MEAM 520

More Jacobians and Singularities

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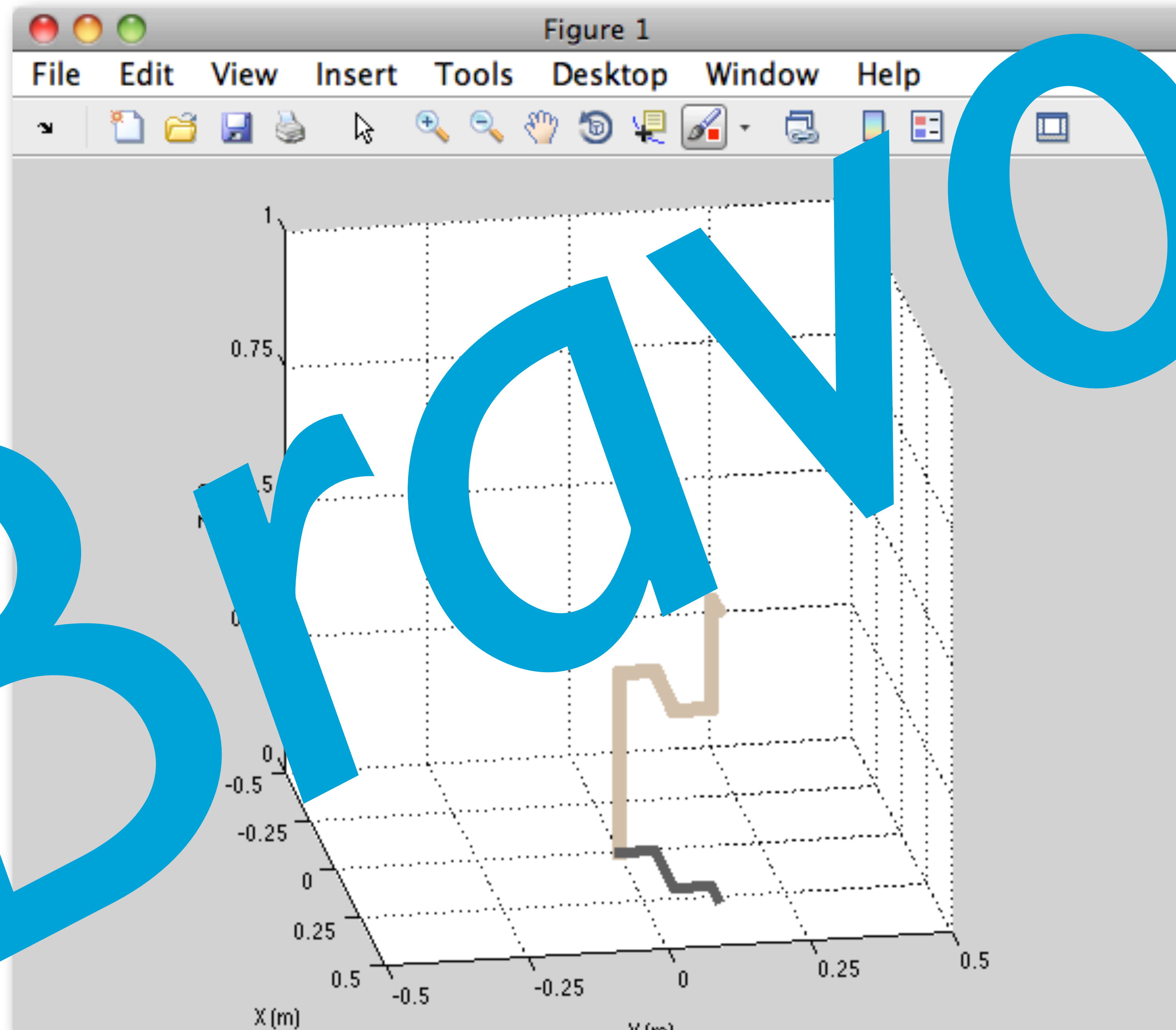
LEGES SINE MORIBUS VANAE

GRASP LABORATORY

Lecture 15: October 22, 2013

Project 1

PUMA Dance



Bravo!

- I have checked all submissions that came in before 6:30 p.m. on Sunday, and I’ll continue in chronological submission order.
- I will send you a brief email after I check yours, letting you know if there seem to be any problems with what you submitted.
- Also, thank you for sharing your comments on the project.

- The TAs and I are meeting tonight to go over all the details of how to operate the robot and the recording equipment.
- Soon we will announce a schedule of one-hour slots when a TA will help you record the real PUMA dancing your dance.
- Each slot can accommodate three teams, so you'll get to see two other dances live.
- We will post videos of all of the dances.
- Any questions or comments?

Robotics-Relevant Courses in Spring 2014

MEAM 513 / ESE 505: Control of Systems (Kothmann)

MEAM 514: Design for Manufacturability (Yim)

MEAM 613: Nonlinear Control Theory (?)

