Name Solutions

Final Exam

MEAM 520, Introduction to Robotics
University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

December 18, 2013

You must take this exam independently, without assistance from anyone else. You may bring in a calculator and four 8.5"×11" sheets of notes for reference. Aside from these pages of notes, you may not consult any outside references, such as the textbook or the Internet. Any suspected violations of Penn's Code of Academic Integrity will be reported to the Office of Student Conduct for investigation.

This exam consists of several problems. We recommend you look at all of the problems before starting to work. If you need clarification on any question, please ask a member of the teaching team. When you work out each problem, please show all steps and box your answer. On problems involving actual numbers, please keep your solution symbolic for as long as possible before converting to numbers at the end; this will make your work easier to follow and easier to grade. The exam is worth a total of 100 points, and partial credit will be awarded for the correct approach even when you do not arrive at the correct answer.

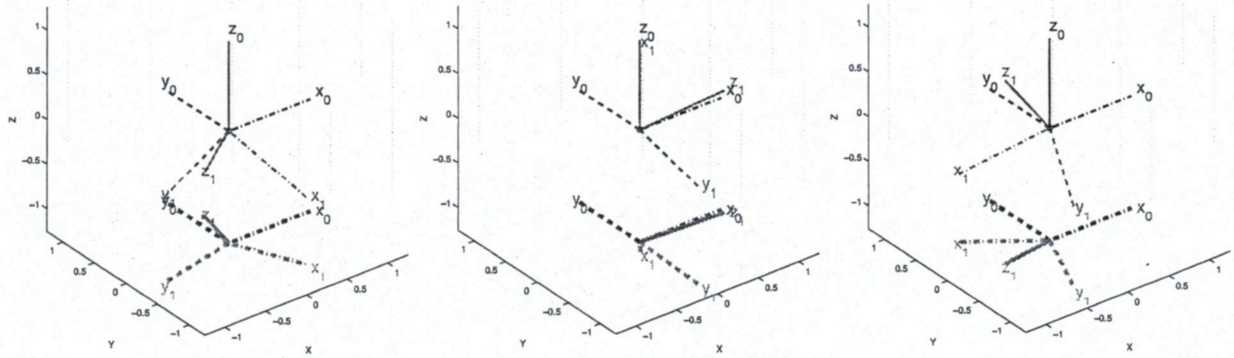
	Points	Score
Problem 1	20	
Problem 2	20	
Problem 3	20	
Problem 4	20	
Problem 5	20	
Total	100	

I agree to abide by the University of Pennsylvania Code of Academic Integrity during this exam. I pledge that all work is my own and has been completed without the use of unauthorized aid or materials.

Signature ______________________________

Date ______________________________

Problem 1: Rotation Matrices and Rotation Parameterizations ***(20 points)***



A *B* *C*

The graphs above depict three different three-dimensional rotation matrices, labeled A, B, and C; each one represents the orientation of a different frame 1 (x_1, y_1, z_1) relative to frame 0 (x_0, y_0, z_0). All frames are right-handed, and frame 0 is always shown in the same orientation. To aid you in perceiving the three-dimensional structure, each graph also shows the shadows (projections straight down) of both frames in a plane parallel to and below the x_0-y_0 plane.

For each question below, choose the graph that best corresponds to the provided rotation matrix or rotation parameterization. **Write your answer (**A**,** B**, or** C**) on each blank line (________).** There is no minimum or maximum number of times that you may use each answer. *(2.5 points per answer)*

$$R_1^0 = \begin{array}{c} \\ x_0 \rightarrow \\ y_0 \rightarrow \\ z_0 \rightarrow \end{array} \begin{array}{c} \begin{array}{ccc} x_1 \downarrow & y_1 \downarrow & z_1 \downarrow \end{array} \\ \begin{bmatrix} -0.704 & -0.358 & -0.614 \\ 0.512 & -0.855 & -0.088 \\ -0.493 & -0.376 & 0.784 \end{bmatrix} \end{array} \quad \underline{\ C\ }$$

