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Latin Hypercube Sampling Monte Carlo Estimation of Average Quality Index for Integrated Circuits

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Abstract. The Monte Carlo method exhibits generality and insensitivity to the number of stochastic variables, but is expensive for accurate Average Quality Measure (AQI) or Parametric Yield estimation of MOS VLSI circuits. In this contribution a new method of *variance reduction* technique, viz. the *Latin Hypercube Sampling (LHS)* method is presented which improves the efficiency of AQI estimation in integrated circuits especially for MOS digital circuits. This method is similar to the *Primitive Monte Carlo (PMC)* method except in samples generation step where the Latin Hypercube Sampling method is used. This sampling method is very simple and does not involve any further simulations. Moreover, it has a smaller variance with respect to the PMC estimator. Encouraging results have thus far been obtained. A 3-dimensional quadratic function, a high pass filter, and a CMOS delay circuit examples are included to demonstrate the efficiency of this technique.

Key Words: Monte Carlo methods, Latin hypercube sampling, average quality index

1. Introduction

During the past decade, the feature sizes of VLSI devices have been scaled down rapidly. Despite the technological progress in patterning fine-line features, the fluctuations in etch rate, gate oxide thickness, doping profiles, and other fabrication steps that are critical to device performances have not been scaled down in proportion. Consequently, the Average Quality Index (AQI) [1] or its special case Parametric Yield is becoming increasingly critical in VLSI design. Circuit designers must ensure their chips will have an acceptable quality or parametric yield under all manufacturing process variations.

The Monte Carlo (MC) method [2], [3] is the most reliable technique in AQI and yield estimation of electrical circuits. The method is applicable to any type of circuit without requiring simplifying assumptions of the forms of the probability distribution of a parameter's value or restrictions on the number of parameters. Nevertheless, it requires a large number of circuit simulations to have a valuable estimation, i.e., to have a low variance estimator.

In the literature to date, several variance reduction techniques [3] which can be applied to the yield estimation (in particular, *importance sampling* [4], [5], [10], *stratified sampling* [5], [8], and *control variates* [5]–[7], [9]) have been studied. Hocevar *et al*. [5] have shown that *importance sampling* is not generally very useful for variance reduction in the MC yield estimation, and the efficiency of stratified sampling is not significant with respect to the complexity of its implementation. The generality and usefulness of the control variate or *shadow model* technique in comparison with alternative methods in MC yield estimation has since been confirmed by Hocevar *et al*. [5], [9] and Soin and Rankin [6], [7]. But this method requires some information about the performance behavior which involves some additional simulation costs. However, all of the variance reduction techniques of the yield estimator are not applicable to the AQI estimator. In contrast to yield estimation, there are not many reports on AQI estimation.

In this paper, we present an efficient method of variance reduction technique for estimating AQI and parametric yield of integrated circuits, especially MOS digital circuits. This method is similar to the Primitive MC (PMC) [2], [3] method except in samples generation where the Latin Hypercube Sampling (LHS) [11] method is used. In contrast to the cited variance reduction technique, this method is very simple and does not involve any further simulations. Moreover, it has a smaller variance with respect to the PMC yield estimator.

In the following section, we describe the statistical quality measures and its special case parametric yield, and briefly review the variance reduction techniques in the MC estimation of AQI. In Section 3, the LHS approach is presented and several theorems related to its properties are given. The applications of LHS will be discussed in Section 4. We give, in Section 5, the successful results of LHS application to a quadratic function and a CMOS circuit. Finally, concluding remarks are made in Section 6.

2. Statistical Quality Measures

The quality of a circuit can be defined in various manners. In a fabrication line, the circuit quality changes from one to another. This is why, one needs to define a Statistical Quality Measure (SQM) for a circuit's production, e.g., AQI definition. One of the important quantities of AQI is the manufacturing parametric yield of a circuit. In this section, the general definition of AQI and parametric yield are discussed. It will be followed by a review of variance reduction techniques for the AQI and yield estimator.

2.1. *Definition of Quality Index Function*

Assume that we have circuit performances $\underline{y} = (y_1, y_2, \ldots, y_m)$. For each performance y_i, a *membership function* $\mu_i = \mu_i(y_i)$ can be defined by using the *fuzzy sets* [12]. μ_i can be interpreted as a quality index measuring the goodness of performance y_i. As an example, for a rough specification $y_i < L_i$, a "sigmoidal" type membership function can be considered. This function is described as

$$\mu(y_i) = \frac{1}{1 + \exp[(y_i - L_i)/\beta]}, \tag{1}$$

where L_i is the value for y_i at which $\mu(y_i)$ takes on the value one-half and β is the parameter of "transition region" (Fig. 1). A good circuit should have a high value of the quality index μ_i for each corresponding performance. The circuit quality index can be defined as

$$\mu(\underline{y}) = i[\mu_1(y_1), \mu_2(y_2), \ldots, \mu_m(y_m)], \tag{2}$$

where $i[.]$ is an appropriate intersection operator [12]. For instance, a general weighted sum intersection is given by [1]

