Name ______________________________

Resubmission of Midterm Exam

MEAM 520, Introduction to Robotics
University of Pennsylvania
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November 8, 2013

If you doubt the correctness of any of the answers you submitted on the in-class midterm, you may rework those problems to try to earn back a proportion of the points you may have lost. You must decide whether to submit new answers for each of the five problems on the exam; if you do any lettered sub-question in a problem, you should resubmit all of them, as they are interrelated. Your final score on each problem will be a weighted average of your in-class score and your resubmission score: $s_{\text{final}} = (1-p)\, s_{\text{in-class}} + p\, s_{\text{resubmission}}$. We anticipate that this proportion p will be about 30%, but it may be raised or lowered to match overall class performance. If you do not submit new answers for a certain problem, your in-class score will be your final score. Completing this exam resubmission is completely optional.

You must do this exam resubmission independently, without talking about it with anyone else. You may use a calculator, your notes, the textbook, the Internet, and any other reference materials you find useful. However, you must not take assistance from any individual (including electronic correspondence of any kind), and you must not give assistance to other students in the class. If you accidentally discuss part of the exam with someone, do not fill out that part of the exam resubmisson. Any suspected violations of Penn's Code of Academic Integrity will be reported to the Office of Student Conduct for investigation.

This resubmission is due **by the start of class (noon) on Tuesday, November 12**. Because we will be discussing the exam in class that day, late resubmissions cannot be accepted. If you need clarification on any question, please post a private note on Piazza. When you work out each problem, please show all steps and $\boxed{\text{box your answer}}$.

	Points	Score
Problem 1	20	
Problem 2	15	
Problem 3	10	
Problem 4	15	
Problem 5	40	
Total	100	

I agree to abide by the University of Pennsylvania Code of Academic Integrity during this exam resubmission. I pledge that all work is my own and has been completed without the use of unauthorized aid or materials.

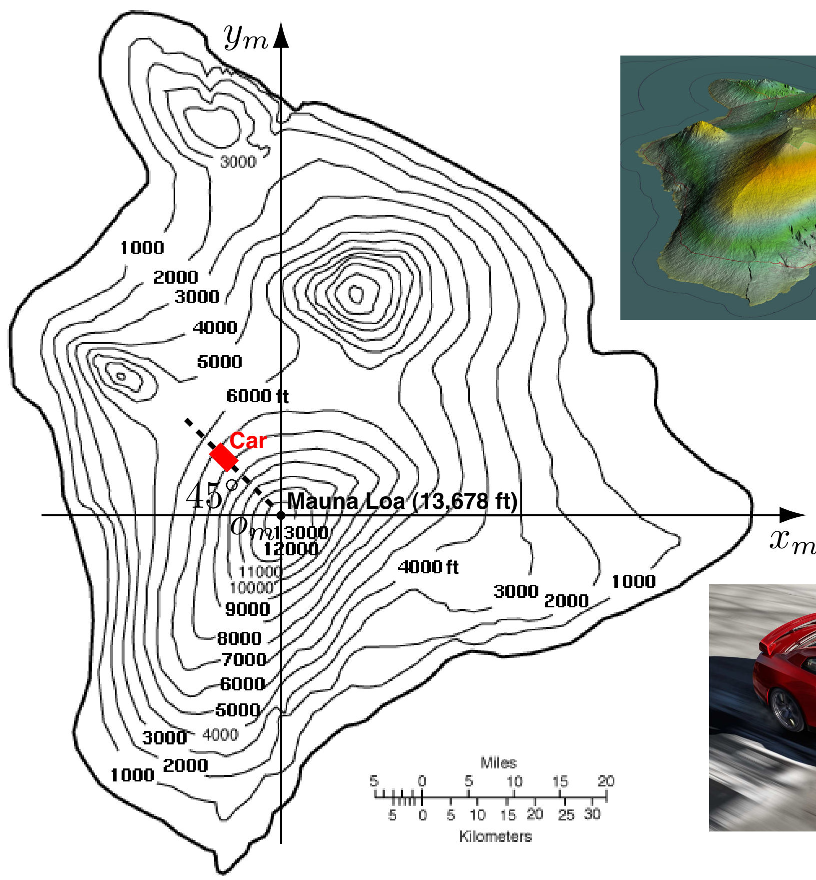
Signature ______________________________ Date ______________

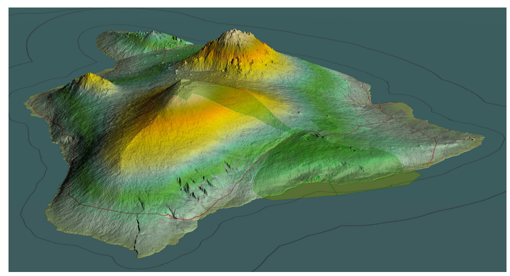
Problem 1: Homogeneous Transformations *(20 points)*