MEAM 620: Advanced Robotics (Daniilidis)

ESE 650: Learning in Robotics (Lee) (ML prereq)

ESE 680: Problems of Robotics (Koditschek)

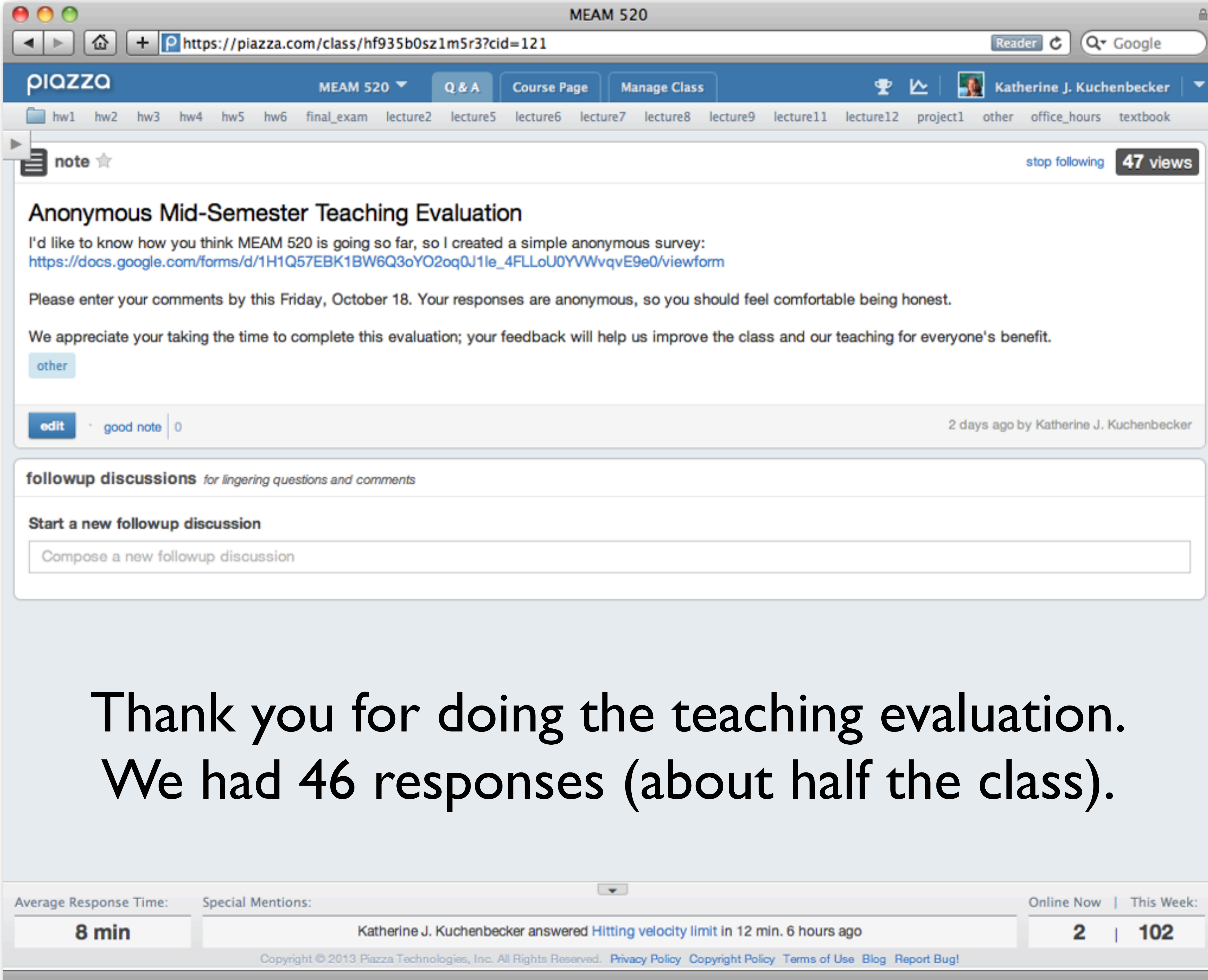
CIS 521: Artificial Intelligence (Marcus)

CIS 563: Physics Based Animation (Kavan)

BE 521: Brain-Computer Interfaces (Litt)

