Name Solutions

Final Exam

MEAM 520, Introduction to Robotics
University of Pennsylvania
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December 19, 2012

You must take this exam independently, without assistance from anyone else. You may bring in a calculator and four 8.5"×11" sheets of notes for reference. Aside from these pages of notes, you may not consult any outside references, such as the textbook or the Internet. Any suspected violations of Penn's Code of Academic Integrity will be reported to the Office of Student Conduct for investigation.

This exam consists of five problems. We recommend you look at all of the problems before starting to work. If you need clarification on any question, please ask a member of the teaching team. When you work out each problem, please show all steps and $\boxed{\text{box your answers}}$. On problems involving actual numbers, please keep your solution symbolic for as long as possible; this will make your work easier to follow and easier to grade. The exam is worth a total of 100 points, and partial credit will be awarded for the correct approach even when you do not arrive at the correct answer.

	Points	Score
Problem 1	20	
Problem 2	20	
Problem 3	20	
Problem 4	20	
Problem 5	20	
Total	100	

I agree to abide by the University of Pennsylvania Code of Academic Integrity during this exam. I pledge that all work is my own and has been completed without the use of unauthorized aid or materials.

Signature ______________________

Date ______________________

Problem 1: Short Answer *(20 points)*

a. Compared to analog angle sensors such as potentiometers, explain **one advantage** of using an **incremental optical encoder** for measuring the angle of a revolute joint. *(2 points)*

Encoders have many advantages over analog sensors, such as less susceptibility to electrical noise, infinite travel range, higher measurement resolution, no friction, and the fact that you don't need an analog-to-digital converter.

b. Compared to analog angle sensors such as potentiometers, explain **one disadvantage** of using a **incremental optical encoder** for measuring the angle of a revolute joint. *(2 points)*

Some disadvantages of encoders include being a relative (rather than absolute) sensor, higher cost, the introduction of quantization noise when you differentiate angle to get angular velocity, and needing encoder counting hardware.

c. Why do roboticists typically use **current-drive amplifiers** instead of voltage-drive amplifiers with DC brushed motors? *(2 points)*

The motor's output torque is proportional to current, not to voltage. Torque is mechanically important.

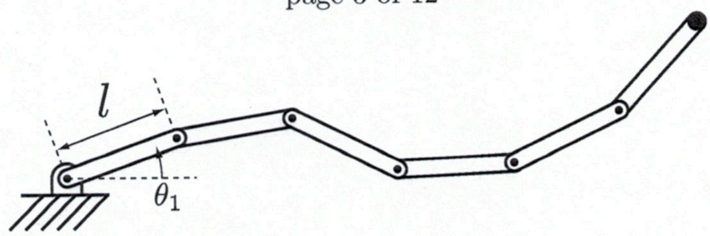
d. The pumaServo.m function imposes a **maximum joint speed** of 1 rad/s. Why did the teaching team include this speed limit? *(2 points)*

The real Puma cannot move much faster than 1 rad/s on some joints, so tracking is best when this limit is obeyed. Slow speeds are also <u>safer</u>.

e. Ask a **good short-answer question and answer it** for yourself. Your question should pertain to this class and should be different from the actual questions on this test. *(2 points)*

Question:

Answer:



The diagram above shows a planar RRRRRR manipulator that resembles a snake. Each link has the same length l. The first angle is measured relative to horizontal, as indicated. The other angles are all measured relative to the previous link, with positive counter-clockwise.

f. How many inverse kinematics solutions exist if I specify the planar position of the snake's tip? Does the number of solutions depend on the commanded position? If so, how? *(3 points)*

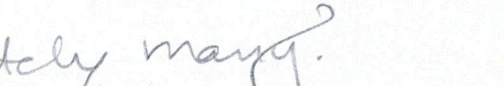
- At the edge of the snake's workspace (a circle of radius $6l$), there is only one solution.
- Outside this circle, there are zero solutions.
- Inside this circle, there are infinitely many solutions.

