MEAM 520

More Inverse Kinematics

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General Robotics, Automation, Sensing, and Perception Lab (GRASP)

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GRASP LABORATORY

Lecture 22: November 19, 2013

Homework 8: Input/Output Calculations for the Phantom Robot

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

November 12, 2013

This assignment is due on **Tuesday, November 19, by midnight (11:59:59 p.m.)**. You should aim to turn the paper part in during class that day. If you don't finish before class, you can turn the paper part in to the bin outside Professor Kuchenbecker's office, Towne 224. Your code should be submitted via email according to the instructions at the end of this document. Late submissions will be accepted until Thursday, November 21, by midnight (11:59:59 p.m.), but they will be penalized by 10% for each partial or full day late, up to 20%. After the late deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down must be your own work, not copied from any other individual or team. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. If you get stuck, post a question on Piazza or go to office hours!

Individual vs. Pair Programming

You may do this assignment either individually or with a partner. If you do this homework with a partner, you may work with anyone in the class. If you are in a pair, you should work closely with your partner throughout this assignment, following the paradigm of pair programming. You will turn in one paper assignment and one set of MATLAB files for which you are both jointly responsible, and you will both receive the same grade. Please follow the pair programming guidelines that have been shared before, and name your files with both of your PennKeys separated by an underscore character. Do not split the assignment in half; both of you should understand all steps of this assignment.

SensAble Phantom Premium 1.0

This entire assignment is focused on a particular robot – the SensAble Phantom Premium 1.0. As shown in the photo below, the Phantom is an impedance-type haptic interface with three actuated rotational joints. Designed to be lightweight, stiff, smooth, and easily backdrivable, this type of robotic device enables a human user to interact with a virtual environment or control the movement of a remote robot through the movement of their fingertip while simultaneously feeling force feedback.

1

Homework 8

Due by midnight today.
Late by midnight Thursday.

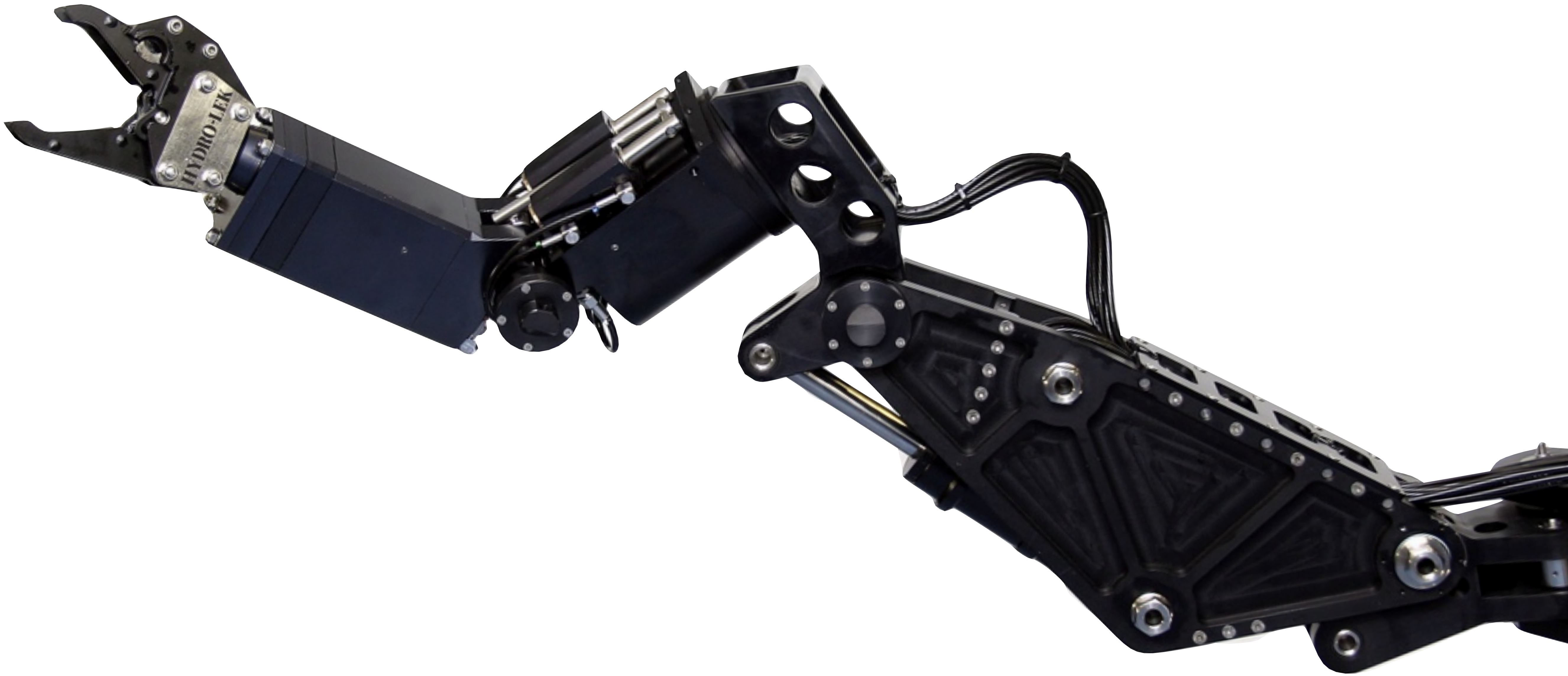
Some extensions were granted; no more requests will be considered.

Has both written and MATLAB parts.

Done individually or with any partner.

All about the SensAble Phantom Premium 1.0

Inverse Position Kinematics

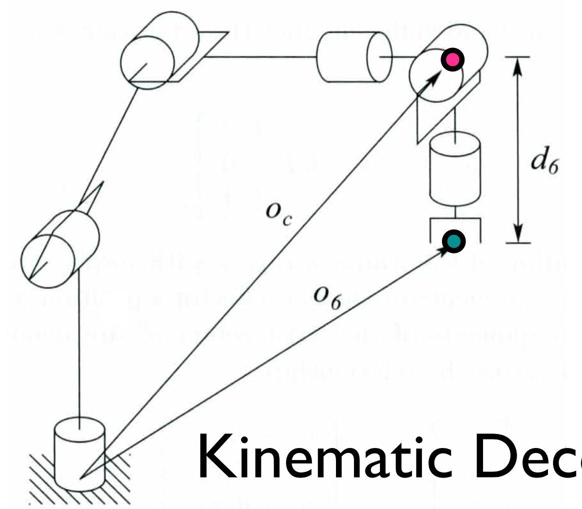


$$\text{given } \mathbf{H} = \begin{bmatrix} \mathbf{R} & o \\ 0 & 1 \end{bmatrix}$$

Inverse Position Kinematics:

finding the joint angles that cause a certain part of a robot to be at a given point in space.

$$\text{find } q_1, \ldots, q_n \text{ such that } \mathbf{T}_n^0(q_1, \ldots, q_n) = \mathbf{H}$$



Kinematic Decoupling

Algebraic Approach

1. Calculate the robot's forward kinematics.
2. If desired, apply kinematic decoupling.
3. Pull the symbolic expressions for the chosen point's position.
4. Use algebra to solve for **sets** of closed-form expressions that give the joint angles in terms of the chosen point's coordinates.

Geometric Approach

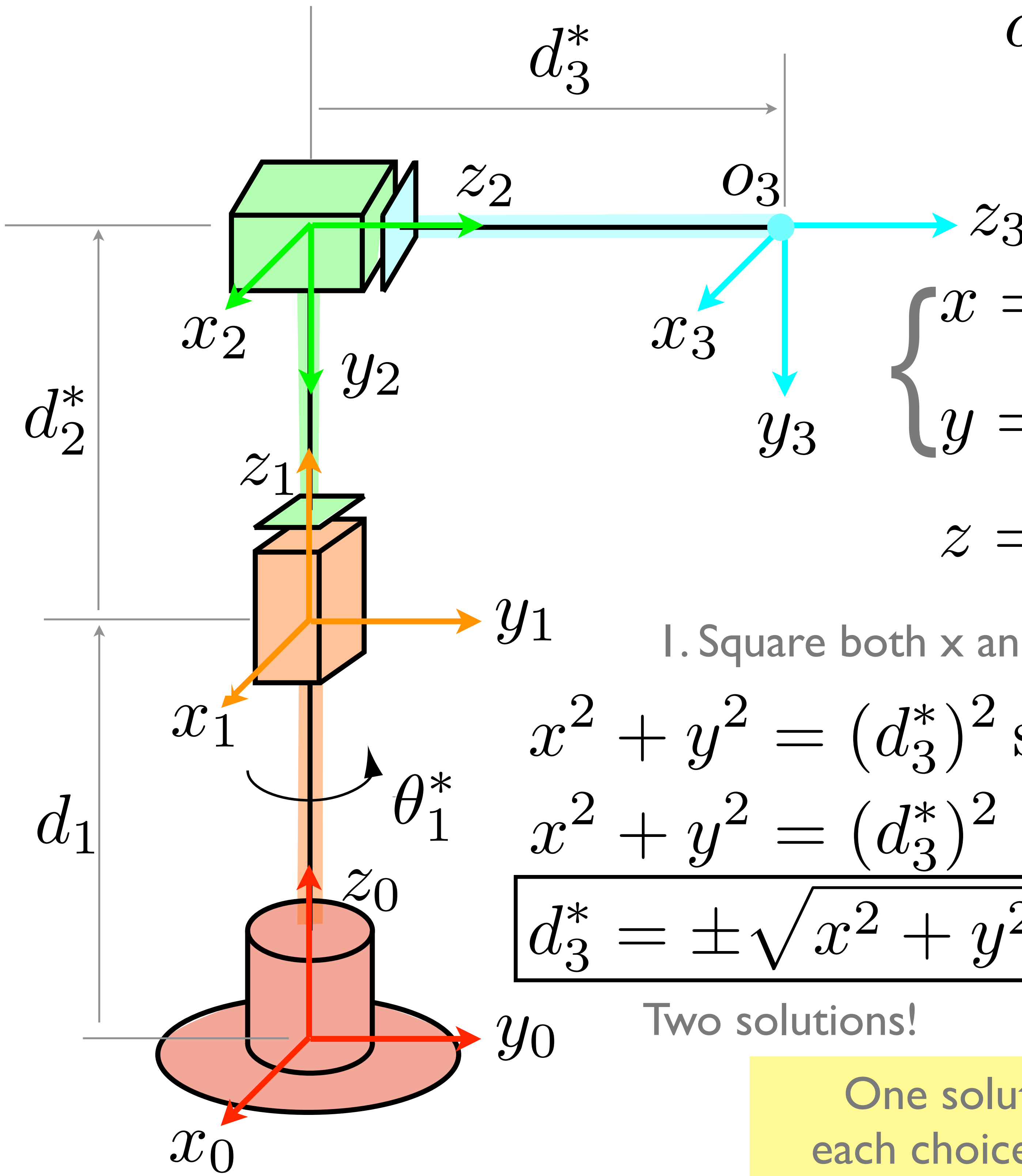
How can you check your IK?

Plug your answers into your FK.

1. If desired, apply kinematic decoupling.
2. For each revolute joint, project the robot onto the plane perpendicular to that joint's axis. For prismatic, look from side.
3. Use geometry to solve for **sets** of closed-form expressions that give the joint angles in terms of the chosen point's coordinates.

The RPP Cylindrical Robot - Algebraic Approach

This slide was corrected to show both solutions for theta1.



$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ignoring joints past RPP

$$\begin{cases} x = -d_3^* \sin(\theta_1^*) \\ y = d_3^* \cos(\theta_1^*) \end{cases}$$

$$z = d_1 + d_2^* \longrightarrow \boxed{d_2^* = z - d_1}$$

1. Square both x and y equations and add them.

$$x^2 + y^2 = (d_3^*)^2 \sin^2(\theta_1^*) + (d_3^*)^2 \cos^2(\theta_1^*)$$

$$x^2 + y^2 = (d_3^*)^2$$

$$\boxed{d_3^* = \pm\sqrt{x^2 + y^2}}$$

Two solutions!

2. Solve for sin and cos of theta1 and take atan2

$$\sin(\theta_1^*) = \frac{-x}{d_3^*} \quad \cos(\theta_1^*) = \frac{y}{d_3^*}$$

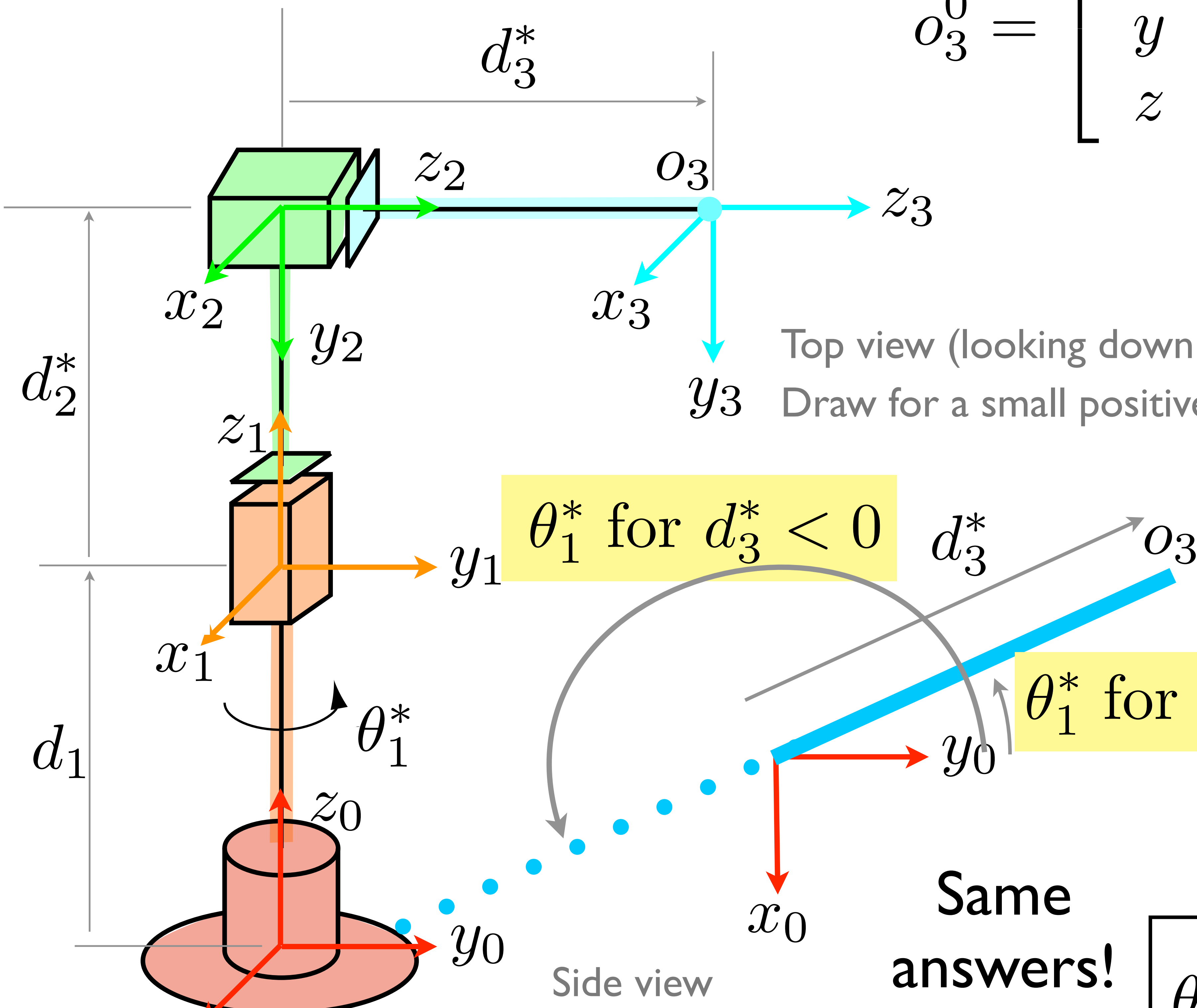
One solution for each choice of d3*

$$\boxed{\theta_1^* = \operatorname{atan2}\left(\frac{-x/d_3^*}{y/d_3^*}\right)}$$

The RPP Cylindrical Robot - Geometric Approach

What is the geometric meaning of the second theta1 solution?

$$o_3^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



Top view (looking down along z0).
Draw for a small positive angle theta1.

$$\theta_1^* = ?$$

$$d_2^* = ?$$

$$d_3^* = ?$$

$$\boxed{d_3^* = \pm\sqrt{x^2 + y^2}}$$

$$\boxed{d_2^* = z - d_1}$$

Same answers!

$$\boxed{\theta_1^* = \operatorname{atan2}\left(\frac{-x/d_3^*}{y/d_3^*}\right)}$$

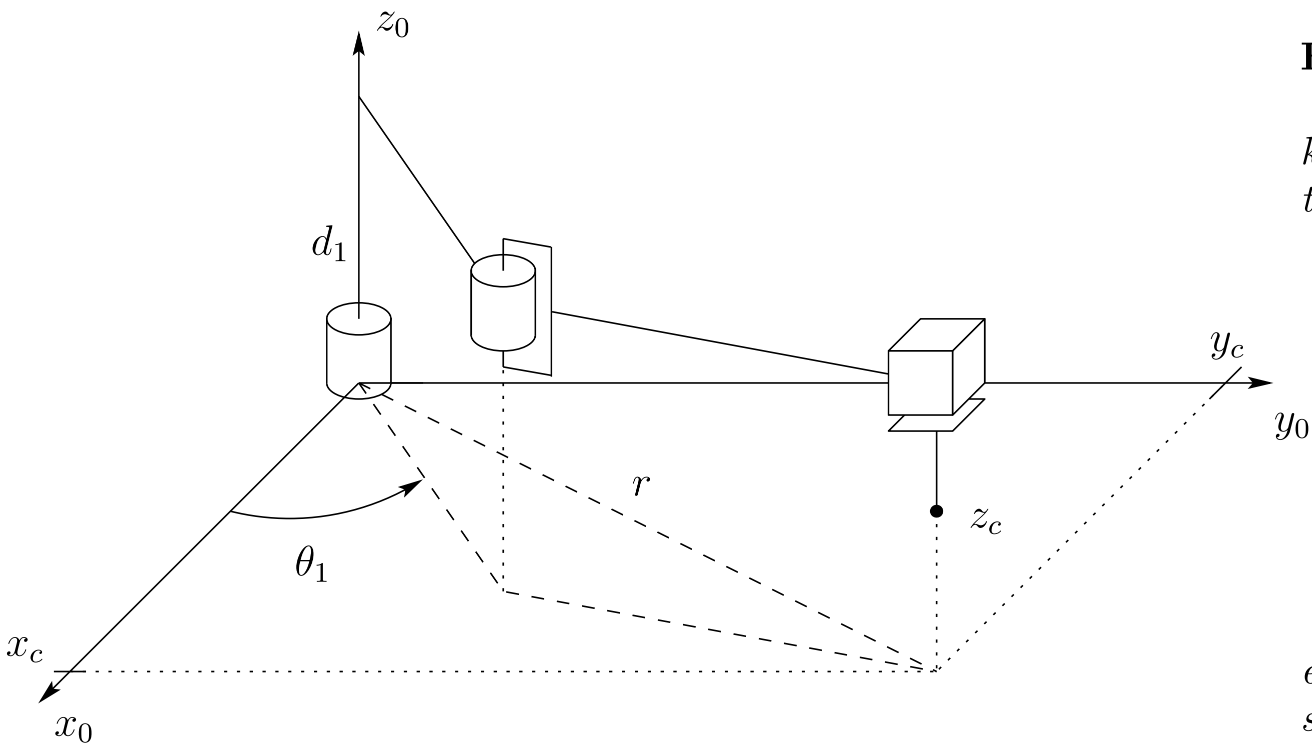
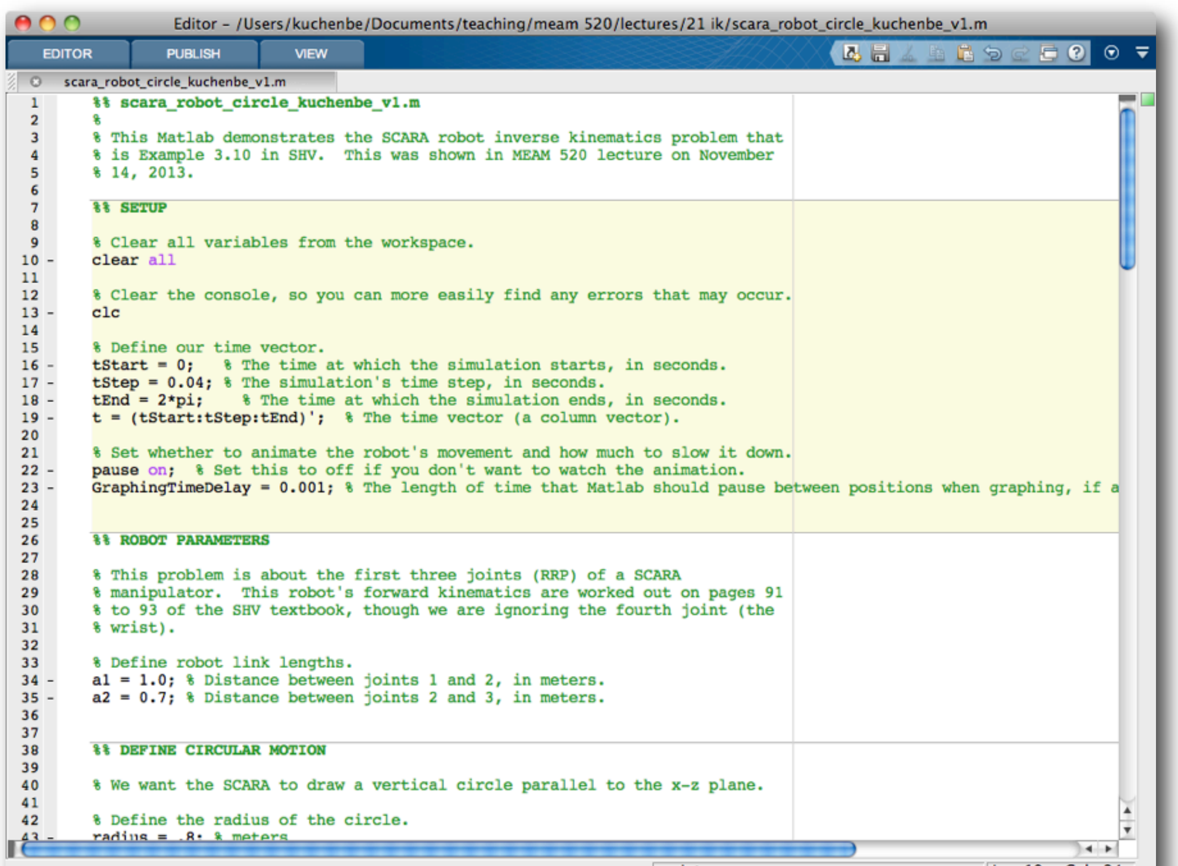


