MEAM 520

Jacobians and Singularities

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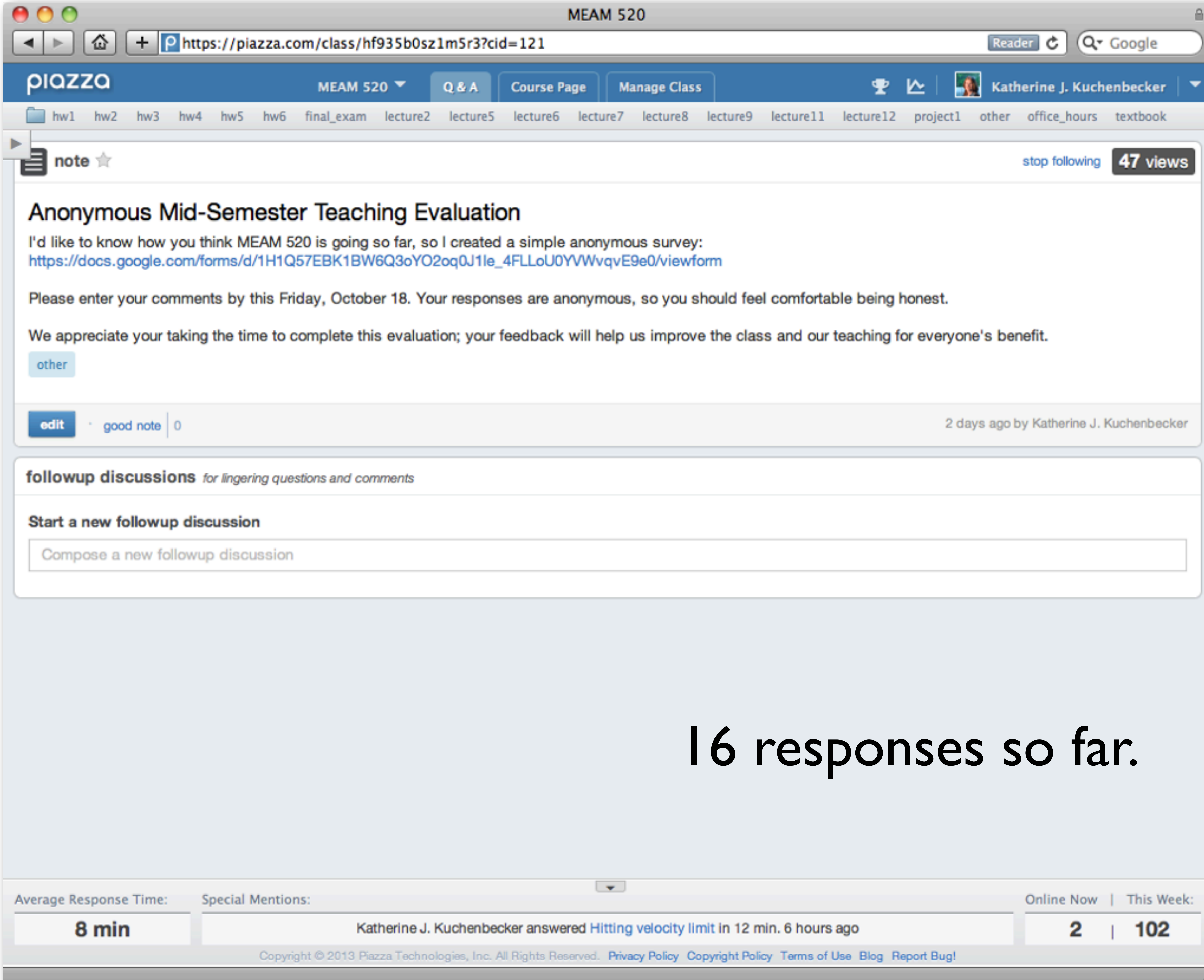
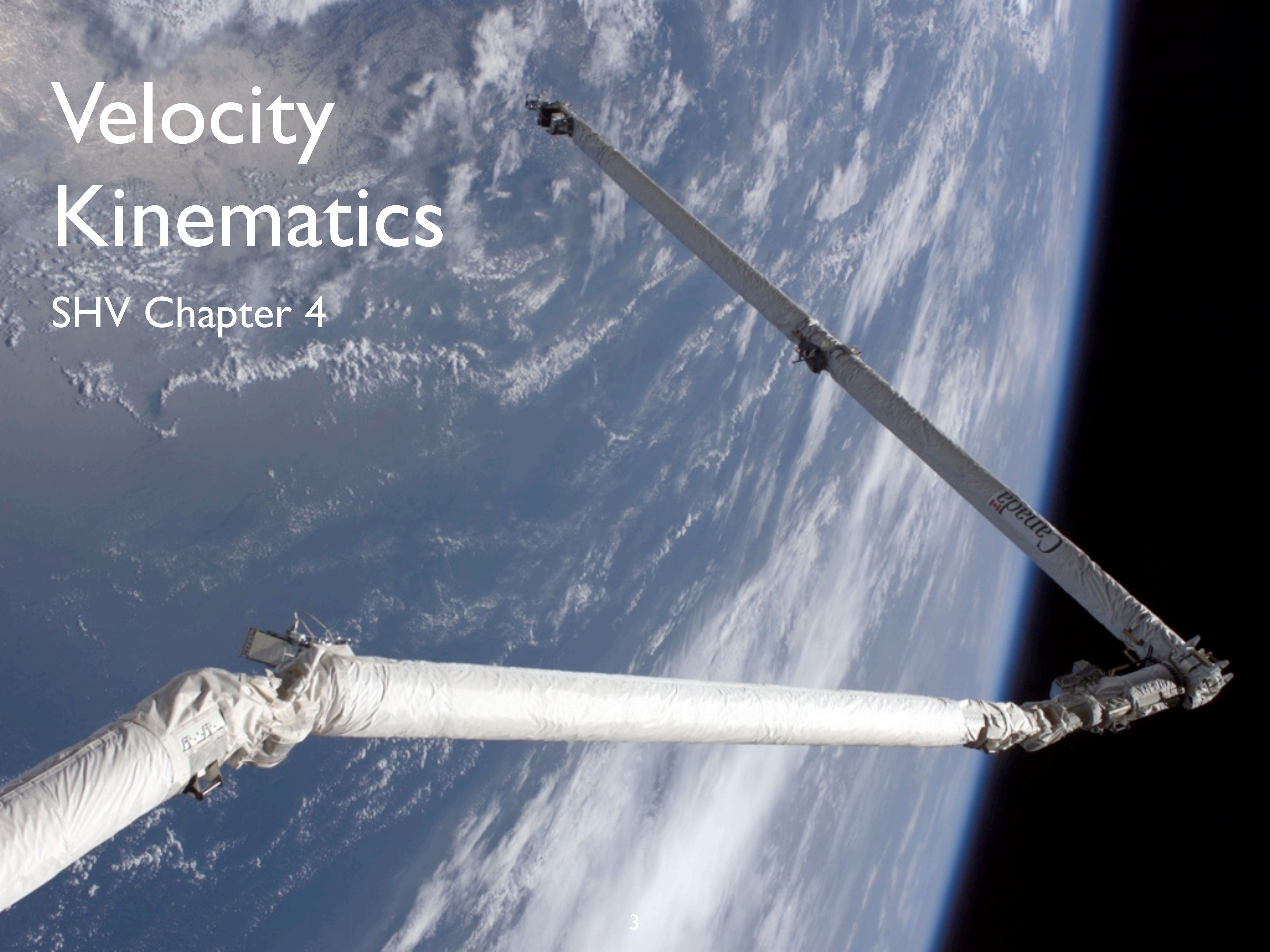
GRASP
LABORATORY

Lecture 14: October 17, 2013



16 responses so far.



Velocity Kinematics

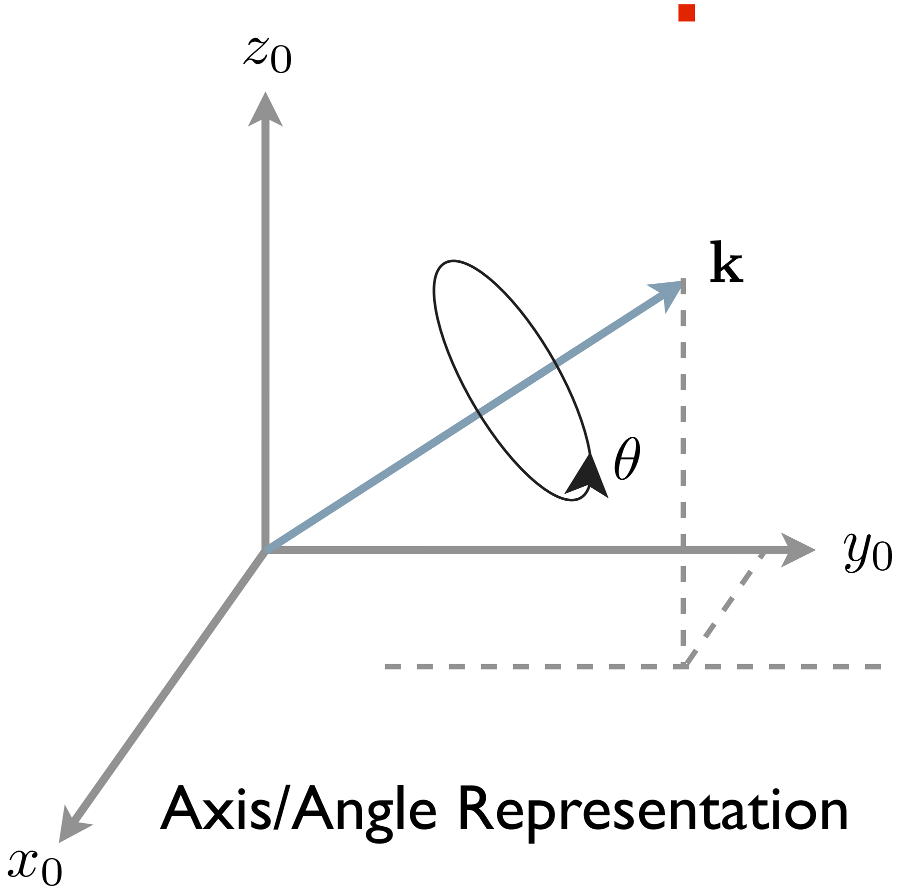
SHV Chapter 4

$$\dot{R} = \frac{dR}{dt} = \frac{dR}{d\theta}\frac{d\theta}{dt}$$

$$S + S^T = 0$$

$$S(\vec{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$S(\vec{a})\vec{p} = \vec{a} \times \vec{p}$$