g. How many inverse kinematics solutions exist if I specify both the planar position and the orientation of the snake's tip? Does the number of solutions depend on the commanded tip configuration? If so, how? *(3 points)*

We can use kinematic decoupling: find for necessary position of final joint. If that is outside a circle of radius $5l$, no solutions. On the edge of this circle, one solution. Inside the circle, infinitely many.

h. Considering both the planar translational velocity and rotational velocity of the snake's tip, explain what singularities this robot has. Calculations should not be needed. *(4 points)*

All joints move this way

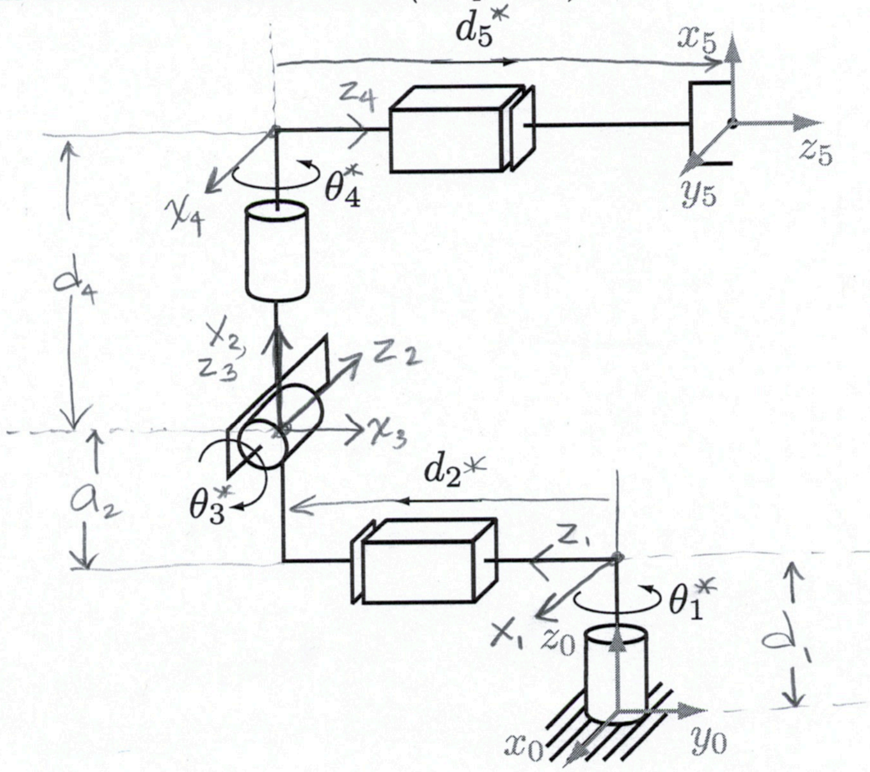


No constraints on θ_1

The robot will be in a singular configuration if all θ_2 to θ_6 equal $0°$ or $180°$. $\begin{bmatrix}\theta_1\\0\\0\\0\\0\\0\end{bmatrix}$ or $\begin{bmatrix}\theta_1\\180\\0\\180\\180\\0\end{bmatrix}$ ... any comb of 0's and 180

The robot is in a straight line, so the tip cannot move along the robot's length.

Problem 2: Forward Kinematics *(20 points)*



The diagram above shows an RPRRP manipulator.

- The positive direction for each joint coordinate is indicated with an arrow.
- The revolute joints are shown at $\theta = 0$.
- The prismatic joints are shown at a positive deflection.
- Frame 0's location is specified on the ground as shown.
- Frame 5's location is specified on the end-effector as shown.

a. Draw **frames 1 through 4** on the above diagram, following the DH convention. You may omit the y-axes if you want. *(4 points)*

b. Fill in the table of **DH parameters** below. Use a **superscript star** to indicate joint variables, e.g., θ_1^*. **Label on the diagram** any DH parameters that you introduce. Be sure to **label the span of** d_2 **and** d_5. *(10 points)*

i	a	α	d	θ
1	0	90°	d_1	θ_1^*
2	a_2	-90°	d_2^*	90°
3	0	90°	0	$\theta_3^* + 90°$
4	0	-90°	d_4	$\theta_4^* - 90°$
5	0	0°	d_5^*	-90°