Fig. 3.22 SCARA manipulator



MATLAB code:
scara_robot_circle_kuchenbe_v1.m

Example 3.10 SCARA Manipulator

As another example, we consider the SCARA manipulator whose forward kinematics is defined by T_4^0 from (3.30). The inverse kinematics solution is then given as the set of solutions of the equation

$$
\begin{aligned}
T_4^0 &= \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & s_{12}c_4 - c_{12}s_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -c_{12}c_4 - s_{12}s_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.81)
\end{aligned}
$$

We first note that, since the SCARA has only four degrees-of-freedom, not every possible H from $SE(3)$ allows a solution of (3.81). In fact we can easily see that there is no solution of (3.81) unless R is of the form

$$
R = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ s_\alpha & -c_\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad (3.82)
$$

and if this is the case, the sum $\theta_1 + \theta_2 - \theta_4$ is determined by

$$
\theta_1 + \theta_2 - \theta_4 = \alpha = \operatorname{atan2}(r_{11}, r_{12}) \qquad (3.83)
$$

Projecting the manipulator configuration onto the $x_0 - y_0$ plane immediately yields the situation of Figure 3.22. We see from this that

$$
\boxed{\theta_2 = \operatorname{atan2}\left(c_2, \pm\sqrt{1 - c_2}\right)} \qquad (3.84)
$$

where

$$
c_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1a_2} \qquad (3.85)
$$

$$
\boxed{\theta_1 = \operatorname{atan2}(o_x, o_y) - \operatorname{atan2}(a_1 + a_2c_2, a_2s_2)} \qquad (3.86)
$$

We may then determine θ_4 from (3.83) as

$$
\begin{aligned}
\theta_4 &= \theta_1 + \theta_2 - \alpha \\
&= \theta_1 + \theta_2 - \operatorname{atan2}(r_{11}, r_{12}) \qquad (3.87)
\end{aligned}
$$

Finally d_3 is given as

$$
\boxed{d_3 = o_z + d_4} \quad d_4 = 0 \qquad (3.88)
$$

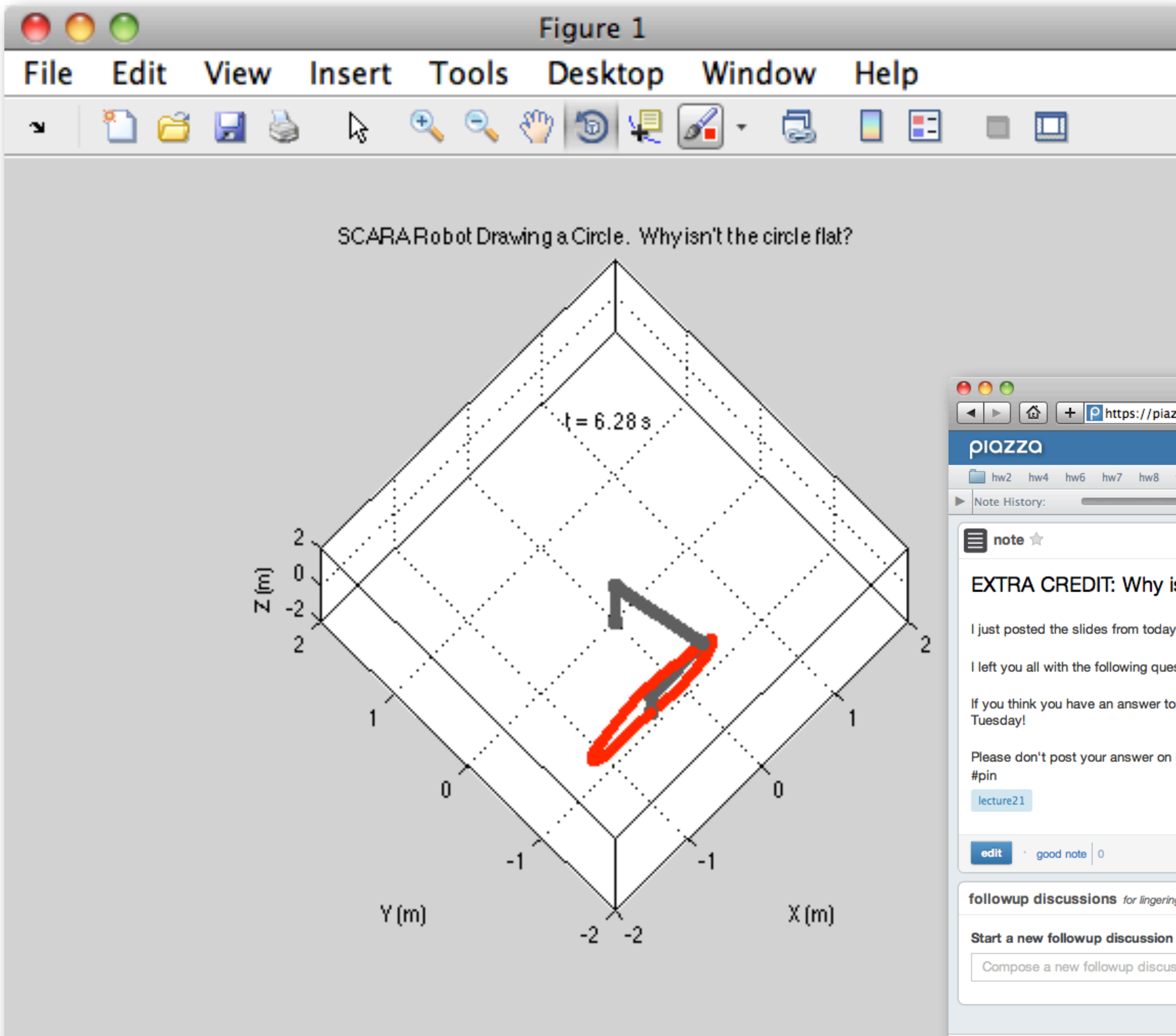
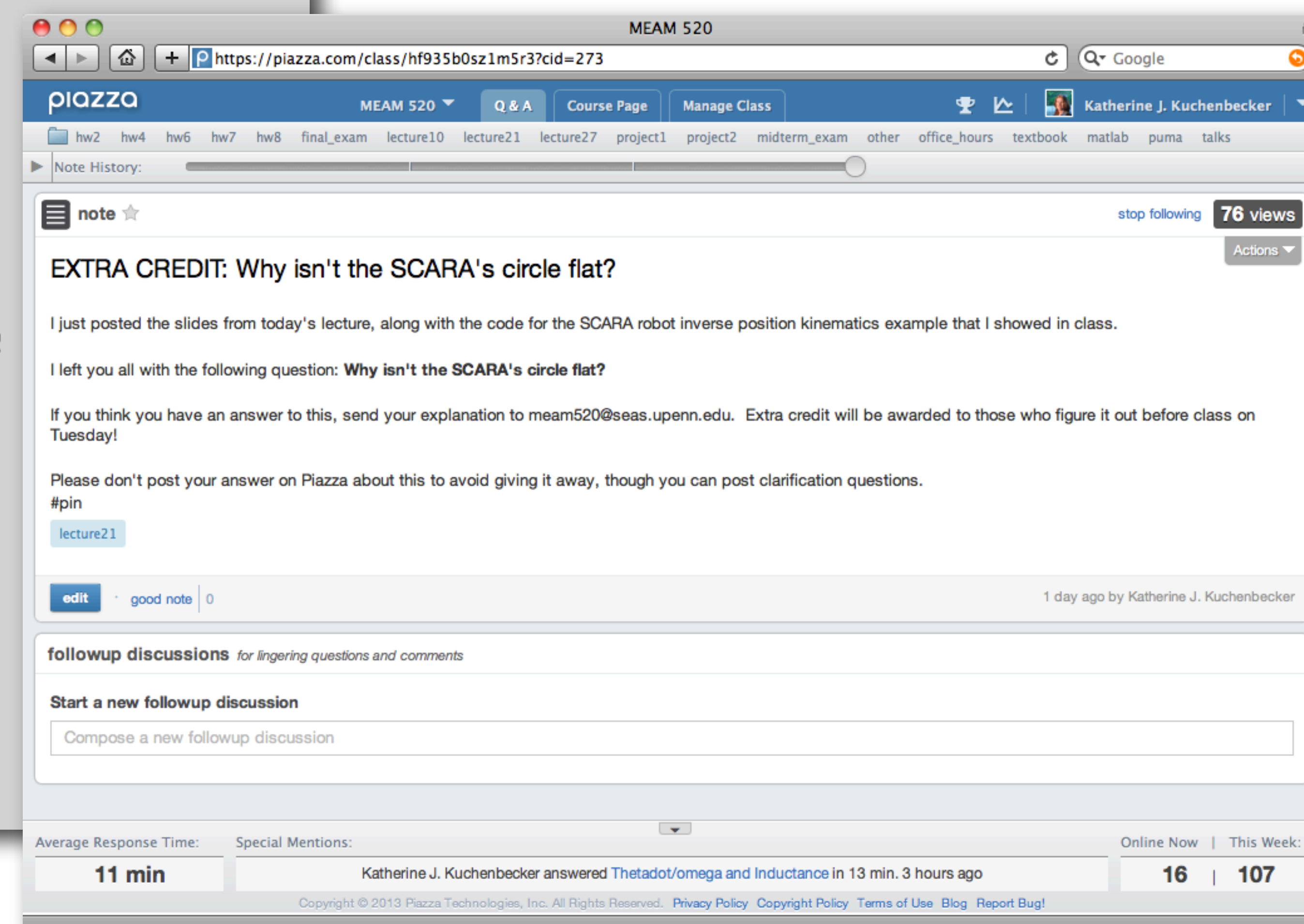


Figure 1
File Edit View Insert Tools Desktop Window Help
SCARA Robot Drawing a Circle. Why isn't the circle flat?
t = 6.28 s
Z (m)
Y (m)
X (m)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EXTRA CREDIT: Why isn't the SCARA's circle flat?
I just posted the slides from today's lecture, along with the code for the SCARA robot inverse position kinematics example that I showed in class.
I left you all with the following question: Why isn't the SCARA's circle flat?
If you think you have an answer to this, send your explanation to meam520@seas.upenn.edu. Extra credit will be awarded to those who figure it out before class on Tuesday!
Please don't post your answer on Piazza about this to avoid giving it away, though you can post clarification questions.
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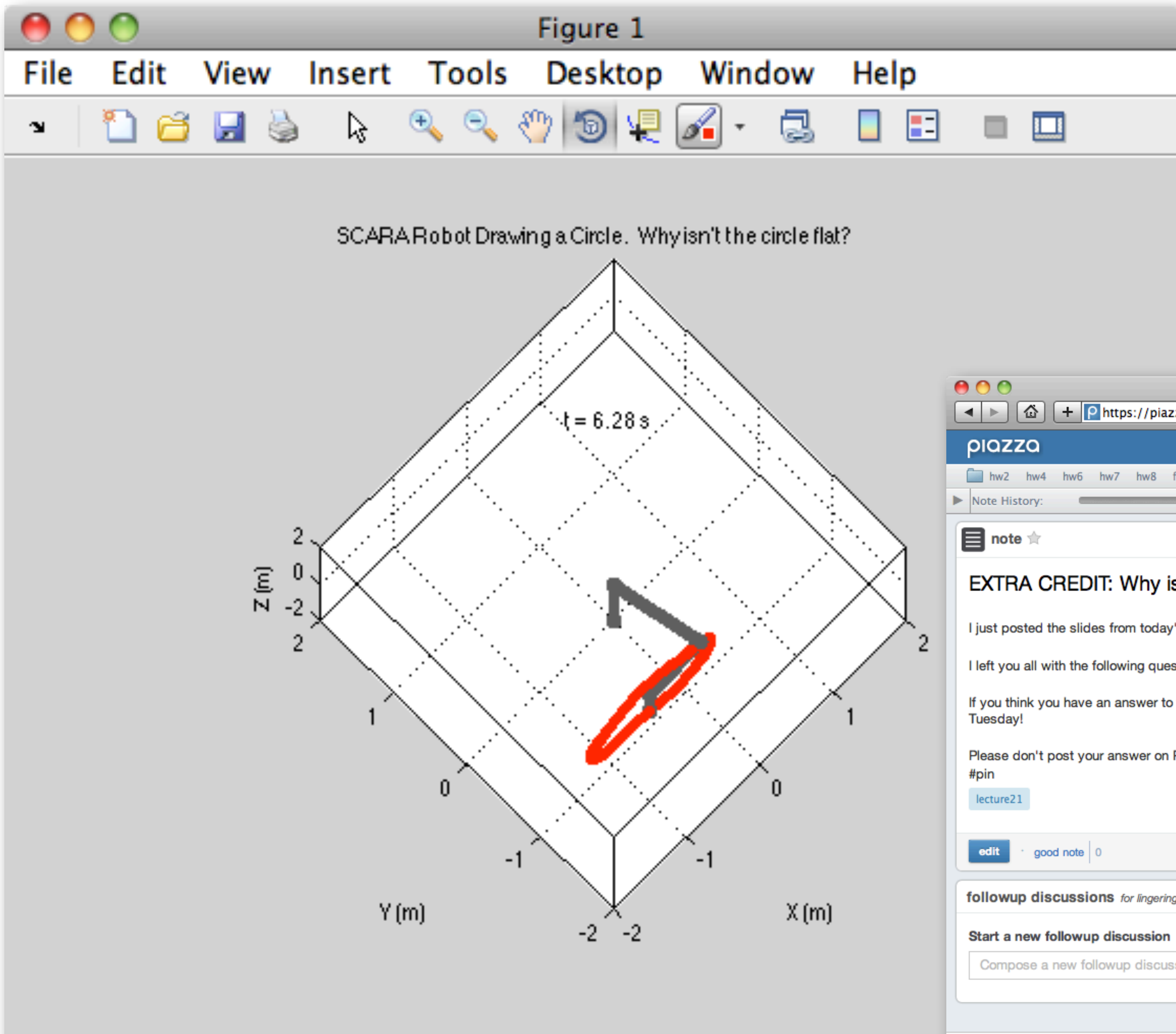
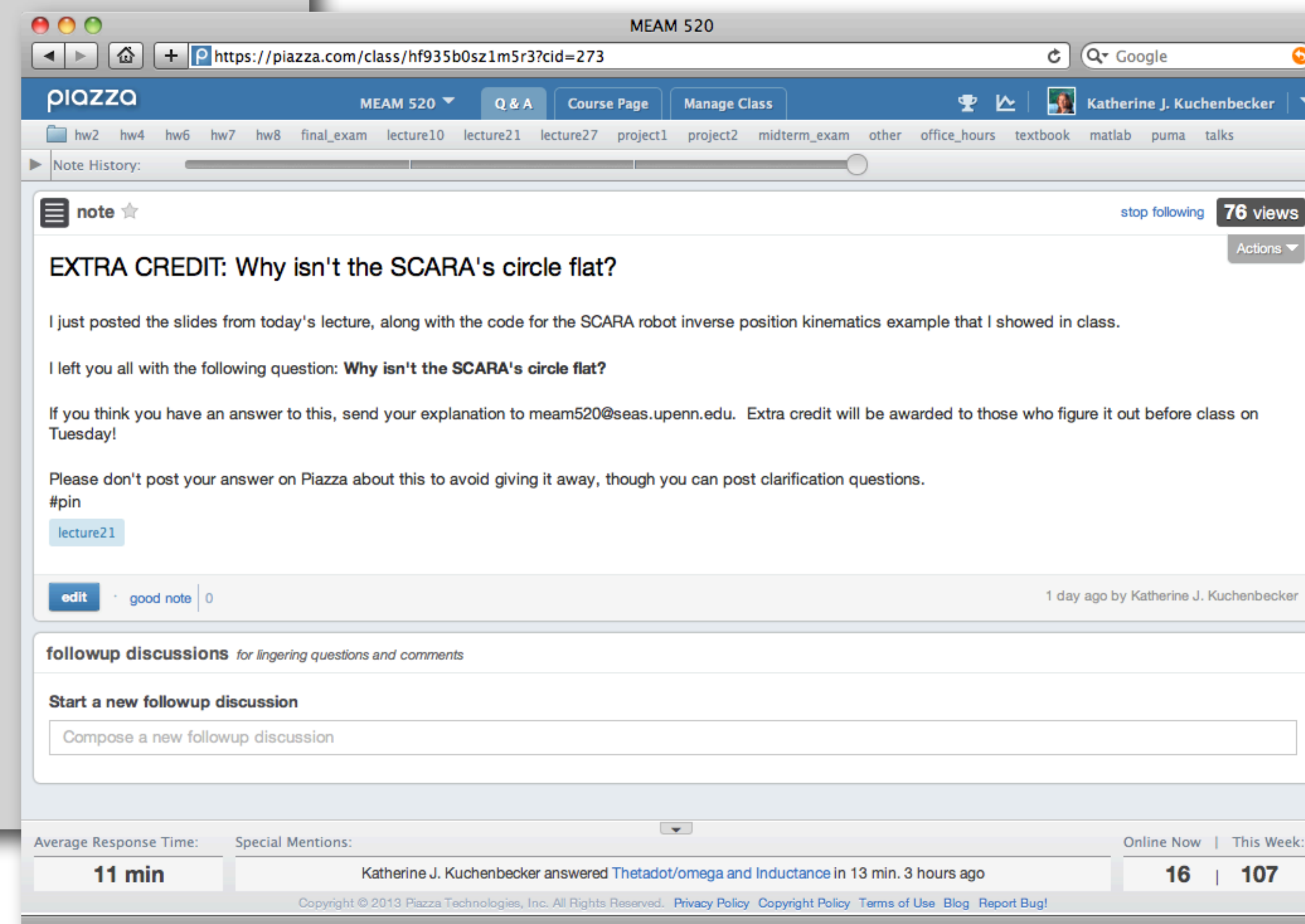
16 | 107

What's wrong?





What's wrong?

The book's IK solution for the SCARA contains mistakes.

Errata: Robot Modeling and Control

Mark W. Spong, Seth Hutchinson, and M. Vidyasagar

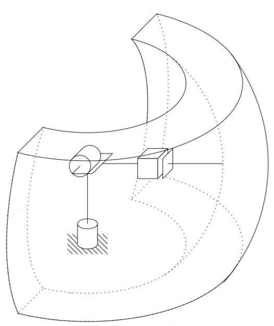
November 5, 2013

Compiled by Katherine J. Kuchenbecker and others affiliated with MEAM 520 at the University of Pennsylvania, expanding on the errata from Seth Hutchinson.

Chapter 1

Page 4 Near the bottom of the page, change “if the joint is the interconnection of links i and $i+1$” to “if the joint is the interconnection of links i and $i-1$”.

