CHAPTER 6

Mechanical Sensors and Actuators

The hand

The hand is the main body organ for interaction with the environment. An actuator as well as a sensor, it is an amazing organ when one really thinks about it. As an actuator it contains 27 bones, of which 14 make up the fingers or digital bones (3 on each finger except the thumb, which has only two), 5 are in the palm (metacarpal bones), and 8 in the wrist (carpal bones). Their structure and interconnections together with a complex series of muscles and tendons give the human hand a flexibility and dexterity not found in any other animal. Apes, monkeys, and lemurs have hands similar to humans, and other animals such as the koala have opposing thumbs, which are useful for climbing, but none are as flexible as the human hand. The hand can perform articulation of the finger bones, between the fingers and the palm, between the palm and the wrist, and between the wrist and the arm. Together with additional articulations at the elbow and shoulder, the hand is a multiaxis actuator capable of surprisingly delicate as well as gross motions. But the hand is also a tactile sensor. The fingertips in particular have the densest nerve endings in the body. They provide feedback for manipulation of objects or sense by direct touch. The hands are controlled by opposing brain hemispheres (left hand by the right hemisphere and right hand by the left hemisphere). This is true of other paired organs, including the eyes and legs.

Sensing and the skin

The skin is the largest organ in the human body, covering the whole body with a layer that averages 2–3 mm in thickness and an average area close to 2 m^2. As with other organs, it is multifunctional, serving as a protection layer for intrusion of organisms into the body, preventing loss of fluid through it, and absorbing vitamin D. It also protects the body from harmful radiation by absorbing ultraviolet radiation in melatonin as well as absorbing oxygen and excreting some chemicals. A critical function is insulation and heat regulation through sweat mechanisms and blood vessels in the dermis (the layer just below the thin, externally visible surface, called the epidermis). But of particular interest here is the function of the skin for sensing. Nerve endings on the skin sense heat, cold, pressure, vibration, and damage (injury), although the sensitivity varies from place to place. Not only can we sense with the skin, but localization of stimuli is very good and quite accurate, allowing us to detect the location of the stimulus on that large surface.

6.1 INTRODUCTION

The class of mechanical sensors includes a fairly large number of different sensors based on many principles, but the four groups of general sensors discussed here—force sensors, accelerometers, pressure sensors, and gyroscopes—cover most of the principles involved in the sensing of mechanical quantities either directly or indirectly. Some of these sensors are used for applications that initially do not seem to relate to mechanical quantities. For example, it is possible to measure temperature through the expansion of gases in a volume (pneumatic temperature sensors are discussed in **Chapter 3**). The expansion can be sensed through the use of a strain gauge, which is a classical mechanical sensor. In this application an indirect use of a strain sensor is made to measure temperature. On the other hand, some mechanical sensors do not involve motion or force. An example of this is the optical fiber gyroscope, which will be discussed later in this chapter.

6.2 | SOME DEFINITIONS AND UNITS

Strain (dimensionless) is defined as the change in length per unit length of a sample. It is given as a fraction (i.e., 0.001) or as a percentage (i.e., 0.1%). Sometimes it is given as microstrain, meaning the strain in micrometers per meter (μm/m). The common symbol for strain is ε. Although this is the same symbol as that of electric permittivity, it should be clear from the context which quantity is implied by its use.

Stress is pressure [N/m^2] in a material. The symbol for stress is σ, and again, this should not be confused with electric conductivity, which uses the same symbol.

Modulus of elasticity is the ratio of stress to strain. This relation, often written as $\sigma = \varepsilon E$, where E is the modulus of elasticity, is called Hooke's law. The modulus of elasticity is often referred to as Young's modulus and has units of pressure [N/m^2].

The **gas constant** or **ideal gas constant** is equivalent to the Boltzmann constant. Whereas Boltzmann's constant expresses energy per temperature increment per particle, the gas constant expresses energy per temperature increment per mole. Denoted as R, its value is 8.3144621 J/mol/K.

The **specific gas constant** is the gas constant divided by the molecular mass of the gas. It is denoted as R_{specific} or R_s and equals 287.05 J/kg/K.

Pressure is force per unit area [N/m^2]. The SI derived unit of pressure is the pascal (1 pascal [Pa] = 1 newton per square meter [N/m^2]). The pascal is an exceedingly small unit and it is much more common to use the kilopascal (kPa = 10^3 Pa) and the megapascal (MPa = 10^6 Pa). Other often used units are the bar (1 bar = 0.1 MPa) and the torr (1 torr = 133 Pa). Also, for some very low pressure uses, the millibar (1 mbar = 1.333 torr = 100 Pa) and the microbar (1 μbar = 0.1 Pa) are employed. In common use one can find the atmosphere (atm). The atmosphere is defined as "the pressure exerted by a 1 m (actually the exact value is 1.032 m) column of water at 4°C on 1 cm^2 at sea level." The use of the atmosphere indicates a totally parallel system of pressure based either on a column of water or a column of mercury. In fact, the torr (named after Evangelista Torricelli) is defined as the pressure exerted by 1 mm of mercury (at 0°C and normal atmospheric pressure). Neither mmHg nor cmH_2O is an SI unit, but they still exist and in some instances are the preferred unit. For example, blood pressure is often measured in mmHg, whereas cmH_2O is used by gas utilities to measure gas pressure. One mmHg is the pressure exerted by a column of mercury 1 mm high at 0°C (the density of mercury is 13.5951 g/cm^3) and assuming the acceleration of gravity is 9.80665. Similarly 1 cmH_2O is the pressure exerted by a column of water 1 cm high at 4°C with density 1.004514556 g/cm^3 and assuming the acceleration of gravity is 9.80665 m/s^2. In the

TABLE 6.1 ■ Main units of pressure and conversion between them

	pascal	atmosphere	torr	bar	psi
pascal	1 Pa	9.869×10^{-6} atm	7.7×10^{-3} torr	10^{-5} bar	1.45×10^{-4} psi
atmosphere	101.325 kPa	1 atm	760 torr	1.01325 bar	14.7 psi
torr	133.32 Pa	1.315×10^{-3} atm	1 torr	1.33×10^{-3} bar	0.01935 psi
bar	100 kPa	0.986923 atm	750 torr	1 bar	14.51 psi
psi	6.89 kPa	0.068 atm	51.68 torr	0.0689 bar	1 psi

Note: Atmospheric (air) pressure is often given in millibars (mbar). The normal atmospheric pressure at sea level is 1013 mbar (1 atm or 101.325 kPa or 14.7 psi). However, none of these units are SI units. The proper unit to use is the pascal.

United States in particular, the common (nonmetric) unit of pressure is pounds per square inch (psi; 1 psi = 6.89 kPa = 0.068 atm).

Table 6.1 shows the main units of pressure in common use and the conversion between them. Note that only the pascal is an SI unit.

In work with pressure and pressure sensors, the concept of a vacuum is often used, sometimes as a separate quantity. Whereas vacuum means lack of pressure, it is usually understood as indicating pressure below ambient. Thus when one talks about so many pascals or psi of vacuum, this simply refers to so many pascals or psi below ambient pressure. While this may be convenient, it is, strictly speaking, not correct and the system of units does not provide for it. Therefore it should be avoided.

6.3 | FORCE SENSORS

6.3.1 Strain Gauges

The main tool in sensing force is the strain gauge. Although strain gauges, as their name implies, measure strain, the strain can be related to stress, force, torque, and a host of other stimuli, including displacement, acceleration, or position. With proper application of transduction methods, it can even be used to measure temperature, level, and many other related quantities.

At the heart of all strain gauges is the change in resistance of materials (primarily metals and semiconductors) due to a change in their length due to strain. To better understand this, consider a length of metal wire L, of conductivity σ, and cross-sectional area A. The resistance of the wire is

$$R = \frac{L}{\sigma A} \quad [\Omega]. \tag{6.1}$$

Taking the log of this expression:

$$\log R = \log\left(\frac{1}{\sigma}\right) + \log\left(\frac{L}{A}\right) = -\log\sigma + \log\left(\frac{L}{A}\right). \tag{6.2}$$

Now, taking the differential on both sides:

$$\frac{dR}{R} = -\frac{d\sigma}{\sigma} + \frac{d(L/A)}{L/A}. \tag{6.3}$$

Thus the change in resistance can be viewed as due to two terms. One is the conductivity of the material and the other (second term on the right-hand side) is due to the deformation of the conductor. For small deformations, both terms on the right-hand side are linear functions of strain, ε. Bundling both effects together (i.e., the change in conductivity and deformation) we can write

$$\frac{dR}{R} = g\varepsilon, \tag{6.4}$$

where g is the sensitivity of the strain gauge, also known as the **gauge factor**. For any given strain gauge this is a constant, ranging between 2 and 6 for most metallic strain gauges and between 40 and 200 for semiconductor strain gauges. This equation is the strain gauge relation and gives a simple linear relation between the change in resistance of the sensor and the strain applied to it.

The change in resistance due to strain increases the resistance of the strain gauge. The resistance of the strain gauge under strain is therefore

$$R(\varepsilon) = R_0(1 + g\varepsilon) \quad [\Omega], \tag{6.5}$$

where R_0 is the no-strain resistance. Before continuing, a bit more on stress, strain, and the connection between them.

Given the conductor discussed above (**Figure 6.1**), and applying a force along its axis, the stress is

$$\sigma = \frac{F}{A} = E\frac{dL}{L} = E\varepsilon \quad [\mathrm{N/m^2}]. \tag{6.6}$$

Note: σ is used here for stress, not for conductivity. The quantities are

$\sigma =$ stress [N/m^2 or Pa], which is in fact pressure.

$E =$ modulus of elasticity or Young's modulus of the material [N/m^2 or Pa]. The modulus of elasticity is the ratio between stress and strain (Hooke's law), as can be seen from **Equation (6.6)**.

$\varepsilon = dL/L =$ strain. Strain is dimensionless and gives the change in length per unit length.

Thus strain is simply a normalized linear deformation of the material, whereas stress is a measure of the elasticity of the material.

Note: σ and ε are used here to denote stress and strain. In Chapter 5 we used the same symbols for electric conductivity and permittivity. These should not be confused.

Since strain gauges are made of metals and metal alloys (including semiconductors), they are also affected by temperature. If we assume that the resistance in **Equation (6.5)** is calculated at a reference temperature T_0, then we can write the resistance of the sensor as a function of temperature using **Equation (3.4)**:

$$R(\varepsilon, T) = R(\varepsilon)(1 + \alpha[T - T_0]) = R_0(1 + g\varepsilon)(1 + \alpha[T - T_0]) \quad [\Omega], \tag{6.7}$$

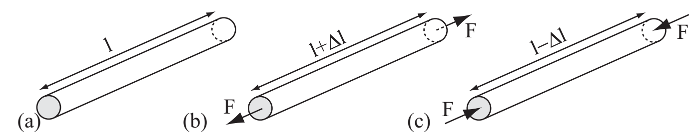


FIGURE 6.1 ■ (a) A wire of length L, cross-sectional area A, and electric conductivity σ. (b, c) Application of force to cause stress and strain in the conductor.

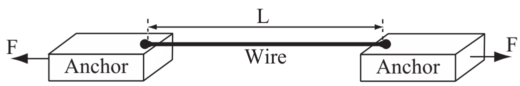


FIGURE 6.2 ■ A rudimentary wire strain gauge (also called unbonded strain gauge).

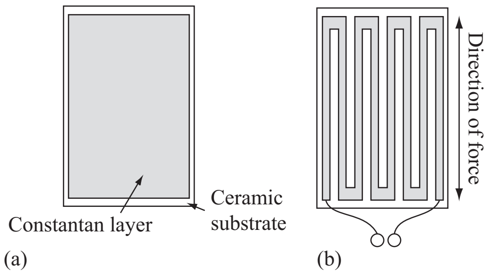


FIGURE 6.3 ■ Common construction of a resistive strain gauge. Constantan is usually used because of its very low TCR.

where α is the TCR of the material (see **Table 3.1**). This clearly shows that the temperature and strain effects are multiplicative and indicates that strain gauges are, necessarily, sensitive to temperature variations.

Strain gauges come in many forms and types. In effect, any material, combination of materials, or physical configuration that will change its resistance (or any other property for that matter) due to strain constitutes a strain gauge. However, we will restrict our discussion here to two types that account for most of the strain gauges in use today: wire (or metal film) strain gauges and semiconductor strain gauges. In its simplest form, a metallic strain gauge can be made of a length of wire held between two fixed posts (**Figure 6.2**). When a force is applied to the posts, the wire deforms, causing a change in the wire's resistance. Although this method was used in the past and is valid, it is not very practical in terms of construction, attachment to the system whose strain needs to be measured, or in terms of the change in resistance (which is necessarily very small). Thus a more practical strain gauge is built out of a thin layer of conducting material deposited on an insulating substrate (plastic, ceramic, etc.) and etched to form a long, meandering wire, as shown in **Figure 6.3**. Constantan (an alloy made of 60% copper and 40% nickel) is the most common material because of its negligible temperature coefficient of resistance (TCR) (see **Table 3.1**). There are other materials in common use, especially at higher temperatures or when special properties are needed. **Table 6.2** shows some of the materials used for strain gauges with their properties, including the gauge factor.

Strain gauges may also be used to measure multiple axis strains by simply using more than one gauge or by producing them in standard configurations. Some of these are shown in **Figure 6.4**. **Figure 6.7** shows two commercial strain gauges.

6.3.2 Semiconductor Strain Gauges

These operate in the same way as conductor strain gauges, but their construction and properties are different. First, the gauge factor for semiconductors happens to be much higher than for metals. Second, the change in conductivity in **Equation (6.1)** due to strain is much larger than in metals. Semiconductor strain gauges are typically smaller

TABLE 6.2 ■ Materials for resistive strain gauges and their properties

Material	Gauge factor	Resistivity [$\Omega \cdot mm^2$/m] at 20°C	TCR [10^{-6}/K]	Expansion coefficient [10^{-6}/K]	Maximum temperature [°C]
Constantan (Cu60Ni40)	2.0	0.5	10	12.5	400
Nichrome (Ni80Cr20)	2.0	1.3	100	18	1000
Manganine (Cu84Mn12Ni4)	2.2	0.43	10	17	
Nickel	−12	0.11	6000	12	
Chromel (Ni65Fe25Cr10)	2.5	0.9	300	15	800
Platinum	5.1	0.1	2450	8.9	1300
Elinvar (Fe55Ni36Cr8Mn0.5)	3.8	0.84	300	9	
Platinum-iridium (Pt80Ir20)	6.0	0.36	1700	8.9	1300
Platinum-rhodium (Pt90Rh10)	4.8	0.23	1500	8.9	
Bismuth	22	1.19	300	13.4	

Notes:

1. There are other specialized alloys often used for the production of strain gauges. These include platinum-tungsten (Pt92W08), isoelastic alloy (Fe55.5Ni36Cr08Mn05), Karma (Ni74Cr20Al03Fe03), Armour D (Fe70Cr20Al10), Manganin (Cu84Mn12Ni4), and Monel (Ni67Cu33).
2. These materials are selected for specific applications. For example, isoelastic alloy is excellent for dynamic strain/stress sensing, although it is particularly highly temperature sensitive. Platinum strain gauges are selected for high-temperature applications.
3. Many of the strain gauges must be temperature compensated.

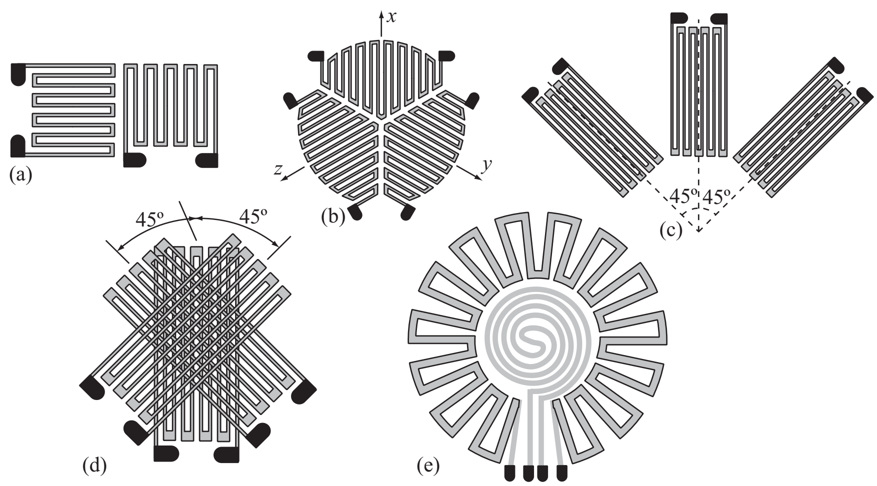


FIGURE 6.4 ■ Various configurations of strain gauges for different purposes. (a) Two-axis. (b) 120° rosette. (c) 45° rosette. (d) 45° stacked. (e) Membrane rosette.

than metal types, but are often more sensitive to temperature variations (hence temperature compensation is often incorporated within the gauge). All semiconductor materials exhibit changes in resistance due to strain, but the most common material is silicon because of its inert properties and ease of production. The base material is doped by diffusion of doping materials (usually boron for *p* type and arsenide for *n* type) to obtain a base resistance as needed. The substrate provides the means of straining the silicon chip and connections are provided by deposition of metal at the ends of the device. **Figure 6.5a** shows the construction of such a device, but a large variation in shapes and types may be found. Some of these, including multiple element gauges are shown in **Figures 6.5b–6.5f**. The range of temperatures for semiconductor strain gauges is limited to less than about 150°C.

One of the important differences between conductor and semiconductor strain gauges is that semiconductor strain gauges are nonlinear devices with typically a quadratic transfer function:

$$\frac{dR}{R} = g_1\varepsilon + g_2\varepsilon^2. \tag{6.8}$$

Although this nonlinearity is problematic in some applications, the higher sensitivity (gauge factor between 40 and 200 or more) is a boon. Also, the fact that *p* and *n* types may be used allows for PTC- and NTC-type behavior as shown in **Figure 6.6**.

The conductivity of semiconductors depends on a number of parameters, including the doping level (concentration or carrier density), type of semiconductor, temperature, radiation, pressure, and light intensity (if exposed), among others. Therefore it is essential that common effects such as temperature variations be compensated for or the errors due to these effects may be of the same order of magnitude as the effects of strain, resulting in unacceptable errors.