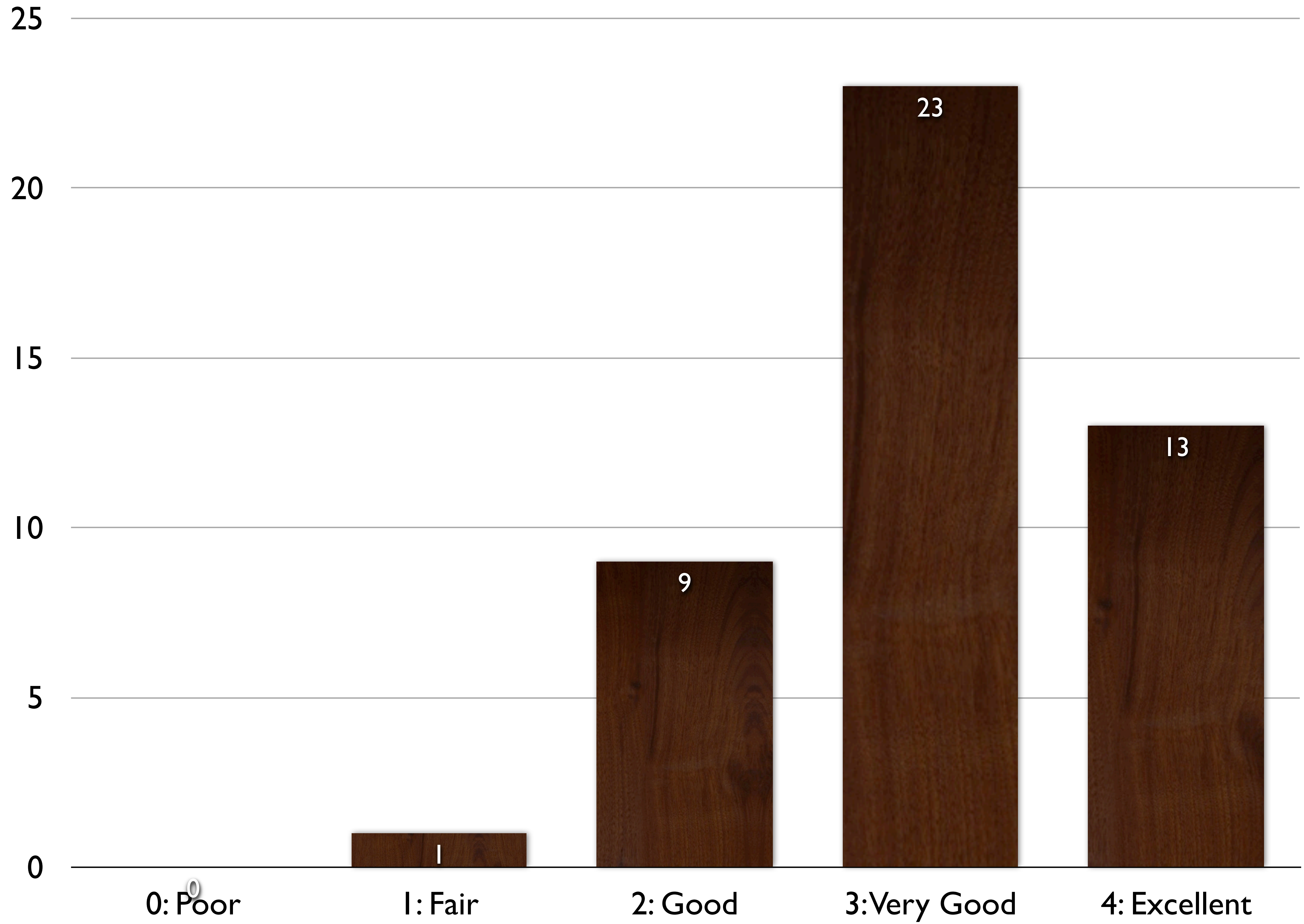
PSYC 739: Bayesian Methods in Perception (Stocker)

I am not teaching a graduate-level course in the spring.



Thank you for doing the teaching evaluation.
We had 46 responses (about half the class).

Overall Rating of MEAM 520



What is going well in this class?

Enthusiastic professor who wants students to learn.

Lectures are clear and engaging.

Concepts are explained well, especially rotation matrices.

Good ordering of material.

Slides and lecture recordings are helpful.

Homework and projects are interesting and reinforce understanding of course concepts.

Good balance of written and programming homework.

Good workload and level of difficulty.

Great TA's!

What specific things could improve this course?

Provide a calendar of upcoming activities.

Talk slower in lecture. Pause. Be calmer.

Reduce clutter on some slides.

Solve more examples and do more exercises in class.

Even out pacing of homework assignments.

Help non-mechanical students more.

Increase difficulty of assignments and course in general.

Provide less starter code.

Allow students to work alone, not in pairs.

Make grading clearer and faster.

Mistakes in textbook are annoying.

More hands-on experience with real robots.

Thank you for the feedback.

The TA's and I will use your ideas to shape the class for the rest of this semester and in the future.

