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Calibration of building energy models for retrofit analysis under uncertainty

Y. Heo^{a,*}, R. Choudhary^b, G.A. Augenbroe^a

- ^a College of Architecture, Georgia Institute of Technology, Atlanta 30332-0155, USA
- ^b Energy Efficient Cities Initiative, Engineering Department, University of Cambridge, Cambridge CB2 1PZ, United Kingdom

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ABSTRACT

Retrofitting existing buildings is urgent given the increasing need to improve the energy efficiency of the existing building stock. This paper presents a scalable, probabilistic methodology that can support large scale investments in energy retrofit of buildings while accounting for uncertainty. The methodology is based on Bayesian calibration of normative energy models. Based on CEN-ISO standards, normative energy models are light-weight, quasi-steady state formulations of heat balance equations, which makes them appropriate for modeling large sets of buildings efficiently. Calibration of these models enables improved representation of the actual buildings and quantification of uncertainties associated with model parameters. In addition, the calibrated models can incorporate additional uncertainties coming from retrofit interventions to generate probabilistic predictions of retrofit performance. Probabilistic outputs can be straightforwardly translated to quantify risks of under-performance associated with retrofit interventions. A case study demonstrates that the proposed methodology with the use of normative models can correctly evaluate energy retrofit options and support risk conscious decision-making by explicitly inspecting risks associated with each retrofit option.

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1. Introduction

In the US and UK, existing buildings account for 39% of the total energy demand [1,2]. While the energy consumption of current buildings is projected to grow annually by 1.7% to 2025 [3], the total floor area of buildings is projected to increase roughly at the rate of 1–2% per year [4]. It is indeed well-accepted that existing buildings will have a critical role in meeting energy and emission reduction targets in developed countries. Mills et al. [5,6], has shown that improving existing buildings will yield median energy savings of 16% in the United States. Furthermore, the study projected that if these median energy savings are applied to the US commercial building stock, potential energy-savings will correspond to monetary savings of approximately \$30 billion by 2030. According to this projection, energy retrofits of existing buildings can play a significant role in achieving national energy reduction targets cost-effectively.

Energy retrofits of existing buildings are important because buildings tend to undergo system degradation, change in use, and unexpected faults over time. It is well known that the efficiency of buildings and their equipment degrades over their service life, and even faster when they are not maintained appropriately. Building components can also under-perform when they are not properly designed or installed. Faults in mechanical systems and lighting equipment alone can account for 2–11% of the total energy consumption for commercial buildings [7]. A meta-analysis of 643 commercial buildings by the Lawrence Berkeley National Laboratory also illustrates the wide variety of problems associated with mechanical equipment, lighting systems, and building envelopes of existing buildings [5,6].

Indeed, owing to the importance of energy retrofits of existing buildings, governments at different levels are investing in policies and initiatives related to improving the efficiency of existing buildings. In the United States, President Obama launched "Better Buildings Initiative" to reduce energy consumption of commercial buildings by 20% by 2020 through cost-effective retrofit interventions [8]. At the federal government level, the U.S. Department of Energy selected 25 innovative projects across the country under the Retrofit Ramp-Up Initiative to support whole-neighborhood building energy retrofits [9,10]. At the city level, the city of Chicago initiated a Chicago Climate Action Plan that targets to retrofit 50% of existing commercial and residential buildings in Chicago for 30% energy reduction by 2020 [11]. Across the pond, in the UK, the government's aim to reduce its carbon emissions by at least 80% by 2050 will require more effort than maintaining stable overall levels of consumption over the years. The Carbon Trust asserts this view in CTC766 [12], requiring all existing commercial buildings to achieve at least an F-rated energy performance certificate by 2020 (most existing commercial buildings are currently G-rated). Recent policies such as the Carbon Reduction Commitment (CRC) scheme,

^{*} Corresponding author. Tel.: +1 4048944885. E-mail address: yeonsook@gatech.edu (Y. Heo).

energy certificates, climate change levy & agreements (CCL), the renewable heat incentive (RHI), and upcoming 'Green Deal' are likewise designed to incentivize commercial building owners to invest in energy efficient retrofits.