Page 18 In Figure 1.17(a) the workspace for the spherical robot is drawn too low. The spherical surface should be centered on the intersection between the two revolute joints, as shown in the image below, which was taken from an early draft of the book:



Page 87 In Figure 3.8, joint 5 is shown at $\theta_5 = -90°$. The end part of the wrist should be straight up to match the given DH parameters. The wrist is drawn correctly later in the chapter.

Page 87 The (3,2) element of A_5 should be $+1$.

Page 90 In Figure 3.10, the origin of frame 6 should be at the end of the gripper, not at the center of the spherical wrist. This placement follows the guidelines for choosing the end-effector frame. If you leave the sixth frame where it is, d_6 should be equal to zero in Table 3.4.

Page 91 In the expressions for r_{11}, the term $-d_2$ should be $-s_1$.

Page 92 In Figure 3.11, Frame $x_0y_0z_0$ should be drawn at the shoulder joint of the robot arm, moved up along z_0 to match the given DH parameters in Table 3.5. Alternatively, you can add d_1 as a constant parameter in the first step of the DH transformations and adjust the matrices A_1, T_4^0.

Page 99 In Figure 3.14, θ_{1} should be θ_1.

Page 109 In Equation (3.70), T_4^1 should be T_4^0.

Page 109 In Equation (3.75), $\sqrt{1-c_2}$ should be $\sqrt{1-c_2^2}$.

Page 109 In Equation (3.78), o_z should be $-o_z$.

Chapter 4

Page 130 In Equation (4.46), the summation $\sum_{i-1}^{n}$ should be $\sum_{i=1}^{n}$.

Page 132 The first line should say “the translational velocity is $\dot{d}_i$” instead of “the magnitude of the translation is $\dot{d}_i$.”

Page 135 In the second sentence, the reference to Equation (4.62) should be Equation (4.63).

Page 140 In the sentence before Equation (4.85), $R = R_{z,\psi}R_{y,\theta}R_{z,\phi}$ should be $R = R_{z,\phi}R_{y,\theta}R_{z,\psi}$.

Page 143 In the second line after Equation (4.90), “that the all possible” should be “that all possible”.

Page 144 In the middle of the first paragraph, θ_4 should be θ_5.

Page 144 In the middle of the first paragraph, “the are unavoidable” should be “they are unavoidable”.

Page 144 In Equation (4.99) the sign of the determinant should be switched.

Page 153 In Equation (4.121), $\xi^T(JJ^T)^1\xi^T$ should be $\xi^T(JJ^T)^{-1}\xi$.

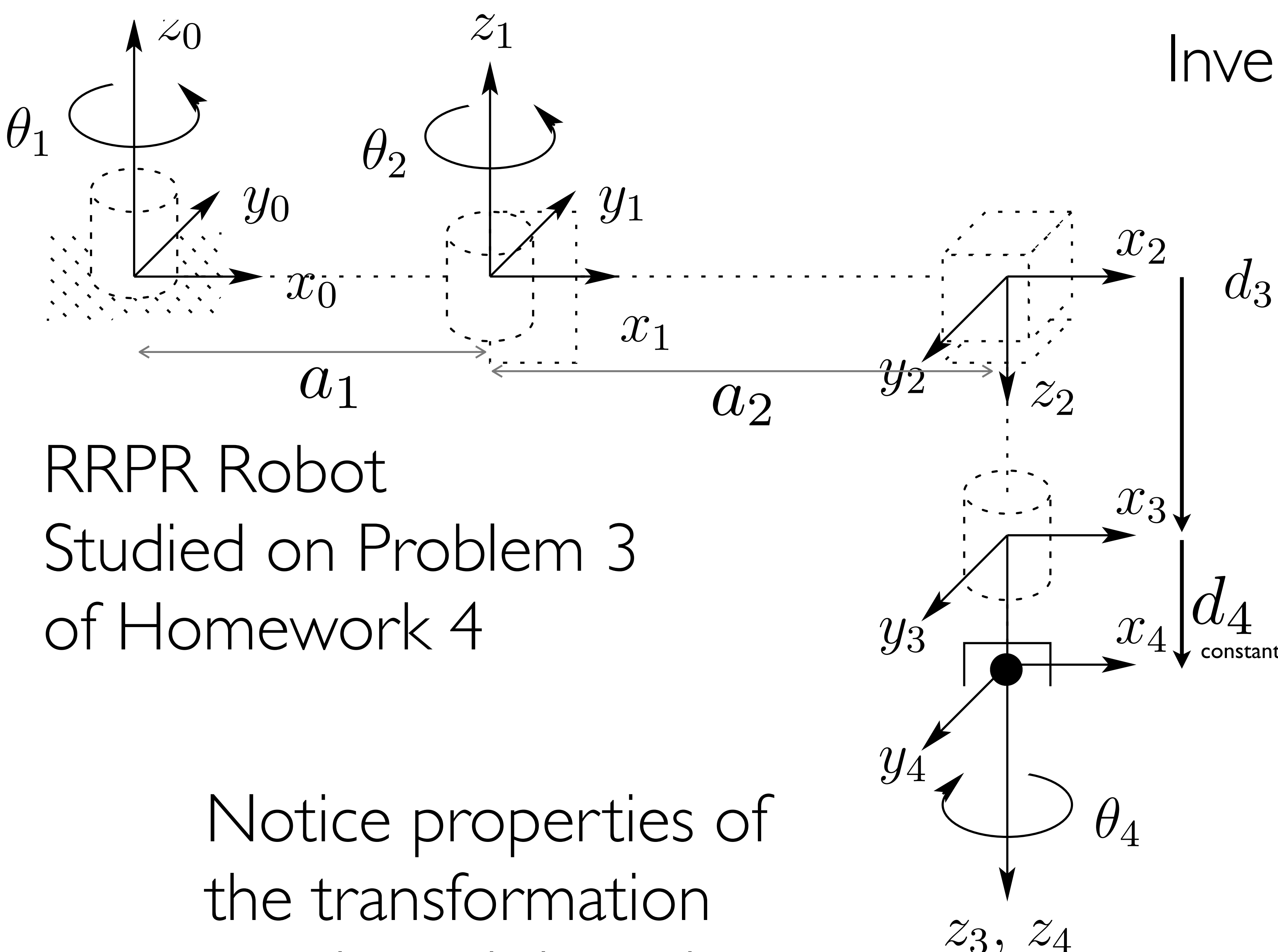
Page 154 After Equation (4.124), $\lambda_1 \geq \lambda_2 \ldots \leq \lambda_m$ should be $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_m$.

Page 158 In problem 4-7, $\phi = \frac{\phi}{2}$ should be $\phi = \frac{\pi}{2}$.

You should all transcribe the errata into your book.

$$T_4^0 = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & s_{12}c_4 - c_{12}s_4 & 0 & \boxed{a_1c_1 + a_2c_{12}} \\ s_{12}c_4 - c_{12}s_4 & -c_{12}c_4 - s_{12}s_4 & 0 & \boxed{a_1s_1 + a_2s_{12}} \\ 0 & 0 & -1 & \boxed{-d_3 - d_4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Position Kinematics



RRPR Robot
Studied on Problem 3
of Homework 4

Notice properties of the transformation matrix and the robot....

$$o_4^0 = \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix}$$

$$\theta_1 = ?$$
$$\theta_2 = ?$$
$$d_3 = ?$$

$$x = a_1c_1 + a_2c_{12}$$

$$y = a_1s_1 + a_2s_{12}$$

$$z = -d_3 - d_4$$

$$x = a_1 c_1 + a_2 c_{12}$$

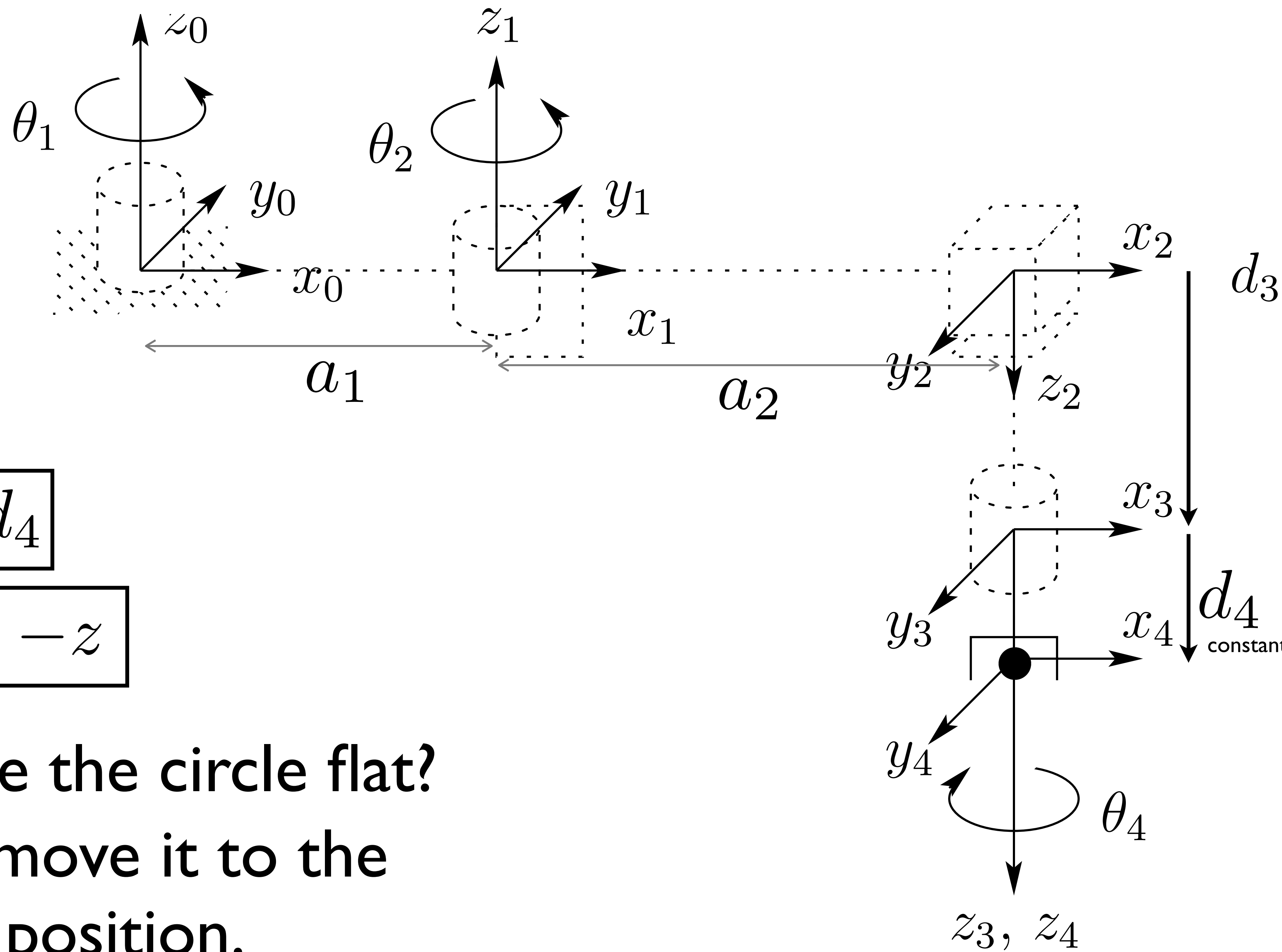
$$y = a_1 s_1 + a_2 s_{12}$$

$$z = -d_3 - d_4$$

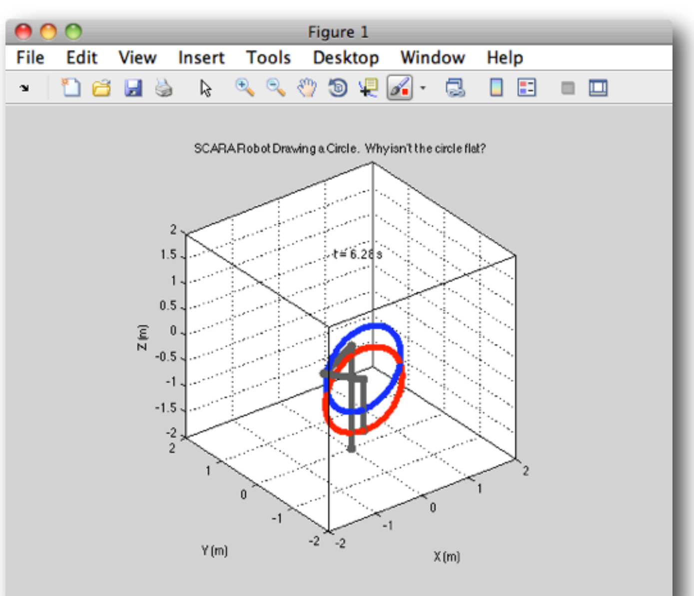
$$-d_3 = z + d_4$$

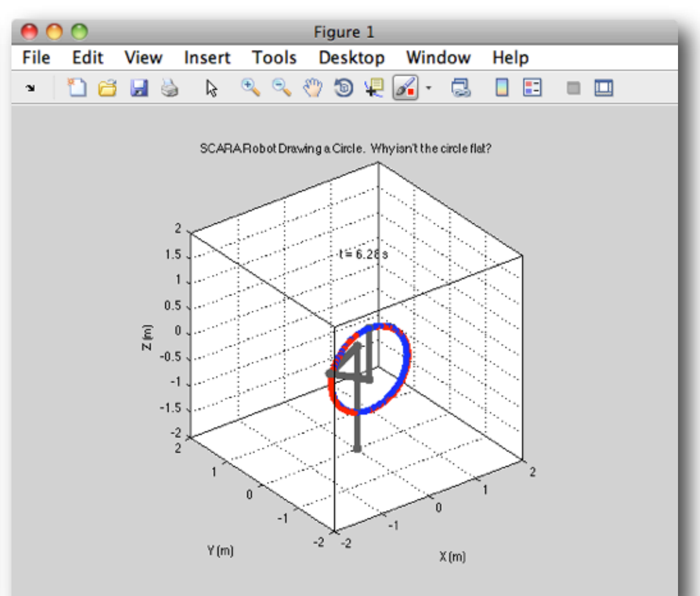
$$\boxed{d_3 = -z - d_4}$$

$$\boxed{\text{if } d_4 = 0, d_3 = -z}$$



Will this fix make the circle flat?
No, but it will move it to the correct z position.





Use the geometric IK method to find theta1 and theta2.

Top view of SCARA, looking along z0 and z1

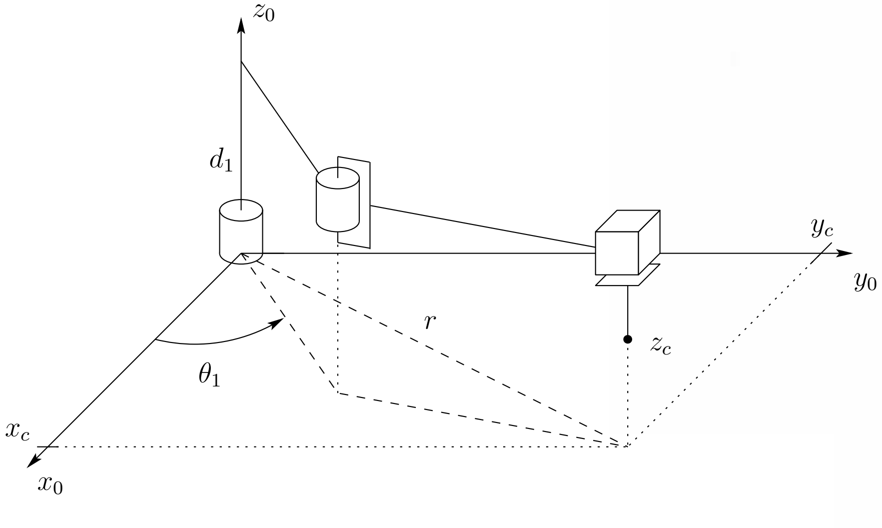
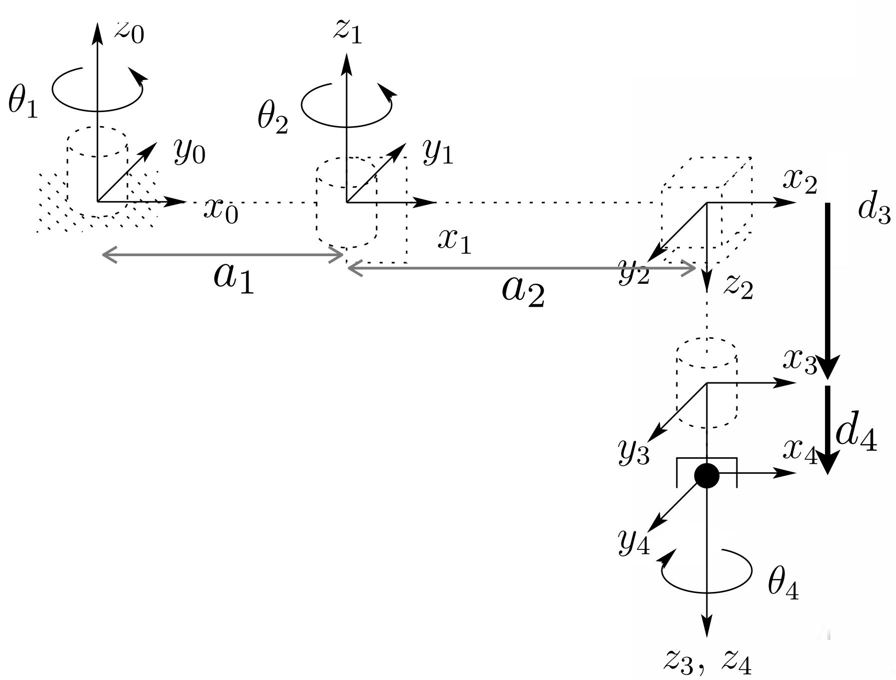
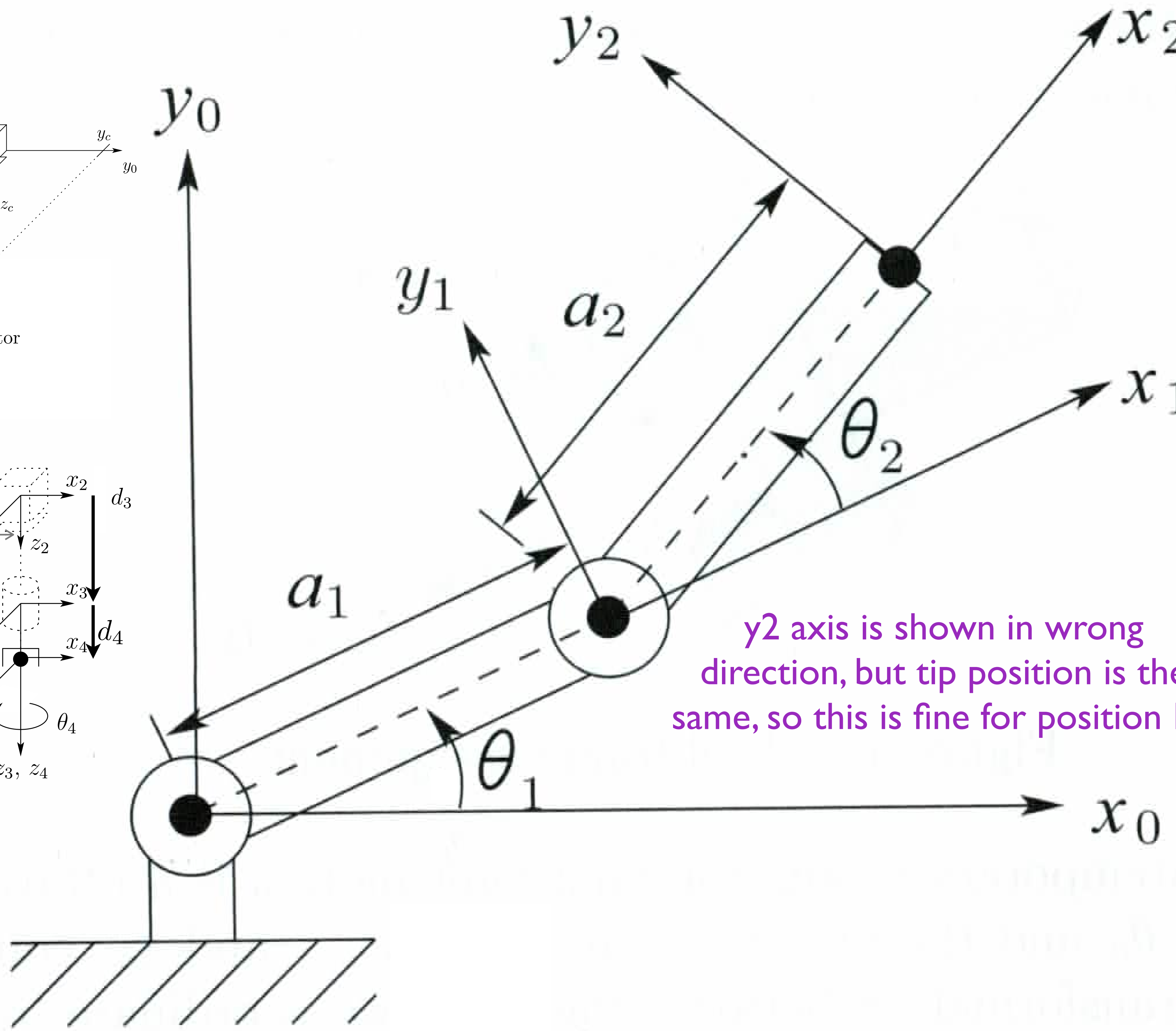


Fig. 3.22 SCARA manipulator





y2 axis is shown in wrong direction, but tip position is the same, so this is fine for position IK.