MEAM 520 Calendar

Ongoing – Film PUMA Dance

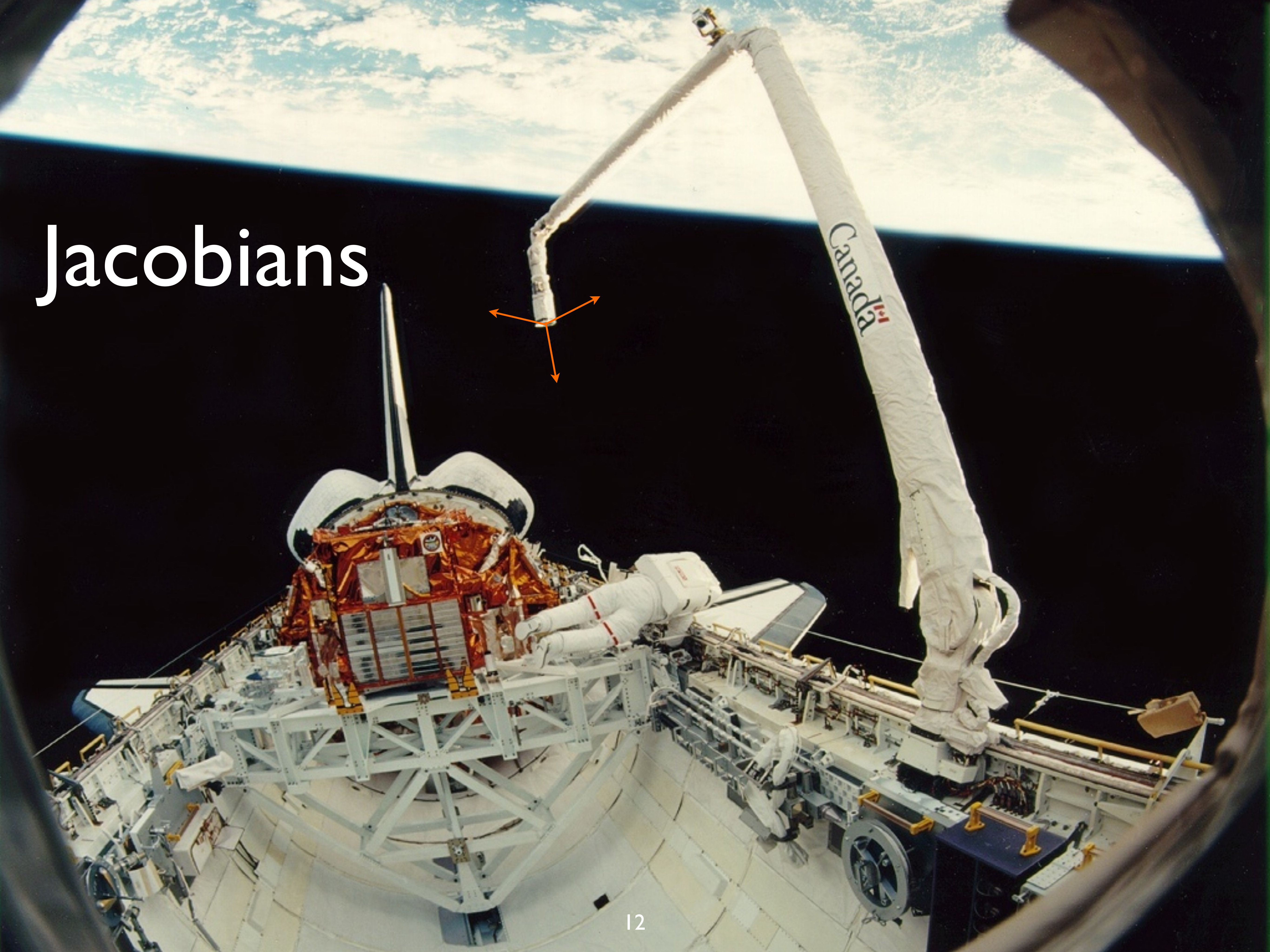
Tuesday 10/22 – Finish Chapter 4
Thursday 10/24 – Start Chapter 6
Sunday 10/27 – Homework 6 Due

Tuesday 10/29 – Chapter 6
Thursday 10/31 – Finish Chapter 6
Sunday 11/3 – Homework 7 Due

Tuesday 11/5 – Midterm Exam
Thursday 11/7 – Start Chapter 3.3 (IK)

Also planning a review session....

Jacobians



$$v_n^0 = J_v \dot{q}$$

(3 × 1) (3 × n)(n × 1)

$$\omega_n^0 = J_\omega \dot{q}$$

(3 × 1) (3 × n)(n × 1)

$$J_v(\vec{q}) = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \cdots & \frac{\partial z}{\partial q_n} \end{bmatrix}$$

$$\omega_{0,n}^0 = \sum_{i=1}^{n} \rho_i (\mathbf{R}_{i-1}^0 \hat{z})\, \dot{\theta}_i$$