The main aim of an energy retrofit is to improve energy efficiency by implementing the most optimal mix of technologies at a reasonable investment. In practice, it has become mainstream to use building simulation software to quantify expected energy savings from retrofit technologies where possible. However, building simulation software are more suitable for predicting energy use of yet-to-be-built projects, in which properties of the building and its systems parameters can be assumed to follow engineering design specifications. Existing buildings come with nuances associated with how buildings and their components are actually operated and these are often difficult to represent in building energy models. Thus, the energy savings output from simulation models of existing buildings need verification and/or calibration. It is in response to this gap that most Energy Service Companies (ESCOs) rely on the the International Performance Measurement and Verification Protocol (IPMVP)[13] for best practice techniques to verify their estimates of energy savings from energy efficiency, water efficiency, and renewable energy projects and to allocate risks appropriately.

Allocation of risks requires uncertainty quantification of projected cost effectiveness of technology options for a given retrofit project. The importance of assessing if certain energy retrofits will be less or most cost effective is critical in context of the ESCOs industry [14–16]. ESCOs undertake retrofits of existing buildings through performance based contracts that typically guarantee savings as part of their service. The expression of a guarantee allows building owners to invest in the retrofits with high confidence, but the structure leads to relatively safe and often less aggressive ambitions towards energy savings. This is due to the fact that ESCOs rely on experts' knowledge and previous successes to estimate investment paybacks of technology choices for building retrofits. An experts' subjective judgement cannot always correctly estimate the uncertainties associated with a combination of technology options. As a result, ESCOs are not likely to invest in high-impact, high-cost technologies, unless the probability of energy savings can be quantified appropriately and associated risks expressed such that comparison between competing technologies is explicit. Yet, there is lack of sufficient research in developing methods that are able to support risk analysis of investment decisions in energy upgrades of buildings.

This paper presents a scalable, probabilistic methodology that can support investments in energy retrofit of buildings while accounting for uncertainty. The methodology is based on Bayesian calibration of normative energy models. Based on CEN-ISO standards, normative energy models are light-weight, quasi-steady state formulations of heat balance equations, which makes them appropriate for modeling large sets of buildings efficiently. Calibration of these models enables improved representation of the actual buildings and quantification of uncertainties associated with model parameters. In addition, the calibrated models can incorporate additional uncertainties associated with retrofit technologies to generate probabilistic predictions of energy savings which can be naturally translated to risks associated with the investment.

2. Energy models of existing buildings

Energy simulation models play a key role in computing potential energy savings from retrofits. In order to reliably predict energy-savings from a set of proposed retrofit technologies, the simulation model must represent a building as operated; that is, the model should capture the building systems as-installed, as-operated, and as-used. Hence, building audits and monitored energy consumption also become integral to the modeling process. If the baseline model can generate outcomes that closely match monitored energy

consumption of a building, then it is more likely to predict reliable estimates of energy-saving from planned retrofit options for that building. To get a good match between outcomes of the baseline model and monitored energy consumption, the analyst or the modeler calibrates the parameters of a simulation model to match their actual counterparts in the building. This is a widely accepted modeling approach for analyzing existing buildings [17–19]. Model parameters can be calibrated to an extent through building audits – at least for those parameters that are physically observable. For non-observable parameters (which can be quite numerous depending on the fidelity of the analysis model), one counts on an expert's knowledge and experience.