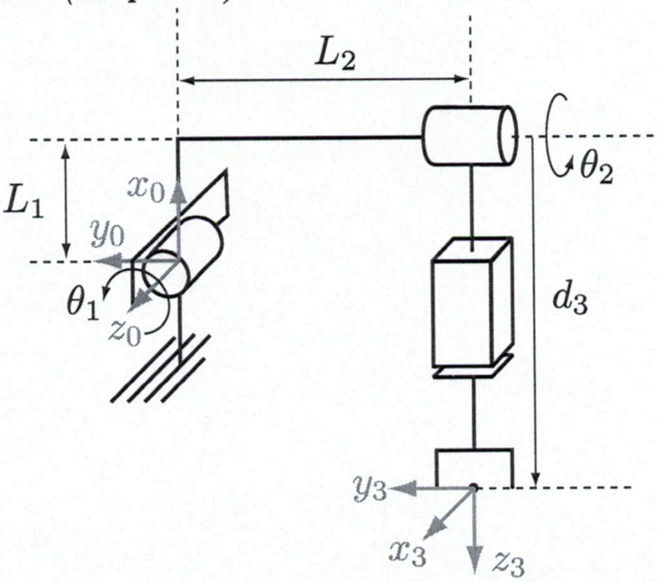
c. Is there **only one correct set** of frame locations and DH parameters for the previous question, or are there **multiple correct sets**? Explain your answer. *(3 points)*

There are multiple correct sets of DH parameters. You always can flip the direction of each x-axis, and you can also flip any z-axis direction (using $-\theta_i$). When successive axes are parallel or coincident, you can also put the origin in infinitely many places (doesn't apply to this robot).

d. Suppose you had a computer with MATLAB installed. How would you **check whether your chosen DH parameters are correct**? Carefully explain what you would do. *(3 points)*

Use forward kinematics to plot all of the origins, connected by lines.
$o_1 = A_1 o$ $o_2 = A_1 A_2 o$ $o_3 = A_1 A_2 A_3 o$
When all $\theta_i^* = 0$ and $d_i^* > 0$, robot should resemble the provided diagram. Then you can move the joints to multiple configurations to see if it's behaving correctly, such as positive directions.

Problem 3: Jacobians *(20 points)*



Consider the RRP spatial manipulator shown above. The two revolute joints are shown at $\theta_1 = 0$ and $\theta_2 = 0$, and the prismatic joint is shown at a positive deflection d_3. The position of the end-effector (the origin of frame 3) in frame 0 is as follows, with s_i and c_i meaning $\sin\theta_i$ and $\cos\theta_i$ respectively:

$$p_3^0 = \begin{bmatrix} L_1 c_1 + L_2 s_1 - d_3 c_1 c_2 \\ L_1 s_1 - L_2 c_1 - d_3 s_1 c_2 \\ -d_3 s_2 \end{bmatrix}$$

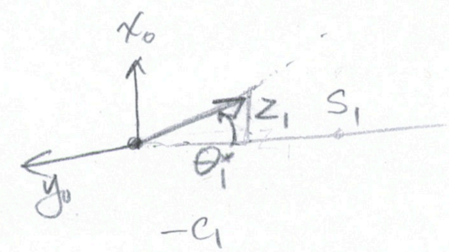
a. Find the translational Jacobian J_v for this manipulator. *(4 points)*

$$\begin{array}{cc} & \begin{array}{ccc} \downarrow \partial/\partial\theta_1 & \downarrow \partial/\partial\theta_2 & \downarrow \partial/\partial d_3 \end{array} \\ \boxed{J_v = \begin{bmatrix} -L_1 s_1 + L_2 c_1 + d_3 s_1 c_2 & d_3 c_1 s_2 & -c_1 c_2 \\ L_1 c_1 + L_2 s_1 - d_3 c_1 c_2 & d_3 s_1 s_2 & -s_1 c_2 \\ 0 & -d_3 c_2 & -s_2 \end{bmatrix}} & \begin{array}{c} \leftarrow x \\ \leftarrow y \\ \leftarrow z \end{array} \end{array}$$