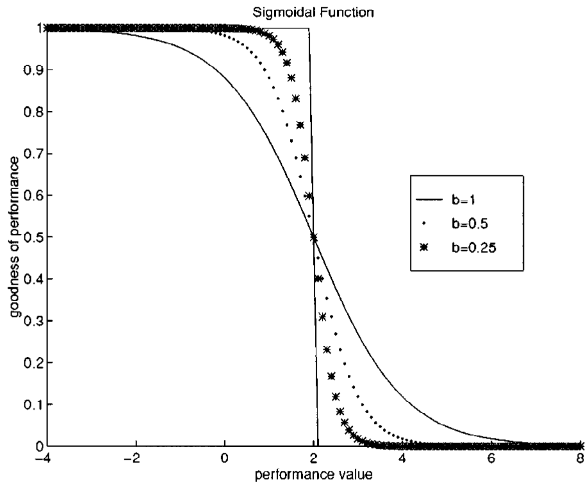

Fig. 1. Crisp and sigmoidal membership function.

$$i_\alpha[\mu_1, \mu_2, \ldots, \mu_m, w_1, w_2, \ldots, w_m] = \left(\sum_{i=1}^{m} w_i \mu_i^\alpha\right)^{1/\alpha}$$

$$0 \le w_i \le 1 \qquad \sum_{i=1}^{m} w_i = 1. \tag{3}$$

Generally, each performance y_i is a function of circuit parameters p. For a given nominal design parameter, the quality index $\mu(\underline{y}(p))$ will be different from one circuit to another, due to process disturbances. Therefore, the goodness of a design should be statistically measured. A robust design should have high AQI [1], defined as follows:

$$Q = \bar{\mu} = \int_{R^d} \mu(\underline{y}(p))\, f_p(p)\, dp = E_p\{\mu(\underline{y}(p))\}, \tag{4}$$

where $f_p(p)$ is the joint probability density function (j.p.d.f.) of circuit parameters and R^d is circuit parameters space. This measure can be considered as the *probability measure of a fuzzy event* [13]. It can be shown that other quality indicies, such as parametric yield, can be derived from AQI.

In the VLSI circuit, the probability distribution of integrated circuit parameters is difficult to model. The parameters p are correlated in a complicated manner, and it is impossible to find an analytic expression for their j.p.d.f. [14].

In the case of integrated circuits, the circuit parameters can be modeled as functions of their deterministic nominal values, p^0, and a set of process disturbances,

ξ, i.e., $p = p(p^0, \xi)$. In addition, the components of ξ can be considered mutually independent [15]. One can formulate the AQI in disturbance space as

$$Q = \bar{\mu} = \int_{R^n} \mu(\underline{y}(p^0, \xi)) f_\xi(\xi)\, d\xi, \tag{5}$$

where $f_\xi(\xi)$ is the j.p.d.f. of the process disturbances, and R^n is disturbance space.

For statistical circuit design we need to calculate the AQI. It can numerically be evaluated using either the quadrature-based, or MC-based [2], [3] methods. The quadrature-based methods have computational costs that explode exponentially with the dimensionality of the statistical space. The MC method is the most reliable technique for the statistical analysis of electrical circuits. The unbiased MC-based estimator of AQI can be expressed as

$$\hat{Q}_{MC}(p^0) = \frac{1}{N} \sum_{i=1}^{N} \mu(p^0, \xi^i), \tag{6}$$

where $\mu(p^0, \xi^i)$ denotes $\mu(\underline{y}(p^0, \xi^i))$, ξ^i's are independently drawn random samples from $f_\xi(\xi)$, and N is the sample size.

2.2. *Parametric Yield Estimation*

One of the most important quality indicies in statistical design is the parametric yield. In order to estimate yield for analysis and design, one first defines a set of performance or response functions, along with constraints for those functions, for each circuit as

$$g_i(p) \leq 0 \qquad i = 1, 2, \ldots, m, \tag{7}$$

where p is the vector of circuit parameters. Also, the acceptability region A_p is defined as

$$A_p = \{p \mid g_i(p) \leq 0 \quad i = 1, 2, \ldots, m\}. \tag{8}$$

The constraint functions are usually only known implicitly via simulations, and thus can be very costly to evaluate.

Mathematically, parametric yield of an integrated circuit is defined as the probability of a circuit meeting the design specifications, i.e.,

$$Y = \int_{R^d} I_p(p) f_p(p)\, dp, \tag{9}$$

where $I_p(p)$ is an indicator function described as

$$I_p(p) = \begin{cases} 1 & p \in A_p \\ 0 & \text{otherwise} \end{cases}. \tag{10}$$

From (4), it is seen that the yield defined by (9) is a special case of AQI, where the quality index function $\mu(\underline{y}(p))$ is replaced by indicator function $I_p(p)$. On the other hand, in AQI the acceptability region A_p is transformed to a fuzzy set, where each circuit belongs to A_p with a *grade* defined by a membership function $\mu(\underline{y}(p))$.