y_1 mostly $-y_0$
$z_1 \perp y_0$

$$R_1^0 = \begin{array}{c} \\ x_0 \rightarrow \\ y_0 \rightarrow \\ z_0 \rightarrow \end{array} \begin{array}{c} \begin{array}{ccc} x_1 \downarrow & y_1 \downarrow & z_1 \downarrow \end{array} \\ \begin{bmatrix} 0.348 & -0.928 & 0.133 \\ -0.784 & -0.210 & 0.584 \\ -0.514 & -0.308 & -0.801 \end{bmatrix} \end{array} \quad \underline{\ A\ }$$

y_1 mostly $-x_0$

ZYZ Euler Angles for R_1^0
Z $\phi = -3.5°$, $\theta = 83.5°$, $\psi = -169.5°$ __B__
almost 0 Y ~90° Z ~180° intermediate
Close to aligned w/ +,− axes.

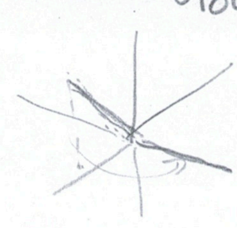
ZYZ Euler Angles for R_1^0 Z
$\phi = -171.8°$, $\theta = 38.3°$, $\psi = -37.4°$ __C__
Z ~180° Y tilts z down 38°
also $\cos^{-1}(0.784) = 38.3°$ from R_1^0's 3,3 element

XYZ Yaw, Pitch, Roll Angles for R_1^0
$\psi = -25.6°$, $\theta = 29.5°$, $\phi = 144.0°$ __C__
X Y Z fixed frame

XYZ Yaw, Pitch, Roll Angles for R_1^0
$\psi = 21.0°$, $\theta = 149.1°$, $\phi = 113.9°$ __A__
X Y almost upside down Z fixed frame

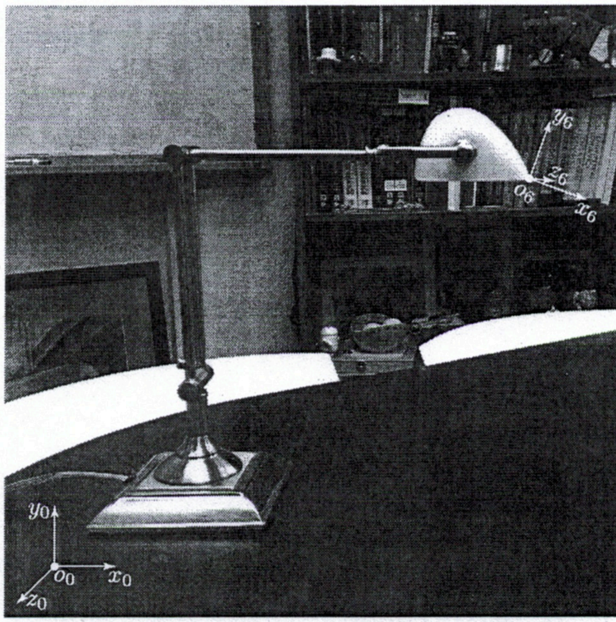
Axis / Angle for R_1^0
$\hat{k}^0 = [-0.802 \quad 0.583 \quad 0.130]^T$ __A__
$\theta = 146.3°$
close to 180° around an axis in x_0-y_0 plane
so flip down.

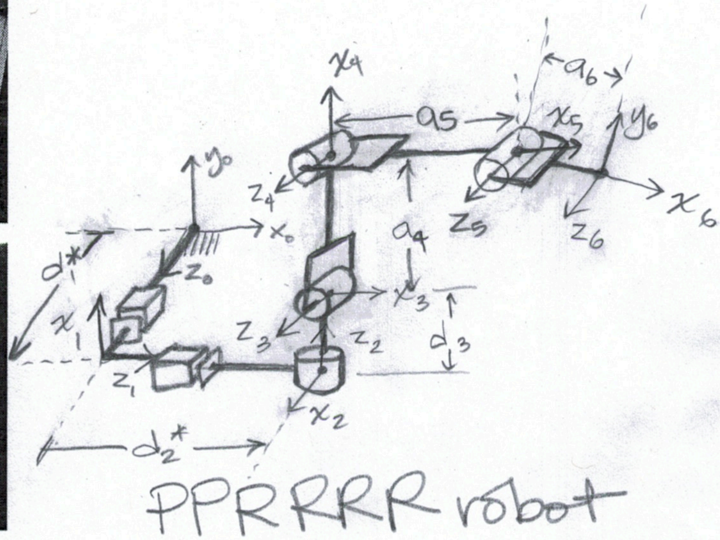
Axis / Angle for R_1^0
$\hat{k}^0 = [\underline{-0.661} \quad 0.081 \quad \underline{-0.746}]^T$ __B__
$\theta = 174.8°$ ~180° around unit vector in x_0-z_0 plane
so y flips.