b. Find the rotational Jacobian J_ω for this manipulator. *(4 points)*

$$\boxed{J_w = \begin{bmatrix} 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ z_0^0 & z_1^0 & \text{prismatic} \end{array}$$



c. Suppose the manipulator is at $\theta_1 = \pi/2$, $\theta_2 = 0$, and $d_3 = 0.3$ m, with $L_1 = 0.15$ m and $L_2 = 0.2$ m. What will the **translational velocity of the end-effector** be if $\dot{\theta}_1 = 1$ rad/s, $\dot{\theta}_2 = -1$ rad/s, and $\dot{d}_3 = 1$ m/s? Express your answer in frame 0 (v_3^0). *(4 points)*

$\theta_1 = \frac{\pi}{2}$ $\quad \theta_2 = 0$ $\quad d_3 = 0.3\,m$

$S_1 = 1$ $\quad C_1 = 0$ $\quad S_2 = 0$ $\quad C_2 = 1$

$L_1 = 0.15\,m$

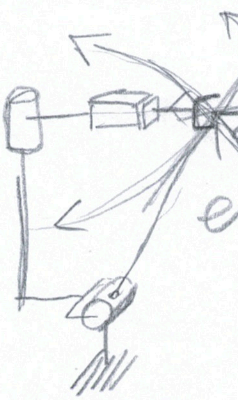
$L_2 = 0.2\,m$

$$V_3^0 = J_v([\theta_1\ \theta_2\ d_3]) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}$$

$$= \begin{bmatrix} -L_1 + d_3 & 0 & 0 \\ L_2 & 0 & -1 \\ 0 & -d_3 & 0 \end{bmatrix} \begin{bmatrix} 1\,rad/s \\ -1\,rad/s \\ 1\,m/s \end{bmatrix} = \begin{bmatrix} 0.15\,m/s \\ 0.2\,m/s - 1\,m/s \\ 0.3\,m/s \end{bmatrix} \quad \boxed{V_3^0 = \begin{bmatrix} 0.15\,m/s \\ -0.8\,m/s \\ 0.3\,m/s \end{bmatrix}}$$

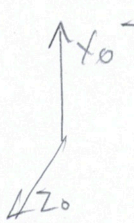
d. Can the robot's end-effector **exert forces** in all Cartesian directions at this same configuration ($\theta_1 = \pi/2$, $\theta_2 = 0$, and $d_3 = 0.3$ m, with $L_1 = 0.15$ m and $L_2 = 0.2$ m)? Explain. *(4 points)*

$\vec{\tau} = J_v^T \vec{F}$ $\qquad \det(J_v) = (-L_1 + d_3)(-d_3) = (+0.15m)(-0.3m) \neq 0$



At this configuration, the J_v matrix full rank (not singular), so yes it can exert forces in all directions. The same can be seen in the diagram at left; arrows show forces from joint torques.

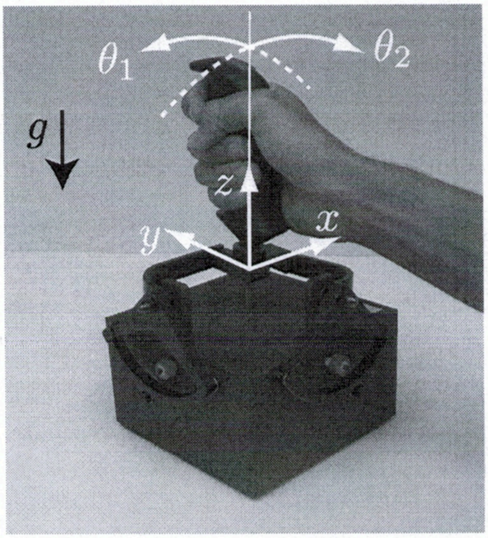
e. What **limitations**, if any, exist on the end-effector's **angular velocity** in this same configuration? *(4 points)*