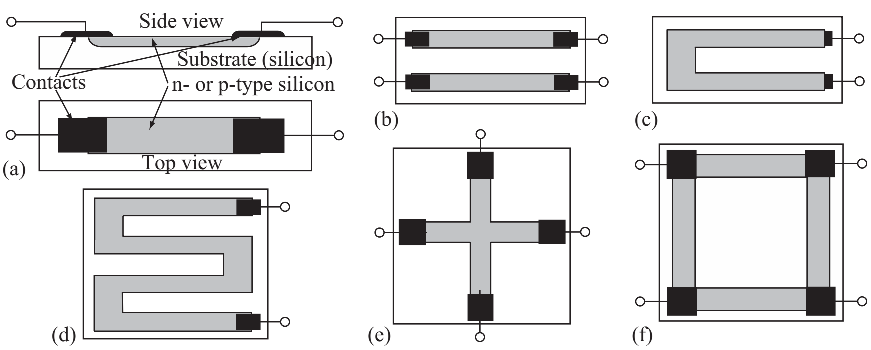


FIGURE 6.5 (a) Construction of a semiconductor strain gauge. (b–f) Various configurations of semiconductor strain gauges.

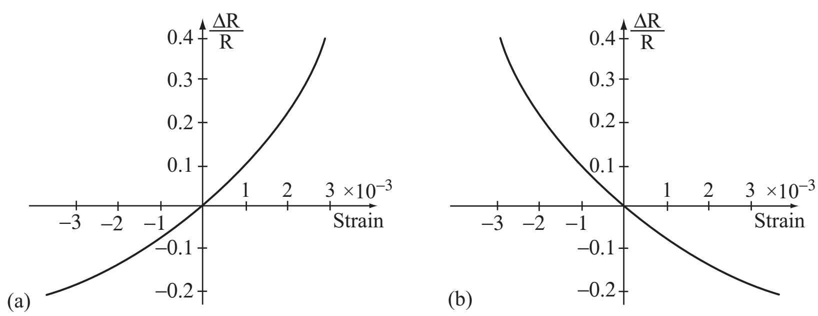


FIGURE 6.6 Transfer functions for *p*- and *n*-type semiconductor strain gauges (PTC and NTC operation).

6.3.2.1 Application

To use a strain gauge as a sensor it must be made to react to force. For this to happen, the strain gauge is attached to the member in which strain is sensed, usually by bonding. Special bonding agents exist for different applications and types of materials and are usually supplied by the manufacturers of strain gauges or by specialized producers. In this mode, they are often used to sense bending strain, twisting (torsional and shear) strain, and longitudinal tensioning/deformation (axial strain) of structures such as engine shafts, bridge loading, truck weighing, and many others. Any quantity related to strain (or force), such as pressure, torque, and acceleration, can be measured directly. Other quantities may be measurable indirectly.

The properties of strain gauges vary by type and application, but most metal gauges have a nominal resistance between 100 and 1000 Ω (lower and higher resistances are available), have a gauge factor between 2 and 5, and have dimensions from less than 3 mm × 3 mm to lengths in excess of 150 mm, but almost any size may be fabricated as necessary. Rosettes (multiple-axis strain gauges) are available with 45°, 90°, and 120° axes, as well as diaphragm and other specialized configurations (see **Figure 6.4**). Typical sensitivities are 5 mΩ/Ω and deformation strain is on the order of 2–3 μm/m. Obviously much higher strains can be measured with specialized gauges.

Semiconductor strain gauges are usually smaller than most metal strain gauges and can be made with higher resistances. Because of the temperature limitations of these gauges, their use is limited to low temperatures, but they can be much less expensive than metal strain gauges and are in common use where applicable. One of the main uses of semiconductor strain gauges is as embedded devices in sensors such as accelerometers and load cells.

6.3.2.2 Errors

Strain gauges are subject to a variety of errors. The first is that due to temperatures, since the resistance, especially in semiconductors, is affected by temperature in the same way as by strain. In some metal gauges, this is small, since care is taken to select materials that have low temperature coefficients of resistance. In others, however, it can be rather large, and in semiconductors, temperature compensation is sometimes provided onboard the device or a separate sensor may be used for this purpose. **Equation (6.7)** gives the general relation for temperature effects (see also **Example 6.2**). Because of this, the nominal resistance of strain gauges is given at a reference temperature T_0 (often the reference temperature is 23°C, but it can be any convenient temperature).

Another source of error is due to lateral strains (i.e., strains in directions perpendicular to the main axis in **Figure 6.3**). These strains, and the change in resistance due to them, affect the overall reading. For this reason, strain gauges are usually built as slender devices with one dimension much larger than the other. Semiconductor strain gauges are particularly good in this respect, as their lateral sensitivity (or cross-sensitivity) is very low because of the very small dimensions of the sensor. A third source of error is due to the strain itself, which, over time, tends to permanently deform the gauge. This error can be eliminated by periodic recalibration and can be reduced by ensuring that the maximum deformation allowed is small and below that recommended for the device. Additional errors are incurred through the bonding process and through thinning of materials (or even breaking) due to cycling. Most strain gauges are rated for a given number of cycles (e.g., 10^6 or 10^7 cycles), maximum strain (3% is typical for conducting strain gauges and 1%–2% for semiconductor strain gauges), and often their temperature

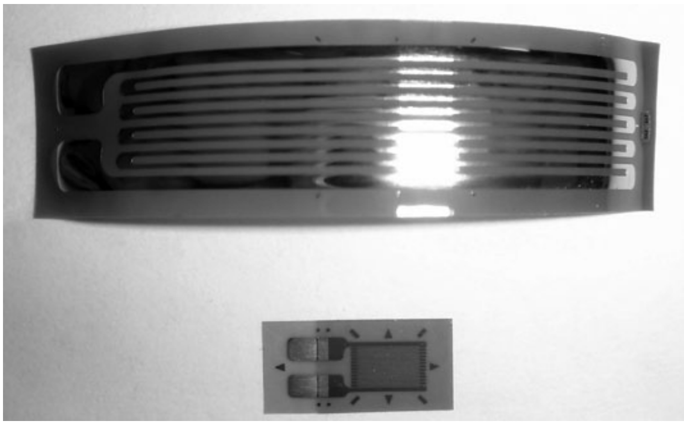


FIGURE 6.7 ■ Two resistive strain gauges. Upper gauge is 25 mm × 6 mm. Lower gauge is 6 mm × 3 mm.

characteristics are specified for use with a particular material (aluminum, stainless steel, carbon steel) for optimal performance when bonded to that material. Typical accuracies when used in bridge configurations are on the order of 0.2%–0.5%.

EXAMPLE 6.1 The strain gauge

A strain gauge similar to that shown in **Figure 6.7** is made with the dimensions shown in **Figure 6.8**. The gauge is 5 μm thick. The sensor is made of constantan to reduce temperature effects.

a. Calculate the resistance of the sensor at 25°C without strain.

b. Calculate the resistance of the sensor if force is applied longitudinally causing a strain of 0.001.

c. Estimate the gauge factor from the calculations in (a) and (b).

Solution:

a. The resistance of the meander strip is calculated from **Equation (6.1)** and the data for constantan in **Table 6.2**. The conductivity at 20°C is 2×10^6 S/m (conductivity is the reciprocal of resistivity). The total length of the strip is

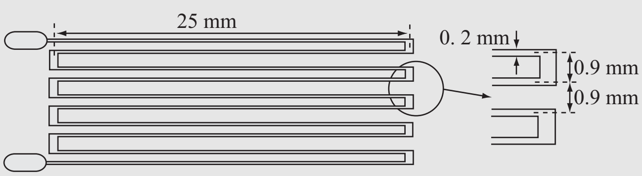


FIGURE 6.8 ■ Dimensions and structure of a strain gauge.

$$L = 10 \times 0.025 + 9 \times 0.0009 = 0.2581 \text{ m}$$

and its cross-sectional area is

$$S = 0.0002 \times 5 \times 10^{-6} = 1.0 \times 10^{-9} \text{ m}^2.$$

At 20°C,

$$R = \frac{L}{\sigma S} = \frac{0.2581}{2 \times 10^6 \times 1 \times 10^{-9}} = 129.05\ \Omega.$$

To calculate the gauge resistance at 25°C we use **Equation (3.4)** with the temperature coefficient of resistance for constantan, which equals 10^{-5} (see **Table 3.1**):

$$R(25°\mathrm{C}) = R_0(1 + \alpha[T - T_0]),$$

where $T_0 = 20°\mathrm{C}$ and R_0 is the resistance calculated at 20°C:

$$R(25°\mathrm{C}) = 129.05\left(1 + 1 \times 10^{-5}[25 - 20]\right) = 129.05 \times 1.00005 = 129.05\ \Omega.$$

The resistance is virtually unchanged because of the small temperature difference and the low coefficient.

b. The strain is the change in length divided by total length:

$$\varepsilon = \frac{\Delta L}{L} = 0.001 \rightarrow \Delta L = 0.001L = 0.001 \times 0.2581 = 0.0002581\ \mathrm{m}.$$

Thus the total length is 0.2583581 m. The cross-sectional area must also change since the volume of material must remain constant. Taking the volume v_0 as LS before deformation we get

$$S' = \frac{v_0}{L + \Delta L} = \frac{LS}{L + \Delta L} = \frac{0.2581 \times 1.0 \times 10^{-9}}{0.2583581} = 9.99 \times 10^{-10}\ \mathrm{m}^2.$$

The resistance of the strain gauge is now

$$R = \frac{L + \Delta L}{\sigma S'} = \frac{0.2583581}{2 \times 10^6 \times 9.99 \times 10^{-10}} = 129.308\ \Omega.$$

This is a small change (0.258 Ω), but is typical of strain gauges.

c. The gauge factor is calculated from **Equation (6.4)** as an approximation:

$$g = \frac{1}{\varepsilon}\frac{dR}{R} = 1000 \times \frac{0.2583581}{129.05} = 2.0.$$

This gauge factor is as expected for conductor strain gauges.

EXAMPLE 6.2 Errors due to temperature variations

To measure strain during testing in a jet engine, a special platinum strain gauge is produced by sputtering the material into a foil and etching the gauge pattern. The sensor has a nominal resistance of 350 Ω at 20°C and a gauge factor of 8.9 (see **Table 6.2**). The platinum grade used has a temperature coefficient of resistance of 0.00385 Ω/°C. The sensor is exposed to temperature variations between −50°C and 800°C during testing.

a. Calculate the maximum resistance expected for a maximum strain of 2%.
b. Calculate the change in resistance due to temperature and maximum error due to temperature changes.

Solution:

a. From **Equation (6.4)**:

$$\frac{dR}{R} = g\varepsilon \rightarrow dR = Rg\varepsilon = 350 \times 8.9 \times 0.02 = 62.3\ \Omega.$$

The maximum resistance will be measured as $62.3 + 350 = 412.3\ \Omega$.

b. The change in resistance of the sensor due to temperature is calculated from **Equation (3.4)**:

$$R(25°\mathrm{C}) = R_0(1 + \alpha[T - T_0]),$$

where R_0 is the resistance of the sensor at T_0. In this case, this is the resistance of the sensor at the given strain. We write

At $-50°$C, minimum strain is

$$R(-50°\mathrm{C}) = 350(1 + 0.00385[-50 - 20]) = 255.675\ \Omega.$$

At $-50°$C, maximum strain is

$$R(-50°\mathrm{C}) = 412.3(1 + 0.00385[-50 - 20]) = 301.185\ \Omega.$$

At 800°C, minimum strain is

$$R(800°\mathrm{C}) = 350(1 + 0.00385[800 - 20]) = 1401.05\ \Omega.$$

At 800°C, maximum strain is

$$R(800°\mathrm{C}) = 412.3(1 + 0.00385[800 - 20]) = 1650.44\ \Omega.$$

Clearly the change in resistance due to temperature is large. Taking the maximum resistance, the error that temperature variations will cause is

At 800°C, maximum strain is

$$error = \frac{1650.44 - 412.3}{412.3} \times 100\% = 300\%.$$

At 800°C, minimum strain is

$$error = \frac{1401.05 - 350}{350} \times 100\% = 300\%.$$

At $-50°$C, maximum strain is

$$error = \frac{301.185 - 350}{350} \times 100\% = -13.95\%.$$

At $-50°$C, minimum strain is

$$error = \frac{255.675 - 350}{350} \times 100\% = -26.95\%.$$

Therefore maximum error occurs at the higher temperature, regardless of strain. This error can be compensated for in properly designed bridge circuits (we shall see this in **Chapter 11**) and the measurement can still be done accurately. This example is extreme, but in many applications of strain gauges, temperature compensation is an essential part of sensing.

6.3.3 Other Strain Gauges

There are a number of strain gauges used for specialized applications. A very sensitive strain gauge can be made from optical fibers. In this type of gauge, the change in length of the fiber changes the phase of the light through the fiber. Measuring the light phase, either directly or by an interferrometric method, can produce readings of minute strain that cannot be obtained in other strain gauges. However, the device and the electronics necessary are far more complicated than standard gauges. There are also liquid strain gauges that rely on the resistance of an electrolytic liquid in a flexible container that can be deformed. Another type of strain gauge that is used on a limited basis is the plastic strain gauge. These are made as ribbons or threads based on graphite or carbon in a resin as a substrate and used in a way similar to other strain gauges. While they have very high gauge factors (up to about 300), they are otherwise difficult to use and inaccurate, as well as unstable mechanically, severely limiting their practical use.

6.3.4 Force and Tactile Sensors

Forces can be measured in many ways, but the simplest and most common method is to use a strain gauge and calibrate the output in units of force. Other methods include measuring acceleration of a mass ($F = ma$), measuring the displacement of a spring under action of force ($x = kF$, where k is the spring constant), measuring the pressure produced by force, and variations of these basic methods. None of these is a direct measure of force and many are more complicated than the use of a strain gauge. The transduction process may mean that the actual, measured quantity is capacitance, inductance, or, as in the case of strain gauges, resistance. The basic method of force sensing is shown in **Figure 6.9**. In this configuration one measures the tensile force by measuring the strain in the strain gauge. The sensor is usually provided with attachment holes and may also be used in compressive mode by prestressing the strain gauge. This type of sensor is often used to measure forces in machine tools, engine mounts, and the like. A common form of force sensor is the load cell. Like the force sensors in **Figure 6.9**, the load cell is instrumented with strain gauges. Usually the load cell is in the form of a cylinder (but a bewildering variety of shapes exist) placed between the two members that apply a force (e.g., the two plates of a press, between the suspension and the body of a vehicle, or between the moving and stationary parts of a truck weight scale).

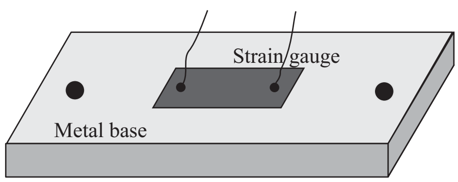


FIGURE 6.9 ■ The basic structure of a force sensor.

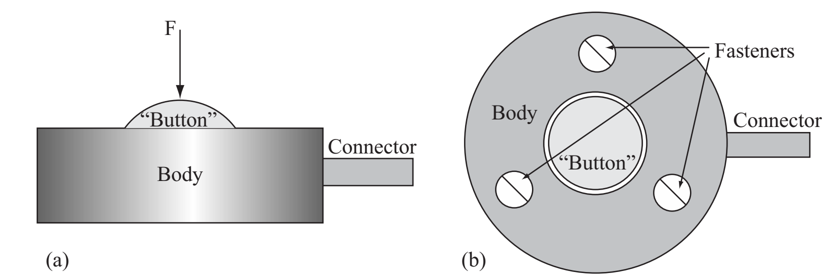


FIGURE 6.10 ■ One type of load cell—the button load cell.

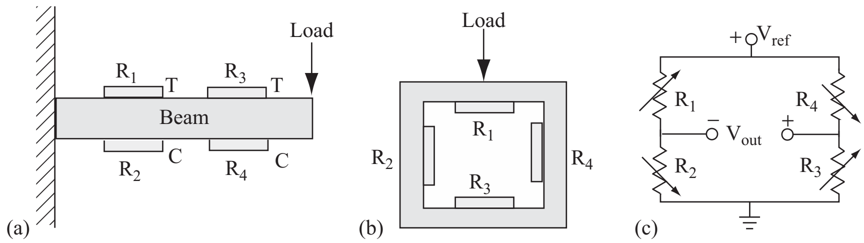


FIGURE 6.11 ■ Structure of load cells. (a) Bending beam load cell. (b) "Ring" load cell. (c) The connection of the strain gauges in a bridge. Arrows pointing up indicate tension, arrows pointing down indicate compression.

Figure 6.10 shows a load cell that operates in compression mode. Note the "button," which transfers the load to the strain gauges. One or more strain gauges may be bonded to this button, which is usually a cylindrical piece, but the button may transfer the load to a beam or any other structure on which the strain gauges are bonded. The strain gauges are prestressed so that under compression the stress is lowered. However, it should be noted that in addition to the basic configuration in **Figures 6.9** and **6.10** there are literally dozens of configurations to suit any need. Load cells exist to sense forces that range from a fraction of a newton to hundreds of thousands of newtons. Although the configurations and shapes of load cells vary wildly, most use four strain gauges, two in compression mode and two in tension mode. Two common configuration are shown in **Figures 6.11a** and **6.11b**. In **Figure 6.11a**, the two gauges under the beam operate in compression mode, whereas the upper two operate in tension mode. In **Figure 6.11b**, the upper and lower members bend inward when a load is applied and hence gauges R_1 and R_3 are under tension. The side members bend outward and the gauges R_2 and R_4 are under compression. The four gauges are connected in a bridge as shown in **Figure 6.11c**. The operation of the bridge will be discussed in **Chapter 11**, including its use for load cells. At this point we simply mention that under no load gauges R_1 and R_3 have identical resistance, whereas gauges R_2 and R_4 have the same resistance (but generally different than that of R_1 and R_3). Under these conditions the bridge is balanced and the output is zero. When a load is applied, the resistance of R_1 and R_3 increases, whereas that of R_2 and R_4 decreases, taking the bridge out of balance and producing an output proportional to the load.

There are also force sensors that do not actually measure force in a quantitative way but, rather, sense the force in a qualitative way and respond to the presence of force above a threshold value. Examples are switches and keyboards, pressure-sensitive polymer mats for sensing presence, and the like.