Axis/Angle Representation

$$\frac{dR}{d\theta} = S(\hat{\omega})\, R$$

The skew-symmetric matrix S defines the axis about which rotation is occurring.

$$\frac{dR}{dt} = S(\vec{\omega})\, R$$

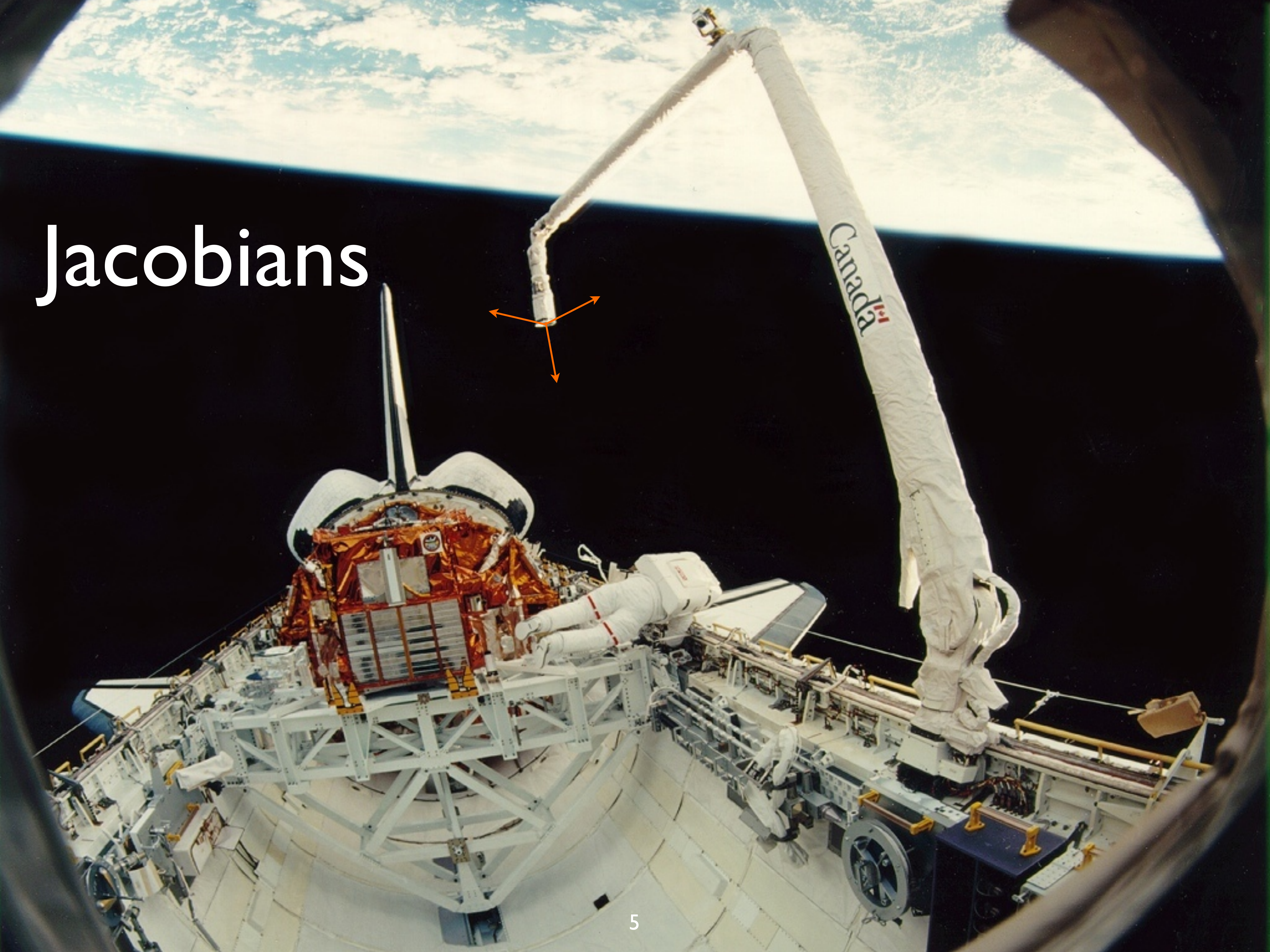
In general, you simply form S from the angular velocity vector and don't need to differentiate the matrix.

$$\vec{\omega}_2^0 = \vec{\omega}_{0,1}^0 + R_1^0\vec{\omega}_{1,2}^1$$

$$\dot{\vec{p}}^0 = S(\vec{\omega}^0)R_1^0\vec{p}^1 + \dot{\vec{o}}_1^0$$

SHV 4.1 – 4.5

Jacobians



explore how **changes** in joint values affect the end-effector movement

could have **N joints**, but only **six** end-effector velocity terms (xyzpts)

The **Jacobian** matrix lets us calculate how joint velocities translate into end-effector velocities (depends on configuration)

look at it in two parts - position and orientation

$$\boxed{v_n^0 = J_v \dot{q}} \qquad \omega_n^0 = J_\omega \dot{q}$$

How do we calculate the linear velocity (position) Jacobian?

Differential Motion

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{p} = f(\vec{q})$$

endpoint position

forward kinematics

nonlinear

joint values

$$\dot{\vec{p}} = J_v(\vec{q})\,\dot{\vec{q}}$$

endpoint velocity

Jacobian matrix

linear approximation of derivative near a given point

joint values

joint velocity

For an n-dimensional joint space and a cartesian workspace, the position Jacobian is a 3 x n matrix composed of the partial derivatives of the end-effector position with respect to each joint variable.

$$J_v(\vec{q}) = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \cdots & \frac{\partial z}{\partial q_n} \end{bmatrix}$$

$$\dot{\vec{p}} = J_v(\vec{q})\,\dot{\vec{q}}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \cdots & \frac{\partial z}{\partial q_n} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

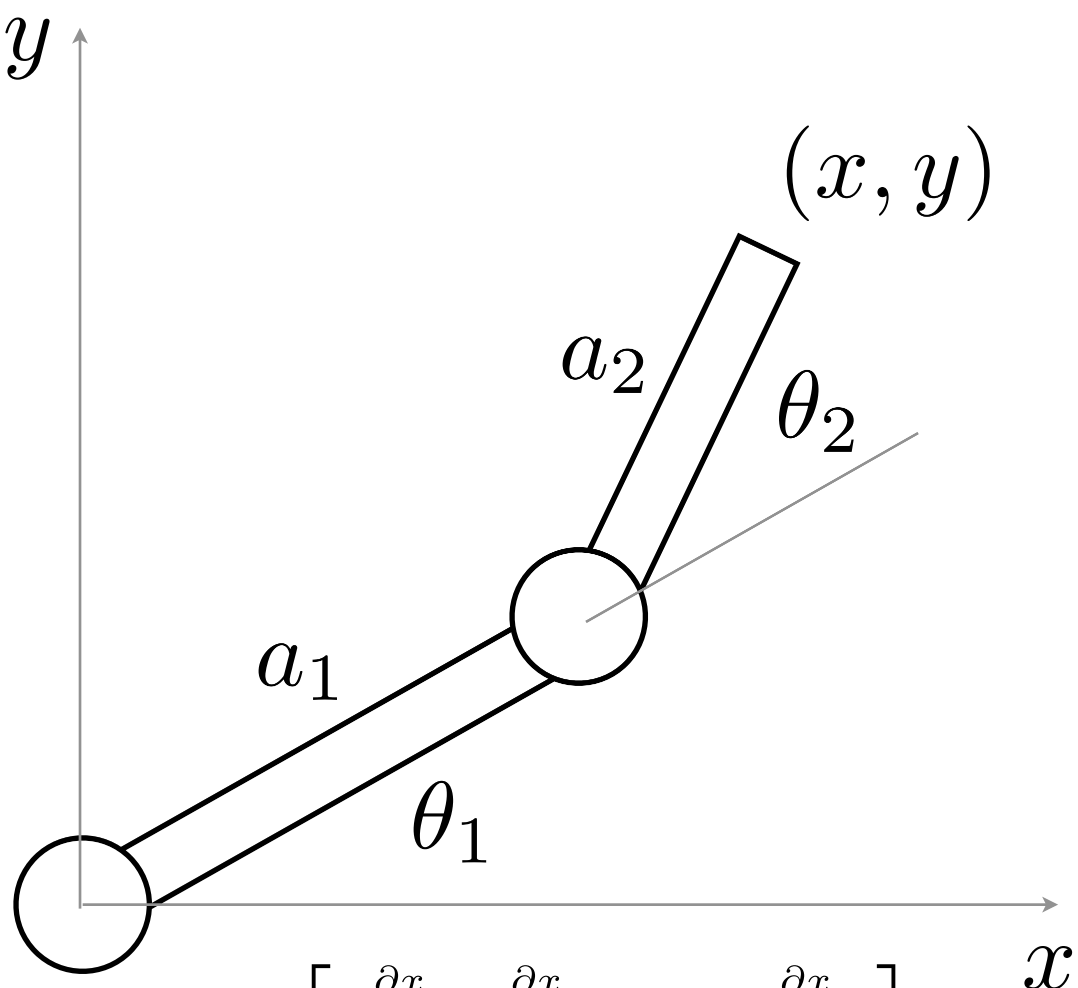
distance/time
such as m/s

revolute: angle/time such as rad/s
prismatic: distance/time such as m/s

Evaluate the Jacobian at the
robot's current pose

What units do the entries of the Jacobian have?