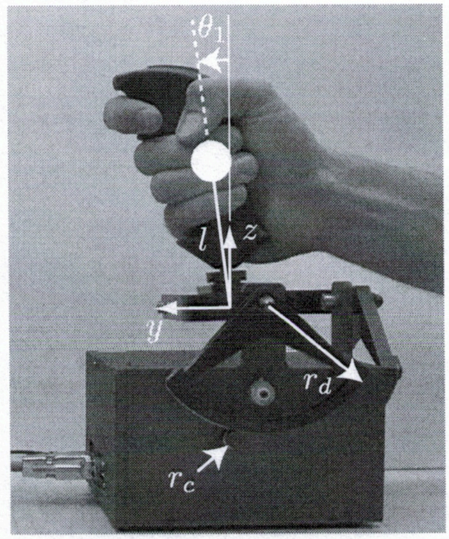


$$J_\omega = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ ← all zeros in second row, so the robot end-effector cannot rotate around the y_0 axis.

$$\omega_3^0 = J_\omega \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}$$

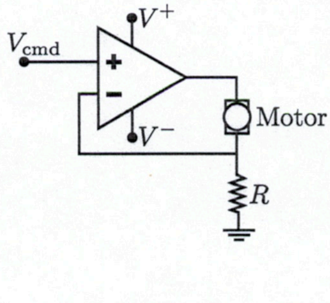
Problem 4: Input/Output Calculations *(20 points)*





The pictures above show an Immersion Impulse Engine 2000 joystick. It has two degrees of freedom: the user can move the handle forward/backward and left/right.

- Each of the joystick's joints is controlled by a Maxon RE025-055-35 118752 DC brushed motor that is located inside the base. This motor's rated voltage is 24 V, its maximum continuous current is 1.23 A, its torque constant is 0.0232 Nm/A, its terminal resistance is 2.32 Ω, its terminal inductance is 0.24 mH, and it does not have a gearhead.
- Attached to the back end of each motor shaft is an Agilent HEDS 5500 A02 incremental optical encoder with a resolution of 500 cycles per revolution (500 slits in the rotating disk).
- The output of each encoder is processed through a quadrature encoder counting circuit.
- There is a cylindrical capstan attached to the output shaft of each motor. Its radius is $r_c = 0.5$ cm.
- Thin stranded cables couple the rotation of each capstan to the rotation of its drum.
- The drum radius is $r_d = 7.0$ cm.
- The joystick's handle is rigidly attached to both drums through simple linkages.
- The distance from the point where the two joint axes intersect to the center of the hand is $l = 9.0$ cm.
- A copy of this device is available for you to inspect in the exam room. One side of the base has been removed so you may look inside to see the motors and circuitry.
- Each motor is driven via the circuit shown below, with $V^+ = 30$ V, $V^- = -30$ V, and $R = 2\ \Omega$. V_{cmd} is the voltage your software commands.



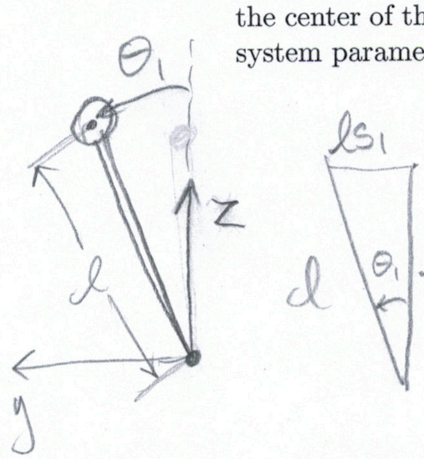
a. Your servo loop reads the present quadrature encoder counts, Q_1 and Q_2, at each time step. You notice that Q_1 increases when you move the handle *forward*, and Q_2 increases when you move the handle *left*. When you hold the handle straight up, $Q_1 = 1040$ counts and $Q_2 = -1762$ counts. The manufacturer does not provide a command for zeroing the quadrature encoder counts directly. Write **formulas for θ_1 and θ_2** such that both joint angles are zero when the handle is straight up, both are measured in radians, and both increase when the handle is moved *forward* and *right*. *(6 points)*

