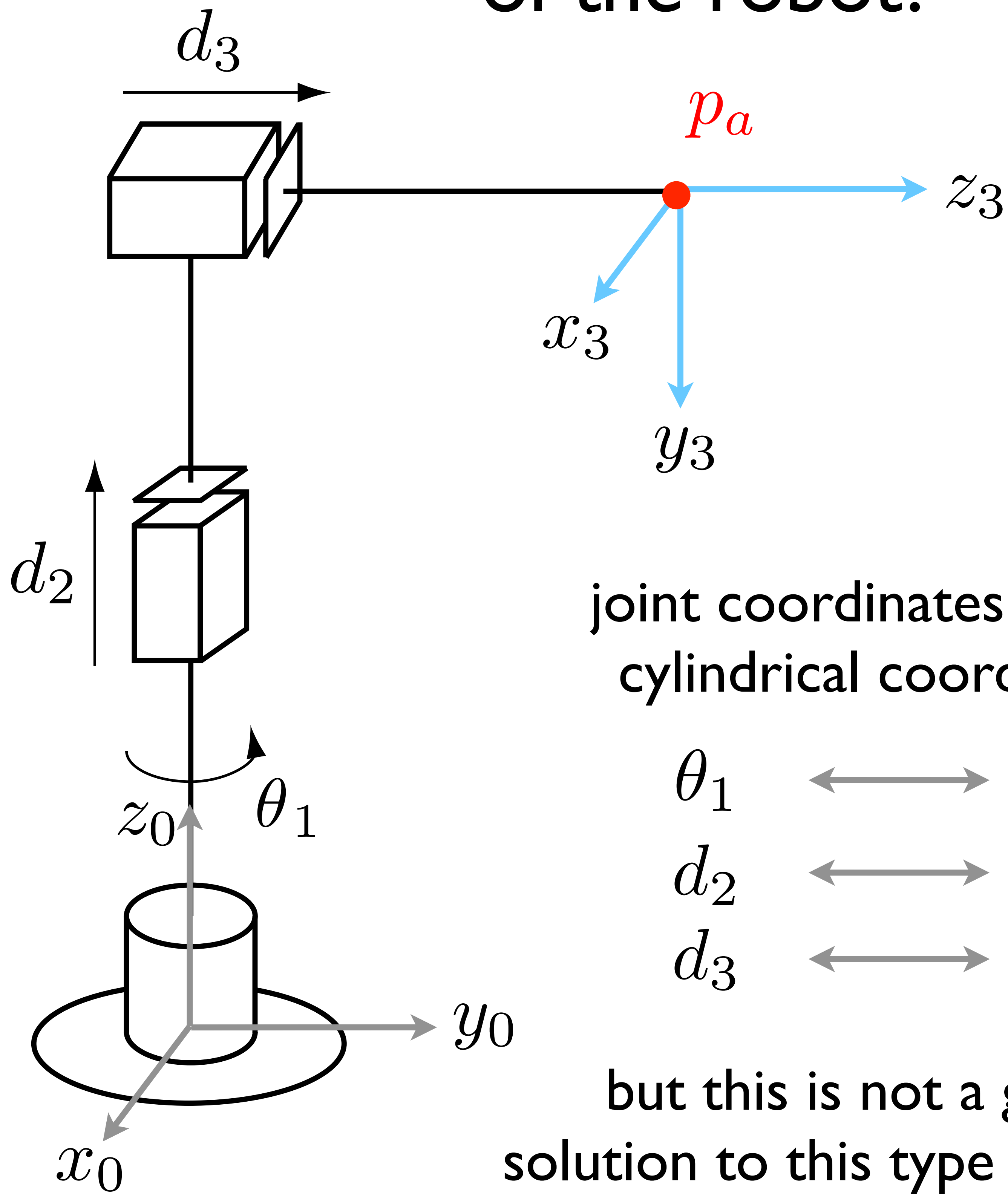
3 joint variables (q_1, q_2, q_3)