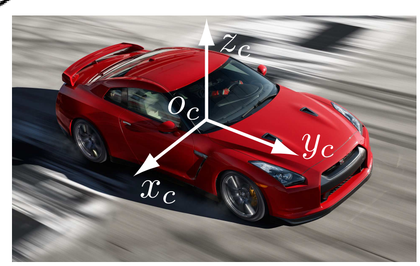
The diagram below depicts a topographic map of the island of Hawaii. As you can see in the upper inset image, this island has a tall mountain named Mauna Loa plus some other peaks.

Each contour line on the topographic map marks a line of constant elevation above sea level, which is the outer border of the island. Each successive line shows an elevation (height) increase of 1000 ft, as marked. We place o_m, the origin of our mountain coordinate frame, at the top of Mauna Loa (elevation of 13,678 ft) with the axes x_m and y_m defined as shown; z_m is positive out of the page. The x and y distance scale is below the diagram; 1000 ft $\approx$ 305 m and 1 mile = 5280 ft.

Imagine you are driving your car straight up Mauna Loa, as shown by the dashed line and the rectangle marked "Car". From overhead, your path makes a 45° angle with the negative x_m axis. We rigidly attach a coordinate frame $o_c x_c y_c z_c$ to your car, as shown in the lower right image.

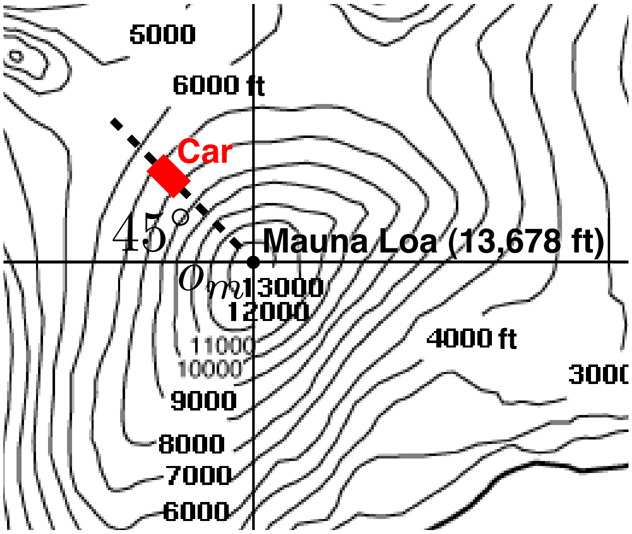


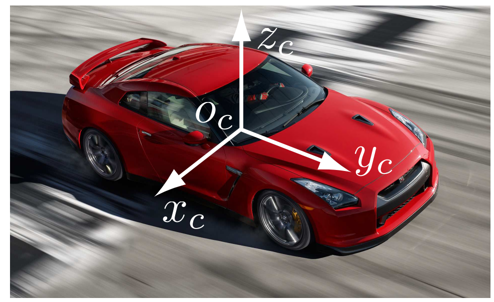




a. What is o_c^m, the **position of the car's center** expressed in the mountain frame? Please use either feet or meters for your calculations. *(6 points)*

b. What is R_c^m, the **orientation of the car's coordinate frame** expressed in the mountain frame? Clearly show how you get to your answer. *(12 points)*

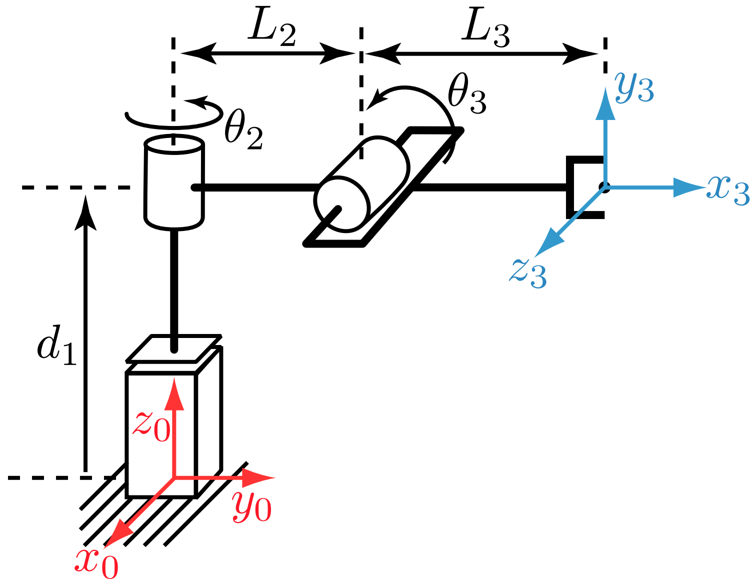




c. What is the **homogeneous transformation** H_c^m? *(2 points)*

Problem 2: DH Parameters *(15 points)*

The diagram below shows a PRR manipulator; this same robot appears in the last problem of this exam. The prismatic joint d_1 is drawn at a positive displacement, and the revolute joints θ_2 and θ_3 are drawn at zero. Positive joint directions are marked with arrows. The base frame is frame 0, and the end-effector frame is frame 3.