Similar to AQI, one can formulate the yield in disturbance space as

$$Y = \int_{R^n} I_\xi(\xi) f_\xi(\xi)\, d\xi, \tag{11}$$

where $I_\xi(\xi)$ is defined in a manner similar to (10). Also, the unbiased MC-based estimator of yield can be expressed as

$$\hat{Y}_{MC} = \frac{1}{N} \sum_{i=1}^{N} I_\xi(\xi), \tag{12}$$

where ξ^i's are independently drawn random samples from $f_\xi(\xi)$, and N is the sample size. It can be easily shown that the variance of $\hat{Y}_{MC}$ is as follows:

$$\sigma_Y^2 = \frac{Y(1-Y)}{N} = \frac{\sigma_B^2}{N}, \tag{13}$$

where σ_B^2 is the variance of the binomial random variable $I_\xi(\xi)$ [16]. The variance of the estimator is independent of the dimension of the parameter space but depends on the square root of the sample size N.

2.3. *Variance Reduction Techniques for Monte Carlo Estimation*

In statistical circuit design we need to estimate AQI. The MC method [2], [3] is the most reliable technique in AQI estimation of electrical circuits. The method is applicable to any type of circuit without requiring simplifying assumptions of the forms of the probability distribution of a parameter values or restrictions on the number of parameters. Nevertheless, it requires a large number of circuit simulations to have a valuable estimation (to have a low variance estimator).

The variance of the MC estimator can be reduced by using the *variance reduction techniques* [2], [3]. Among these techniques, *importance sampling* [4], [5],

[10], *stratified sampling* [5], [8], and *control variates* [5]–[7], [9] have been studied for yield estimation. The applications of *importance sampling* have shown that this technique is not generally very useful for variance reduction in the MC yield estimation [5]. The variance of the stratified sampling estimator is smaller than PMC estimator [5], but its efficiency is not very considerable and the implementation of sampling strategy is not simple. In addition, for these methods we need to have some knowledge about the acceptability region of the circuit.

The generality and usefulness of the control variate or *shadow model* technique in comparison with alternative methods in MC yield estimation has since been confirmed by Hocevar *et al*. [5], [9] and Soin and Rankin [6], [7]. The type of control model has an important role in the efficiency of control variate method. It can be a simplified circuit or any function approximation (e.g., *Response Surface Methodology* (RSM) [17]). Therefore, we need some supplementary knowledge about the circuit responses that involves some additional computational costs.

All of the variance reduction techniques of the yield estimator are not applicable to the AQI estimator. For example, the *Sectional Weighting* method [5], which is a type of importance sampling, uses an approximation of acceptability region that is not defined in AQI method. In contrast to the yield estimator, there are not many reports on the AQI estimator.

3. Latin Hypercube Sampling Monte Carlo (LHSMC)

The method of *LHS* is an extension of *quota sampling* [18], and can be viewed as an n-dimensional extension of *Latin square sampling* [19]. This method first was used in "*Uncertainty Analysis*" by selecting input values $X = (x_1, x_2, \ldots, x_n)$ (random variable) of a function $y = h(X)$, in order to estimate the cumulative distribution function (c.d.f.) and mean value of y [11], [20]–[22]. Then, it has been widely used for base *point generation* in the construction of Design Matrix [17] (*Design of Experiments*) in function approximation of circuit performances (RSM) [23]–[26].

This sampling approach ensures that each of the input variables has all portions of its range represented. LHS is computationally cheap to generate and can cope with many input variables. In the following text the generation and application of LHS in MC yield estimation is presented.

Table I. A 10-run Latin hypercube sample for 3 parameters

Run	ξ_1	ξ_2	ξ_3
1	0.34	−0.73	0.87
2	−0.12	−0.53	0.52
3	−0.97	0.17	−0.38
4	0.72	−0.82	0.08
5	0.85	−0.14	−0.53
6	−0.25	0.35	−0.65
7	0.59	0.62	0.79
8	−0.68	0.93	0.31
9	−0.48	0.52	−0.92
10	0.06	−0.24	−0.18

3.1. *Samples Generation*

The LHS method [11] is a type of stratified MC sampling [5]. The sampling region is partitioned into a specific manner by dividing the range of each component of ξ. We will only consider the case where the components of ξ are independent or can be transformed to independent bases. Moreover, the sample generation for correlated components with Gaussian distribution can be easily achieved [21].