In this paper we use the term operational adjustments to refer to the process of auditing a building to determine appropriate values for the observable parameters of a building simulation model. It typically includes site visits, interviews with building managers, field measurements to determine physical properties of the building (such as geometry, location of blinds, installed systems, etc.), occupancy patterns, plug loads, and control settings. This is an important part of the calibration process since the actual building operation often deviates from specifications assumed and documented during design and construction. The term parameter estimation is used to define the process of setting values for the non-observable simulation parameters. Most simulation exercises on retrofit analysis employ a heuristic method for the parameter estimation process: The expert selects a set of parameters that are likely to influence the outputs of the simulation model most, and are also likely to require adjustments on a building-to-building basis. He/she then sets the most appropriate combination of values for those parameters by running the simulation model iteratively with different parameter values until differences between computed and actual energy use are reasonably small. Indeed, although the calibration process is manual, it essentially resembles deterministic optimization, resulting in an optimum set of parameter values to be used for exercising the simulation model for retrofit analysis.

Thus, building a reliable model of an existing building asks for a fundamentally different approach than simulating the behavior of a building that is yet to be constructed. The former is essentially an empirical exercise that relies on good observations to infer values of model parameters, and the latter is based on embodying the physics of the important components of the proposed building to appropriate level of fidelity so that the overall model is a good enough representation of its main sub-systems and their interactions. For both cases, high fidelity and high resolution models are generally accepted as more reliable in the simulation community. Indeed, most practices tend to use dynamic or transient simulation models for analyzing buildings – especially when a single building is in focus.

Many inefficiencies can be noted in the modelling approach for building retrofits: First, the quality of the model relies heavily on the subjective judgment of an expert; Second, identifying a single combination of parameter values that result in a good fit between monitored and computed energy consumption does not guarantee that the parameter values represent reality. Despite being quite labor intensive and time consuming, the process does not derive a set of parameter values which can be used to evaluate relative cost-benefits of different retrofit options with confidence. Most importantly though, uncertainties regarding parameter values always exist in any model, and are left unquantified even after being considered during the parameter estimation process. If uncertainties in parameter values are not quantified, one cannot compute their cumulative impact on the reliability of the model outputs. We thus argue for improving the parameter estimation step substantially by using statistical inference. We propose and demonstrate a model calibration process based on a Bayesian approach, and are hence able to quantify energy consumption in probabilistic form. Our approach is particularly suitable for the case of modeling existing buildings because we can leverage the information that is anyways needed in the modelling process to its fullest advantage (monitored energy consumption and observable parameters). We also elicit a wider pool of 'expert knowledge' when collecting initial estimates of parameter values – through onsite surveys, interviews with building managers/experts, published literature, etc. Probabilistic predictions of energy saving potential is much more informative when competing retrofit technologies are considered, and in addition, the outcome is better aligned with the risk-averse attitude towards decision-making within the ESCOs (most retrofit jobs require guaranteed savings contracts) [20].

While considering a Bayesian approach to calibrate simulation models of existing buildings, we question the level of model fidelity necessary to serve the purpose of evaluating retrofit options for their energy saving potential. No doubt, several commercial transient simulation models have been well-used over the last decade (e.g. energyplus, eQuest, IES-VE, TRNSYS, etc.), and they have earned the confidence of the simulation user community in the industry. These models emulate the energy consumption of a building by solving the full set of dynamic heat balance equations using numerical methods over a time step and for the specified simulation period (typically, 1 year). They can be used to model a building and its control systems to a high degree of detail, which is quite beneficial when detailed design and sizing of specific systems need to be evaluated within the context of overall energy consumption of the building. To compare cost-benefits of competitive retrofit technologies at the macro-level, this level of detail is often not necessary, and in fact tends to burden the modelling process with excessive number of parameters. Quantifying the uncertainty of each sub-system or component level parameter individually can be time-consuming and the subsequent calibration process is computationally expensive. These features have a high cost in routine practice. Thus, incorporating uncertainty quantification in the modelling process on a regular basis becomes challenging.

We argue that our approach can potentially allow the use of lower resolution models to be used without compromising the degree-of-confidence in the outcomes. We consider the use of normative models, which are quasi-steady state models designed to calculate the energy consumption by main end-uses in a building. The model is an approximate representation of main energy flows in a building at the macro level. In many instances this can be a reasonable level of model resolution, especially when the goal is to evaluate macro-level retrofits at the building level rather than to finetune its sub-systems and their components. A well-accepted normative method is defined in the CEN-ISO standards for energy performance calculation [21,22]. The normative energy calculation methods can make the calibration process much faster: (a) the level of information required from building audits is much less. This is an extremely useful advantage since gathering detailed specifications can be extremely time-consuming, if not impossible, (b) modelling effort and computational run-time is significantly reduced, and (c) aggregation of a set of parameters in one representative value of a building sub-system rather than component level allows uncertainty quantification for a reduced set of parameters.