we must place coordinate frames

Forward Kinematics of the RPP Cylindrical Robot

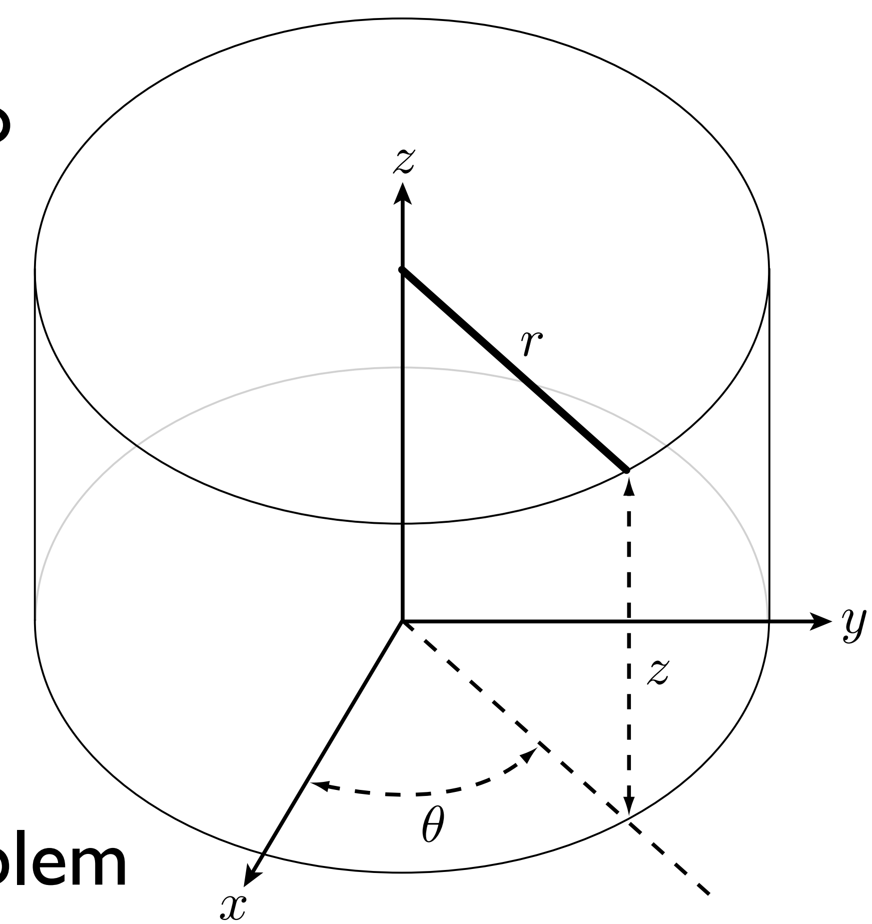
Ignore final (vertical) joint.

Given (q_1, q_2, q_3), where is the tip of the robot?