EXAMPLE 6.3 **Force sensor for a truck scale**

A truck scale is made of a platform and four compression force sensors, one at each corner of the platform. The sensor itself is a short steel cylinder, 20 mm in diameter. A single stain gauge is prestressed to 2% strain and bonded on the outer surface of the cylinder. The strain gauges have a nominal resistance (before prestressing) of 350 Ω and a gauge factor of 6.9. The steel used for the cylinders has a modulus of elasticity (Young's modulus) of 30 GPa.

a. Calculate the maximum truck weight that the scale can measure.
b. Calculate the change in resistance of the sensors for maximum weight.
c. Calculate the sensitivity of the scale assuming the response of the strain gauges is linear.

Solution:
a. The relation between pressure and strain is given in **Equation (6.6)**:

$$\frac{F}{A} = \varepsilon E \quad [\mathrm{Pa}],$$

where A is the cross-sectional area of the cylinder, F is the force applied, ε is the strain, and E is the modulus of elasticity. Since there are four sensors, the total force is

$$F = 4\varepsilon AE = 4 \times 0.02 \times \pi \times 0.01^2 \times 30 \times 10^9 = 753{,}982 \text{ N}.$$

This is 753,982/9.81 = 76,858 kg of force or 76.86 tons.

b. We need to relate the force to the resistance of the sensor. To do so we use **Equation (6.4)**:

$$\frac{dR}{R_0} = g\varepsilon \rightarrow dR = g\varepsilon R_0.$$

However, since the gauge is prestressed, its rest resistance is

$$R = R_0 + dR = R_0(1 + g\varepsilon) = 350(1 + 6.9 \times 0.02) = 398.3\ \Omega,$$

where $R_0 = 350\ \Omega$ is the nominal (unstressed) resistance. As the sensor is compressed, the resistance goes down until at maximum allowable strain its resistance is R_0. Thus the change in resistance is $-48.3\ \Omega$.

c. The sensitivity is the output (resistance) divided by the input (force). For any of the sensors the force is 76.86/4 = 19.215 tons and the change in resistance is $-48.3\ \Omega$. Thus

$$S_o = -\frac{48.3}{19.215} = -2.514\ \Omega/\text{ton}.$$

Tactile sensors are force sensors, but because the definition of "tactile" action is broader, the sensors are also more diverse. If one views a tactile action as simply sensing the presence of force, then a simple switch is a tactile sensor. This approach is commonly used in keyboards where membranes or resistive pads are used and the force is applied against the membrane or a silicon rubber layer. In applications of tactile sensing it is often important to sense a force distribution over a specified area (such as the "hand" of a robot). In such cases either an array of force sensors or a distributed sensor may be used. These are usually made from piezoelectric films that generate an electrical signal in response to deformation (passive sensors). An example is shown in **Figure 6.12**. The polyvinylidene fluoride (PVDF) film is sensitive to deformation. The lower film is driven with an AC signal and therefore it contracts and expands mechanically and periodically. This deformation is transferred to the upper film through the compression layer acting somewhat like a transformer and thus establishing a signal at the output. When the upper film is deformed by a force, its signal changes from normal and the amplitude and/or phase of the output signal is now a measure of deformation (force). Since the compression layer is thinner, when a force is applied, the output is higher and proportional (but not necessarily linearly) to the applied force. The PVDF films can be long narrow ribbons for a linear sensor or sheets of various sizes for tactile sensing over an area.

Another example is shown in **Figure 6.13**. In this case the output is normally zero. When a force is applied, the strain in the film gives rise to an output proportional to stress (force) and changing with the force. Because of this the output can be used to sense not only force, but also variations in force. This idea has been used to sense minute changes due to breathing patterns in babies, primarily in hospitals, but it can be used to sense under other conditions. In a sensor of this type, a sheet of PVDF is placed under the patient and its output is monitored for an expected pattern in the signal as the center of gravity of the body shifts with breathing. The issue of piezoelectricity and the associated piezoelectric force sensors will be revisited in **Chapter 7** in conjunction with ultrasonic sensors.

The simplest tactile sensors are made of conductive polymers or elastomers or with semiconducting polymers and are called piezoresistive sensors or force sensitive resistive (FSR) sensors. In these devices, the resistance of the material is pressure dependent and is shown schematically in **Figure 6.14**. A very simple tactile sensor can be made from conducting foam and two electrodes. The resistance of FSR sensors is a nonlinear function of force (**Figure 6.14c**), but the change in resistance is quite high (large dynamic range) and hence the sensor is rather immune to noise and easily interfaced

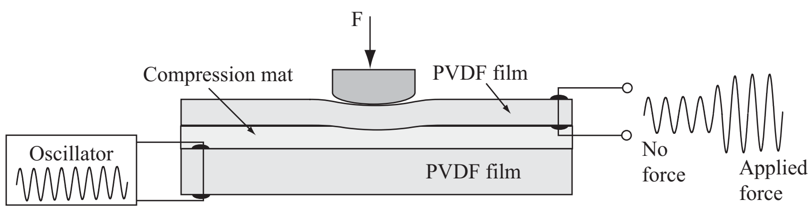


FIGURE 6.12 ■ A piezoelectric film tactile sensor. The compression due to force changes the coupling between the lower and upper PVDF layers and hence the amplitude of the output.

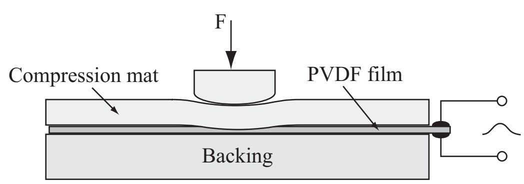


FIGURE 6.13 ■ A piezoelectric film sensor used to detect sliding motion due to breathing. The output is monitored for a pattern consistent with the breathing pattern and the shift in the center of gravity as a consequence.

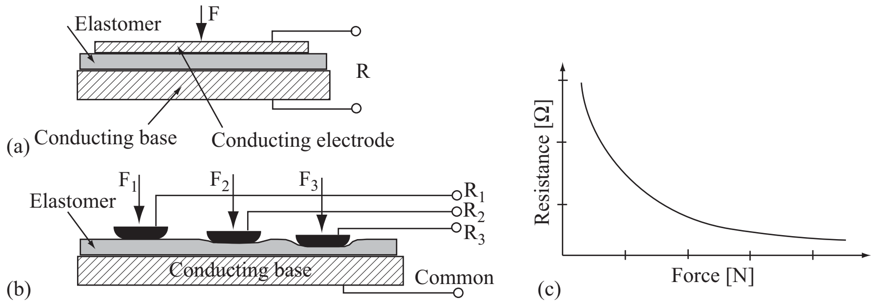


FIGURE 6.14 ■ FSR tactile sensor using conducting elastomers. (a) Principle and structure. (b) An array of tactile sensors. (c) The transfer function of FSRs.

with microprocessors. Either DC or AC sources can be used and the device can be as large or as small as needed. An array of sensors can be built by using one large electrode on one side of the film and multiple electrodes on the other side. In this configuration, sensing occurs over an area or a line (**Figure 6.14b**).

EXAMPLE 6.4 Evaluation of a force sensor

A force sensor (FSR) is evaluated experimentally. To do so, the resistance of the sensor is measured for a range of forces as follows:

F [N]	50	100	150	200	250	300	350	400	450	500	550	600	650
R [Ω]	500	256.4	169.5	144.9	125	100	95.2	78.1	71.4	65.8	59.9	60	55.9

Calculate the sensitivity of the sensor throughout its range.

Solution: Sensitivity is the slope of the resistance versus force curve and is clearly a nonlinear quantity. However, we recall from **Chapter 2** (see **Example 2.16**) that force resistive sensors have a linear relation between force (F) and conductance ($1/R$). Therefore it is simpler to first calculate the conductance.

F [N]	50	100	150	200	250	300	350	400	450	500	550	600
$1/R$ [1/Ω]	.002	.0039	.0059	.0069	.008	.01	.0105	.0128	.014	.0152	.0167	.0179

Now we have two options. We can calculate the sensitivity as a local quantity as

$$S = \frac{\Delta(1/R)}{\Delta F} \quad \left[\frac{1}{\Omega \cdot \mathrm{N}}\right]$$

or start with the resistance and calculate

$$S = \frac{\Delta R}{\Delta F} \quad [\Omega/\mathrm{N}].$$

A better approach is to find a linear fit for the conductance, then write the resistance as a function of force and take the derivative of that.

Using the linear fit in **Equation (2.19)**, we write the conductance $C = 1/R = aF + b$, where

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \left\{\sum_{i=1}^{n} x_i\right\}\left\{\sum_{i=1}^{n} y_i\right\}}{n\sum_{i=1}^{n} x_i^2 - \left\{\sum_{i=1}^{n} x_i\right\}^2}$$

and

$$b = \frac{\left\{\sum_{i=1}^{n} x_i^2\right\}\left\{\sum_{i=1}^{n} y_i\right\} - \left\{\sum_{i=1}^{n} x_i\right\}\left\{\sum_{i=1}^{n} x_i y_i\right\}}{n\sum_{i=1}^{n} x_i^2 - \left\{\sum_{i=1}^{n} x_i\right\}^2},$$

In this case, $n = 12$ is the number of points, x_i is the force at point i, and y_i is the conductance at point i. From the points in the table above, we get

$$a = 0.00014182, b = 0.0010985.$$

The conductance is

$$C = 0.00014182F + 0.0010985 \quad [1/\Omega].$$

Now we can write the resistance as

$$R = \frac{1}{0.00014182F + 0.0010985} \quad [\Omega].$$

The sensitivity is

$$\begin{aligned}\frac{dR}{dF} &= \frac{dR}{dF}(0.00014182F + 0.0010985)^{-1} \\ &= -\frac{0.00014182}{(0.00014182F + 0.0010985)^2} \quad [\Omega/\mathrm{N}].\end{aligned}$$

Clearly sensitivity goes down with force, as can be seen from the table above as well. The negative sign simply indicates that increasing the force reduces the resistance. The method used here applies to other sensors for which a reciprocal function happens to be linear or may be approximated as such.

6.4 | ACCELEROMETERS

By virtue of Newton's second law ($F = ma$), a sensor may be made to sense acceleration by simply measuring the force on a mass. At rest, acceleration is zero and the force on the mass is zero. At any acceleration a, the force on the mass is directly proportional to mass and acceleration. This force may be sensed with any method of sensing force (see above) but, again, the strain gauge will be representative of direct force measurement.

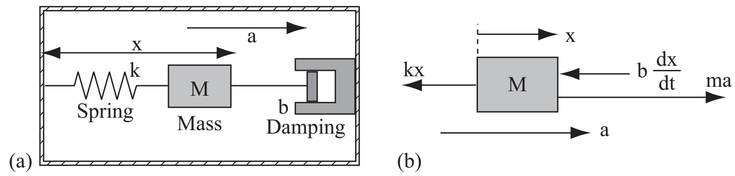


FIGURE 6.15 ■ (a) Mechanical model of an accelerometer based on sensing the force on a mass. (b) Free body diagram of the accelerometer in (a).

There are, however, other methods of sensing acceleration. Magnetic methods and electrostatic (capacitive) methods are commonly used for this purpose. In their simplest forms, the distance between the mass and a fixed surface, which depends on acceleration, can be made into a capacitor whose capacitance increases (or decreases) with acceleration. Similarly a magnetic sensor can be used by measuring the change in field due to a magnetic mass. The higher the acceleration, the closer (or farther) the magnet from a fixed surface and hence the larger or lower the magnetic field. The methods used in **Chapter 5** to sense position or proximity can now be used to sense acceleration. There are other methods of acceleration sensing including thermal methods. Velocity and vibrations may also be measured by similar methods and these will also be discussed in this section.

To understand the method of acceleration sensing it is useful to look at the mechanical model of an accelerometer based on sensing the force on a mass shown in **Figure 6.15**. The mass, which can move under the influence of forces, has a restoring force (spring) and a damping force (which prevents it from oscillating). Under these conditions, and assuming the mass can only move in one direction (along the horizontal axis), Newton's second law may be written as

$$ma = kx - b\frac{dx}{dt} \quad [\mathrm{N}]. \tag{6.9}$$

This assumes that the mass has moved a distance x under the influence of acceleration, k is the restoring (spring) constant, and b is the damping coefficient. Given the mass m and the constants k and b, a measurement of x gives an indication of the acceleration a. The mass is often called an inertial mass or proof mass.

Therefore, for a useful acceleration sensor it is sufficient to provide an element of given mass that can move relative to the sensor's housing and a means of sensing this movement. A displacement sensor (position, proximity, etc.) can be used to provide an appropriate output proportional to acceleration.

6.4.1 Capacitive Accelerometers

In this type of accelerometer, one plate of a small capacitor is fixed and connected physically to the body of the sensor. The second, which serves as the inertial mass of the sensor is free to move and connected to a restoring spring. Three basic configurations are shown in **Figure 6.16**. In these, the restoring force is provided by springs (**Figure 6.16a**, **c**) or by a cantilever beam (**Figure 6.16b**). In **Figures 6.16a** and **6.16b**, the distance between the plates changes with acceleration. In **Figure 6.16c**, the effective area of the capacitor plates changes while the distance between the plates stays constant. In either case, acceleration either increases the capacitance or decreases it, depending on

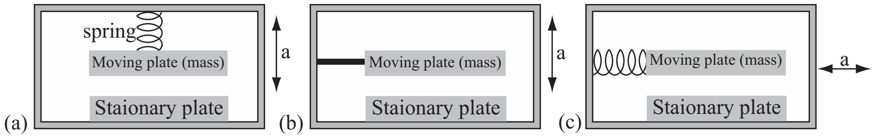


FIGURE 6.16 ■ Three basic capacitive acceleration sensors. (a) Moving plate against a spring. (b) Beam-suspended plate. (c) Sideways moving plate against a spring.

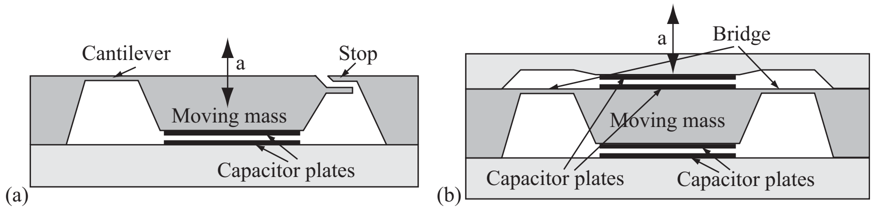


FIGURE 6.17 ■ Two basic forms of producing accelerometers. (a) Cantilever (supported on the left). (b) Bridge.

the direction of motion. Of course, for a practical accelerometer, the plates must be prevented from touching by stoppers and a damping mechanism must be added to prevent the springs or the beam from oscillating. Some of these issues are addressed in the structures in **Figure 6.17**, but regardless of the specific arrangement, the capacitance changes proportional to acceleration and the capacitance is thus a measure of acceleration. It should be noted, however, that the changes in capacitance are very small and therefore, rather than measuring these changes directly, indirect methods such as using the capacitor in an *LC* or *RC* oscillator are often used. In these configurations, the frequency of oscillation is a direct measure of acceleration. The frequency can be easily converted into a digital reading at the output. Accelerometers of this type can be produced as semiconductor devices by etching both the mass, fixed plate, and springs directly into silicon. By doing so, microaccelerometers can be produced quite easily. Two structures are shown in **Figure 6.17**. The first is a cantilever structure. The second is similar to **Figure 6.16a** and relies on etched bridges to provide the springs. In the latter structure, the mass moves between two plates and forms an upper and lower capacitor. By doing so, a differential mode may be obtained since at rest the two capacitors are the same (see **Section 5.3.1** and **Figure 5.10**). In both of these structures, limit stops are provided.

EXAMPLE 6.5 Capacitive accelerometer

Consider a simplified design of a capacitive accelerometer to be used in a car. Its ultimate function is to deploy an airbag in case of collision. Suppose the configuration in **Figure 6.16a** is used and the sensor is mounted so that in case of collision the spring elongates and the plates get closer together, increasing capacitance. Airbags are supposed to deploy when a deceleration of 60 g is detected (equivalent to crashing into a barrier at 23 km/h). The sensor has a fixed plate and a moving plate of mass 20 g. The two plates are separated a distance 0.5 mm apart, producing a capacitance of 330 pF at rest. To trigger airbag deployment, the capacitance must double. To ensure that the capacitance doubles at a deceleration of 60 *g*, the spring constant must be selected carefully. Find the necessary spring constant to accomplish this.

Solution: The capacitance of a parallel plate capacitor is given as

$$C = \frac{\varepsilon A}{d} \quad [\mathrm{F}].$$

This means that to double the capacitance, the distance d must be halved since everything else remains constant with acceleration. That is, the airbag will trigger when the plates are 0.25 mm apart. Under these conditions, the spring has elongated a distance $x = 0.25$ mm. The force equation now requires that the force due to deceleration equal the force on the spring:

$$ma = kx \rightarrow k = \frac{ma}{x} \quad [\mathrm{N/m}],$$

where k is the spring constant, m is the mass of the moving plate, a is deceleration, and x is the displacement of the plate. Thus we get

$$k = \frac{ma}{x} = \frac{20 \times 10^{-3} \times 60 \times 9.81}{0.25 \times 10^{-3}} = 47,088 \ \mathrm{N/m}.$$

Note: The calculation here is rather simple and does not address issues such as keeping the plates parallel. Nevertheless, the calculation indicates how a sensor might be designed. Some acceleration sensors used for this purpose are contact sensors, that is, when the necessary pressure has been achieved, a contact is closed (such as, e.g., the two plates touching each other). This avoids the need to actually measure capacitance and thus decreasing the response time of the sensor.

6.4.2 Strain Gauge Accelerometers

The structures in **Figures 6.16** and **6.17** can also be fitted with strain gauges to measure the strain due to acceleration. A strain gauge accelerometer is shown in **Figure 6.18**. The mass is suspended on a cantilever beam and a strain gauge senses the bending of the beam. A second strain gauge may be fitted under the beam to sense acceleration in both directions. Also, by fitting (or manufacturing) strain gauges on the bridge or cantilever beam in **Figure 6.17**, the capacitive sensor is transformed into a strain gauge sensor. In this configuration the strain gauges are usually semiconductor strain gauges, whereas in **Figure 6.18** they may be bonded metallic gauges. The operation remains the same as for capacitive accelerometers, only the means of sensing the force changes. Strain gauge sensors can be as sensitive as capacitive sensors and in some cases may be easier to work with since the measurement of resistance is typically simpler than that of capacitance. On the other hand, strain gauges are temperature sensitive and must be properly compensated.