Need to account for zero, direction, resolution, of 500 cycles per rev $\therefore \frac{2000 \text{ counts}}{1 \text{ rev.}}$ $\frac{2000 \text{ counts}}{2\pi \text{ rad}}$ and gear ratio.

$$\boxed{\theta_1 = +(Q_1 - 1040 \text{ counts}) \cdot \frac{2\pi \text{ rad}}{2000 \text{ counts}} \cdot \frac{0.5\text{cm}}{7.0\text{cm}}}$$

$$\boxed{\theta_2 = \underbrace{-}_{\text{direction}}\underbrace{(Q_2 + 1762 \text{ counts})}_{\text{enc offset}} \cdot \underbrace{\frac{2\pi \text{ rad}}{2000 \text{ counts}}}_{\text{encoder resolution w/ quadrature}} \cdot \underbrace{\frac{0.5\text{ cm}}{7.0\text{cm}}}_{\text{gear ratio}}}$$

b. Imagine you hold θ_2 perfectly at zero. Write **formulas for the x, y, and z coordinates** of the center of the hand (the white circle in the diagram) as a function of θ_1 and any necessary system parameters. Use the depicted coordinate frame. *(4 points)*



$$\boxed{\begin{aligned} x &= 0 \text{ cm} \\ y &= (9.0\text{cm}) \cdot \sin\theta_1 \\ z &= (9.0\text{cm}) \cdot \cos\theta_1 \end{aligned}}$$

$l = 9.0\text{cm}$

c. The joystick was designed to have low friction and be lightweight. For your application, though, you added a new handle that is quite heavy. Unfortunately, this new handle tends to fall toward the extremes of its workspace when moved away from vertical. What are **two distinct approaches you could use to prevent this falling behavior**? *(4 points)*

① Use the motors to exert the torque needed to hold the handle up at every angle. Need to know τ_{max} and structure ($\sin\theta_1, \sin\theta_2$). Could test + tune.

② Add a mechanical counterbalance somewhere so gravity pulls handle up.

d. Imagine your code has already calculated the two **joint torques** $\tau_1 = -0.1$ **Nm and** $\tau_2 = 0.8$ **Nm** that you want to output with this joystick. Both of these are in newton-meters (Nm) and are defined to be positive in the same directions as θ_1 and θ_2. During testing, you noticed that a positive command voltage makes each motor rotate its joint in the positive direction. What **command voltages** $V_{\text{cmd},1}$ and $V_{\text{cmd},2}$ should you output to each of the drive circuits? *(6 points)*

$\tau_1 = -0.1\,\text{Nm}$ $\tau_2 = 0.8\,\text{Nm}$

Need to account for gear ratio, motor torque constant, and current amp gain.

$V = iR$ w/ $R = 2\Omega$

r_c/r_d k_t $2\Omega = R$

$$V_{cmd,1} = -0.1\,\text{Nm} \cdot \frac{0.5\,\text{cm}}{7.0\,\text{cm}} \cdot \frac{1\,\text{A}}{0.0232\,\text{Nm}} \cdot \frac{2\,\text{V}}{1\,\text{A}} = -0.616\,\text{V}$$

k_t $2\Omega = R$

$$V_{cmd2} = 0.8\,\text{Nm} \cdot \frac{0.5\,\text{cm}}{7.0\,\text{cm}} \cdot \frac{1\,\text{A}}{0.0232\,\text{NM}} \cdot \frac{2\,\text{V}}{1\,\text{A}} = \cancel{4.926\,\text{V}}$$

This is quite high.

1.23A

$i_2 = 2.463\,\text{A} \geq I_{cont,max}$

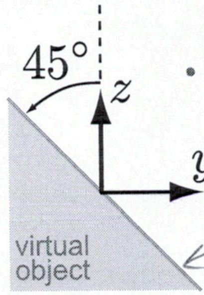
so output max $1.23\,\text{A} \cdot \frac{2\,\text{V}}{\text{A}} = 2.46\,\text{V}$

$$\boxed{\begin{aligned} V_{cmd,1} &= -0.616\,\text{V} \\ V_{cmd,2} &= 2.46\,\text{V} \end{aligned}}$$

4.926V is correct except for current limit.