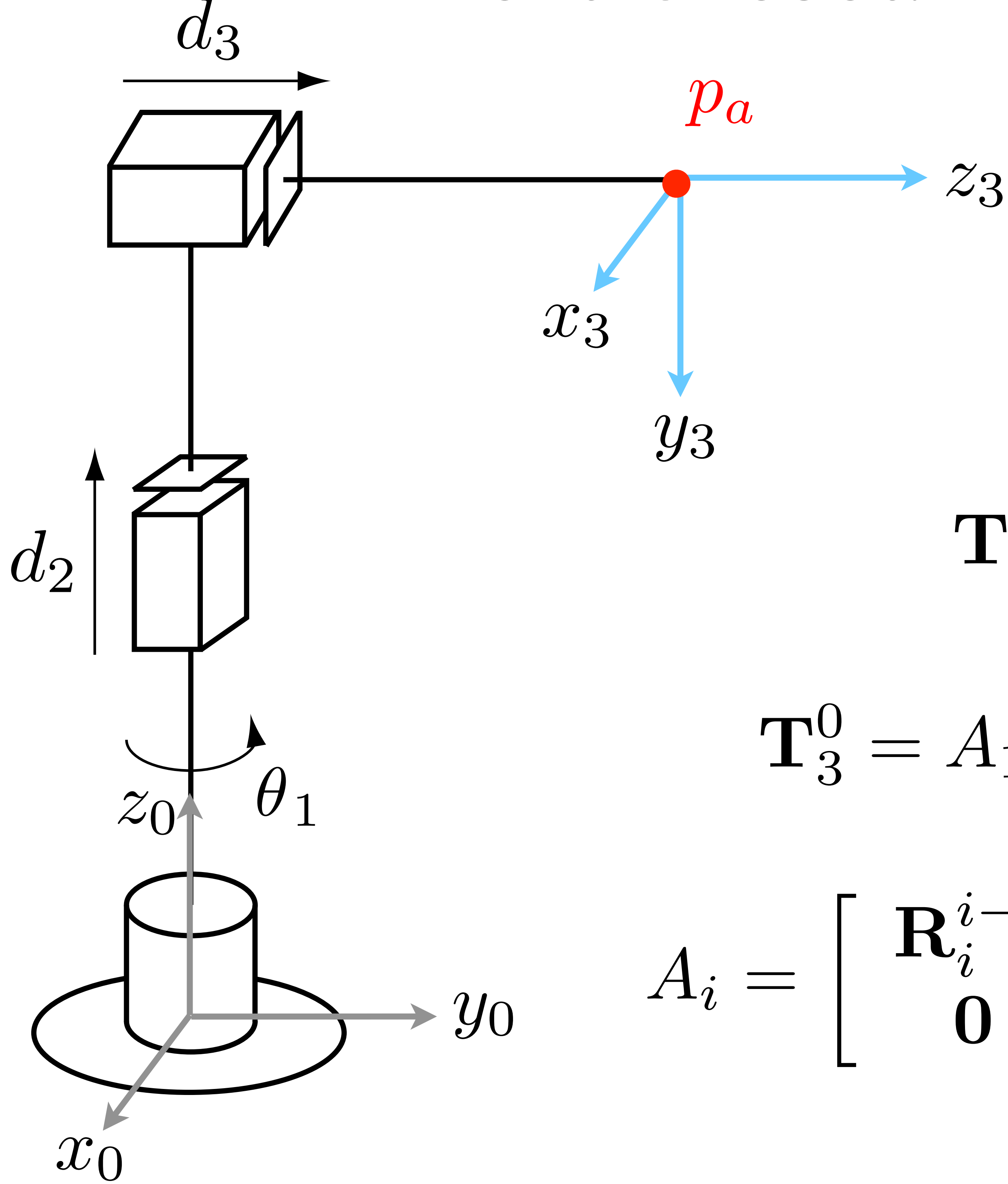
joint coordinates map to cylindrical coordinates

$$\theta_1 \longleftrightarrow \theta$$
$$d_2 \longleftrightarrow z$$
$$d_3 \longleftrightarrow r$$



but this is not a general solution to this type of problem

Given (q_1, q_2, q_3), where is the tip of the robot?