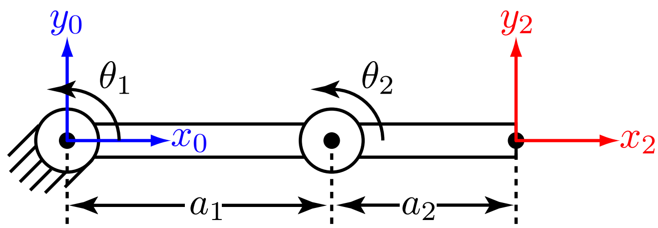
a. Follow the Denavit-Hartenberg conventions to **draw your intermediate frames** on the diagram above. *(3 points)*

b. Then fill in the table of **DH parameters** below. *(12 points)*

i	a	α	d	θ
1				
2				
3				

Problem 3: More DH Parameters *(10 points)*

The diagram below shows a planar manipulator with two revolute joints. Both θ_1 and θ_2 are measured relative to the robot's previous link; the angles are zero in the depicted pose and increase when the links are rotated in the depicted directions. The base frame and the end-effector frame are defined in the drawing, with the base frame (frame 0) being stationary and the end-effector frame (frame 2) moving with the manipulator's distal link. Both z_0 and z_2 are positive out of the page. One correct set of DH parameters is listed below.



i	a	α	d	θ
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2

a. Write **another correct set of DH parameters** for this manipulator. They should yield the same T_2^0 matrix as the parameters above but differ in a non-trivial way. *(5 points)*

i	a	α	d	θ
1				
2				

b. Write a **third correct set of DH parameters** for this manipulator. They should yield the same T_2^0 matrix but differ in a new non-trivial way from both sets above. *(5 points)*

i	a	α	d	θ
1				
2				

Problem 4: Calculating a Trajectory *(15 points)*

We seek a trajectory $d(t)$ that will move a prismatic robot joint from position $d_0 = 4$ cm at time $t_0 = 0$ s to be at position $d_f = 8$ cm at time $t_f = 2$ s. The trajectory should have zero initial and final velocity. To avoid colliding with an obstacle in the environment, this trajectory must move through an intermediate point $d_i = 0$ cm at time $t_i = 1$ s. The trajectory must be continuous in position, velocity, and acceleration at the midpoint $t_i = 1\,\mathrm{s}$.

We will create this trajectory using two different cubic polynomials. Let the trajectory be described as follows:

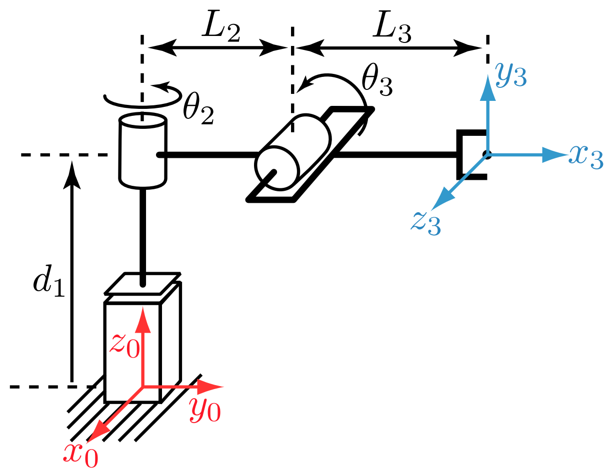
$$\text{for } 0\,\mathrm{s} \leq t \leq 1\,\mathrm{s}, \quad d(t) = \alpha(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\text{for } 1\,\mathrm{s} \leq t \leq 2\,\mathrm{s}, \quad d(t) = \beta(t) = b_0 + b_1(t - 1\,\mathrm{s}) + b_2(t - 1\,\mathrm{s})^2 + b_3(t - 1\,\mathrm{s})^3$$

a. Write down all of the **independent equations** that the unknown coefficients ($a_0, a_1, a_2, a_3, b_0, b_1, b_2, b_3$) must satisfy to obey the constraints listed above. *(8 points)*

b. Given the additional information that $a_3 = 11\,\mathrm{cm/s^3}$, $b_1 = 3\,\mathrm{cm/s}$, and $b_3 = -13\,\mathrm{cm/s^3}$, **solve for the other coefficients**. Include appropriate units. *(7 points)*