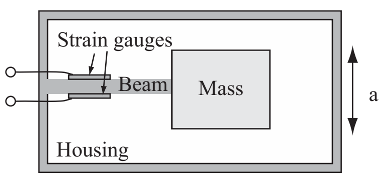


FIGURE 6.18 ■ An accelerometer in which the beam bending is sensed by two strain gauges to sense acceleration in both vertical directions.

6.4.3 Magnetic Accelerometers

A simple magnetic accelerometer can be built as a variable inductance device in which the mass, or a rod connected to and moving with the mass, links magnetically to a coil. The inductance of the coil is proportional to the position of the mass and increases the further the ferromagnetic rod penetrates into the coil (**Figure 6.19**). This configuration is a simple position sensor calibrated for acceleration. Instead of the coil, an LVDT can be used for an essentially linear indication of position. A different approach is to use a permanent magnet as a mass on a spring or cantilever beam and to sense the field of the permanent magnet using a Hall element or a magnetoresistive sensor (**Figure 6.20**). The reading of the Hall element is now proportional to the magnetic field, which is proportional to acceleration. It is also possible to bias the Hall element with a small magnet and use a ferromagnetic mass. In this configuration, the proximity of the mass changes the flux density, providing an indication of acceleration.

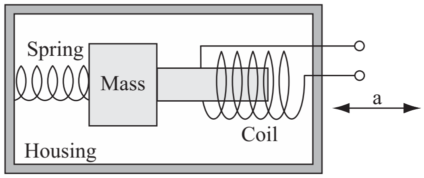


FIGURE 6.19 ■ An inductive accelerometer in which the horizontal motion of the mass is sensed by a change in the inductance of a coil.

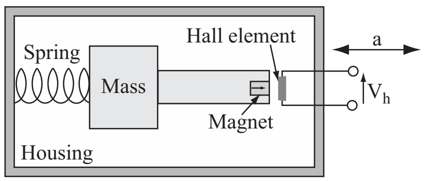


FIGURE 6.20 ■ An accelerometer in which the position of the mass is sensed by a Hall element.

EXAMPLE 6.6 **Magnetic accelerometer**

A magnetic accelerometer is built as in **Figure 6.21** with the mass being a cylinder of diameter $d = 4$ mm and some length l. The mass is 10 g and a spring with spring constant $k = 400$ N/m holds the mass in place. The mass is made of silicon steel with a relative permeability of 4000. The coil has $n = 1$ turn/mm and its inductance is sensed as a measure of the position of the mass. As the mass moves in or out of the coil, its inductance increases or decreases accordingly. In a long coil, the inductance per unit length can be approximated as (**see Equation (5.28)**)

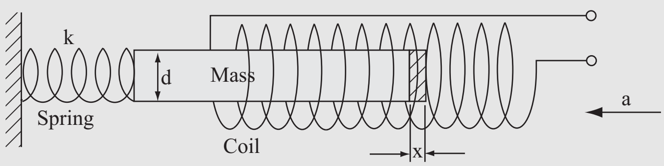


FIGURE 6.21 ■ Magnetic accelerometer.

$$L = \mu n^2 S \quad [\text{H/m}],$$

where n is the number of turns per unit length and S is the cross-sectional area of the coil. Calculate the change in voltage on the coil for an acceleration of $\pm 10\ g$ if the coil is driven with a sinusoidal current of amplitude 0.5 A and a frequency of 1 kHz.

Solution: As the mass moves a distance x into the coil, the inductance changes according to the position. As long as the distances are small, the change in inductance is linear and can be calculated as

$$\Delta L = Lx = \mu n^2 Sx \quad [\text{H}].$$

The maximum distance the mass moves in either direction is determined by the acceleration and the spring constant. That is,

$$ma = kx \rightarrow x = \frac{ma}{k} = \frac{10 \times 10^{-3} \times 10 \times 9.81}{400} = 2.4525 \text{ mm}.$$

The mass moves in or out a maximum of 2.453 mm. The change in inductance is therefore

$$\begin{aligned}\Delta L &= Lx = \mu n^2 Sx \\ &= 4000 \times 4\pi \times 10^{-7} \times 1000^2 \times \pi \times \left(2 \times 10^{-3}\right)^2 \times 2 \times 2.4525 \times 10^{-3} \\ &= 0.00031\,\text{H}.\end{aligned}$$

That is, the inductance changes by $\pm 310\ \mu$H.

The voltage across an inductor is related to its current as (see **Equation (5.29)**)

$$V = L\frac{dI(t)}{dt} \quad [\text{V}].$$

The change in voltage due to the change in inductance is therefore

$$\begin{aligned}\Delta V = \Delta L\frac{dI(t)}{dt} &= 310 \times 10^{-6} \times \frac{d}{dt}(0.5\sin 2\pi \times 1000t) \\ &= 310 \times 10^{-6} \times 0.5 \times 2 \times \pi \times 1000\cos 2\pi \times 1000t \\ &= 0.974\cos 2\pi \times 1000t \quad [\text{V}].\end{aligned}$$

The voltage across the coil changes by ± 0.974 V, a change sufficient for sensing.

Note that this can be increased if necessary by increasing the frequency or by increasing the current in the coil. Also, we have assumed here a linear relation based on the small travel distance and the fact that the coil is long. Friction and damping were neglected.

6.4.4 Other Accelerometers

There are many other types of accelerometers, but all employ a moving mass in one form or another. A good example of the range of principles used for this purpose is the heated gas accelerometer shown in **Figure 6.22**. In this device the gas in a cavity is heated to an equilibrium temperature and two (or more) thermocouples are provided

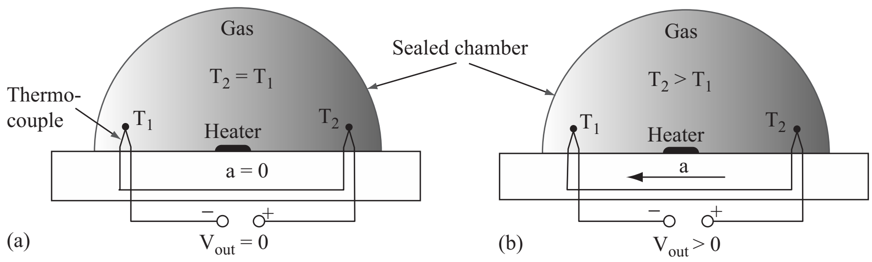


FIGURE 6.22 ■ The heated gas accelerometer.

equidistant from the heater. Under rest conditions, the two thermocouples are at the same temperature and hence their differential reading (one thermocouple is the sense thermocouple, the second is the reference thermocouple) is zero. When acceleration occurs, the gas shifts to the direction opposite of the motion (the gas is the inertial mass), causing a temperature rise that can be calibrated in terms of acceleration.

Other accelerometers use optical means (by activating a shutter by means of the moving mass), optical fiber accelerometers that use an optical fiber position sensor, vibrating reeds whose vibration rate changes with acceleration, and many more.

Finally, it should be noted that multiple-axis accelerometers can be built by essentially using single-axis accelerometers with axes perpendicular to each other. These can be fabricated as two- or three-axis accelerometers, or two or three single-axis accelerometers may be attached appropriately. Although this may seem cumbersome with regular devices, it is entirely practical to do so with microdevices. We will see that this is done routinely in MEMs (**Chapter 10**).

The uses of accelerometers are vast and include airbag deploying sensors, weapons guidance systems, vibration and shock measurement and control, and other similar applications. They can also be found in consumer devices such as telephones and computers, as well as in toys.

EXAMPLE 6.7 A seismic sensor

Detection of seismic activity, such as earthquakes, can be (and often is) undertaken through accelerometers by detecting the motion caused by earthquakes. To do so, an accelerometer is built as follows: A steel bar, 10 mm × 10 mm in cross section, is fixed in a concrete slab and extends vertically 50 cm above the slab. A 12 kg mass is welded to the top of the steel bar. To detect acceleration due to motion of the earth, a semiconductor strain gauge with a nominal resistance of 350 Ω and a gauge factor of 125 is fixed on one of the surfaces of the steel bar at the point where the bar emerges from the concrete slab. Assume the distance between the center of the mass and sensor is exactly 50 cm. Assume as well that the strain gauge is temperature compensated and the minimum change in resistance of the strain gauge that can be reliably measured is 0.01 Ω. Calculate the minimum acceleration the seismic sensor can detect. The modulus of elasticity for steel is 200 GPa.

Solution: The strain gauge relation in **Equation (6.5)** can be used to calculate the strain needed to cause a change in resistance of 0.01 Ω. Then we use the basic relations for beam bending to find the acceleration that will produce that strain. From **Equation (6.5)**:

$$R(\varepsilon) = R(1 + g\varepsilon) = 350 + 125\varepsilon \quad [\Omega].$$

Therefore

$$125\varepsilon = 0.01\ \Omega$$

or

$$\varepsilon_{\min} = \frac{0.01}{125} = 0.00008.$$

That is, an 80 microstrain will produce a change of 0.01 Ω in the resistance of the strain gauge.

As the soil moves, the acceleration a produces a force on the mass ($m = 12$ kg):

$$F = ma \quad [\mathrm{N}].$$

The force bends the beam, causing a bending moment:

$$M = Fl = mal \quad [\mathrm{N \cdot m}],$$

where $l = 50$ cm is the distance between the mass and the sensor.

To calculate the strain at the surface of the beam (where the strain gauge is located) we write

$$\varepsilon = \frac{M(d/2)}{EI} \quad [\mathrm{m/m}],$$

where M is the bending moment, E is the modulus of elasticity, I is the moment of area of the beam, and d is the thickness of the beam. E is given and I is

$$I = \frac{bh^3}{12} = \frac{d^4}{12} \quad [\mathrm{m^4}],$$

where b is the width and h is the height of the beam cross section. In this case $b = h = d = 0.01$ m and we get the strain in the bar:

$$\varepsilon = \frac{mal(d/2)}{Ed^4/12} = \frac{6mal}{Ed^3} \quad [\mathrm{m/m}].$$

The minimum acceleration detectable is

$$a = \frac{\varepsilon Ed^3}{6ml} \quad [\mathrm{m/s^2}].$$

For the numerical values given,

$$a = \frac{0.00008 \times 200 \times 10^9 \times (0.01)^3}{6 \times 12 \times 0.5} = 0.444\ \mathrm{m/s^2}.$$

This is a small acceleration (about 0.045 g). Note as well that the accelerometer can be made more sensitive in a number of ways. First and foremost one can use a less "stiff" bar, that is, a bar with a lower modulus of elasticity. A bigger mass and longer bar will do the same thing. Similarly one can increase sensitivity by reducing the cross section of the bar, but of course one must come up with a reasonable compromise. For example, one cannot use a much thinner bar for the mass given, or alternatively, as one increases the mass the cross section of the bar must also be adjusted to support the mass. Finally, we note that the calculation here assumes acceleration is perpendicular to the surface on which the strain gauge is placed. Since one cannot predict the direction of acceleration in the case of earthquakes, it is necessary that at least two surfaces of the bar be equipped with strain gauges on perpendicular surfaces and the acceleration calculated from the two perpendicular components of acceleration.

6.5 | PRESSURE SENSORS

Sensing of pressure is perhaps only second in importance to sensing of strain in mechanical systems (and strain gauges are often used to sense pressure). These sensors are used either in their own right, that is, to measure pressure, or to sense secondary quantities such as force, power, temperature, or any quantity that can be related to pressure. One of the reasons for their prominence in the realm of sensors is that in sensing in gases and fluids, direct measurement of force is not an attractive option—only pressure can be measured and related to properties of these substances, including the forces they exert. Another reason for their widespread use and of exposure of most people to them is their use in cars, atmospheric weather prediction, heating and cooling, and other consumer-oriented devices. Certainly the "barometer" hanging on many a wall and the use of atmospheric pressure as an indication of weather conditions has helped popularize the concept of pressure.

The sensing of pressure, which is force per unit area, follows the same principle as the sensing of force—that of measuring the displacement of an appropriate member of the sensor in response to pressure. Therefore it is not surprising to find that the same principles used for sensing of force are used to sense pressure. Any device that will respond to pressure either by direct displacement or equivalent quantities (such as strain) is an appropriate means of sensing pressure. Thus the range of methods is quite large and includes thermal, mechanical, as well as magnetic and electrical principles.

6.5.1 Mechanical Pressure Sensors

Historically the sensing of pressure started with purely mechanical devices that did not require electrical transduction—a direct transduction from pressure to mechanical displacement was used. As such, these devices are actuators that react to pressure and, perhaps surprisingly, are as common today as ever. Some of these mechanical devices have been combined with other sensors to provide electrical output, while others are still being used in their original form. Perhaps the most common is the bourdon tube, shown in **Figure 6.23**. This sensor has been used for more than 150 years in pressure gauges, in which the dial indicator is connected directly to the tube (invented by Eugene Bourdon in 1849). This type of sensor, in different forms, is still the most common pressure gauge used today, and because it does not need additional components, it is simple and inexpensive. However, it is only really useful for relatively high pressures. It is typically used for gases, but it can also be used to sense fluid pressure.

Other methods of sensing pressure mechanically are the expansion of a diaphragm, the motion of a bellows, and the motion of a piston under the influence of pressure.

FIGURE 6.23 ■ The bourdon tube pressure sensor. The bourdon tube (C-shaped portion) expands with pressure, turning the dial (below the bezel, not seen) through a leverage arm and gear mechanism.

FIGURE 6.24 ■ The diaphragm pressure sensor.

The motion produced can be used to directly drive an indicator or can be sensed by a displacement sensor (LVDT, magnetic, capacitive, etc.) to provide a reading of pressure. A simple diaphragm pressure sensor used in wall barometers is shown in **Figure 6.24**. It is essentially a sealed metal can with relatively flexible walls. One side is held fixed (in this case by the small screw that also serves to adjust or calibrate it) while the other moves in response to pressure. This particular device is hermetically sealed at a given pressure so that any pressure below the internal pressure will force the diaphragm to expand and any higher pressure will force it to contract. While very simple and trivially inexpensive, it is easy to see its drawbacks, including the possibility of leakage and the inevitable dependence on temperature. A bellows is a similar device that can be used for direct reading or to activate another sensor. The bellows, in various forms, can also be used as an actuator. One of its common uses is in "vacuum motors," used in vehicles to activate valves and to move slats and doors, particularly in heating and air conditioning systems and in speed controls. They have survived in these roles into the modern era, mostly in vehicles, because of their simplicity, quiet operation, and the availability of a

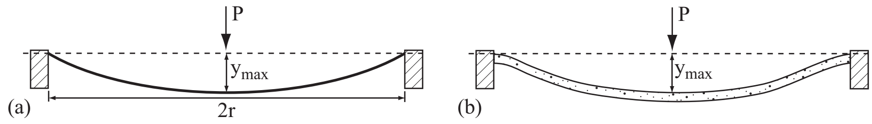


FIGURE 6.25 ■ (a) The thin plate. (b) The membrane.

source of low pressure (hence the use of the name vacuum) in internal combustion engines.

These mechanical devices indicate the need for a mechanism that can deflect under the influence of pressure. By far the most common structures used for this purpose are the thin plate and the diaphragm or membrane. In simple terms, a membrane is a thin plate with negligible thickness, whereas a plate has a finite thickness. Their behavior and response to pressure is very different. In relation to **Figure 6.25a**, the deflection of the center of a membrane (maximum deflection) that is under radial tension S and the stress in the diaphragm are given as

$$y_{\max} = \frac{r^2 P}{4S} \quad [\mathrm{m}], \quad \sigma_m = \frac{S}{t} \quad [\mathrm{N/m^2}], \tag{6.10}$$

where P is the applied pressure (actually, the pressure difference between the top and bottom of the membrane), r is its radius, and t is its thickness. Strain can be calculated by dividing the strain by the modulus of elasticity (Young's modulus).

If, on the other hand, the thickness t is not negligible, the device is a thin plate (**Figure 6.25b**) and the behavior is given as

$$y_{\max} = \frac{3(1-\nu^2)r^4 P}{16Et^2} \quad [\mathrm{m}], \quad \sigma_m = \frac{3r^2 P}{4t^2} \quad [\mathrm{N/m^2}], \tag{6.11}$$

where E is Young's modulus and v is Poisson's ratio.

In either case, the displacement is linear with pressure, hence the widespread use of these structures for pressure sensing. The displacement $y_{\max}$ or the stress σ_m (or the equivalent strain) are measured depending on the type of sensor used. In modern sensors it is actually more common to measure strain using either a metal or, even more commonly, a semiconductor strain gauge or a piezoresistor. One advantage of using strain gauges is that the displacement needed is very small, allowing for very rugged construction and sensing of very high pressures. If displacement must be measured, this can be done capacitively, inductively, or even optically.

Pressure sensors come in four basic types, defined in terms of the pressure they sense. These are

Absolute pressure sensors (**PSIA**): pressure is sensed relative to absolute vacuum.

Differential pressure sensors (**PSID**): the difference between two pressures on two ports of the sensor is sensed.

Gauge pressure sensors (**PSIG**): senses the pressure relative to ambient pressure.

Sealed gauge pressure sensor (PSIS): the pressure relative to a sealed pressure chamber (usually 1 atm at sea level or 14.7 psi) is sensed.

The most common sensors are gauge sensors, but differential sensors are often used, as are sealed gauge sensors.

EXAMPLE 6.8 A piston-based mechanical pressure sensor

In a manner similar to the diaphragm, a piston acting against a spring can serve as a simple pressure sensor. This mechanical sensor/actuator is commonly used to measure pressure in tires and is shown schematically in **Figure 6.26**. A typical gauge rated at 700 kPa (100 psi) is made as a short cylinder, about 15–20 cm long and 10–15 mm in diameter. A valve at the bottom allows one-way gas entry to pressurize the cylinder. The inner stem moves against the spring and the graduations on the stem then indicate the pressure. The stem typically extends a maximum of about 5 cm so that it operates in the linear range of the spring. For a typical inner diameter of 10 mm, a pressure of 700 kPa generates a force on the piston of

$$F = PS = 700 \times 10^3 \times \pi \times \left(5 \times 10^{-3}\right)^2 = 54.978 \text{ N}.$$

The spring must compress 50 mm. This means the spring constant k, must be

$$F = kx \rightarrow k = \frac{F}{x} = \frac{54.978}{0.05} = 1100 \text{ N/m}.$$

The pressure is read on the graduation and these are at 14 kPa/mm (approximately 2 psi/mm). This, of course, is the sensitivity of the sensor.