As originally described, LHS operates in the following manner to generate a sample size N from the n variables $\xi_1, \xi_2, \ldots, \xi_n$. The range of each variable is partitioned into N non overlapping intervals on the basis of equal probability size $1/N$. One value from each interval is selected at random with respect to the probability density in the interval. The N values thus obtained for ξ_1 are paired in a random manner with the N values of ξ_2. These N pairs are combined in a random manner with the N values of ξ_3 to form N triplets, and so on, until a set of N n-tuples is formed. This set of n-tuples is the Latin hypercube sample. Thus, for given values of N and n, there exist $(N!)^{n-1}$ possible interval combinations for a LHS. A 10-run LHS for 3 normalized parameters (range $[-1, 1]$) with the uniform p.d.f. is listed in Table I. In this case the equal probability spaced values are $-1, -0.8, \ldots, 0.8, 1$.

3.2. *Efficiency of LHSMC*

Consider the case that ξ denotes an n-vector random variable with j.p.d.f. $f_\xi(\xi)$ for $\xi \in S$. Let h denote an objective function given by $h = q(\xi)$. Consider now the following class of estimators

$$T = \frac{1}{N}\sum_{i=1}^{N} g(h^i), \tag{14}$$

where $g(.)$ is an arbitrary known function and $h^i = q(\xi^i)$. If $g(h) = h$ then T represents an estimator of $E(h)$. If $g(h) = h^r$ one obtains the r^{th} sample moment. By choosing $g(h) = u(c-h)$ ($u(.)$ is a step function), one achieves the empirical distribution function of h at the point c. Now consider the following theorem.

THEOREM 1 *If ξ^i's are generated by the LHS method. Then, the statistic T (14) is an unbiased estimator of the mean of $g(h)$. That is,*

$$E[T] = E[g(h)]. \tag{15}$$

Proof: This is a special case of *Theorem 1* in [20]. It should be emphasized that even if the variables are correlated the LHS estimator will be unbiased.

Let T_R denote estimator (14) with standard random sampling of ξ, and T_L denote the estimator with the LHS generator of ξ. Now consider the following theorem related to the variances of T_L and T_R.

THEOREM 2 *If $h = q(\xi_1, \xi_2, \ldots, \xi_n)$ is monotonic at least in $(n-1)$ of its arguments, and if $g(h)$ is a monotonic function of h, then the variance of LHSMC estimator is less than that of PMC, i.e., $Var(T_L) \le Var(T_R)$ [27].*

The goodness of an unbiased estimator of yield can be measured by the size of its variance. From *Theorem 2*, it is seen that for the monotonic function $q(.)$ for $n-1$ variables and a monotonic $g(.)$ the LHSMC method gives a better estimate than that of the random sampling, without any significant additional computational costs.

THEOREM 3 *If $h = 1(\xi_1, \xi_2, \ldots, \xi_n)$ is monotonic in each of its arguments and if $g(h)$ is a monotonic function of h, then a lower bound of the variance differences between the LHSMC and the PMC estimators is [27]*

$$Var(T_L) - Var(T_R) \le -\frac{1}{N}\max_{i\in[1,\ldots,n]}\{Var[E_{\bar{I}_i}(\mu_c)]\}, \tag{16}$$

where $\bar{I}_i = I_{1,\ldots,i-1,i+1,\ldots,n}$ and $\mu_c(I_1, I_2, \ldots, I_n) = \int_{\xi\in cell\,I} g(h)\, f_\xi(\xi)\, d\xi$ in which I_i presents the ith component of cell I (the interval number in the direction ξ_i).

It should be emphasized that the monotonicity conditions of *Theorem 2* and *Theorem 3* are a sufficient condition and are not necessary. Consider now the following theorem with no assumption of monotonicity in the two-dimensional space.

THEOREM 4 *If $h = q(\xi_1, \xi_2)$ and $g(h)$ are arbitrary functions, then the difference of variances between the LHSMC and the PMC estimators is [27]*

$$Var(T_L) - Var(T_R) = \frac{1}{N(N-1)}\left[Var(\mu_c) - N\sum_{i=1}^{2} Var(E_{I_i}(\mu_c))\right]. \tag{17}$$

In order to compare two different estimation methods, an efficiency measure is introduced as the product of the ratio of the respective variances with the ratio of the respective computation times [5]

$$\eta = \frac{\sigma_R^2 \tau_R}{\sigma_L^2 \tau_L}, \tag{18}$$

where τ_R and σ_R^2 denote the computation time and the variance of the PMC estimator. τ_L and σ_L^2 are respectively the computation time and the variance of the LHSMC estimator.

4. Applications of LHSMC

Iman and Helton [22] applied the LHS approach to cumulative distribution function (c.d.f.) estimation of three computer models: 1) environmental radionuclide movement, 2) multicomponenet aerosol dynamics, and 3) salt dissolution in bedded salt formations. They reported good agreement of c.d.f. estimations. In this section, the application of LHSMC to MOS digital circuits as well as general multi-performance circuits will be discussed.