Problem 2: Kinematics of a Desk Lamp Robot *(20 points)*

The photo below shows a desk lamp that has a base that can slide along and rotate on the table, along with three lockable revolute joints. All joint angles are shown at zero. The base and end-effector frames are defined as shown. In the zero pose, z_0 and z_6 both point straight out of the page, and the angle between x_0 and x_6 is 20°. This lamp is available at the front of the exam room.





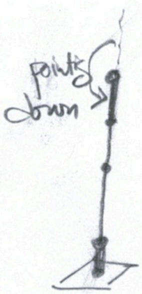
a. Following the book's conventions for drawing serial-chain robots in three dimensions, use the empty space above right to **sketch the lamp as a six-jointed robot in its zero pose**; draw all revolute joints at zero (the angles shown in the photo), and draw any prismatic joints at a positive displacement. *(2 points)*

b. Follow the DH conventions to **draw your intermediate frames** on your diagram. *(2 points)*

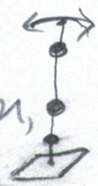
c. Fill in the lamp's table of **DH parameters** below. **Label** any symbolic parameters that you introduce on your diagram. *(9 points)*

i	a	α	d	θ
1	0	90°	d_1^*	90°
2	0	90°	d_2^*	90°
3	0	90°	d_3	$\theta_3^*+90°$
4	a_4	0°	0	$\theta_4^*+90°$
5	a_5	0°	0	$\theta_5^*-90°$
6	a_6	0°	0	$\theta_6^*-20°$

d. What are this robot's **translational velocity singularities**, if any? Here we are considering the tip of the glass shade as the end-effector. You should be able to answer this question without any calculations. *(3 points)*



The two prismatic joints enable the robot tip always to move in the plane of the table. The vertical (y_0) direction is the only direction in which translational velocity singularities could occur. The singularity will be whenever all links are vertical



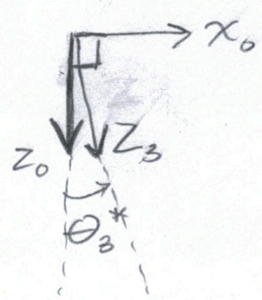
All arm revolute joints make horizontal motion, and the first revolute causes no motion. Arm could fold back on itself too, as long as all segments vertical

e. What is this robot's J_ω **matrix**? Because the desk lamp robot has six joints, this matrix should have three rows and six columns. *(4 points)*

$$J_\omega = \begin{bmatrix} 0 & 0 & 0 & \sin\theta_3^* & \sin\theta_3^* & \sin\theta_3^* \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta_3^* & \cos\theta_3^* & \cos\theta_3^* \end{bmatrix}$$

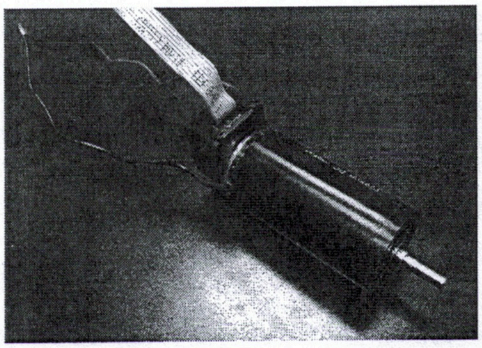
$\uparrow$ $\uparrow$

prismatic



Problem 3: Motor Dynamics *(20 points)*

The Maxon RE 35 285785 brushed DC motor has graphite brushes, is 35 mm in diameter, and has an assigned power rating of 90 Watts. A table of additional data for this motor is shown below:



Motor Data		
Values at nominal voltage		
1 Nominal voltage	V	15
2 No load speed	rpm	7180
3 No load current	mA	247
4 Nominal speed	rpm	6500
5 Nominal torque (max. continuous torque)	mNm	73.1
6 Nominal current (max. continuous current)	A	4
7 Stall torque	mNm	929
8 Starting current	A	47.8
9 Max. efficiency	%	83
Characteristics		
10 Terminal resistance	Ω	0.314
11 Terminal inductance	mH	0.085
12 Torque constant	mNm/A	19.4
13 Speed constant	rpm/V	491
14 Speed / torque gradient	rpm/mNm	7.93
15 Mechanical time constant	ms	5.65
16 Rotor inertia	gcm²	68.1

a. From the other data listed in the table, show that this motor's **starting current** is 47.8 A *(3 points)*

Apply nominal voltage to motor

$$V_{nom} = i_a R_a + L\cancel{\frac{di_a}{dt}} + K_m \cancel{\omega}$$

$V_{nom} = 15V$; $R_a = 0.314\Omega$; Not changing at very start. ; not turning

$$i_a = \frac{V_{nom}}{R_a} = \frac{15V}{0.314V}$$

$$\boxed{i_a = 47.77A}$$

b. Based on the no-load test data reported in the table, what is B_m, this motor's **frictional damping coefficient** in Nm/(rad/s)? *(9 points)*

$V = 15V$

$$J\underbrace{\cancel{\alpha}}_{0} = K_t i - B_m \omega + \underbrace{\cancel{\tau_{load}}}_{0}$$

$$K_t i = B_m \omega \qquad B_m = \frac{K_t i_{noload}}{\omega_{noload}}$$

$$\omega_{no\,load} = 7180 \frac{rev}{min} \cdot \frac{2\pi\, rad}{1\, rev} \cdot \frac{1\, min}{60\, s} = 751.88\, rad/s$$

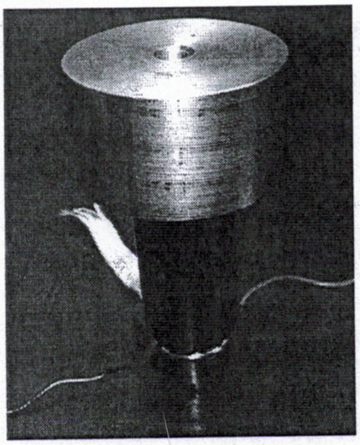
$i_{no\,load} = 0.247\, A$

$K_t = 0.0194 \frac{Nm}{A}$

$$\tau_{no\,load} = 0.247A \cdot 0.0194 \frac{Nm}{A} = 0.0047918\, Nm$$

$$B_m = \frac{\tau_{no\,load}}{\omega_{no\,load}} = \frac{0.0047918\, Nm}{751.88\, rad/s} = \boxed{0.000006373\, Nm/(rad/s)}$$

$$\boxed{B_m = 6.373\times10^{-6}\, Nm/(rad/s)}$$

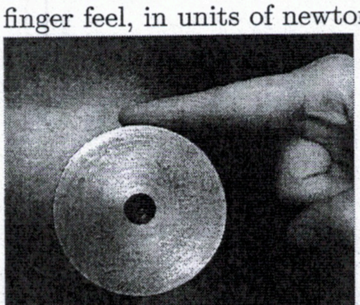


Motor Data		
Values at nominal voltage		
1 Nominal voltage	V	15
2 No load speed	rpm	7180
3 No load current	mA	247
4 Nominal speed	rpm	6500
5 Nominal torque (max. continuous torque)	mNm	73.1
6 Nominal current (max. continuous current)	A	4
7 Stall torque	mNm	929
8 Starting current	A	47.8
9 Max. efficiency	%	83
Characteristics		
10 Terminal resistance	Ω	0.314
11 Terminal inductance	mH	0.085
12 Torque constant	mNm/A	19.4
13 Speed constant	rpm/V	491
14 Speed / torque gradient	rpm/mNm	7.93
15 Mechanical time constant	ms	5.65
16 Rotor inertia	gcm²	68.1

c. Imagine we rigidly attached a 70 mm-diameter steel wheel to this motor's output shaft, as shown in the image above. You hold the body of the motor still on a table so it cannot spin, and the wheel can spin freely in the air. If we then applied 15 V to the motor, **how would the resulting motor motion differ from what occurs during a standard no-load test, if at all?** Explain. *(3 points)*