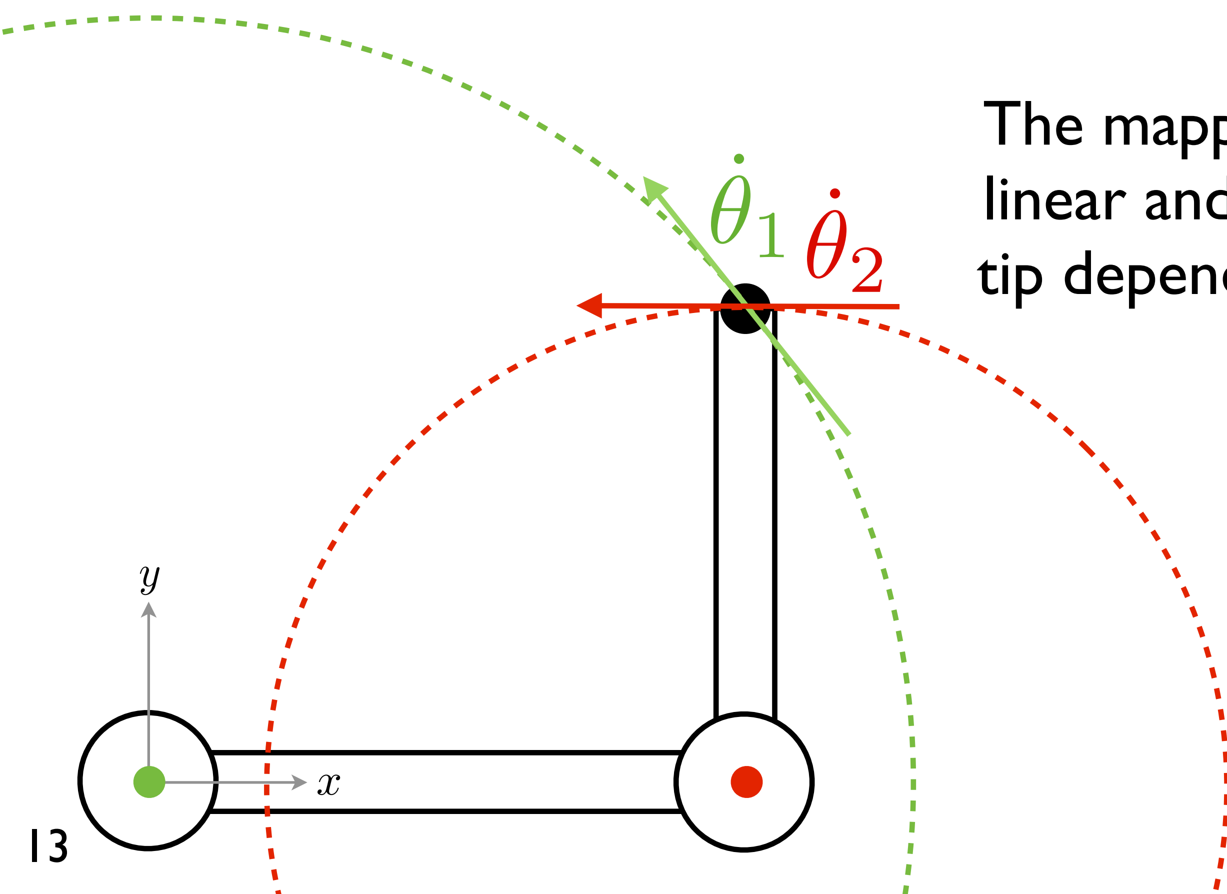
$$\rho_i = \begin{cases} 0 \text{ for prismatic} \\ 1 \text{ for revolute} \end{cases}$$

Prismatic $J_{v_i} = z_{i-1}$

Revolute $J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$

Prismatic $J_{\omega_i} = 0$

Revolute $J_{\omega_i} = z_{i-1}$



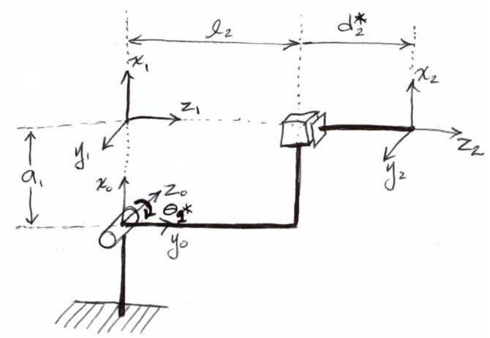
The mapping from joint velocities to the linear and angular velocity of the robot's tip depends on the robot's current pose!

Questions ?

A robot loses the ability to move its end-effector in certain directions when $\det(J_v) = 0$

Singular configurations or singularities

Two-Link Planar RP Arm With Offset (SHV 3-4)



Link	a_i	α_i	d_i	θ_i
1	a_1	-90°	0	θ_1^*
2	0	0°	$l_2 + d_2^*$	0°

$$T_2^0 = \begin{bmatrix} c_1 & 0 & -s_1 & a_1 c_1 - (l_2 + d_2^*)\, s_1 \\ s_1 & 0 & c_1 & a_1 s_1 + (l_2 + d_2^*)\, c_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