4.1. MOS Digital Circuits

It has been shown that the principal independent factors in disturbance space of an MOS VLSI circuit consist of the *geometrical parameters* (length reduction, width reduction, and oxide thickness) and the *electrical parameter* (flat band voltage [28]). The study of performance approximation revealed that for MOS digital circuits the performance constraints which define the yield body can be approximated by linear functions of the four cited statistical variables [29], [30]. Therefore the monotonicity of performances in MOS digital circuits is a realistic hypothesis.

In order to use the direct conclusion of *Theorem 2*, assume that we want to estimate the yield with respect to one performance, e.g., the delay between output and input voltages or the delay skew [1], and consider the quality index which is defined by (1). Therefore, the condition of monotonicity is satisfied.

The LHSMC estimator is always an unbiased estimator (see *Theorem 1*). Additionally, by using the class estimator (14), one can efficiently estimate AQI and yield of the circuits and cumulative distribution function and standard deviation of each performances in MOS digital integrated circuits. Consequently, the LHSMC estimators is more efficient than the PMC estimators for MOS digital circuits.

4.2. General Multi-Performance Circuits

Suppose that we have a circuit with several performance functions. For each performance, a quality index can be defined. By using an intersection operator, the circuit quality can be expressed as a quality index function. Generally speaking, the quality index function of a circuit is not a monotone function of each of the parameters or process disturbances.

The LHSMC approach can be used for general multi-performance circuits, e.g., a filter circuit. This is an unbiased estimator even if the parameters are correlated or the monotonicity conditions are not satisfied. Moreover, the monotonicity conditions are sufficient conditions and are not necessary. As an example, *Theorem 2* says that the monotonicity is not required for all of the variables. In *Theorem 4*, it can be experimentally shown that the variance of LHSMC estimator is less than the PMC estimator for practical functions.

Additionally, in a statistical design environment the visualization of c.d.f. and an estimate of standard deviation of each performance can be useful for understanding the functionality of a circuit under statistical variation of parameters, and the results can be used for redesign procedure.

5. Examples and Results

We now illustrate the effectiveness of the proposed method in three yield estimation examples: a quadratic performance function, a CMOS delay circuit [1], and a high pass filter circuit [31].

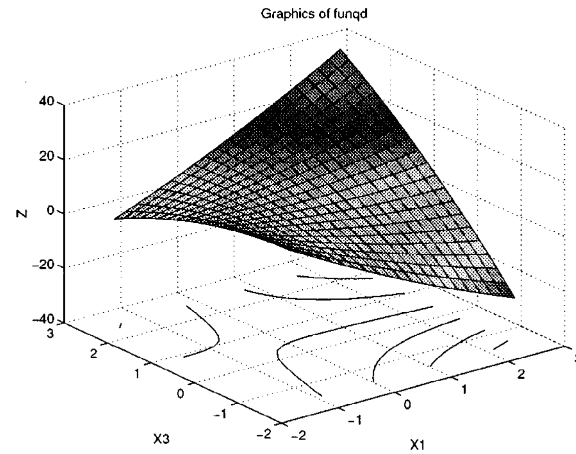

Fig. 2. A 2-dimensional slice of the quadratic performance.

In order to compare two different estimation methods, the efficiency measure (18) is used. In the following examples we set $\tau_R = \tau_L$ to indicate that the number of circuit simulations for the two methods is the same.

5.1. Quadratic Performance Function

Suppose that the behavior of a circuit performance can be expressed as a 3-dimensional quadratic function. The function for this example is taken as

$$h(\xi) = a_0 + J.\xi + \frac{1}{2}\xi^T.H.\xi, \tag{19}$$

where $a_0 = 3$, and the matrices J and H are as follows:

$$J = \begin{bmatrix} -1 \\ 10 \\ 2 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & -4 & 5 \\ -4 & 3 & 4 \\ 5 & 4 & -2 \end{bmatrix}.$$

The disturbances are considered to be independent with the Gaussian p.d.f. over the following region of tolerance

$$R_T = \{\xi \mid |\xi_i - \xi_i^0| \le t_i \quad i = 1, 2, 3\}, \tag{20}$$

where $\xi^0 = [0.5, 0.5, 0.5]^T$ and $t = [1, 1, 1]^T$ is the tolerance vector of disturbances. It can be shown that the quadratic function (19) is monotonic over the region of tolerance (20) with respect to each of the disturbances.

A two-dimensional slice of this function is shown in Fig. 2. In addition, assume that the crisp constraint for this performance is $h(\xi) \le h_{thr}$.