Adding this wheel greatly increases the (J_m) inertia of the spinning part of the motor. The motor will thus accelerate from rest much more slowly than usual (without wheel), but it will reach the same no-load speed and no-load current as in the table.

d. Now we connect this motor to a current-drive circuit. You push one finger into the side of the 70 mm-diameter wheel so it cannot rotate relative to the motor body, as shown in the top-view photo below. You then apply 2 A to the motor. What **tangential force** will your finger feel, in units of newtons? *(5 points)*



$$\tau = k_t i = \left(0.0194 \frac{Nm}{A}\right)(2A)$$

$$\tau = 0.0388 \text{ Nm}$$

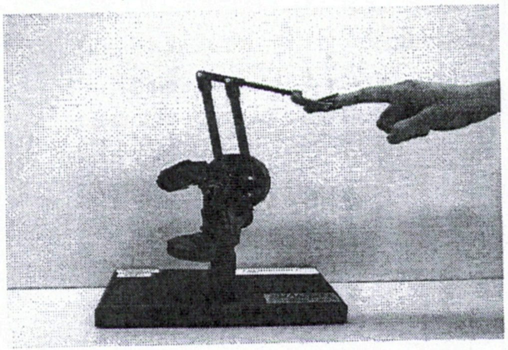
$$F \cdot r = \tau \qquad F = \frac{\tau}{r} \qquad r = 0.035 m$$

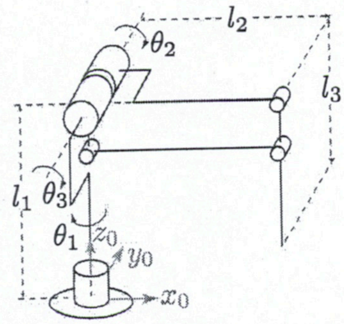
$$F = \frac{0.0388 Nm}{0.035 m} = 1.10857 N$$

$$\boxed{F = 1.11 N}$$

The finger counteracts the motor torque with a tangential force applied at 35mm radius.

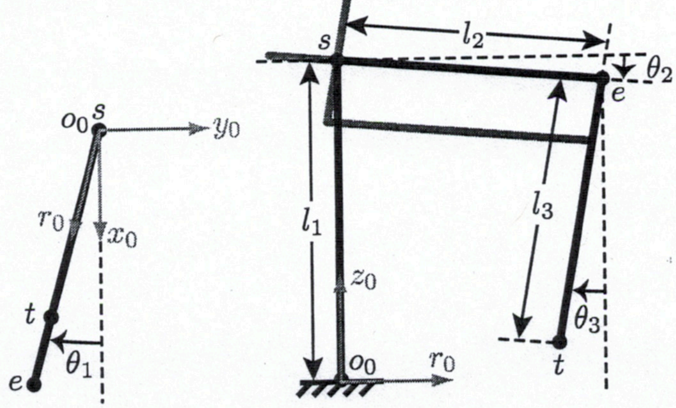
Problem 4: Phantom Inverse Kinematics *(20 points)*





As discussed in class, the SensAble Phantom Premium 1.0 is a small RRR manipulator commonly used for haptic rendering. The axis locations, positive directions, and zero configuration of the three joint angles, θ_1, θ_2, and θ_3, are defined by the schematic above. The schematic also shows the right-handed base frame x_0, y_0, z_0, and the three link lengths, l_1, l_2, and l_3.

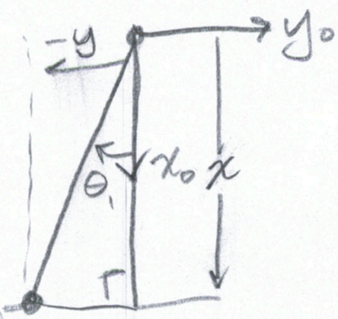
Below are two diagrams that were used to derive the robot's forward kinematics. The point labeled t is the robot's tip, while s and e are the shoulder and elbow, respectively. Given the joint angles, the x, y, z position of the Phantom's tip in the base frame can be calculated as follows:



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (l_2 \cos\theta_2 - l_3 \sin\theta_3)\cos\theta_1 \\ -(l_2 \cos\theta_2 - l_3 \sin\theta_3)\sin\theta_1 \\ l_1 - l_2 \sin\theta_2 - l_3 \cos\theta_3 \end{bmatrix}$$

a. Imagine you have been given the x, y, and z coordinates where the Phantom's tip should be located, and you know that point is in the Phantom's workspace. **What value of θ_1 is required** to put the tip at the desired location? Proved a closed-form equation. *(5 points)*

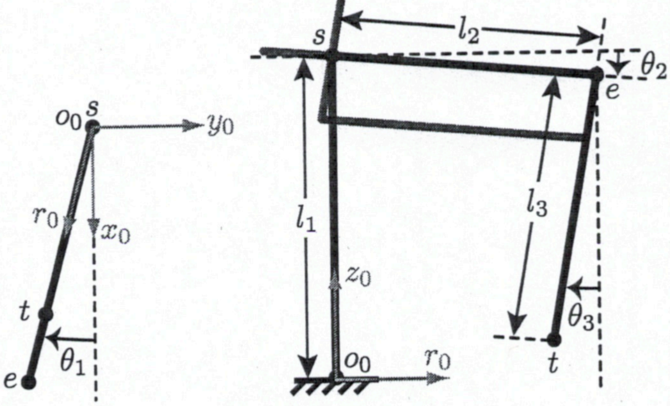
The overhead view is the relevant one here.



$$\tan(\theta_1) = \frac{-y}{x}$$

$$\boxed{\theta_1 = \text{atan2}\left(\frac{(-y)}{x}\right)}$$ ← numerator ← denominator

in Matlab, $\theta_1 = \text{atan2}(-y, x)$

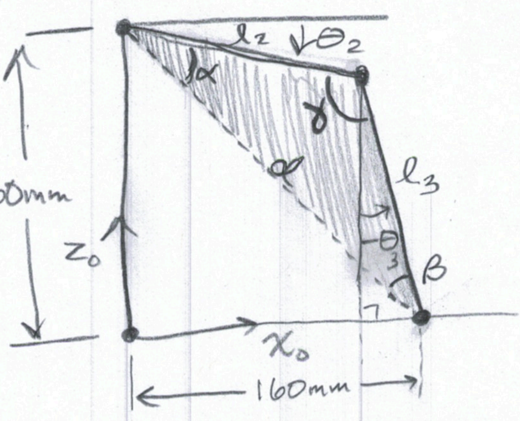


$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (l_2 \cos\theta_2 - l_3 \sin\theta_3)\cos\theta_1 \\ -(l_2 \cos\theta_2 - l_3 \sin\theta_3)\sin\theta_1 \\ l_1 - l_2 \sin\theta_2 - l_3 \cos\theta_3 \end{bmatrix}$$

$l_1 = 160$ mm
$l_2 = l_3 = 140$ mm

b. Now imagine you have been asked to solve for the joint angles that will put the Phantom's tip at $x = 160$ mm, $y = 0$ mm, and $z = 0$ mm. You can immediately see that $\theta_1 = 0°$ for this location, but **what should θ_2 and θ_3 be?** Provide any valid solution. *(15 points)*
As you work to solve this problem, recall (1) the Law of Cosines, (2) the fact that the sum of the internal angles of a triangle is 180°, and (3) the properties of isosceles triangles.
While we encourage you to check your answers using the provided forward kinematics equations, please solve this problem algebraically and/or geometrically, not numerically.

The side view matters here. Draw at approximate pt.



Use Law of Cosines on shaded tri.

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

for us: $d^2 = l_2^2 + l_3^2 - 2 l_2 l_3 \cos\gamma$

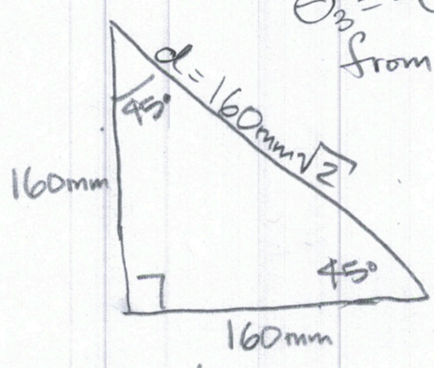
$l_2 = l_3 = l$

$d^2 = 2l^2 - 2l^2\cos\gamma$

$2l^2\cos\gamma = 2l^2 - d^2$

$\theta_3 = -\theta_2$ from symmetry.