Same as a planar RR!

SCARA

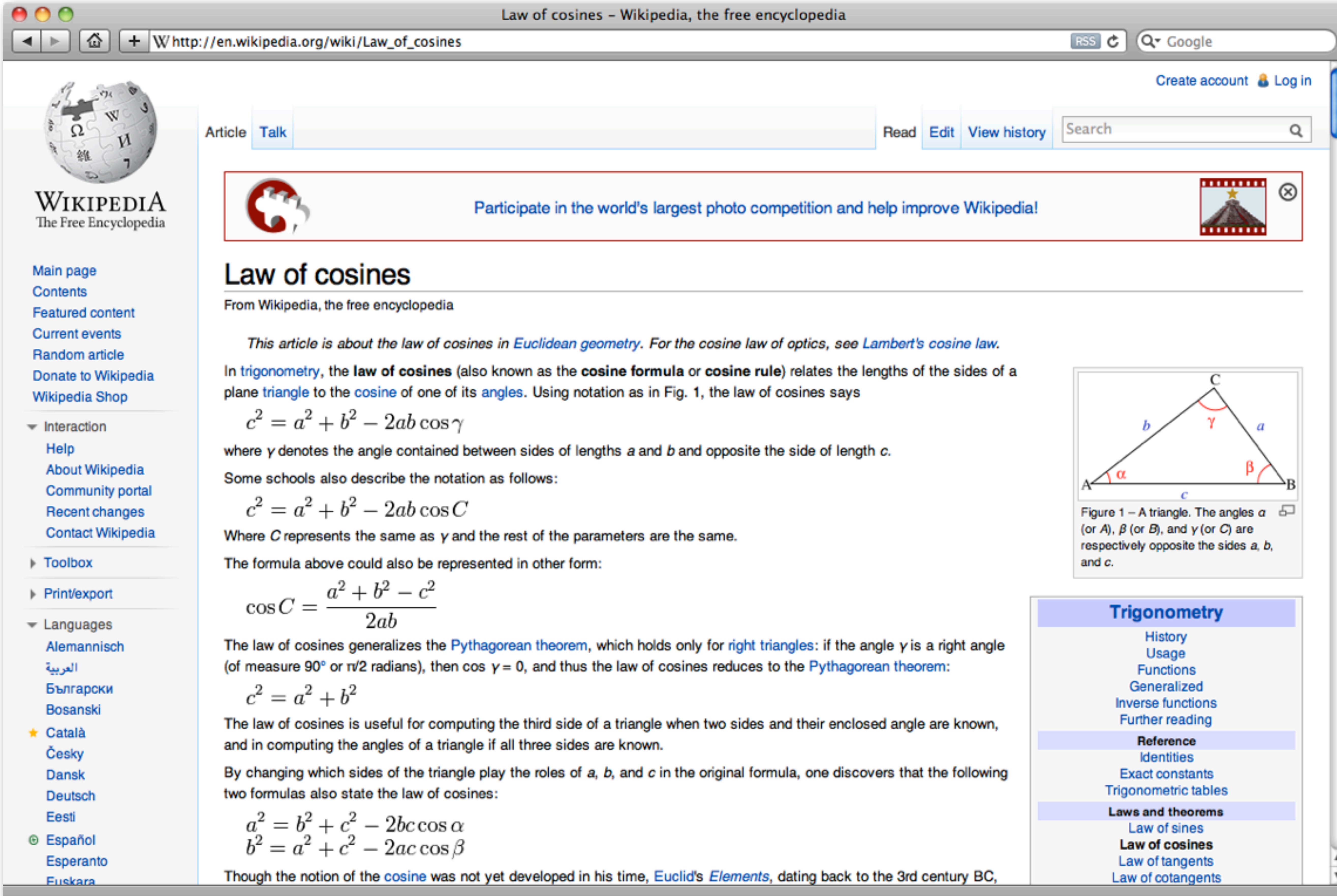
$$T_4^0 = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & s_{12}c_4 - c_{12}s_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -c_{12}c_4 - s_{12}s_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Planar RR

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_1^0 &= A_1 \\ T_2^0 &= A_1A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$


Law of cosines - Wikipedia, the free encyclopedia

http://en.wikipedia.org/wiki/Law_of_cosines

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Law of cosines

From Wikipedia, the free encyclopedia

This article is about the law of cosines in Euclidean geometry. For the cosine law of optics, see Lambert's cosine law.

In trigonometry, the **law of cosines** (also known as the **cosine formula** or **cosine rule**) relates the lengths of the sides of a plane triangle to the cosine of one of its angles. Using notation as in Fig. 1, the law of cosines says

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

where γ denotes the angle contained between sides of lengths a and b and opposite the side of length c.

Some schools also describe the notation as follows:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Where C represents the same as γ and the rest of the parameters are the same.

The formula above could also be represented in other form:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

The law of cosines generalizes the Pythagorean theorem, which holds only for right triangles: if the angle γ is a right angle (of measure 90° or $\pi/2$ radians), then $\cos\gamma = 0$, and thus the law of cosines reduces to the Pythagorean theorem:

$$c^2 = a^2 + b^2$$

The law of cosines is useful for computing the third side of a triangle when two sides and their enclosed angle are known, and in computing the angles of a triangle if all three sides are known.

By changing which sides of the triangle play the roles of a, b, and c in the original formula, one discovers that the following two formulas also state the law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

Though the notion of the cosine was not yet developed in his time, Euclid's *Elements*, dating back to the 3rd century BC,

Figure 1 – A triangle. The angles α (or A), β (or B), and γ (or C) are respectively opposite the sides a, b, and c.

Trigonometry
History
Usage
Functions
Generalized
Inverse functions
Further reading
Reference
Identities
Exact constants
Trigonometric tables
Laws and theorems
Law of sines
Law of cosines
Law of tangents
Law of cotangents

Page 354 In Equation (10.62), the left side of the last two terms should be changed to $L_{ad_f^2}(g)T_1$ and $L_{ad_f^3}(g)T_1$, respectively.

Pages 355-6 All occurrences of MgL should be changed to $Mg\ell$.

Page 359 In Equation (10.83), change $\dot{x}_1$ and $\dot{x}_3$ to x_1 and x_3, respectively.

Page 359 In Equation (10.86), change $T_1(x_1)$ to $T_1(x)$ in the first equation.

Page 362 Remove the semicolon in Equation (10.100).

Page 367 After Equation (10.113), change g_2 *it follows* to g_2. *It follows.*

Page 368 In the third sentence of Definition 10.11, change $\bar{\Delta}$ *is an involutive distribution such that* to $\bar{\Delta}$ *is an involutive distribution containing* Δ *such that.*

Page 375 In Problem 10-21, change rank 3 to rank 2.

Chapter 11

Page 387 In the second paragraph, change "Likewise, if half or the pixels" to "Likewise, if half of the pixels".

Chapter 12

Page 426 In the first row vector in Equation (12.21), change the first term L_{v_z} to L_{v_x}.

Appendix A

Page 436 In the Law of Cosines, change cb^2 to b^2 to give $c^2 = a^2 + b^2 - 2ab\cos\theta$.

Appendix D

Page 452 In Equation (D.7), change $x_i x_i$ to $x_i x_j$.

6

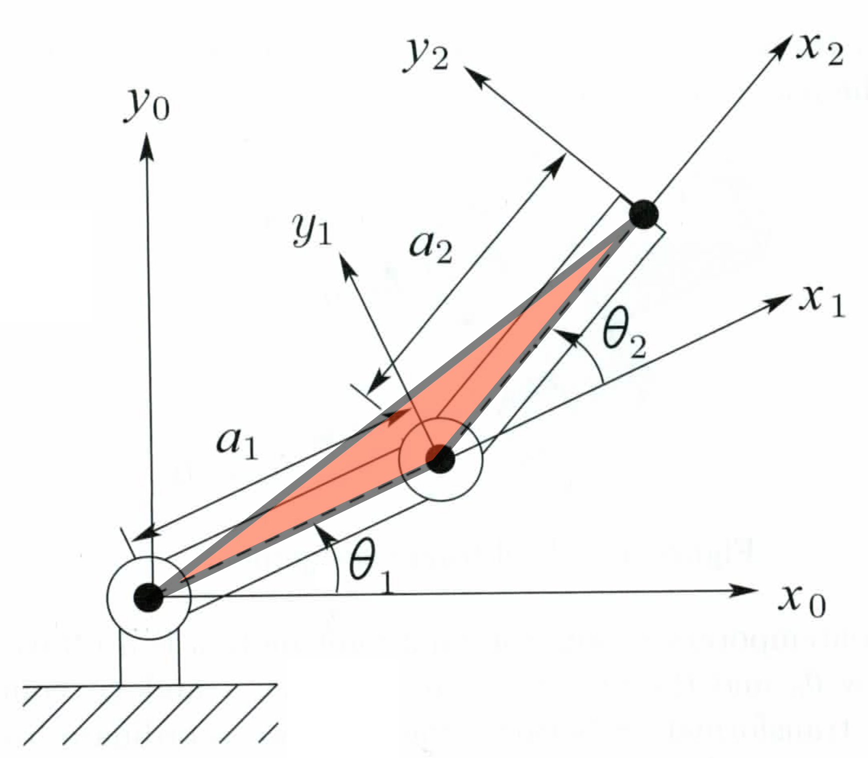
Law of Cosines is wrong in SHV Appendix A.

They say:

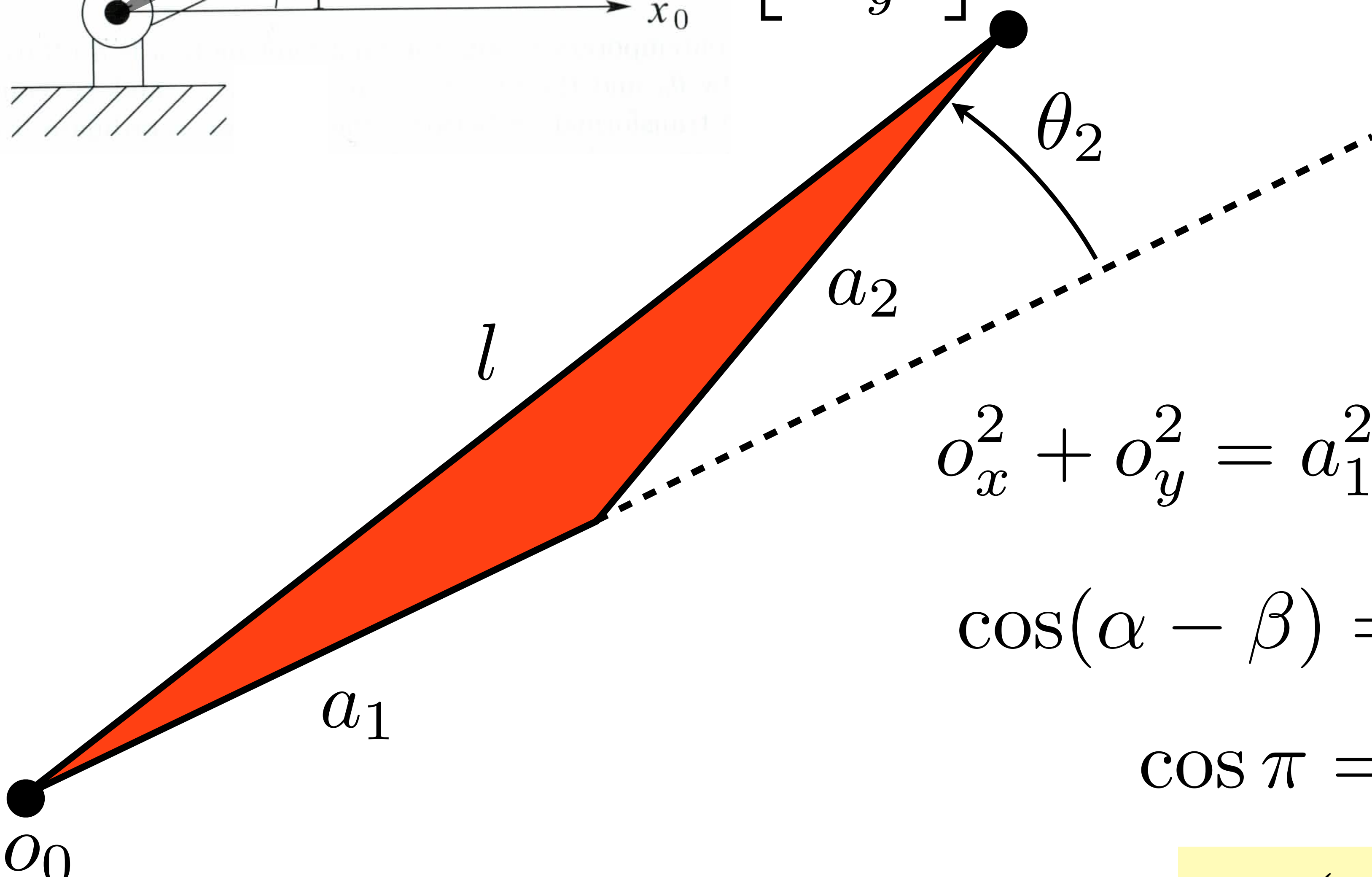
$$c^2 = a^2 + cb^2 - 2ab\cos\gamma$$

It should be:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$



$$\begin{bmatrix} o_x \\ o_y \end{bmatrix}$$



$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$l^2 = a_1^2 + a_2^2 - 2a_1a_2\cos(\pi - \theta_2)$$

$$l^2 = o_x^2 + o_y^2$$

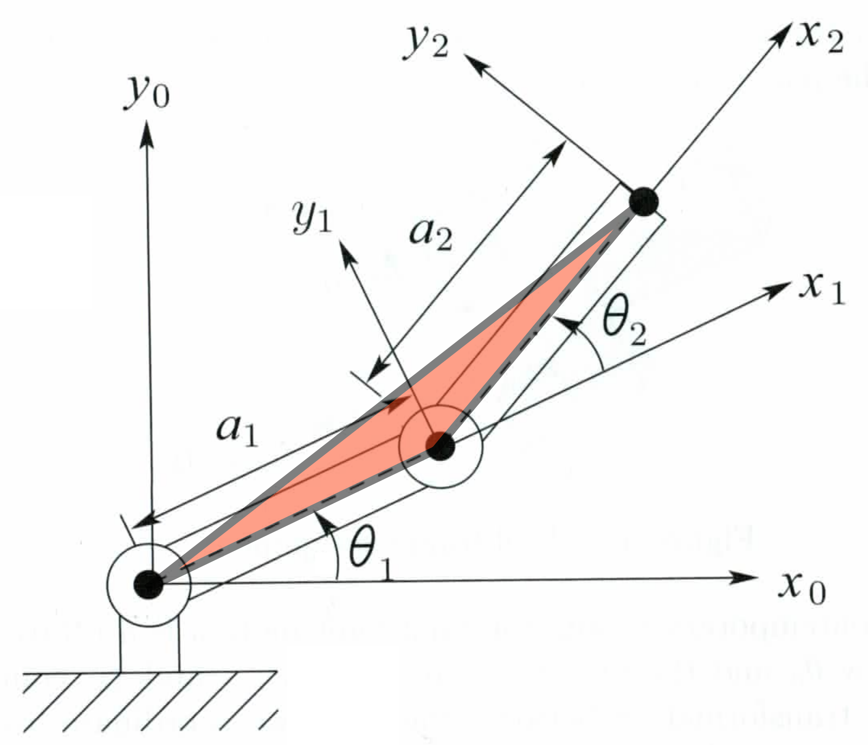
$$o_x^2 + o_y^2 = a_1^2 + a_2^2 - 2a_1a_2\cos(\pi - \theta_2)$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos\pi = -1 \qquad \sin\pi = 0$$

$$\cos(\pi - \theta_2) = -\cos\theta_2$$

$$o_x^2 + o_y^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\theta_2$$

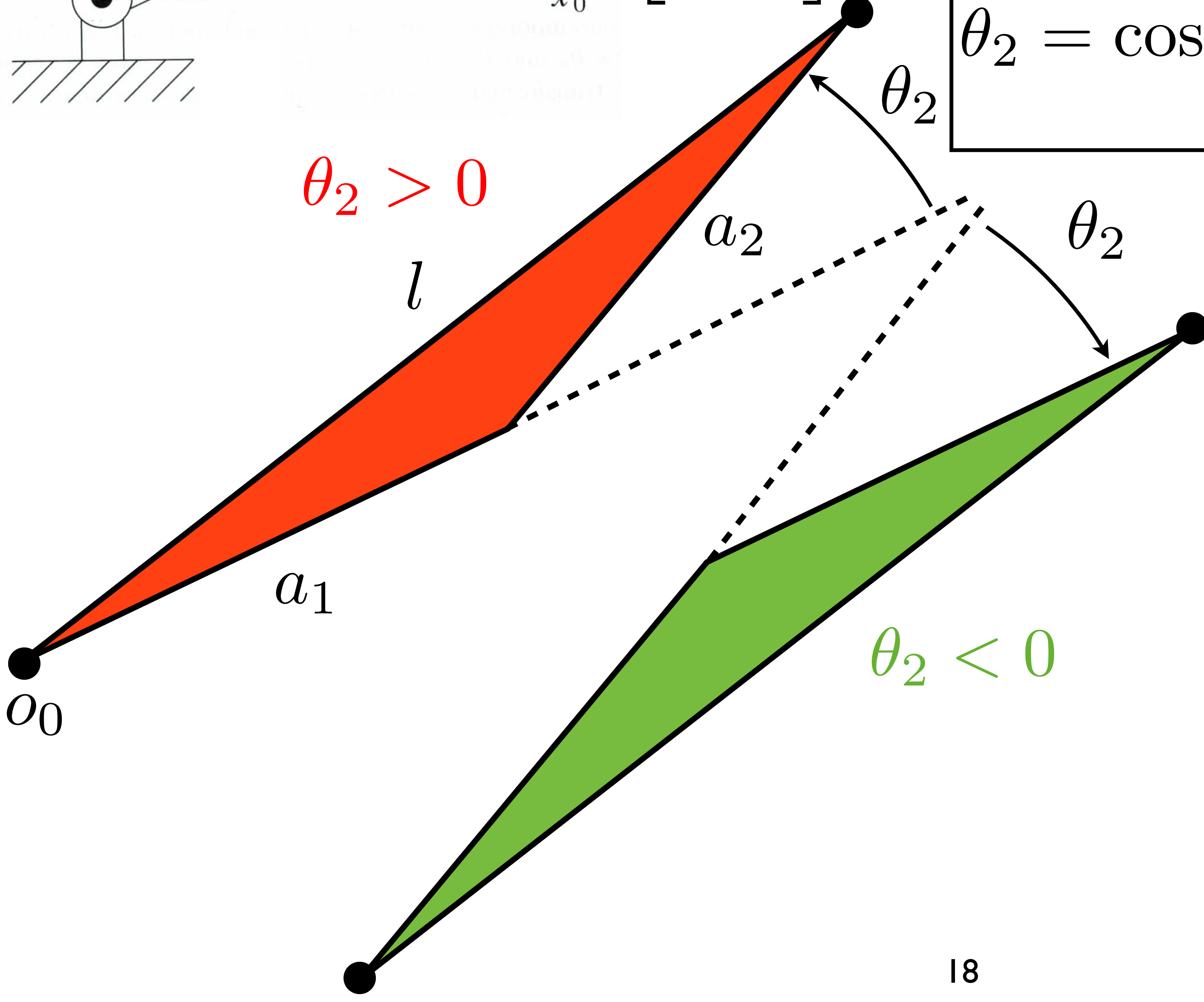


$$o_x^2 + o_y^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\theta_2$$

$$\cos\theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$\boxed{\theta_2 = \cos^{-1}\left(\frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1 a_2}\right)}$$

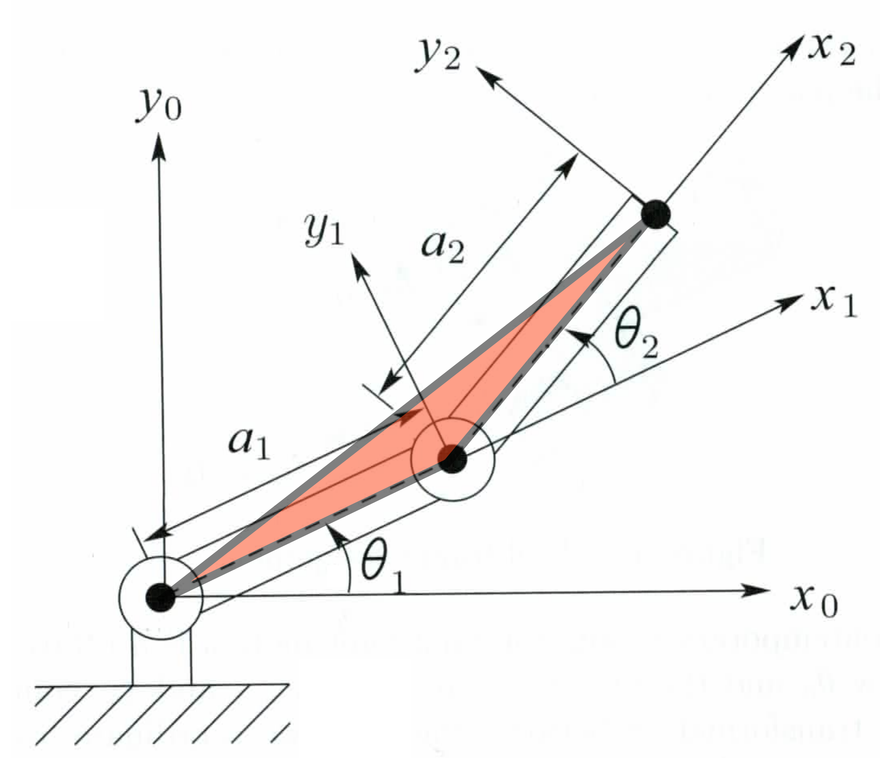
$$\begin{bmatrix} o_x \\ o_y \end{bmatrix}$$



How many solutions are there?

