Homework 6:
Velocity Kinematics and Jacobians

MEAM 520, University of Pennsylvania
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This paper-based assignment is due on **Sunday, October 27, by midnight (extended)** to the bin outside Professor Kuchenbecker's office, Towne 224. Late submissions will be accepted until Wednesday, October 30, by midnight (11:59:59 p.m.), but they will be penalized by 10% for each partial or full day late, up to 30%. After the late deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down must be your own work, not copied from any other individual or a solution manual. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. If you get stuck, post a question on Piazza or go to office hours!

These problems are loosely based on problems that appear in the printed version of the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV); all of the needed instructions are included in this document. Write in pencil, show your work clearly, $\boxed{\text{box your answers}}$, and staple together all pages of your assignment. This assignment is worth a total of 20 points.

1. Skew-Symmetric Matrices *(6 points)*

a. We define $\hat{u} = [x \;\; y \;\; z]^T$ to be a unit vector. What is $S(\hat{u})$, the skew-symmetric matrix associated with this unit vector?

b. Now define $\vec{v} = [0 \;\; 10 \;\; 0]^T$ and calculate $S(\hat{u})\vec{v}$.

c. What is the geometric meaning of the result you obtained in the last step? Draw a sketch with an arbitrarily chosen unit vector $\hat{u}$ to explain. Think about both the magnitude and the direction of the result.

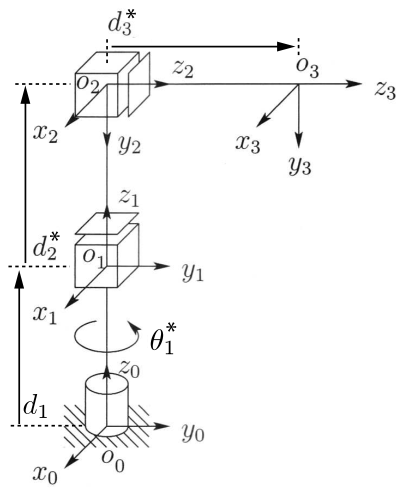
d. Show that $S^3(\hat{u}) = -S(\hat{u})$.

e. What is the geometric meaning of the equation $S^3(\hat{u}) = -S(\hat{u})$? Explain using words and a sketch.

f. $R_{\hat{u},\theta}$ is a rotation matrix representing rotation by the time-varying angle θ about the constant unit vector $\hat{u}$. By considering equation (2.43) in the book, one can show that $R_{\hat{u},\theta} = I + S(\hat{u})\sin\theta + S^2(\hat{u})\operatorname{vers}\theta$, where the versine $\operatorname{vers}\theta = 1 - \cos\theta$. Note that you do not need to show this equivalence. Instead, use this equivalence and the equation from the previous step to show that $\frac{dR_{\hat{u},\theta}}{d\theta} = S(\hat{u})R_{\hat{u},\theta}$.

g. What is the intuitive meaning of the equation $\frac{dR_{\hat{u},\theta}}{d\theta} = S(\hat{u})R_{\hat{u},\theta}$?

2. Three-link Cylindrical Manipulator *(7 points)*



$$T_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Above are the DH diagram and the corresponding transformation matrix T_3^0 for a three-link cylindrical manipulator. The diagram shows θ_1 at zero, and d_2 and d_3 are shown at positive displacements. These materials are adapted from the derivation on pages 85 and 86 of the book.

a. Use the position of the end-effector in the base frame to calculate the 3×3 linear velocity Jacobian J_v for this robot.

b. Use the positions of the origins o_i and the orientations of the z-axes z_i to calculate the 3×3 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.

c. Find the 3×3 angular velocity Jacobian J_ω for the same robot.

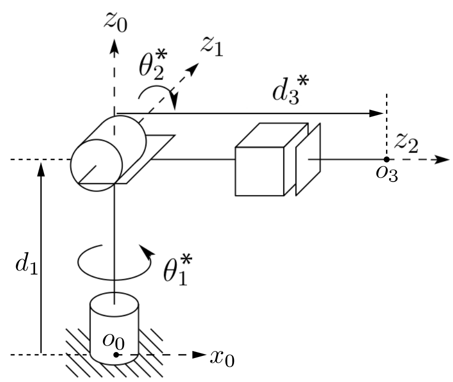
d. Imagine this robot is at $\theta_1 = \pi/2$ rad, $d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1$ rad/s, $\dot{d}_2 = 0.25$ m/s, and $\dot{d}_3 = -0.05$ m/s. What is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

e. For the same situation, what is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

f. Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

g. Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

3. Three-link Spherical Manipulator ***(7 points)***



$$d_3^0 = \begin{bmatrix} c_1 c_2 d_3 \\ s_1 c_2 d_3 \\ d_1 - s_2 d_3 \end{bmatrix}$$

Above are the DH diagram and the corresponding tip position vector d_3^0 for a three-link spherical manipulator. The diagram shows θ_1 and θ_2 at zero, and d_3 is shown at a positive displacement.

a. Calculate the 3×3 linear velocity Jacobian J_v for this manipulator. You may use any method you choose.

b. Find the 3×3 angular velocity Jacobian J_ω for the same robot.

c. Imagine this robot is at $\theta_1 = \pi/4$ rad, $\theta_2 = 0$ rad, and $d_3 = 1$ m. What is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Make sure to provide units for any coefficients in these equations, if needed.

d. For the same configuration described in the previous step, what is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Provide units for any coefficients in these equations, if needed.

e. What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous steps and I want its tip to move at $v_3^0 = [0 \text{ m/s} \quad 0.5 \text{ m/s} \quad 0.1 \text{ m/s}]^T$? Make sure to provide units with your answer.

f. Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

g. Sketch the spherical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

h. Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.