$\cos\gamma = \frac{(2l^2 - d^2)}{2l^2}$ -0.30612



$\gamma = \cos^{-1}\left(\frac{2l^2 - d^2}{2l^2}\right)$

$\theta_2 = 45° - \alpha$
$\theta_3 = -\theta_2$

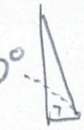
$\gamma = 107.8257°$

$\boxed{\theta_2 = 8.91° \quad \theta_3 = -8.91°}$

$\alpha + \beta + \gamma = 180°$ $\alpha = \beta$

$2\alpha = 180° - \gamma$ $\alpha = (180° - \gamma)/2 = 36.087°$

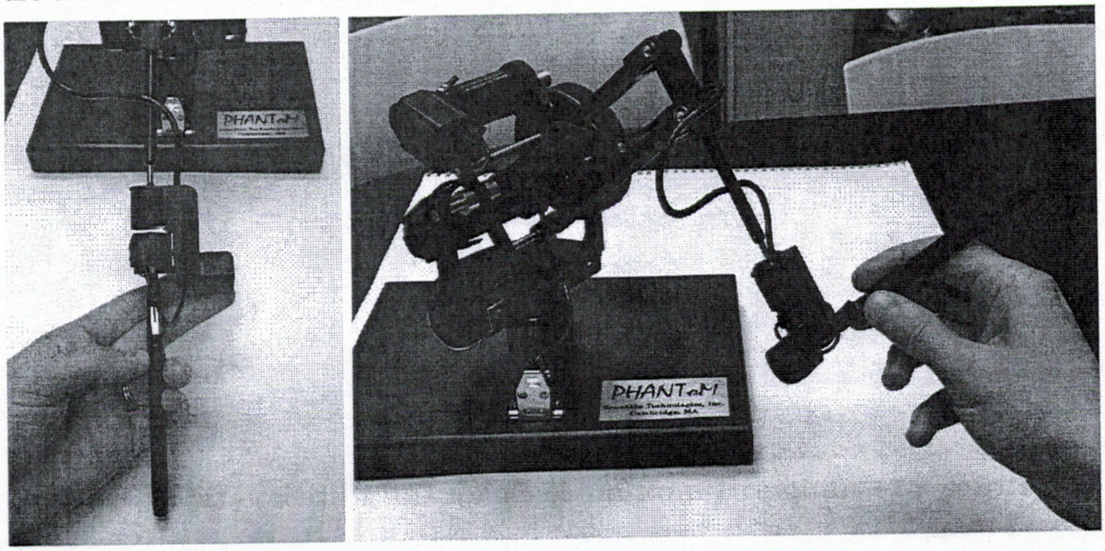
θ_3 alternative:
$90° + 45° + \beta - \theta_3 = 180°$



$\theta_3 = -45° + \beta = -45° + 36.087°$
$= -8.91°$

Problem 5: Encoded Stylus for the Phantom Premium *(20 points)*

In addition to the non-sensed gimbaled thimble that you used in Homework 9 (shown in the Phantom photograph earlier in this exam), SensAble sells a pen-like stylus that attaches to the end of the Phantom through three revolute joints, each of which includes an incremental optical encoder. The photos below show two views of this encoded stylus mounted to a Phantom; the black cylinders are the three encoders. This Phantom is also available at the front of the exam room.



a. Consider the design of this encoded gimbal. What **potential problems** do you anticipate may be encountered in its use? Discuss at least two. *(5 points)*

- The main issue with this gimbal is the same problem that plagues spherical wrists: when the first and third axes align, the end effector cannot rotate around the axis perpendicular to the coincident axes and the second axis. The rotational motion becomes constrained near the singularity and can pop, or flip around quickly.
- Another possible issue is that the wires for the encoders may get caught or pulled or interfere with the stylus motion.
- Another potential problem is that the encoded stylus is rather heavy, so the user may get tired of holding it up.

b. Imagine you connected the Phantom with the encoded stylus to the computer in Towne B2. You zero all the encoders and then rotate the final stylus joint by about 180°. Read via the computer, the count for that encoder changes from 0 to −198. If you opened the encoder up, **how many slots** would you expect to see cut into its disk? *(5 points)*