$$\mathbf{T}_3^0 = \ ?$$

$$\mathbf{T}_3^0 = A_1(q_1)A_2(q_2)A_3(q_3)$$

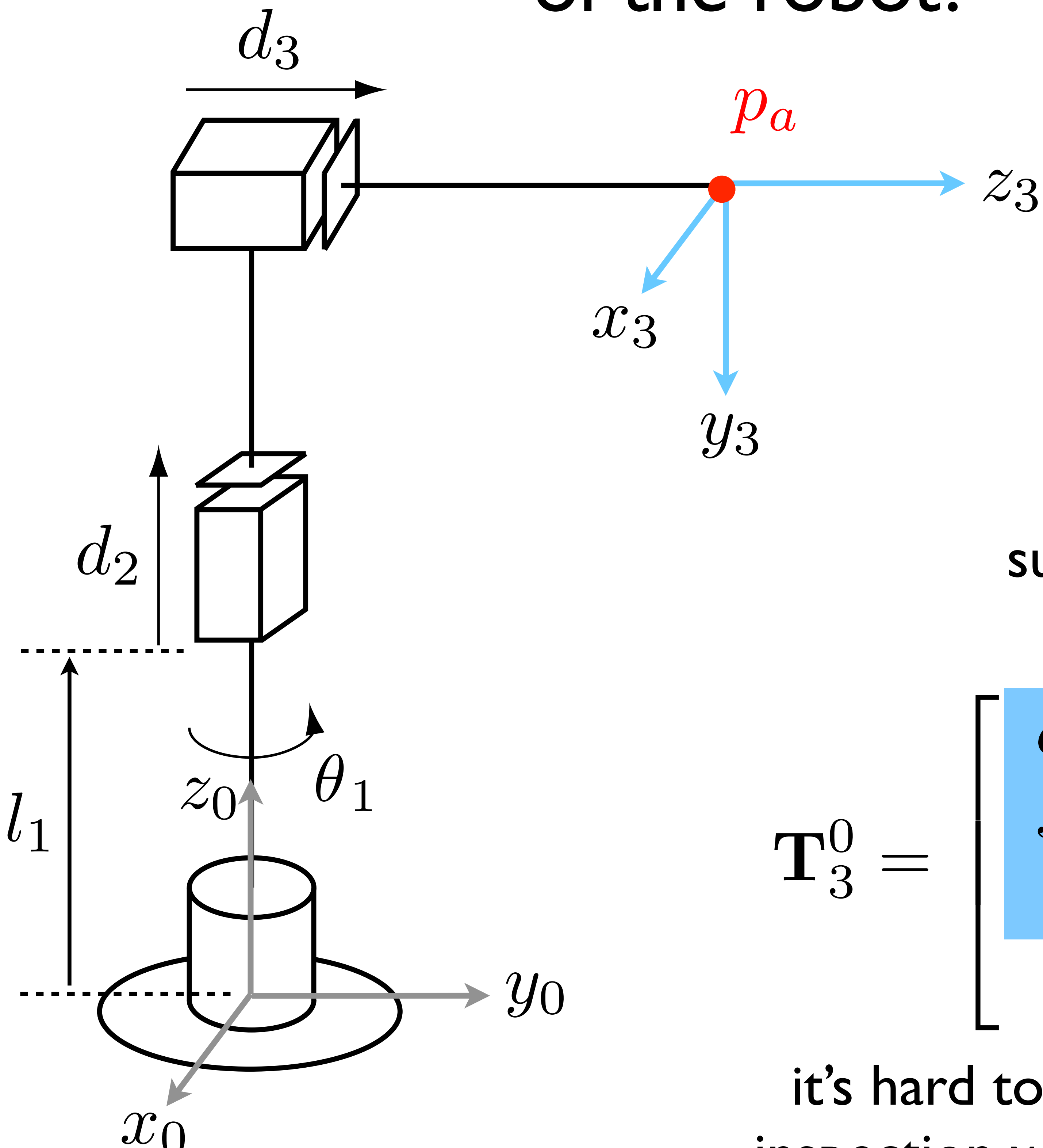
$$A_i = \begin{bmatrix} \mathbf{R}_i^{i-1} & \mathbf{d}_i^{i-1} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{P}_a^0 = \mathbf{T}_3^0 \mathbf{P}_a^3$$

Forward Kinematics of the RPP Cylindrical Robot

Ignore final (vertical) joint.