Two: one positive, and one negative.

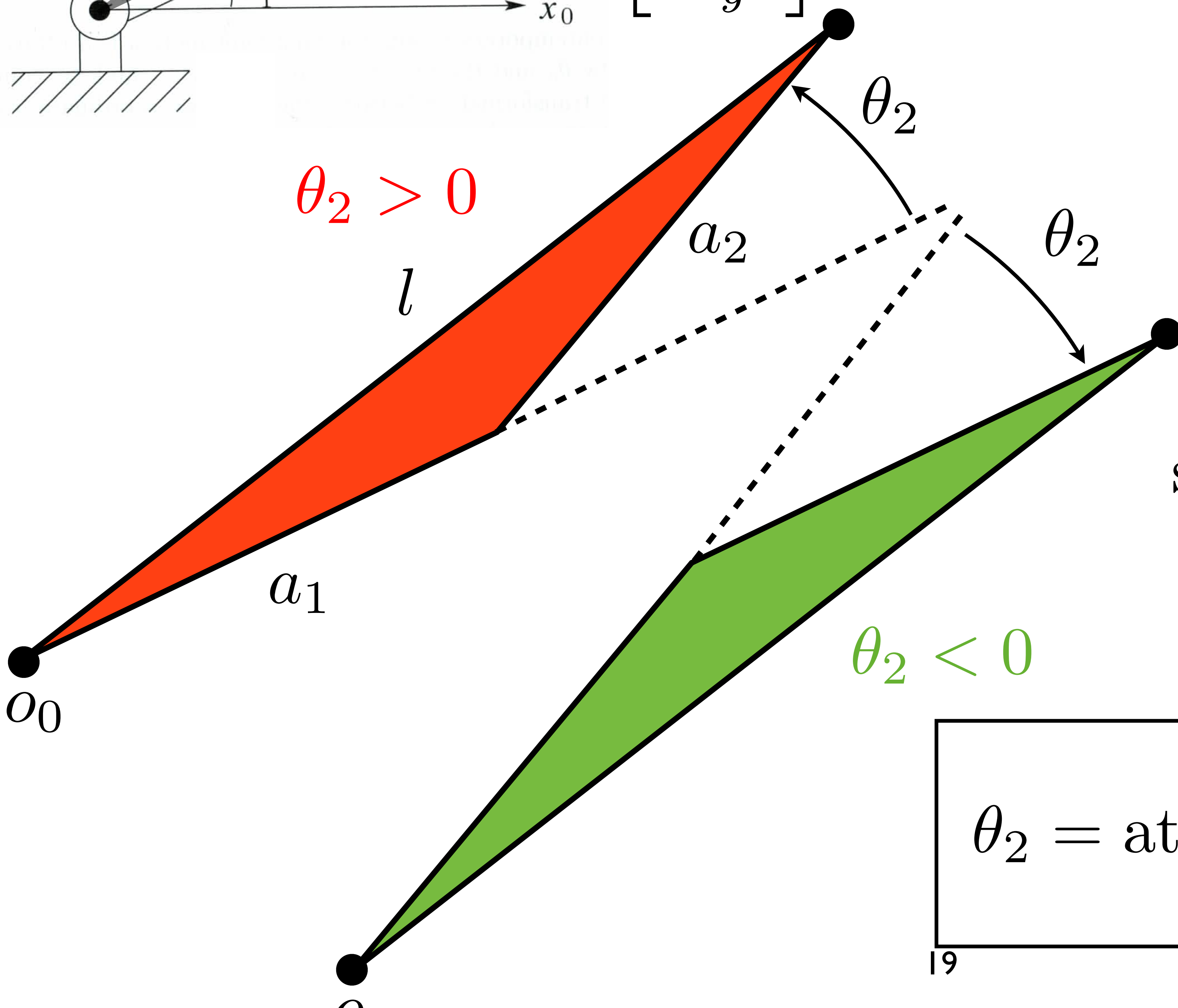
These two solutions are commonly called "elbow down" and "elbow up"



$$\cos\theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$\begin{bmatrix} o_x \\ o_y \end{bmatrix}$$

The book reaches the solution a different way.

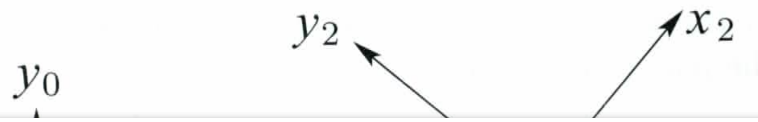


$$\sin^2\theta_2 + \cos^2\theta_2 = 1$$

$$\sin^2\theta_2 = 1 - \cos^2\theta_2$$

$$\sin\theta_2 = \pm\sqrt{1 - \cos^2\theta_2}$$

$$\boxed{\theta_2 = \operatorname{atan2}\left(\frac{\pm\sqrt{1-\cos^2\theta_2}}{\cos\theta_2}\right)}$$



Example 3.10 SCARA Manipulator

As another example, we consider the SCARA manipulator whose forward kinematics is defined by T_4^0 from (3.30). The inverse kinematics solution is then given as the set of solutions of the equation

$$T_4^0 = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & s_{12}c_4 - c_{12}s_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -c_{12}c_4 - s_{12}s_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.81}$$

We first note that, since the SCARA has only four degrees-of-freedom, not every possible H from $SE(3)$ allows a solution of (3.81). In fact we can easily see that there is no solution of (3.81) unless R is of the form

$$R = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ s_\alpha & -c_\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{3.82}$$

and if this is the case, the sum $\theta_1 + \theta_2 - \theta_4$ is determined by

$$\theta_1 + \theta_2 - \theta_4 = \alpha = \text{atan2}(r_{11}, r_{12}) \tag{3.83}$$

Projecting the manipulator configuration onto the $x_0 - y_0$ plane immediately yields the situation of Figure 3.22. We see from this that

$$\boxed{\theta_2 = \text{atan2}\left(c_2, \pm\sqrt{1 - c_2}\right)} \tag{3.84}$$

where

$$c_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1a_2} \tag{3.85}$$

$$\boxed{\theta_1 = \text{atan2}(o_x, o_y) - \text{atan2}(a_1 + a_2c_2, a_2s_2)} \tag{3.86}$$

We may then determine θ_4 from (3.83) as

$$\begin{aligned} \theta_4 &= \theta_1 + \theta_2 - \alpha \\ &= \theta_1 + \theta_2 - \text{atan2}(r_{11}, r_{12}) \end{aligned} \tag{3.87}$$

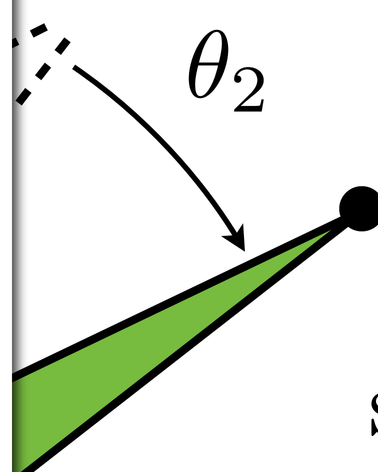
Finally d_3 is given as

$$\boxed{d_3 = o_z + d_4} \tag{3.88}$$

c2 should be squared

$$\cos\theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

The book reaches the solution a different way.



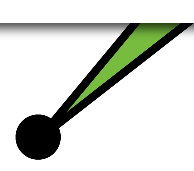
$$\sin^2\theta_2 + \cos^2\theta_2 = 1$$

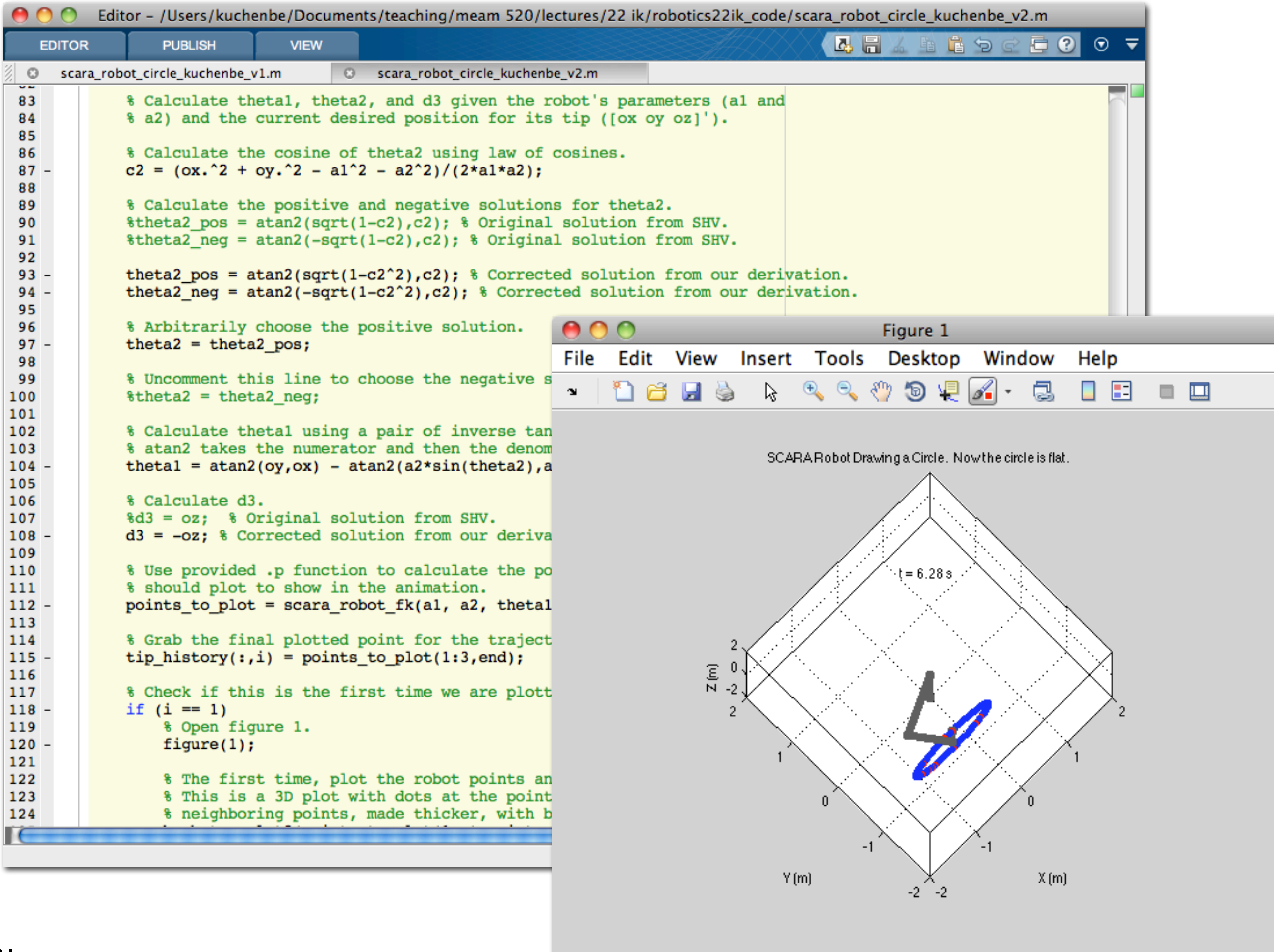
$$\sin^2\theta_2 = 1 - \cos^2\theta_2$$

$$\sin\theta_2 = \pm\sqrt{1 - \cos^2\theta_2}$$

< 0

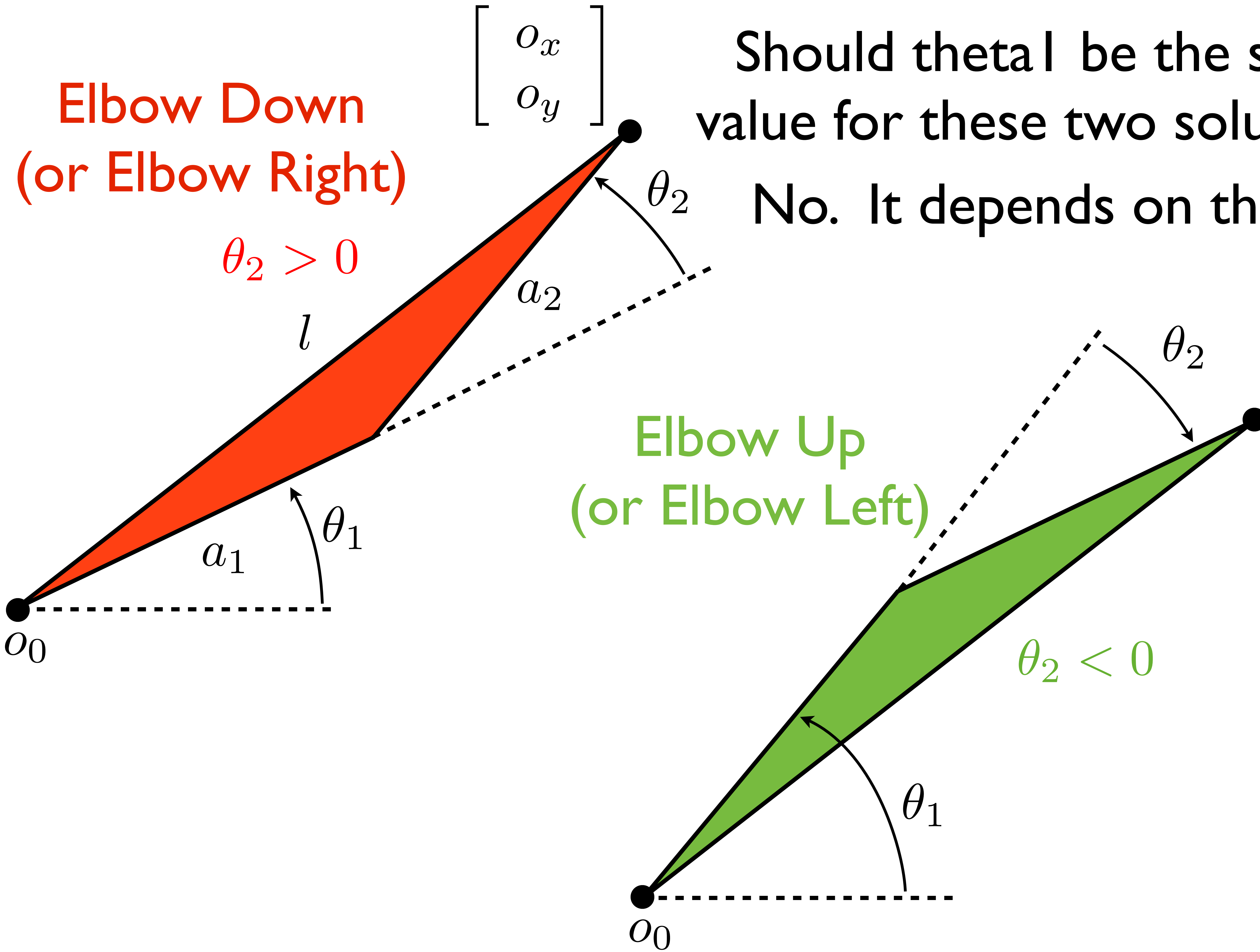
$$\boxed{\theta_2 = \text{atan2}\left(\frac{\pm\sqrt{1 - \cos^2\theta_2}}{\cos\theta_2}\right)}$$

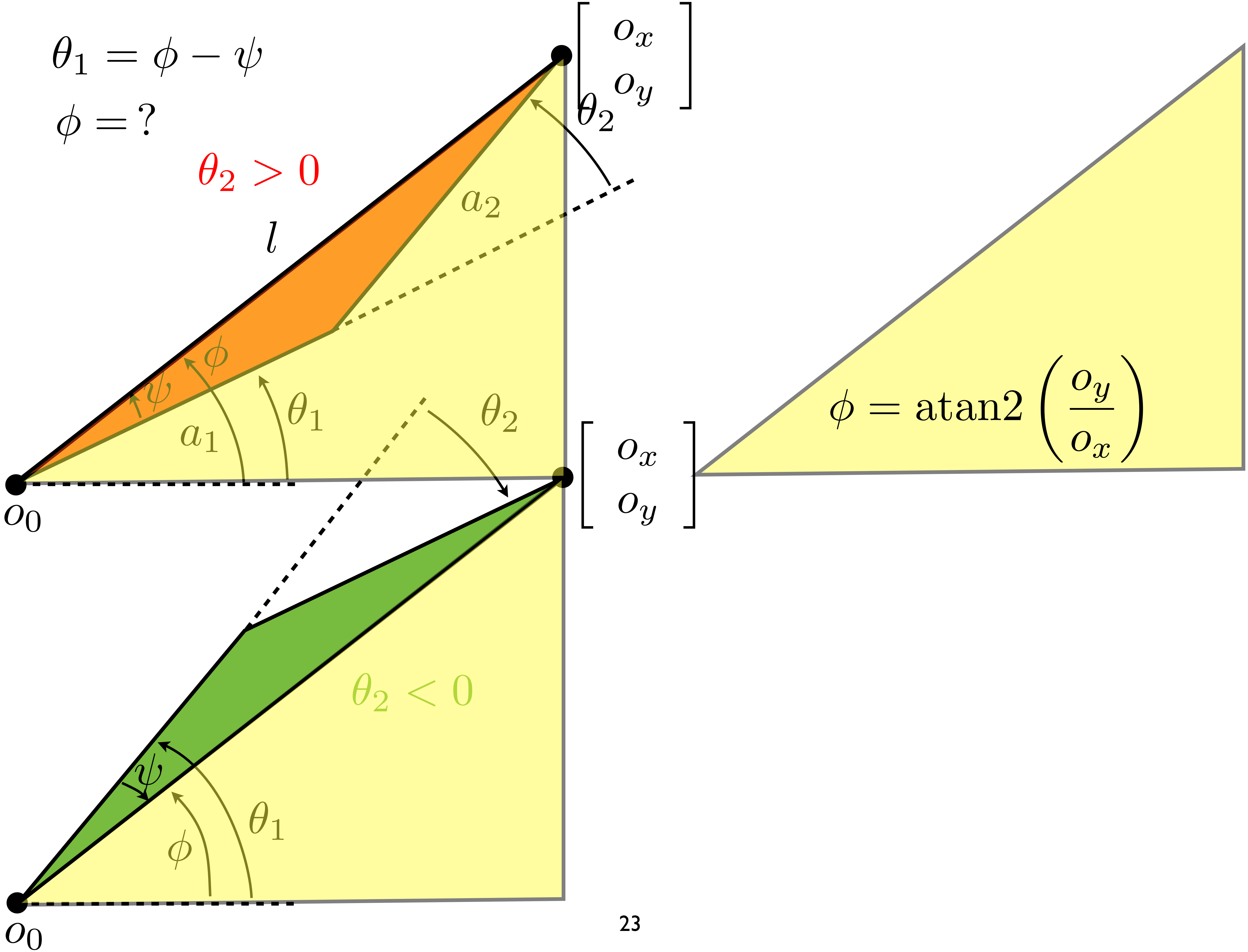



Editor – /Users/kuchenbe/Documents/teaching/meam 520/lectures/22 ik/robotics22ik_code/scara_robot_circle_kuchenbe_v2.m
EDITOR
PUBLISH
VIEW
scara_robot_circle_kuchenbe_v1.m
scara_robot_circle_kuchenbe_v2.m
% Calculate theta1, theta2, and d3 given the robot's parameters (a1 and
% a2) and the current desired position for its tip ([ox oy oz]').
% Calculate the cosine of theta2 using law of cosines.
c2 = (ox.^2 + oy.^2 - a1^2 - a2^2)/(2*a1*a2);
% Calculate the positive and negative solutions for theta2.
%theta2_pos = atan2(sqrt(1-c2),c2); % Original solution from SHV.
%theta2_neg = atan2(-sqrt(1-c2),c2); % Original solution from SHV.
theta2_pos = atan2(sqrt(1-c2^2),c2); % Corrected solution from our derivation.
theta2_neg = atan2(-sqrt(1-c2^2),c2); % Corrected solution from our derivation.
% Arbitrarily choose the positive solution.
theta2 = theta2_pos;
% Uncomment this line to choose the negative s
%theta2 = theta2_neg;
% Calculate theta1 using a pair of inverse tan
% atan2 takes the numerator and then the denom
theta1 = atan2(oy,ox) - atan2(a2*sin(theta2),a
% Calculate d3.
%d3 = oz; % Original solution from SHV.
d3 = -oz; % Corrected solution from our deriva
% Use provided .p function to calculate the po
% should plot to show in the animation.
points_to_plot = scara_robot_fk(a1, a2, theta1
% Grab the final plotted point for the traject
tip_history(:,i) = points_to_plot(1:3,end);
% Check if this is the first time we are plott
if (i == 1)
% Open figure 1.
figure(1);
% The first time, plot the robot points an
% This is a 3D plot with dots at the point
% neighboring points, made thicker, with b
Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
SCARA Robot Drawing a Circle. Now the circle is flat.
t = 6.28 s
Z (m)
Y (m)
X (m)

Should theta1 be the same value for these two solutions?