Hence, normative models promise to be good candidates for building audits and ensuing calibration processes as they replace the complex and expensive transient simulation models. Although they are widely used for benchmarking buildings against a reference case (for example, UK Energy Performance Certificate scheme), their suitability for evaluating energy retrofits is limited and untested. Since normative models represent energy flows in a building through a relatively small set of macro-level parameters, it needs to be tested whether the sub-system level uncertainty is able to capture the cumulative effect of its components. For example, in

the normative model the uncertainty quantified for the efficiency of a heat generation system represents the cumulative uncertainty in the efficiency of all pumps, heating plants, control settings, etc. Therefore, this paper will also examine whether suitably calibrated normative models are adequate for retrofit analysis of buildings.

Thus, the primary aim of this paper is to outline the model calibration process based on a Bayesian approach through an illustrative study and demonstrate its advantages. The second aim of the paper is to test our hypothesis regarding model resolution. To do so, we compare the predictions from a calibrated normative model with two cases: (a) uncalibrated transient simulation model and (b) calibrated transient simulation model. We expect that predictions from a calibrated normative model of a building can in fact be more accurate than an uncalibrated transient simulation model and competitive with a calibrated transient simulation model.

3. Bayesian calibration

We employ a Bayesian approach to calibrate uncertain parameters θ in energy simulation models given observed data y and building specifications. The calibration process aims to derive a set of values for θ that will give the closest possible match between the output of the energy simulation model and observed data.

We follow the mathematical formulation of Bayesian calibration developed by Kennedy and O'Hagan [23]. The statistical formula captures three types of uncertainties: (a) parameter uncertainty in the energy simulation model, (b) discrepancy between the model and the true behavior of the building, and (c) observation errors. We quantify these uncertainties with respect to known conditions x under which the observations are taken. The relationship between observations and model outputs follows:

$$y(x) = \eta(x, \theta) + \delta(x) + \varepsilon(x) \tag{1}$$

Observations are denoted by y(x); $\eta(x,\theta)$ denotes building energy model outputs computed at known conditions x (e.g. external temperature, known occupancy, etc.) and calibration parameters θ . The energy model may not capture the actual consumption of the building even with the best possible values of the calibration parameters. Indeed, building energy models are based on approximations of the heat transfer processes occurring in a building. This discrepancy between the model and the true physical behavior of the building is represented by $\delta(x)$. This term prevents over-estimation of calibration values, and describes how the energy model falls short. Any errors in recording observations (energy consumption in this case) are denoted by $\varepsilon(x)$.

In the Bayesian paradigm uncertain parameters are assigned prior distributions $p(\theta)$ based on some expert judgment, which could be derived from a pool of sources (experiments, surveys, expert knowledge, industry standards, etc.). Prior distributions are updated using observations through a formal set up in which the likelihood of obtaining observations from the building energy model drives the updating process. As a result, we collect plausible distributions of calibration parameters, also known as posterior distributions.

4. Calibration of the normative energy model

This section describes the calibration of the normative energy model for an existing building. As an illustrative example, we apply the calibration methodology on a building located in Cambridge, UK. Since the building serves the purpose of demonstrating the calibration process, it was selected based on availability of information for the energy model, good quality observations (utility data), and access to the site for operational adjustments of model parameters. The building is representative of a large set of office buildings

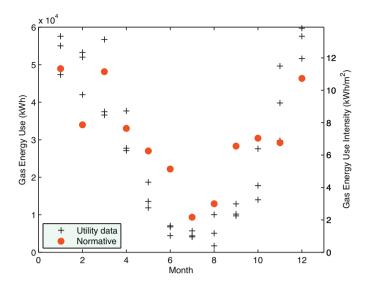


Fig. 1. Monitored gas consumption against model predictions.