Given (q1, q2, q3), where is the tip of the robot?



superscript * marks joint angles, which vary over time

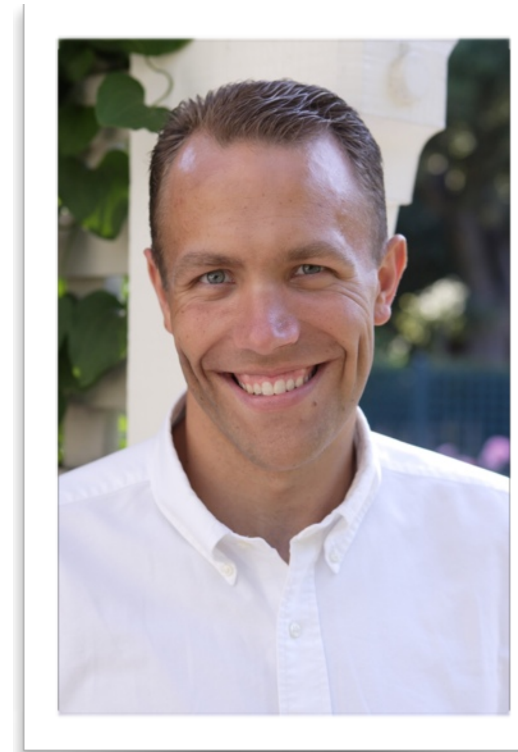
$$
\mathbf{T}_3^0 = \begin{bmatrix} c_1^* & 0 & -s_1^* & -d_3^* s_1^* \\ s_1^* & 0 & c_1^* & d_3^* c_1^* \\ 0 & -1 & 0 & d_2^* + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

it's hard to write transformation matrices by inspection when the robot is more complicated

This is the general idea of forward kinematics for manipulators.

There are many choices one must make regarding placement of intermediate frames, which means there are many equally good ways to reach the same final solution.

The robotics community has agreed on a set of conventions to ensure uniformity.



Slides created by
Jonathan Fiene

Denavit-Hartenberg Parameters (DH)

The **Denavit-Hartenberg convention** defines four parameters and some rules to help characterize arbitrary kinematic chains

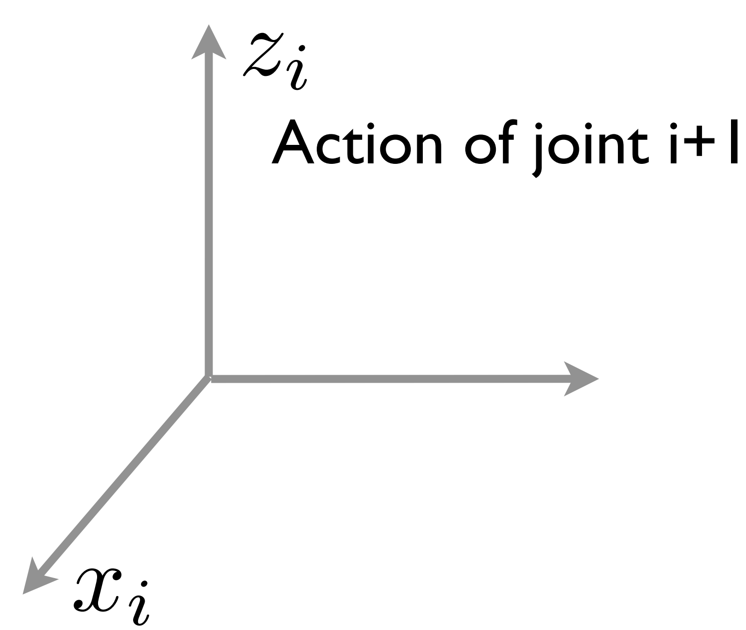
Start by drawing a schematic of the robot in its zero pose.