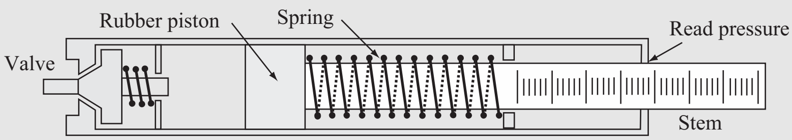


FIGURE 6.26 ■ Piston-based mechanical pressure sensor.

EXAMPLE 6.9 The sealed gauge pressure sensor as a simple barometer

The barometer shows air pressure usually by mechanical means on a simple dial. A simple sealed gauge pressure sensor similar in principle to the one shown in **Figure 6.24** can be used for this purpose. To see in more detail how this can be accomplished, consider the device in **Figure 6.27**, which consists of a chamber, sealed by a piston. The chamber is sealed at a standard atmospheric pressure, $P_0 = 101{,}325$ Pa (1.013 bar or 1013.25 mbar). The air pressure is indicated on a linear scale with reference to normal pressure. As the external pressure increases, the piston is pushed down, compressing the air in the chamber. When the external pressure decreases, the air expands, allowing the piston to move up. Air pressure in the atmosphere varies by small amounts (the highest pressure ever recorded was 1086 mbar and the lowest pressure ever recorded was 850 mbar) so a scale between 800 mbar and 1100 mbar is sufficient. Calculate the range of motion of the piston (i.e., the length of the scale).

Solution: The compression of the sealed air is governed by Boyle's law (under constant temperature conditions):

$$P_1 V_1 = P_2 V_2.$$

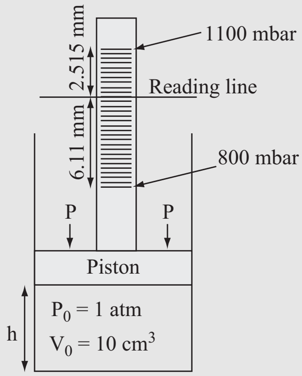


FIGURE 6.27 ■ A sealed chamber barometer.

That is, as pressure changes, the volume changes accordingly to keep the product constant. Now, taking the volume at the nominal pressure $P_0 = 1013.25$ mbar to be $V_0 = 10\ \text{cm}^3$, we write for the volumes at minimum and maximum pressure

$$P_{\min}V_{\min} = P_0V_0 \rightarrow V_{\min} = \frac{P_0V_0}{P_{\min}} = \frac{1013.25 \times 10}{850} = 11.92\ \text{cm}^3$$

and

$$P_{\max}V_{\max} = P_0V_0 \rightarrow V_{\max} = \frac{P_0V_0}{P_{\max}} = \frac{1013.25 \times 10}{1100} = 9.21\ \text{cm}^3.$$

The scale is found from the height of the displaced air column:

At low pressure,

$$V_{\min} = \pi\frac{d^2}{4}h_{\min} = 11.92 \rightarrow h_{\min} = \frac{4 \times 11.92}{\pi d^2} = \frac{4 \times 11.92}{\pi \times 2^2} = 3.7942\ \text{cm}.$$

At high pressure,

$$V_{\max} = \pi\frac{d^2}{4}h_{\max} = 9.21 \rightarrow h_{\max} = \frac{4 \times 9.21}{\pi \times 2^2} = 2.9316\ \text{cm}.$$

At nominal pressure the height is

$$h_0 = \frac{4 \times 10}{\pi \times 2^2} = 3.1831\ \text{cm}.$$

That is, given the position of the line at nominal pressure, the lowest pressure line is 6.11 mm below and the highest pressure point is 2.515 mm above the nominal pressure line. The whole range of motion is 8.625 mm.

Note that reducing the diameter by a factor of two increases the range by a factor of four (in this case, to 34.5 mm). In the fluid barometer, the piston is replaced by a fluid—usually water or oil—that serves not only as a "piston," but also as a direct indication of pressure. In other barometers the motion of the piston, or its equivalent, turns a dial.

6.5.2 Piezoresistive Pressure Sensors

Although a piezoresistor is simply a semiconductor strain gauge, and can always be replaced with a conductor strain gauge, most modern pressure sensors use it rather than the conductor-type strain gauge. Only when higher temperature operation is needed or for specialized applications are conductor strain gauges preferred. In addition, the diaphragm itself may be fabricated of silicon, a process that simplifies construction and allows for additional benefits such as onboard temperature compensating elements, amplifiers, and conditioning circuitry. The basic structure of a sensor of this type is shown in **Figure 6.28**. In this case, the two gauges are parallel to one dimension of the diaphragm. The change in resistance of the two piezoresistors is

$$\frac{\Delta R_1}{R_1} = -\frac{\Delta R_2}{R_2} = \frac{1}{2}\pi\left(\sigma_y - \sigma_x\right), \tag{6.12}$$

where σ_x and σ_y are the stresses in the transverse directions. Although other types of arrangements of the piezoresistors will result in different values for the change in resistance (e.g., R_2 in **Figure 6.29** can be placed perpendicular to R_1), this formula is representative of the expected values. In the device in **Figure 6.29**, both the piezoresistors and the diaphragm are fabricated of silicon. In this case, a vent is provided, making this a gauge sensor. If the cavity under the diaphragm is hermetically sealed and the pressure in it is P_0, then the sensor becomes a sealed gauge pressure sensor sensing the pressure $P - P_0$. A differential sensor is produced by placing the diaphragm between two chambers, each vented through a port, as shown in **Figure 6.29**.

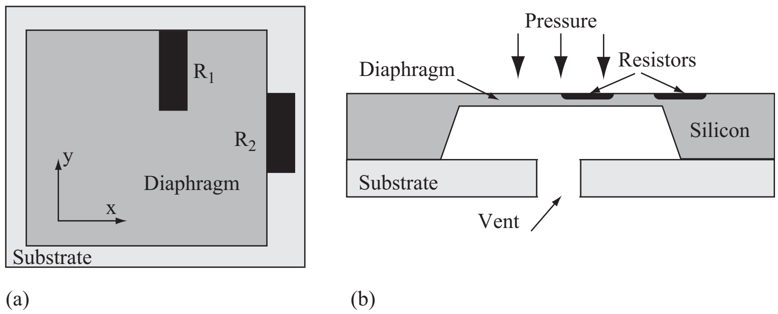


FIGURE 6.28 ■ A piezoresistive pressure sensor. (a) Placement of the piezoresistances. (b) Construction showing the diaphragm and vent hole (for gauge pressure sensors).

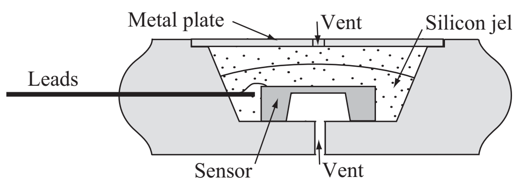


FIGURE 6.29 ■ Construction of a differential pressure sensor. The diaphragm is placed between the two ports.

A different approach is to use a single strain gauge, as in **Figure 6.30**, with a current passing through it and pressure applied perpendicular to the current. The voltage across the element is measured as an indication of the stress, and thus pressure.

There are many variations on these basic types of sensors with different materials and processes, different sensitivities, etc., but these do not constitute separate types of sensors and will not be discussed separately.

Although the most common method of sensing is through the use of semiconductor strain gauges, the construction of the body of the sensor and, in particular, that of the diaphragm varies based on applications. Stainless steel, titanium, and ceramics are used in corrosive environments and other materials, including glass, can be used for coatings.

Figures 6.31 and **6.32** show a number of pressure sensors of various constructions, sizes, and ratings.

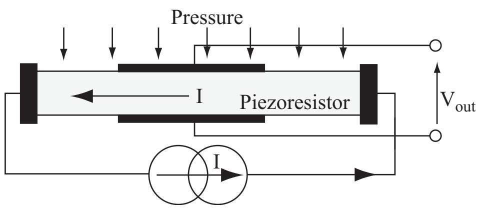


FIGURE 6.30 ▪ A single piezoresistance pressure sensor. The potential across the resistor is a measure of pressure. Pressure is applied perpendicular to the current.

(a) (b) (c)

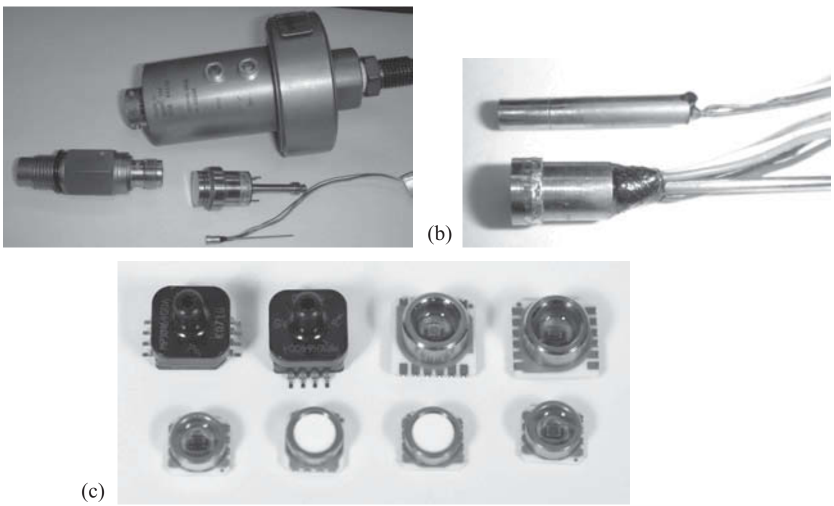
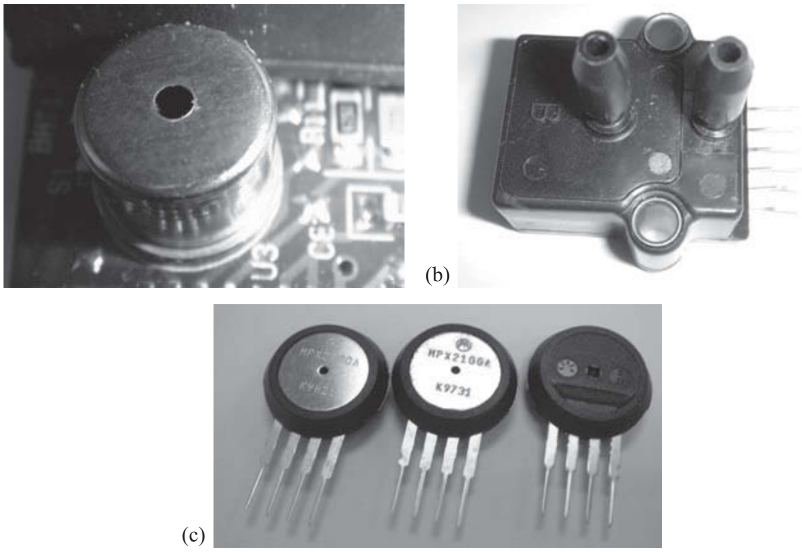


FIGURE 6.31 ▪ Various pressure sensors. (a) Pressure sensors of various sizes. The smallest is 2 mm in diameter, the largest is 30 mm in diameter. Note the connectors. All are sealed gauge pressure sensors. (b) Small sensors in stainless steel housings (absolute pressure sensors). (c) Miniature surface-mount digital pressure sensors (from top left, clockwise: 14 bar, 7 bar, 1 bar, and 12 bar) sealed gauge sensors.



(a) (b) (c)

FIGURE 6.32 ■ Various pressure sensors. (a) A 100 psi absolute pressure sensor in a metal can. (b) A 150 psi differential pressure sensor intended for automotive applications. (c) Three 15 and 30 psi gauge pressure sensors.

EXAMPLE 6.10 Water depth sensor

A depth sensor for an autonomous diving vehicle is built as a thin disk of stainless steel of radius 6 mm, 0.5 mm thick, supported by a ring with an inner radius of 5 mm and an outer radius of 6 mm, as shown in **Figure 6.33**. The top is open to the water and the bottom is sealed at atmospheric pressure (1 atm) before lowering the vehicle into the water. A radially oriented strain gauge is attached to the lower part of the disk and senses the strain in the disk. Find a relation for the resistance of the strain gauge with depth if it has nominal resistance of 240 Ω and a gauge factor of 2.5. Assume atmospheric pressure of 1 atm, the modulus of elasticity of stainless steel is 195 GPa, and the average water density is 1025 kg/m^3.

Solution: The sensor is in effect a sealed gauge pressure sensor (PSIG) with a thin plate as a transducer. The pressure in **Equation (6.11)** is the water pressure on the top of the disk minus the sealed pressure of 1 atm. Thus we first need to calculate the pressure as a function of depth. The latter is rather simple. The pressure at the surface is 1 atm. It increases by 1 atm for every 1.032 m of depth (see section on units). Given that 1 atm equals 101.325 kPa, the pressure in water as a function of depth from the surface can be written as

$$P = \frac{d}{1.032} \times 101.325 + 101.325 \quad [\text{kPa}],$$

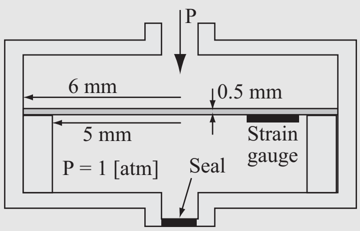


FIGURE 6.33 ■ A water depth sensor. This is in effect a sealed gauge pressure sensor measuring the difference between the pressure of the water and the pressure at the surface of the ocean (1 atm).

where d is the depth in meters. The pressure sensed by the sensor described here is

$$P = \frac{d}{1.032} \times 101{,}325 \quad [\text{Pa}]$$

because of the sealed pressure of 1 atm. This means that the sensor will measure zero pressure at the surface of the water. Therefore the depth is measured directly as

$$d = \frac{1.032P}{101325} = 10^{-5}P \quad [\text{m}].$$

To calculate the change in resistance of the sensor we use **Equation (6.5)**, which in turn requires the strain in the disk. Therefore we first calculate the stress using **Equation (6.11)** and then divide stress by the modulus of elasticity to find the strain. The stress in the disk is

$$\sigma_m = \frac{3r^2P}{4t^2} = \frac{1.032 \times 3r^2dP}{1.032 \times 4t^2} = 10^5 \frac{r^2d}{t^2}\frac{3}{4} \quad [\text{N/m}^2].$$

Dividing by the modulus of elasticity we get the strain:

$$\varepsilon_m = \frac{\sigma_m}{E} = 7.5 \times 10^4 \frac{r^2d}{Et^2} \quad [\text{m/m}].$$

Now we substitute this in **Equation (6.5)**:

$$R(\varepsilon) = R(1 + g\varepsilon) = R\left(1 + 7.5 \times 10^4 \frac{gr^2d}{Et^2}\frac{3}{4}\right) \quad [\Omega],$$

where R is the nominal resistance of the strain gauge and g is the gauge factor. Therefore the change in resistance due to depth is

$$\Delta R = 7.5 \times 10^4 \frac{gRr^2d}{Et^2}\frac{3}{4} \quad [\Omega].$$

Another way to write this is

$$d = \frac{Et^2}{7.5 \times 10^4 gRr^2}\frac{4}{3}\Delta R \quad [\text{m}].$$

In this form the depth is immediately available as a function of ΔR, the difference in resistance between the sensed resistance and the nominal resistance of the strain gauge. For the values given here,

$$d = \frac{Et^2}{7.5 \times 10^4 gRr^2}\frac{4}{3}\Delta R = \frac{195 \times 10^9 \times (0.0005)^2 \times 4}{7.5 \times 10^4 \times 2.5 \times 240 \times (0.005)^2 \times 3}\Delta R = 57.78\Delta R \quad [\text{m}].$$

That is, for every 1 m depth, the strain gauge resistance will change by 1/57.78 = 0.0173 Ω. This is a simple calibration curve, and as long as the strain gauge is temperature compensated, the depth can be measured accurately.

Note: A higher gauge factor strain gauge (such as a semiconductor strain gauge) can increase the sensitivity. For example, a gauge factor of 125 (not unusual for semiconductor strain gauges) will increase the change in resistance to 0.865 Ω per meter of depth.

6.5.3 Capacitive Pressure Sensors

The deflection of the diaphragm in any of the structures described above with respect to a fixed conducting plate constitutes a capacitor in which the distance between the plates is pressure sensitive. The basic structure in **Figure 6.17** may be used or a similar configuration devised. These sensors are very simple and are particularly useful for sensing of very low pressures. At low pressure, the deflection of the diaphragm may be insufficient to cause large strain but can be relatively large in terms of capacitance. Since the capacitance may be part of an oscillator, the change in its frequency may be sufficiently large to make for a very sensitive sensor. Another advantage of capacitive pressure sensors is that they are less temperature dependent and, because stops on motion of the plate can be incorporated, they are less sensitive to overpressure. Usually overpressures two to three orders of magnitude larger than rated pressure may be easily tolerated without ill effects. The sensors are linear for small displacements, but at greater pressures the diaphragm tends to bow, causing nonlinear output.

6.5.4 Magnetic Pressure Sensors

A number of methods are used in magnetic pressure sensors. In large deflection sensors an inductive position sensor or an LVDT attached to the diaphragm can be used. However, for low pressures, the so-called variable reluctance pressure sensor is more practical. In this type of sensor the diaphragm is made of a ferromagnetic material and is part of the magnetic circuit shown in **Figure 6.34a**. The reluctance of a magnetic circuit is the magnetic equivalent of resistance in electric circuits and depends on the sizes of the magnetic paths, their permeability, and their cross-sectional areas (see **Section 5.4** for a discussion of magnetic circuits and **Equation (5.32)** for a definition of magnetic reluctance). **Figure 5.23b** shows the equivalent circuit where $\mathcal{R}_f$ indicates paths in iron, $\mathcal{R}_g$ indicates gaps and $\mathcal{R}_d$ indicates paths in the diophragm. If the magnetic core (the E-shaped path in **Figure 6.34**) and the diaphragm are made of high-permeability ferromagnetic materials, their reluctance is negligible. In this case the reluctance is directly proportional to the length of the air gap between the diaphragm and the E-core. As pressure changes, this gap changes and the inductance of the two coils changes accordingly. This inductance can be sensed directly, but more often the current in the circuit made of a fixed impedance and a variable impedance due to the motion of the diaphragm is measured, as shown in **Figure 6.34c**. The advantage of a sensor of this type is that a small deflection can cause a large change in the inductance of the circuit, making for a very sensitive device. In addition, magnetic sensors are almost devoid of temperature sensitivity, allowing operation at elevated or variable temperatures.