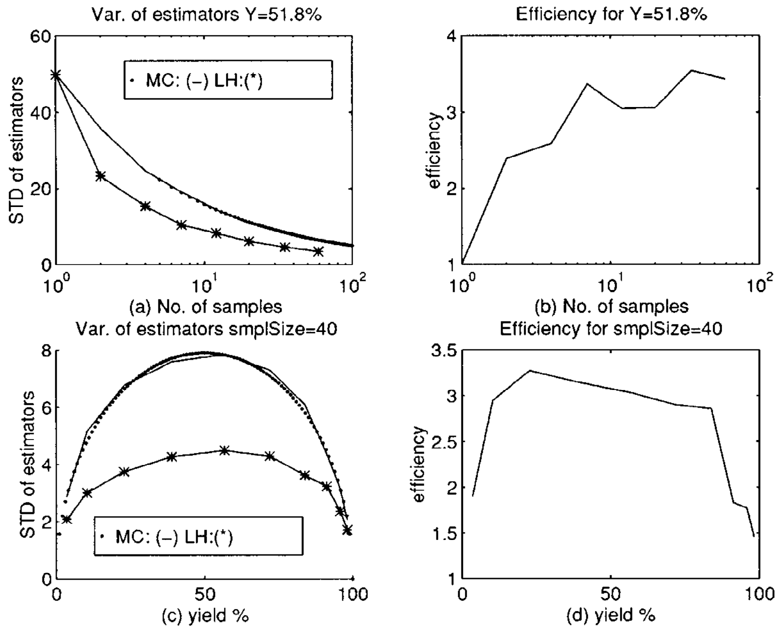

Fig. 3. Comparison of Latin hypercube sampling and primitive sampling in Monte Carlo yield estimation.

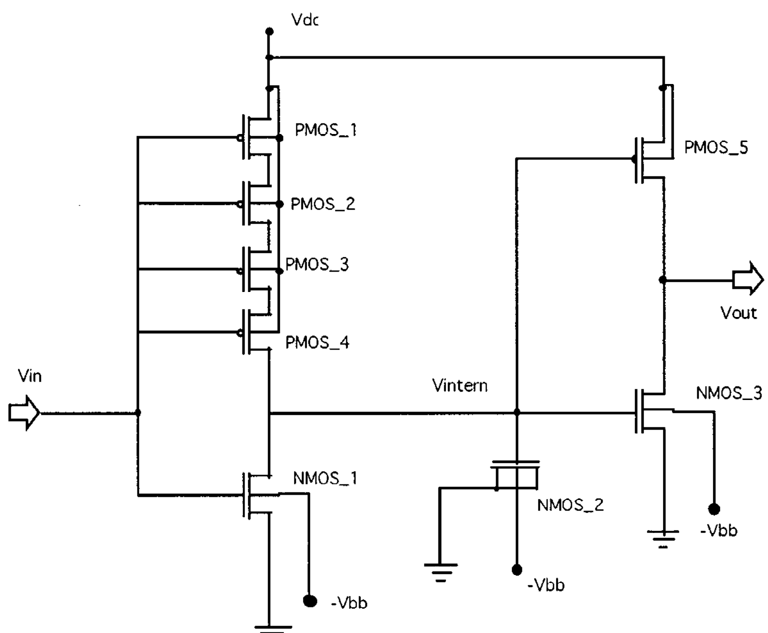

Fig. 4. A CMOS delay circuit.

The results of simulations are shown in Fig. 3. In order to estimate the variance of each estimator, we repeated the estimation process 200 times. In Fig. 3(a) and 3(c), it is seen that the variance of the yield esti-

Table II. Process noise factors

i	ξ_i	$E(\xi_i)$	sigma σ_i	Description
1	ξ_1	0.4 μm	0.04 μm	PMOS Width Reduction
2	ξ_2	0.05 μm	0.004	PMOS Length Reduction
3	ξ_3	−0.822 V	0.06 V	PMOS Threshold Voltage
4	ξ_4	27.5 nm	1.4 nm	Oxide Thickness
5	ξ_5	0.4 μm	0.04 μm	NMOS Width Reduction
6	ξ_6	0.05 μm	0.004	NMOS Length Reduction
7	ξ_7	0.822 V	0.06 V	NMOS Threshold Voltage

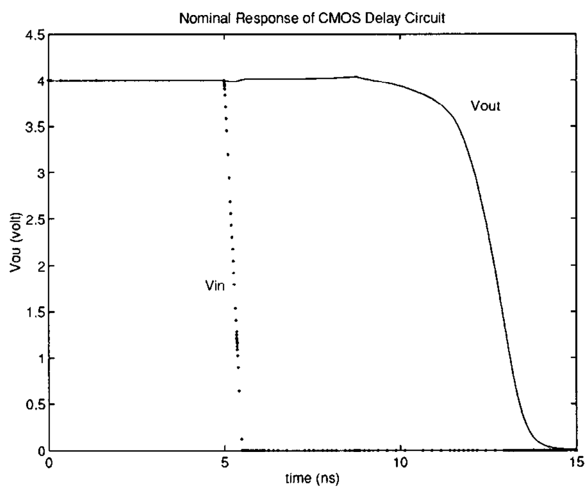

Fig. 5. Nominal response of CMOS delay circuit.