in the UK, mostly housing offices and meeting rooms. These spaces are heated by radiators, to which hot water is supplied by a condensing gas boiler. The building is naturally ventilated through operable windows. No mechanical cooling is provided. Other end uses include lighting, nominal plug loads, and domestic hot water, all of which use electricity. The predominant demand is for spaceheating, and indeed, gas consumption of the building far exceeds its electricity demand. Therefore, the energy model is built for spaceheating only, and calibrated with total monthly gas consumption of the building.

We follow the standard process of consulting construction documents and design specifications to build the normative energy model of the building. The main parameters of the energy model can be broadly summarized within following groups: (1) thermal properties of construction materials (e.g. thermal transmittance, emissivity, solar absorptance) (2) dynamic heat capacity of the building envelope (2) internal loads (plug-in appliances, lighting, occupants, etc.), (3) properties of the heating system, (4) properties describing ventilation and infiltration, and (5) external environment (weather data). Initial values assigned to the model parameters have been adjusted to align with the actual building following audits and interviews with the building manager. Fig. 1 shows that there is considerable difference between the outputs from this model and actual gas consumption of the building. The main steps of the model calibration process are detailed in the following sections.

4.1. Prior uncertainty quantification

As the first step, we need to estimate the uncertainties in model parameters by reviewing values reported in published literature and industry standards. For some parameters it is more straightforward to draw upon literature. For example, the uncertainty in thermal properties of materials largely arises from variations in how they are measured in laboratories and/or differences in manufacturing, rather than due to differences in how a building is specifically used or constructed. Thus, we are able to use uncertainties as quantified in Macdonald [24] for thermal transmittance $U(W/m^2 K)$, solar absorptance S, and emissivity E of wall and roof materials. For impermeable materials, the standard deviation of uncertainties in thermal transmittance, density, and specific heat is 5%, 1%, and 12.25% respectively. For solar absorptance and emissivity, the uncertainty range quantified as standard deviation differs depending on the type of materials, which is well summarized in

Macdonald [24]. Using the standard deviation values, we use the 95% confidence interval as the minimum and the maximum values.

In addition to these, the normative energy model uses an aggregate parameter called effective heat capacity $C(kJ/m^2K)$, which approximates the dynamic heat storage (thermal mass) of the building envelope as a whole. Starting from the internal surface, it is calculated as an area weighted function of density $\rho(kg/m^3)$ and specific heat capacity c(J/kgK) of building materials up to the first insulating layer, the maximum thickness 10 cm, or the middle of the wall and roof assembly, either of which comes first. The resulting value of C for concrete buildings typically varies between 160 and 275 (kJ/m^2K) . Lower values correspond to light-weight buildings with low thermal mass. For this particular case, we use the value recommended in EN ISO 13786:2007 [25] for a heavy-weight building as the base $(260 \, kJ/m^2\, K)$.

Parameters describing ventilation and infiltration rate vary from one building to another. In energy models, they are quantified by estimating the volumetric flow rate of outside air into a building $\dot{V}(m^3/h)$, or as number of air changes per hour $n h^{-1}$. It is generally understood that the infiltration rate (air tightness) of a building is a function of its age (which can cause cracks at joints), its construction quality (air tightness), and weather conditions (pressure difference between the outside and inside of the building), but researchers have not been able to quantify the correlation among these parameters precisely [26,27]. ATTMA [28] and CIBSE [29] provide normal and best practice infiltration rates for different buildings; for naturally ventilated offices, the best practice is 3.0-5.0 m³/h and the normal values are estimated to fall between 7.0 and 10.0 m³/h per unit area at 50 Pa. These values correspond to pressurization tests at specific conditions and are translated into an average annual air change rate n using an empirically derived correction factor from CIBSE [30] for a naturally ventilated office building up to 5 storeys. In addition, we collected the measured air-tightness data of 10 UK office buildings that are naturally ventilated from Perera et al. [31]. The measured values ranged between $8.3 \,\mathrm{m}^3/\mathrm{h}$ and $32.0 \,\mathrm{m}^3/\mathrm{h}$ per unit area at $50 \,\mathrm{Pa}$ with their mean value of 17.9 m³/h per unit area at 50 Pa. Disparity between the measured data and the standard values indicates that actual infiltration rates of existing buildings are often much higher than the values recommended in the standards. Hence, based on both the standard and the measured data, we quantify the minimum and maximum value as 0.10 and $1.25 \,h^{-1}$ respectively with $0.50 \,h^{-1}$ as the base value.