if joint *i* is revolute, column *i* is in distance/angle such as m/rad or m
if joint *i* is prismatic and units match, column *i* is unitless



$$J_v(\vec{q}) = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \cdots & \frac{\partial z}{\partial q_n} \end{bmatrix}$$

From the forward kinematics, we can extract the position vector from the last column of the transform matrix:

$$d_2^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

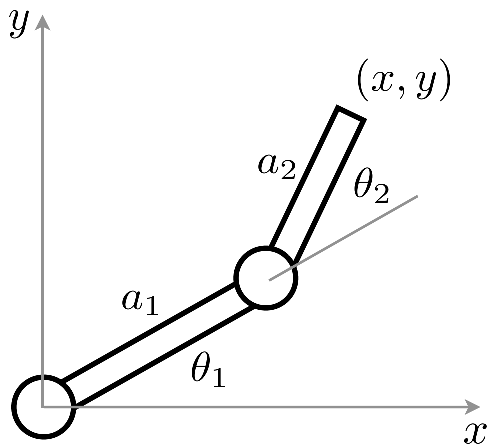
Taking the partial derivative with respect to each joint variable produces the Jacobian:

$$J_v(\vec{q}) = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix}$$

which relates instantaneous joint velocities to endpoint velocities

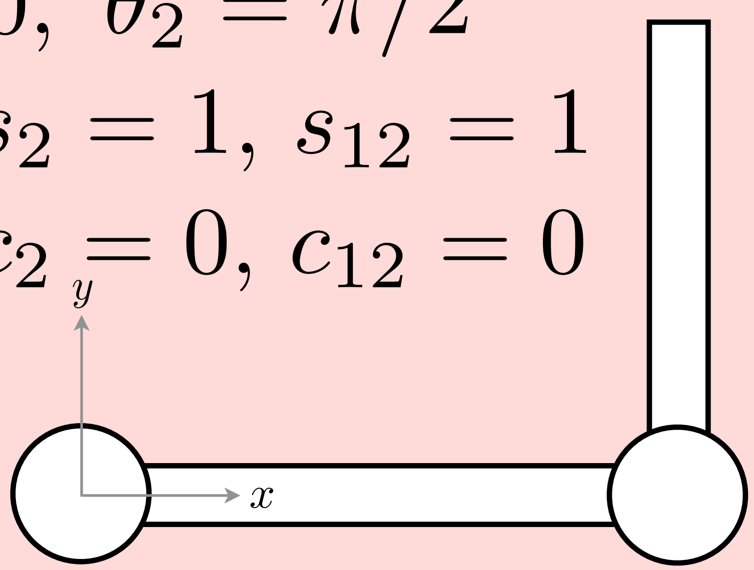
This mapping depends on the robot's current pose!

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$



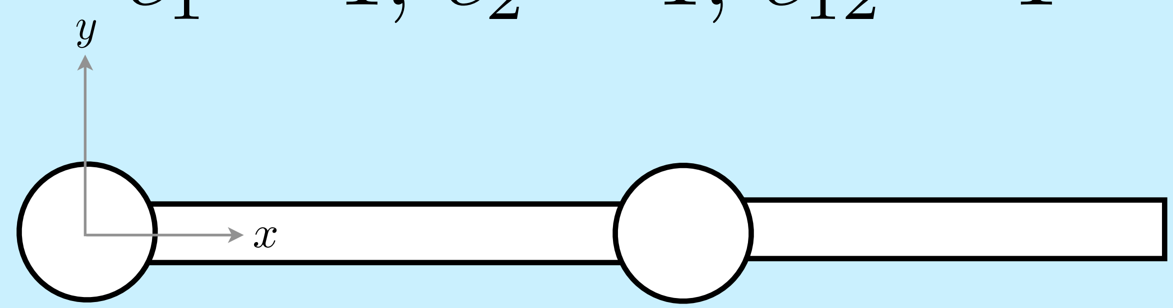
$$J_v(\vec{q}) = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix}$$

$$\theta_1 = 0,\ \theta_2 = \pi/2$$
$$s_1 = 0,\ s_2 = 1,\ s_{12} = 1$$
$$c_1 = 1,\ c_2 = 0,\ c_{12} = 0$$



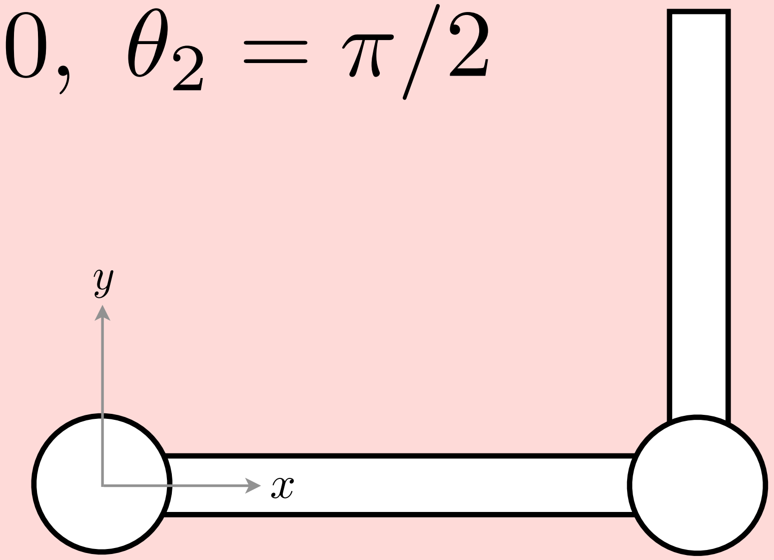
$$J_v([0 \ \ \pi/2]^T) = \begin{bmatrix} -a_2 & -a_2 \\ a_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\theta_1 = 0,\ \theta_2 = 0$$
$$s_1 = 0,\ s_2 = 0,\ s_{12} = 0$$
$$c_1 = 1,\ c_2 = 1,\ c_{12} = 1$$



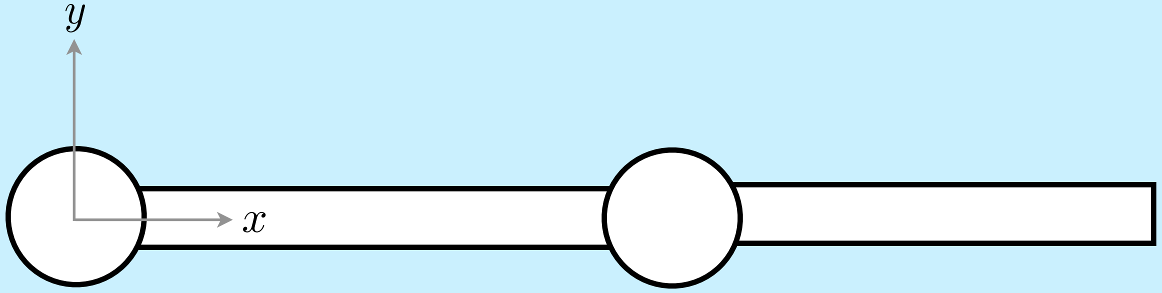
$$J_v([0 \ \ 0]^T) = \begin{bmatrix} 0 & 0 \\ a_1 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$\theta_1 = 0,\ \theta_2 = \pi/2$$



$$J_v([0 \quad \pi/2]^T) = \begin{bmatrix} -a_2 & -a_2 \\ a_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\theta_1 = 0,\ \theta_2 = 0$$



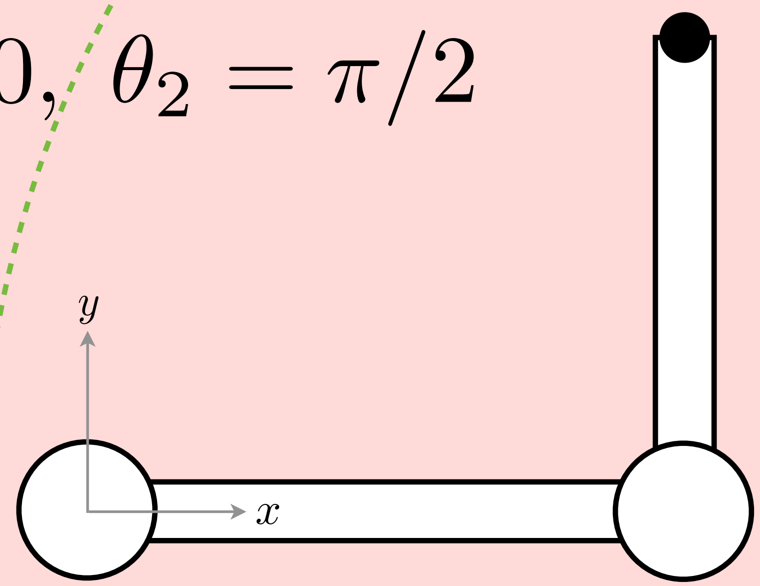
$$J_v([0 \quad 0]^T) = \begin{bmatrix} 0 & 0 \\ a_1 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$\dot{\vec{p}} = J_v(\vec{q})\,\dot{\vec{q}}$$

