CHAPTER 7

Acoustic Sensors and Actuators

The ear

The ear is a sensor and actuator in more than one way. Essentially a mechanochemical sensor, it includes a moving mechanism on the hearing side of the structure. But the ear also features a gyroscope, the inner ear, responsible for stability and sense of position. The ear itself is made of the outer and inner ear. The external ear is no more than a means of concentrating and guiding the sound toward the tympanic membrane (eardrum). In humans, the external ear is a relatively small, static feature, but in some animals it is both large and adjustable. The fenec fox, for example, has external ears that are larger than its head. At the bottom of the ear canal, the tympanic membrane moves in response to sound and, in the process, moves an assembly of three bones, the malleus (connected to the eardrum), the incus (an intermediate flexural bone), and the stapes. The latter, the smallest bone in the body, transmits the motion to the cochlea in the inner ear. The three bones not only transmit the sound, but also amplify it through lever advantage afforded by their structure and dimensions. The cochlea is a spiral tube filled with a fluid. The stapes move like a piston, moving the fluid that in turn moves a series of hairlike structures lining the cochlea. These are the actual sensors that release a chemical onto the auditory nerve to affect hearing.

The inner ear also contains three semicircular canals arranged at 90° to each other, with two roughly vertical and one horizontal. They have a similar structure to the cochlea, including a series of hairlike structures affected by the fluid in the canals based on the position of the body. These serve to maintain balance and provide information on the position and attitude of the body. The effect of motion on these structures can be immediately seen if the body rotates, as, for example, on a merry-go-round. We temporarily loose, the ability to keep our balance.

The ear is a uniquely sensitive structure. It can sense pressures as low as 2×10^{-5} Pa (or 10^{-12} W/m^2; that is on the order of one-billionth of the atmospheric pressure) and can function at levels 10^{13} times higher. That means the dynamic range is about 130 dB. The nominal frequency response is between 20 Hz and 20,000 Hz, although most humans have a much narrower range. But the ear is also very sensitive to pitch and can distinguish very small changes in pitch and frequency. A 1 Hz difference between two sounds is easily detectable. The hearing in humans is binaural and the brain uses that to detect the direction of sources of sound. Many animals use the mechanical motion of the outer ear to accomplish the same function, but much better than we do. It should be noted as well that many animals have much more sensitive hearing than humans, with ears that respond to higher frequencies and to a wider range of frequencies.

7.1 INTRODUCTION

The term acoustics can mean sound or the science of sound. It is in the latter sense that it is used here. Acoustics thus covers all aspects of sound waves, from low-frequency sound waves to ultrasound waves and beyond to what are simply called acoustic waves.

The distinction between acoustics and ultrasonics is based on the span of the human ear. The common, stated range of the human ear is 20 Hz–20 kHz and is based on the ability of our ears to distinguish differences in pressure (usually not only in the atmosphere, but also in water). This is called the audio or audible range or span. It should also be noted that most humans can only hear on a portion of this span (about 50 Hz–14 kHz) and that the whole span is not necessary for transmission of audio information (e.g., telephones use the range between 300 Hz and 3 kHz and an AM radio station has a frequency bandwidth of 10 kHz).

Vibrations from any source cause variations in pressure and these propagate in the substance in which they are generated at a velocity that depends on the substance. The waves are understood to be elastic waves, meaning that they can only be generated in elastic substances (gases, solids, liquids), but not in vacuum or plastic substances (plastic media absorb waves; the term plastic here indicates a material that is not rigid). Above 20 kHz, the same vibrations generate variations in pressure (in air or any other material) and these are called ultrasonic waves. Below 20 Hz, elastic waves are called infrasound. There is no specific range for ultrasound—any acoustic wave above 20 kHz qualifies, and often ultrasonic waves at frequencies well above 100 MHz can be generated and are useful in a variety of applications. Acoustic waves can be and often are generated at much higher frequencies, well above 1 GHz.

Acoustic waves, in the more general sense, cover ultrasonic and infrasonic waves and have roughly the same properties. That is, their general behavior is the same even though certain aspects of the waves change with frequency. For example, the higher the frequency of a wave, the more "direct" its propagation, that is, the less likely it is to diffract (bend) around corners and edges.

As a means of sensing and actuation, acoustic waves have developed in a number of directions. The most obvious is the use of sound waves in the audible range for the sensing of sound (microphones, hydrophones, dynamic pressure sensors) and for actuation using loudspeakers. Another direction that has contributed greatly to the development of sensors and actuators is the extensive work in sonar—the generation and detection of acoustic energy (including infrasound and ultrasound) in water, initially for military purposes and later for the study of oceans and life in the oceans, and even down to fishing aids. Out of this work has evolved the newer area of ultrasonics, which has found applications in the testing of materials, material processing, ranging, and medicine. The development of surface acoustic wave (SAW) devices has extended the range of ultrasonics well into the gigahertz region and for applications that may not seem directly connected to acoustics, such as oscillators in electronic equipment. SAW devices are important not only in sensing, especially in mass and pressure sensing, but also in a variety of chemical sensors.

Because acoustic waves are involved, it should not be surprising to find that the interest in them and their properties is not new. It is impossible to assume that ancient man did not observe that sounds propagate farther in cold dense air than in warm, thin air, or that sounds seem to be louder underwater. In fact, Leonardo da Vinci wrote in 1490 that by using a hydrophone (a tube inserted in water), he could detect noise from ships at great distances. The images from movies where someone presses his ear to the ground to detect an incoming rider are probably familiar to many. Of course, it is altogether a different issue to quantify the speed of propagation of sound and to define its relation to material properties—these came much later (starting around 1800).

7.2 | UNITS AND DEFINITIONS

Perhaps more than any other area of sensing and actuation, the issue of units, measurement, and definitions in acoustics seems confusing at times. Part of the reason is that many units have been developed from work with sound in conjunction with the span of the human ear. There are even units based on perceived quantities and, again, because of the range of human hearing, the use of logarithmic scales is very common. In the audio range, the most common way of describing the propagation of acoustic waves is through the use of sound pressure—newtons per square meter (N/m^2) or pascals (Pa)—since acoustic waves are elastic waves and their effect, especially on the human ear, manifests itself in changes in pressure. However, pressure can be directly related to power density (watts per square meter [W/m^2]), especially when the pressure acts on a diaphragm such as the eardrum, a microphone, or is generated by a loudspeaker. Thus there are two equivalent methods of describing acoustic behavior: one in terms of pressure and the other in terms of power density. Because much of the work in audio relates to hearing, the threshold of hearing holds a unique place and often pressure and power density are related to the threshold of hearing. The threshold of hearing is taken as 2×10^{-5} Pa in terms of pressure or 10^{-12} W/m^2 in terms of power density. A second point on the scale is the threshold of pain, typically taken as 100 Pa or 10 W/m^2, indicating the level beyond which damage to the ear can occur. It should be mentioned that these values are subjective and different sources will use different values for both the threshold of hearing and threshold of pain.

Because the range is so large, sound pressure level and power density are often given in decibels (dB). For the sound pressure level (SPL),

$$SPL_{dB} = 20\log_{10}\frac{P_a}{P_0} \quad [\text{dB}], \tag{7.1}$$

where P_0 is the threshold of hearing and is viewed as a reference pressure and P_a is the acoustic pressure. Thus the threshold of hearing is 0 dB and the threshold of pain is 134 dB (for the values given above). Normal speech is between approximately 45 dB and 70 dB.

For power density,

$$PD_{dB} = 10\log_{10}\frac{P_a}{P_0} \quad [\text{dB}], \tag{7.2}$$

where P_0 is 10^{-12} W/m^2 and P_a is the acoustic power density being sensed. Using the values above, the threshold of hearing is 0 dB and the threshold of pain is 130 dB. Although the numbers look similar to the SPL values, one should be very careful not to confuse the two, as they indicate different quantities.

Acoustic actuators are often specified in terms of power (e.g., the power specification of loudspeakers). The data can be given as average power, peak power (or even peak-to-peak power) based on sinusoidal excitation. It can sometimes be given as maximum power during a specific, typically short, period of time. Whereas these specifications serve mostly marketing purposes, it is important to recognize that the power specified for an actuator is almost always the electric, dissipated power, that is, the power the actuator can dissipate without being damaged. This is very different than the acoustic power the actuator can couple into the space around it. Typically the acoustic power is a very small fraction of the input electric power to the actuator. Most of that power is lost as heat in the actuator itself.

In the ultrasound range, when acoustic waves propagate in materials (other than gases) they are perceived as producing stress in materials and hence stress and strain play a significant role in analysis. Pressure and power density can still be used, but it is more common to describe behaviors in terms of displacement and strain as measures of the ultrasonic signal. When decibel scales are used, the reference pressure (or power density) is taken as 1 since now the thresholds of hearing and pain have no meaning.

EXAMPLE 7.1 Pressure and power density during normal speech

Normal speech ranges from approximately 45 dB to 70 dB, typically measured at a distance of 1 m from the speaker. What are the ranges in terms of pressure and power density at the eardrums of the listener.

Solution: Using **Equation 7.1**, we have for the lower range

$$20\log_{10}\frac{P_a}{P_0} = 45 \text{ dB},$$

that is,

$$\log_{10}\frac{P_a}{P_0} = \frac{45}{20} = 2.25 \rightarrow P_a = P_0 10^{2.25} = 2\times10^{-5}\times10^{2.25} = 3.556\times10^{-3} \text{ Pa}.$$

At the higher range,

$$20\log_{10}\frac{P_a}{P_0} = 70 \text{ dB},$$

or

$$\log_{10}\frac{P_a}{P_0} = \frac{70}{20} = 3.5 \rightarrow P_a = P_0 10^{3.5} = 2\times10^{-5}\times10^{3.5} = 6.325\times10^{-2} \text{ Pa}.$$

The range is between 0.0035565 Pa and 0.06325 Pa.

The power density is obtained from **Equation (7.2)**. At the lower range,

$$10\log_{10}\frac{P_a}{P_0} = 45 \text{ dB},$$

or

$$\log_{10}\frac{P_a}{P_0} = \frac{45}{10} = 4.5 \rightarrow P_a = P_0 10^{4.5} = 10^{-12}\times10^{4.5} = 3.162\times10^{-8} \text{ W/m}^2.$$

At the higher range,

$$10\log_{10}\frac{P_a}{P_0} = 70 \text{ dB},$$

or

$$\log_{10}\frac{P_a}{P_0} = \frac{70}{10} = 7 \rightarrow P_a = P_0 10^{7} = 10^{-12}\times10^{7} = 10^{-5} \text{ W/m}^2.$$

The range is between 31.62 nW/m^2 and 10 μW/m^2.

The properties of acoustic waves are defined by the media through which the waves propagate. Some of the properties that affect the behavior of acoustic wave include the following.

Bulk modulus (K) is the ratio of volume stress per unit of volume strain. It may be viewed as the ratio of the rate of increase in pressure to the resulting relative decrease in volume or the ratio of the rate of increase in pressure to the relative increase in density:

$$K = -\frac{dP}{dV/V} = \frac{dP}{d\rho/\rho} \quad [\mathrm{N/m^2}]. \tag{7.3}$$

Note that the bulk modulus has units of pressure. The bulk modulus is an indication of the resistance of the material to compression. The reciprocal, $1/K$, may be viewed as a measure of the compressibility of the material. Bulk moduli of materials are available in tables based on experimental data.

Shear modulus (G) is the ratio of shear stress and shear strain. It is viewed as a measure of the rigidity of the material or its resistance to shear deformation:

$$G = \frac{dP}{dx/x} \quad [\mathrm{N/m^2}], \tag{7.4}$$

where dx is the shear deformation or shear displacement. Another way to understand the definition is as the ratio of the change in pressure to the relative change in shear deformation.

The bulk and shear moduli, together with the modulus of elasticity defined in **Chapter 6**, describe the elastic properties of materials. The difference between the shear modulus and the modulus of elasticity is that the modulus of elasticity defines the linear or longitudinal deformation, whereas the shear modulus defines the transverse or shear deformation of the material.

The **ratio of specific heats** of a gas is the ratio of the specific heat capacity at constant pressure (P_c) to specific heat capacity at constant volume (V_c). It is also known as the **isentropic expansion factor** and is denoted by γ. The specific heat capacity is the amount of heat (in joules [J]) needed to raise the temperature of a unit mass (in kilograms [kg]) by 1°C (see **Section 3.1.1**).

In acoustics, some of the terms used are based on subjective measures rather than on absolute scales because of the intricate link between sound and hearing, and hence with the perception of sound by the human brain. One of these terms is **loudness**, defined as an attribute of the auditory perception that ranks sounds on a scale ranging from quiet to loud. The sensation of sound by the human brain depends on a variety of terms, including intensity (amplitude) and frequency. To measure loudness one employs two basic units: the phon and the sone.

The **phon** is a unit of loudness that measures the intensity of sound in decibels above a reference tone having a frequency of 1000 Hz and a root mean square (RMS) sound pressure of 20×10^{-6} Pa. An alternative definition is "a unit of apparent loudness, equal in number to the intensity in decibels of a 1000 Hz tone perceived to be as loud as the sound being measured."

The **sone** is a unit of perceived loudness equal to the loudness of a 1000 Hz tone at 40 dB above the threshold of hearing.

Another subjective term in common use is **tone**, which is used to describe the quality or character of a sound. Obviously it cannot be measured on any objective scale, but it is an important aspect of acoustics, especially as it relates to music.

7.3 | ELASTIC WAVES AND THEIR PROPERTIES

Sound waves are longitudinal elastic waves; that is, a pressure wave, as it propagates, changes the pressure along the direction of its propagation. Thus an acoustic wave impinging on our eardrums will push and pull on the eardrum to affect hearing. Waves, including acoustic waves, have three fundamental properties that are of special importance.

First, they have a frequency (or a range of frequencies). The frequency, f, of a wave is the number of variations of the wave per second, measured in hertz (Hz, or cycles per second). This is normally defined for harmonic waves and is understood to be the number of cycles of the harmonic wave per second. For example, if we were to count the number of crests in an ocean wave passing through a fixed point in 1 s, the result would be the frequency of that wave.

The second property is the wavelength, λ, which is related to frequency and is the distance in meters (m) a wave propagates in one cycle of the wave. In the example of the ocean waves, the wavelength is the distance between two crests (or two valleys).

The speed of propagation of a wave, c, is the speed (in meters per second [m/s]) with which the front of a wave propagates. These three quantities are related as

$$\lambda = \frac{c}{f} \quad [\mathrm{m}]. \tag{7.5}$$

The relation between frequency and wavelength can be seen in **Figure 7.1**. Although this relation may seem minor, one of the most important aspects of acoustic waves is the short wavelength they exhibit. In fact, this property is responsible for the relatively high resolution of ultrasonic tests such as tests for defects in materials or tests for medical purposes. As a rule, the resolution one can expect from any test using a wave is dependent on the wavelength. The shorter the wavelength, the higher the resolution. We will see later in this chapter that this property is taken full advantage of in SAW devices.

Waves can be transverse waves, longitudinal waves, or a combination of the two. Transverse waves are those waves that cause a change in amplitude in directions transverse to the direction of propagation of the wave. Waves produced by a tight string are of this type. When we pluck a string, it vibrates perpendicular to the length of the string while the wave itself propagates along the string. This is shown schematically in **Figure 7.2**. The figure also shows that the wave propagates away from the source, in this case in two directions. The ocean wave mentioned above and electromagnetic waves are also transverse waves.

Acoustic waves in gases and liquids are longitudinal waves. In solids they can also be transverse waves. Transverse acoustic waves are often called shear waves. To avoid confusion, and because in most cases we will encounter longitudinal waves, the discussion that follows relates to longitudinal waves. Whenever the need arises to discuss

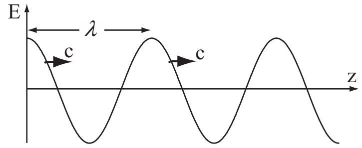


FIGURE 7.1 ■ Relation between frequency, wavelength, and speed of propagation for a generic time-harmonic wave.

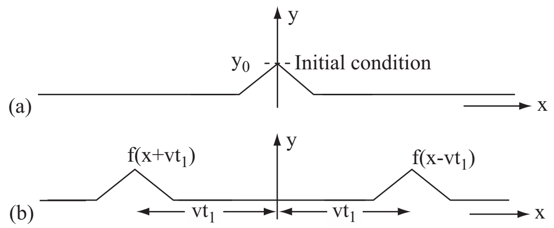


FIGURE 7.2 ■ Wave propagation along a tight string. (a) The string is plucked at $x = 0$. (b) The string and the disturbance after a time t_1.

shear waves, and later, surface waves, these will be indicated explicitly to distinguish them from longitudinal waves.

7.3.1 Longitudinal Waves

The speed of an acoustic wave is directly related to the change in volume and the resulting change in pressure (say, due to the motion of the piston in **Figure 7.3**):

$$c = \sqrt{\frac{\Delta p V}{\Delta V \rho_0}} \quad [\mathrm{m/s}]. \tag{7.6}$$

Note that $\Delta p/(\Delta V/V)$ is in fact the bulk modulus, so one can write **Equation (7.6)** as

$$c = \sqrt{\frac{K}{\rho_0}} \quad [\mathrm{m/s}], \tag{7.7}$$

where ρ_0 is the density of the undisturbed fluid, ΔV is the change in volume, Δp is the change in pressure, and V is the volume. In gases, this simplifies to the following:

$$c = \sqrt{\frac{\gamma p_0}{\rho_0}} \quad [\mathrm{m/s}], \tag{7.8}$$

where p_0 is the static pressure and γ is the ratio of specific heats for the gas. Thus the speed of acoustic waves in materials is pressure and temperature dependent. In solids, the speed of sound depends on the "elasticity" of the solid—more specifically, on the shear and bulk moduli of the medium. **Table 7.1** gives the speed of sound in a number of materials for longitudinal waves. These values are experimental and will vary somewhat depending on the source. For example, the speed of sound in air at 20°C is quoted as 343 m/s to 358 m/s by various sources. The speed of sound also varies with pressure

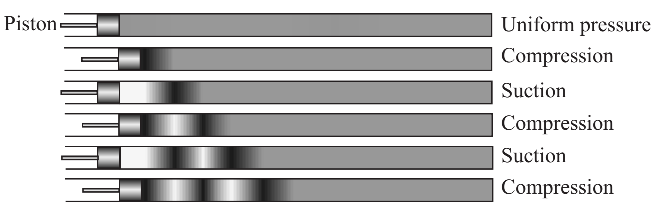


FIGURE 7.3 ■ Generation of a longitudinal wave by motion of a piston. The particles of the substance are displaced longitudinally to create local variations in pressure.

TABLE 7.1 ■ Speed of sound for longitudinal waves in some materials at given temperatures

Material	Speed [m/s]	Temperature [°C]
Air	331	0
Freshwater	1486	20
Seawater	1520	20
Muscle tissue	1580	35
Fat	1450	35
Bone	4040	35
Rubber	2300	25
Granite	6000	25
Quartz	5980	25
Glass	6800	25
Steel	5900	20
Copper	4600	20
Aluminum	6320	20
Beryllium	12,900	25
Titanium	6170	20
Brass	3800	20

and relative humidity. In solids, especially in metals, dependency on temperature is lower than in gases or liquids.

A longitudinal wave changes its amplitude along the direction of propagation. A simple example is the mechanical wave generated by a piston in a tube. As the piston moves back and forth, it compresses and decompresses the gas ahead of it. This motion then propagates along the tube, as shown in **Figure 7.3**. Acoustic waves are of this type.

Assuming for simplicity that we have a harmonic longitudinal wave of frequency f, it may be written in general terms as

$$p(x,t) = P_0 \sin(kx - \omega t) \quad [\mathrm{N/m^2}], \tag{7.9}$$

where $p(x,t)$ is the time- and position-dependent pressure in the medium, P_0 is the pressure amplitude of the wave, and k is a constant. The wave propagates in the positive x direction (in this case) and $\omega = 2\pi f$ is its angular frequency.

The amplitude of the wave is

$$P_0 = k\rho_0 c^2 y_m \quad [\mathrm{N/m^2}], \tag{7.10}$$

where y_m is the maximum displacement of a particle during compression or expansion in the wave. The constant k is called the wavenumber or the phase constant and is given as

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \quad [\mathrm{rad/m}]. \tag{7.11}$$

Waves carry energy. A shockwave (like that generated by an earthquake or a sonic boom) can cause damage, while a loud sound can hurt our ears or shatter a window. A wave is said to be a propagating wave if it carries energy from one point to another.

A wave can propagate in an unbounded medium with or without attenuation (losses). Attenuation of a wave depends on the medium in which it propagates and this attenuation reduces the amplitude of the wave. Attenuation of waves is exponential, with an exponent that depends on the material properties. An **attenuation constant**, α, is defined for each material and the amplitude of the wave (pressure), as it propagates, changes as follows:

$$p(x,t) = P_0 e^{-\alpha x} \sin(kx - \omega t) \quad [\mathrm{N/m^2}]. \tag{7.12}$$

This attenuation causes a loss of energy as the wave propagates and eventually dissipates all the energy in the wave. In addition, unless the wave propagates in a perfectly collimated beam, it spreads into space so its energy is spread over an increasingly larger area. Under this condition, the amplitude decreases at any point in space regardless of attenuation. The attenuation constant, α, has units of neper per meter (Np/m), where 1 Np/m = 8.686 dB/m. It should also be noted here that power, as opposed to amplitude (force, pressure, displacement), attenuates with a constant 2α.

The attenuation constant in a number of materials is given in **Table 7.2**. The attenuation constant is itself dependent on temperature, but the most striking feature of the attenuation constant in materials is its dependency on frequency. In air it also depends on relative humidity and pressure, and is roughly proportional to f^2, especially at higher frequencies. Attenuation is typically given in decibels per kilometer (dB/km), decibels per meter (dB/m), or decibels per centimeter (dB/cm). Because of the complex nature of the attenuation constant and its dependency on many parameters, its values are typically available in tables. **Table 7.3** shows some of these and their dependency on frequency for air and other materials. In some cases the properties of sound waves are available in formulas, usually based on fitting to experimental data. For example, the attenuation constant in water can be calculated from the following formula:

$$\alpha_{\text{water}} = 0.00217 f^2 \quad [\mathrm{dB/cm}], \tag{7.13}$$

where f is frequency (in megahertz [MHz]). Similar formulas with differing coefficients exist for other fluids, but unfortunately the behavior in other materials is not as simple. Also, the formula for water does not apply below about 1 MHz.