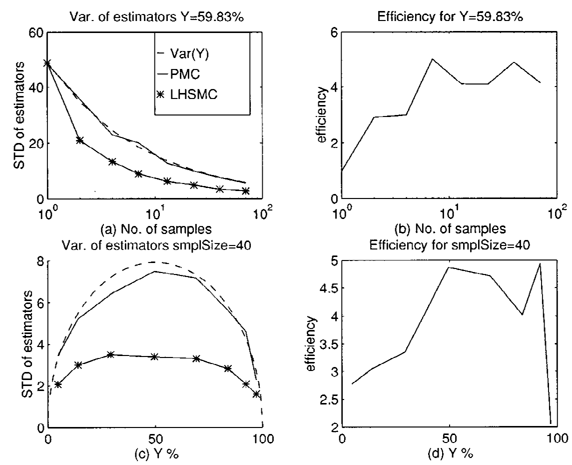

Fig. 6. Results of the CMOS delay circuit for the variance of the estimators.

mation using LHSMC is less than the PMC estimator with respect to sample size and yield value, respectively. The dotted line is the theoretical value of the PMC yield estimator standard deviation. Also, the efficiency of LHSMC is shown in Figs. 3(b) and 3(d). Therefore, the results of this example confirm that the LHSMC method in yield estimation gives better estimation than the PMC estimator.

Moreover, the same results have been obtained for the uniform p.d.f. for the disturbance space of the quadratic function.

5.2. CMOS Delay Circuit

A CMOS delay circuit [1] is shown in Fig. 4. The delay between V_{out} and V_{in} is defined as the circuit performance of interest. The design specification is that the delay should be smaller than a certain value.

OMEGA [32] is an open electric simulator which was developed at institute "Ecole Superieure d'Electricite" (*SUPELEC*). OMEGA was used as the circuit simulator with BSIM transistor models. In addition, Matlab [33] is considered as our programming environment. The interactions between OMEGA and Matlab are done by *Interprocess Communications* [34].

The circuit response for nominal value of parameters is shown in Fig. 5. The model parameters used to characterize CMOS manufacturing process disturbances are listed in Table II. These variables are considered independent with Gaussian probability distribution.

The results of simulations are shown in Fig. 6. In order to estimate the variance of each estimator, 100 times of yield estimation were carried out. In Fig. 6(a) and 6(c), one can see that the variance of the yield

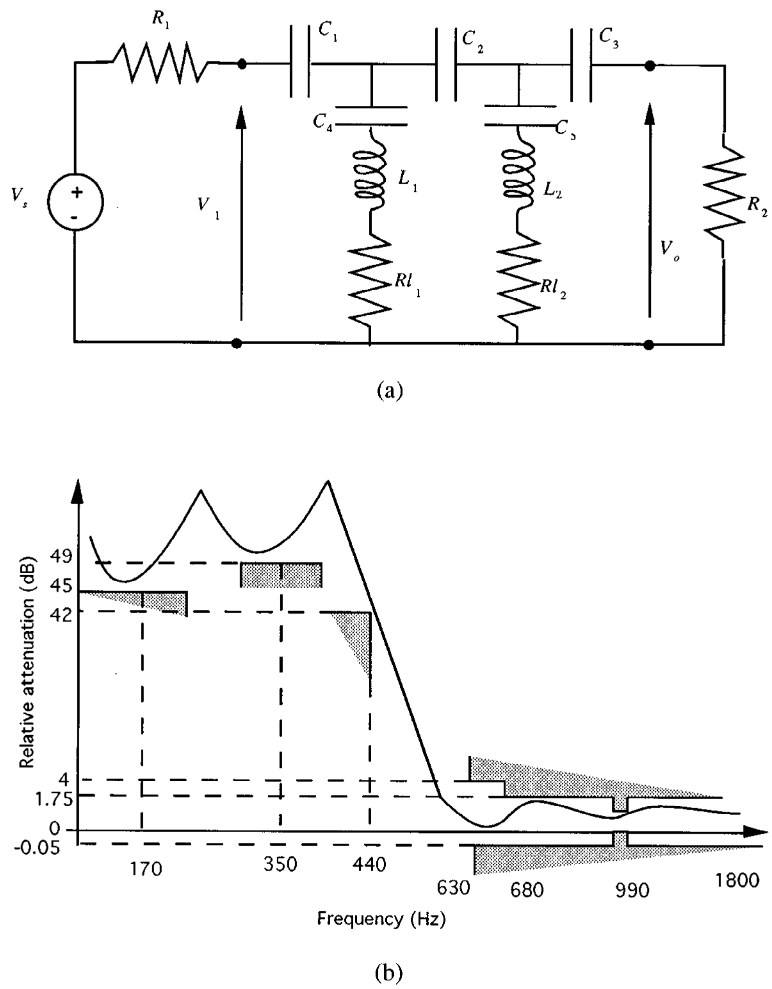

Fig. 7. High Pass Filter circuit.

estimator using LHSMC is less than PMC estimator vs. sample size and yield value, respectively. The dotted line corresponds to the theoretical value of the PMC yield estimator. Moreover, the efficiency of LHSMC is shown in Figs. 6(b) and 6(d). It is seen that for a certain value of yield the computational time for yield estimation may be reduced by four. This is a very important point in VLSI circuits where one transient simulation takes a considerable time. Consequently, the results of this circuit confirm that the use of LHSMC method in yield estimation gives a better estimator than the PMC estimator. In addition, the results of LHSMC estimator show that it is an unbiased yield estimator that confirms the theoretical results.