Then attach one frame to each link:

the joint variable for joint i+1 acts along/around z_i

the orientation of z_i determines the joint angle's positive direction

the axis x_i is perpendicular to, and intersects, z_{i-1}



Takes you from previous (i-1) frame to this (i) frame

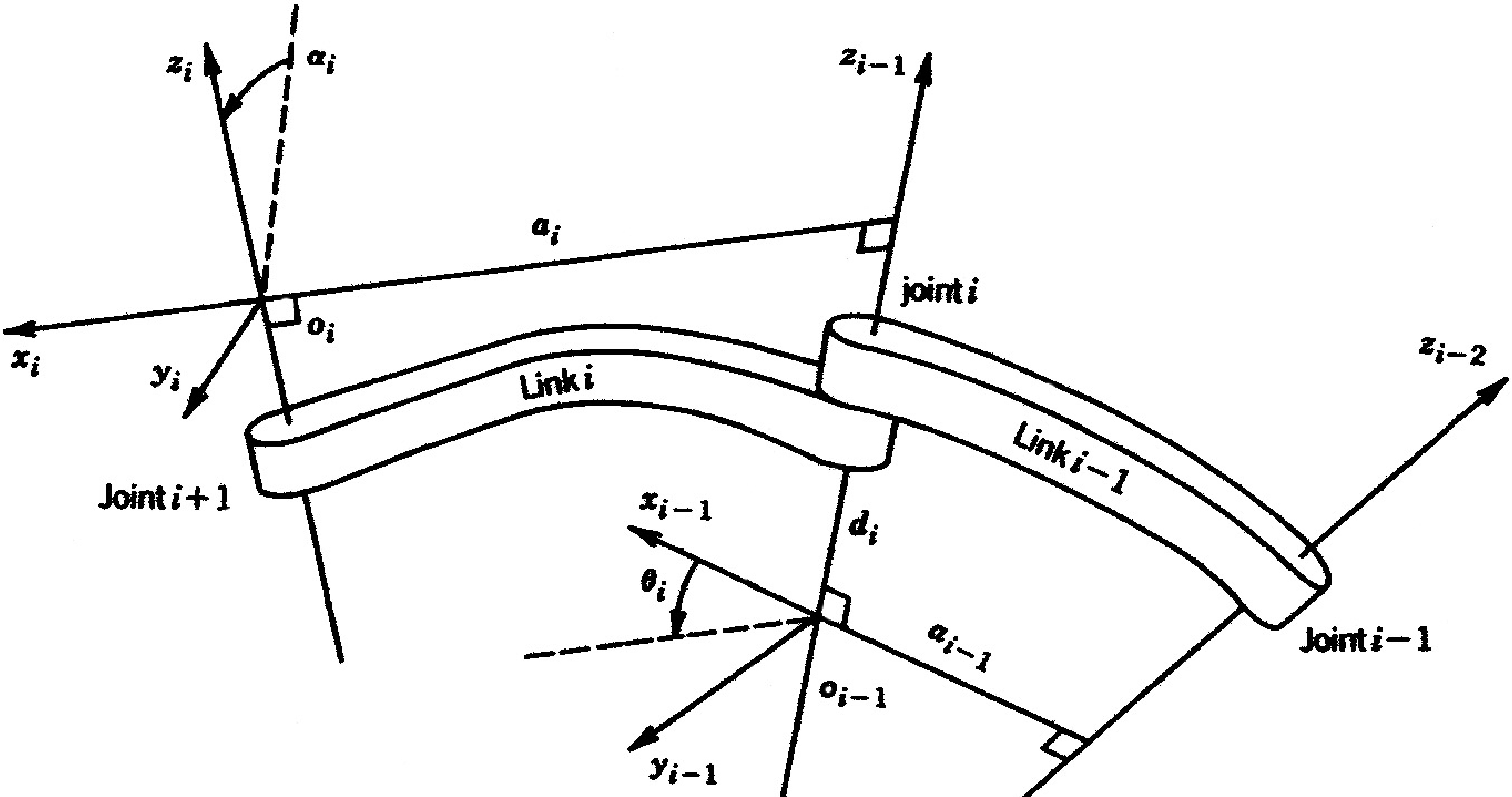
Must also choose a location for the base (0) frame:

Origin must be on z_0.

x_0 and y_0 are chosen for convenience.