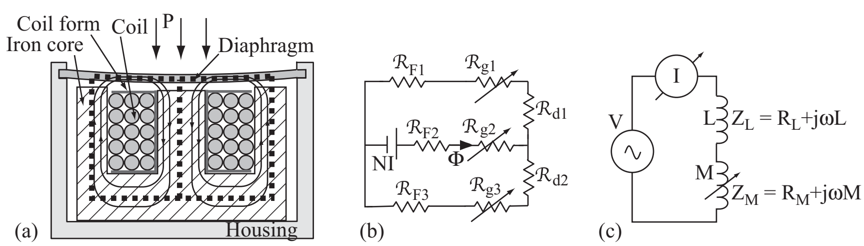


FIGURE 6.34 ■ A variable reluctance pressure sensor. (a) Structure and operation. (b) Equivalent circuit in terms of reluctances. (c) Operation with an AC source. The core and diaphragm are circular.

There are a number of other types of pressure sensors that rely on diverse principles. Optoelectronic pressure sensors use the principle of a Fabri–Perot optical resonator to measure exceedingly small displacements. In a resonator of this type, light reflected from a resonant optical cavity is measured by a photodiode to produce a measure of the sensed pressure. Another, very old method of sensing low pressures (hence they are often called vacuum sensors) is the Pirani gauge. It is based on measuring the heat loss from gases, which is dependent on pressure. The temperature of a heated element in the gas flow is sensed and correlated to pressure, usually in an absolute pressure sensor arrangement.

The properties of pressure sensors vary considerably depending on construction and on the principles used. Typically semiconductor-based sensors can only operate at low temperatures ($-50°C$ to $+150°C$). Their temperature-dependent errors can be high unless properly compensated either externally or internally. The range of sensors can exceed 300 GPa (50,000 psi) and can be as small as a few pascals. Impedance is anywhere between a few hundred ohms to about 100 kΩ, again depending on the type of device. Linearity is between 0.1% and 2% and response time is typically less than 1 ms. The maximum pressure, burst pressure, and proof pressure (overpressure) are all part of the specifications of the device, as is its electrical output, which can be either direct (no internal circuitry and amplification) or after conditioning and amplification. Digital outputs are also available. As indicated above, the materials used (silicon, aluminum, titanium, stainless steel, etc.) and compatibility with gases and liquids are specified and must be followed to avoid damage and incorrect readings. Other specifications are the port sizes and shapes, connectors, venting ports, and the like. The cycling of pressure sensors is also specified, as are hysteresis (usually less than 0.1% of full scale) and repeatability (typically less than 0.1% of full scale).

6.6 | VELOCITY SENSING

Velocity sensing is actually more complicated than acceleration sensing and often requires indirect methods. This is understandable from the fact that velocity is relative and thus requires a reference. Of course, one can always measure something proportional to velocity. For example, we can infer the velocity of a car from the rotation of the wheels (or the transmission shaft—a common method of velocity measurement in cars) or count the number of rotations of a shaft per unit time in an electric motor, or, indeed, use GPS for that purpose. In other applications, such as aircraft, velocity may be inferred from pressure or from temperature sensors measuring the cooling effect of moving air. However, a free-standing sensor that measures velocity directly is much more difficult to produce. One approach that may be used is the induction of emf in a coil due to a moving magnet. However, this requires that the coil be stationary and, if the velocity is constant (no acceleration), the magnet cannot move relative to the coil since the coil must have a restoring force (spring). For changing velocity (when acceleration is not zero), the principle in **Figure 6.35** may be useful. The emf induced in the coils is governed by Faraday's law:

$$emf = -N\frac{d\Phi}{dt} \quad [\mathrm{V}], \tag{6.13}$$

where N is the number of turns and Φ is the flux in the coil. The time derivative indicates that the magnet must be moving to produce a nonzero change in flux.

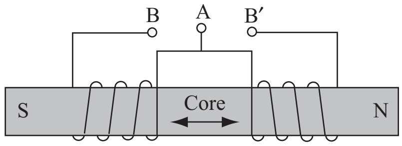


FIGURE 6.35 ■ A velocity sensor. The induced emf in the coils is proportional to the velocity of the magnet.

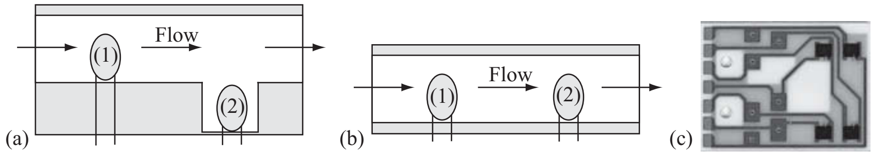


FIGURE 6.36 ■ Flow velocity sensor. (a) The downstream temperature sensor (2) is shielded from the flow but measures the air (or fluid) temperature. (b) The downstream sensor (2) is also in the flow, but is cooled less because of the heat transfer from the upstream sensor (1). (c) A fluid velocity sensor showing four deposited thermistors on a ceramic substrate (right-hand side of the picture). Flow is from top to bottom and the sensors are connected in a bridge configuration. A reference thermistor is placed on the reverse side of the substrate along with a temperature sensor.

Thus the most common approach to velocity sensing is to use an accelerometer and integrate its output using an integrating amplifier. Since velocity is the time integral of acceleration, the velocity is easily obtained, but, as before, constant velocity cannot be sensed (zero acceleration). Fortunately, in many instances, the velocity may be measured directly without the use of specific sensors. The speed of vehicles is one example, where the speed relative to the stationary ground can be measured in many ways.

The velocity of fluids and gasses can be sensed quite easily. The velocity of watercraft and aircraft can also be measured relative to stationary or moving fluids. However, the methods used to do so are all indirect. One simple method of fluid velocity sensing is to sense the cooling of a thermistor relative to a thermistor that is not exposed to the fluid flow. This is particularly useful for air flow or sensing of air velocity in an aircraft (**Figure 6.36a, b**). The downstream sensor (2) can be shielded from the flow, as in **Figure 6.37a**, or it can be in the flow, as in **Figure 6.36b**. In the first case, the downstream sensor remains at a constant temperature that only depends on the static temperature (independent of flow), whereas in the second, the upstream sensor is cooled by the flow while the downstream sensor is cooled much less because of the heated fluid downstream caused by the upstream sensor. In either case, the temperature difference can be related to fluid velocity (or, if needed, to fluid mass flow). A similar method is used to measure air mass in the intake of vehicles (will be discussed in **Chapter 10**). **Figure 6.36c** shows a fluid flow sensor that uses four thermistors, two downstream and two upstream, arranged in a bridge configuration. The sensors are deposited on a ceramic substrate about 15 mm × 20 mm in size. In essence, the temperature difference between the upstream and downstream sensors is measured and correlated with fluid velocity or fluid flow. Since the transfer function depends on the actual temperature of the fluid, an additional thermistor (on the other side of the substrate) measures the fluid temperature directly, and since this sensor is in a stagnant section of the fluid, it is not affected by the flow. In some

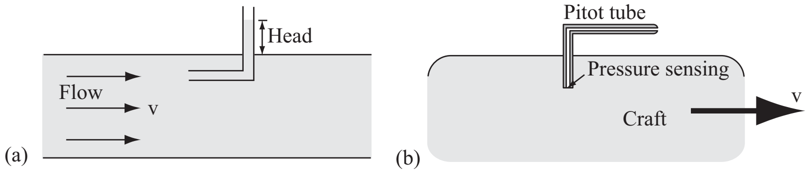


FIGURE 6.37 ■ The Pitot tube. (a) The original use was to measure water velocity and flow rates in rivers. (b) The modern adaptation to measure airspeed in an aircraft or the relative speed in a fluid. What is measured is the total (stagnant) pressure in the tube.

applications, especially at high temperatures, thermistors may be replaced with RTDs. In others, two transistors or two diodes can serve the same purpose (see **Chapter 3**).

Another common method of sensing speed is based on differential pressure: the change in pressure due to motion in the fluid gives an indication of the speed. This is a standard method of speed sensing in modern aircraft (including commercial aircraft) and is based on one of the oldest sensors in existence, the Pitot tube. The basic method, that due to Henry Pitot, dates to 1732 and was initially used to measure water speed in rivers. The principle is shown in **Figure 6.37a**. As water velocity increases, the total pressure in the tube increases and the water head rises, indicating fluid speed (or, if properly calibrated, flow rate). The modern pitot tube as used in aircraft consists of a bent tube with its opening facing forward (parallel to the body of the aircraft), shown in **Figure 6.37b**. The tube is either sealed at the interior end and the pressure at that point is measured or the pressure is allowed to act against a mechanical indicator (such as the diaphragm pressure sensor in **Figure 6.24** or even a bourdon tube) or a pressure sensor can be used. Under these conditions, the total pressure in the tube (also called the stagnant pressure since the fluid is constrained from moving) is given by Bernouli's principle:

$$P_t = P_s + P_d \quad [\mathrm{Pa}], \tag{6.14}$$

where P_t is the total (or stagnant) pressure, P_s is the static pressure, and P_d is the dynamic pressure. In the case of the aircraft, P_s is the pressure one would measure if the aircraft were stationary (i.e., the atmospheric pressure), whereas P_d is the pressure due to the motion of the aircraft (or, in water, the motion of a boat). The latter is given as

$$P_d = \rho\frac{V^2}{2} \quad [\mathrm{Pa}], \tag{6.15}$$

where ρ is the density of the fluid (air, water) and V is the velocity of the craft. Since the interest here is measurement of velocity:

$$V = \sqrt{\frac{2(P_t - P_s)}{\rho}} \quad [\mathrm{m/s}]. \tag{6.16}$$

The density, ρ, can be measured separately or may be known (such as in the case of water). For flight speed purposes it is important to remember that density (and pressure) varies with altitude. The density may be deduced from pressure as (approximately)

$$\rho = \frac{P_t}{RT} \quad [\mathrm{kg/m^3}], \tag{6.17}$$

where R is the specific gas constant (equal to 287.05 J/kg/K for dry air) and T [K] is the absolute temperature. More accurate relations take into account humidity through the use of vapor pressure, but this approximation is often sufficient in "dry air" conditions. The static pressure at height h in the atmosphere can be calculated from the altitude (or the altitude can be calculated from static pressure) using the following relation:

$$P_s = P_0\left(1 - \frac{Lh}{T_0}\right)^{\frac{gm}{RL}} \quad [\text{Pa}], \tag{6.18}$$

where P_0 is the standard pressure at sea level (101,325 Pa), L is the temperature change with elevation, also called temperature lapse rate (0.0065 K/m), h is the altitude in meters, T_0 is the standard temperature at sea level (288.15 K), $g = 9.80665\ \text{m/s}^2$ is the gravitational acceleration, m is the molar mass of dry air (0.0289644 kg/mol), and R is the gas constant (8.31447 J/mol/K). A simpler relation, often called the barometric equation, is given as follows:

$$P_s(h) = P_0 e^{-\frac{Mgh}{RT_0}} \quad [\text{Pa}]. \tag{6.19}$$

Although this formula overestimates the pressure at height h, it is commonly used, and in fact serves as the basis of many altimeters, including those in aircraft.

Because the difference between the total pressure and the static pressure is needed to measure speed, the Pitot tube has been modified to measure static pressure independently. This is known as the Prandtl tube (although it is most often called simply a Pitot tube or a pitot-static tube) and is shown in **Figure 6.38**. In this unique sensor there is an additional port open to the side of the tube to measure the static pressure. A differential pressure sensor measures the pressure difference between the total pressure (forward-facing opening) and the static pressure (side-facing opening). Now the velocity can be measured directly from **Equation (6.16)**. It is important to recognize that the sensor measures the relative fluid velocity, so in the case of aircraft, the sensor gives the aircraft velocity relative to air (the airspeed). The Pitot tube (or the Prandtl tube) has narrow openings that, particularly in aircraft, are prone to icing. This is a very dangerous situation since the engine speed is regulated using the airspeed provided by the tube. Icing has been blamed for many airplane crashes, so to minimize this possibility, the sensors are heated to prevent ice buildup. The exact same idea can be used in water for surface watercraft or underwater for submarines, again measuring the speed relative to the fluid.

Other methods of sensing speed include ultrasonic, electromagnetic, and optical methods relying on reflections from the moving object and measuring the time of flight of the wave to and from the moving object. We will encounter the ultrasonic speed

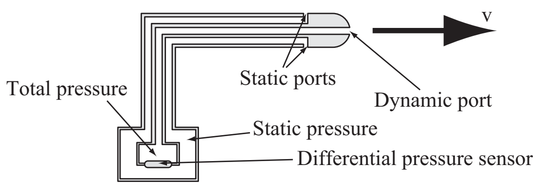


FIGURE 6.38 ■ The Prandtl tube. A differential pressure sensor measures the difference between the total pressure and the static pressure. The tube moves to the right at a velocity v in a fluid (air).

sensor in **Chapter 7**. Velocity can also be measured/sensed using the Doppler effect through use of ultrasonic, electromagnetic, or light waves. However, the Doppler method is not really a sensor, but rather a system that measures the change in frequency of a reflected wave due to the relative velocity between the source of the wave and the body whose velocity is being sensed. Although the system is somewhat complicated, it has important applications in weather prediction (detection and analysis of tornadoes and hurricanes), in space applications and science (measurement of moving objects, including the recession of stars), and in law enforcement (speed detection of cars, anticollision systems, and more).

EXAMPLE 6.11 **Water pressure in a river**

The Pitot tube can also be used to measure dynamic pressure rather than speed. Suppose a Pitot tube is immersed in the flow of a river and the dynamic pressure needs to be measured. To do so, the water head above the surface of the water is measured, as in **Figure 6.37a** (alternatively, the Prandtl tube can be used, as in **Figure 6.38**, to measure the differential pressure to which the static pressure can then be added). Assume water density is 1000 kg/m^3. Neglect the effect of temperature.

a. Given the water speed as $V_0 = 5$ m/s, and an ambient pressure equal to 101.325 kPa (1 atm), calculate the dynamic pressure due to flow just below the surface.
b. What is the dynamic pressure at a depth of 3 m?

Solution: At the surface the pressure is essentially that of the atmosphere, which we will take here as 101.325 kPa. Static water pressure under the surface increases by 101.325 kPa (1 atm) for every 1.032 m, but the dynamic pressure only depends on speed.

a. The dynamic pressure can be calculated directly from **Equation (6.15)**:

$$P_d = \rho \frac{V_0^2}{2} = 1000 \frac{5^2}{2} = 12{,}500 \text{ Pa.}$$

b. The dynamic pressure remains the same as long as the speed and density remain constant. At greater depths the density changes somewhat (increases) and the dynamic pressure at a constant velocity increases as well.

6.7 | INERTIAL SENSORS: GYROSCOPES

Gyroscopes come to mind usually as stabilizing devices in aircraft and spacecraft in such applications as automatic pilots or, more recently, in stabilizing satellites so they point in the right direction. However, they are much more than that and much more common than one can imagine. Just like the magnetic compass is a navigational tool, the gyroscope is a navigational tool. Its purpose is to keep the direction of a device or vehicle or to indicate attitude. As such, they are used in all satellites, in smart weapons, and in all other applications that require attitude and position stabilization. Their accuracy has made them useful in such unlikely applications as tunnel construction and mining. As they become smaller, one can expect them to find their way into consumer products such as cars. They have already found their way into toys such as remotely controlled model aircraft.

The basic principle involved is the principle of conservation of angular momentum: "In any system of bodies or particles, the total angular momentum relative to any point in space is constant, provided no external forces act on the system."

The name gyroscope comes from concatenation of the Greek words *gyro* (rotation or circle) and *skopeein* (to see), coined by Leon Foucault, who used it to see or demonstrate the rotation of the earth around 1852. The principle was known at least since 1817, when it was first mentioned by Johann Bohnenberger, although it is not clear whether he discovered it or was the first to use it.

6.7.1 Mechanical or Rotor Gyroscopes

The mechanical gyroscope is the best known of the existing gyros and the easiest to understand, although its heyday has passed (it still exists, but in miniaturized forms). It consists of a rotating mass (heavy wheel) on an axis in a frame. The spinning mass provides an angular momentum (**Figure 6.39**). So far this is merely a rotating wheel. However, if one tries to change the direction of the axis by applying a torque to it, a torque is developed in a direction perpendicular to the axis of rotation and that of the applied torque, which forces a precession motion. This precession is the output of the gyroscope and is proportional to the torque applied to its frame and the inertia of the rotating mass. Using **Figure 6.39**, if a torque is applied to the frame of the gyroscope around the input axis, the output axis will rotate as shown. This precession now becomes a measure of the applied torque and can be used as an output to, for example, correct the direction of an airplane or the position of a satellite antenna. Application of torque in the opposite direction reverses the direction of precession. The relation between applied torque and the angular velocity of precession Ω is

$$T = I\omega\Omega \quad [\mathrm{N \cdot m}], \tag{6.20}$$

where T is the applied torque [N · m], ω is the angular velocity [rad/s], I is the inertia of the rotating mass [kg · m^2], and Ω is the **angular velocity of precession** [1/rad · s], also called the **rotational rate**. $I\omega$ is the angular momentum [kg · m^2 · rad/s]. Clearly then, Ω is a measure of the torque applied to the frame of the device:

$$\Omega = \frac{T}{I\omega} \quad [1/\mathrm{rad \cdot s}]. \tag{6.21}$$

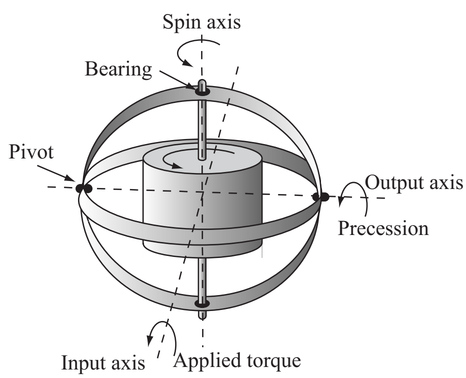


FIGURE 6.39 ■ The rotating mass gyroscope.