TABLE 7.2 ■ Typical attenuation constants for some representative materials

Material	Attenuation constant [dB/cm]	Frequency
Steel	0.429	10 MHz
Quartz	0.02	10 MHz
Rubber	3.127	300 kHz
Glass	0.173	10 MHz
PVC	0.3	350 kHz
Water	See **Equation (7.13)**	
Aluminum	0.27	10 MHz
Copper	0.45	1 MHz

TABLE 7.3 ■ Attenuation constant (in dB/cm) and its dependency on frequency

	1 kHz	10 kHz	100 kHz	1 MHz	5 MHz	10 MHz
Air	1.4×10^{-4}	1.9×10^{-3}	0.18	1.7	40	170
Water	See **Equation (7.13)**					
Aluminum				0.008	0.078	0.27
Quartz				0.002	0.01	0.02

TABLE 7.4 ■ Acoustic impedance of some materials

Material	Acoustic impedance [kg/m^2/s]
Air	415
Fresh water	1.48×10^6
Muscle tissue	1.64×10^6
Fat	1.33×10^6
Bone	7.68×10^6
Rubber	1.54×10^6
Quartz	14.5×10^6
Rubber	1.74×10^6
Steel	45.4×10^6
Aluminum	17×10^6
Copper	42.5×10^6

Another example of well-established relations is the variation of the speed of sound in pure water as a function of temperature, given as an nth-order polynomial and designed for specific temperature ranges. The formulas range from second- to fifth-order polynomials in which the coefficients are calculated from experimental data. An example is the following:

$$c_{\text{water}} = 1405.03 + 4.624T - 0.0383T^2 \quad [\text{m/s}], \tag{7.14}$$

where T is the temperature (in °C). The formula is designed for the normal range of temperatures of bodies of water (10°C–40°C), but it may be used beyond this range with increased error. An approximate formula for the dependency of the speed of sound in air on temperature also exists:

$$c_{\text{air}} = 331.4 + 0.6T \quad [\text{m/s}]. \tag{7.15}$$

The waves also posses a property called wave impedance, although in the case of acoustic waves it is often called acoustic impedance. The **wave impedance** or **acoustic impedance** is the product of density (ρ) and velocity (c):

$$Z = \rho c \quad [\text{kg/m}^2/\text{s}]. \tag{7.16}$$

Acoustic impedance is an important parameter of the material and is useful in a number of acoustic applications, including reflection and transmission of waves, and hence in the testing of materials and the detection of objects and conditions using ultrasound. In general, elastic materials have high acoustic impedance, whereas "soft" materials tend to have low acoustic impedance. The differences can be orders of magnitude, as can be seen in **Table 7.4**. For example, the acoustic impedance of air is 415 kg/m^2/s, whereas that of steel is 4.54×10^7 kg/m^2/s. These large differences affect the behavior of the acoustic waves and their usefulness in various application.

EXAMPLE 7.2 Tsunami detection system

A tsunami detection and warning system consists of a number of shore stations that detect earthquakes using essentially accelerometers. The system consists of a number of basic components, including the sensors themselves and the detection stations where the accelerometers are located. A number of sensors, located at fixed positions, detect earthquakes. They determine the strength and, by triangulation, the location (epicenter) of the earthquake. This provides the distance and the likelihood that a tsunami will be generated (based on strength, location, and depth). Then the system determines how much time will elapse until the tsunami arrives at various locations. This is based on the speed of propagation of compression waves. In the earth's crust, the speed of propagation of seismic waves is approximately 4 km/s, whereas in water it is 1.52 km/s. For this reason, as well as for practical reasons of installation, the detection of seismic waves is done on land. Tsunamis travel at speeds of up to 1000 km/h.

Suppose an earthquake occurs 250 km from a city located on the seashore. The earthquake is detected at a station in a different location, 700 km from the epicenter of the earthquake. If the detection system determines that a tsunami is likely, how long do people in the city have to evacuate before the tsunami hits?

Solution: To detect the earthquake requires a time t_1:

$$t_1 = \frac{700}{4} = 175 \text{ s},$$

which is approximately 3 min.

The tsunami requires a time t_2 to travel a distance of 250 km:

$$t_2 = \frac{250}{1000} = 0.25 \text{ h},$$

which is one-quarter of an hour, or 15 min. Since one needs 3 min for detection, the city has at most 12 min to prepare. This assumes of course that the warning is issued without delay. This is one reason that tsunamis are so dangerous—the time available for preparation and evacuation is typically short except at large distances from the epicenter.

EXAMPLE 7.3 Attenuation of acoustic waves in air

Acoustic waves attenuate in air at a rate that depends on a number of factors, including temperature, pressure, relative humidity, and the frequency of the wave. All of these have a significant effect on attenuation, but to understand the propagation of ultrasound in air we will look at the effect of frequency alone. The following data are available for sound propagation in air:

Attenuation at 1 kHz, 20°C, 1 atm at sea level, 60% relative humidity, 4.8 dB/km.

Attenuation at 40 kHz, 20°C, 1 atm at sea level, 60% relative humidity, 1300 dB/km.

Attenuation at 100 kHz, 20°C, 1 atm at sea level, 60% relative humidity, 3600 dB/km.

Given a sound wave amplitude (sound pressure) of 1 Pa, calculate the sound pressure at a distance $d = 100$ m from the source at the three frequencies.

Solution: Since the attenuation is given in decibels per kilometer (dB/km), we need first to convert it into neper per meter (Np/m) so we can use **Equation (7.12)**. To do so we write the following:

At 1 kHz,

$$4.8 \text{ dB/km} = \frac{4.8}{8.686} \text{ Np/km} = \frac{4.8}{8.686 \times 1000} = 5.526 \times 10^{-4} \text{ Np/m}.$$

At 40 kHz,

$$1300\ \mathrm{dB/km} = \frac{1300}{8.686 \times 1000} = 0.1497\ \mathrm{Np/m}.$$

At 100 kHz,

$$3600\ \mathrm{dB/km} = \frac{3600}{8.686 \times 1000} = 0.4145\ \mathrm{Np/m}.$$

With these, the amplitude at a distance d, which we denote as P_d, is written in terms of the source pressure P_0 as

$$P_d = P_0 e^{-\alpha d} \quad [\mathrm{Pa}].$$

At 1 kHz,

$$P_d = 1e^{-5.526\times 10^{-4}\times 100} = 0.9994\ \mathrm{Pa}.$$

At 40 kHz,

$$P_d = 1e^{-0.1497\times 100} = 3.15 \times 10^{-7}\ \ \mathrm{Pa}.$$

At 100 kHz,

$$P_d = 1e^{-0.4145\times 100} = 9.96 \times 10^{-19}\ \ \mathrm{Pa}.$$

These results reveal that in air, ultrasound can only be used for short-range applications. Indeed, most ultrasound applications in air use either 24 kHz or 40 kHz and are intended for ranges of less than about 20 m. The lower the frequency, the longer the range. Clearly at 1 kHz the sound has attenuated very little. It is perhaps for this reason that the human voice has evolved to use low frequencies. Lower frequencies (i.e., those sounds below our own hearing limit [infrasound]) propagate for very long distances and are used by some animals (such as elephants and whales). It should be noted that in water and in solids the attenuation is much lower and sound waves can propagate for long distances (see, e.g., **Problem 7.7**).

When a propagating wave encounters a discontinuity in the unbounded space (an object such as a wall, a change in air pressure, etc.), part of the wave is reflected and part of it is transmitted through the discontinuity. Thus we say that **reflection** and **transmission** occur, and reflected and transmitted waves can propagate in directions other than that of the original wave. The transmitted wave is understood as being the **refraction** of the wave across the discontinuity. To simplify the discussion, we define a transmission coefficient and a reflection coefficient. In the simplest case, when the propagating wave impinges on the interface perpendicularly ($\theta_i = 0$ in **Figure 7.4**), propagating from material 1 into material 2, the reflection coefficient (R) and transmission coefficient (T) are defined as

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \quad T = \frac{2Z_2}{Z_2 + Z_1}, \tag{7.17}$$

where Z_1 and Z_2 are the acoustic impedances of medium 1 and medium 2, respectively.

The reflection coefficient multiplied by the amplitude of the incident wave gives the amplitude of the reflected wave. The transmission coefficient multiplied by the

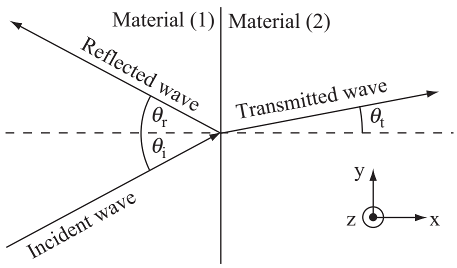


FIGURE 7.4 ■ Reflection, transmission, and refraction of a wave.

amplitude of the incident wave gives the amplitude of the wave transmitted from medium 1 to medium 2. That is, the reflected and transmitted wave amplitudes (say, for pressure) are

$$P_r = P_i R \quad [\mathrm{N/m^2}], \quad P_t = P_i T \quad [\mathrm{N/m^2}], \tag{7.18}$$

where P_i is the incident pressure, P_r is the reflected pressure, and P_t is the transmitted pressure. Note that the reflection coefficient can be negative and varies from -1 to $+1$, whereas the transmission coefficient is always positive and varies from 0 to 2.

In acoustics, and in particular in ultrasonics, the quantities of interest are often power or energy rather than pressure. Since power and energy are related to pressure squared, the transmitted and reflected power or energy are related to the reflection and transmission coefficient squared. For example, assuming a collimated ultrasound beam with total incident power W_i, the reflected and transmitted power will be

$$W_r = W_i R^2 \quad [\mathrm{W}], \quad W_t = W_i T^2 \quad [\mathrm{W}]. \tag{7.19}$$

The refraction of the wave is defined in **Figure 7.4**. The reflected wave is reflected at an angle equal to the angle of incidence ($\theta_r = \theta_i$, defined between the direction of the propagating wave and the normal to the surface on which the wave reflects). The transmitted wave propagates in material 2 at an angle θ_t, which may be calculated from the following

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i, \tag{7.20}$$

where c_2 is the speed of propagation of the wave in the medium into which the wave transmits and c_1 is the speed in the medium from which the wave originates.

The reflected waves propagate in the same medium as the propagating wave and therefore can interfere with the propagating wave to the extent that their amplitude can add (constructive interference) or subtract (destructive interference). The net effect is that the total wave can have amplitudes smaller or larger than the original wave. This phenomenon is well known and leads to the idea of a standing wave. In particular, suppose that the wave is totally reflected so that the amplitudes of the reflected and incident waves are the same. This will cause some locations in space to have zero amplitudes, whereas others will have amplitudes twice as large as the incident wave. This is called a standing wave because the locations of zero amplitude (called nodes) are fixed in space, as are the locations of maxima. **Figure 7.5** shows this and also the fact that the nodes of the standing wave are at distances of $\lambda/2$, whereas maxima occur at $\lambda/4$ on either side of a node. A good example of standing waves can be seen in vibrating tight strings in which reflections occur at the locations where the strings are attached. The vibration at various

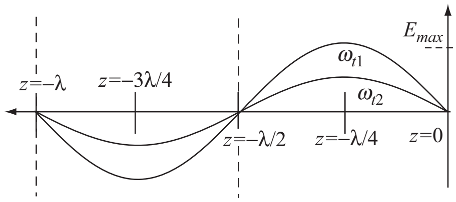


FIGURE 7.5 ■ Standing waves. The waves oscillate vertically (in time) but are stationary in space.

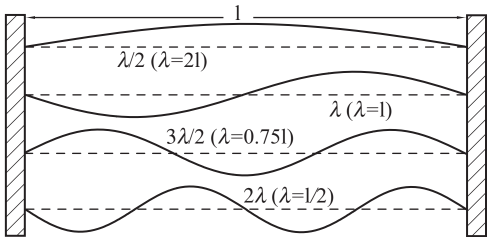


FIGURE 7.6 ■ Modes of a vibrating string. Note that the nodes of the standing wave are at equal distances from each other ($\lambda/2$) and occur at fixed locations for each mode.

wavelengths and its interaction with the air accounts for the music we perceive when a violin plays. **Figure 7.6** shows the first few modes of a vibrating string. For each mode, the nodes (zero displacement) occur at fixed physical locations.

The reflection of sound waves is also responsible for scattering. In essence, **scattering** is reflection of the waves in all directions caused by anything in the path of the waves. Dispersion of sound waves is another important property. **Dispersion** is the propagation of various frequency components at different speeds causing distortion in the received sound wave.

EXAMPLE 7.4 Properties of waves: resolution

Waves are often used for a variety of sensing and actuation functions. However, not all waves are the same and equally useful. The use of ultrasound for imaging is well established, including imaging of the body for medical purposes and use in the testing of materials. Animals as well use ultrasound in much the same way. Bats and dolphins use it for echolocation—for the identification of prey and avoidance of danger. But ultrasound is used in actuation as well. Dolphins stun fish with an intense ultrasound burst, whereas we use it to break up kidney stones, for ultrasonic cleaning, and for descaling of equipment. Electromagnetic waves, including light waves, are similarly used for imaging, as well as for echolocation, speed sensing, and a whole host of other applications. These functions are possible because of the interaction of the waves with materials, and one of the critical issues in this interaction is the wavelength. If the wavelength is long, the wave is useful in identifying large obstructions. The shorter the wavelength, the smaller the objects it can identify and hence the higher the resolution. Consider the following examples:

Ultrasound in air: A bat transmits ultrasound at 40 kHz in air. With the speed of sound in air equal to 331 m/s, the wavelength is (from **Equation (7.5)**)

$$\lambda = \frac{c}{f} = \frac{331}{40{,}000} = 8.275 \times 10^{-3}\ \text{m}.$$

This is a mere 8.275 mm, sufficiently small to hunt for insects.

Ultrasound in water: A dolphin transmits ultrasound at 24 kHz in water. With the speed of sound equal to 1500 m/s, the wavelength is

$$\lambda = \frac{c}{f} = \frac{1500}{24{,}000} = 62.5 \times 10^{-3} \text{ m}.$$

At a wavelength of 62.5 mm, the dolphin is well equipped for fishing.

Imaging with ultrasound: An ultrasound wave at 2.75 MHz is used to monitor the condition of a human heart. Assuming the speed of sound to be the same as in water, the wavelength is

$$\lambda = \frac{c}{f} = \frac{1500}{2.75 \times 10^6} = 5.455 \times 10^{-4} \text{ m}.$$

The test is capable of distinguishing features at submillimeter levels (less than 0.5 mm), sufficient for diagnostics of conditions such as deteriorating valves, blood vessel wall thickness, and more.

By way of comparison, the frequency of visible light varies between 480 THz (red) and 790 THz (violet). Its wavelength varies between 380 nm (violet) and 760 nm (red). The resolution possible with optical means is of that order of magnitude. Anything much smaller than that will not be seen using optical means (i.e., microscopes) and will require lower wavelengths (e.g., the use of electron microscopes).

7.3.2 Shear Waves

As mentioned above, solids can support shear or transverse waves in addition to longitudinal waves. In shear waves, the displacement (i.e., vibration of molecules) is perpendicular to the direction of propagation. Most of the properties defined for longitudinal waves, as well as properties such as reflection and transmission, are the same for shear waves. Other properties are different. In particular, the speed of propagation of shear waves is slower than for longitudinal waves. While the speed of propagation of longitudinal waves depends on the bulk modulus, that of shear waves depends on the shear modulus:

$$c = \sqrt{\frac{G}{\rho_0}} \quad [\text{m/s}]. \tag{7.21}$$

Since the shear modulus is lower than the bulk modulus, the speed of propagation of shear waves is lower (by about 50%).

The acoustic impedance in **Equation (7.16)** applies to shear waves as well, but since the speed is lower, so is the acoustic impedance.

7.3.3 Surface Waves

Acoustic waves can also propagate on the surface between two media and, in particular, at the interface between an elastic medium and vacuum (or air). This applies in particular to propagation on the surface of solids. Surface waves are also called Rayleigh waves and propagate on the surface of an elastic medium with little effect on the bulk of the medium and have properties that are significantly different than those of either longitudinal waves or shear waves. The most striking difference is their slower speed of propagation. The speed of propagation of a surface wave can be written as

$$c = g\sqrt{\frac{G}{\rho_0}} \quad [\mathrm{m/s}], \tag{7.22}$$

where g is a constant that depends on the particular material but is around 0.9. This means that surface waves propagate slower than shear waves and propagate much slower than longitudinal waves.

In addition, the propagation of surface waves in ideal, elastic, flat surfaces is non-dispersive, that is, their speed of propagation is independent of frequency. In reality there is some dispersion but it is lower than for other types of acoustic waves. This, combined with the slow speed of propagation, has found an important application in SAW devices. They also have uses in seismology and the study of earthquakes. The exact definition of a Rayleigh wave is "a wave that propagates at the interface between an elastic medium and vacuum or rarefied gas (air, for example) with little penetration into the bulk of the medium."

7.3.4 Lamb Waves

In addition to longitudinal, shear, and surface waves (sometimes designated as *L*-waves, *P*-waves, and *S*-waves, respectively), acoustic waves propagate in thin plates in a unique way dominated by modes of propagation that depend on the thickness of the plates. These are called Lamb waves (named after Horace Lamb). A plate will support an infinite number of modes that depend on the relationship between the thickness of the plate and the wavelength of the acoustic wave.

7.4 | MICROPHONES

We start the discussion on acoustic devices with the better known of these—audio sensors and actuators. Microphones and loudspeakers are familiar, at least to a certain extent. These are common devices, but like any other area in sensing and actuation, exhibit considerable variability in construction and applications. Microphones are differential pressure sensors where the output depends on the pressure difference between the front and back of a membrane. Since under normal conditions the two pressures are the same, the microphone can only sense changes in pressure and hence may be viewed as a dynamic pressure sensor. It may also be used to sense vibrations or any quantity that generates variations in pressure in air or in a fluid. Microphones designed to work in water or other fluids are called hydrophones.

7.4.1 The Carbon Microphone

The very first microphones and loudspeakers (or earphones) were devised and patented for use in telephones. In fact, the first patent of the telephone is not really a patent of a telephone, but rather, that of a microphone. Alexander Graham Bell patented the first variable resistance microphone in 1876, although in its early form it was a very inconvenient device. It was built as in **Figure 7.7** and used a liquid solution. The resistance

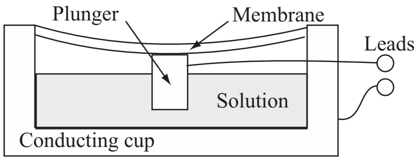


FIGURE 7.7 ■ Bell's microphone relied on changes in resistance in a solution.

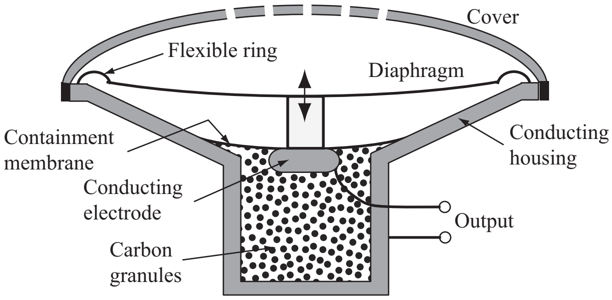


FIGURE 7.8 ■ The construction of the carbon microphone.

FIGURE 7.9 ■ The carbon microphone used in a telephone handset.

between the plunger and the body of the microphone depends on sound pressure (which pushes the plunger into the solution). This microphone worked, but was not practical and was soon replaced by others more suited for the job. The first practical microphone was invented by Thomas Edison and was essentially the same construction as Bell's microphone, but the solution was replaced with carbon or graphite particles—hence its name, the carbon microphone. Although it has many problems, it has been in continuous use in telephones ever since its invention. Because of its rather poor performance (noise, limited frequency response, dependence on position, and distortion), it has not been used since the late 1940s except in telephones. Nevertheless, it is a somewhat unique device, particularly because of the fact that it is an "amplifying" device (it can modulate large currents). In that capacity it was, and is still being used, to drive an earpiece directly without the need for an amplifier. Its structure is shown in **Figure 7.8** and a picture of a carbon microphone is shown in **Figure 7.9**. As the diaphragm moves, the resistance between the conducting electrode and the conducting housing changes, and when connected in a circuit, this change in resistance changes the current in the circuit, producing sound in the earpiece (see **Figure 1.5**). In modern telephones the carbon microphone has been largely replaced by better microphones, albeit microphones that require electronic circuits for amplification.

7.4.2 The Magnetic Microphone

The magnetic microphone, better known as the moving iron microphone, together with its cousin, the moving iron gramophone pickup, have largely disappeared and have been replaced by better devices. Nevertheless, it is worth looking at its structure since that structure is quite common in sensors (we have seen a similar device used as a pressure sensor in **Chapter 6** called the variable reluctance pressure sensor). The basic principle is shown in **Figure 7.10**. The operation is straightforward. As the armature moves (a piece of iron that moves due to the action of sound, or a needle in the case of a record pickup), it decreases the gap toward one of the poles of the iron core. This changes the reluctance in the magnetic circuit. If the coil is supplied with a constant voltage, the current in it depends on the reluctance of the circuit. Hence the current in the coil depends on the position of the armature (sound level). The moving iron microphone was a slight improvement over the carbon microphone, and perhaps the only real advantage it had was that its operation was reversible—the moving iron armature could be made to move under the influence of a current, and by doing so it could serve as an earpiece or loudspeaker. This microphone was quickly replaced by the so-called moving coil microphone, shown in **Figure 7.11**, also known as the dynamic microphone. This was the first microphone that could reproduce the whole range of the human voice, and it has survived into our time even though newer, simpler devices have been developed. The operation of the moving coil microphone is based on Faraday's law. Given a coil moving in a magnetic field, it will produce an emf as follows:

$$V = -N\frac{d\Phi}{dt} \quad [\text{V}], \tag{7.23}$$

FIGURE 7.10 ■ The construction of a moving armature magnetic microphone.