In a naturally ventilated building, the volumetric flow rate $q_v(m^3/s)$ through single-sided open windows can be estimated from the following expression [32]:

$$q_{\nu} = C_D \cdot A_e \cdot \left(\frac{2|\Delta P|}{\rho}\right)^{0.5} \tag{2}$$

where C_D is the flow discharge coefficient, A_e is the effective window area, ΔP is the pressure difference across the opening (Pa), and ρ is the density of the air (kg/m³). The pressure difference ΔP represents the sum of direct wind pressure, thermal buoyancy, and fluctuations. Since wind pressure is the dominant parameter in our building, we ignore the terms representing thermal buoyancy and fluctuations. Following the relationships derived in Larsen and Heiselberg [32], ΔP across the window opening can be thus approximated by the ratio of local air velocity at the window and a reference mean velocity of outside air, at different wind directions, and for a given configuration of the building. Since ΔP is derived mainly as a function of external weather conditions, we do not consider it as an uncertain parameter in this study. C_D represents airflow loss due to shape of the windows. We use the empirically derived values of C_D found in de Wit [33], which estimates their values to fall in the range of 0.6-0.75 for rectangular windows. A_e represents half the window opening area for ventilation, and is therefore a proportion of the total window area A of the building:

$$A_e = f_a \cdot \frac{A}{2} \tag{3}$$

where f_a is the percentage of windows open at a given time. Quantifying f_a for a building can be quite difficult, especially when the act of opening or closing windows is controlled by a diverse set of occupants in their own offices or meeting rooms. Studies have shown that occupants can have significantly different levels of activeness in relation to opening and closing windows. The act of opening or closing a window, and the duration for which they are left in one state or the other is triggered by a whole range of physical, environmental, and psychological factors. In an extensive and seminal study, Borgeson and Brager [34] study a large set of variables potentially influencing occupant control of windows. This work also summarizes existing body of work in this area, many of which are empirical studies for quantifying f_a as a function of local environmental conditions. From these, we consider a logistic regression model derived from a study of fifteen office buildings in UK [35]. The empirical model is able to compute the probability f_a of a window being open as a function of outdoor air temperature by the logit link function:

$$f_a = \frac{e^{b \cdot T + c}}{1 + e^{b \cdot T + c}} \tag{4}$$

where T is the outdoor temperature, b the regression coefficient for T, and c is the constant in the regression equation. This study also derived that the bounds of the intercept c will vary from -3.80 to -2.09, depending on occupant behaviour in a building (see Table 4 in Rijal et al. [35]) with 2.92 as the base value. Coefficient b is also uncertain, but the range of values reported in Rijal et al. [35] seems small enough for us to ignore them.

The heating system of the building is parameterized by its two main components: the efficiency of the heat generating equipment and the losses in the distribution system. The efficiency of the heat generating equipment (a condensing gas boiler in this case) can vary depending on its thermal efficiency, operation under full or partial loads, and temperature of the return water to the boiler. Thermal efficiency of a heat source is typically documented by manufacturers. For condensing boilers, the effects of partial loads on the system efficiency are known to be negligible [36,37]. Hence, the main source of uncertainty about the boiler efficiency is due to the fact that the return water temperature is not known precisely. Provided the boiler efficiency is steady over partial loads, Lazzarin and Schibuola [37] developed three empirical relationships that estimate average, minimum, and maximum performance efficiency as a function of the return water temperature. We follow these relationships to quantify the bounds of uncertainty for the seasonal efficiency of condensing boilers to be in the range of 95-98%.

