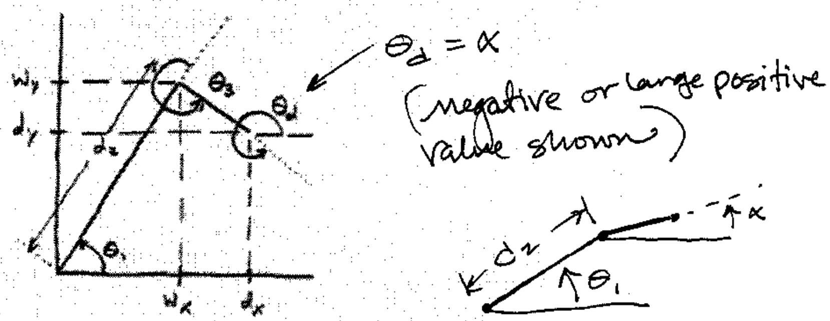

1. Given a desired position $d = [d_x, d_y]^T$ of the end-effector only, we can write the coordinates of the end-effector as two equations in three unknowns.

$$\begin{aligned} d_x &= d_2\cos(\theta_1) + a_3\cos(\theta_1+\theta_3) \\ d_y &= d_2\sin(\theta_1) + a_3\sin(\theta_1+\theta_3) \end{aligned}$$

Therefore, this problem is underconstrained. In general, there are infinitely many solutions to the inverse kinematics problem. More specifically,

$$\text{there are} \begin{cases} \infty & \text{solutions if } d \text{ is inside workspace} \\ 1 & \text{solution if } d \text{ is on workspace boundary} \\ 0 & \text{solutions if } d \text{ is outside workspace.} \end{cases}$$

2. Given a desired position $d = [d_x, d_y]^T$ and orientation θ_d of the end-effector, we can write the coordinates of the wrist center $[w_x, w_y]^T$

$$\begin{aligned} w_x &= d_x - a_3\cos(\theta_d) \\ w_y &= d_y - a_3\sin(\theta_d). \end{aligned}$$

Now we have reduced the problem to finding a solution for the first two links that will reach the wrist center. Solving the geometric problem, we find

$$\begin{aligned} \theta_1 &= \text{Atan2}(w_x, w_y) \\ d_2 &= \sqrt{(w_x^2 + w_y^2)} \\ \theta_3 &= \theta_d - \theta_1 \end{aligned}$$

atan2 $\left(\frac{w_y}{w_x} \right)$ numerator denominator.

$$\text{There are} \begin{cases} \infty & \text{solutions if the wrist center is the origin} \\ 1 & \text{solution if the wrist center is on or inside the 2-link workspace boundary} \\ 0 & \text{solutions if the wrist center is outside the 2-link workspace.} \end{cases}$$