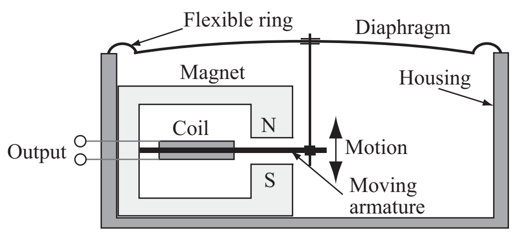
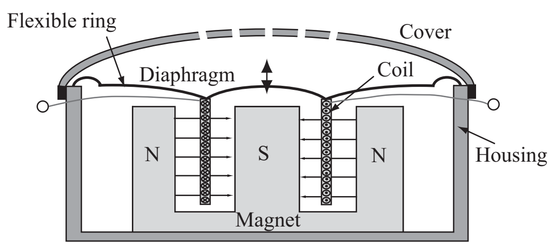


FIGURE 7.11 ■ A moving coil microphone.



where $\mathbf{\Phi}$ is the flux in the coil and N is the number of turns. This relation also explains the term dynamic. One property that should be emphasized is that this is a passive device—it generates its own output and requires no source of power.

As the coil vibrates in the magnetic field, a voltage with appropriate polarity is generated that can then be amplified for audio reproduction. The emf, when connected in a circuit, will generate a current and both of these are proportional to the velocity of the coil. These microphones have excellent characteristics, with relatively low noise and high sensitivity. They can be connected directly to many low-input impedance amplifiers and are still in use today. Note also that the structure in **Figure 7.11** is not fundamentally different from that of a common loudspeaker or the voice coil actuator discussed in **Section 5.9.1** except that in microphones the structure is modified to increase the change in flux as the diaphragm moves and, of course, the dimensions are smaller. Therefore any small magnetic loudspeaker can serve as a dynamic microphone, and the dynamic microphone, just like the moving iron microphone, is a dual device capable of serving as a loudspeaker or earphone (with the appropriate changes in dimensions, coil size, etc.).

This also means that an alternative way of looking at the result in **Equation (7.23)** is to start with the motion of the coil in the magnetic field and the force on the charge q moving at a velocity $\mathbf{v}$ as given in **Equation (5.21)**:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}. \tag{7.24}$$

From the fact that the force on a charge can always be written as $\mathbf{F} = q\mathbf{E}$, we conclude that $\mathbf{E} = \mathbf{v} \times \mathbf{B}$ is an electric field intensity. Integrating the field around the circumference of a loop of the coil and multiplying by the number of loops we get the emf produced in the coil:

$$emf = N\int_{loop} \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l} \ [\mathrm{V}]. \tag{7.25}$$

This emf relates to the velocity of the coil in the magnetic field but produces exactly the same result as **Equation (7.23)**.

EXAMPLE 7.5 The moving coil microphone

To understand the operation of a moving coil microphone it is sufficient to note that as the coil moves in and out of the magnetic field, the total flux through the coil changes and hence the emf induced in the coil changes. Exact calculation is not simple, as the motion of the coil under pressure depends on the mechanical properties of the diaphragm, the uniformity of the magnetic field, and the coil itself. But we can get an idea by assuming that the change in flux is proportional to the amplitude of sound and hence the position of the coil within the magnetic structure. Because of this, microphones are rated in terms of a sensitivity factor, k [mV/Pa]. For a pressure at amplitude P_0, the microphone's output emf is

$$emf = kP_0 \sin \omega t \quad [\mathrm{mV}],$$

where $\omega = 2\pi f$ and f is the frequency of the pressure wave, indicating that only changes in signals can be detected. Sensitivities of 10–20 mV/Pa are common (much more sensitive microphones exist).

The limit of human auditory threshold is 2×10^{-5} Pa. At that pressure, a microphone with a sensitivity of 20 mV/Pa will produce an emf of 0.4 μV. This will likely be below the noise level, meaning that the signal is not usable at or near the threshold level. But at the normal speech level of about 2 Pa, the output would be 40 mV, a signal that can easily be amplified.

7.4.3 The Ribbon Microphone

Another microphone in the same class as moving armature and moving coil microphones is the ribbon microphone. This is shown in **Figure 7.12** and is a variation of the moving coil microphone. The ribbon is a thin metallic foil (aluminum, for example) between the two poles of a magnet. As the ribbon moves, an emf is induced across it based on Faraday's law in **Equation (7.23)**, except that in this case $N = 1$. The current produced by this emf is the output of the microphone. These simple microphones have wide, flat frequency responses because of the very small mass of the ribbon. However, the small mass also makes them susceptible to background noise and vibration, and often they require elaborate suspension to prevent these effects. Because of their qualities, they are often used in studio sound recordings. The impedance of these microphones is very low, typically less than 1 Ω, and they must be properly interfaced for operation with amplifiers.

7.4.4 Capacitive Microphones

Early on in the development of audio reproduction, in the early 1920s, it became apparent that the motion of a plate in a parallel plate capacitor could be used for this purpose and hence the introduction of the capacitive or "condenser" microphone (condenser is the old name for a capacitor). The basic structure in **Figure 7.13** can be

FIGURE 7.12 ■ The ribbon microphone.

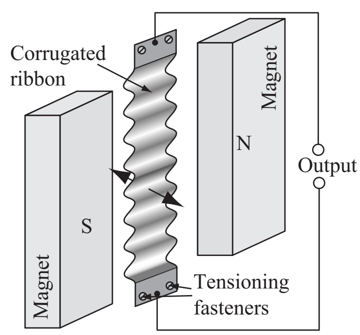
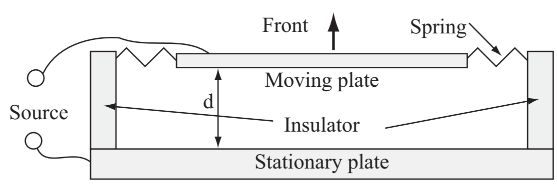


FIGURE 7.13 ■ The basic idea of a capacitive microphone.



used to understand the principle. The operation is based on the two basic equations of the parallel plate capacitor:

$$C = \frac{\varepsilon A}{d} \quad \text{and} \quad C = \frac{Q}{V} \ \rightarrow \ V = Q\frac{d}{\varepsilon A} \quad [\text{V}]. \tag{7.26}$$

This may look simple, but it also reveals a flaw in the whole idea of a simple parallel plate microphone: to produce an output voltage proportional to the distance d between the plates, a source of charge must be available. Sources of charge are not easy to come by except from external sources. Nevertheless, a solution has been found in the form of the **electret microphone**.

To understand what an electret is, it may be useful to consider first the idea of a permanent magnet. To produce a permanent magnet, a "hard" magnetic material, say, samarium-cobalt, is used and made into the shape needed. Then the material is magnetized by subjecting it to a very large external magnetic field. This moves the magnetic domains and sets up a permanent magnetization vector inside the material. When the external field is disconnected, the internal magnetization is retained by the material and that magnetization sets up the permanent magnetic field of the permanent magnet. One needs an equal or larger field to demagnetize it. An equivalent process can be done with an electric field. If a special material (it would be appropriate at this point to call it an electrically hard material) is exposed to an external magnetic field, a polarization of the atoms inside the material occurs. In these materials, when the external electric field is removed, the internal electric polarization vector is retained and this polarization vector sets up a permanent external electric field. Electrets are usually made by applying the electric field while the material is heated, to increase the atom energy and allow easier polarization. As the material cools, the polarized charges remain in this state. Materials used for this purpose are Teflon FEP (fluorinated ethylene propylene), barium titanate (BaTi), calcium titanate oxide ($CaTiO_3$), and many others. Some materials can be made into electrets by simply bombarding the material in its final shape by an electron beam.

The electret microphone is thus a capacitive microphone made of the same two conducting plates discussed above, but with a layer of an electret material under the upper plate, as shown in **Figure 7.14a**. The electret here is made of a thin film to provide the flexibility and motion necessary.

The electret generates a surface charge density σ_s on the upper metallic surface and an opposite sign surface charge density $-\sigma_s$ on the lower metal backplane. This generates an electric field intensity in the gap, s_1. The voltage across the two metallic plates, in the absence of any outside stimulus (sound), is

$$V = \frac{\sigma_s s_1}{\varepsilon_0 s + \varepsilon s_1} \quad [\text{V}]. \tag{7.27}$$

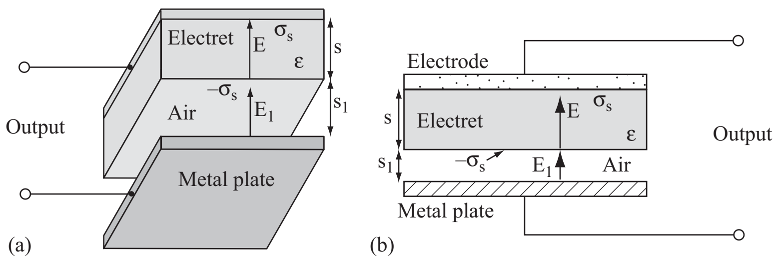


FIGURE 7.14 ■ The construction of an electret capacitive microphone.

If sound is applied to the diaphragm, the electret will move down a distance Δs and a change in voltage occurs as

$$\Delta V = \frac{\sigma_s \Delta s}{\varepsilon_0 s + \varepsilon s_1} \quad [\mathrm{V}]. \tag{7.28}$$

This voltage, which is the true output of the sensor, can be directly related to the sound pressure by first calculating the change in the gap length:

$$\Delta s = \frac{\Delta p}{\gamma p_0 s_1 + 8\pi T/A} \quad [\mathrm{m}], \tag{7.29}$$

where A is the area of the membrane, T is the tension, γ is the specific heat ratio of air, p_0 is the ambient pressure (or, in a more general sense, the pressure in the gap between the plate and the electret), and Δp is the change in pressure above ambient pressure due to sound. Thus the change in output voltage due to sound waves is obtained by substituting Δs in **Equation (7.28)**:

$$\Delta V = \frac{\sigma_s}{\varepsilon_0 s + \varepsilon s_1}\left(\frac{\Delta p}{\gamma p_0 s_1 + 8\pi T/A}\right) \quad [\mathrm{V}]. \tag{7.30}$$

This voltage can now be amplified as necessary.

Electret microphones are very popular because they are simple and do not require a source (they are passive devices). However, their impedance is very high and they require special circuits for connection to instruments. Typically a field-effect transistor (FET) preamplifier is required to match the high impedance of the microphone to the lower input impedance of the amplifier. In terms of construction, the membrane is typically made of a thin film of electret material on which a metal layer is deposited to form the movable plate.

In many ways the electret microphone is almost ideal. By proper choice of dimensions and materials, the frequency response can be totally flat from zero to a few megahertz. These microphones have very low distortions and excellent sensitivities (a few millivolts per microbar [mV/μbar]). Electret microphones are usually very small (some no more than 3 mm in diameter and about 3 mm long) and are inexpensive. Electret microphones can be found everywhere, from recording devices to cell phones. A sample of electret microphones is shown in **Figure 7.15**.

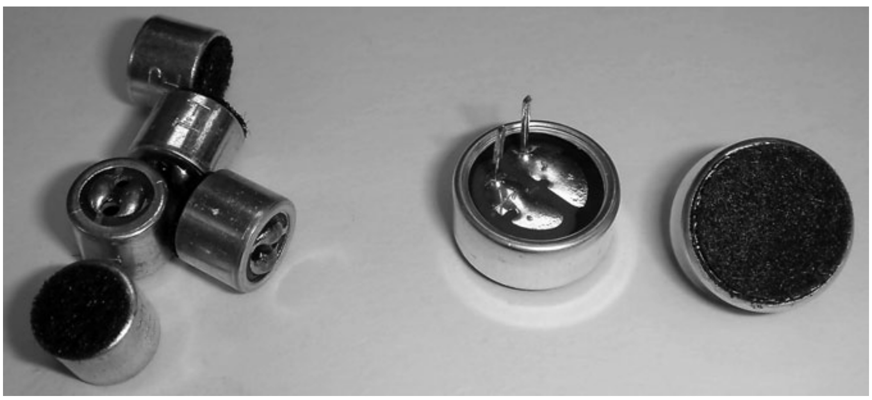


FIGURE 7.15 ■ Common electret microphones.

EXAMPLE 7.6 The electret microphone: design considerations

Consider the design of a small electret microphone for use in cellular phones, made in the form of a cylinder 6 mm in diameter and 3 mm long to fit in a slim telephone. Internally the designer has considerable flexibility as to the choice of materials and dimensions, as long as they fit within the external dimensions. Assuming the protective external structure requires a thickness of 0.5 mm, the diaphragm cannot be larger than 5 mm in diameter. The thickness of the diaphragm depends on the material used. Assuming a polymer, a reasonable thickness is 0.5 mm and a tension of 2 N can be easily supported by the structure. Polymers have relatively low permittivities, so we will assume a relative permittivity of 6. The gap between the electret and the lower conducting plate will be taken as 0.2 mm (the smaller the gap, the more sensitive the microphone). The ratio of specific heat in air is 1.4 (varies somewhat with temperature, but we will neglect that since the variation is rather small). The polymer can be charged at various levels, but the surface charge density cannot be very high. We will assume a charge density of 0.2 μC/m^2. With these values, and assuming an ambient pressure of 101,325 Pa (1 atm), we obtain a transfer function between the output voltage and the change in pressure using **Equation (7.30)**:

$$\Delta V = \frac{\sigma_s}{\varepsilon_0 s + \varepsilon s_1}\left(\frac{1}{\gamma p_0 s_1 + 8\pi T/A}\right)\Delta p.$$

In this relation, if p_0 is in pascals, Δp must also be in pascals. Numerically,

$$\begin{aligned}\Delta V &= \frac{0.2\times 10^{-6}}{8.854\times 10^{-12}\times 0.5\times 10^{-3} + 6\times 8.854\times 10^{-12}\times 0.2\times 10^{-3}}\\ &\quad\times\left(\frac{1}{1.4\times 0.2\times 10^{-3}\times 101{,}325 + 8\pi\times 2/(\pi\times(0.0025)^2)}\right)\Delta p\\ &= 5.19\Delta p \quad [\mathrm{V}]\end{aligned}$$

This is a sensitivity of 5.134 mV/Pa.

For a normal level of speech (45 dB to 70 dB) the pressure is 3.5×10^{-3} Pa to 6.3×10^{-2} Pa (see **Example 7.1**), producing an output of 18 to 327 mV, the microphone will produce a change in voltage of 10.268–20.536 mV, a level that is not only sufficient, but also rather high, for a microphone of the dimensions assumed here. Note, however, that this change is on top of the existing voltage given in **Equation (7.27)**.

The sensitivity can be improved by increasing the surface charge density or the area of the diaphragm, or decreasing the gap, the permittivity, the thickness of the electret, or the tension in the diaphragm.

7.5 | THE PIEZOELECTRIC EFFECT

The piezoelectric effect is the generation of electric charge in crystalline materials upon application of a mechanical stress. The opposite effect, often called electrostriction, is equally useful: the application of a charge across the crystal causes mechanical deformation in the material. The piezoelectric effect occurs naturally in materials such as quartz (silicon oxide) and has been used for many decades in so-called crystal oscillators. It is also a property of some ceramics and polymers, of which we have already met

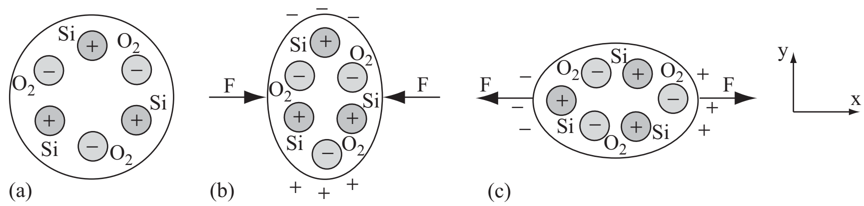


FIGURE 7.16 ■ The piezoelectric effect in a quartz crystal. (a) Undisturbed. (b) Strain applied in one direction. (c) Strain applied in the opposite direction.

the piezoresistive materials of **Chapter 5** (PZT is the best known) and the piezoresistive polymers PVF and PVDF. The piezoelectric effect has been known since 1880 and was first used in 1917 to detect and generate sound waves in water for the purpose of detecting submarines (sonar). The piezoelectric effect can be explained in a simple model by deformation of crystals. Starting with a neutral crystal (**Figure 7.16a**), a deformation in one direction (**Figure 7.16b**) displaces the molecular structure so that a net charge occurs as shown. In this case, the net charge on top is negative. Deformation on a perpendicular axis (**Figure 7.16c**) generates charges on the perpendicular axis. These charges can be collected on electrodes deposited on the crystal and measurement of the charge (or voltage) is then a measure of the displacement or deformation. This model uses the quartz crystal (SiO_2), but other piezoelectric materials behave in a similar manner. In addition, the behavior of the crystal depends on how the crystal is cut, and different cuts are used for different applications.

The polarization vector in a medium (polarization is the electric dipole moment of atoms per unit volume of the material) is related to stress through the following simple relation:

$$P = d\sigma \quad [\mathrm{C/m^2}], \tag{7.31}$$

where d is the **piezoelectric constant** and σ is the stress in the material. In reality, the polarization is direction dependent in the crystal and may be written as

$$P = P_{xx} + P_{yy} + P_{zz}, \tag{7.32}$$

where x, y, and z are the standard axes in the crystal. The relation above now becomes

$$P_{xx} = d_{11}\sigma_{xx} + d_{12}\sigma_{yy} + d_{13}\sigma_{zz}, \tag{7.33}$$

$$P_{yy} = d_{21}\sigma_{xx} + d_{22}\sigma_{yy} + d_{23}\sigma_{zz}, \tag{7.34}$$

$$P_{zz} = d_{31}\sigma_{xx} + d_{32}\sigma_{yy} + d_{33}\sigma_{zz}. \tag{7.35}$$

Now d_{ij} are the **piezoelectric coefficients** along the orthogonal axes of the crystal. Clearly then the coefficient depends on how the crystal is cut. To simplify the discussion, we will assume that d is single valued, but that depends on the type of piezoelectric material and how it is cut and excited (see the tables below for additional explanation of the indices). The inverse effect is written as

$$e = gP, \tag{7.36}$$

where e is strain (dimensionless) and g is the constant coefficient. The constant coefficient is related to the piezoelectric coefficient as

$$g = \frac{d}{e} \quad \text{or} \quad g_{ij} = \frac{d_{ij}}{e_{ij}}. \tag{7.37}$$

Normally the notation for stress is ε (see **Chapter 6**), but here the notation e is used to avoid confusion with permittivity, which is also denoted by ε. This relation also shows that the various coefficients are related to the electrical anisotropy of materials.

A third important coefficient is called the **electromechanical coupling coefficient** and is a measure of the efficiency of the electromechanical conversion:

$$k^2 = dgE \quad \text{or} \quad k_{ij}^2 = d_{ij}g_{ij}E_{ij}, \tag{7.38}$$

where E is the modulus of elasticity (Young's modulus). The electromechanical coupling coefficient is simply the ratio of the electrical and mechanical energies per unit volume. Some of these properties are listed in **Tables 7.5–7.7** for some crystals and ceramics often used in piezoelectric sensors and actuators. These tables also list some properties of polymers, materials that are becoming increasingly useful in piezoelectric (and piezoresistive) sensors.

Piezoelectric devices are often built as simple capacitors, as shown in **Figure 7.17**. Assuming that force is applied on the x-axis in this figure, the charge generated is

$$Q_x = d_{ij}F_x \quad [\mathrm{C}]. \tag{7.39}$$

Taking the capacitance of the device to be C, the voltage developed across it is

$$V = \frac{Q_x}{C} = \frac{d_{ij}F_x}{C} = \frac{d_{ij}F_x d}{\varepsilon_{ij}A} \quad [\mathrm{V}], \tag{7.40}$$

where d is the thickness of the piezoelectric material and A is its area. Thus the thicker the device, the larger the output voltage. A smaller area has the same effect. The output is directly proportional to force (or pressure). Note also that the pressure generates the

TABLE 7.5 ■ Piezoelectric coefficients and other properties in monocrystals

Crystal	Piezoelectric coefficient, d_{ij}, $\times 10^{-12}$ [C/N]	Relative permittivity, ε_{ij}	Coupling coefficient, k_{max}
Quartz (SiO_2)	$d_{11} = 2.31$, $d_{14} = 0.7$	$\varepsilon_{11} = 4.5$, $\varepsilon_{33} = 4.63$	0.1
ZnS	$d_{14} = 3.18$	$\varepsilon_{11} = 8.37$	0.1
CdS	$d_{15} = -14$, $d_{33} = 10.3$, $d_{31} = -5.2$	$\varepsilon_{11} = 9.35$, $\varepsilon_{33} = 10.3$	0.2
ZnO	$d_{15} = -12$, $d_{33} = 12$, $d_{31} = -4.7$	$\varepsilon_{11} = 8.2$	0.3
KDP (KH_2PO_4)	$d_{14} = 1.3$, $d_{36} = 21$	$\varepsilon_{11} = 42$, $\varepsilon_{33} = 21$	0.07
ADP ($NH_4H_2PO_4$)	$d_{14} = -1.5$, $d_{36} = 48$	$\varepsilon_{11} = 56$, $\varepsilon_{33} = 15.4$	0.1
$BaTiO_3$	$d_{15} = 400$, $d_{33} = 100$, $d_{31} = -35$	$\varepsilon_{11} = 3000$, $\varepsilon_{33} = 180$	0.6
$LiNbO_3$	$d_{31} = -1.3$, $d_{33} = 18$, $d_{22} = 20$, $d_{15} = 70$	$\varepsilon_{11} = 84$, $\varepsilon_{33} = 29$	0.68
$LiTaO_3$	$d_{31} = -3$, $d_{33} = 7$, $d_{22} = 7.5$, $d_{15} = 26$	$\varepsilon_{11} = 53$, $\varepsilon_{33} = 44$	0.47

TABLE 7.6 ■ Piezoelectric coefficients and other properties in ceramics

Ceramic	Piezoelectric coefficient, d_{ij}, $\times 10^{-12}$ [C/N]	Relative permittivity, ε	Coupling coefficient, k_{max}
$BaTiO_3$ (at 120°C)	$d_{15} = 260$, $d_{31} = -45$, $d_{33} = -100$	1400	0.2
$BaTiO_3 + 5\%CaTiO_3$ (at 105°C)	$d_{31} = 43$, $d_{33} = 77$	1200	0.25
Pb(Zr0.53Ti0.47)O_3 + (0.5–3%) La_2O_2 or Bi_2O_2 or Ta_2O_5 (at 290°C)	$d_{15} = 380$, $d_{31} = 119$, $d_{33} = 282$	1400	0.47
(Pb0.6Ba0.4)Nb_2O_6 (at 300°C)	$d_{31} = 67$, $d_{33} = 167$	1800	0.28
(K0.5Na0.5)NbO_3 (at 240°C)	$d_{31} = 49$, $d_{33} = 160$	420	0.45
PZT ($PbZr_{0.52}Ti_{0.48}O_3$)	$d_{15} = d_{24} = 584$, $d_{31} = d_{32} = -171$, $d_{33} = 374$	1730	0.46

TABLE 7.7 ■ Piezoelectric coefficients and other properties in polymers

Polymer	Piezoelectric coefficient, d_{ij}, $\times 10^{-12}$ [C/N]	Relative permittivity, ε [F/m]	Coupling coefficient, k_{max}
PVDF	$d_{31} = 23$, $d_{33} = -33$	106–113	0.14
Copolymer	$d_{31} = 11$, $d_{33} = -38$	65–75	0.28

Note: The indices *i,j* of the coefficients indicate the relation between input (force) and output (strain). Thus an index of 3,3 indicates that a force applied along the 3-axis produces a strain in that direction. An index 3,1 indicates a strain on the 1-axis when force is applied in the direction of the 3-axis of the crystal.