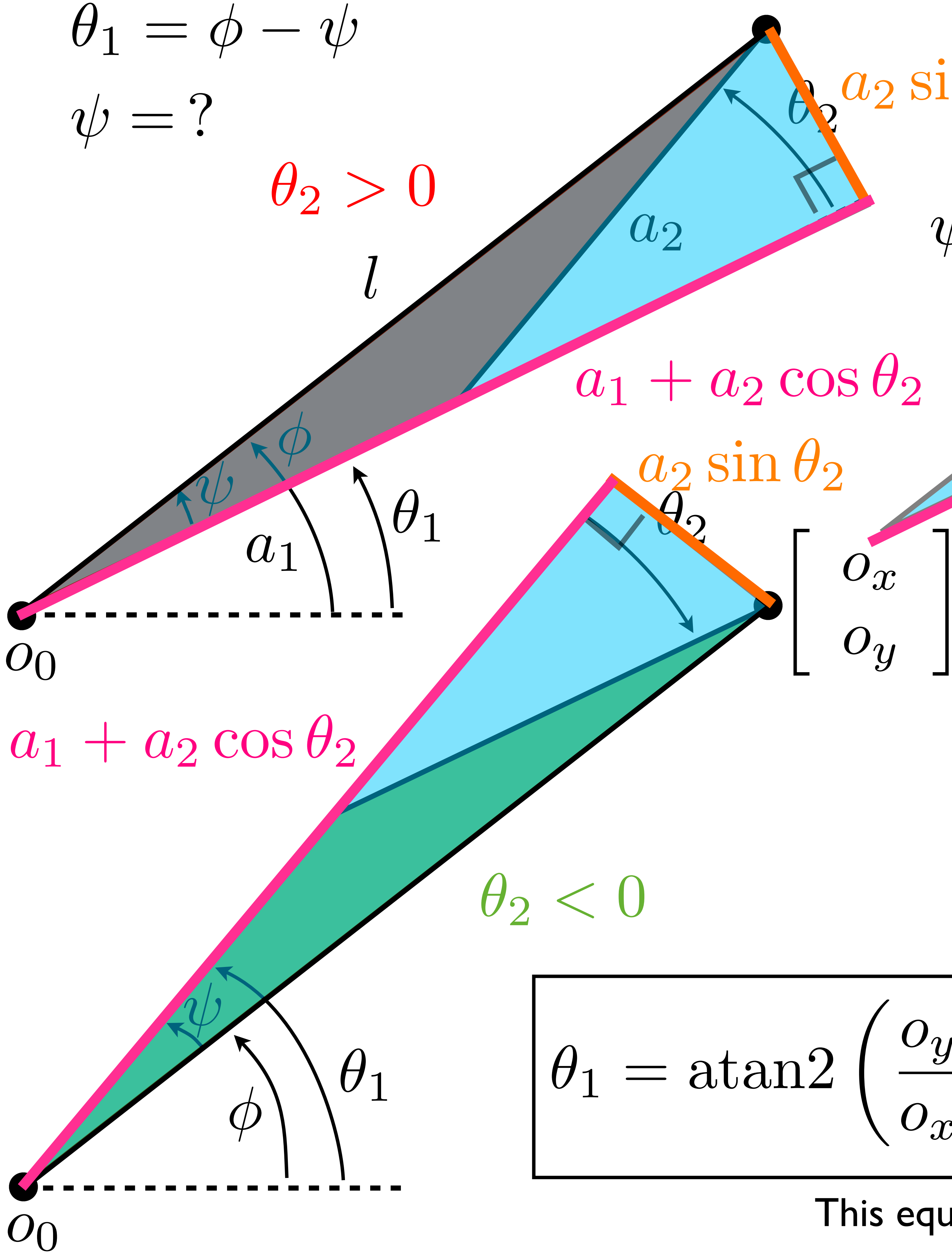
No. It depends on theta2.

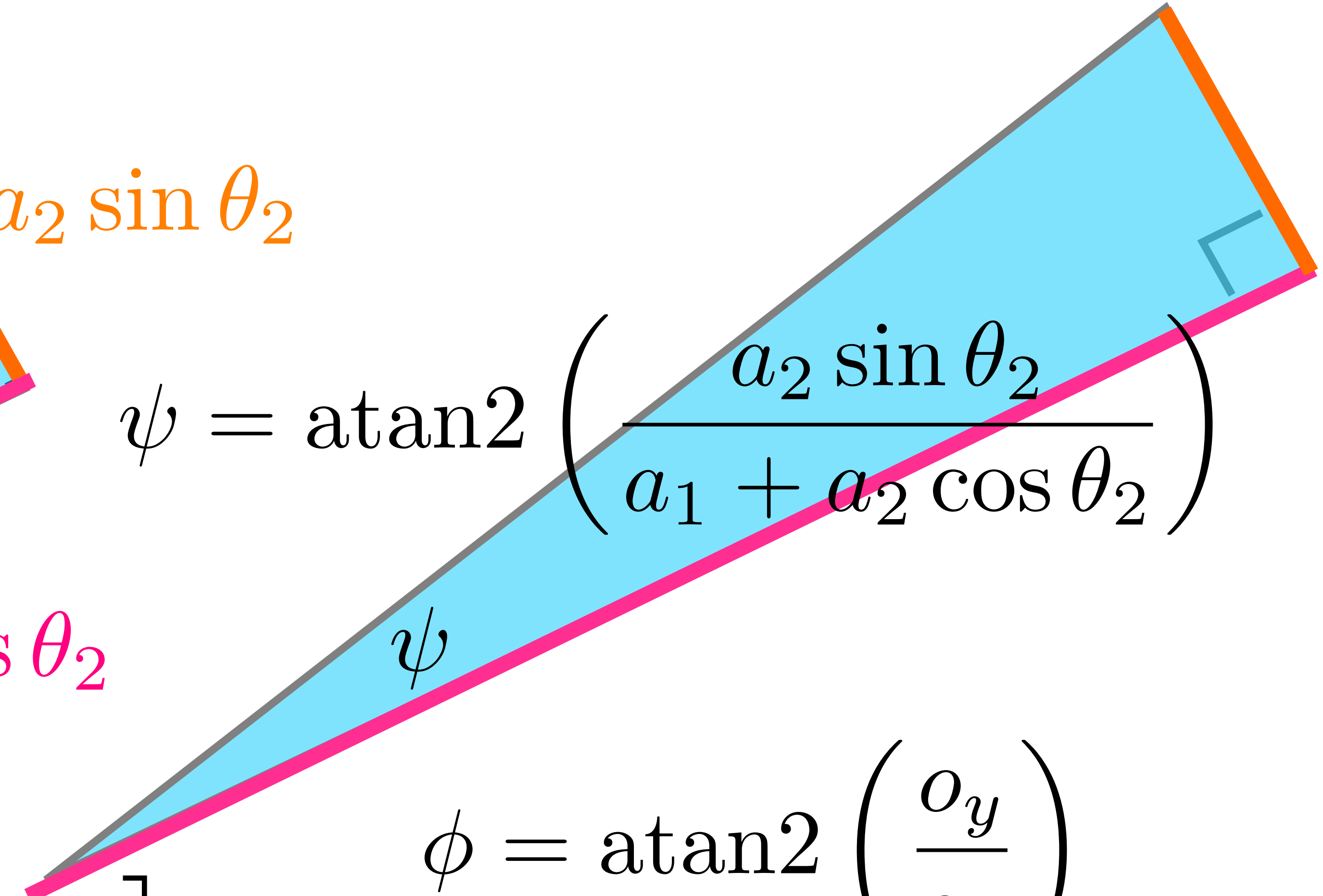



$\theta_1 = \phi - \psi$
$\phi = ?$
$\theta_2 > 0$
l
a_2
θ_2
$\begin{bmatrix} o_x \\ o_y \end{bmatrix}$
ϕ
ψ
a_1
θ_1
θ_2
o_0
$\begin{bmatrix} o_x \\ o_y \end{bmatrix}$
$\phi = \operatorname{atan2}\left(\frac{o_y}{o_x}\right)$
$\theta_2 < 0$
ψ
θ_1
ϕ
o_0

$$\theta_1 = \phi - \psi$$

$$\psi = ?$$





$$\psi = \operatorname{atan2}\left(\frac{a_2 \sin \theta_2}{a_1 + a_2 \cos \theta_2}\right)$$

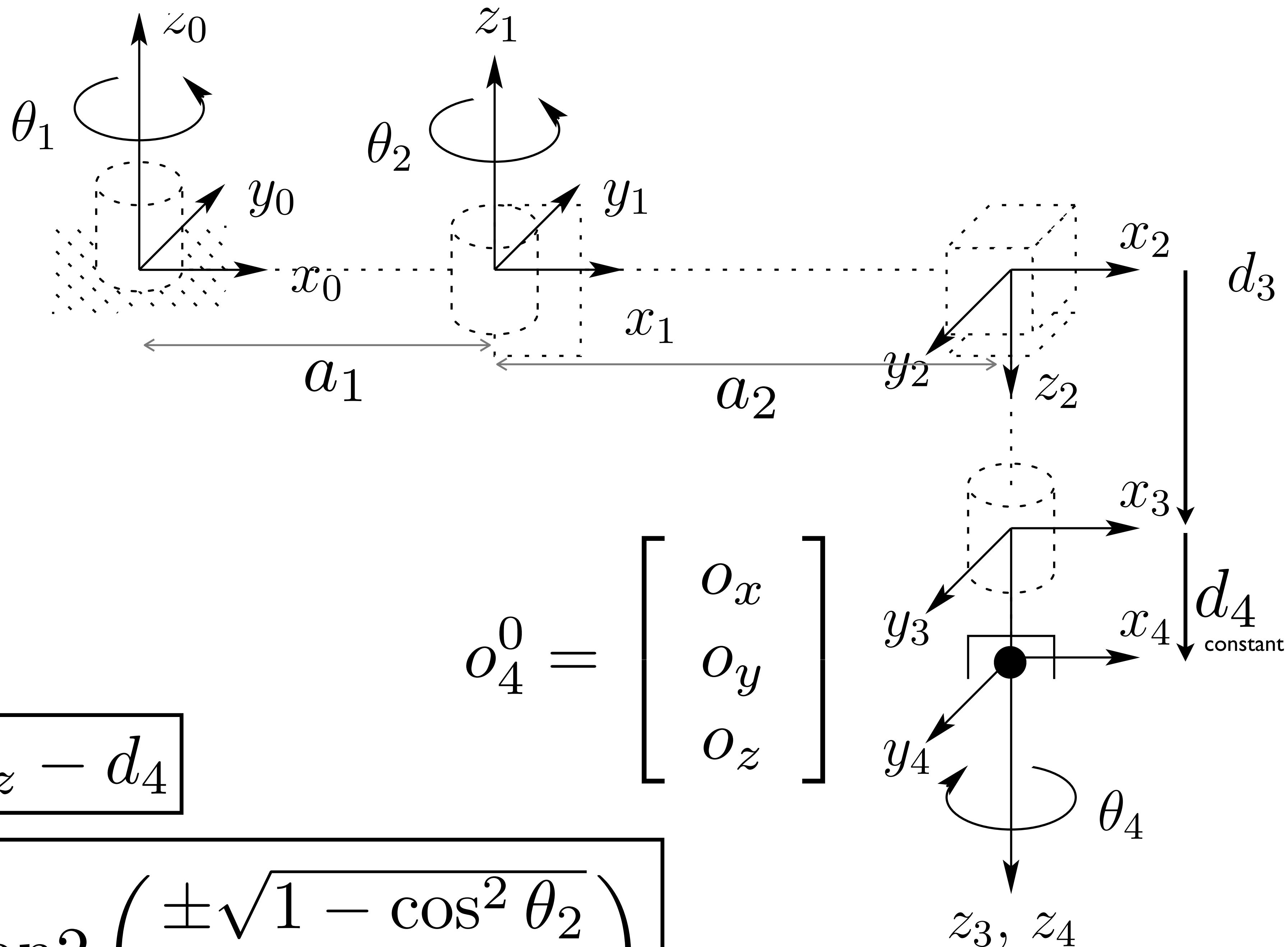
$$\phi = \operatorname{atan2}\left(\frac{o_y}{o_x}\right)$$

$$\theta_1 = \phi - \psi$$

$$\boxed{\theta_1 = \operatorname{atan2}\left(\frac{o_y}{o_x}\right) - \operatorname{atan2}\left(\frac{a_2 \sin \theta_2}{a_1 + a_2 \cos \theta_2}\right)}$$

This equation is correct in the book.

Inverse Position Kinematics for the SCARA



$$o_4^0 = \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix}$$

$$\boxed{d_3 = -o_z - d_4}$$

$$\boxed{\theta_2 = \operatorname{atan2}\left(\frac{\pm\sqrt{1-\cos^2\theta_2}}{\cos\theta_2}\right)}$$

$$\boxed{\theta_1 = \operatorname{atan2}\left(\frac{o_y}{o_x}\right) - \operatorname{atan2}\left(\frac{a_2\sin\theta_2}{a_1 + a_2\cos\theta_2}\right)}$$

What questions do you have ?

Name ______________________________

Resubmission of Midterm Exam

MEAM 520, Introduction to Robotics
University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

November 8, 2013

If you doubt the correctness of any of the answers you submitted on the in-class midterm, you may rework those problems to try to earn back a proportion of the points you may have lost. You must decide whether to submit new answers for each of the five problems on the exam; if you do any lettered sub-question in a problem, you should resubmit all of them, as they are interrelated. Your final score on each problem will be a weighted average of your in-class score and your resubmission score: $s_{\text{final}} = (1-p)\ s_{\text{in-class}} + p\ s_{\text{resubmission}}$. We anticipate that this proportion p will be about 30%, but it may be raised or lowered to match overall class performance. If you do not submit new answers for a certain problem, your in-class score will be your final score. Completing this exam resubmission is completely optional.

You must do this exam resubmission independently, without talking about it with anyone else. You may use a calculator, your notes, the textbook, the Internet, and any other reference materials you find useful. However, you must not take assistance from any individual (including electronic correspondence of any kind), and you must not give assistance to other students in the class. If you accidentally discuss part of the exam with someone, do not fill out that part of the exam resubmisson. Any suspected violations of Penn's Code of Academic Integrity will be reported to the Office of Student Conduct for investigation.

This resubmission is due **by the start of class (noon) on Tuesday, November 12**. Because we will be discussing the exam in class that day, late resubmissions cannot be accepted. If you need clarification on any question, please post a private note on Piazza. When you work out each problem, please show all steps and $\boxed{\text{box your answer}}$.

	Points	Score
Problem 1	20	
Problem 2	15	
Problem 3	10	
Problem 4	15	
Problem 5	40	
Total	100	

I agree to abide by the University of Pennsylvania Code of Academic Integrity during this exam resubmission. I pledge that all work is my own and has been completed without the use of unauthorized aid or materials.

Signature ______________________________ Date ______________

1

86 of 94 students did the resubmission.

Most of the 8 students who did not resubmit did quite well on the in-class exam.

On average, students who resubmitted correctly answered 56% of the points they had missed in class.

Resubmission especially seemed to help with calculations and just having time to do all parts of the test. It didn't help with most conceptual confusions.

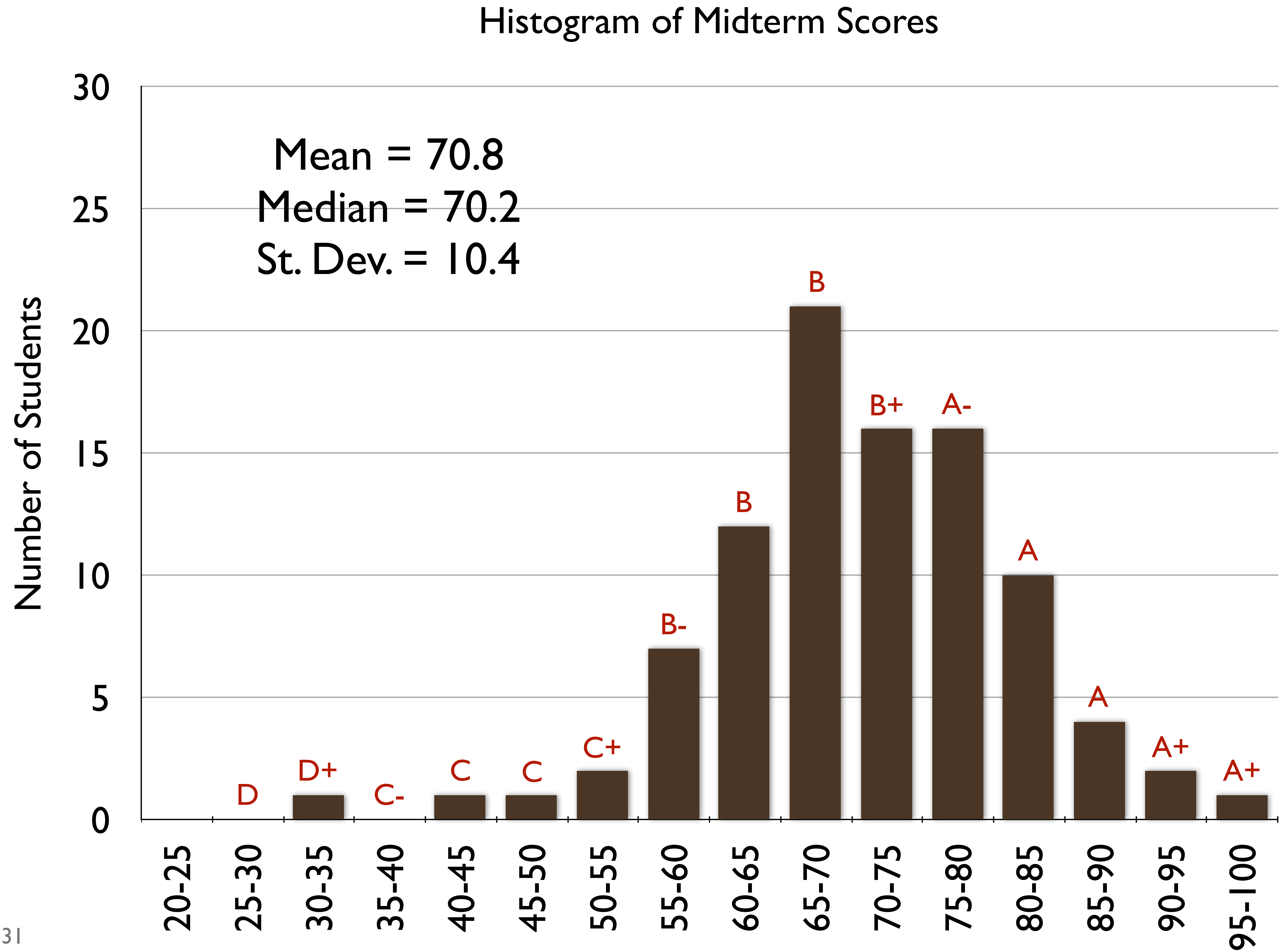
Canvas will soon show resubmission scores for everyone, separated out by problem.