Problem 5: Haptic Rendering and Teleoperation *(20 points)*

a. Imagine you are using a PHANToM Premium 1.0 to create the three-dimensional haptic virtual environment shown below. The x-axis (not shown) is positive out of the page.



$\cdot\ \vec{h} = [h_x \ \ h_y \ \ h_z]^T$

$y = -z$ or $z = -y$
or $y + z = 0$

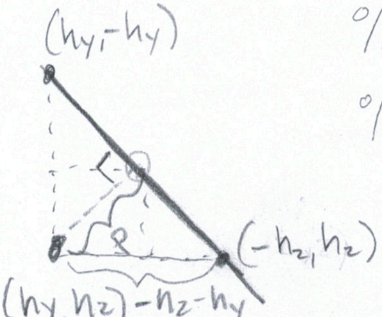
Your goal is to let the user feel an infinite frictionless flat plane that intersects the origin and is tilted 45° around the x-axis to face up and to the right. At each time step, you know the position of the haptic interface's tip ($\vec{h} = [h_x \ \ h_y \ \ h_z]^T$). Write pseudocode that specifies appropriate values for the **force vector components** F_x, F_y, and F_z to give the user the illusion they are touching the depicted plane. The force components are defined to be positive in the same directions as their associated axes. *(6 points)*

% Check Collision or $(h_y + h_z < 0)$ or $(h_y < -h_z)$

if $(h_z < -h_y)$

% User is contacting the surface.

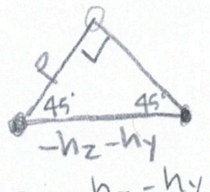
% Calculate total penetration distance



$p = (-h_y - h_z)/\sqrt{2}$;

% Calculate total force.

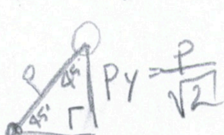
$F = kp$;



$p = \dfrac{-h_z - h_y}{\sqrt{2}}$

$F_y = k \cdot p/\sqrt{2} = k(-h_y - h_z)/2$;

$F_z = k(-h_y - h_z)/2$; % same as F_y



$F_x = 0$;

$p_x = \dfrac{p}{\sqrt{2}}$

else

$F_x = F_y = F_z = 0$

end

b. When Professor Kuchenbecker tested all of the haptic damping programs submitted for project 2, she found that most felt very smooth, like moving a spoon in honey. But a few submissions generated **a small vibration at about 33 Hz** that she could feel. The vibration was strongest when she moved fast, and it was not detectable when holding still. What do you think caused these vibrations? Be specific. *(6 points)*

The code was not executing at an even rate, and these groups assumed constant Δt instead of measuring it every iteration using toc. The velocity estimate was thus spiky, making vibrations. Period $= \frac{1}{33\text{Hz}}$ $= 0.03$s

Answering the velocity is not filtered enough is not a great answer. This is the graphics loop period.

c. Imagine we replaced the PUMA's tri-color LED end-effector with a small paintbrush, and we attached a cup of paint and a large piece of paper to the table within the PUMA's workspace. Carefully describe a **controller** that would enable you **to teleoperate the 6-DOF PUMA with the 3-DOF PHANToM Premium 1.0** and paint a picture. Be thorough. *(8 points)*

- Use forward kinematics to calculate Phatom tip position. Use scaling to calculate the desired position for the puma (from Phantom pos.)
- Use IK on Puma to get joint angles to put wrist center at desired position choosing solution closest to current pos.
- Do orientation IK on Puma to point brush ↓.
- Run PID controller on each Puma joint (puma Servo) plus gravity compensation
- Send actual position of Puma back, invert scale to calculate Phantom's des. pos.
- Use Cartesian PD, J^T to pull Phantom to des pos.