5.3. High Pass Filter Circuit

As an example of multi-performance circuit, we consider a fifth-order high pass filter circuit [31] shown in Fig. 7(a) which has served as a test example for many tolerance design methods. The nominal circuit response and the performance specifications are shown in Fig. 7(b). The following nominal values are obtained by nominal circuit optimization:

$$Rl_1 = Rl_2 = 10\Omega,\ L_1 = 3.8H,\ L_2 = 3.11H,$$
$$C_1 = 15.4nF,\ C_2 = 12.6nF,\ C_3 = 8.9nF,$$
$$C_4 = 38nF,\ C_5 = 244nF.$$

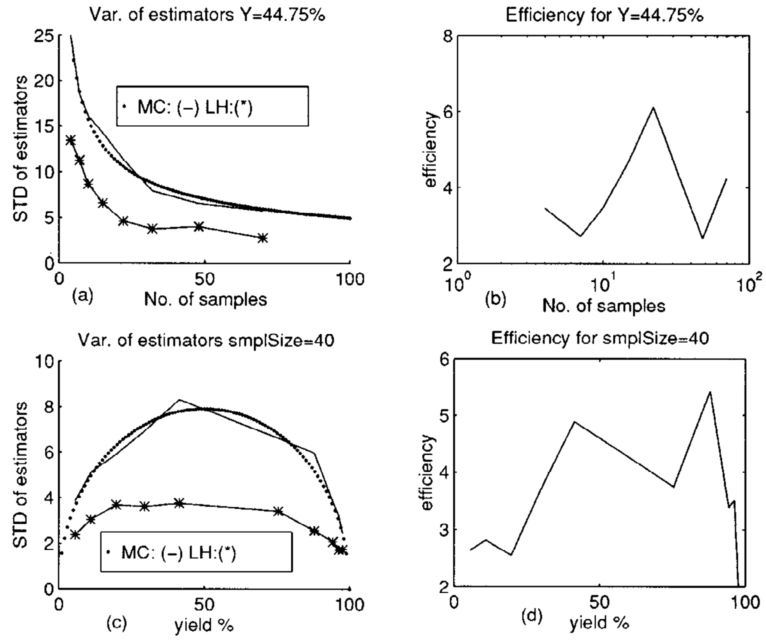

Fig. 8. Results of the High Pass Filter circuit for the variance of the estimators.

In this example, we suppose that C_1, C_2, and C_3 are subjected to Gaussian statistical variations with a tolerances of 20%. Here, the quality index is an indicator function (10). It is observed that this function is not monotone with respect to the parameter space. The results of simulations are shown in Fig. 8. Although the conditions of monotonicity are not satisfied in this case, it is seen that the efficiency of LHSMC is more than unity.

6. Conclusions

In this paper, a new method of variance reduction technique in Monte Carlo AQI and yield estimation for VLSI CMOS digital circuits was presented. The transient simulation of CMOS digital circuits is a very time consuming procedure. In order to have an accurate AQI estimation by Primitive Monte Carlo [2], [3] a large number of simulations are required. Several variance reduction techniques have been developed for reducing the computational cost [3]–[10]. These methods generally require some knowledge about the circuit responses that involves further computational cost, and some of them based on the acceptability region which is not applicable to the AQI estimator. In addition, the efficiency of these methods depends on the quantity of *a priori* knowledge about the circuit responses.

The proposed method is similar to PMC except in sample generation. The samples are generated by the Latin Hypercube Sampling method [11]. This sampling approach ensures that each of the input variables has all portions of its range represented. The LHS approach is computationally cheap to generate and can cope with many input variables. Under certain assumptions, it can be mathematically shown that the efficiency of this method in AQI and yield estimation is always greater than unity. These assumptions are sufficient conditions in the related *Theorems* and are not necessary. We have also given expressions for the minimum variance reduction between the two mentioned methods. The validity of these assumptions are discussed for CMOS digital circuits. The results of application to a 3-dimensional quadratic performance function, a CMOS delay circuit, and a high pass filter circuit showed good efficiency of the LHSMC method. Moreover, this method can also be used as a quality or yield estimator in a Quality or Yield Optimization inner loop. But to have an accelerated version, one should adapt it to the optimization algorithm.

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