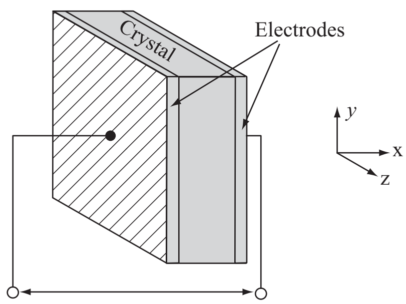


FIGURE 7.17 ■ The basic structure of a piezoelectric device.

stress in the material and hence the output may also be viewed as being proportional to stress or strain in the material. Piezoelectric sensors are often made of ceramics such as lead zirconite titanium oxide (PZT) and polymer films such as polyvinylidene fluoride (PVDF). Barium titanate ($BiTiO_3$) in crystal or ceramic form, as well as crystalline quartz, are also used for some applications.

One important development is the use of thin-film piezoelectric materials. Polymers are natural candidates for these films but they are fairly weak mechanically. Other materials such as PZT and zinc oxide (ZnO) are often used for this purpose because they have better mechanical and piezoelectric properties.

7.5.1 Electrostriction

It should be noted that the piezoelectric coefficient, with units of coulombs per newton (C/N) can also be viewed as having units of meters per volt (m/V) since 1 N/C = 1 V/m. This in turn may be viewed as follows:

$$d_{ij} \rightarrow \left[\frac{\mathrm{C}}{\mathrm{N}}\right] = \left[\frac{\mathrm{m}}{\mathrm{V}}\right] = \left[\frac{\mathrm{m/m}}{\mathrm{V/m}}\right] = \left[\frac{\textit{strain}}{\textit{electric field intensity}}\right].$$

Therefore the piezoelectric coefficient is strain developed per unit electric field intensity applied. This strain may be parallel or perpendicular to the applied force depending on the axes *i,j* involved. Note, however, that the strain produced per unit electric field intensity is small.

This gives rise to the electrostriction property. That is, when an electric field intensity is applied on a piezoelectric material, its dimensions change (strain). For example, the piezoelectric coefficient d_{33} for PZT is 374×10^{-12} C/N. That is, a sample of PZT, 1 m long will change its length by 374 pm per 1 V/m. This means that the electric field intensity must be high to produce any significant change in dimensions of the medium. Fortunately, very thin samples can be made and a large electric field intensity on the order of 1–2 million V/m can be applied to them, producing displacements on the order of hundreds of micrometers. In the example shown here, an electric field intensity of 2×10^6 V/m will produce a strain of 748 μm/m, or alternatively, 0.748 μm/mm. This is a reasonably large strain and is sufficient for many applications. Note again that this means that a 1 mm sample will increase in dimensions by 0.748 mm when an electric field intensity of 2×10^6 V/m is applied. To produce that electric field intensity across a 1 mm sample requires a voltage of 2000 V.

High voltages are typical in piezoelectric devices. For this reason, and to allow operation at more convenient, lower voltages, many applications use very thin samples. This is certainly the case for most electrostrictive actuators in which a voltage is applied to produce displacement. However, the piezoelectric effect can be used to generate high voltages by applying a strain. In this case the piezoelectric crystal should be thick to produce the required voltage (see **Problems 7.31** and **7.32**).

7.5.2 Piezoelectric Sensors

One of the most common piezoelectric sensors is the **piezoelectric microphone**, a device useful in both acoustic and ultrasound applications. The device in **Figure 7.17** can serve as a microphone by applying a force (due to sound pressure) on its surface. Given this structure, and a change in pressure Δp, the change in voltage expected (from **Equation (17.40)**) is

$$\Delta V = \frac{d_{ij}(\Delta p A)d}{\varepsilon_{ij}A} = \frac{d_{ij}d}{\varepsilon_{ij}}\Delta p \quad [\mathrm{V}]. \tag{7.41}$$

A linear relation is therefore available to sense the sound pressure. A common structure for the microphone is shown in **Figure 7.18**. The fact that capacitance is involved also indicates that piezoelectrics are high-impedance materials and thus require impedance matching networks.

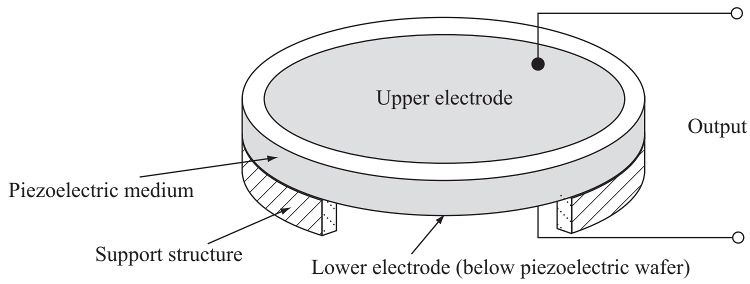


FIGURE 7.18 ■ The structure of a piezoelectric microphone.

One overriding property of these devices is that they can operate at high frequencies, hence their use as ultrasonic sensors. In addition, the piezoelectric microphone can be used as a piezoelectric actuator, and it is just as efficient. In other words, whereas there is a big difference between a magnetic (or capacitive) microphone and a loudspeaker, the piezoelectric microphone and piezoelectric actuator are essentially the same in all respects including dimensions. This complete duality is unique to piezoelectric transducers and, to a smaller extent, to magnetostrictive transducers.

Typical construction consists of films (PVDF or copolymers) with metal coatings for electrodes or disks of various piezoelectric crystals. These can be round, square, or almost any other shape. One particularly useful form is a tubelike electrode, which is usually used in hydrophones. These elements can be connected in series to cover a larger area, as is sometimes required in hydrophones.

The output of piezoelectric microphones is relatively low in the range of human speech because of the low pressures produced. Normal sensitivities are on the order of 10 μV/Pa, so in the range of normal speech one can expect voltages on the order of 100 μV, depending on the properties of the material involved and the distance of the microphone from the source of the sound.

Piezoelectric microphones have exceptional qualities and a flat frequency response. For this reason they are used in many applications, chief among them as pickups in musical instruments and for detection of low-intensity sounds such as the flow of blood in the veins. Other applications include voice-activated devices and hydrophones.

EXAMPLE 7.7 The piezoelectric microphone

A piezoelectric microphone is made of lithium titanate oxide ($LiTiO_3$) in the form of a disk that is 10 mm in diameter and 0.25 mm thick. Two electrodes, 8 mm in diameter, are coated on the opposite surfaces of the $LiTiO_3$ crystal. The crystal is cut on the 3-3 axis and is used to record speech at a distance of 1 m from a person. The sound pressure produced by normal speech at that distance is approximately 60 dB above the threshold of hearing. If the person were to shout, the sound pressure would increase to about 80 dB above the threshold of hearing. The threshold of hearing is 2×10^{-5} Pa taken at a reference of 0 dB. Calculate the range in voltages produced by the microphone under these conditions.

Solution: It is common to provide sound pressure in decibels rather than in pascals or newtons per square meter. However, we need the sound pressure in actual units so we can use **Equation (7.41)**. We therefore start by converting the given values using the relation

$$P(\mathrm{dB}) = 20\,\log_{10}P \quad [\mathrm{dB}].$$

However, because we need a zero reference at a pressure of 2×10^{-5} Pa, we must add this reference in decibels to any conversion. Thus we get

$$P_0 = 20\log_{10} 2 \times 10^{-5} = -94\ \ \mathrm{dB}.$$

For normal level speech, $P = 60 - 94 = -34$ dB:

$$-34\ \mathrm{dB} = 20\log_{10} P \rightarrow \log_{10} P = -1.7 \rightarrow P = 10^{-1.7} = 0.02\ \mathrm{Pa}.$$

At an elevated level, $P = 80 - 94 = -14$ dB:

$$-14\ \mathrm{dB} = 20\log_{10} P \rightarrow \log_{10} P = -0.7 \rightarrow P = 10^{-0.7} = 0.2\ \mathrm{Pa}.$$

Since these pressure levels are above the ambient pressure, we can take them as changes in pressure due to speech. We use **Equation (7.41)** with the relative permittivity ε_{33}.

At normal speech,

$$\Delta V_l = \frac{d_{33}d}{\varepsilon_{33}}\Delta p = \frac{7 \times 10^{-12} \times 0.25 \times 10^{-3}}{44 \times 8.854 \times 10^{-12}} 0.02 = 89.84 \times 10^{-9}\ \mathrm{V}.$$

At an elevated level,

$$\Delta V_e = \frac{d_{33}d}{\varepsilon_{33}}\Delta p = \frac{7 \times 10^{-12} \times 0.25 \times 10^{-3}}{44 \times 8.854 \times 10^{-12}} 0.2 = 8.984 \times 10^{-7}\ \mathrm{V}.$$

The output of the microphone changes from 89.84 nV to 0.8984 μV as the voice rises from normal to shouting. This output is consistent with piezoelectric microphones that produce low outputs (because voice pressures are low).

7.6 | ACOUSTIC ACTUATORS

Among these we shall discuss two types. First is the classic loudspeaker as is used in audio reproduction. We have already discussed its basic properties in **Chapter 5** in the section on voice coil actuators. Here we will discuss other properties as these relate to the audio range. Second, we introduce the use of piezoelectric actuators for the purpose of sound generation. These devices, sometimes referred to as buzzers, are quite common in electronic equipment where audible signals (rather than voice or music) are needed. They are also much simpler, more rugged, and less expensive than classic loudspeakers. The issue of mechanical actuation using piezoelectric means will be discussed separately later in this chapter.

7.6.1 Loudspeakers

The basic structure of a loudspeaker driving mechanism is shown in **Figure 7.19**. The magnetic field in the gap is radial and acts on the coil (see **Figure 7.20**). For a

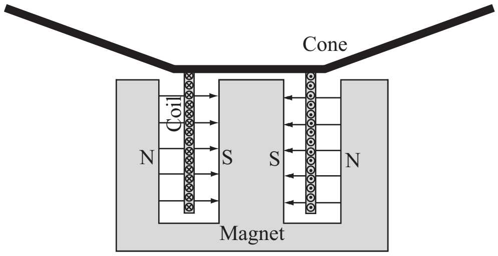


FIGURE 7.19 ■ Structure of a magnetic loudspeaker. The radial magnetic field is produced by a permanent magnet. The current in the coil is also shown.

(a) (b)

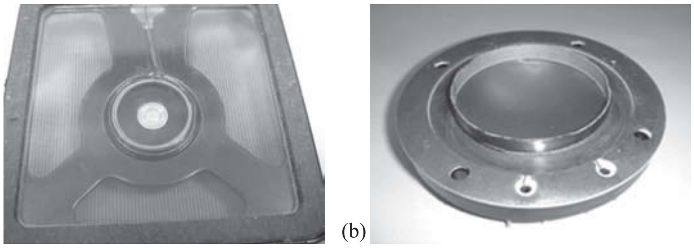


FIGURE 7.20 ■ (a) A small loudspeaker with a transparent cone (or diaphragm) showing the coil and the magnet at the center as well as the lead wires to the coil and the frame. (b) The coil of a public address loudspeaker. In this case the diaphragm is behind the coil and is made of titanium. The coil is wound on a short paper tube bonded to the diaphragm.

current-carrying loop, the force is given by the Lorentz force (see **Section 5.4**, and in particular **Equations (5.21)–(5.26)**, as well as **Section 5.9.1**). With N turns, the force is $NBIL$, where L is the circumference of the loop and we assume a uniform magnetic field. The field is only approximately uniform and the force is slightly nonlinear at the very end of travel of the coil, as was discussed in conjunction with voice coil actuators in **Section 5.9.1**. This is also the range in which most of the distortions occur.

Loudspeakers come in many varieties with various methods of construction, but as a rule the driving coil is round and the magnetic field in the gap is radial. Some old loudspeakers used electromagnets to generate the magnetic field, but all modern loudspeakers use permanent magnets for this purpose. The magnets are made as strong as possible and the gap as narrow as possible to ensure maximum force for a given current, thus reducing the power dissipated in the speaker. In most cases the coils are simple varnish-insulated copper wires wound tightly in a vertical spiral, usually in a single layer and supported by a backing of paper, Mylar, or fiberglass, depending on the speaker (see **Figure 7.20b**). The diaphragm supplies the restoring force and keeps the coil centered. The paper support for the coil and the method of leading the coils out of the device are shown as well. The cone is usually made of paper (in very small speakers they may be made of Mylar or some other reasonably stiff material; see **Figure 7.20a** and **Figure 7.22**) and is suspended on the rim of the speaker, which in turn is made as stiff as possible to avoid vibrations. The operation of a loudspeaker is essentially the motion of the coil in response to variations of current through it that in turn change the pressure in front of (and behind) the cone, thus generating a longitudinal wave in the air. The same principle can be used to generate waves in fluids or even in solids.

The power rating of a speaker is usually defined as the power in the coil, that is, the voltage across the coil multiplied by the current in the coil. This power can be specified as average or peak power, but it is not the power radiated by the cone. The radiated power is a portion of the total power supplied to the loudspeaker and is the difference between the total power and the dissipated power. The efficiency of loudspeakers is not particularly high.

The power handling capacity of the loudspeaker is the power the loudspeaker can handle without damage to its coil. The acoustic radiated power is quite different and depends on the electrical and mechanical properties of the loudspeaker. Assuming an unimpeded diaphragm connected to a coil of radius r and N turns in a magnetic field B, the radiated acoustic power is

$$P_r = \frac{2I^2B^2(2\pi rN)2Z}{R_{ml}^2 + X_{ml}^2} \quad [\mathrm{W}], \tag{7.42}$$

where Z is the acoustic impedance of air, R_{ml} is the total mechanical resistance, and X_{ml} is the total mass reactance seen by the diaphragm. However, these quantities are not easy to obtain and are often either estimated or measured for a particular loudspeaker, as they depend on the speaker and its construction. The radiated power may also be estimated from calculations of the magnetic force on the coil and the velocity of travel of the diaphragm in the magnetic field (see **Example 7.8**). However, this method is not very accurate since it does not take the mechanical properties of the loudspeaker into account.

A simplified approach to calculating the radiated power is based on the pressure generated by a piston of area A. Assuming uniform pressure across the area of the loudspeaker, the radiated acoustic power may be approximated as

$$P_{ac} = \frac{p^2{}_{ac}A}{Z} \quad [\mathrm{W}], \tag{7.43}$$

where P_{ac} is the pressure and A is the area of the loudspeaker (i.e., the circular area at the top of the cone, not the surface area of the cone). This relation may also be used to estimate the acoustic power in a buzzer, where the flat diaphragm is a better approximation of a piston.

These relations only give a rough idea of the power radiated. **Equation (7.42)** indicates that power is proportional to current, magnetic flux density, and the size (both physical and the number of turns) of the coil, whereas **Equation (7.43)** looks at power from a pressure point of view, which in turn is generated by the forces produced by the current. There are other issues that have to be taken into account, including reflections from the speaker's body, vibration of the structure, and damping due to the suspension of the cone, but the relations above are sufficient for a general understanding of radiated acoustic power.

EXAMPLE 7.8 Radiated and dissipated power in a loudspeaker

A loudspeaker is made as in **Figure 7.19**, with the following parameters: the coil is 60 mm in diameter, has 40 copper turns, and each turn is 0.5 mm in diameter, with a magnetic flux density produced by a permanent magnet equal to 0.85 T. The loudspeaker is fed with a sinusoidal current of amplitude 1 A at a frequency of 1 kHz. The coil and the diaphragm have a total mass of 25 g. Use an electric conductivity of 5.8×10^7 S/m for copper.

a. Estimate the power loss in the coil.
b. Estimate the radiated power of the loudspeaker.

c. Estimate the maximum travel of the diaphragm.

d. Discuss the approximations needed to get the results above.

Solution: The power loss can be calculated directly from the resistance of the wires (see part (d)). The power radiated by the loudspeaker is the mechanical power calculated from the product Fv, where F is the force and v is the velocity of the coil.

a. We will calculate here the DC resistance of the wires using a total length of wire:

$$L = 2\pi rn \quad [\text{m}],$$

where r is the radius of the coil and n is the number of turns. The wire cross-sectional area S is

$$S = \pi \frac{d^2}{4} \quad [\text{m}^2],$$

where d is the diameter of the wire. Given the conductivity of copper, the DC resistance of the coil is

$$R = \frac{L}{\sigma S} = \frac{2\pi rn}{\pi \frac{d^2}{4}} = \frac{8rn}{\sigma d^2} = \frac{8 \times 0.03 \times 40}{5.8 \times 10^7 \times (0.0005)^2} = 0.662\ \Omega.$$

To calculate the power dissipated we need the current. The current is sinusoidal at a frequency of 1 kHz:

$$I(t) = 1\sin(2\pi \times 1000t) = 1\sin(6283t) \quad [\text{A}].$$

Power is an averaged value. Given the RMS value of the current is $I/\sqrt{2}$, where I is the amplitude (peak value) of the current, the power dissipated is

$$P = \frac{I^2R}{2} = \frac{1 \times 0.662}{2} = 0.332\ \text{W}.$$

b. To calculate power radiated we start by calculating the force the magnetic field exerts on the coil. We note that the magnet produces a uniform magnetic flux density in the gap. Therefore the loops of the coil are in a uniform radial field. Using **Equation (5.26)** for the force on a length of wire, the peak magnetic force on the coil is

$$F = B(nI)L = 2\pi rnBI = 2\pi \times 0.03 \times 40 \times 0.9 \times 1 = 6.786\ \ \text{N},$$

where r is the radius of the coil, n is the number of turns, B is the magnetic flux density, and I is the current in the coil. This force moves the coil in or out depending on the phase of the current. We assume the speaker's diaphragm moves the coil in tandem with the current (otherwise the speaker cannot be expected to produce sound of any fidelity to the source that drives it). The time-dependent force is

$$F(t) = 6.786\sin(6283t) \quad [\text{N}]$$

From this we can calculate the acceleration of the coil:

$$F = ma \rightarrow a = \frac{F}{m} = \frac{6.786\sin(6283t)}{30 \times 10^{-3}} = 226.2\sin(6283t) \quad [\text{m/s}^2].$$

We integrate the acceleration to obtain the velocity of the coil:

$$v(t) = \int a_0 \sin(\omega t)dt = -\frac{a_0}{\omega}\cos\omega t = -\frac{226.2}{6283}\cos(6283t) = 0.036\cos(6283t) \quad [\mathrm{m/s}].$$

Now we can write the instantaneous power as

$$P(t) = F(t)v(t) = 6.786\sin(6283t) \times 0.036\cos(6283t) = 0.244\sin(12566t) \quad [\mathrm{W}].$$

The averaged radiated power is half the amplitude of the instantaneous power:

$$P_{\text{avg}} = 0.122 \text{ W}.$$

Note: This power may not seem very high, but it is sufficient for normal listening. A higher power would necessitate a larger number of turns, a larger current, and/or a larger magnetic field. When these parameters are changed, the power dissipation changes as well. The efficiency of the loudspeaker shown here is about 73%, an excellent figure for loudspeakers.

c. The maximum displacement of the diaphragm is calculated from the equation of motion of the diaphragm. Since we know that the diaphragm is at its rest position when the current is zero (i.e., at $t = 0$), it reaches its positive maximum in ¼ cycle ($T/4$), where $T = 1$ ms. That is, the maximum force occurs after 1/4000 s. Thus we write the distance as

$$L = \int_{t=0}^{T/4} a\frac{t^2}{2} \quad [\mathrm{m}].$$

This would give an exact solution. A simpler way is to use the RMS value of acceleration and write

$$L = a_{\text{RMS}}\frac{t^2}{2} = \frac{226.2}{\sqrt{2}} \times \frac{(1/4000)^2}{2} = 5.7\ \mu\text{m}.$$

The diaphragm moves only ±5.7 μm at that frequency. The lower the frequency, the larger the displacement for the same force. The displacement obtained here may seem unreasonably small, but in fact the diaphragm only has 1 ms to move from rest to maximum displacement then to maximum displacement in the opposite direction and back to rest. For this reason smaller, lighter cones will respond to higher frequencies, whereas heavier cones will not. In the end, for any given speaker, its frequency response is to a large extent a function of this ability to follow the current.

d. We have made a number of assumptions, both explicit and implicit. The first is the use of DC resistance for the speaker. This is convenient because it is simple, but the AC resistance of conductors is frequency dependent and increases with frequency. Therefore the power loss we calculated is the minimum possible—essentially power loss at zero frequency. Second, we assumed a uniform magnetic flux density, which in actuality may not be uniform and may not be the same for loops close to the top of the magnet. More importantly, we have not taken into account mechanical issues such as forces needed to act against the restoring spring action that keeps the diaphragm at its starting position. In addition, any effects due to heat dissipation, such as a change in resistance of the coil with temperature, have been neglected.