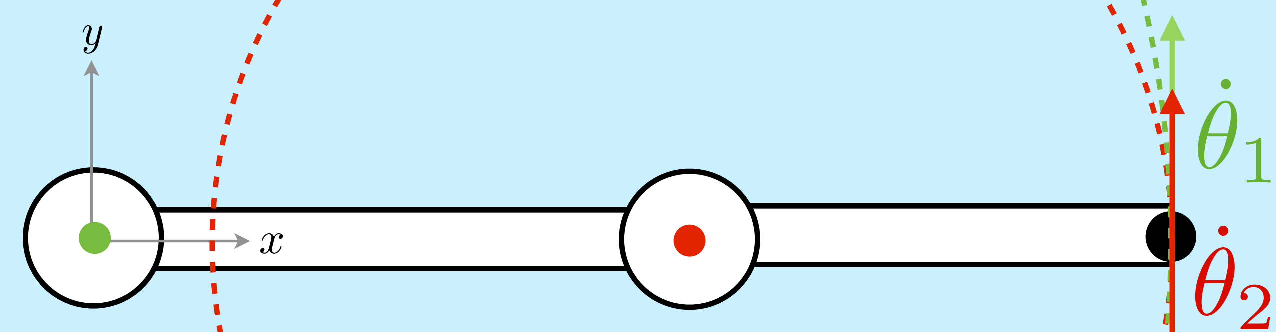
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a_2\dot{\theta}_1 - a_2\dot{\theta}_2 \\ a_1\dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ (a_1 + a_2)\dot{\theta}_1 + a_2\dot{\theta}_2 \\ 0 \end{bmatrix}$$

$$\theta_1 = 0, \ \theta_2 = \pi/2$$



$$\theta_1 = 0, \ \theta_2 = 0$$



$$J_v([0 \ \ \pi/2]^T) = \begin{bmatrix} -a_2 & -a_2 \\ a_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_v([0 \ \ 0]^T) = \begin{bmatrix} 0 & 0 \\ a_1 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$\boxed{\dot{\vec{p}} = J_v(\vec{q})\, \dot{\vec{q}}}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a_2\dot{\theta}_1 - a_2\dot{\theta}_2 \\ a_1\dot{\theta}_1 \\ 0 \end{bmatrix}$$

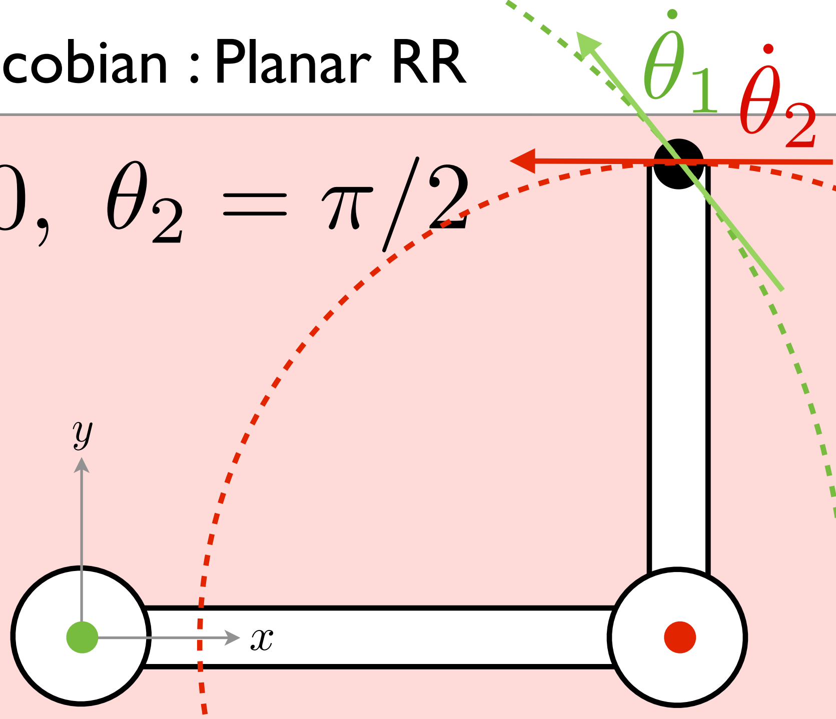
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ \boxed{(a_1 + a_2)\dot{\theta}_1} \boxed{+ a_2\dot{\theta}_2} \\ 0 \end{bmatrix}$$

Notice anything else about this configuration?

The robot's tip cannot move in the x or z directions…

The Position Jacobian : Planar RR

$$\theta_1 = 0,\ \theta_2 = \pi/2$$

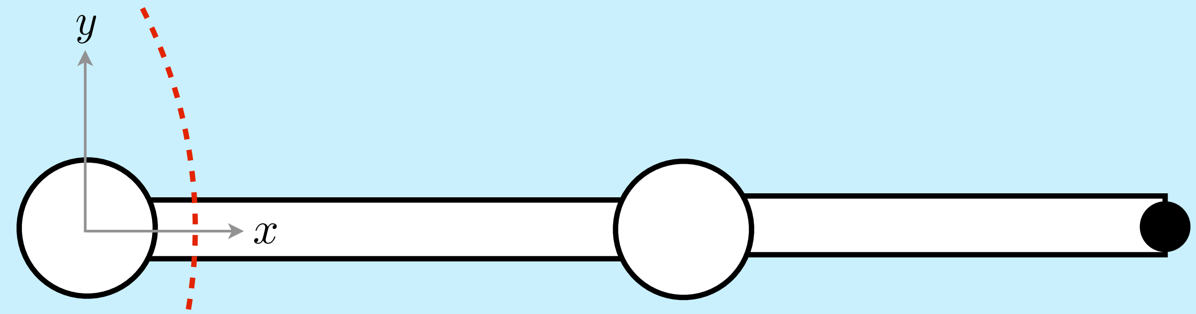


$$J_v([0 \quad \pi/2]^T) = \begin{bmatrix} -a_2 & -a_2 \\ a_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot x \\ \dot y \\ \dot z \end{bmatrix} = \begin{bmatrix} \boxed{-a_2\dot\theta_1}\boxed{-\ a_2\dot\theta_2} \\ \boxed{a_1\dot\theta_1} \\ 0 \end{bmatrix}$$

The robot's tip cannot move in the z direction, but it can move in both x and y directions...

$$\theta_1 = 0,\ \theta_2 = 0$$



$$J_v([0 \quad 0]^T) = \begin{bmatrix} 0 & 0 \\ a_1 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

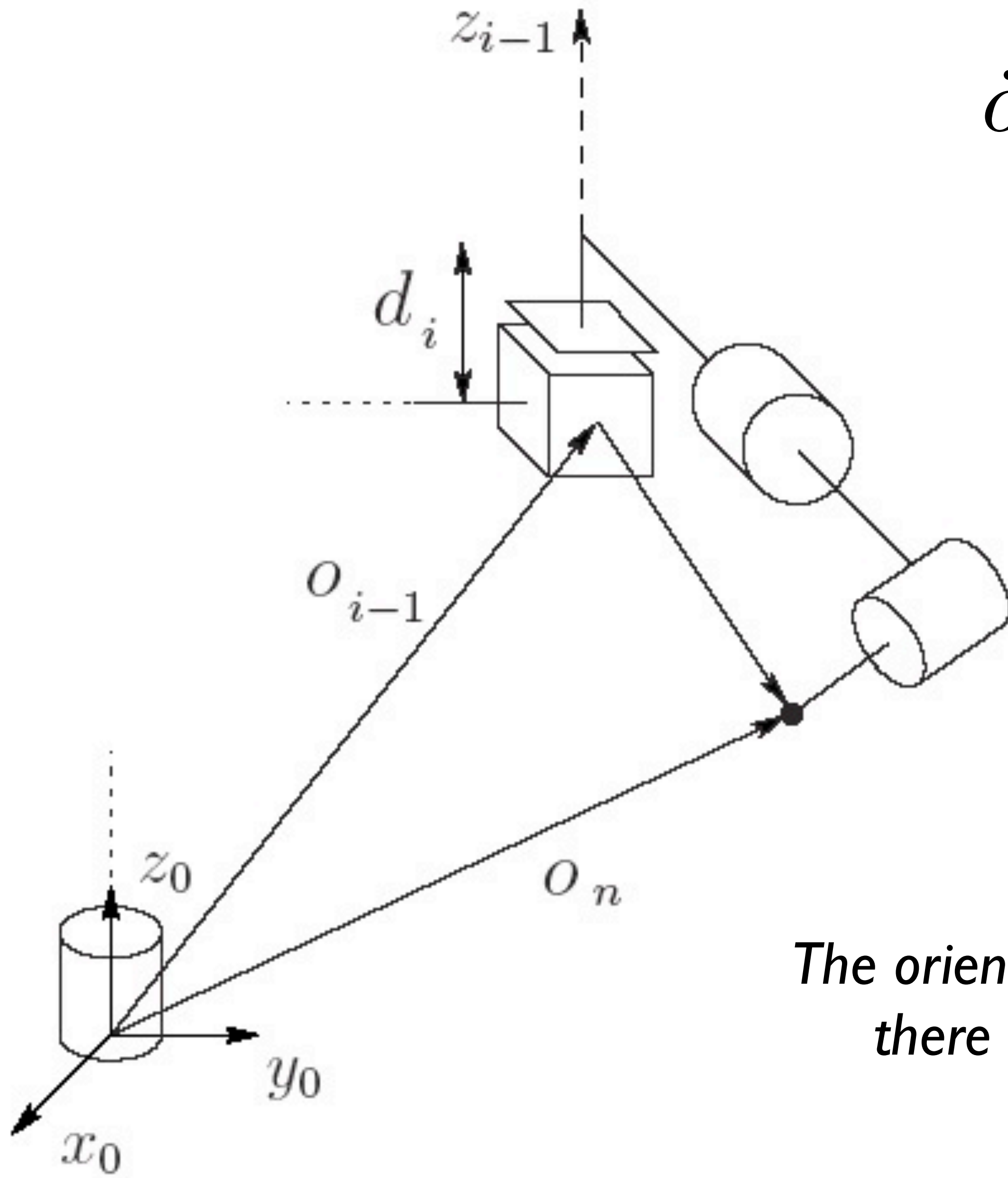
$$\begin{bmatrix} \dot x \\ \dot y \\ \dot z \end{bmatrix} = \begin{bmatrix} 0 \\ (a_1 + a_2)\dot\theta_1 + a_2\dot\theta_2 \\ 0 \end{bmatrix}$$