Denavit & Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," ASME Journal of Applied Mechanics, June 1955

Iterative process of defining frame i using frame i-1



The **Denavit-Hartenberg convention** defines four parameters and some rules to help characterize arbitrary kinematic chains (see page 80 for parameter definitions)

DH parameters are usually written in this order, but I prefer the opposite order.

---

a_i

Link Length

the distance between z_{i-1} and z_i, measured along x_i

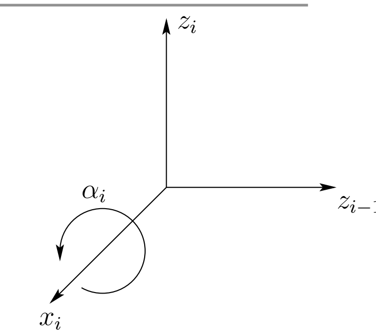
To be continued....

α_i

Link Twist

the angle between z_{i-1} and z_i , measured in the plane normal to x_i

(right-hand rule around x_i)



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d_i

Link Offset

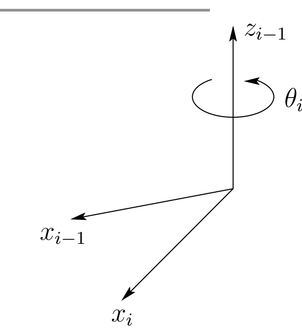
the distance between x_{i-1} and x_i, measured along z_{i-1}

θ_i

Joint Angle

the angle between x_{i-1} and x_i, measured in the plane normal to z_{i-1}

(right-hand rule around z_{i-1})



Denavit & Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," ASME Journal of Applied Mechanics, June 1955

Homework 3:
Flying Box

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

September 12, 2013

This assignment is due on **Friday, September 20, by midnight (11:59:59 p.m.)** Your code should be submitted via email according to the instructions at the end of this document. Late submissions will be accepted until Sunday, September 22, by midnight (11:59:59 p.m.), but they will be penalized by 10% for each partial or full day late, up to 20%. After the late deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down should be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. If you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

You may do this assignment either individually or with a partner, according to your personal preference. Read the assignment to decide which option is right for you. If you do this homework with a partner, you may work with anyone you choose, even someone with substantial MATLAB experience. If you are looking for a partner, consider using the "Search for Teammates!" tool on Piazza.

If you are in a pair, you should work closely with your partner throughout this assignment, following the paradigm of pair programming. You will turn in one MATLAB script for which you are both jointly responsible, and you will both receive the same grade. Please follow these pair programming guidelines, which were adapted from "All I really need to know about pair programming I learned in kindergarten," by Williams and Kessler, *Communications of the ACM*, May 2000:

- Start with a good attitude, setting aside any skepticism and expecting to jell with your partner.
- Don't start writing code alone. Arrange a meeting with your partner as soon as you can.
- Use just one computer, and sit side by side; a desktop computer with a large monitor is better for this than a laptop. Make sure both partners can see the screen.
- At each instant, one partner should be driving (using the mouse and keyboard or recording design ideas) while the other is continuously reviewing the work (thinking and making suggestions).
- Change driving/reviewing roles at least every thirty minutes, *even if one partner is much more experienced than the other.* You may want to set a timer to help you remember to switch.
- If you notice a bug in the code your partner is typing, wait until they finish the line to correct them.
- Stay focused and on-task the whole time you are working together.
- Recognize that pair programming usually takes more effort than programming alone, but it produces better code, deeper learning, and a more positive experience for the participants.
- Take a break periodically to refresh your perspective.
- Share responsibility for your project; avoid blaming either partner for challenges you run into.

Homework 3

Due Friday 9/20
by midnight.

Late deadline is Sunday
9/22 at midnight.

Done individually or in
pairs – your choice.