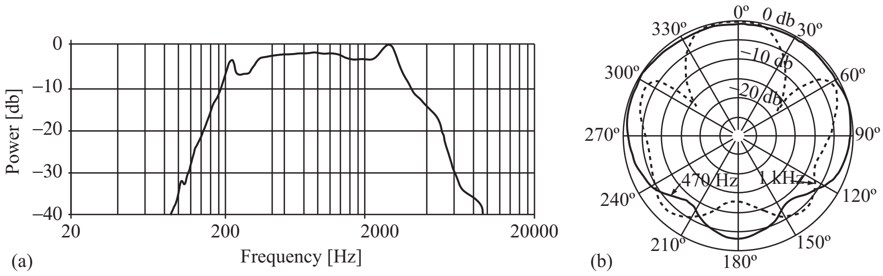


FIGURE 7.21 ■ Frequency response of a midrange loudspeaker. (a) Rectangular plot of power versus frequency. (b) Polar plot of normalized power at 470 Hz and 1 KHz.

In addition to radiated power and dissipated power, speakers are characterized by other properties such as dynamic range, maximum displacement of the diaphragm, and distortion. However, two other properties are of paramount importance. One is the frequency response of the speaker, the other its directional response (also called the radiation pattern or coverage pattern). The frequency response of a loudspeaker over its useful span is shown in **Figure 7.21a**. It shows power as a function of frequency in decibels, normalized to 1. This particular speaker shows a response between 90 Hz and 9 kHz with a bandwidth between about 200 Hz and 3.5 kHz (half power points). Also to be noted are the peaks or resonances at 220 Hz and 2.7 kHz. These are usually associated with the mechanical structure of the speaker. This speaker is obviously a general purpose speaker and others will have better responses at lower frequencies (woofers) or higher frequencies (tweeters), usually associated with the physical size of the speakers.

The directional response indicates the relative power density in different directions in space. **Figure 7.21b** shows such a plot at two frequencies indicating where in space one can expect larger or smaller power densities and the general coverage. In particular, note that the power density behind the speaker is lower than in front of it, as expected. When measuring the spatial response of loudspeakers the measured quantity may be pressure or, as in this case, power density. **Figures 7.22** and **7.23** show a number of speakers, some very small, some larger, but these only cover the "conventional" range. Many other types and shapes exist, some of them truly large.

FIGURE 7.22 ■ Some small loudspeakers. The smallest is 15 mm in diameter, the largest is 50 mm. The largest has a paper cone, whereas the others feature Mylar cones.

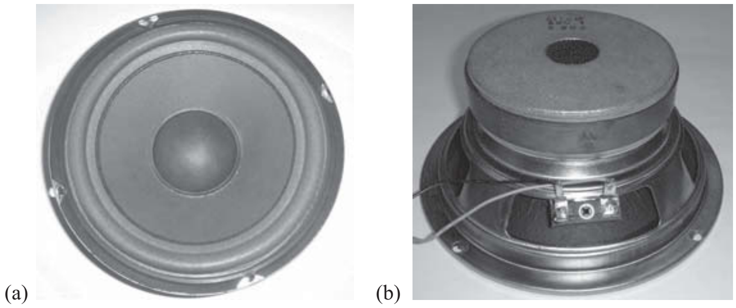


FIGURE 7.23 ■ A medium-size loudspeaker used for low-frequency reproduction (woofer). (a) View of the cone (front). (b) View of the cone (back) showing the magnet on top, the frame, and the connections. This loudspeaker is 16 cm in diameter.

7.6.2 Headphones and Buzzers

The loudspeakers in **Figures 7.20a**, **7.22**, and **7.23** represent the common structures of loudspeakers. Instead of moving the coil, one can conceive the opposite—moving the magnet while keeping the coil fixed. An adaptation of this idea is the moving diaphragm actuator shown in **Figure 7.24**. This is not used in loudspeakers, but has been used in the past in headphones and is in use today as earpieces in land telephones and in magnetic warning devices called buzzers. These magnetic actuators come in two basic varieties. One is simply a coil and a suspended membrane, as in **Figure 7.24**. Current in the coil attracts the membrane and variations in current move it back and forth with respect to the coil depending on the magnitude and direction of the current. A permanent magnet may also be present, as shown, to bias the device and keep the membrane in place. In this form the device acts as a small loudspeaker, but of a fairly inferior quality. It does have one advantage over conventional loudspeakers, especially in its use in telephones: because the coil is fairly large (many turns), its impedance is relatively high, so it can be connected directly in a circuit and driven by a carbon microphone without the need for an amplifier. However, for all other sound reproduction systems, it is not acceptable. Instead, modern magnetic headphones use small loudspeakers for much better sound quality.

7.6.2.1 The Magnetic Buzzer

The magnetic earpiece, mentioned above, has evolved into the modern magnetic buzzer. In this form, sound reproduction is not important, but rather the membrane is made to vibrate at a fixed frequency, say 1 kHz, to provide an audible warning for circuits, machinery, fire alarms, and the like. This can be done by driving the basic circuit in **Figure 7.24** with a square wave, usually directly from the output of a microprocessor or

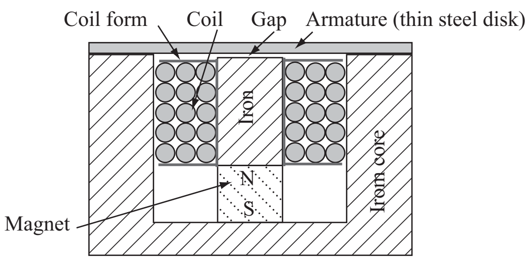


FIGURE 7.24 ■ The moving armature actuator: the buzzer.

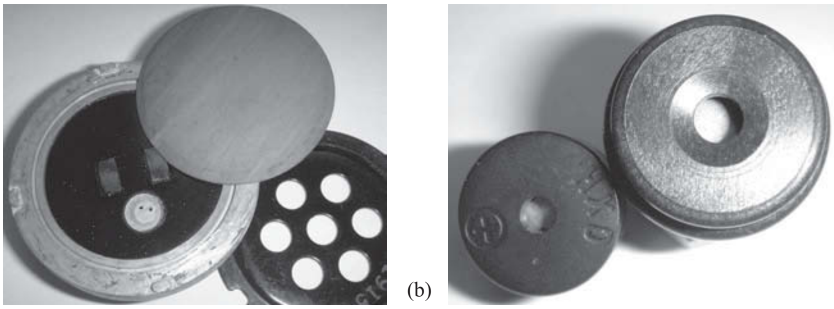
(a) (b)


FIGURE 7.25 ■ (a) A World War II era earpiece made as a moving diaphragm element. The idea has survived in the modern magnetic buzzer. (b) Two modern magnetic buzzers based on the same principles. The one on the left is 12 mm in diameter, the one on the right is 15 mm.

through a suitable oscillator. In some devices the circuitry necessary for oscillation is internal to the device and the only external connections are to power. **Figure 7.25a** shows a World War II era earpiece showing the magnetic yoke at the center, the steel diaphragm, and the cover. **Figure 7.25b** shows two modern magnetic buzzers, 12 and 15 mm in diameter, respectively, based on the same basic structure. In the structure in **Figure 7.24**, given a coil of N turns carrying a current I, the magnetic flux density in the gap between the coil and the diaphragm can be approximated as

$$B = \frac{\mu_0 NI}{d} \quad [\mathrm{T}], \tag{7.44}$$

where d is the gap length and μ_0 is the permeability of air. For this approximation to be valid the permeability of the iron structure and the diaphragm must be large. This field generates a force on the diaphragm (see **Example 7.9**) forcing it to move. If, for example, the current were sinusoidal or a square wave, the diaphragm would move back and forth at the frequency of the signal, generating a pressure wave at that frequency. It is this property that makes the device useful as a warning device or as a means of producing simple sounds (such as the optional clicks used as feedback when typing on a keypad). The force on the diaphragm can be approximated by first calculating the energy per unit volume in the gap. This was calculated in **Equation (5.62)** as magnetic energy:

$$w_m = \frac{B^2}{2\mu_0} \quad [\mathrm{J/m^3}]. \tag{7.45}$$

Now we use the fact that a force F, moving the armature a distance dl, produces a change in volume dv and a change in work $dW_m = Fdl$:

$$dW_m = Fdl \quad [\mathrm{J}]. \tag{7.46}$$

The force becomes

$$F = \frac{dW_m}{dl} \quad [\mathrm{J}]. \tag{7.47}$$

For this relation to be useful, we define a small motion of the plate and calculate the change in energy due to that motion (which changes the volume in which the energy density exists by dv). Then the change in energy per change in distance due to that

FIGURE 7.26 ■ The piezoelectric diaphragm earpiece. The piezoelectric disk is shown in the center of the diaphragm.

motion gives the force. This method is called the virtual displacement method and is a common method of calculating forces in magnetic circuits (see **Example 7.9**).

7.6.2.2 The Piezoelectric Headphone and Piezoelectric Buzzer

Both the headphone and the buzzer also exist as piezoelectric devices in which a piezoelectric disk is physically bonded to a diaphragm. The piezoelectric element is a disk, as shown in **Figure 7.18**, and connection to a voltage source will cause a mechanical motion in the disk. When an AC source is applied, the variations in motion of the disk generate a sound at the applied source's frequency. An earphone of this type is shown in **Figure 7.26** together with its piezoelectric element, seen as the smaller disk at the center of the diaphragm.

The earpiece in **Figure 7.26** can be used as a buzzer by driving it with an AC source. However, for incorporation in an electronic circuit, these devices often come either as a device with a third connection, which when appropriately driven forces the diaphragm to oscillate at a fixed frequency, or has the necessary circuit to do so incorporated in the device. **Figure 7.27** shows a piezoelectric buzzer and, separately, its diaphragm shown from underneath. The piezoelectric element has two sections. A large circular section and a smaller finger-shaped section. The latter, when properly driven, causes local distortion in the diaphragm and the interaction of these distortions and those of the main element

FIGURE 7.27 ■ A piezoelectric buzzer showing the structure and the diaphragm with the piezoelectric disk on it.

FIGURE 7.28 ■ Piezoelectric buzzers of various sizes (13 mm–28 mm).

cause the device to oscillate at a set frequency that depends on the sizes and shapes of the two piezoelectric elements. These buzzers are very popular since they use little power, can operate down to about 1.5 V, and are rather loud, making them useful as directly driven devices in microprocessors. A device like this can be used for audible feedback or as a warning device (e.g., for a moving robot or as a backup warning in trucks and heavy equipment). **Figure 7.28** shows a number of piezoelectric buzzers of different sizes.

EXAMPLE 7.9 Pressure generated by a magnetic buzzer

A magnetic buzzer is made as shown in **Figure 7.29**. The structure is circular with an outer radius $a = 12.5$ mm and an inner radius $b = 11$ mm. The inner cylinder supporting the coil has a radius $c = 12$ mm. Assume the whole structure, including the diaphragm, is made of a high-permeability material so that any magnetic field generated by the coil is contained within the structure and the gap between the coil and the diaphragm. The gap is $d = 1$ mm. Given a coil with $N = 400$ turns and a current I at 1 kHz and an amplitude of 200 mA, calculate the maximum pressure generated by the diaphragm. Neglect mechanical losses in the system.

Solution: Since the configuration described here is essentially that of **Figure 7.24**, the magnetic flux density in the gap is (from **Equation (7.44)**):

$$B = \frac{\mu_0 NI}{d} \quad [\mathrm{T}].$$

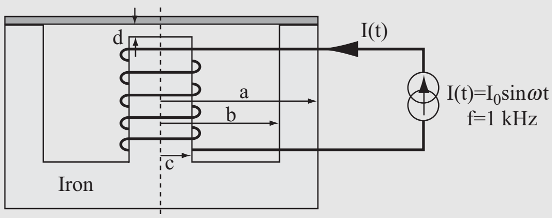


FIGURE 7.29 ■ Structure and dimensions of a magnetic buzzer.

The energy density in the gap between the core and diaphragm is

$$w_m = \frac{B^2}{2\mu_0} = \frac{\mu_0 N^2 I^2}{2d^2} \quad [\text{J/m}^3].$$

Now suppose the diaphragm moves a very small distance dx so that it either reduces the gap or increases the gap. The change in energy in the gap is

$$dW = dw_m S dx = \frac{\mu_0 N^2 I^2}{2d^2} S dx \quad [\text{J}],$$

where S is the cross-sectional area of the coil core. The latter is πc^2. Thus the force may be written as

$$F = \frac{dW}{dx} = \frac{\mu_0 N^2 I^2}{2d^2} \pi c^2 \quad [\text{N}].$$

Since this is the force acting on the diaphragm, the pressure produced must be the force divided by the area of the diaphragm, πb^2:

$$P = \frac{F}{\pi b^2} = \frac{\mu_0 N^2 I^2 c^2}{2d^2 b^2} \quad [\text{N/m}^2].$$

This is better defined as a change in pressure to indicate that it is above (or below) the ambient pressure. It should also be noted that this is dynamic pressure, that is, it only exists while the diaphragm is moving (and hence sound is produced only during that time). Once the diaphragm settles into a fixed position (such as if we apply DC instead of AC) the pressure is the ambient pressure and no sound is produced.

For the values given,

$$P = \frac{\mu_0 N^2 I^2 c^2}{2d^2 b^2} = \frac{4\pi \times 10^{-7} \times 400^2 \times 0.2^2 \times 0.006^2}{2 \times 0.001^2 \times 0.011^2} = 1196.4 \text{ N/m}^2.$$

That is, 1196.4 Pa. Since buzzers are often rated in decibels, we calculate the rating of this buzzer as follows:

$$20 \log_{10} \frac{1194.6}{2 \times 10^{-5}} = 155.5 \text{ dB}.$$

In other words, a very loud sound, sure to get one's attention since it is above the threshold of pain. Recall that normal speech is on the order of a few pascals or about 50 dB. Note, however, that this is the sound level at the diaphragm. At a distance from the diaphragm the sound level is reduced by attenuation and by spreading of the acoustic power.

7.7 | ULTRASONIC SENSORS AND ACTUATORS: TRANSDUCERS

Ultrasonic sensors and actuators are, in principles of operation, identical to the acoustic sensors and actuators discussed above, but they are somewhat different in construction and very different in terms of materials used and their range of frequencies. However, because the ultrasonic range starts where the audible range ends, the two, in effect, overlap. It is therefore quite reasonable to assume that an ultrasonic sensor (i.e., microphone or, as is more often the nomenclature, transducer or receiver) or actuator for the near ultrasound range should be quite similar to an acoustic sensor or actuator. In fact they are, at least at first glance. **Figure 7.30** shows an ultrasonic transmitter (left) and an ultrasonic receiver (right) designed for operation in air at 24 kHz. The first thing to note is that the two are of the same size and essentially the same construction. This is typical of piezoelectric devices, in which the same exact device can be used for both purposes, as explained above. Both use an identical piezoelectric disk similar to the one in **Figure 7.18**. The only visible difference is in the slight difference in the construction of the cone. **Figure 7.31** shows a closer view of another device, this time operating at 40 kHz, also designed to operate in air, in which the piezoelectric device is square, seen at the center below the brass supporting member (the second connecting wire is underneath). These devices operate exactly as microphones and as speakers.

These ultrasonic sensors are very common in applications in air (typical frequencies are 24 kHz and 40 kHz) for range finding and obstacle avoidance in robots. Other

FIGURE 7.30 ■ An ultrasonic transmitter (left) and an ultrasonic receiver (right) designed to operate in air at 24 kHz.

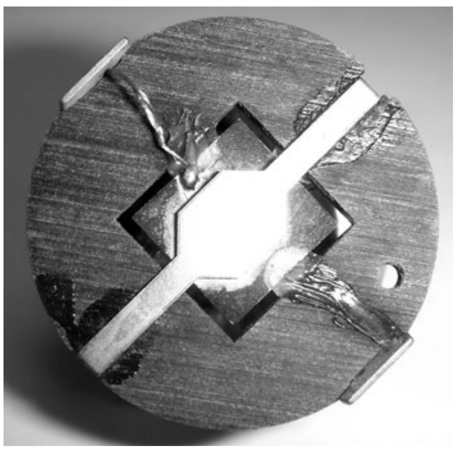


FIGURE 7.31 ■ A 40 kHz ultrasonic sensor (transducer) for operation in air.

applications are for presence detection in alarm systems and for safety in cars, where they are used for intrusion alarms and for collision avoidance when backing up. In some of these applications, higher frequencies are often used. The main difficulty with the propagation of ultrasound in air is that the attenuation of high-frequency ultrasound in air is high, so that the range of these devices is relatively short. On the other hand, the use of ultrasound is very attractive both because it is relatively simple (at these low frequencies) and because ultrasound, just like sound, tends to spread and cover a relatively large area. At higher frequencies the propagation can be much more direct and focused, but at the low frequencies used for ultrasound in air it is not. **Figure 7.32** shows a transmitter–receiver pair typically used for distance ranging in robots.

The scope of ultrasonic sensing is much wider than what is implied by the previous paragraphs. Its use for sensing and actuation is much more common and perhaps more important in materials other than air and at higher frequencies. In particular, when viewing **Table 7.1**, it is clear that ultrasound is better suited for use in solids and liquids where ultrasound propagates at higher velocities and lower attenuation. Also, solids support waves other than longitudinal, a property that allows additional flexibility in the use of ultrasonic waves: shear waves (these are transverse waves that can only exist in solids) and surface waves are two types often used in addition to longitudinal waves (see **Section 7.3**).

Ultrasonic sensors exist at almost any frequency and can certainly be made in frequencies exceeding 1 GHz. For practical applications, most sensors operate below 50 MHz, but a unique class of sensors based on SAW principles uses higher frequency to achieve a number of sensing and actuation functions. Most ultrasonic sensors and actuators are based on piezoelectric materials, but some are based on magnetostrictive materials since, in effect, what is needed is a means of converting an electrical signal into strain (for a transmitter) or strain into an electrical signal (receiver).

One particularly important property of piezoelectric materials that makes them indispensable in the design of ultrasonic sensors and actuators is their ability to oscillate at a fixed, sharply defined resonant frequency. The resonant frequency of a piezoelectric crystal (or ceramic element) depends on the material itself and its effective mass, strain, and physical dimensions, and is also influenced by temperature, pressure, and other environmental conditions such as humidity. To understand resonance, it is useful to look at the equivalent circuit of a piezoelectric device sandwiched between two electrodes, shown in **Figure 7.33a**. This circuit has two resonances: a parallel resonance and a series

FIGURE 7.32 ■ A 40 kHz transmitter–receiver pair for distance ranging in robots.

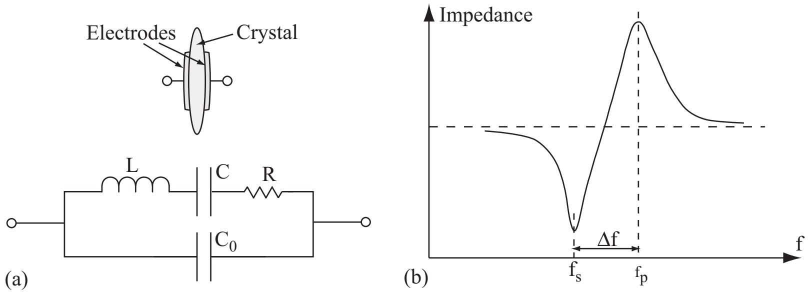


FIGURE 7.33 ■ The piezoelectric resonator. (a) The resonator and its equivalent circuit. (b) The two resonances.

resonance (also called antiresonance), shown in **Figure 7.33b**. These two resonant frequencies are given as

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad [\text{Hz}] \text{ (series resonance)} \tag{7.48}$$

and

$$f_p = \frac{1}{2\pi\sqrt{LC[C_0/(C+C_0)]}} \quad [\text{Hz}] \text{ (parallel resonance).} \tag{7.49}$$

In most applications a single resonance is desirable, and for these applications materials or geometries for which the two resonant frequencies are widely separated are used. To identify the frequency separation between the two resonances, a capacitance ratio is defined as

$$m = \frac{C}{C_0}. \tag{7.50}$$

With this, the relation between the two frequencies becomes

$$f_p = f_s(1+m) \quad [\text{Hz}] \tag{7.51}$$

Thus the larger the ratio m, the larger the separation between the two resonant frequencies.

The resistance R in the equivalent circuit does not figure in the resonance, but acts as a damping (loss) factor. This is associated with the quality factor (Q-factor) of the piezoelectric device, given as

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \quad [\text{C}]. \tag{7.52}$$

The Q-factor tends to infinity for zero resistance and is, by definition, the ratio between stored and dissipated energy in the crystal.

The importance of resonance is two-fold. First, at resonance, the amplitude of mechanical distortion is highest (in transmit mode), whereas in receive mode, the signal generated is largest, meaning that the sensor is most efficient at resonance. The second reason is that the sensors operate at clear and sharp frequencies and hence the parameters of propagation, including reflections and transmissions, are clearly defined, as are other properties such as wavelength.

The construction of a piezoelectric transducer intended for operation in solids or liquids is shown in **Figure 7.34**. The piezoelectric element is rigidly attached to the front of the sensor so that vibrations can be transmitted to and from the sensor. The front of the sensor is often just a thin flat metal surface, or it may be prismatic, conical, or spherical to focus the acoustic energy. **Figure 7.34a** shows a flat, nonfocusing coupling element. **Figure 7.34b** shows a concave, focusing coupling element. The damping chamber

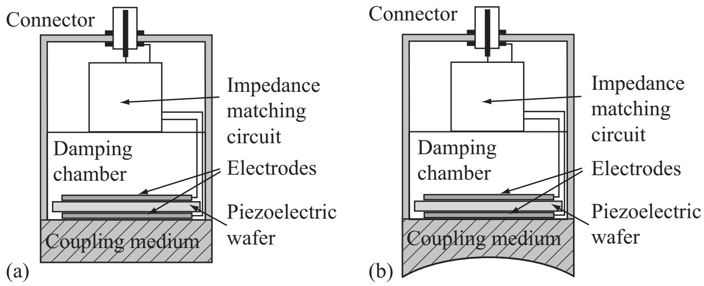


FIGURE 7.34 ■ Construction of an ultrasonic sensor. (a) Flat, nonfocusing sensor. (b) Concave, focusing sensor.

prevents ringing of the device, while the impedance matching circuit (not always present, sometimes it is part of the driving supply) matches the source with the piezoelectric element. Every sensor is specified for a resonant frequency, power, and for operational environment (solids, fluids, air, harsh environments, etc.). **Figure 7.35** shows a number of ultrasonic sensors for various applications and operating at various frequencies.