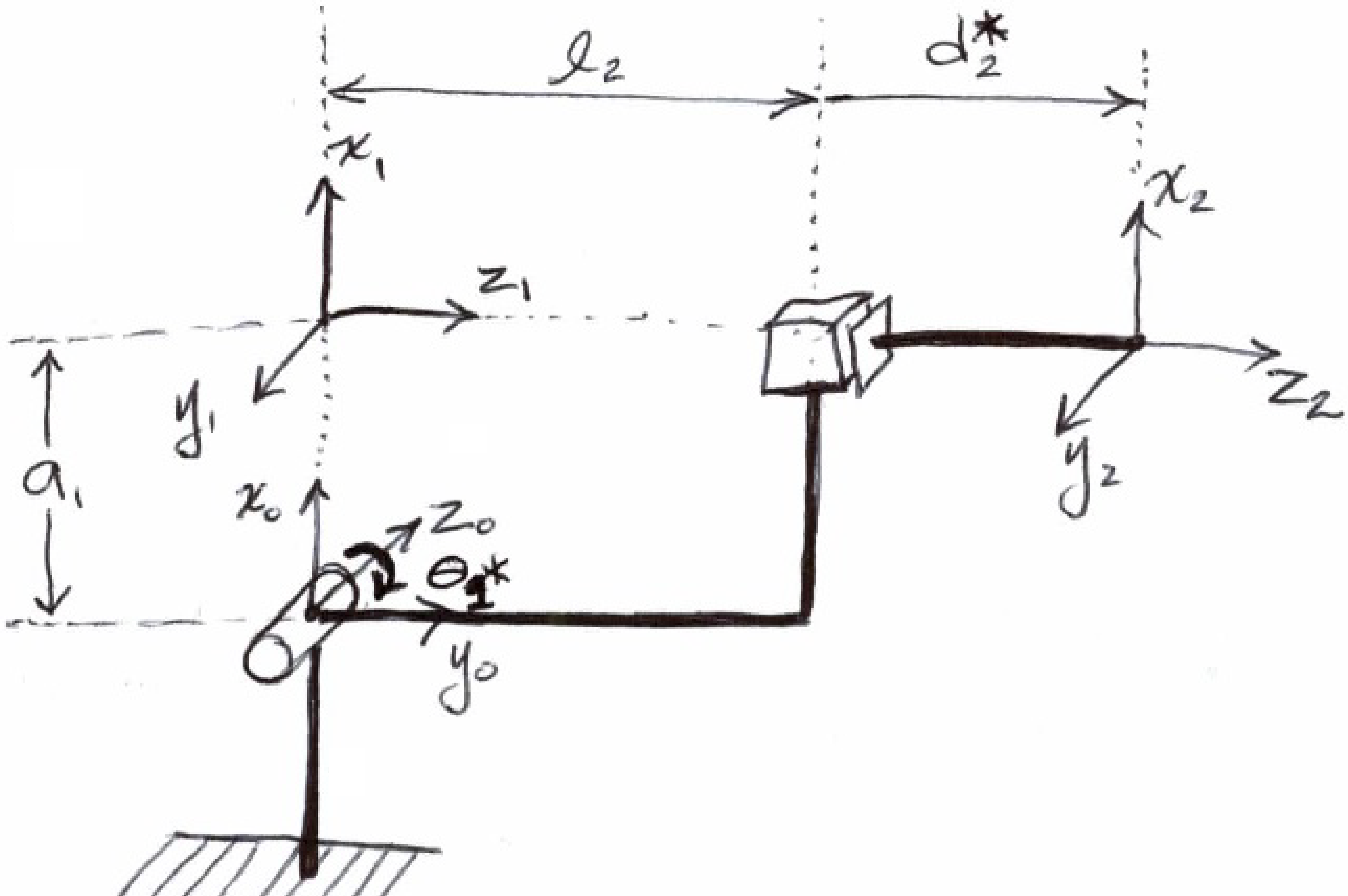
where $c_1 = \cos\theta_1^*$ and $s_1 = \sin\theta_1^*$

$J_v = ?$

$J_\omega = ?$

What are this robot's linear velocity singularities, if any?

Work with a partner.


l_2
d_2^*
x_1
z_1
y_1
a_1
x_0
z_0
θ_1^*
y_0
x_2
z_2
y_2

Today: several more topics relevant to Jacobians.

(3 x n) linear velocity Jacobian

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

(3 x n) angular velocity Jacobian

(6 x n) Jacobian

a.k.a. manipulator Jacobian

a.k.a. geometric Jacobian

$$\xi = J(q)\dot{q}$$

(n x 1) joint velocities

(6 x 1) body velocity

(6 x n) Jacobian

Notice that the body velocity is not the time derivative of a body position vector because of the angular velocity.

$$\begin{bmatrix} v_n^0 \\ \omega_n^0 \end{bmatrix} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \dot{q}$$

SHV Section 4.8 – The Analytical Jacobian

The Analytical Jacobian is an alternative to the Geometric Jacobian; it uses a different representation for orientation.

Instead of enabling you to calculate the angular velocity of the end-effector's frame, it lets you calculate the time derivatives of three values that represent the orientation of the end-effector frame.

$$\dot{X} = \begin{bmatrix} \dot{d} \\ \dot{\alpha} \end{bmatrix} = J_a(q)\dot{q}$$

Euler angles are the most commonly used minimal representation.

$$\mathbf{R} = \mathbf{R}_{z,\phi}\, \mathbf{R}_{y,\theta}\, \mathbf{R}_{z,\psi}$$

$$\alpha = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

$$\omega = \begin{bmatrix} c_\psi s_\theta & -s_\psi & 0 \\ s_\psi s_\theta & c_\psi & 0 \\ c_\theta & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = B(\alpha)\dot{\alpha}$$

$$J_a(q) = \begin{bmatrix} I & 0 \\ 0 & B^{-1}(\alpha) \end{bmatrix} J(q)$$

We won't use the analytical Jacobian in this class, but you may encounter it elsewhere.