The device in **Figure 6.39** is a single-axis gyroscope. Two- or three-axis gyroscopes can be built by duplicating this structure with rotation axes perpendicular to each other.

This type of gyroscope has been used for many decades in aircraft, but it is a fairly large, heavy, and complex device, not easily adapted to small systems. It also has other problems associated with the spinning mass. Obviously the faster the spin and the larger the mass, the larger the angular momentum ($I\omega$) and the lower the frequency of precession for a given appiled torque. But fast rotation adds friction and requires delicate balancing of the rotating disk, as well as precision machining. This has led to many variations, including rotation in a vacuum, magnetic and electrostatic suspension, use of high-pressure gas bearings, cryogenic magnetic suspension, and more. However, none of these can ever make this device a low-cost, general-purpose sensor. Some modern gyroscopes still use the spinning mass idea, but the mass is much smaller, the motor is a small DC motor, and the whole unit is relatively small. These devices compensate for the smaller mass by using high-speed motors and sensitive sensors to sense the torque. But other types of gyroscopes designed for reliable operation at low cost have been developed. Some gyroscopes are only gyroscopes in the equivalent operation and bear no resemblance to the spinning mass gyroscope. Nevertheless, they are gyroscopes, and are highly useful at that.

Instead of the conventional gyroscope, the idea of Coriolis acceleration has been used to devise much smaller and more cost-effective gyroscopic sensors. These are built in silicon by standard etching methods and thus can be produced inexpensively. We shall discuss Coriolis acceleration-based gyroscopes in more detail in **Chapter 10**, but for completeness we outline here the basics behind Coriolis force gyroscopes. The idea is based on the fact that if a body moves linearly in a rotating frame of reference, an acceleration appears at right angles to both motions, as shown in **Figure 6.40**. The linear motion is typically supplied by the vibration of a mass, usually in a harmonic motion, and the resulting Coriolis acceleration is used for sensing. Under normal conditions the Coriolis acceleration is zero and the force associated with it is zero as well. If the sensor is rotated in the plane perpendicular to the linear vibration, an acceleration is obtained, proportional to the angular velocity, Ω.

6.7.2 Optical Gyroscopes

One of the more exciting developments in gyroscopes is the optical gyroscope, which, unlike the rotating mass gyroscope or the vibrating mass gyroscope, has no moving elements. These modern devices are used extensively for guidance and control and are based on the Sagnac effect. The Sagnac effect is based on the propagation of light in optical fibers (or in any other medium) and can be explained using **Figure 6.41**. Suppose first that the optical fiber ring is at rest and two laser beams travel the length of the ring, one in the clockwise (CW) direction, the other in the counterclockwise (CCW) direction, both produced by the same laser (so that they are at the same frequency and phase). The time it takes either beam to travel the length of the ring is $\Delta t = 2\pi Rn/c$, where n is the

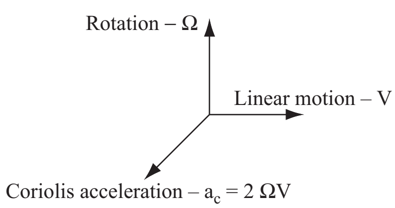


FIGURE 6.40 ■ The relation between linear velocity V, angular velocity Ω, and Coriolis acceleration a_c.

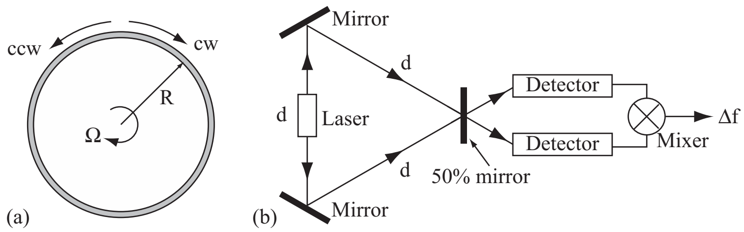


FIGURE 6.41 ■ (a) The Sagnac effect in an optical fiber ring rotating at an angular frequency Ω. (b) Implementation of the ring resonator using mirrors to "close" the ring.

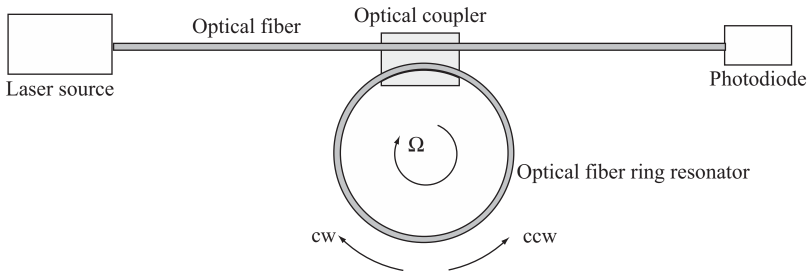


FIGURE 6.42 ■ A resonating ring optical fiber gyroscope.

index of refraction of the optical fiber and c is the speed of light in vacuum (i.e., c/n is the speed of light in the fiber).

Now suppose that the ring rotates clockwise at an angular velocity Ω [rad/s]. The beams now travel different paths in each direction. The CW beam will travel a distance $2\pi R + \Omega R \Delta t$ and the CCW beam will travel a distance $2\pi R - \Omega R \Delta t$. The difference between the two paths is

$$\Delta l = \frac{4\pi \Omega R^2 n}{c} \quad [\mathrm{m}]. \tag{6.22}$$

Note that if we divide this distance by the speed of light in the fiber, we get Δt as follows:

$$\Delta t = \frac{\Delta l n}{c} = \frac{4\pi \Omega R^2 n^2}{c^2} \quad [\mathrm{s}]. \tag{6.23}$$

Equations (6.22) and **(6.23)** provide linear relations between Ω (the stimulus in this case) and the change in the length traveled or the change in the time needed. The challenge is to measure either of these quantities. This can be done in a number of ways. One method is to build an optical resonator. A resonator is an optical path that has a dimension equal to multiple half wavelengths of the wave. In this case, a ring is built as shown in **Figure 6.42**. Light is coupled through the light coupler (beam splitter). At resonance, which occurs at a given frequency depending on the circumference of the ring, maximum power is coupled into the ring and minimum power is available at the detector. The incoming beam frequency is tuned to do just that. If the ring rotates at an angular velocity Ω, the light beams in the ring change in frequency (wavelength) to compensate for the change in the apparent length of the ring. The relation between frequency, wavelength, and length is

$$-\frac{df}{f} = \frac{d\lambda}{\lambda} = \frac{dl}{l}, \tag{6.24}$$

where the negative sign simply indicates that an increase in length decreases the resonant frequency. In effect, the wavelength of light increases in one direction and decreases in the other. The net effect is that the two beams generate a frequency difference. To show this we write

$$-\frac{\Delta f}{f}=\frac{\Delta l}{l}\rightarrow \Delta f=-f\frac{\Delta l}{l}. \tag{6.25}$$

Substituting from **Equation (6.22)**,

$$\Delta f=-f\frac{4\pi\Omega R^2 n}{lc}=-\frac{4\pi R^2}{\lambda l}\Omega\quad[\mathrm{Hz}], \tag{6.26}$$

where $\lambda = c/fn$ is the wavelength in the optical fiber. Alternatively, since $\lambda = \lambda_0 n$, where λ_0 is the wavelength in vacuum, we can write

$$\Delta f=-\frac{4\pi R^2}{\lambda_0 nl}\Omega=-\frac{4S}{\lambda_0 nl}\Omega\quad[\mathrm{Hz}], \tag{6.27}$$

where S is the area of the loop regardless of its shape. Since $l = 2\pi R$, we finally get for the circular loop,

$$\Delta f=-\frac{2R}{\lambda_0 n}\Omega\quad[\mathrm{Hz}]. \tag{6.28}$$

In all of these relations it was assumed that the detector is at the same location as the source and hence the beam travels the circumference of the ring. If the detector is, say, at the bottom of the ring and the source at the top (**Figure 6.41a**), each beam only travels half the circumference, and hence the relations must be halved. For example, the frequency shift in **Equation (6.28)** would be half of that shown.

This frequency shift is measured in the detector by mixing and filtering (we shall discuss these methods in **Chapter 11**) and is an indication of the angular velocity of precession, also called the **rotation rate**, Ω. In most cases **Equation (6.27)** is most convenient, whereas **Equation (6.28)** is only suitable for circular loops. It should be noted in particular that the larger the loop area, the larger the change in frequency, and hence the greater the sensitivity. In optical fiber gyroscopes it is a simple matter to loop the fiber N times and by doing so increase the output by a factor of N.

Figure 6.41b shows a common implementation of the Sagnac ring sensor using a set of mirrors to implement the ring. A cavity laser is used since a cavity laser generates two equal-amplitude beams traveling in opposite directions and hence the issue of splitting the beam is trivial. The two beams arriving at mirror M are directed into a detector that generates the frequency difference. This type of sensor is often called a loop or mirror gyroscope.

A different and more sensitive implementation is shown in **Figure 6.43**. Here an optical fiber is wound in a coil to increase its length and fed from a polarized light source through a beam splitter to ensure equal intensity and phase (the phase modulator adjusts for any variations in phase between the two beams). The beams propagate in opposite directions and, when returning to the detector, are at the same phase in the absence of rotation. If rotation exists, the beams will induce a phase difference at the detector that is dependent on the angular frequency of precession (rotation rate) Ω.

These devices are not cheap, but they are orders of magnitude cheaper than the spinning mass gyroscope, much smaller and lighter, and do not have the mechanical

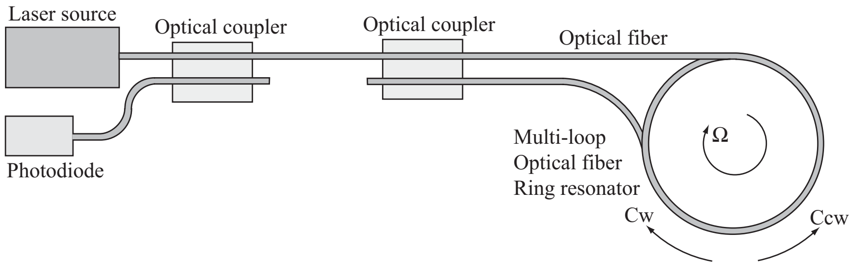


FIGURE 6.43 ■ A coil optical fiber gyroscope.

problems a rotating mass has. They have a very large dynamic range (as high as 10,000) and so they can be used for sensing rotation rates over a large span. In addition, optical fiber gyroscopes are immune to electromagnetic fields as well as to radiation and thus can be used in very hostile environments, including space. A ring gyroscope can measure the rotation of a fraction of a degree per hour. In many cases these are the devices of choice in aerospace applications, and the loop gyroscope can be produced as a microsensor.

There are other types of gyroscopes, often referred to as angular rate sensors, and some of these will be described in **Chapter 10**.

EXAMPLE 6.12 **The optical gyroscope**

A ring resonator is built as in **Figure 6.41a** with a radius of 10 cm. The source is a red laser operating at 850 nm in an optical fiber with an index of refraction $n = 1.516$. Calculate the output frequency for a rotation rate of 1°/h.

Solution: We first calculate the rate Ω in radians per second rather than degrees per hour:

$$\Omega = 1^\circ/\text{hr} \to \Omega = \frac{1^\circ}{180} \times \pi \times \frac{1}{3600} = 4.848 \times 10^{-6}\ \text{rad/s}.$$

The wavelength of the laser in vacuum is 850 nm, therefore we have, from **Equation (6.29)**,

$$\Delta f = -\frac{2R}{\lambda_0 n}\Omega = -\frac{2 \times 0.1}{850 \times 10^{-9} \times 1.516} \times 4.848 \times 10^{-6} = 0.752\ \text{Hz}.$$

This is not a very large shift in frequency, but it is measurable. By increasing the number of loops, say to 10, one obtains a shift of 7.52 Hz/degree/h.

6.8 | PROBLEMS

Strain gauges

6.1 **Wire strain gauge.** A strain gauge is made in the form of a simple round platinum-iridium wire of length 1 m and diameter 0.1 mm, used to sense strain on an antenna mast due to wind loading. Calculate the change in resistance of the sensor per pro-mil strain (1 pro-mil is 0.001 or 0.1% strain).

6.2 **NTC semiconductor strain gauge.** The following measurements are given for an NTC strain gauge:

Nominal resistance (no strain): 1 kΩ
Resistance at −3000 microstrains: 1366 Ω
Resistance at −1000 microstrains: 1100 Ω
Resistance at +3000 microstrains: 833 Ω

a. Find the transfer function of the strain gauge. Compare with **Figure 6.6b**.

b. Find the resistance of the strain gauge at 0.1% strain and at −0.1% strain.

6.3 **PTC semiconductor strain gauge:** The following measurements are given for a PTC strain gauge:

Nominal resistance (no strain): 1 kΩ
Resistance at −3000 microstrains: 833 Ω
Resistance at +1000 microstrains: 1100 Ω
Resistance at +3000 microstrains: 1366 Ω

a. Find the transfer function of the strain gauge. Compare with **Figure 6.6a**.

b. Find the resistance of the strain gauge at 0.1% strain and at −0.1% strain.

6.4 **Semiconductor strain gauges in a bridge configuration.** The strain gauge configuration in **Figure 6.5f** is used to sense the strain on a square metal sheet under tension with two opposing forces, as shown in **Figure 6.44**. Assume the material has a Young's modulus E and the strain gauges have gauge factors $g_1 = g$ and $g_2 = h$. Assume as well that the deformation of the material is elastic (i.e., the strain is not high enough to permanently deform the material) and that the strain does not exceed the maximum strain for the strain gauges used.

a. Find the resistances as a function of force of the four strain gauges of nominal resistance R_0.

b. Find the sensitivity of the four sensors to force.

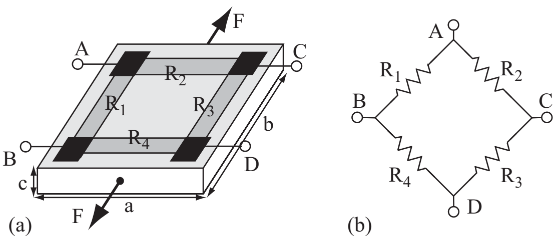


FIGURE 6.44 ■ Semiconductor strain gauge in a bridge configuration.

6.5 **Semiconductor strain gauges in a bridge configuration.** Repeat **Problem 6.4** but with the forces shown in **Figure 6.45**.

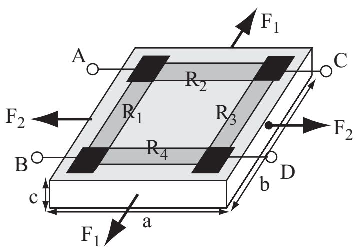


FIGURE 6.45 ■ Semiconductor strain gauges in a bridge configuration—two-axis forces.

Force and tactile sensors

6.6 **Basic force sensor.** The ceiling of a large building is supported by 16 vertical steel tubes, each with an inner diameter of 100 mm and an outer diameter of 140 mm. Each tube is equipped with a 240 Ω strain gauge (nominal, unstressed resistance at 20°C), prestressed to 2.5% strain. The Young's modulus for steel is 200 GPa. The strain gauge has a gauge factor of 2.2.

a. If the maximum strain measured is 2% in each tube, what is the maximum weight of the roof?

b. What is the change in the resistance of the strain gauge and what is the actual resistance reading for maximum allowable weight?

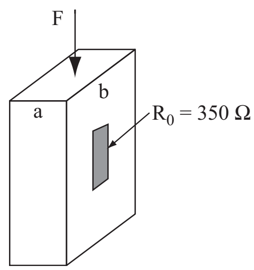
c. If the expected temperature range is 0°C–50°C, what is the error in reading of the maximum weight, assuming the sensor is not temperature compensated and it is made of constantan?

6.7 **Force sensor.** A force sensor is made of a strip of steel with cross-sectional area of $a = 40$ mm by $b = 10$ mm with a platinum, 350 Ω strain gauge (nominal, unstressed resistance) bonded to one surface, as shown in **Figure 6.46**. The sensor is intended for use as a compression force sensor.

a. Given the modulus of elasticity of 200 GPa, calculate the range of resistance of the sensor if it cannot exceed a strain of 3%. What is the range of forces that can be applied?

b. Calculate the sensitivity of the sensor assuming it is prestressed to 3%.

FIGURE 6.46 ■ A simple compression force sensor.



6.8 **Compensated force sensor.** A proposed sensing strategy for force is shown in **Figure 6.47**. The force applied to the beam is sensed by measuring the voltage across the upper sensor (marked as R_1). The lower sensor is prestressed to 3%, whereas the unstressed, nominal resistance of the strain gauges is 240 Ω at 20°C. Both strain gauges have a gauge factor of 6.4. The strain gauges are attached to a steel beam whose dimensions are given in **Figure 6.47**. The modulus of elasticity for the material used is given as 30 GPa. The two sensors are connected in series as shown.

a. Calculate the output voltage as a function of the applied force.

b. What is the maximum force that can be sensed?

c. Show that any change in temperature will have no effect on the output as long as the sensors are made of the same materials and are at the same temperature.

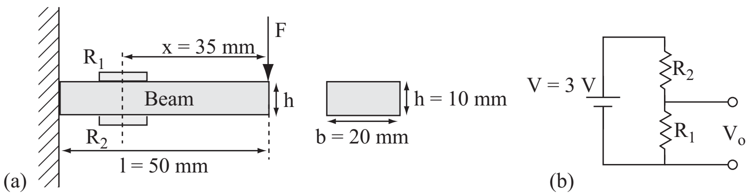


FIGURE 6.47 ■ A compensated force sensor.

6.9 **Capacitive force sensor.** A capacitive force sensor is made as shown in **Figure 6.48**. The three plates are identical, each $w = 10$ mm wide and $L = 20$ mm long. The two outer plates are fixed, whereas the center plate is suspended on a spring so that at zero force the three plates are aligned. Between the plates there is a separation sheet made of Teflon with relative permittivity of 2.25 and thickness $d = 0.1$ mm (one on each side of the center plate).

a. If the spring has a constant of 100 N/m, find the transfer function of the sensor and its sensitivity.

b. What is the maximum possible span of the sensor?

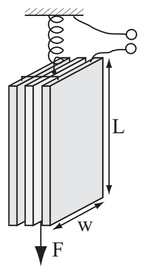


FIGURE 6.48 ■ A capacitive force sensor.