FIGURE 7.35 ■ A number of ultrasonic sensors. (Left to right) An industrial ultrasonic sensor operating at 175 kHz, a medical grade sensor operating at 2.25 MHz, an immersible sensor operating at 3.5 MHz, and a 15 MHz sensor with focusing lens used for testing of materials.

7.7.1 Pulse-Echo Operation

All ultrasonic sensors are dual—they can transmit or receive. In many applications, as, for example, in range finding, one can use two sensors (see **Figure 7.32**). In other applications the same sensor is used to transmit and to receive by switching between transmit and receive modes. That is, the sensor is driven to transmit an ultrasonic burst and then switched into receive mode to receive the echo reflected from any object the sound beam encounters. This is a common mode for operation in medical applications and in the testing of materials. The method is based on the fact that any discontinuity in the path of the acoustic wave causes reflection or scattering of the sound waves (see **Section 7.3.1**). The reflection is received and becomes an indication of the existence of the discontinuity and the amplitude of the reflection is a function of the size of the discontinuity. The exact location of the discontinuity can be found from the time it takes the waves to propagate to and from the discontinuity. This time is called time of flight. **Figure 7.36a** shows an example of finding the location/size of a defect in a piece of metal. The front and back surfaces manifest themselves as large reflections, whereas the defect usually produces a

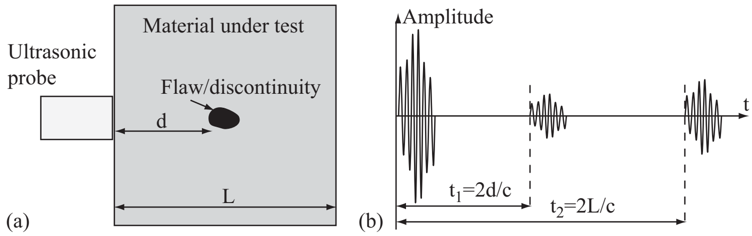


FIGURE 7.36 ■ Ultrasonic testing of materials. The echoes from various discontinuities can be detected and evaluated.

smaller signal. Its location can be easily detected. The same idea can be used to create an image of a baby in the womb, to sense a heartbeat, to measure blood vessel thickness and condition, for position sensing in industry, or in range finding. Using the configuration in **Figure 7.36**, the time it takes for the acoustic wave to reach the flaw is

$$t_1 = \frac{d}{c} \quad [\mathrm{s}]. \tag{7.53}$$

Since the ultrasonic probe receives the reflection after an additional time t_1 needed for the reflection to traverse the distance d, the signal is received back after a time $t_f = 2t_1$ and in the process the acoustic wave traverses a distance $2d$. The location of the flaw is calculated as

$$d = \frac{t_f}{2c} \quad [\mathrm{m}]. \tag{7.54}$$

Thus the location of the flaw or, for that matter, the thickness of the material may be inferred from the time of flight of the acoustic wave.

In addition to these important applications, ultrasonic sensors are useful in sensing other quantities, such as the velocity of a fluid. For this purpose there are three effects that can be used. One is the fact that sound velocity is relative to the fluid in which it travels. For example, voice carries downwind faster (by the wind velocity) than in still air. This speed difference can be measured as the time it takes the sound to get from one point to another since the speed of sound is constant and known. The second effect is based on the phase difference caused by this change in speed. The third is the Doppler effect—the frequency of the wave propagating downwind is higher than the frequency in still air or in a stagnant fluid. An example of a fluid speed sensor is shown in **Figure 7.37**. In this case, the distance and angle of the sensors are known and the transmit time downstream is

$$t = \frac{d}{c + v_f \cos\theta} \quad [\mathrm{s}], \tag{7.55}$$

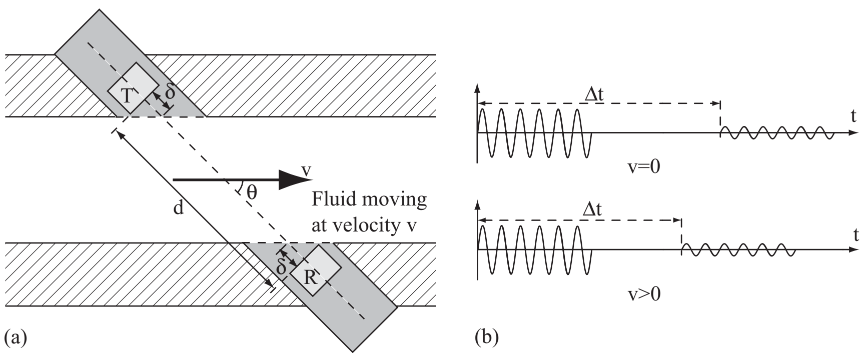


FIGURE 7.37 ■ A fluid velocity sensor. (a) The locations of the sensors. (b) The relation between transmitted and received signals.

where c is the sound velocity in the fluid and v_f is the speed of the fluid. The fluid velocity is

$$v_f = \frac{d}{t\cos\theta} - \frac{c}{\cos\theta} \quad [\mathrm{m/s}]. \tag{7.56}$$

All terms in **Equation (7.56)** except t are known constants, so by measuring the time of flight t, the velocity is immediately available. An alternative method that is often used is based on the Doppler effect. We will discuss the Doppler effect again in **Chapter 9** in connection with radar, but the effect can be used with ultrasound as well. The basic idea is that as the wave propagates in the direction of the flow, the net speed of the wave increases by a velocity Δv. Therefore the signal arrives at the receiver sooner than it would otherwise. In effect, this means that the frequency is higher. Assuming a signal at a fixed frequency f is transmitted, the received frequency is

$$f' = \frac{f}{1 - v_f \cos\theta/c} \quad [\mathrm{Hz}]. \tag{7.57}$$

The fluid velocity is

$$v_f = c\frac{f' - f}{f' \cos\theta} \quad [\mathrm{m/s}]. \tag{7.58}$$

As can be seen, the change in frequency is a direct measure of fluid velocity.

Naturally, if the receiver were to be placed upstream rather than downstream, the frequency would be lower (the negative sign in **Equation (7.57)** becomes positive). The advantage of the Doppler method is in the fact that frequency is easier to measure accurately. With a constant frequency f, the method can be very accurate.

EXAMPLE 7.10 Doppler ultrasound sensing of water flow

To see the frequency levels and changes in frequency involved in a Doppler ultrasound fluid velocity sensor, consider **Figure 7.37** as a guide. The transmitter is upstream and the receiver downstream. The transmitter operates at 3.5 MHz and the sensor is at 45° to the flow. The sound velocity in water is 1500 m/s.

a. Calculate the change in frequency of the sensor for a fluid speed of 10 m/s.
b. Calculate the sensitivity of the sensor in hertz per meter per second (Hz/m/s).

Solution:
a. From **Equation (7.57)**, the change in frequency is

$$\Delta f = f' - f = \frac{f}{1 - v_f \cos\theta/c} - f = \frac{3.5 \times 10^6}{1 - 10\cos 45°/1500} - 3.5 \times 10^6$$

$$= 3.516577 \times 10^6 - 3.5 \times 10^6 = 16{,}577 \text{ Hz}.$$

This is a relatively large change in frequency and is easily measurable by a number of means, including a microprocessor (see **Chapter 12**).

b. The sensitivity is the change in frequency (output) over the change in fluid velocity. We write

$$\frac{df'}{dv_f} = \frac{d}{dv_f}\left(\frac{f}{1 - v_f \cos\theta/c}\right) = \frac{d}{dv_f} f\left(1 - v_f \cos\theta/c\right)^{-1}$$
$$= -f\left(1 - v_f \cos\theta/c\right)^{-2}(-\cos\theta/c) = \frac{f \cos\theta/c}{\left(1 - v_f \cos\theta/c\right)^2} \quad [\mathrm{Hz/m/s}]$$

Note that this relation looks nonlinear and seems to increase with velocity. However, the term $v_f \cos\theta/c$ is very small and hence the term in brackets in the denominator calculated for a fluid velocity of 10 m/s is

$$1 - v_f \cos\theta/c = 1 - 10\frac{\sqrt{2}}{2 \times 1500} = 0.9953.$$

This means that we can calculate a very good numerical approximation that would work for all velocities except for the unlikely case of fluid velocities that approach the sound velocity in the fluid. Taking the value above, the sensitivity is

$$\frac{df'}{dv_f} = \frac{f \cos\theta/c}{\left(1 - v_f \cos\theta/c\right)^2} = \frac{3.5 \times 10^6 \times \left(\sqrt{2}/2\right)/1500}{(0.9953)^2} = 1665.54 \ \mathrm{Hz/m/s}.$$

This result is consistent with the result in (a), but it is only an approximation. By multiplying this by 10 we get 16,655 Hz instead of the 16,577 Hz we got in (a), for an error of 0.4%. Of course, the general result is more accurate than the numerical approximation.

The properties described above have also been used for other important applications. For example, the sonar used by surface ships and submarines is essentially a pulse-echo ultrasound method. The main difference is that the power involved is very large to allow long-distance sensing. It also relies on the very good propagation qualities of water. In medical applications, ultrasound is often used to sense motion, such as the motion of veins (blood pressure) or of heart valves, to detect abnormal conditions. Another useful application is to break apart kidney stones. In this case high-intensity bursts are applied to the body while it is immersed in water (the transducer is an actuator). The stones are pulverized and can then pass with the urine.

7.7.2 Magnetostrictive Transducers

For operation in air or in fluids, piezoelectric sensors seem to be the best. However, in solids there is an alternative method, based on magnetostriction, that can be much more effective. One can imagine that by applying a pulse to a magnetostrictive bar, it constricts and expands alternately to “bang” on the solid just like a hammer. These sensors are collectively called magnetostrictive ultrasonic sensors and they are used at lower frequencies (about 100 kHz) to generate higher intensity waves.

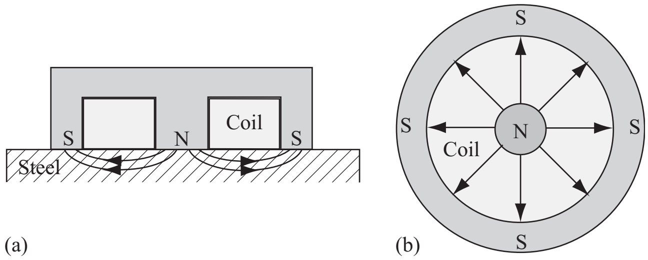


FIGURE 7.38 ■ Construction of an EMAT. (a) Side view showing the permanent magnet field. (b) Bottom view.

If, however, the ultrasound is to be coupled into a magnetostrictive material, all that is necessary is to attach a coil to the material and drive it at the required frequency. The field generated in the material itself generates stresses in the material, which in turn generates an ultrasonic wave (just like generating stress in the earth's crust generates earthquakes). The reason this type of actuator is important is that iron is magnetostrictive and hence the method can be used to generate ultrasound waves in iron and steel products for the purpose of integrity testing.

This principle is implemented as follows: A coil driven by AC (or pulses) generates induced electric currents (eddy currents) in the magnetostrictive material. A magnetic field produced by an external permanent magnet produces a force acting on these currents. The interaction between the magnetic field and the eddy currents generates stresses and an acoustic wave ensues. These devices are called electromagnetic acoustic transducers (EMATs). As with other acoustic transducers, they are dual function and can sense acoustic waves as well as generate them. **Figure 7.38** shows a schematic EMAT. EMATs are commonly used for nondestructive testing and evaluation of steel because of their simplicity, but they tend to operate at low frequencies ($<$100 kHz) and have relatively low efficiencies.

7.8 | PIEZOELECTRIC ACTUATORS

We have seen that piezoelectric sensors can act as actuators as used in transmitters for ultrasound. But piezoelectric devices can be used in more direct types of actuators to affect motion. One of the first types of piezoelectric actuator has been in use in analog clocks for decades. It consists of a crystal that oscillates at a fixed frequency to serve as a time reference for the clock. But other actuators have been designed that can move much larger distances and apply significant forces as well. One such device is shown in **Figure 7.39**. It is a thin, stiff plate, with the piezoelectric material bonded to it

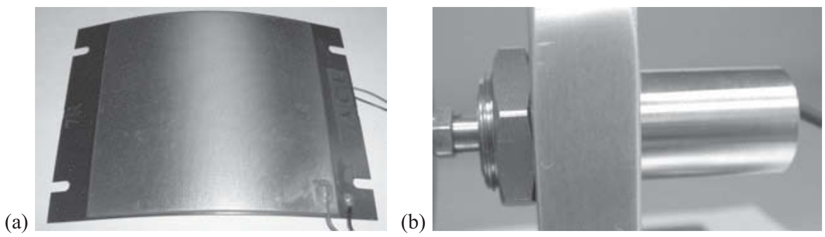


FIGURE 7.39 ■ (a) A large-displacement, rectangular, piezoelectric actuator. (b) A cylindrical, stacked, piezoelectric actuator. It can only move about 0.05 mm but can deliver about 40 N of force. The moving shaft is seen on the left pushing against a workpiece.

(gray patch). When a voltage is applied across the piezoelectric element, one edge moves relative to the other (one edge, say the left edge, must be fixed). The motion is accompanied by force and this force can be utilized for actuation. Note, however, the high voltage needed. Although some piezoelectric sensors and actuators can operate at lower voltages, high voltages are typical of piezoelectric actuators and are one serious limitation for their widespread use.

Other approaches to piezoelectric actuators are to stack individual elements, each with its own electrodes to produce stacks of varying lengths. In such devices, the displacement is anywhere between 0.1% and 0.25% of the stack length, but this is still a small displacement. One of the advantages of these stacks is that the forces are larger than those achievable by bending plates such as the one in **Figure 7.39a**. A small actuator, capable of a displacement of about 0.05 mm and a force of about 40 N, is shown in **Figure 7.39b**.

EXAMPLE 7.11 The ultrasonic motor

The ultrasonic motor is an interesting and useful actuator, originally developed for autofocus lenses in cameras. It consists of a simple metal disk (the rotor) with a second piezoelectric disk below it. The piezoelectric disk is toothed, allowing it to flex, and as it does so it moves the rotor. To generate the wave motion of the stator disk it is necessary to generate two standing waves of equal amplitude (a standing wave is a motion, say up and down of the ring, similar to an ocean wave). A standing wave cannot generate motion just like an ocean wave, which can only move a body up and down. However, if two standing waves, 90° out of phase in space and time are generated, their sum is a travelling wave whose direction of motion depends on the frequency of the two waves and on the mode of excitation. **Figure 7.40** shows the operation in three steps, highlighting a single tooth. The wave propagates to the right (counterclockwise [CCW]) and the marked tooth first touches the rotor with the back edge since it is slightly inclined to the right (**Figure 7.40a**). As the wave propagates, the tooth straightens (**Figure 7.40b**), pushing the rotor to the left (clockwise [CW]). In **Figure 7.40c** the tooth bends to the left, pushing the disk further to the left (CW). The tooth therefore describes an elliptical path as it moves up and down, making contact with the disk during part of the cycle and causing the rotation of the disk. In this configuration, with the undulation propagating to the right, the rotor rotates in the CW direction (viewed from the top). Changing the direction of undulation reverses the direction of rotation.

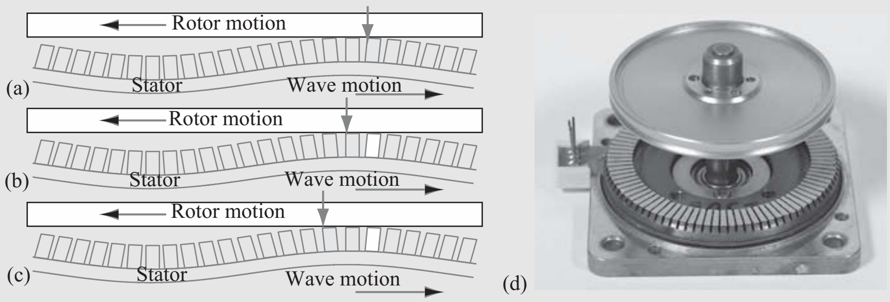


FIGURE 7.40 ■ Sequence of motion in an ultrasonic motor. (a) The leading edge of a tooth touches the rotor. (b) The motion of the rotor moves the stator to the left. (c) The trailing edge of the third tooth disengages from the rotor. The sequence has moved the rotor to the left. (d) A commercial ultrasonic motor showing the rotor (lifted above the stator) and the toothed stator. The piezoelectric segments are bonded to the bottom of the stator.

The advantages of this motor are many, including very small size, rotation speed that can be controlled directly through the propagating wave, and significant torque. It is a friction-driven motor and hence it has considerable holding torque. The ultrasonic standing waves are generated by applying high-frequency electric fields to piezoelectric strips bonded to the bottom surface of the stator. The piezoelectric strips are driven in sequence to generate the standing waves. The motor is small in size, relatively fast, does not require gearing (direct drive), and is quiet, some of the properties that made it so useful in autofocus lenses as well as other applications.

The implementation and control of motion is rather simple. The two waves are generated by applying an electric field at two or more opposite locations on the stator (i.e., the out of phase in space requirement) and, at the same time, the two locations are driven with electric fields that are 90° out of phase in time. The two waves look as follows:

$$u_1(\theta, t) = A \cos \omega t \cos n\theta$$

$$u_2(\theta, t) = A \cos(\omega t + \pi/2) \cos(n\theta + \pi/2),$$

where n is the nth mode of oscillation of the stator (n is the number of peaks in the standing wave pattern produced in the stator, can be any integer from 1 to infinity and can be controlled by the number of locations at which the stator is excited). These two waves add up to

$$\begin{aligned} u_1(\theta, t) + u_2(\theta, t) &= A \cos \omega t \cos n\theta + A \cos(\omega t + \pi/2) \cos(n\theta + \pi/2) \\ &= A \cos \omega t \cos n\theta + A[\cos(\omega t) \cos(\pi/2) - \sin(\omega t)\sin(\pi/2)] \\ &\quad \times [\cos(n\theta) \cos(\pi/2) - \sin(n\theta)\sin(\pi/2)] \\ &= A \cos \omega t \cos n\theta + A[-\sin(\omega t)][-\sin(n\theta)] \\ &= A \cos \omega t \cos n\theta + A\sin(\omega t)\sin(n\theta) = A \cos(\omega t - n\theta). \end{aligned}$$

The speed of propagation of this wave (more precisely, its phase velocity) is $v = \omega/n$ [m/s], as can be seen from **Equation (7.11)**:

$$v = \frac{\omega}{n} = \frac{2\pi f}{n} \quad [\text{m/s}].$$

The rotation of the rotor is due to the vibration of the stator teeth that make contact with the rotor and generate motion (see **Figure 7.40**). The velocity of the rotor is not the same as the phase velocity of the wave and depends on displacement of the teeth (and hence on the current in the piezoelectric elements), on the load, and on the mode of vibration. In general, the higher the mode and the larger the radius of the stator, the slower the rotational speed of the motor.

Multiplying the velocity by 1 s gives the distance travelled by the wave in 1 s. Dividing this result by the circumference of the motor gives the number of rotations per second (rps):

$$v_r = \frac{2\pi f \times (1\ \text{s})}{2\pi r n} = \frac{1}{rn} \quad [\text{rps}],$$

where r is the radius of the stator. Note that the speed of the motor is independent of the speed of the wave—it depends on the vibration speed, which tends to be constant. As an example, a motor of radius 2 cm operating in the fundamental mode ($n = 1$) at a frequency of 30 Hz will rotate at a speed of

$$v_r = \frac{1}{0.02} = 50\ \text{rps}.$$

That is, 3000 rpm. The same motor, with eight pairs of excitation locations would operate in the eighth mode and would rotate at 375 rpm.

This means that one can control the speed of rotation by changing the mode of oscillation through introduction of additional generation points on the circumference of the stator. Note also that if we change the phase from $+\pi/2$ to $-\pi/2$, the wave will propagate in the opposite direction and the motor will turn in the opposite direction.

EXAMPLE 7.12 Linear piezoelectric actuator

A simple linear actuator is made by stacking alternating conducting disks and piezoelectric disks as shown in **Figure 7.41**. There are N piezoelectric disks, each of thickness t and radius a, and $N+1$ conducting disks ($N = 5$ in the figure). The conducting disk's whole purpose is to apply an external voltage to generate an electric field intensity in the piezoelectric disks. Given the properties of the piezoelectric material (relative permittivity ε_{ii} and piezoelectric constant d_{ii}),

a. Calculate the displacement of the stack for an applied voltage V.
b. Calculate the force the stack can generate for an applied voltage V.
c. Calculate the displacement and force for a 3-3 cut barium titanate oxide ($BaTiO_3$) piezoelectric with dimensions $a = 10$ mm, $t = 1$ mm, and $V = 36$ V for a stack with $N = 40$ disks.
d. What is the maximum possible displacement and force if the breakdown electric field in the crystal is 32,000 V/mm whereas the breakdown voltage in air is 3000 V/mm?