$$\dot{\vec p} = J_v(\vec q)\,\dot{\vec q}$$

Questions ?

Another Way to Calculate the Jacobian Covered in SHV 4.6.2

Prismatic Joints



$$\dot{o}_n^0 = \dot{d}_i R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \dot{d}_i z_{i-1}^0$$

$$J_{v_i} = z_{i-1}$$

The orientation of a z-axis depends on the robot's pose if there are any revolute joints before it in the chain.

Figure 4.1: Motion of the end effector due to primsmatic joint i.

Revolute Joints

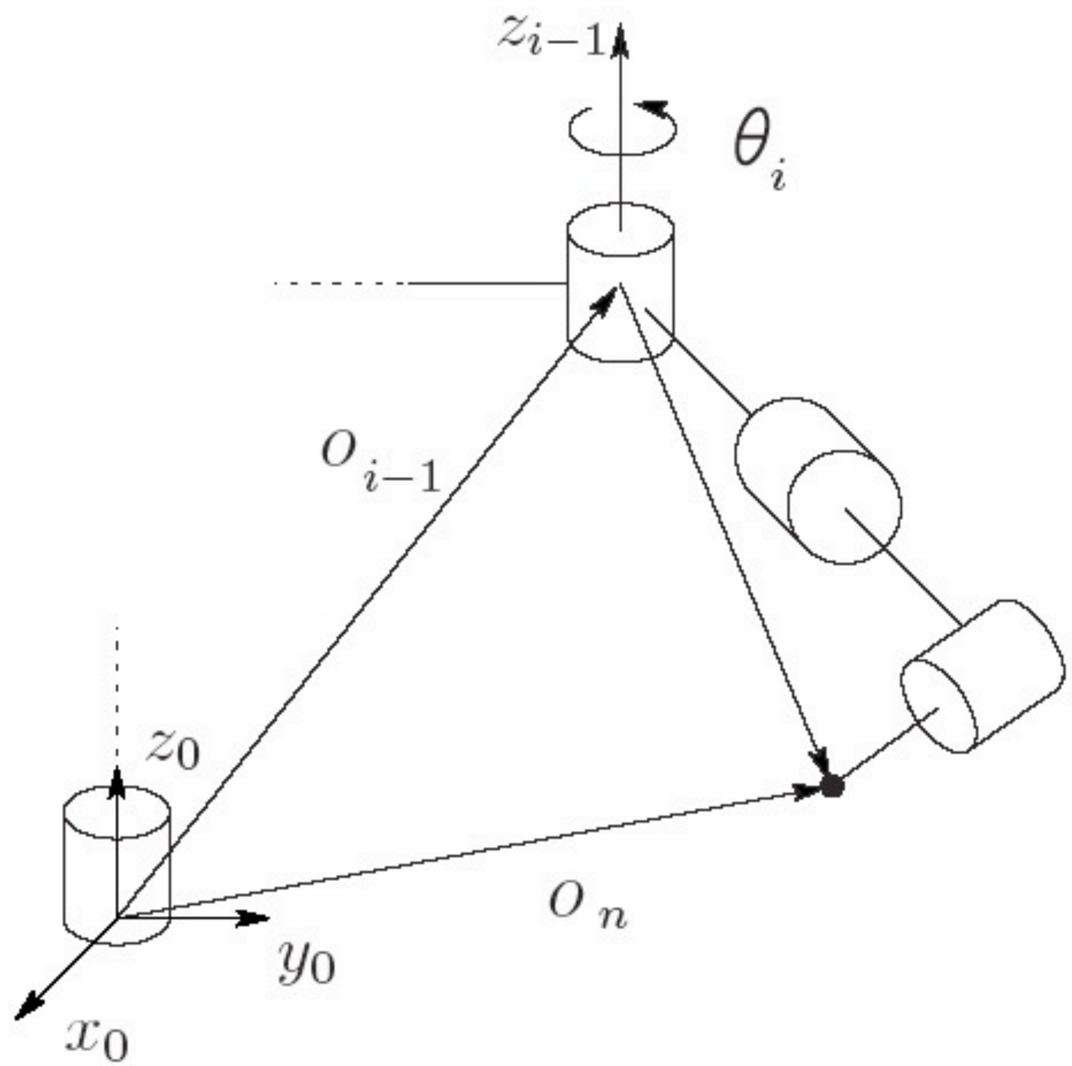


Figure 4.2: Motion of the end effector due to revolute joint i.

$$v = \omega \times r$$

$$\omega = \dot{\theta}_i z_{i-1}$$

$$r = o_n - o_{i-1}$$

$$J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$$

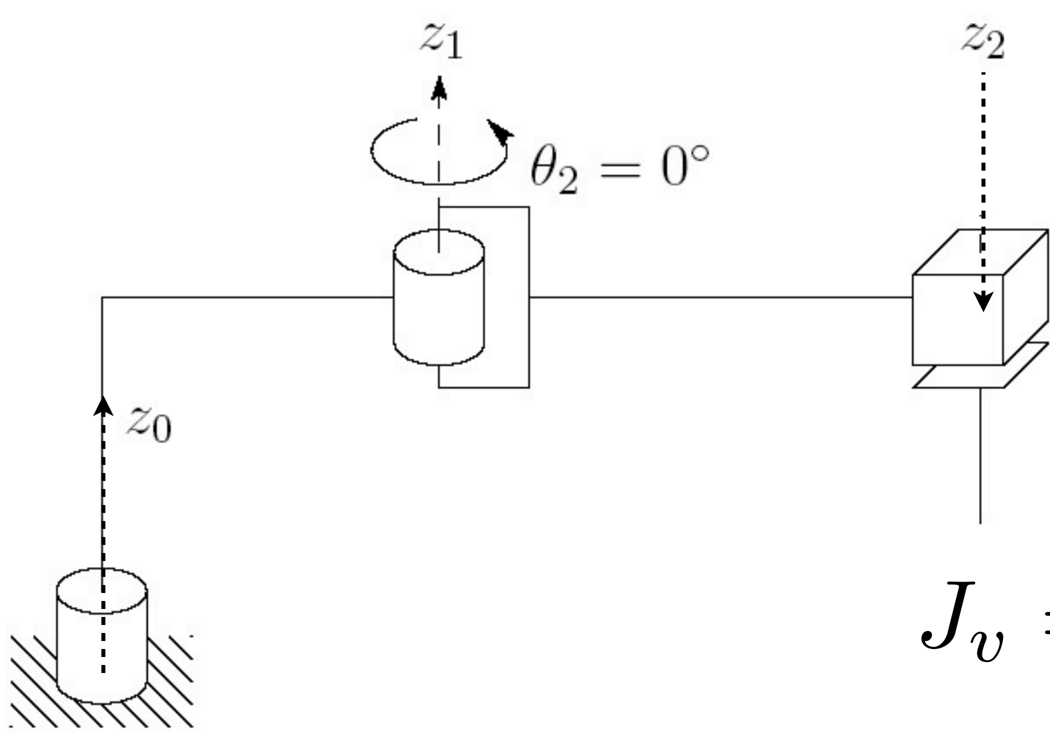
Another way to construct the position Jacobian.

Prismatic Joints

$$J_{v_i} = z_{i-1}$$

Revolute Joints

$$J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$$



What is the SCARA's Jacobian?

$$J_v = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

I prefer to calculate position Jacobians by differentiating the end-effector position, but both approaches are valid, enabling you to check your work and increase your intuition.

Questions ?

$$v_n^0 = J_v \dot{q}$$

What joint velocities should I choose to cause a desired end-effector velocity?
(inverse velocity kinematics)

$$\dot{q} = J_v^{-1} v_n^0$$

Can a robot always achieve all end-effector velocities?

No. This works only when the Jacobian is square and invertible (non-singular).