If you did not resubmit a problem, the score listed there is just your in-class score for that problem.

Your grade on the midterm is a weighted average of your in-class and resubmission scores.

$$s_{\text{final}} = 0.7\, s_{\text{in-class}} + 0.3\, s_{\text{resubmission}}$$

Overall Resubmission Scores
In Class
New Grade
Resubmission
100
75
50
25
0
47
97
Students


Histogram of Midterm Scores
Mean = 70.8
Median = 70.2
St. Dev. = 10.4
Number of Students
0
5
10
15
20
25
30
D
D+
C-
C
C
C+
B-
B
B
B+
A-
A
A
A+
A+
20-25
25-30
30-35
35-40
40-45
45-50
50-55
55-60
60-65
65-70
70-75
75-80
80-85
85-90
90-95
95-100

Handing back resubmissions
in alphabetical order by last name

Naomi has last names A – K

Dr. K had last names L – Z

Graded homework assignments and exams
are also attached to some resubmissions.

Look over your exam and your resubmission; compare both with the solution (on Piazza).

Check the grades posted on Canvas as well.

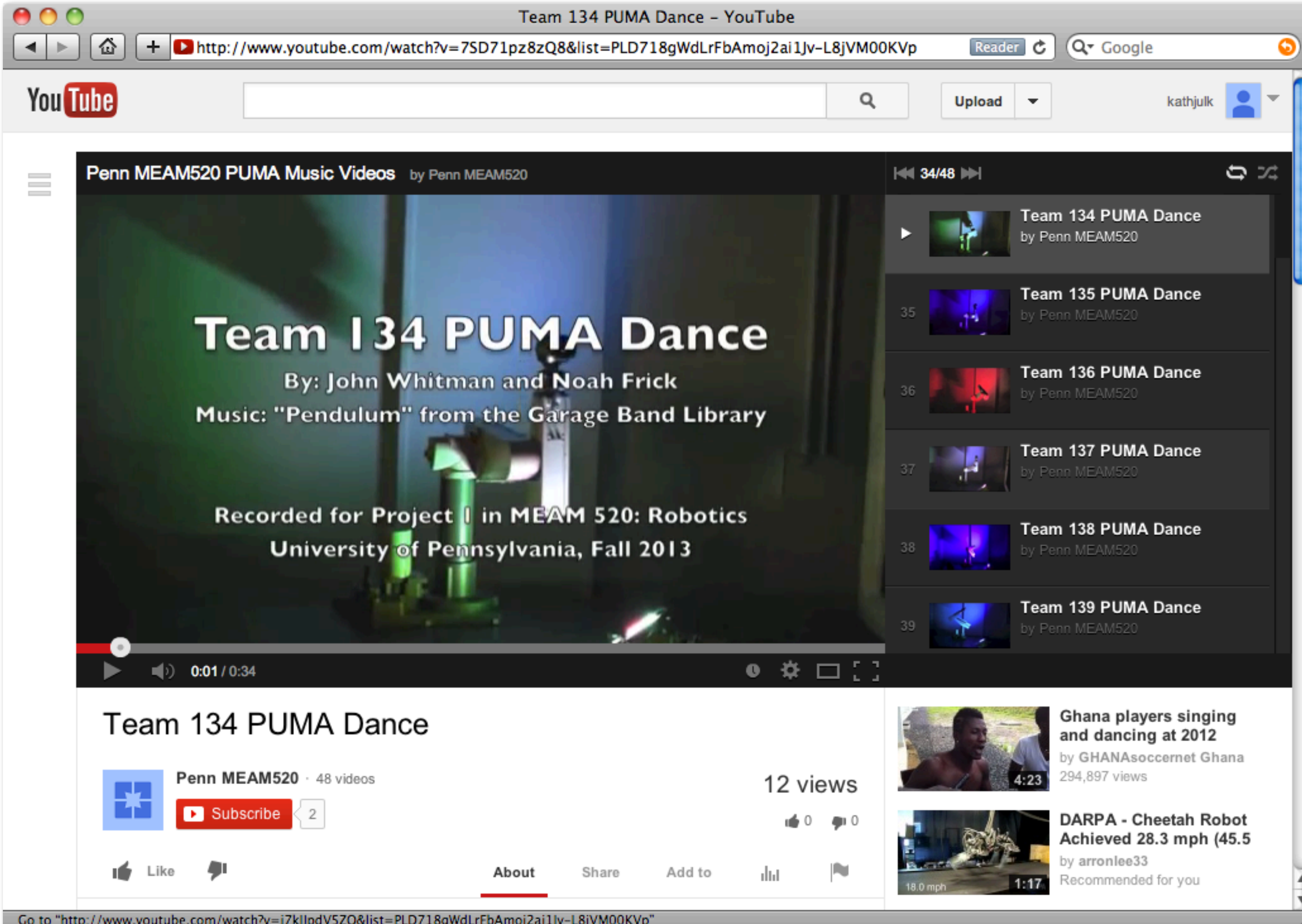
If you think we made a mistake in grading your test, write out an explanation on a separate piece of paper; do not write on or modify your exam at all.

Attach your written inquiry to your test and give it to Naomi.

We will correct any grading mistakes.

What questions do you have?

Let's watch Teams 134–140.

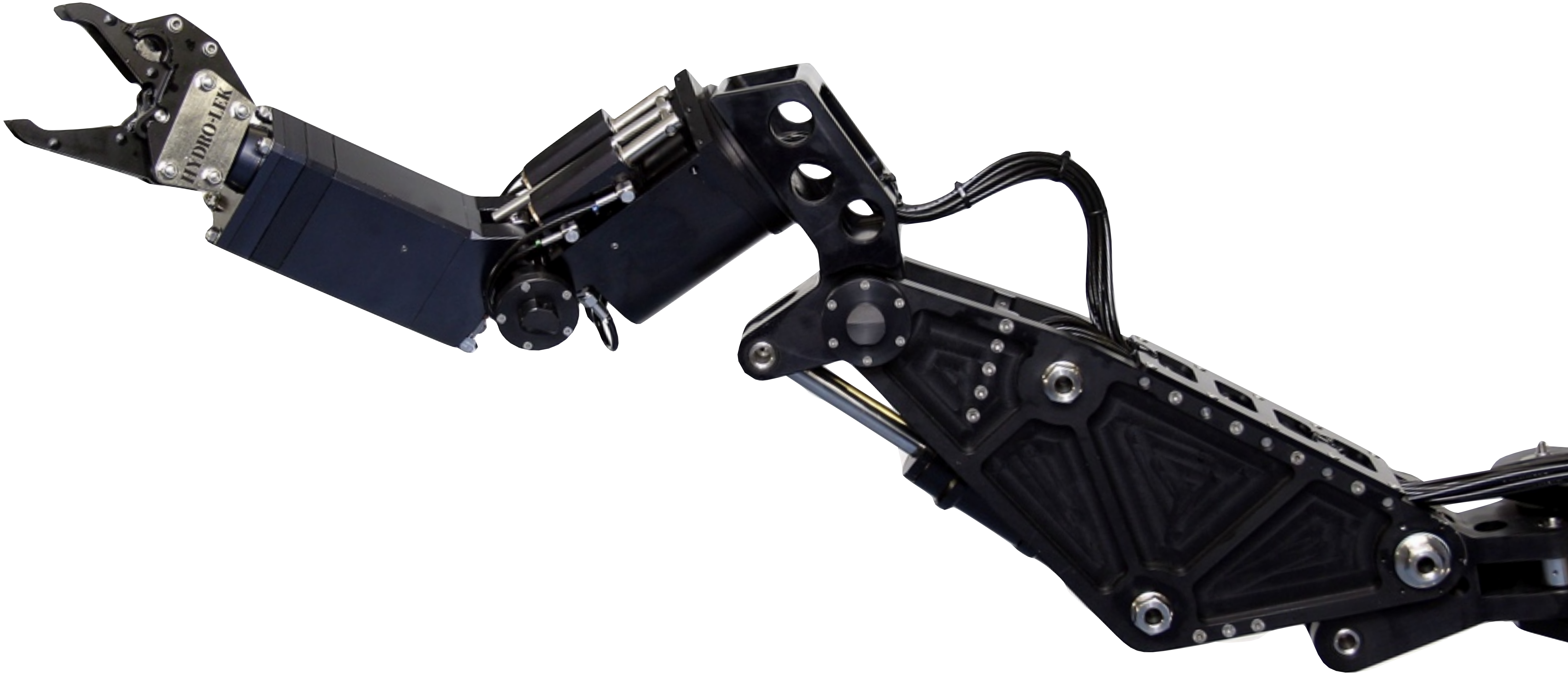


Project 2

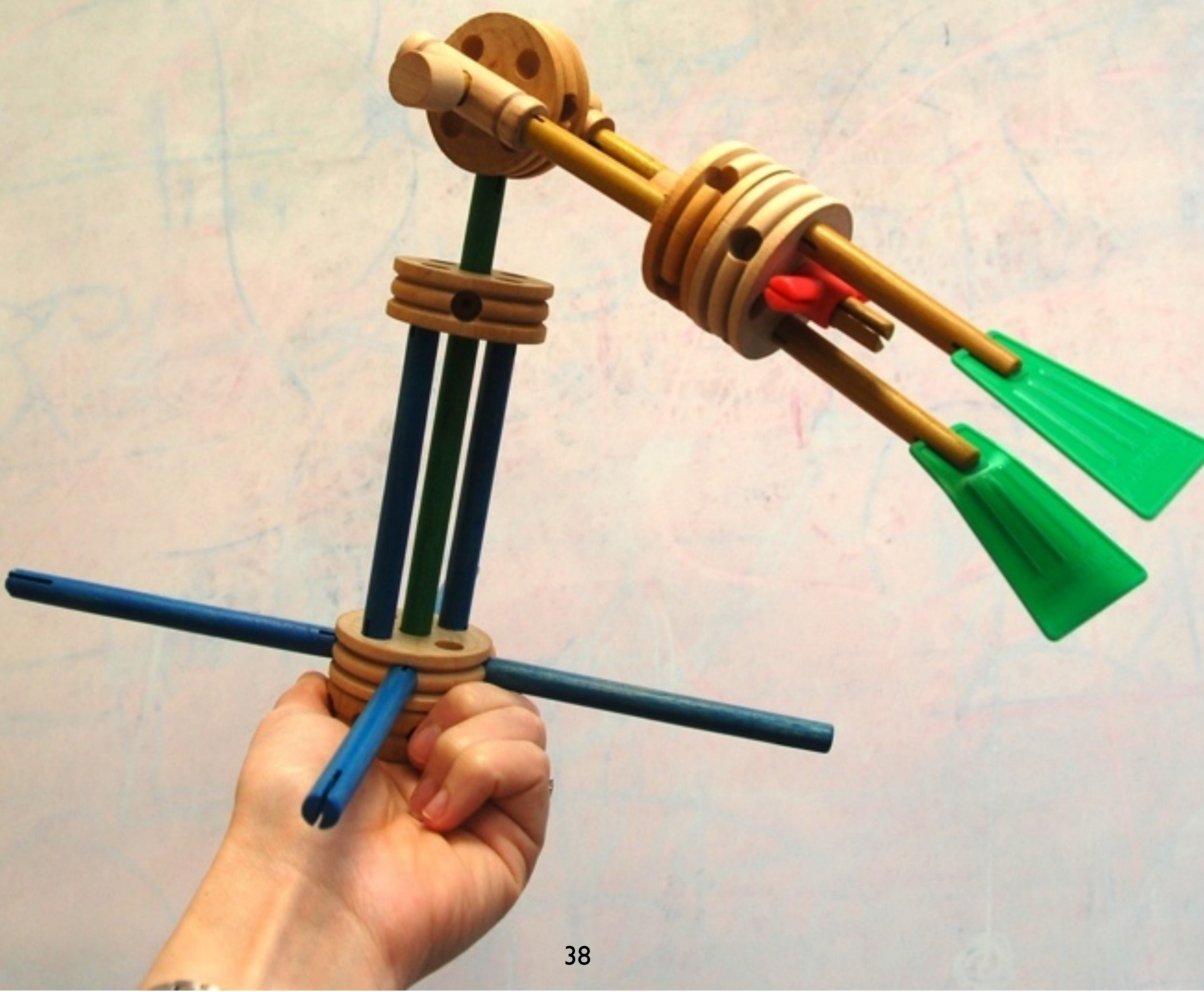
PUMA Light Painting

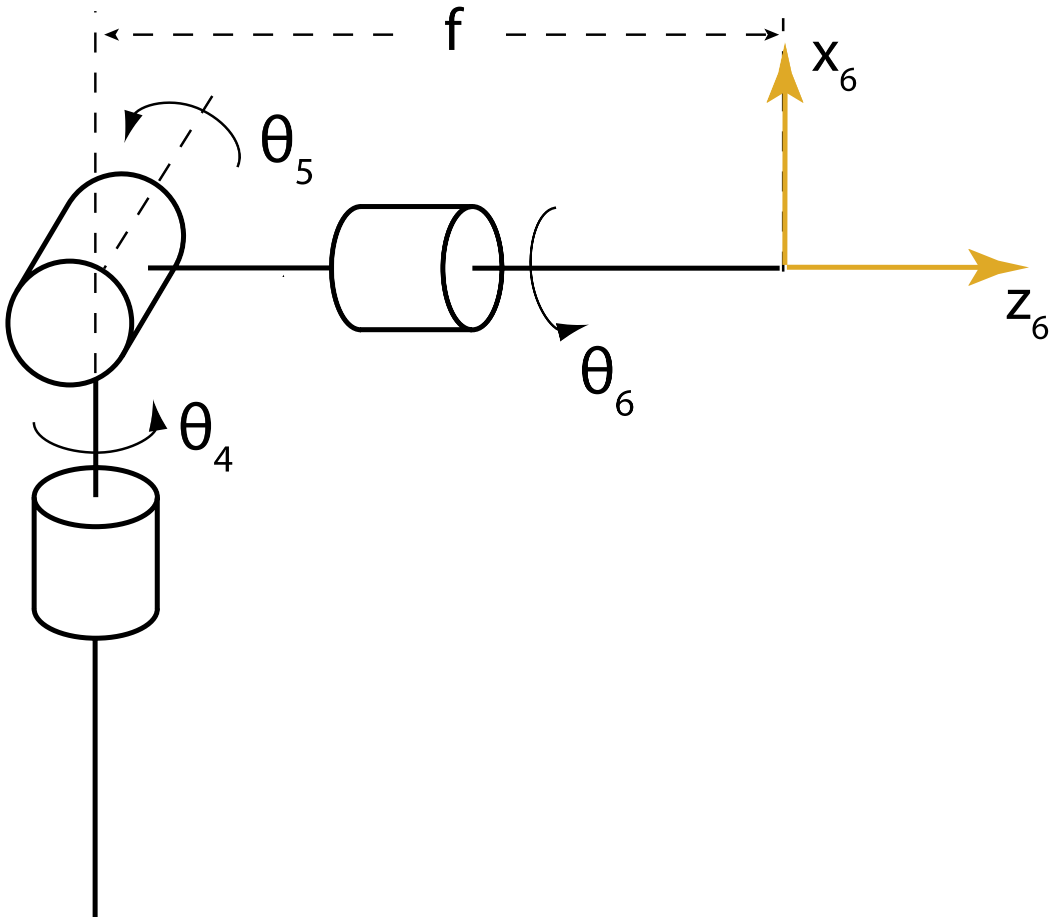
- You will work in teams of three students.
- Because we have 94 students, there will be one team of four students or two teams of two students.
- When you choose your team, send an email to meam520@seas.upenn.edu. The subject should be "Project 2 Team Selection". List the three full names in the body of the email. Only one person needs to submit the team. We will respond with a team number and post it on Piazza.
- As another alternative, we are happy to assign you to a random partner or two. To ask us to do this, send an email to meam520@seas.upenn.edu with the subject "Project 2 Partner Request" with your full name (or two full names if in a pair) in the body of the email.
- If possible, please do this by midnight (11:59 p.m.) on Thursday, November 21.

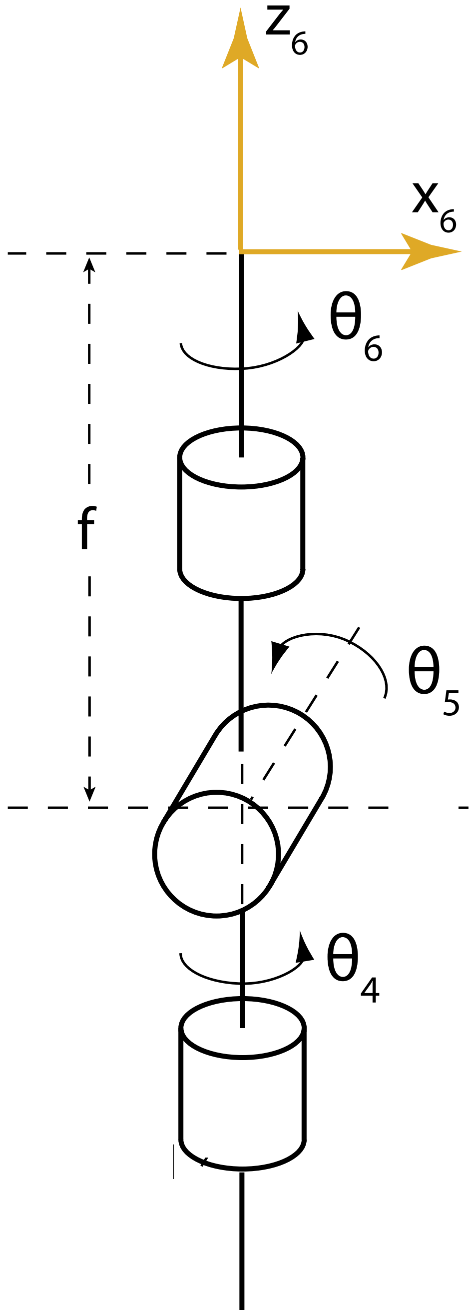
Inverse Orientation Kinematics



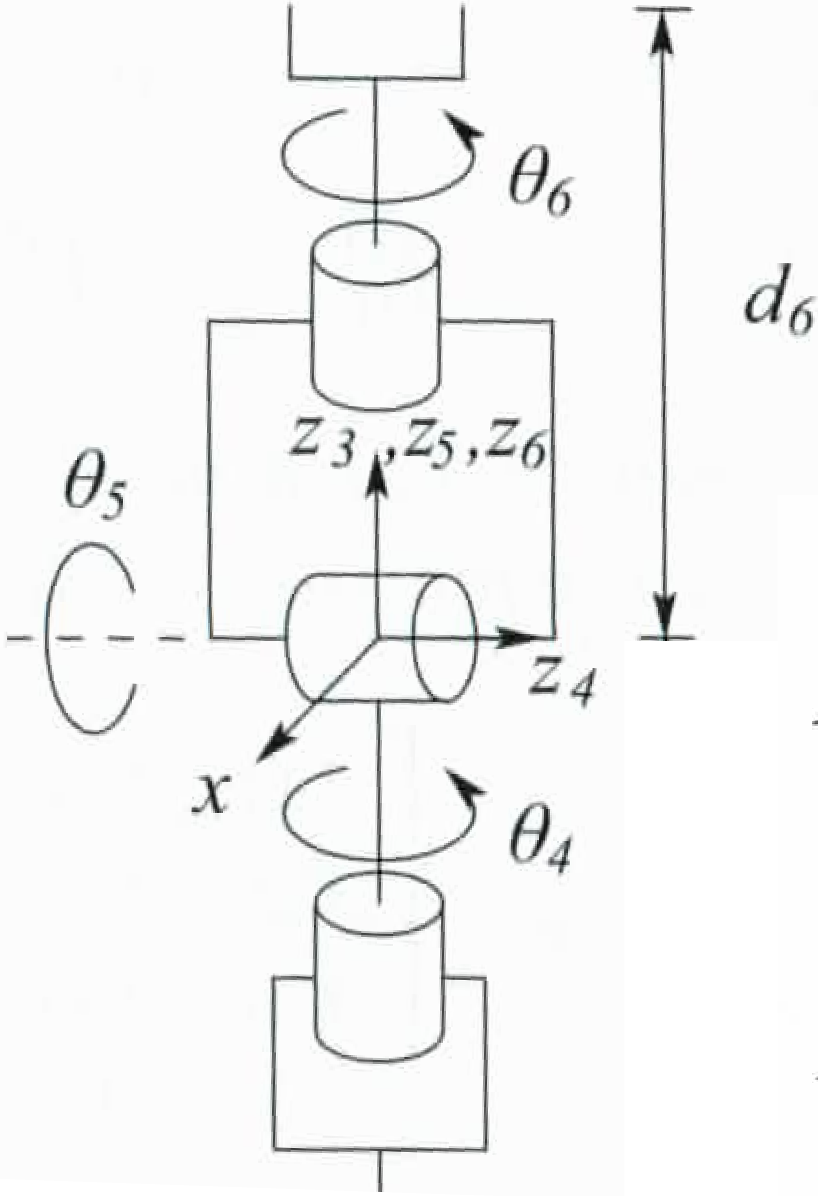
Spherical Wrist
RRR with middle joint perpendicular to the other two.




f
x6
z6
θ5
θ6
θ4


z6
x6
θ6
f
θ5
θ4

Given rotation matrix **R**, find the joint angles that put the end-effector in the desired orientation.