Solution: The displacement is calculated directly from the piezoelectric coefficient d_{ii}, whereas the force is calculated from **Equation (7.40)**. We start with the displacement.

a. By definition, the piezoelectric constant is the strain per unit electric field and strain is the ratio of displacement divided by length. We write for a disk of thickness t,

$$\frac{dt}{t} = d_{ii}E = d_{ii}\frac{V}{t} \quad [\text{m/m}],$$

where $E = V/t$ is the electric field intensity in the disk produced by the potential difference on the disk, V. Therefore the displacement, or the change in thickness of the disk, is dt:

$$dt = d_{ii}V \quad [\text{m}].$$

The total change in the length of the stack is therefore Ndt:

$$\Delta l = Ndt = Nd_{ii}V \quad [\text{m}]$$

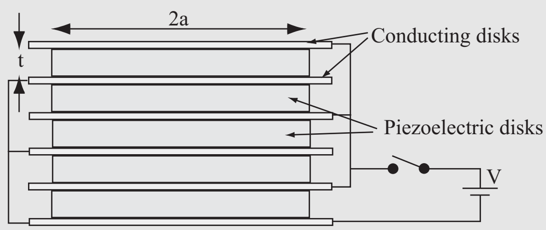


FIGURE 7.41. ■ A piezoelectric stack actuator.

b. The force is calculated from **Equation (7.40)** after rewriting the equation:

$$F = \frac{\varepsilon_{ii} A V}{t d_{ii}} \quad [\mathrm{N}],$$

where A is the surface area of the base of the disk (πa^2) and ε_{ii} is its permittivity. This is the force produced by one disk. All other disks produce an identical force, but since the disks are in series, the total force of the N disk stack is the same as that of a single disk.

c. For the given properties and dimensions, we have

$$\Delta l = N d_{ii} V = 40 \times 100 \times 10^{-12} \times 36 = 0.144\ \mu\mathrm{m}$$

and

$$F = \frac{\varepsilon_{ii} A V}{t d_{ii}} = \frac{180 \times 8.854 \times 10^{-12} \times \pi \times 0.01^2 \times 36}{10^{-3} \times 100 \times 10^{-12}} = 180.25\ \mathrm{N}.$$

Note the typical characteristics of piezoelectric actuators: small displacement, but large forces.

d. The maximum electric field intensity is the breakdown electric field intensity. In this case it is 32,000 V/mm and requires a potential difference of 32,000 V. However, in air the electric field intensity is only 3000 V/mm. It is impossible to raise the voltage of the stack above 3000 V/mm because at that voltage difference there will be breakdown in air. Therefore the maximum electric field intensity is 3000 V/m or 3×10^6 V/m. We have for the maximum possible displacement,

$$\Delta l_{\max} = N d_{ii} V = 40 \times 100 \times 10^{-12} \times 3 \times 10^6 = 12,000\ \mu\mathrm{m}.$$

This is 12 mm, and as a theoretical result it looks reasonable. But in fact this would require a strain of 30%, and that is not possible in a real material. Perhaps 1/10 of that or a total displacement of 1.2 mm seems reasonable (a strain of 3% is certainly possible). To obtain this will require a 300 V potential difference between the plates.

The maximum theoretical force possible is

$$F = \frac{\varepsilon_{ii} A V}{t d_{ii}} = \frac{180 \times 8.854 \times 10^{-12} \times \pi \times 0.01^2 \times 3000}{10^{-3} \times 100 \times 10^{-12}} = 15{,}020\ \mathrm{N}.$$

Again, 1/10 of this, or 1500 N, may be more reasonable, obtainable with a 300 V potential difference.

7.9 | PIEZOELECTRIC RESONATORS AND SAW DEVICES

In **Section 7.7** we discussed ultrasonic sensors and in **Section 7.3** the theory of sound waves. Most of this was based on the idea of the generation and propagation of longitudinal waves and their interaction with materials and the environment. Sound waves in air and in fluids are essentially longitudinal waves, but under appropriate conditions other waves

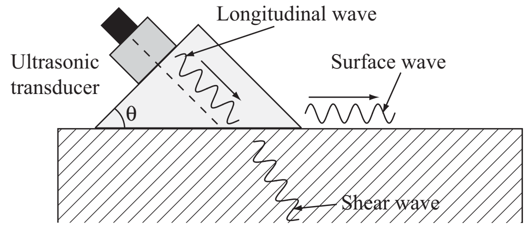


FIGURE 7.42 ■ Conversion of a longitudinal wave into a surface wave by means of a wedge.

may be generated. We saw that solids can support shear waves (**Section 7.3.2**) and the surface between solids and air can support surface waves (**Section 7.3.3**). Surface waves are of particular interest because of their slow speed of propagation and low dispersion. Under most conditions the slow propagation speed of surface waves would seem to be a disadvantage, but looking at the wavelength alone as the ratio of velocity and frequency, $\lambda = c/f$, it is clear that the lower the velocity of the wave, the shorter the wavelength in that medium. This means, for example, that if a device must be, say, one-half wavelength in size, then the same device utilizing surface waves will be physically smaller than if it were to utilize longitudinal waves. It is this property that is at the heart of SAW devices.

Surface waves can be generated in a number of ways. In a thick sample, one can set up a surface wave by a process of wave conversion. Essentially, a longitudinal wave device is used and energy coupled through a wedge at an angle to the surface. At the surface of the medium there will be both a shear wave and a surface wave (**Figure 7.42**). This is an obvious solution, but not necessarily optimal. A much more efficient method of generating surface waves, one that is almost ideally suited for fabrication is to apply metallic strips on the surface of a piezoelectric material in an interdigital fashion (a comblike structure) as shown in **Figure 7.43**. This establishes a periodic structure of metallic strips. When an oscillating source is connected across the two sets of electrodes, a periodic electric field intensity is established in the piezoelectric material, equal to the periodicity of the electrodes and parallel to the surface. (The period is equal to the distance between each two electrodes and the latter are designed so that each strip is $\lambda/4$ wide and the gap between strips is also $\lambda/4$ wide.) Because of this electric field, an equivalent, periodic stress pattern is established on the surface of the piezoelectric medium. This generates a stress wave (sound wave) that now propagates away from the electrodes in both directions. The generation is most efficient when the period of the surface wave equals the interdigital period. For example, in the structure in **Figure 7.44**, suppose the frequency of the source is 400 MHz. The speed of propagation in a piezoelectric is on the order of 3000 m/s. This gives a wavelength of 7.5 μm. Making each strip in the structure $\lambda/4$ means each strip is 1.875 μm wide and the distance between neighboring strips is 1.875 μm. This calculation shows first that the dimensions required are very small (a device at the same frequency, based on electromagnetic waves, has a

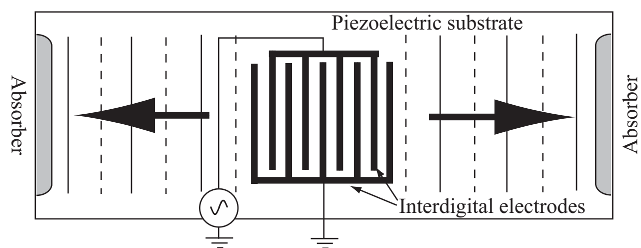


FIGURE 7.43 ■ Generation of a SAW by a series of periodic surface electrodes driven from a resonant source.

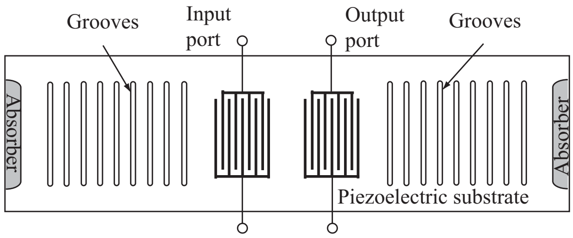


FIGURE 7.44 ■ Construction of a SAW resonator.

wavelength of 750 mm). Second, it shows that production of these devices can be accomplished using lithographic techniques that are compatible with semiconductor production methods.

Returning now to the basic structure, just as the comblike structure generates surface acoustic waves, and hence stress in the piezoelectric medium, an acoustic wave in the piezoelectric medium produces a signal in a comblike structure because of the surface stress produced by the acoustic wave. Thus the structure can be used both for generation and sensing of surface waves. That, in turn, means that the device can be used for sensing or actuation.

By far the most common use of SAW principles is in SAW resonators, filters, and delay lines. A SAW resonator is shown in **Figure 7.44**. The sections marked as *In* and *Out* are used as the input and output ports of the resonator (i.e., the external connections of the resonator). The parallel lines on each side are grooves etched in the quartz piezoelectric. The input port establishes a surface wave, which is reflected by the grooves on each side. These reflections interfere with each other, establishing a resonance that depends on the separation of the grooves in the grating. Only those signals that interfere constructively will establish a signal in the output port, the others cancel.

This device has quickly become popular as the basic element for oscillators in communication systems since a very small device can easily operate at low frequencies and at the other extreme can operate at frequencies above the limit of conventional oscillators, including crystal oscillators. **Figure 7.46** shows a numbere of SAW resonators used in low power transmitters. The device in **Figure 7.44** may also be viewed as a very narrow bandpass filter and this is in fact another of its uses.

The configuration in **Figure 7.45** is a SAW delay line. The comb on the left generates a surface wave and this is detected after a delay in the comb on the right. The delay depends on the distance between the combs and, because the wavelength is usually small, the delay can be relatively long.

In addition to these important applications, SAW devices can be used for sensing of almost any quantity, taking advantage of the properties of the piezoelectric medium. For example, the application of stress to the piezoelectric changes the speed of sound in the

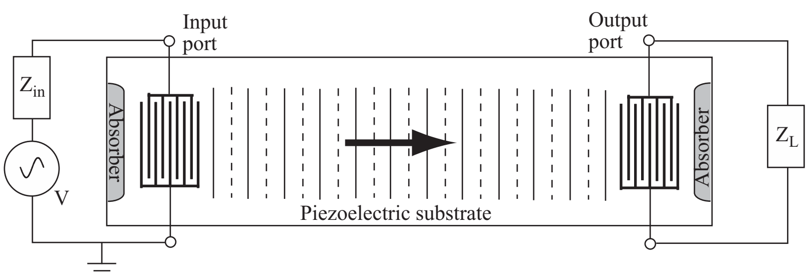


FIGURE 7.45 ■ A SAW delay line.

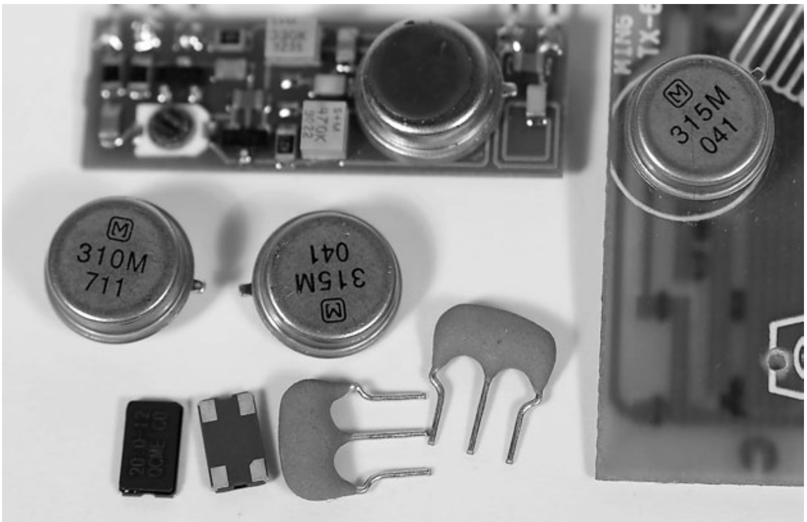


FIGURE 7.46 ■ SAW resonators used in transmitters and receivers. The device covered with the dark label resonates at 433.92 MHz (shown soldered in a transmitter). The metal can resonators are for 310 and 315 MHz applications. The surface mount devices (bottom left) and the three-pin devices (bottom center) resonate at 433.92 MHz.

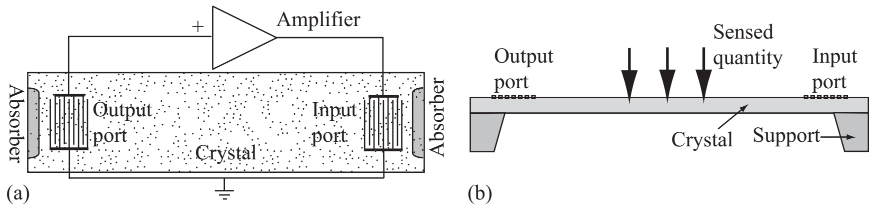


FIGURE 7.47 ■ The basic structure of a SAW sensor based on the delay line.

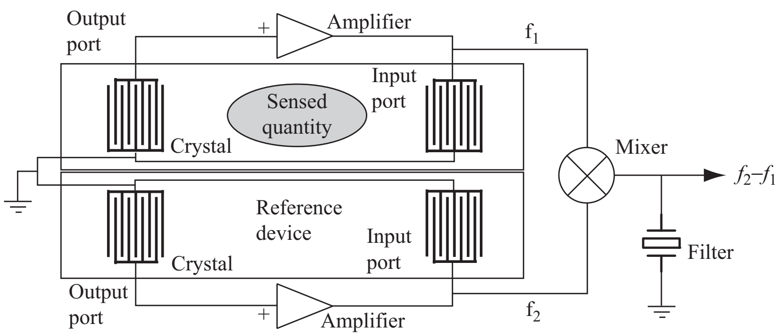


FIGURE 7.48 ■ A compensated SAW resonator. One delay line is used for sensing, the second for compensation of common mode effects such as temperature or pressure.

material. This in turn will change, say, the resonant frequency of the device in **Figure 7.44** or the delay in **Figure 7.45** and by so doing sense force, pressure, acceleration, mass, and a number of other related quantities.

The basic SAW sensor is shown in **Figure 7.47** and is based on a delay line in which the delay is influenced by the stimulus. An essentially identical sensor is shown in **Figure 7.48**. It has two identical delay lines and the output is differential. One line is used as the proper sensor, the second as a reference to cancel common mode effects such as temperature. In most cases the delay time is not measured, but rather a feedback amplifier (**Figure 7.48**) is connected (positive feedback) that causes the device to resonate at a frequency established by the time delay between the two ports. The sensed quantity is measured through measurement of the frequency of resonance.

The stimuli that can be measured are many. First, the speed of sound is temperature dependent. Temperature changes both the physical length of the delay line and the sound speed as follows:

$$L = L_0[1 + \alpha(T - T_0)] \quad [\mathrm{m}], \quad c = c_0[1 + \delta(T - T_0)] \quad [\mathrm{m/s}], \tag{7.59}$$

where α is the coefficient of linear expansion and δ is the temperature coefficient of sound velocity.

Both the length and the speed of sound increase with temperature, hence the delay and oscillator frequency are a function of the difference between them. In fact, the change in frequency with temperature is

$$\frac{\Delta f}{f} = (\delta - \alpha)\Delta T. \tag{7.60}$$

This is linear and a SAW sensor has a sensitivity of about 10^{-3}°C.

In sensing pressure, the delay in propagation is due to stress in the piezoelectric, as indicated above. Measurement of displacement, force, and acceleration are done by measuring the strain (pressure) produced in the sensor. Many other stimuli can be measured, including radiation (through the temperature increase), voltage (through the stress it produces through the electric field), and so on. **Equations 7.59** and **7.60** indicate a linear relationship between the change in frequency and the change in length (in this case due to temperature). That means that if the length increases, say, by 1%, the frequency must necessarily decrease by 1%. This can be used to sense any quantity that would change the length of the sensor. Given a change in length Δl, we can write

$$\frac{\Delta f}{f} = \frac{\Delta l}{l}, \tag{7.61}$$

where f is the frequency at length l. Note that the right-hand side of **Equation (7.61)** is the strain in the medium and that it can be related to pressure, force, acceleration, or mass.

We will discuss additional applications of SAW devices in chemical sensing in **Chapter 8**.

EXAMPLE 7.13 Sensitivity of a SAW pressure sensor

A pressure sensor is built with a SAW resonator serving as a beam (see **Figure 7.47b**) that resonates at 500 MHz. The maximum strain allowed in the beam is 1000 microstrain, corresponding to 10^6 Pa (9.87 atm).

a. Calculate the maximum change in frequency of the sensor.
b. Suppose that the temperature changes by 1°C. Calculate the error introduced by the change in temperature at 100 kPa if the temperature sensitivity is 10^{-4}/°C.

Solution:

a. The strain, by definition, is the change per unit length. That is,

$$\frac{\Delta l}{l} = 1000 \times 10^{-6} = 10^{-3}.$$

Therefore the change in frequency is

$$\frac{\Delta f}{f} = \frac{\Delta l}{l} = 10^{-3} \rightarrow \Delta f = 500 \times 10^{6} \times 10^{-3} = 500 \times 10^{3} \text{ Hz}.$$

This is a very large change in frequency with a sensitivity of 500 Hz/microstrain.

b. Since the sensor is linear and the temperature sensitivity is 10^{-3}/°C, we can write from **Equation (7.61)**:

$$\Delta f = 500 \times 10^{6} \times 10^{-4} = 5 \times 10^{4} \text{ Hz}.$$

At 100 kPa, the change in frequency due to pressure is only 50 kHz. That is, a change of 1°C causes a change in frequency equal to the change due to pressure. Obviously, unless the sensor is temperature compensated, it cannot be used for sensing. It is for this reason that the configuration in **Figure 7.48** is so important.

7.10 | PROBLEMS

Units

7.1 Acoustic pressure at a distance from a source. A jet engine on the ground generates a sound power density level of 155 dB at a distance of 10 m from the engine. Assume that the sound travels uniformly in all directions in the space above the ground. Neglect attenuation of sound waves in air.

a. What is the shortest safe distance for an operator without hearing protection?

b. What is the shortest safe distance if the operator uses hearing protection rated at 20 dB?

7.2 Stress and strain produced by an ultrasonic actuator. An ultrasonic actuator is used to test steel for cracks. To do so it produces a pressure of 1000 Pa on the surface of the steel. Calculate the stress and strain produced by the actuator at the contact surface if the modulus of elasticity of steel is 198 GPa. Comment on the effect this might have on the material.

Elastic waves and their properties

7.3 Speed of propagation of sound waves. Popular wisdom says that when you see lightning, start counting slowly (it is assumed that each count is a second). The number you reach by the time you hear the thunder is three times the distance to the lightning strike in kilometers. How accurate is this estimate?

7.4 Ultrasonic testing of materials. Ultrasonic testing of materials for flaws is an established method often used in industry, especially in metals, looking for flaws, cracks, thinning due to corrosion, inclusions, and other effects that can be detrimental to the functioning of a structure. It relies on reflection of ultrasonic waves from discontinuities within the structure. The resolution, that is, the smallest detail that can be "seen" by the waves, depends on frequency. For that reason, ultrasonic testing is done at relatively high frequencies. For flaws to be detectable they must be on the order of the wavelength of the acoustic wave. Consider the testing of titanium blades in jet engines for small cracks. Since these tend to initiate larger flaws followed by failure, the test must detect cracks smaller than 0.5 mm.

a. Given the speed of propagation of sound waves in titanium as 6172 m/s, what is the lowest sound frequency that will detect these flaws?
b. Discuss the consequences of using higher frequencies in ultrasonic sensors, especially for very small flaws.

7.5 **Range of normal speech—the ear as a sensor.** The sound intensity of human speech varies from about 10^{-12} W/m^2 (very faint whisper) to about 0.1 W/m^2 (loud scream). The human ear can detect sounds as faint as 10^{-12} W/m^2, but to understand a conversation requires at least 10^{-10} W/m^2. Normal conversation is considered to be 10^{-6} W/m^2. Suppose a person speaks normally, producing a power density of 10^{-6} W/m^2 at a distance of 1 m.

a. What is the absolute longest possible distance a person can be heard and understood assuming that the sound can be directed to the listener without spreading in space (such as speaking through a tube) and there are no losses in the path other than attenuation of sound waves in air? Use the attenuation data in **Example 7.3**.
b. What is the distance the person can be heard and understood assuming the sound waves propagate uniformly in all directions and neglecting attenuation?

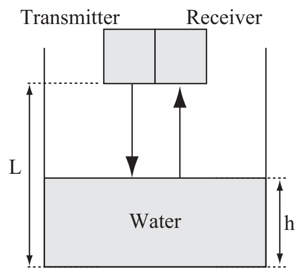
7.6 **Fishing sonar.** The attenuation constant for sound waves in water increases with frequency. Consider the sonar used for fishing. The reflection of sound from fish schools is used for sports and commercial fishing. Fish produce relatively large reflections because of their air sack, the reflection being due to the interface between flesh and air, whereas the reflection at the interface between flesh and water is small. The signal is generated by an ultrasound actuator and detected by an ultrasound sensor. Suppose the actuator produces a power of 0.1 W at 50 kHz (a common frequency in fishing sonars) that spreads in a 30° cone as it propagates into water. Assuming that 20% of the sound reaching a fish school is reflected and that reflected sound is scattered uniformly over a half-sphere, calculate the required sensitivity of the ultrasonic sensor (in W/m^2) to detect a fish school at a depth of 20 m. Attenuation in water at 50 kHz is 15 dB/km.

7.7 **Attenuation of ultrasonic waves in water.** Much work with ultrasound is conducted in water or in conjunction with water (sonar, diagnostics in the body, ultrasonic cleaning, treatment for kidney stones, and others). The selection of the frequency of operation is critical for the successful implementation of ultrasonic systems and one parameter of importance is the attenuation at the selected frequency. As a rule, the higher the frequency, the higher the attenuation, and also the better the resolution. The attenuation of ultrasound in water at frequencies above 1 MHz is approximately: $\alpha = 0.217f^2$ [dB/m], where f is the frequency in megahertz. Suppose an ultrasonic test is applied for diagnostics deep into the body and requires a spatial resolution of 1 mm. The spatial resolution of ultrasound is approximately 1 wavelength. Assuming that the waves should be able to image an artifact 2 mm in diameter at 15 cm depth and the speed of sound in water is approximately 1500 m/s, calculate the lowest frequency that can be used and the minimum power the ultrasonic transmitter must transmit to receive back 10 μW. The artifact is a small bone fragment embedded in the soft tissue due to a fracture. Assume the beam produced by the transmitter is collimated in a cylinder of diameter 20 mm, equal to the diameter of the sensor, and the reflected waves scatter in all directions equally.

7.8 **Water level detection.** Water level can be detected and accurately measured using ultrasonic waves based on the reflection of waves at the surface. Given a fixed ultrasonic transmitter above the surface of the water, the measurement of time of flight is sufficient to sense the surface of the water (**Figure 7.49**). The properties of ultrasonic waves in air and water are given in **Table 7.1**.

a. Given the amplitude of the pulse transmitted as A, show that the amplitude of the pulse received by the receiver, B, only depends on the distance traveled in air and its properties and is not affected by the properties of water. Find the relation needed to sense water level based on the measurement of amplitude.
b. Find a relation between the water level h and the time of flight of the ultrasonic wave measuring the start of the pulse in the transmitter and that in the receiver.
c. Explain why the method in (b) is preferable.