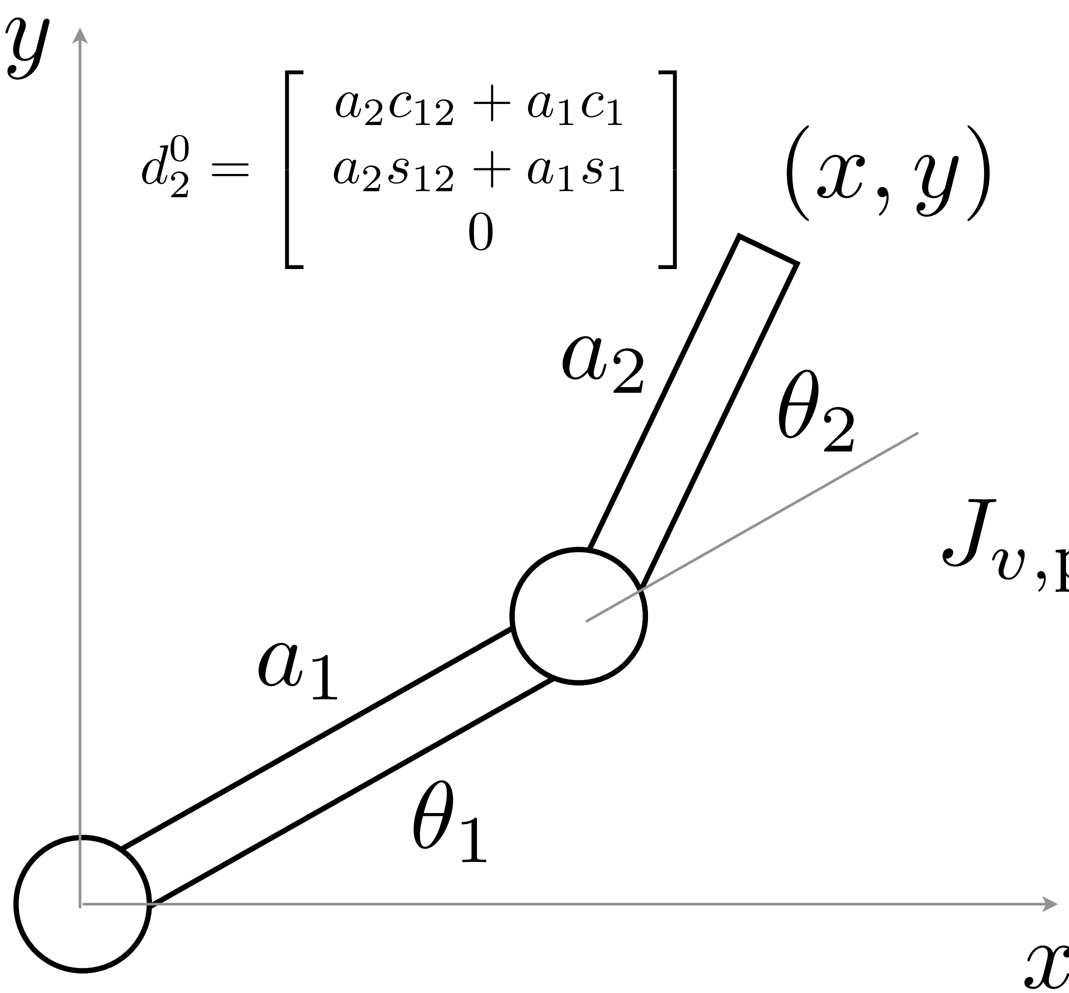
SHV 4.11 explains what to do when the Jacobian is not square:
rank test (v is in range of J)
use J^+ (right pseudoinverse of J)
when the robot has extra joints, there are many solutions

Singularities are points in the configuration space where infinitesimal motion in a certain direction is not possible and the manipulator loses one or more degrees of freedom

Mathematically, singularities exist at any point in the workspace where the Jacobian matrix loses rank.

a matrix is singular if and only if its determinant is zero:

$$\det(J_v) = 0$$



$$J_v(\vec{q}) = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \\ 0 & 0 \end{bmatrix}$$

$$J_{v,\text{planar}}(\vec{q}) = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

$$\det(J_{v,\text{planar}}(\vec{q})) = ?$$

$$\det(J_{v,\text{planar}}(\vec{q})) = a_1 a_2 (\underline{c_1 s_{12} - s_1 c_{12}})$$

When does $\det(\mathbf{J}) = 0$?

$$\det(\mathbf{J}) = 0 \text{ when } \theta_2 = 0$$

Any other times?

$$\det(\mathbf{J}) = 0 \text{ when } a_1 = 0 \text{ or } a_2 = 0$$

$$\text{if } \theta_2 = 0, c_1 s_{12} - s_1 c_{12} = c_1 s_1 - s_1 c_1 = 0$$

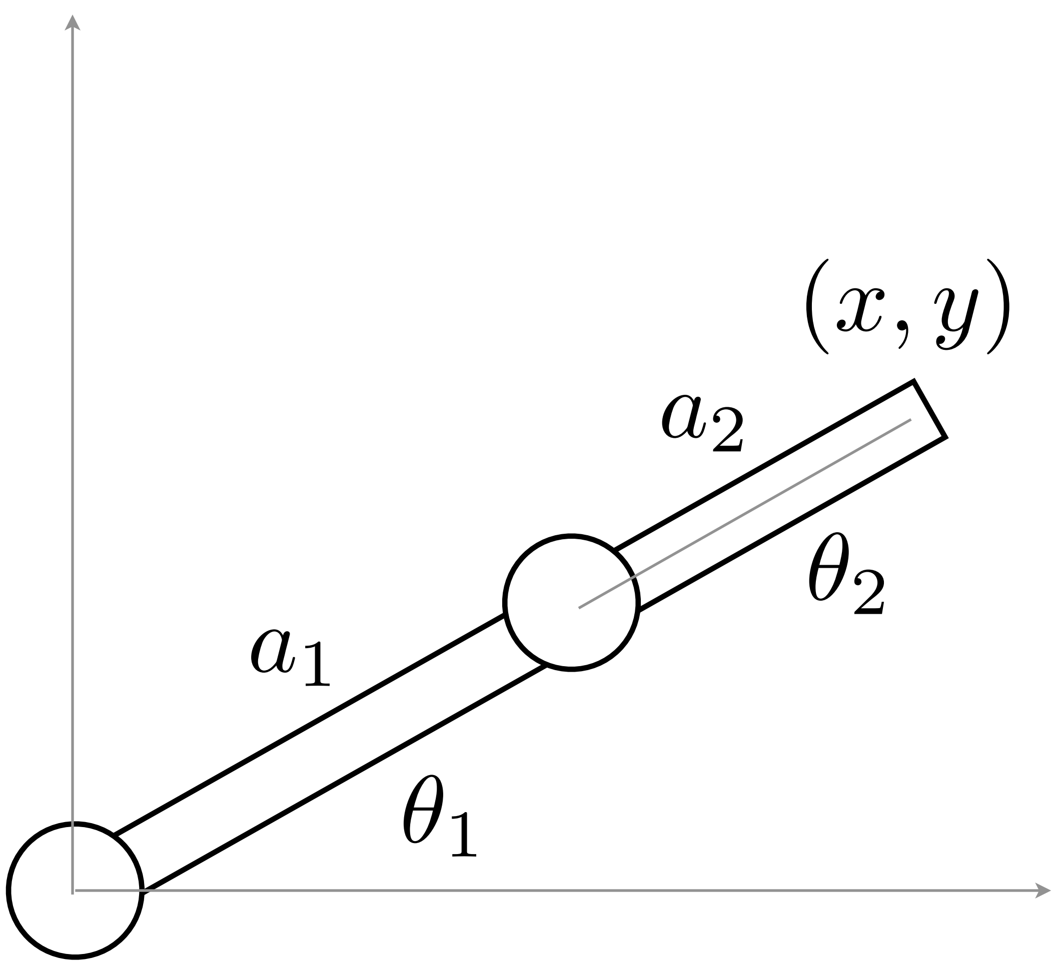
Is that the only time?

No... $\det(\mathbf{J}) = 0$ when $\theta_2 = \ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots$

For $\theta_2 = 0$

The Jacobian collapses to have linearly dependent rows

$$\mathbf{J}_{\theta_2=0} = \left[\begin{array}{cc} -a_1 s_1 - a_2 s_1 & -a_2 s_1 \\ a_1 c_1 + a_2 c_1 & a_2 c_1 \end{array} \right]$$



This means that actuating either joint causes motion in the same direction

We often try to avoid singularities.

Questions ❓

explore how **changes** in joint values affect the end-effector movement

could have **N joints**, but only **six** end-effector velocity terms (xyzpts)

The **Jacobian** matrix lets us calculate how joint velocities translate into end-effector velocities (depends on configuration)

look at it in two parts - position and orientation

$$v_n^0 = J_v \dot{q}$$

$$\omega_n^0 = J_\omega \dot{q}$$

How do we calculate the orientation Jacobian?