Problem 5: Velocity Kinematics and Singularities *(40 points)*



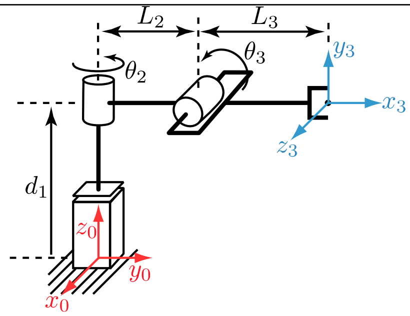
$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin\theta_2(L_2 + L_3\cos\theta_3) \\ \cos\theta_2(L_2 + L_3\cos\theta_3) \\ d_1 + L_3\sin\theta_3 \end{bmatrix}$$

$$= \begin{bmatrix} -s_2(L_2 + L_3 c_3) \\ c_2(L_2 + L_3 c_3) \\ d_1 + L_3 s_3 \end{bmatrix}$$

The diagram above shows a PRR manipulator; this same robot appears in the second problem of this exam. The prismatic joint d_1 is drawn at a positive displacement, and the revolute joints θ_2 and θ_3 are drawn at zero. Positive joint directions are marked with arrows. The end-effector frame is frame 3; the position of the end-effector's origin in frame 0 is written above.

a. What is the **angular velocity Jacobian** J_ω for this robot? *(4 points)*

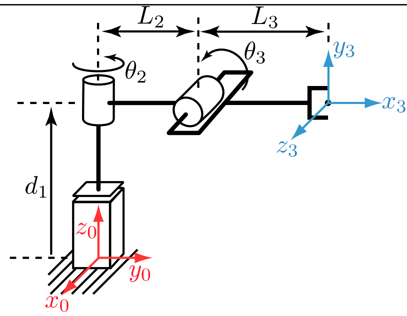
b. The robot is in the pose depicted above with $\dot{d}_1 = 0.1\,\mathrm{m/s}$, $\dot{\theta}_2 = -0.2\,\mathrm{rad/s}$, and $\dot{\theta}_3 = 0.3\,\mathrm{rad/s}$. What is $\omega_{0,3}^0$, the **angular velocity of the end-effector frame** relative to frame 0, expressed in frame 0? *(3 points)*



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -s_2(L_2 + L_3 c_3) \\ c_2(L_2 + L_3 c_3) \\ d_1 + L_3 s_3 \end{bmatrix}$$

c. For the same conditions (the pose depicted above with $\dot{d}_1 = 0.1\,\text{m/s}$, $\dot{\theta}_2 = -0.2\,\text{rad/s}$, and $\dot{\theta}_3 = 0.3\,\text{rad/s}$), what is $\frac{dR_3^0}{dt}$, the **time derivative of the rotation matrix** that represents the orientation of the end-effector frame relative to the base frame? *(6 points)*

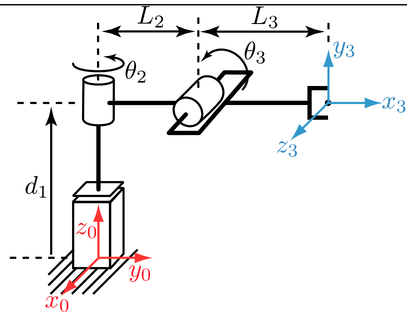
d. Calculate the **linear velocity Jacobian** J_v for this robot. *(8 points)*



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -s_2(L_2 + L_3 c_3) \\ c_2(L_2 + L_3 c_3) \\ d_1 + L_3 s_3 \end{bmatrix}$$

e. Imagine the robot is at $d_1 = 1\,\mathrm{m}$, $\theta_2 = \pi\,\mathrm{rad}$, and $\theta_3 = \pi/2\,\mathrm{rad}$. What is v_3^0, the **linear velocity of the end-effector**, in terms of $\dot{d}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$, L_2, and L_3? **Sketch** the manipulator in this configuration to check your answer graphically. *(4 points)*

f. Imagine you pre-multiplied J_v by R_0^3. **What could you calculate with the resulting matrix** $K = R_0^3 J_v$? *(3 points)*



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -s_2(L_2 + L_3 c_3) \\ c_2(L_2 + L_3 c_3) \\ d_1 + L_3 s_3 \end{bmatrix}$$

g. Use your answers from above to derive the **singular configurations** of the arm, if any, assuming $L_3 > L_2 > 0\,\text{m}$. **Sketch** the manipulator in each singular configuration that you find, and explain what effect the singularity has on the robot's motion in that configuration. *(12 points)*