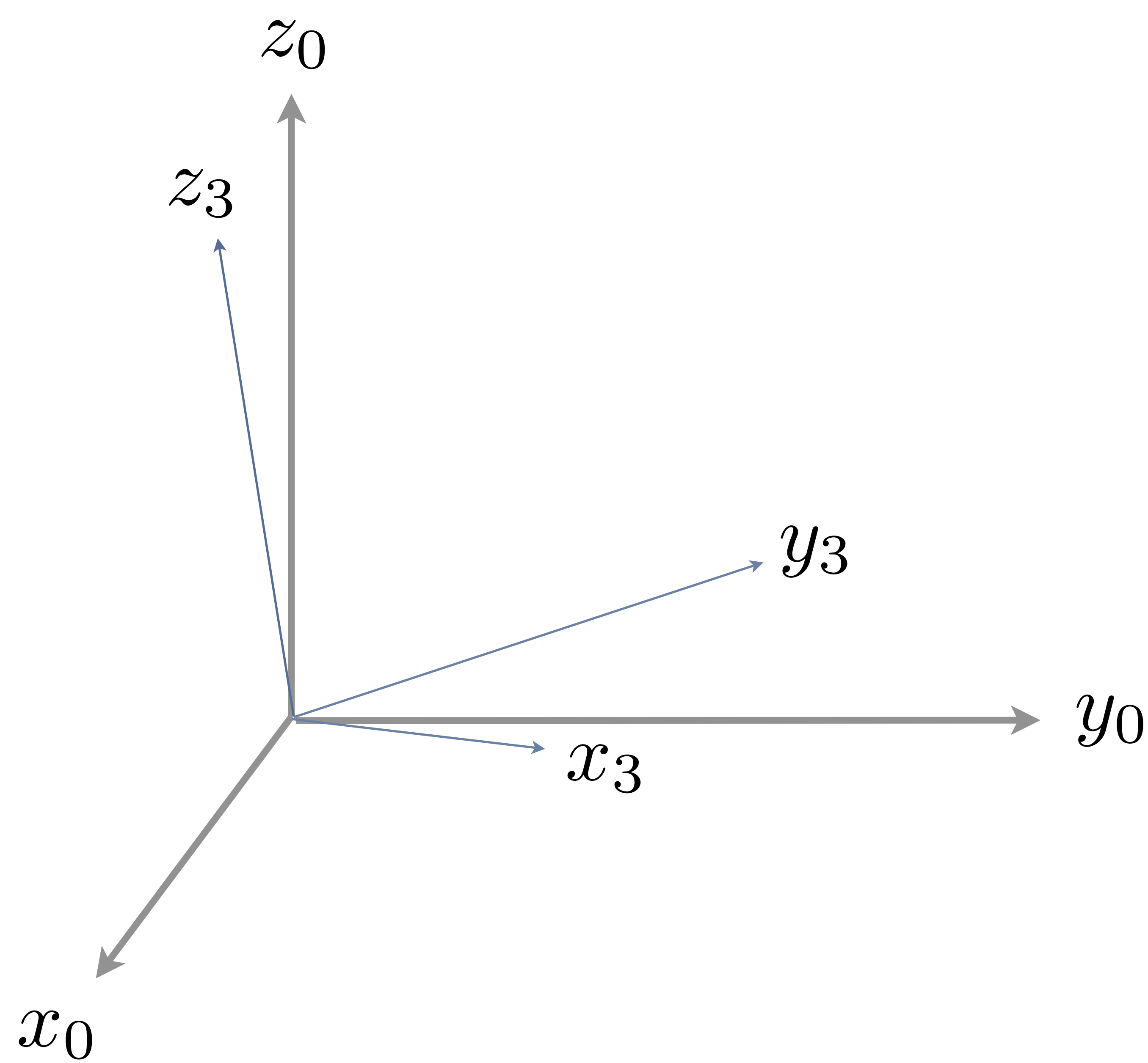
Link	a_i	α_i	d_i	θ_i
4	0	-90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6	θ_6^*

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

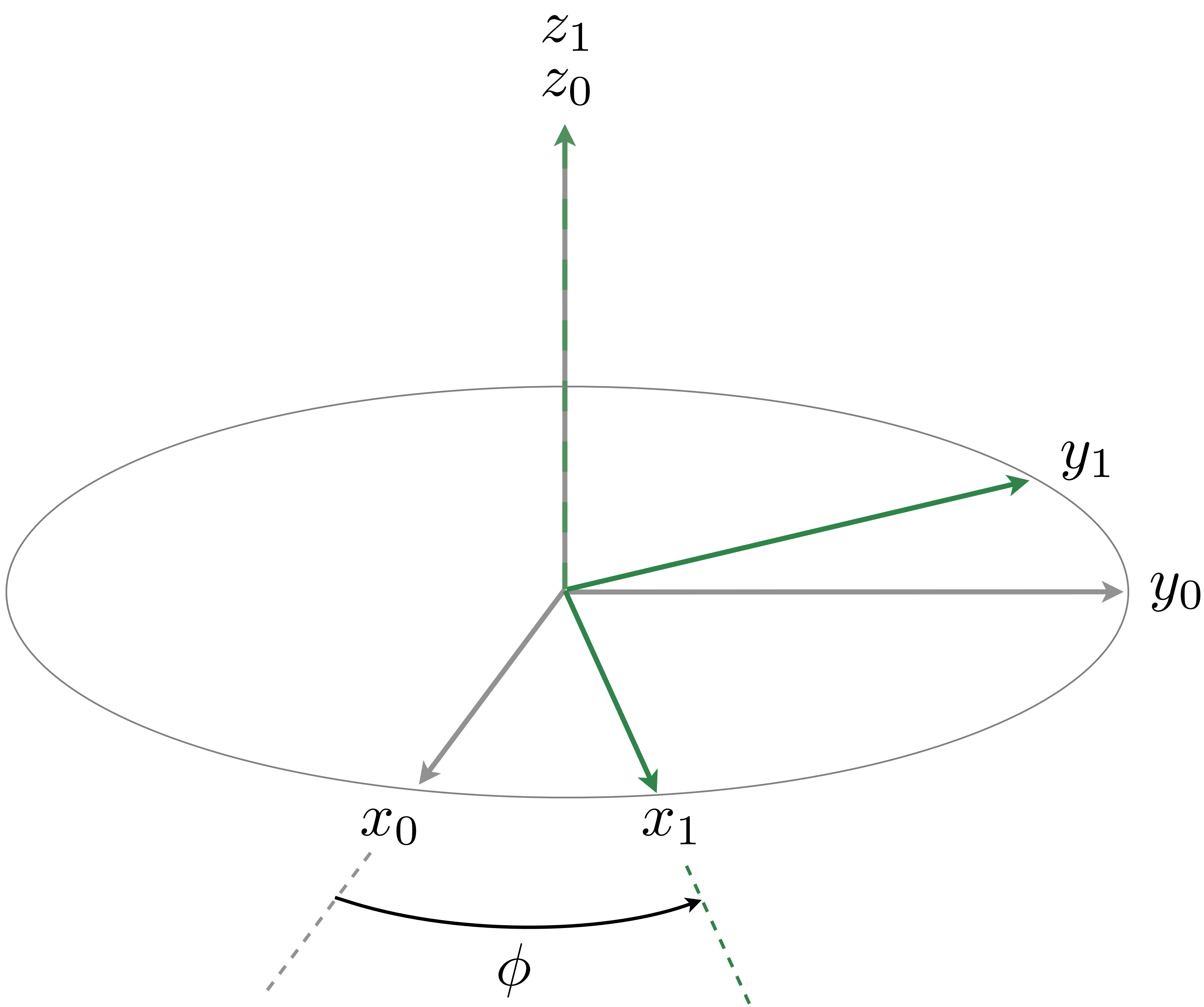
$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_6^3 &= A_4 A_5 A_6 \\ &= \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

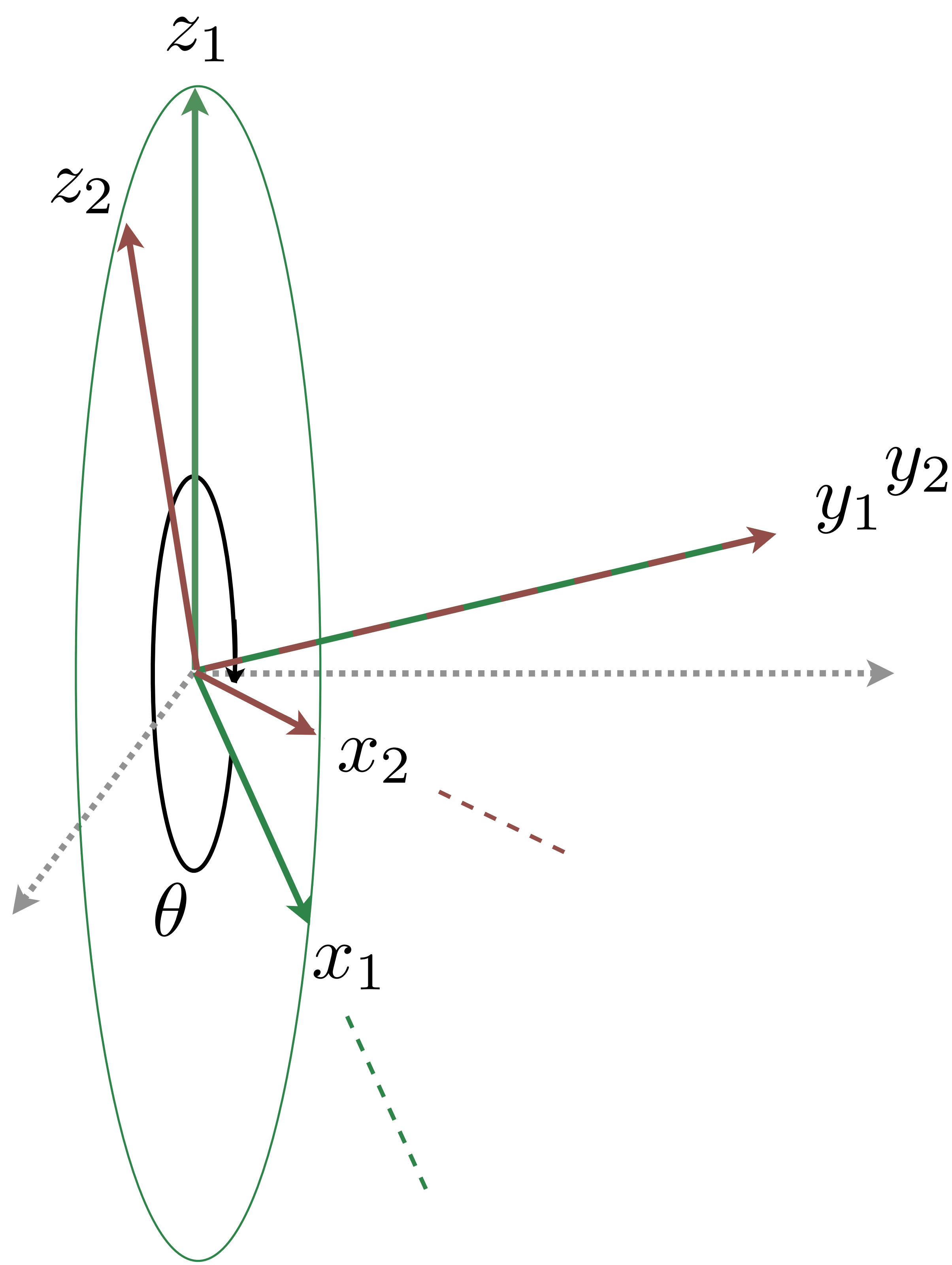
Define a set of three **intermediate** angles, ϕ, θ, ψ , to go from $0 \rightarrow 3$



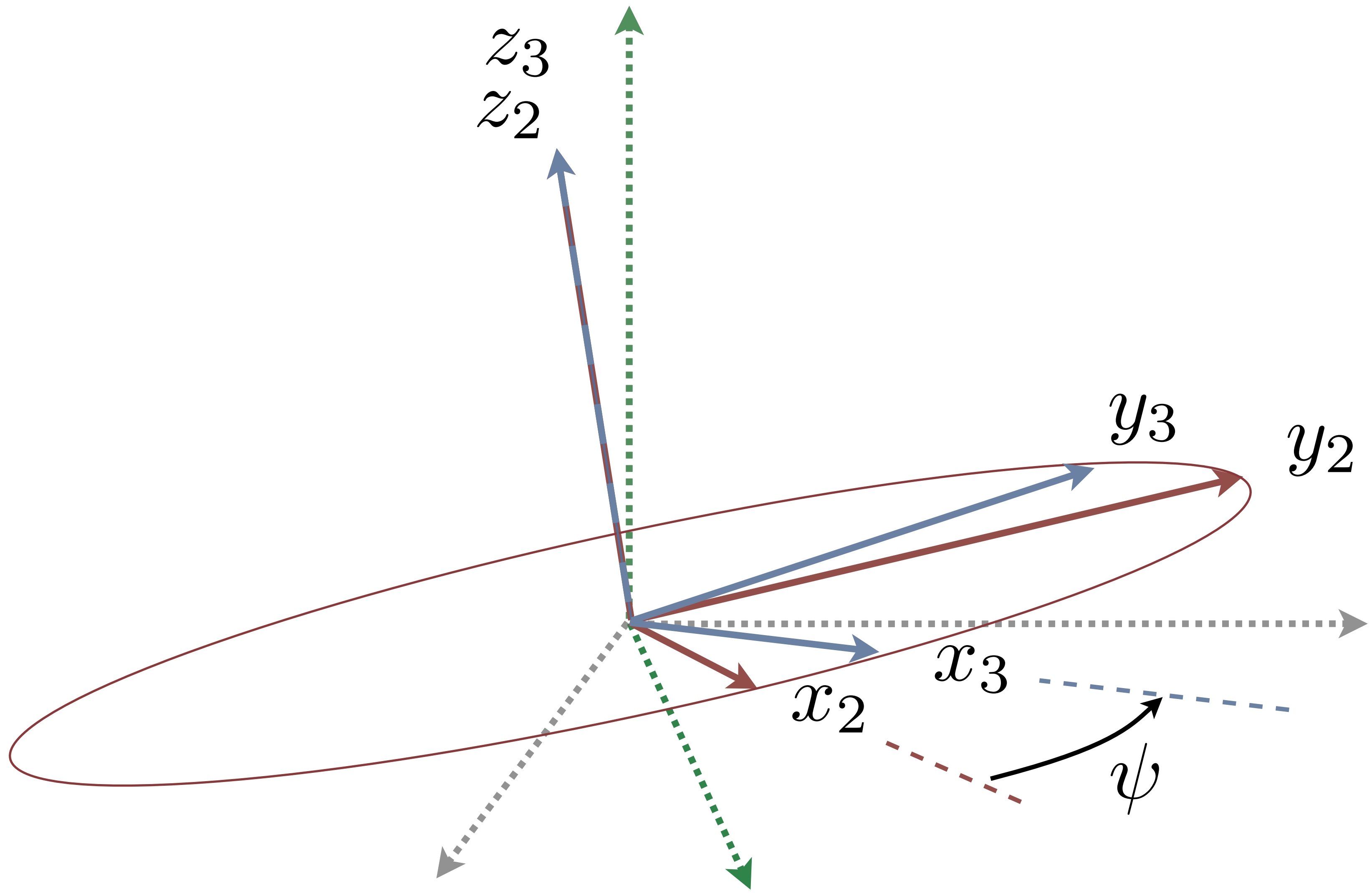
step 1: rotate by ϕ about z_0



step 2: rotate by θ about y_1



step 3: rotate by ψ about z_2



Euler Angles to Rotation Matrices

(**post**-multiply using the **basic rotation matrices**)

$$\mathbf{R} = \mathbf{R}_{z,\phi}\, \mathbf{R}_{y,\theta}\, \mathbf{R}_{z,\psi}$$

$$= \begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

$$
\begin{aligned}
T_6^3 &= A_4 A_5 A_6 \\
&= \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$\theta_4 = \phi \qquad \theta_5 = \theta \qquad \theta_6 = \psi$$

$$
= \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}
$$

The book explains how to calculate the three angles given **R**: see SHV pages 55-56

Rotation Matrix to Euler Angles?

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Use atan2 to determine ϕ for both θ options

$$= \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Use atan2 to determine ψ for both θ options

Two solutions for θ because sign of s_θ is not known.

$$\phi = ? \quad \theta = ? \quad \psi = ?$$

Special case for cos(theta2) = +1, −1

To find a solution for this problem we break it down into two cases. First, suppose that not both of r_{13}, r_{23} are zero. Then from Equation (2.26) we deduce that $s_\theta \neq 0$, and hence that not both of r_{31}, r_{32} are zero. If not both r_{13} and r_{23} are zero, then $r_{33} \neq \pm 1$, and we have $c_\theta = r_{33}$, $s_\theta = \pm\sqrt{1 - r_{33}^2}$ so

$$\theta = \text{Atan2}\left(r_{33}, \sqrt{1 - r_{33}^2}\right) \tag{2.28}$$

or

$$\theta = \text{Atan2}\left(r_{33}, -\sqrt{1 - r_{33}^2}\right) \tag{2.29}$$

where the function Atan2 is the **two-argument arctangent function** defined in Appendix A.

If we choose the value for θ given by Equation (2.28), then $s_\theta > 0$, and

$$\phi = \text{Atan2}(r_{13}, r_{23}) \tag{2.30}$$
$$\psi = \text{Atan2}(-r_{31}, r_{32}) \tag{2.31}$$

If we choose the value for θ given by Equation (2.29), then $s_\theta < 0$, and

$$\phi = \text{Atan2}(-r_{13}, -r_{23}) \tag{2.32}$$
$$\psi = \text{Atan2}(r_{31}, -r_{32}) \tag{2.33}$$

Thus, there are two solutions depending on the sign chosen for θ.

If $r_{13} = r_{23} = 0$, then the fact that R is orthogonal implies that $r_{33} = \pm 1$, and that $r_{31} = r_{32} = 0$. Thus, R has the form

$$R = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix} \tag{2.34}$$

If $r_{33} = 1$, then $c_\theta = 1$ and $s_\theta = 0$, so that $\theta = 0$. In this case, Equation (2.26) becomes

$$\begin{bmatrix} c_\phi c_\psi - s_\phi s_\psi & -c_\phi s_\psi - s_\phi c_\psi & 0 \\ s_\phi c_\psi + c_\phi s_\psi & -s_\phi s_\psi + c_\phi c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{\phi+\psi} & -s_{\phi+\psi} & 0 \\ s_{\phi+\psi} & c_{\phi+\psi} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, the sum $\phi + \psi$ can be determined as

$$\phi + \psi = \text{Atan2}(r_{11}, r_{21}) = \text{Atan2}(r_{11}, -r_{12}) \tag{2.35}$$

Since only the sum $\phi + \psi$ can be determined in this case, there are infinitely many solutions. In this case, we may take $\phi = 0$ by convention. If $r_{33} = -1$, then $c_\theta = -1$ and $s_\theta = 0$, so that $\theta = \pi$. In this case Equation (2.26) becomes

$$\begin{bmatrix} -c_{\phi-\psi} & -s_{\phi-\psi} & 0 \\ s_{\phi-\psi} & c_{\phi-\psi} & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{2.36}$$

The solution is thus

$$\phi - \psi = \text{Atan2}(-r_{11}, -r_{12}) \tag{2.37}$$

As before there are infinitely many solutions.

What questions do you have ?