And now, angular velocity

$$\omega = \mathbf{J}_\omega(q)\ \dot{\mathbf{q}}$$

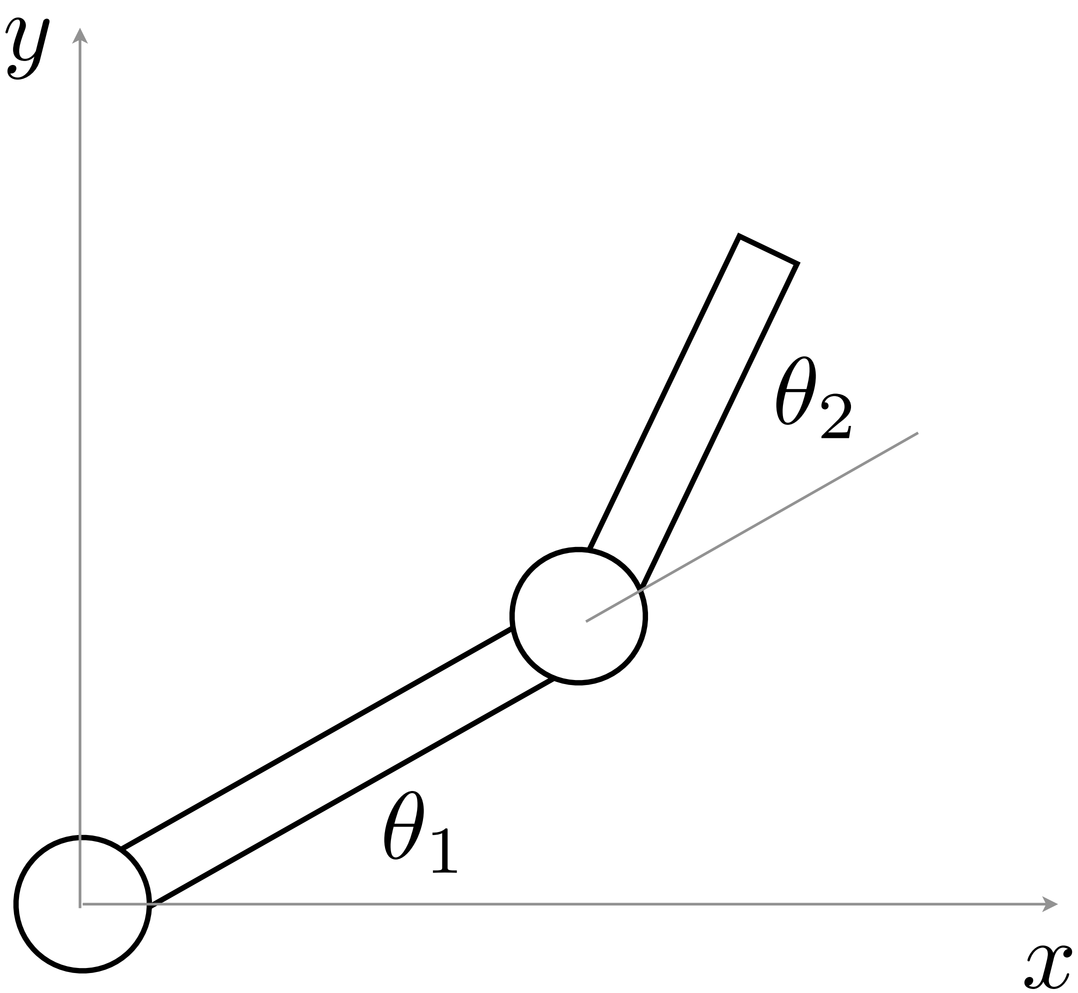
final frame angular velocity

joint velocities

$$\omega_{i,j}^{k}$$

this is the angular velocity of frame **j**
with respect to frame **i**,
expressed in frame **k**

SHV 4.1 gives a good explanation of angular velocity for fixed-axis rotation. SHV 4.2-4.5 go into greater detail.



$$\omega_{0,1}^{0} = 0\,\hat{x}_0 + 0\,\hat{y}_0 + \dot{\theta}_1\,\hat{z}_0$$

$$\omega_{1,2}^{1} = 0\,\hat{x}_1 + 0\,\hat{y}_1 + \dot{\theta}_2\,\hat{z}_1$$

$$\omega_{1,2}^{0} = \mathbf{R}_1^0\,\omega_{1,2}^{1}$$

$$\begin{aligned}\omega_{0,2}^{0} &= \omega_{0,1}^{0} + \mathbf{R}_1^0\,\omega_{1,2}^{1}\\ &= 0\,\hat{x}_0 + 0\,\hat{y}_0 + (\dot{\theta}_1 + \dot{\theta}_2)\,\hat{z}_0\end{aligned}$$

$$\omega_{0,n}^{0} = \sum_{i=1}^{n} \mathbf{R}_{i-1}^0\,\omega_{i-1,i}^{i-1}$$

$$\omega_{0,n}^{0} = \sum_{i=1}^{n} (\mathbf{R}_{i-1}^0 \hat{\mathbf{z}})\,\dot{\theta}_i$$

note: this holds for revolute joints only (by definition, a prismatic joint cannot create angular velocity)

And now, for that Jacobian!

$$\omega_{0,n}^{0} = \sum_{i=1}^{n} \rho_i (\mathbf{R}_{i-1}^{0} \hat{z})\, \dot{\theta}_i \qquad \rho_i = \begin{matrix} \text{0 for prismatic} \\ \text{1 for revolute} \end{matrix}$$

$$\omega_{0,n}^{0} = [\rho_1 \hat{\mathbf{z}} \quad \rho_2 \mathbf{R}_1^0 \hat{\mathbf{z}} \quad \rho_3 \mathbf{R}_2^0 \hat{\mathbf{z}} \quad \cdots \quad \rho_n \mathbf{R}_{n-1}^0 \hat{\mathbf{z}}] \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

$$\omega = \mathbf{J}_\omega(q)\, \dot{\mathbf{q}}$$

(3 x n) cartesian velocity Jacobian

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_p \\ \mathbf{J}_\omega \end{bmatrix}$$

(6 x n) Jacobian

a.k.a. manipulator Jacobian

a.k.a. geometric Jacobian

(3 x n) angular velocity Jacobian

The Jacobian is easily constructed from the manipulator's forward kinematics.

What do you need from the forward kinematics?

4.6.3 Combining the Linear and Angular Velocity Jacobians

As we have seen in the preceding section, the upper half of the Jacobian J_v is given as

$$J_v = [J_{v_1} \cdots J_{v_n}] \tag{4.56}$$

in which the i^{th} column J_{v_i} is

$$J_{v_i} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & \text{for revolute joint } i \\ z_{i-1} & \text{for prismatic joint } i \end{cases} \tag{4.57}$$

The lower half of the Jacobian is given as

$$J_\omega = [J_{\omega_1} \cdots J_{\omega_n}] \tag{4.58}$$

in which the i^{th} column J_{ω_i} is

$$J_{\omega_i} = \begin{cases} z_{i-1} & \text{for revolute joint } i \\ 0 & \text{for prismatic joint } i \end{cases} \tag{4.59}$$

Questions ?

Homework 6:
Velocity Kinematics and Jacobians

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

October 17, 2013

This paper-based assignment is due on **Thursday, October 24, by midnight (11:59:59 p.m.)** You should aim to turn it in during class that day. If you don't finish until later in the day, you can turn it in to Professor Kuchenbecker's office, Towne 224, in the bin or under the door. Late submissions will be accepted until Sunday, September 29, by midnight (11:59:59 p.m.), but they will be penalized by 10% for each partial or full day late, up to 30%. After the late deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down must be your own work, not copied from any other individual or a solution manual. Any submissions suspected of violating Penn's Code of Academic Integrity will be reported to the Office of Student Conduct. If you get stuck, post a question on Piazza or go to office hours!

These problems are loosely based on problems that appear in the printed version of the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV); all of the needed instructions are included in this document. Write in pencil, show your work clearly, box your answers, and staple together all pages of your assignment. This assignment is worth a total of 20 points.

1. Skew-Symmetric Matrices *(6 points)*

a. We define $\hat{u} = [x \;\; y \;\; z]^T$ to be a unit vector. What is $S(\hat{u})$, the skew-symmetric matrix associated with this unit vector?

b. Now define $\vec{v} = [0 \;\; 10 \;\; 0]^T$ and calculate $S(\hat{u})\vec{v}$.

c. What is the geometric meaning of the result you obtained in the last step? Draw a sketch with an arbitrarily chosen unit vector $\hat{u}$ to explain. Think about both the magnitude and the direction of the result.

d. Show that $S^3(\hat{u}) = -S(\hat{u})$.

e. What is the geometric meaning of the equation $S^3(\hat{u}) = -S(\hat{u})$? Explain using words and a sketch.

f. $R_{\hat{u},\theta}$ is a rotation matrix representing rotation by the time-varying angle θ about the constant unit vector $\hat{u}$. By considering equation (2.43) in the book, one can show that $R_{\hat{u},\theta} = I + S(\hat{u})\sin\theta + S^2(\hat{u})\mathrm{vers}\,\theta$, where the versine $\mathrm{vers}\,\theta = 1 - \cos\theta$. Note that you do not need to show this equivalence. Instead, use this equivalence and the equation from the previous step to show that $\frac{dR_{\hat{u},\theta}}{d\theta} = S(\hat{u})R_{\hat{u},\theta}$.

g. What is the intuitive meaning of the equation $\frac{dR_{\hat{u},\theta}}{d\theta} = S(\hat{u})R_{\hat{u},\theta}$?