$180° \cong 198$ counts (negative is irrelevant here)

$180° \cdot \frac{2\pi \text{ rad}}{360°} = \pi \text{ rad} = 0.5 \text{ rev}$

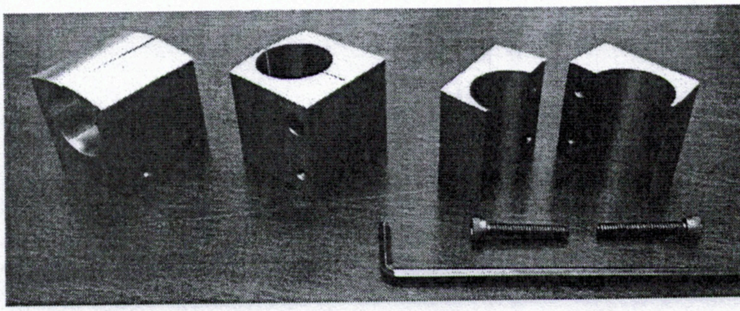
$\frac{198 \text{ counts}}{0.5 \text{ rev}} = \frac{396 \text{ counts}}{\text{rev}} = 4 \cdot n$

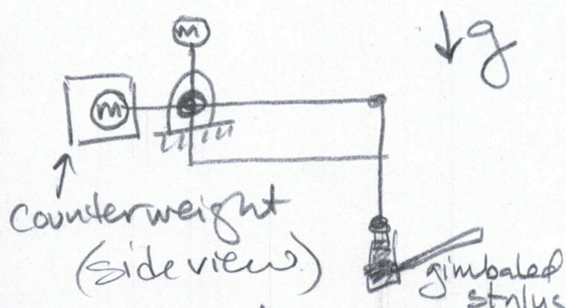
↑ quadrature ↖ #slots

$n \cong \frac{396 \text{ counts}}{4 \text{ counts/line}}$ $\boxed{\cong 100 \text{ lines}}$

probably an even number

c. Everyone who buys an encoded stylus from SensAble also receives one of the metal objects shown in the photo below; the photo shows three of these, with the rightmost one being disassembled into its two pieces. They are also available at the front of the exam room. **What do you think this metal object is, and how would one use it?** *(5 points)*





This metal object is a counterweight. It can be clamped on the lower motor on the upper drum (the shoulder motor) to offset the weight of the stylus. The radius of the cutout fits perfectly on the motor.

d. Now imagine the user is holding the stylus and has moved the Phantom to a certain set of joint angles $\vec{q} = [\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6]^T$. Imagine you have also been given a certain Cartesian direction in the base frame, specified by the unit vector $\hat{u}^0 = [u_x\ u_y\ u_z]^T$. Given this pose and this direction, **explain how to calculate the maximum force magnitude that the Phantom can continuously exert in that direction from that pose.** You do not need to do the calculation, but carefully explain each step. *(5 points)* Ignore $\theta_4, \theta_5, \theta_6$ (no force output)

- Derive the 3x3 translational velocity Jacobian (J_v) for this robot (from FK), and evaluate it at $\theta_1, \theta_2, \theta_3$
- Multiply $J_v^T \hat{u}^0$ to get $\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$ vector of joint torques needed to create a unit torque in $\hat{u}^0$ dir'n.
- Look up maximum continuous current for these three joints. Multiply by torque constant and the gear ratio for each joint to get max joint torques
- Divide each τ_i by $\tau_{i\,max}$. Choose the largest resulting absolute value — it limits the force output b/c it will hit first. that is the max scale.
- Maximum force is (1N/max scale). For example, if 1N force requires 40% of max torque on joint 1 and less than that on joints 2 and 3, max force = 1N/0.40 = 2.5N.

e. EXTRA CREDIT: Given a certain robot pose, **what shape would you see if you graphed the maximum force vectors for all possible directions?** *(up to 3 extra credit points)*

You will see a parallelepiped — a six-sided object like a reangled and stretched rectangular solid. The 8 corners are the max current points on all three axes combined (pos/neg^3). The edges are 2 axes maxing simultaneously, and the faces are with just one saturating. It is the surface made from basis vectors of 3 max torques