6.10 **Load sensing in a bridge.** A foot bridge is made as a simple deck, 4 m long, with a cross section 2 m (wide) and 20 cm thick, and made of wood. The deck is supported at the two ends. The maximum load allowable on the bridge is 10 tons, provided it is uniformly distributed on the deck. To sense this load, a strain gauge is placed at the center of the bridge and its resistance is monitored. If the sensor has a nominal resistance of 350 Ω and a gauge factor of 3.6, what is the reading of the strain gauge at maximum load? The modulus of elasticity for the wood used in the construction is 10 GPa.

6.11 **Overload sensing in elevators.** Most elevators are rated for a certain load, typically by specifying the number of persons allowed or by specifying maximum weight (or both). Modern elevators will not move if the maximum load is exceeded. There are many ways to sense this load, but the simplest to understand and one of the most accurate is to use a plate as the floor of the elevator and support it on load cells. Consider an elevator rated at 1500 kg (force) and its floor is supported on four load cells. Each load cell is equipped with a strain gauge with a nominal resistance of 240 Ω and a gauge factor of 5.8. Calculate the reading of each strain gauge at maximum load if the cross-sectional area of the load cell button (on which the strain gauge is mounted; see **Figure 6.10**) is 0.5 cm^2. Assume the buttons are made of steel with a modulus of elasticity of 60 GPa and the strain gauges are prestressed to 0.5%.

6.12 **Linear array capacitive tactile sensor.** A simple capacitive tactile sensor can be built as a linear array of simple plates as shown in **Figure 6.49**. The plates are covered with a thin dielectric with relative permittivity equal to 4 and thickness of 0.1 mm. The capacitance between each two plates is 6 pF. To sense presence or position, a finger slides over the dielectric layer. Calculate the maximum change in capacitance between any two neighboring plates as the finger passes above them, assuming the finger is a conductor and that it completely covers the two neighboring plates.

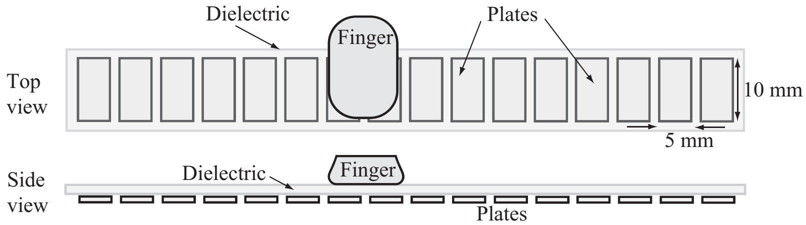


FIGURE 6.49 ■ A linear tactile sensor.

6.13 **Capacitive tactile sensor: the touch pad.** Touch pads can be made as two-dimensional arrays of capacitive sensors, as shown in **Figure 6.50**. A lower set of strips is covered with a dielectric and on top another set of strips is laid at 90° to the lower strips. The overlapping sections form capacitors with a capacitance that depends on the size of the strips and the material between them. The pad senses the position of a finger by the fact that as the finger slides it presses down on the top layer and pushes the strips underneath it closer together, increasing the capacitance between them. Consider a touch pad with strips 0.2 mm wide separated 0.02 mm apart with a dielectric with relative permittivity of 12. Assume that the fingers press down so that the distance between the dielectric compresses by 20%. In the process, the permittivity also increases locally by 10%. Calculate the rest capacitance at any strip intersection and the change in capacitance due to pressure from fingers.

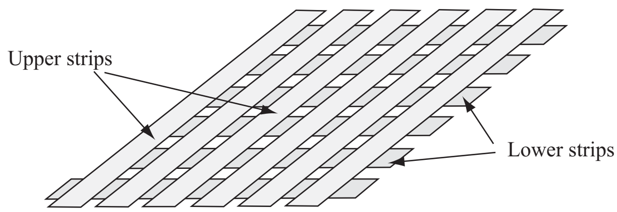


FIGURE 6.50 ■ A tactile touch pad.

Capacitive accelerometers

6.14 **Force, pressure, and acceleration sensor.** A force sensor is built as a capacitor with one plate fixed while the other can move against a spring with constant k [N/m]. By pressing on the movable plate, the distance the plate moves is directly proportional to the force. The plates have an area S and are separated by a distance d. The permittivity of the material between the plates is that of free space. Using **Figure 6.51**, and assuming the plate has moved a distance x from the rest position:

a. Find the relation between measured force and the capacitance of the sensor.

b. Plot a calibration curve for the following values: $k = 5$ N/m, $S = 1$ cm^2, $d = 0.02$ mm.

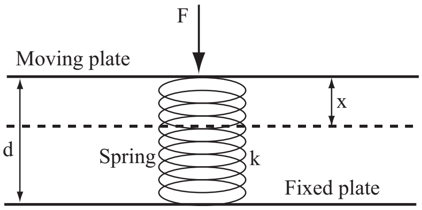


FIGURE 6.51 ■ A simple force, acceleration, or pressure sensor.

c. How can this device be used to measure pressure? Calculate the relation between pressure and capacitance.

d. How can this device be used to measure acceleration? Calculate the relation between acceleration and capacitance. Assume the mass of each plate is m [kg] and that the mass of the spring is negligible.

Note: The force necessary to compress a spring is $F = kx$. The spring in the figure represents a restoring force and is not necessarily a physical spring.

6.15 **Capacitive accelerometer.** An accelerometer is made as shown in **Figure 6.52**. The mass is small in size and together with the upper plate of the capacitor has mass of 10 g. The beam is $e = 1$ mm thick and $b = 2$ mm wide and it is made of silicon. The total length of the beam (from the center of mass to the fixed point) is $c = 20$ mm. With plates separated a distance $d = 2$ mm apart and capacitor plates 10 mm $\times$ 10 mm in area, and assuming the maximum strain in silicon cannot exceed 1%, calculate:

a. The span of acceleration the sensor is capable of. Use a modulus of elasticity of 150 GPa.

b. The range of capacitances corresponding to the span calculated in (a). Assume there is a 0.1 mm thick stop that prevents the plates from approaching each other at less than 0.1 mm.

c. The sensitivity of the sensor in pF/m/s^2.

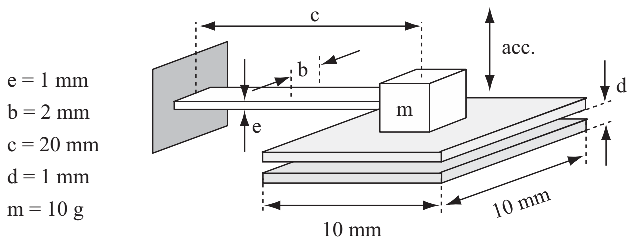


FIGURE 6.52 ■ A capacitive accelerometer.

6.16 **Strain gauge accelerometer.** The sensor in **Figure 6.52** is given again, but now the capacitor plates are removed and instead two strain gauges are placed one on the top surface of the beam and one on the bottom surface. The strain gauges are silicon gauges, very small with a nominal resistance of 1000 Ω. Both are prestressed to 1.5% strain and have a maximum range of 3% strain. The strain gauges are bonded in the middle of the beam (5 mm from the fixed position). For the mass, dimensions, and Young's modulus in **Problem 6.15**:

a. Calculate the span of

b. Calculate the sensitivity of the sensor if the gauge factor for the strain gauges is 50.

c. Repeat (a) and (b) if the strain gauges are moved to the fixed location of the beam.

6.17 **Two-axis accelerometer.** A two-axis accelerometer is made as shown in **Figure 6.53**. The mass, $m = 2$ g, is attached to the center of four beams, each $e = 4$ mm long and with a square cross section $b = 0.2$ mm by $c = 0.2$ mm. A semiconductor strain gauge with a gauge factor of 120 and nominal (unstressed) resistance of 1 kΩ is built onto each beam, across from the attachment point.

a. Calculate the range of the accelerometer if the strain gauges can handle $\pm 2\%$ strain and can work only under tension. The sensor is made of silicon with a modulus of elasticity of 150 GPa.

b. Calculate the range of resistance of the strain gauges.

c. Calculate the sensitivity of the sensor.

d. Discuss how the sensitivity of the sensor can be increased.

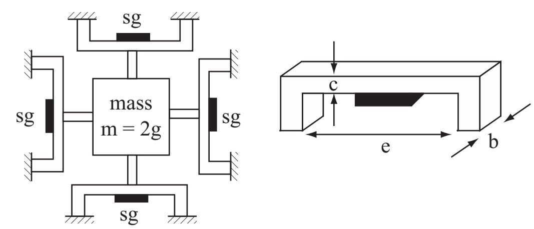


FIGURE 6.53 ■ A two-axis accelerometer.

Magnetic accelerometers

6.18 **Magnetic accelerometer.** A magnetic accelerometer is made as shown in **Figure 6.54**. A coil with $N = 500$ turns carries a current $I_c = 10$ mA, wound around the moving mass but allowing the mass full freedom to move. A spring that can act either in tension or in compression keeps the mass in place with a gap of 2 mm between its surface and the surface of the Hall sensor on the stationary pole. The latter is made of iron, 15 mm in diameter, has high relative permeability so that the reluctance of iron can be neglected, and the mass of the moving core is 10 g.

a. If the sensor must be capable of sensing accelerations up to 100 g in each direction, what is the spring constant?

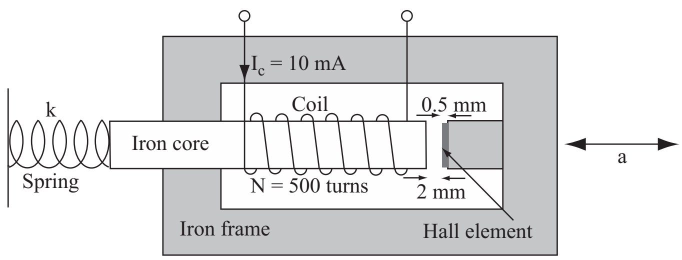


FIGURE 6.54 ■ A magnetic accelerometer.

b. If the Hall element used is connected to a current I and is d mm thick, calculate the sensitivity of the sensor as a function of acceleration and the current in the Hall element. Assume the Hall coefficient is known.

c. A silicon Hall element, 0.5 mm thick with a Hall coefficient of $-0.01\ m^3/A \cdot s$ is used, driven with a 5 mA current. Calculate the span of the output of the sensor and its sensitivity over the span.

6.19 **LVDT-based accelerometer.** LVDTs have a number of advantages over other sensors, including linearity and high output. Because of the moving core, they can be used in a number of ways, including for sensing of acceleration, although they must be modified for the purpose. Consider an LVDT designed to sense position in the range -10 mm to $+10$ mm for which it produces an output of ± 5 V (rms) (for each direction; i.e., it produces 5 V for a displacement of 10 mm). The core is free to move and a spring is attached to each end of the core to restore it to its zero position (**Figure 6.55**).

a. With a mass of 40 g, calculate the spring constant needed to sense acceleration in the range $-2\ m/s^2$ to $+2\ m/s^2$.

b. What is the sensitivity of the accelerometer?

c. The output of the LVDT is measured using a digital voltmeter with a resolution of 0.01 V. What is the resolution of the sensor?.

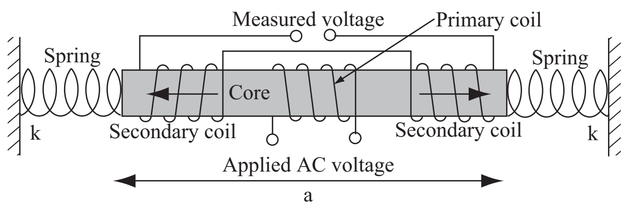


FIGURE 6.55 ■ Use of an LVDT to sense acceleration.

Pressure sensors

6.20 **The altimeter.** Most altimeters use a pressure sensor to measure height, using the barometric equation as the basis.

a. What type of pressure sensor can be used for this purpose?

b. Show how the barometric equation can be used to calibrate a pressure sensor in terms of elevation above sea level.

c. What is the required resolution of a pressure sensor to produce a 1 m resolution altimeter at sea level and at 10,000 m?

d. Calculate the span required of a pressure sensor to serve as an altimeter for mountain climbing purposes with an elevation range of 10 km (the highest mountain on earth is 8800 m high).

6.21 **The depth meter.** Depth meters are essential tools for divers and for submarines. It is proposed to use a pressure sensor and calibrate it in meters. Since pressure in water is produced by the column of water above the point at which the pressure is sensed, the relation between pressure and depth is relatively simple, and because water density can be considered constant the relation is accurate. Given the density of seawater as 1025 kg/m^3:

a. Calculate the span of a sealed pressure gauge pressure sensor to sense pressure down to 100 m. The sealed pressure is 101,325 Pa (1 atm).

b. Calculate the required resolution of the sensor for a resolution of 0.25 m of the depth meter.

c. The water density in freshwater is 1000 kg/m^3. What is the error in the reading of a depth pressure sensor if used in freshwater without recalibration?

Note: Water density does vary with temperature, but this is neglected here.

Velocity sensing

6.22 **Water speed and flow volume sensing.** To measure the speed of water in a channel and the flow volume it is suggested to use a Pitot tube and measure the head above water, as in **Figure 6.37a**.

a. Find a relation between the speed of water and the head.

b. If a difference of 0.5 cm is practical, what is the sensitivity of the device?

c. Calculate the water flow in cubic meters per second [m^3/s] as a function of velocity and as a function of the head if the cross-sectional area is S and the flow velocity is uniform throughout the channel.

6.23 **Speed sensing in a boat.** A Pitot tube can be installed in the prow or on the side of a boat to measure its speed. If a pressure sensor with a resolution of 1000 Pa and a range from 0 to 50,000 Pa is used, and assuming a pressure of 101.325 kPa (1 atm) at the surface of the water and a density of water of 1025 kg/m^3:

a. Calculate the resolution in terms of speed that can be measured, neglecting the effects of static pressure and assuming the sensor is calibrated to zero output at zero speed.

b. Calculate the range of the sensor.

6.24 **Sensing airspeed in an aircraft.** A passenger aircraft uses two Pitot tubes for speed sensing. One tube is aligned parallel to the airplane and the other is perpendicular to it, each equipped with a pressure sensor.

a. For an aircraft flying at 11,000 m, calculate the readings of the pressure sensors in each of the tubes and the differential pressure if the airplane flies at 850 km/h. The temperature at that elevation is $-40°$C. Neglect any effect the speed may have on air density inside the tube.

b. Suppose that the lateral tube becomes blocked with ice at 11,000 m and now the aircraft climbs to 12,000 m. What is the error in the speed reading, assuming the airplane has not changed speed and the temperature remains the same as at 11,000 m?

6.25 **Speed and depth sensing in submarines.** The Pitot and Prandtl tubes are equally effective underwater. Suppose a submarine is equipped with a forward-pointing Prandtl tube. Two independent sensors are used, one to sense static pressure and the other to sense total pressure. The density of water is assumed constant with depth, equal to 1025 kg/m^3 and independent of temperature.

a. If the submarine is expected to descend to 1000 m, what is the span required from each pressure sensor? The maximum speed of the submarine is 25 knots (1 knot = 1.854 km/h).

b. Show that these two measurements are sufficient to provide both the velocity and the depth of the submarine.

6.26 **A resistive pressure sensor.** Conducting polymers can be used to sense pressure by measuring the resistance of the polymer. A pressure sensor is proposed as follows: A small hollow spherical ball is made of a polymer with a given conductivity. The inner and outer surfaces of the ball are plated with a conducting surface to form an inner and outer electrode, as shown in **Figure 6.56**. The pressure inside the ball is sensed by measuring the resistance between the inner and outer electrodes. At a reference temperature P_0, the inner radius is $r = a$ and the thickness is $t = t_0$. Also, you may assume $a \gg t_0$ and, in general, $r \gg t$. The relation between pressure and radius is

$$r = \alpha\sqrt{P}.$$

That is, as the pressure increases, the radius increases with α a constant value. The conductivity of the material is σ. This relation holds throughout the pressure range, including the reference pressure. Find the relation between pressure and resistance between the electrodes.

Note: The fact that $r \gg t$ means that the shell is thin with respect to the radius. Use this approximation to simplify the calculations.

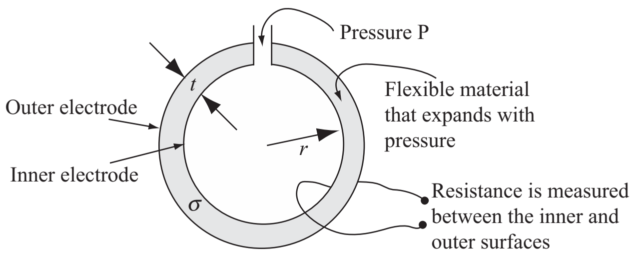


FIGURE 6.56 ■ A resistive pressure sensor.

Optical gyroscopes

6.27 **The mechanical gyroscope**. A miniature mechanical gyroscope contains a wheel of mass 50 g, radius 40 mm, and length 20 mm, rotating at 10,000 rpm.

a. Calculate the sensitivity of the gyroscope to torque perpendicular to its axis.

b. What is the lowest torque it can sense if the frequency of precession can be measured to within 0.01 rad/s?

6.28 **Ring gyroscope.** A Sagnac gyroscope is implemented as in **Figure 6.41b**.

a. With the side of the triangle $a = 5$ cm and using a green laser operating at 532 nm in a vacuum, calculate the sensitivity of the sensor (in Hz/°/s).

b. If a frequency can be reliably measured down to 0.1 Hz, what is the lowest rate that can be sensed?

6.29 **Optical fiber loop gyroscope.** A small optical fiber gyroscope is designed for high sensitivity. For reliable reading the output frequency resolution is set at 0.1 Hz. How many loops are required to sense a rate of 10°/h if the loop is 10 cm in diameter and an infrared LED at 850 nm is used as the source? The index of refraction of the fiber is 1.85.

6.30 **Ring gyroscope.** A Sagnac gyroscope is built as in **Figure 6.57**. For $a = 40$ mm and using a red laser at 680 nm, calculate the output expected for a rate of 1°/s and the sensitivity of the gyroscope.

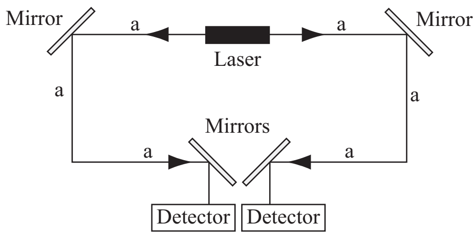


FIGURE 6.57 ■ Implementation of a ring gyroscope.