Questions ❓

$$\xi = J(q)\dot{q}$$

It is mathematically challenging to find all of the singularities for a 6-DOF manipulator; the determinant of the Jacobian gets very complicated!

For a 6-DOF manipulator with a spherical wrist, we can decouple the determination of singular configurations into two simpler problems.

$$\xi = J(q)\dot{q}$$

$$J = [J_{\text{arm}} \mid J_{\text{wrist}}]$$

(the book calls this $J = [J_P \mid J_O]$)

$$J = [J_{\text{arm}} \mid J_{\text{wrist}}] = \left[\begin{array}{c|c} J_{11} & J_{12} \\ \hline J_{21} & J_{22} \end{array}\right]$$

$$J_{\text{wrist}} = \begin{bmatrix} z_3 \times (o_6 - o_3) & z_4 \times (o_6 - o_4) & z_5 \times (o_6 - o_5) \\ z_3 & z_4 & z_5 \end{bmatrix}$$

if we choose $o_4 = o_5 = o_6$

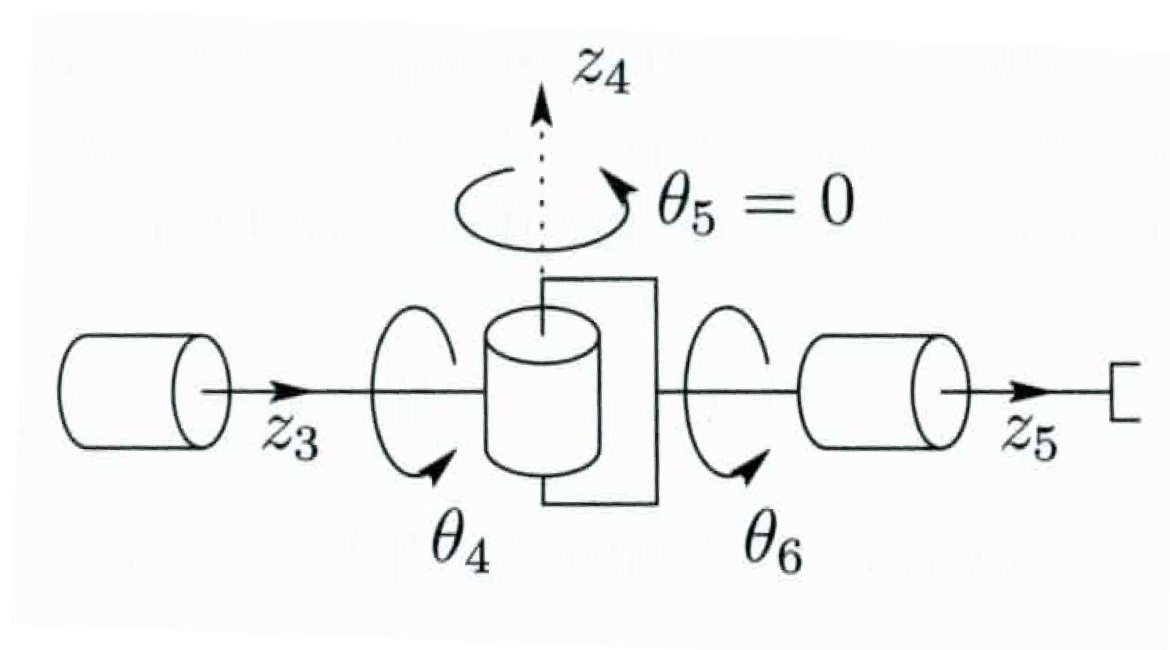
$$J_{\text{wrist}} = \begin{bmatrix} 0 & 0 & 0 \\ z_3 & z_4 & z_5 \end{bmatrix}$$

$$J = \left[\begin{array}{c|c} J_{11} & 0 \\ \hline J_{21} & J_{22} \end{array}\right] \qquad \det(J) = \underbrace{\det(J_{11})}_{\text{arm}} \, \underbrace{\det(J_{22})}_{\text{wrist}}$$

$$\det(J) = \det(J_{11})\det(J_{22})$$

$$J_{22} = \begin{bmatrix} z_3 & z_4 & z_5 \end{bmatrix}$$

Singular when any two wrist axes align



$z_3 \perp z_4$ $z_4 \perp z_5$

z_3 can become $||$ z_5

$\theta_5 = 0, \pi$ are singular configurations

Questions ?

Homework 6:
Velocity Kinematics and Jacobians

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

October 17, 2013

This paper-based assignment is due on **Sunday, October 27, by midnight (extended)** to the bin outside Professor Kuchenbecker's office, Towne 224. Late submissions will be accepted until Wednesday, October 30, by midnight (11:59:59 p.m.), but they will be penalized by 10% for each partial or full day late, up to 30%. After the late deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down must be your own work, not copied from any other individual or a solution manual. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. If you get stuck, post a question on Piazza or go to office hours!

These problems are loosely based on problems that appear in the printed version of the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV); all of the needed instructions are included in this document. Write in pencil, show your work clearly, $\boxed{\text{box your answers}}$, and staple together all pages of your assignment. This assignment is worth a total of 20 points.