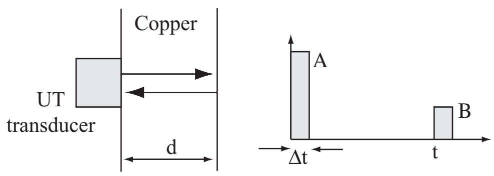
FIGURE 7.49 ■ Water level sensor.



7.9 **Pulse-echo ultrasonic testing.** In a pulse-echo ultrasound test, the transmitter serves as the receiver during the time the pulse is off. The transmitter sends a pulse of width Δt, at the end of which it is ready to receive. The test is applied to measure the thickness of a copper slab by detecting the reflected pulse from the far surface (**Figure 7.50**). The properties of copper are as follows: speed of propagation 4600 m/s, attenuation constant 0.45 dB/cm, acoustic impedance 42.5×10^6 kg/m^2/s. The pulse generated during transmission is 200 ns wide.

a. What is the thinnest slab that can be tested using the pulse-echo method described here and what is the maximum frequency of a train of pulses intended to repeat the process indefinitely?

FIGURE 7.50 ■ Pulse-echo testing of materials.



b. Calculate the amplitude of the reflected pulse received by the transducer for the conditions in (a), given an amplitude of the transmitted wave, V_0.
Note: The application of a pulse to an ultrasonic transducer generates a series of sinusoidal waveforms as the transducer oscillates at its resonant frequency. However, for simplicity we will assume here that the pulse propagates and reflects as a pulse.

c. Find a relation between the thickness of the slab and the amplitude of the reflected pulse.

d. Find a relation between the thickness of slab and the time it takes to receive the pulse.

7.10 Stealth submarines. The reflection of sound waves off the hull of a submarine is one of the most important methods of their detection (another is the disturbance they create in the local terrestrial magnetic field). To avoid detection, submarines are coated with rubber (or rubberlike substances). A sonar operates at 10 kHz and produces a signal of power $P_0 = 1$ kW transmitted uniformly in a 5° circular cone, used to detect a submarine at a depth of 300 m. Assume the reflection off the submarine also propagates back in a 5° circular cone and that the power that penetrates into the rubber coating is dissipated within the coating. The top surface area of the submarine is 65 m^2.

a. Calculate the ratio of power received by the sonar from a steel hull submarine and that of a rubber-coated submarine. Is this an effective way of reducing the visibility of submarines?

b. In theory, the reflection off submarines can be reduced to zero. Explain the requirements of the coating to achieve this.

Resistive and magnetic microphones

7.11 A resistive microphone. Consider the microphone in **Figure 7.51**. It consists of conducting particles suspended in a lightweight foam. The particles are made of low-conductivity particles with a conductivity $\sigma_c = 1$ S/m, whereas the foam can be considered nonconducting. When no pressure is applied, the particles occupy 50% of the total volume. The foam has a restoring constant (spring constant) of 0.1 N/m. The conductivity of the combined foam and particles equals the conductivity of carbon multiplied by the ratio of the volume of carbon to total volume:

$$\sigma = \frac{\sigma_c V_c}{V_t} \quad [\text{S/m}].$$

To operate, a current is established through the microphone as shown and the current is then a measure of sound pressure. Normal speech produces pressures

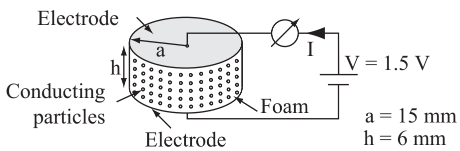


FIGURE 7.51 ■ A simple resistive microphone.

varying between 40 dB (very soft) and 70 dB (very loud). Calculate the range of current in the circuit for the pressure range in normal speech.

7.12 **Loudspeaker used as a dynamic microphone.** A small loudspeaker is used both as a microphone and as a loudspeaker in an intercom system. The loudspeaker has a cone 60 mm in diameter, an 80 turn coil of radius 15 mm, and a magnet that produces a radial magnetic flux density of 1 T (see **Figure 7.19** for the geometry and field configuration). The coil and cone have a mass of 8 g. Calculate the output of the loudspeaker as a microphone (emf) when a 60 dB sinusoidal sound pressure at 1 kHz is applied on the cone.

Capacitive microphones

7.13 **The electret microphone: sensitivity to variations in properties.** One of the parameters involved in the output of the electret microphone is the permittivity of the electret.

a. Calculate the sensitivity of the output to the relative permittivity of the electret.

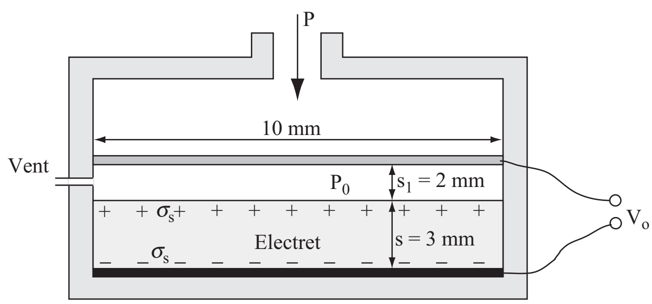
b. What is the error in the expected reading if the permittivity of the electret decreases by 5% due to aging or due to variations in manufacturing? Use the data in **Example 7.6**.

7.14 **The electret pressure sensor.** The electret microphone can serve as a pressure sensor. In particular, any electret microphone can serve as a differential pressure sensor, measuring the pressure $P - P_0$, where P_0 is the air pressure, provided a way is found to apply pressure to one of the electrodes. A pressure sensor of this type is shown in **Figure 7.52**. The thickness of the electret is 1 mm and the plate is at a tension of 100 N. The distances are shown in the figure and the relative permittivity of the electret is 4.5. The ratio of specific heats in air is 1.4 and a surface charge density on the electret of 0.6 μC/m^2 may be assumed. The sensor is cylindrical, with an internal diameter of 10 mm.

a. Given the dimensions and properties, calculate and plot the output as a function of external pressure starting at 0.1 atm (assuming the metal diaphragm can withstand that pressure). The ambient pressure is 1 atm. What is the maximum pressure the sensor can respond to? What is the sensitivity of the sensor?

b. The gap between the electret and plate is now evacuated and the vent is then sealed, keeping the pressure inside the gap between the electret and the plate at 50,000 Pa (approximately 0.5 atm) when no external pressure is applied.

FIGURE 7.52 ■ An electret pressure sensor.



Calculate and plot the sensor output for external pressures between 10,000 Pa (0.1 atm) and the maximum pressure to which the sensor responds. Does the sensitivity change due to the change in the pressure in the gap?

The piezoelectric microphone

7.15 Dynamic pressure sensor. Piezoelectric devices cannot be used to measure true static pressure because once the charge has been generated on the electrodes, it will discharge through the internal impedance of the sensor and through external impedances. They are, however, well suited to measure dynamic pressure such as that due to vibrations, detonations, engine knock and ignition, and many others. In that role it is, in essence, a modified microphone. Consider a piezoelectric pressure sensor designed to sense pressure in the cylinders of a diesel engine. The normal pressure in a diesel engine at the peak of piston travel is around 4 MPa. Calculate the output expected from a lead zirconium ceramic sensor with a piezoelectric coefficient of 120×10^{-12} C/N, a capacitance of 5000 pF, and an active surface area of 1 cm^2 (i.e., the area on which the pressure acts).

7.16 Sound intensity sensor. Consider a flat, round piezoelectric microphone with a piezoelectric disk 25 mm in diameter and a thickness of 0.8 mm made of zinc oxide (ZnO) cut on the 3-3 axis. Two electrodes are plated on the disk, one on each side. The microphone is used to measure sound intensity to alert workers of damaging noise levels. The sound pressure the human ear responds to is in the range 2×10^{-5} Pa (0 dB) and 200 Pa (threshold of pain).

a. Calculate the output of the microphone over the entire range.
b. From a practical point of view, what is the approximate useful range of the microphone? Explain in terms of practical output voltage levels.

7.17 The magnetic buzzer. Small buzzers are common in portable equipment as well as fixed installations, where they provide audible feedback or warning signals. In many cases they are driven directly from microprocessors, as they require little power to be effective. A magnetic buzzer for use with a microprocessor is built as in **Figure 7.24** with an iron core of radius 6 mm containing 150 turns and with the magnet replaced by iron. The gap between the core and the diaphragm is 1 mm and the iron core as well as the diaphragm have very high permeability. The diaphragm itself has an effective radius of 12 mm. Because the coil is driven directly from a microprocessor, its maximum current is 25 mA and it cannot operate at currents below 5 mA.

a. Calculate the range of sound pressures it can generate.
b. Calculate the range of acoustic power transmitted by the buzzer and the corresponding efficiencies if the microprocessor operates at 3.3 V.

Acoustic actuators

7.18 Force and pressure in a loudspeaker. Consider the loudspeaker structure in **Figure 7.19** with the following specifications: $I = 2\sin 2\pi ft$, number of turns in the coil $N = 100$, radius of the coil $a = 40$ mm, and magnetic flux density $B = 0.8$ T. The radius of the cone is $b = 15$ cm. Assuming that the magnetic flux density in the coil is constant at all times,

a. Calculate the maximum force on the coil.

b. If the cone has a restoring constant $k = 750$ N/m (this restoring constant is due to the attachment of the cone to the body of the loudspeaker and acts exactly as a spring constant to return the cone to its centered position), calculate the maximum displacement of the cone.

c. Calculate the maximum pressure the cone can apply.

7.19 **The electrostatic loudspeaker.** Most loudspeakers are magnetic devices because the forces that can be generated are significant. However, capacitive loudspeakers, also called electrostatic loudspeakers, exist in which the diaphragm is moved by electrostatic forces. The structure of the capacitive microphone in **Figure 7.13** can be used for this purpose, with the obvious change in dimensions and the application of an AC voltage as input (see also **Section 5.3.3** on capacitive actuators). In the device in **Figure 7.13**, the moving plate is a disk 10 cm in diameter separated from the fixed, lower plate by a distance of 6 mm when the input voltage is zero. The mass of the plate is 10 g. The input is a sinusoidal voltage varying between 3000 V and 0 V at a frequency of 1 kHz (**Figure 7.53**), that is, the sinusoidal signal is centered at 1500 V.

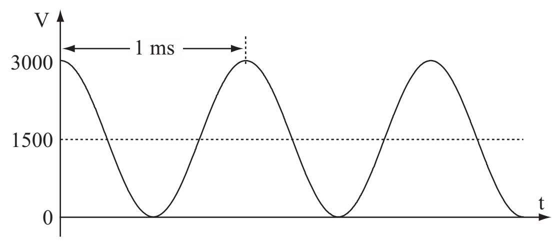
Note: Electrostatic loudspeakers require high voltages to work and produce measurable pressures.

a. Calculate the restoring spring constant needed to ensure the moving plate does not move closer than 2 mm from the fixed plate.

b. Estimate the sound pressure generated by the loudspeaker.

c. Loudspeakers are usually characterized by their power. Estimate the average power of this loudspeaker.

FIGURE 7.53 ■ Input voltage to the electrostatic loudspeaker.



Ultrasonic sensors

7.20 **Ultrasonic evaluation of structures.** In an ultrasonic test for delamination effects in steel plates, a pulse is transmitted and the signals received are detected

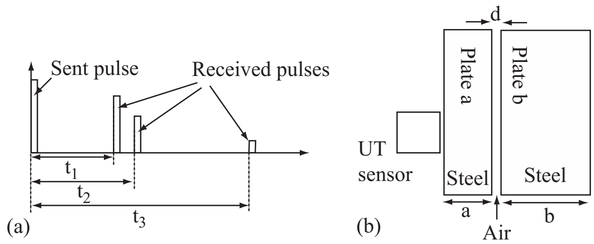


FIGURE 7.54 ■ Ultrasonic testing. (a) Signals recorded at the sensor. (b) The structure and configuration that produces the signals in (a).

on an oscilloscope. The timing of these signals is given in **Figure 7.54a**. Since delamination is suspected, it is assumed that the flaw is air-filled (a sketch of the expected configuration is shown in **Figure 7.54b**). From the signal received and the speed of sound in air, c_a, and steel, c_s, calculate the thicknesses of the two sheets and the width of the delamination.

Note: The transmitted and received signals look like those in **Figure 7.36b**, but are shown here as simple pulses for simplicity.

7.21 **Doppler ultrasound sensing of fluid velocity**. To measure fluid velocity in a channel it is proposed to use the configuration in **Figure 7.55**. The velocity in the channel is measured by placing both sensors at the top of the channel. The wave transmitted by the ultrasound transmitter (actuator) on the left reflects off the bottom of the channel and is received by the sensor on the right. The frequency of the ultrasound wave is 3 MHz and the speed of propagation in water is 1500 m/s.

a. Calculate the sensitivity of the system for a frequency f, fluid velocity v_f, and angle θ. Show that it is exactly the same as that for the sensor in **Figure 7.37** if the frequency, velocity, and angle are the same.
b. Calculate the frequency shift for $f = 3$ MHz and $\theta = 30°$ in water moving at a speed of 3 m/s.
c. Suppose the location of the receiver and the transmitter are interchanged so that the receiver is upstream. Now what are the answers to (a) and (b)?

7.22 **Time of flight method of speed sensing.** In **Problem 7.21**, instead of measuring the frequency shift, the time required for the wave to traverse the distance between the transmitter and receiver is measured.

a. If the depth of the channel is h, find a relation between the speed of flow and time. Assume a flow velocity v and a sensor angle of θ.
b. Calculate the sensitivity of the sensor.

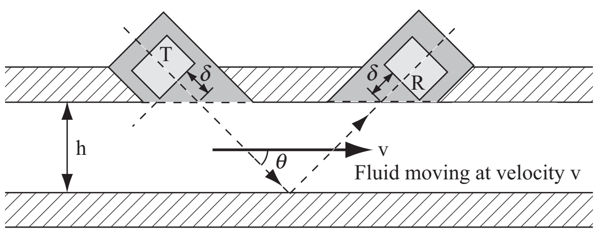


FIGURE 7.55 ■ Fluid velocity sensor adapted to sense velocity in a channel.

c. The depth of the channel is $h = 1$ m and the sensors are at an angle of 30°. A time of flight of 2.6 ms is measured. What is the fluid velocity if the speed of sound in water is 1500 m/s?

d. Suppose the location of the receiver and the transmitter are interchanged so that the receiver is upstream. What are now the answers to (a) and (b)?

7.23 **Single-transducer fluid flow sensor.** In an attempt to reduce the cost of a Doppler speed sensor an engineer proposes to use a single ultrasonic transducer and operate it in a pulsed-echo mode in which the transmitter sends a pulse then switches into receive mode to receive the reflection. That is, in **Figure 7.37a** the receiver is replaced with a reflector (a metal plate).

Note: The sensor is driven by a pulse that generates harmonics, but ultrasonic sensors are resonant, meaning they will only receive the fundamental frequency.

a. Calculate the exact shift in frequency at the receiver. Show that this is smaller than would be obtained with the configuration in **Figure 7.37a**.

b. Calculate the frequency shift for a sensor operating in water moving at a velocity $v = 5$ m/s, inclined at an angle $\theta = 60°$, and the mirror at a distance $d = 10$ cm (see **Figure 7.37a**). The sensor resonates at 3.5 MHz and the measurement is made in water, with a speed of sound of 1500 m/s. Compare this with the frequency shift that would be obtained using the configuration in **Figure 7.37a**.

c. What is the maximum pulse width the transducer can generate and still perform as intended for the sensor and fluid properties in (b)?

Piezoelectric actuators

7.24 **Sound intensity produced by a piezoelectric buzzer.** The sound intensity produced by a buzzer can be estimated from the mechanical power produced by the buzzer's piezoelectric disk. A piezoelectric buzzer is driven from a 1 kHz (square wave) source with an amplitude of 12 V, a current of amplitude 1 mA, and a 50% duty cycle. If the power efficiency of the device is 30%, and assuming the mechanical power of the buzzer is converted into sound, calculate the peak sound intensity produced by the device in power/area [W/m^2] and in decibels, considering the fact that the reference value for sound is taken as 10^{-12} W/m^2 (the threshold of hearing). The piezoelectric element is a disk 30 mm in diameter.

7.25 **Piezoelectric actuator.** Application of a voltage across a piezoelectric element causes a force to develop according to **Equation (7.40)**. This means that a strain is generated in the element, which in turn changes its length in the direction of the field produced by the potential. An actuator is made as a stack of $N = 20$ piezoelectric disks of radius $a = 10$ mm and thickness $d = 1$ mm each. The disks are made of PZT in a 3-3 cut. A voltage $V = 120$ V is applied across the stack. Calculate the change in length of the stack.

7.26 **Quartz SAW resonator.** A quartz SAW resonator is made as shown in **Figure 7.44**. It consists of 45 reflecting grooves on each side of the ports and the grooves are separated a distance 20 μm apart. What is the resonant frequency of the device? The speed of sound in quartz is 5900 m/s.

7.27 **Strain produced by an ultrasonic actuator.** An ultrasonic transducer is used to test thick aluminum billets for defects. To do so a transducer capable of generating 6 W of acoustic power operating at 10 MHz is used. The transducer is circular with a diameter of 30 mm. Assume that propagation of the ultrasonic wave is in a 15° cone and that the power density is uniform across the cross section of the cone. The acoustic properties of aluminum are given in **Tables 7.1–7.4**. The coefficient of elasticity of aluminum is 79 GPa.

a. Calculate the strain in the material at the surface at the location of the transducer.
b. Calculate the strain in the material at a depth of 60 mm.
c. Comment on the results and on the use of ultrasonic diagnostics in the body.

7.28 **SAW resonator temperature sensor.** The SAW resonator can be used to sense any quantity that will affect its resonant frequency, including temperature. The sound velocity of quartz is 5900 m/s at 20°C. The speed of sound is temperature dependent and increases by 0.32 mm/s/°C, and the coefficient of thermal expansion is 0.557 μm/m/°C.

a. Sketch a compensated sensor that will measure temperature but not be affected by other quantities such as ambient pressure. Can this be done in practice?
b. Calculate the sensitivity to temperature for a SAW sensor operating at 400 MHz. Comment on the practicality of this sensor.

7.29 **SAW resonator as a pressure sensor.** The structure in **Figure 7.56** is used as a pressure sensor. The sensor is made of quartz and consists of a number of grooves separated 10 μm apart. The area on which the pressure operates is $w = 2$ mm wide, $L = 10$ mm long, and $d = 0.8$ mm thick. Assume the quartz chip bends under pressure as a simply supported beam of thickness 0.8 mm with its supports at the edges of the device. Quartz has a modulus of elasticity of 71.7 GPa.

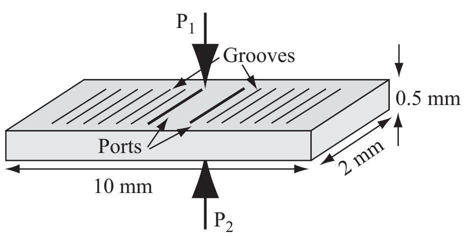


FIGURE 7.56 ■ The SAW resonator as a pressure sensor.

a. Calculate the sensitivity to pressure and the shift in resonant frequency for a pressure of 1 atm above ambient if the pressure P_1 is applied on the upper surface ($P_2 = 1$ atm).

b. Calculate the resonant frequency of the sensor for $P_1 = 120{,}000$ Pa and $P_2 = 150{,}000$ Pa.

7.30 **SAW mass sensor.** A SAW resonator made as in **Figure 7.48** has a length $a = 4$ mm, width $w = 2$ mm, and thickness $t = 0.2$ mm. The sensor is made of fused quartz and oscillates at 120 MHz without a stimulus. The modulus of elasticity for quartz is 71.7 GPa.

a. If the sensor is used to sense mass, what is the sensitivity of the sensor (in g/Hz)? Assume the mass is uniformly distributed on the sensor.

b. The maximum strain allowable in quartz is 1.2%. Calculate the range and span of the sensor.

c. If the frequency is measured by a frequency counter and the lowest frequency change that can be distinguished is 10 Hz, what is the resolution of the instrument?

7.31 **Piezoelectric ignition device.** A unique and common actuator makes use of a piezoelectric device to generate sufficiently high voltages that can generate sparks to ignite gas. This device can be found in all cigarette lighters and in ignition switches for gas kitchen stoves, furnaces, and other applications. The device uses a small, typically cylindrical crystal and a spring-loaded hammer that delivers a fixed, known force when hitting the crystal. Consider the device used in cigarette lighters. The crystal is 2 mm in diameter and 10 mm long (**Figure 7.57**).

a. Assuming the crystal is $BaTiO_3$ with a 3-3 cut and the required voltage to produce a spark of the appropriate size is 3200 V, calculate the impact force required. Describe the approximations needed and their validity.

b. Describe how that force can be generated using a spring with a spring constant $k = 2000$ N/m.

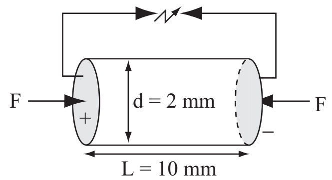


FIGURE 7.57 ■ Piezoelectric gas ignition device.

7.32 **Improved piezoelectric ignition device.** An important parameter in piezoelectric igniters is the energy that the device can supply. The higher the energy, the more likely ignition will occur. Consider again **Problem 7.31**. Using the same crystal and the same material, suppose the configuration is changed to that shown in **Figure 7.58**.

a. Using the dimensions shown and assuming a $BaTiO_3$ crystal with a 3-3 cut, calculate the force required to produce a voltage of 3200 V across the spark gap.

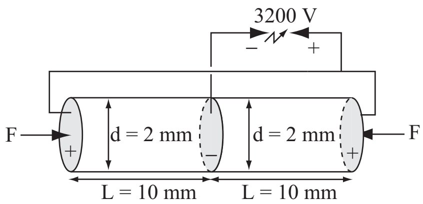


FIGURE 7.58 ■ Improved piezoelectric gas ignition device.

b. Show in general terms that the energy supplied by this device is twice the energy supplied by the device in **Figure 7.57**.

c. Where does the extra energy come from?

d. Show how the energy available from the ignition device can be further increased without increasing the force and discuss the practicality of your approach.