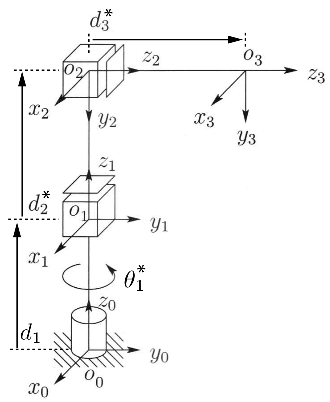
1

Homework 6

Written Assignment

Due next Thursday

2. Three-link Cylindrical Manipulator *(7 points)*



$$T_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Above are the DH diagram and the corresponding transformation matrix T_3^0 for a three-link cylindrical manipulator. The diagram shows θ_1 at zero, and d_2 and d_3 are shown at positive displacements. These materials are adapted from the derivation on pages 85 and 86 of the book.

a. Use the position of the end-effector in the base frame to calculate the 3×3 linear velocity Jacobian J_v for this robot.

b. Use the positions of the origins o_i and the orientations of the z-axes z_i to calculate the 3×3 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.

c. Find the 3×3 angular velocity Jacobian J_ω for the same robot.

d. Imagine this robot is at $\theta_1 = \pi/2$ rad, $d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1$ rad/s, $\dot{d}_2 = 0.25$ m/s, and $\dot{d}_3 = -0.05$ m/s. What is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

e. For the same situation, what is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

f. Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

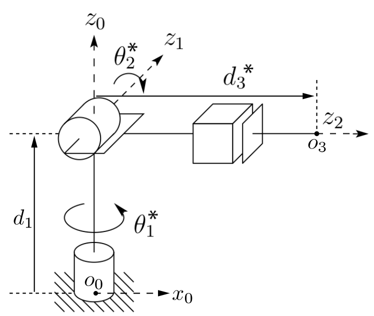
g. Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

Homework 6

Written Assignment

Due next Thursday

3. Three-link Spherical Manipulator *(7 points)*



$$d_3^0 = \begin{bmatrix} c_1 c_2 d_3 \\ s_1 c_2 d_3 \\ d_1 - s_2 d_3 \end{bmatrix}$$

Above are the DH diagram and the corresponding tip position vector d_3^0 for a three-link spherical manipulator. The diagram shows θ_1 and θ_2 at zero, and d_3 is shown at a positive displacement.

a. Calculate the 3×3 linear velocity Jacobian J_v for this manipulator. You may use any method you choose.

b. Find the 3×3 angular velocity Jacobian J_ω for the same robot.

c. Imagine this robot is at $\theta_1 = \pi/4$ rad, $\theta_2 = 0$ rad, and $d_3 = 1$ m. What is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Make sure to provide units for any coefficients in these equations, if needed.

d. For the same configuration described in the previous step, what is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Provide units for any coefficients in these equations, if needed.

e. What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous steps and I want its tip to move at $v_3^0 = [0 \text{ m/s} \quad 0.5 \text{ m/s} \quad 0.1 \text{ m/s}]^T$? Make sure to provide units with your answer.

f. Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

g. Sketch the spherical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

h. Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.

3

Homework 6

Written Assignment

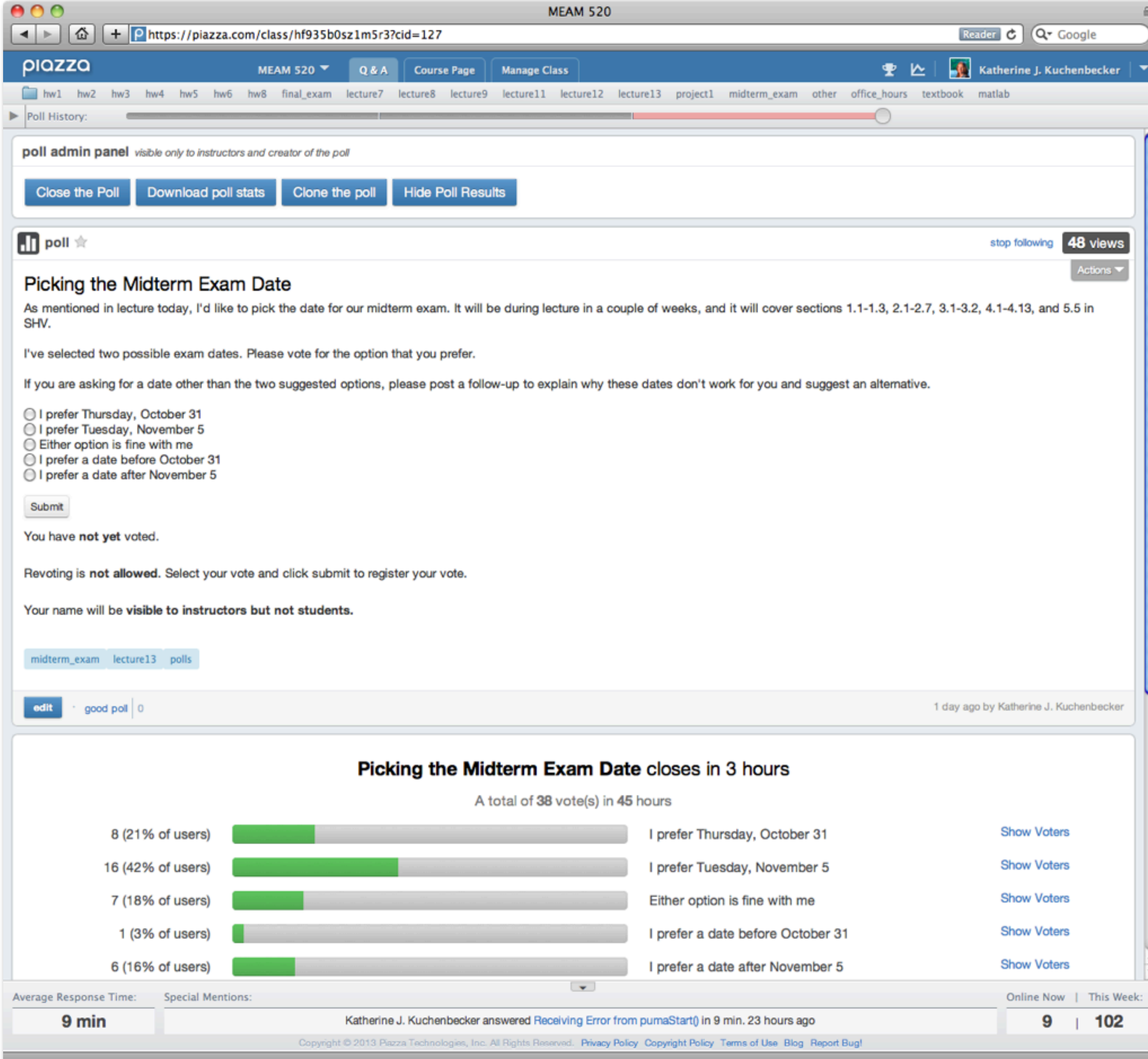
Due next Thursday

Proposed Midterm Dates

Thursday, October 31, in class

or

Tuesday, November 5, in class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Q & A
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Katherine J. Kuchenbecker
hw1 hw2 hw3 hw4 hw5 hw6 hw8 final_exam lecture7 lecture8 lecture9 lecture11 lecture12 lecture13 project1 midterm_exam other office_hours textbook matlab
Poll History:
poll admin panel visible only to instructors and creator of the poll
Close the Poll
Download poll stats
Clone the poll
Hide Poll Results
poll
stop following
48 views
Actions
Picking the Midterm Exam Date
As mentioned in lecture today, I'd like to pick the date for our midterm exam. It will be during lecture in a couple of weeks, and it will cover sections 1.1-1.3, 2.1-2.7, 3.1-3.2, 4.1-4.13, and 5.5 in SHV.
I've selected two possible exam dates. Please vote for the option that you prefer.
If you are asking for a date other than the two suggested options, please post a follow-up to explain why these dates don't work for you and suggest an alternative.
I prefer Thursday, October 31
I prefer Tuesday, November 5
Either option is fine with me
I prefer a date before October 31
I prefer a date after November 5
Submit
You have not yet voted.
Revoting is not allowed. Select your vote and click submit to register your vote.
Your name will be visible to instructors but not students.
midterm_exam lecture13 polls
edit
good poll 0
1 day ago by Katherine J. Kuchenbecker
Picking the Midterm Exam Date closes in 3 hours
A total of 38 vote(s) in 45 hours
8 (21% of users) I prefer Thursday, October 31 Show Voters
16 (42% of users) I prefer Tuesday, November 5 Show Voters
7 (18% of users) Either option is fine with me Show Voters
1 (3% of users) I prefer a date before October 31 Show Voters
6 (16% of users) I prefer a date after November 5 Show Voters
Average Response Time:
9 min
Special Mentions:
Katherine J. Kuchenbecker answered Receiving Error from pumaStart() in 9 min. 23 hours ago
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9 | 102
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MEAM 520

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hw1 hw2 hw3 hw4 hw5 hw6 final_exam lecture2 lecture5 lecture6 lecture7 lecture8 lecture9 lecture11 lecture12 project1 other office_hours textbook

note

stop following 47 views

Anonymous Mid-Semester Teaching Evaluation

I'd like to know how you think MEAM 520 is going so far, so I created a simple anonymous survey:
https://docs.google.com/forms/d/1H1Q57EBK1BW6Q3oYO2oq0J1le_4FLLoU0YVWvqvE9e0/viewform

Please enter your comments by this Friday, October 18. Your responses are anonymous, so you should feel comfortable being honest.

We appreciate your taking the time to complete this evaluation; your feedback will help us improve the class and our teaching for everyone's benefit.

other

edit · good note | 0

2 days ago by Katherine J. Kuchenbecker

followup discussions *for lingering questions and comments*

Start a new followup discussion

Compose a new followup discussion

Average Response Time: 8 min

Special Mentions: Katherine J. Kuchenbecker answered Hitting velocity limit in 12 min. 6 hours ago

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