1. Skew-Symmetric Matrices *(6 points)*

a. We define $\hat{u} = [x \;\; y \;\; z]^T$ to be a unit vector. What is $S(\hat{u})$, the skew-symmetric matrix associated with this unit vector?

b. Now define $\vec{v} = [0 \;\; 10 \;\; 0]^T$ and calculate $S(\hat{u})\vec{v}$.

c. What is the geometric meaning of the result you obtained in the last step? Draw a sketch with an arbitrarily chosen unit vector $\hat{u}$ to explain. Think about both the magnitude and the direction of the result.

d. Show that $S^3(\hat{u}) = -S(\hat{u})$.

e. What is the geometric meaning of the equation $S^3(\hat{u}) = -S(\hat{u})$? Explain using words and a sketch.

f. $R_{\hat{u},\theta}$ is a rotation matrix representing rotation by the time-varying angle θ about the constant unit vector $\hat{u}$. By considering equation (2.43) in the book, one can show that $R_{\hat{u},\theta} = I + S(\hat{u})\sin\theta + S^2(\hat{u})\text{vers}\,\theta$, where the versine $\text{vers}\,\theta = 1 - \cos\theta$. Note that you do not need to show this equivalence. Instead, use this equivalence and the equation from the previous step to show that $\frac{dR_{\hat{u},\theta}}{d\theta} = S(\hat{u})R_{\hat{u},\theta}$.

g. What is the intuitive meaning of the equation $\frac{dR_{\hat{u},\theta}}{d\theta} = S(\hat{u})R_{\hat{u},\theta}$?

1

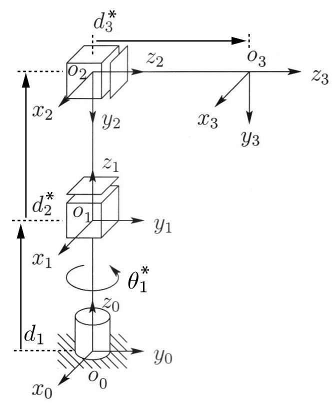
Homework 6

Written Assignment on Velocity Kinematics, Jacobians, and Singularities

Due Sunday 10/27 (extended)

Any questions?

2. Three-link Cylindrical Manipulator *(7 points)*



$$T_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Above are the DH diagram and the corresponding transformation matrix T_3^0 for a three-link cylindrical manipulator. The diagram shows θ_1 at zero, and d_2 and d_3 are shown at positive displacements. These materials are adapted from the derivation on pages 85 and 86 of the book.

a. Use the position of the end-effector in the base frame to calculate the 3×3 linear velocity Jacobian J_v for this robot.

b. Use the positions of the origins o_i and the orientations of the z-axes z_i to calculate the 3×3 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.

c. Find the 3×3 angular velocity Jacobian J_ω for the same robot.

d. Imagine this robot is at $\theta_1 = \pi/2$ rad, $d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1$ rad/s, $\dot{d}_2 = 0.25$ m/s, and $\dot{d}_3 = -0.05$ m/s. What is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

e. For the same situation, what is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

f. Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

g. Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

2

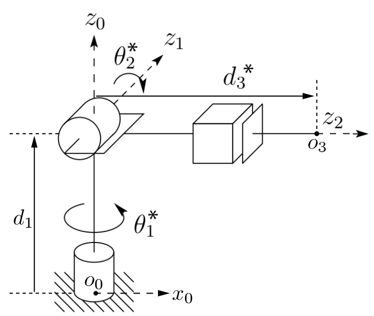
Homework 6

Written Assignment on Velocity Kinematics, Jacobians, and Singularities

Due Sunday 10/27 (extended)

Any questions?

3. Three-link Spherical Manipulator *(7 points)*



$$d_3^0 = \begin{bmatrix} c_1 c_2 d_3 \\ s_1 c_2 d_3 \\ d_1 - s_2 d_3 \end{bmatrix}$$

Above are the DH diagram and the corresponding tip position vector d_3^0 for a three-link spherical manipulator. The diagram shows θ_1 and θ_2 at zero, and d_3 is shown at a positive displacement.

a. Calculate the 3×3 linear velocity Jacobian J_v for this manipulator. You may use any method you choose.

b. Find the 3×3 angular velocity Jacobian J_ω for the same robot.

c. Imagine this robot is at $\theta_1 = \pi/4$ rad, $\theta_2 = 0$ rad, and $d_3 = 1$ m. What is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Make sure to provide units for any coefficients in these equations, if needed.

d. For the same configuration described in the previous step, what is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Provide units for any coefficients in these equations, if needed.

e. What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous steps and I want its tip to move at $v_3^0 = [0 \text{ m/s} \quad 0.5 \text{ m/s} \quad 0.1 \text{ m/s}]^T$? Make sure to provide units with your answer.

f. Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

g. Sketch the spherical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

h. Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.

3

Homework 6

Written Assignment on Velocity Kinematics, Jacobians, and Singularities

Due Sunday 10/27 (extended)

Any questions?