Heat generated by the boiler is delivered through a distribution system, and part of this heat energy is lost during delivery. Heat losses in water-based distribution systems $Q_{H,dis,ls}$ can be quantified by the following relationship [38]:

$$Q_{H,dis,ls} = \sum_{j=1}^{n} \Psi_{L,j} \cdot (\Theta_m - \Theta_j) \cdot L_j \cdot t_{op}$$
(5)

where $(\Psi_{L,j})$ is the linear thermal transmittance of the pipes in zone j, (Θ_m) is the supply water temperature, (Θ_j) is the temperature of the surrounding spaces, (L_j) is the total length of pipes in zone j, and (t_{op}) is the number of hours zone j is heated. While most of these parameters are observable, it is quite difficult to obtain them even from the most thorough building audit. In fact, most energy models tend to ignore heat losses in the distribution system.

Table 1Preliminary estimates of uncertainties in model parameters.

Model parameters	Initial value	Min	Max
Thermal properties			
Roof <i>U</i> -value (W/m ² K)	0.19	0.17	0.21
Roof solar absorptance	0.68	0.60	0.76
Roof emissivity	0.91	0.87	0.95
Wall <i>U</i> -value (W/m² K)	0.32	0.29	0.36
Wall solar absorptance	0.63	0.43	0.83
Wall emissivity	0.91	0.87	0.95
Window <i>U</i> -value (W/m ² K)	2.62	2.36	2.88
Window solar transmittance	0.77	0.76	0.79
Window emissivity	0.84	0.75	0.92
Envelope heat capacity (kJ/m ² K)	260	160	275
Internal loads			
Appliance power density (W/m ²)	15	12	22
Lighting power density (W/m ²)	13	11	15
Occupant heat gain (W/m ²)	4	3	7
Control			
Indoor temperature (°C)	22	20	24
Ventilation			
Infiltration rate (h ⁻¹)	0.50	0.10	1.25
Discharge coefficient	0.68	0.60	0.75
Intercept c	-2.92	-3.80	-2.09
Heating system			
Heating generation efficiency	0.97	0.95	0.98
Heating distribution loss factor	0.08	0.06	0.16

Therefore, we use our best estimates for parameters described in Eq. (5) to derive that the value of $Q_{H,dis,ls}$ will fall between 6 and 16%.

Internal loads are produced by occupants, lighting, plug-in appliances, and any other heat generating equipment within a building. They are generally expressed as heat input per unit area of the building (W/m^2) . Occupant heat gains depend on their activity level. We estimate heat gains from occupants to be within $3-7(W/m^2)$ based on Macdonald [24]. Heat gains from plug-in appliances are estimated to be within $8-16(W/m^2)$. This range is based on the survey of 30 UK buildings by Dunn and Knight [39] that suggests that plug-in equipment loads range between 124 and 229 (W) per person in a building. Table 1 summarizes the uncertainties around the initial values assigned to model parameters by listing the minimum and maximum limit of values that can be assigned to them. We translate these into prior uncertainty distributions $p(\theta)$ by assigning a triangle shaped distribution to parameters. The initial value is the top of the triangle, and its probability decays linearly to zero at the edges of the assigned interval.

So far we have discussed uncertainties about parameters that describe the physical and operational characteristics of the building. Parameters describing the external environment, or the climate, are typically considered to be known conditions or 'scenarios' under which model is calibrated. For example, following Eq. (1), we use monthly values of gas consumption as observations y(x) taken under known weather conditions x to calibrate the energy model using parameters θ . This does not imply that the scenario themselves cannot be uncertain. Indeed, weather conditions (e.g. local ambient temperature, cloud cover, local wind speed) can differ from the TMY climate data used in the model. In such cases, it would be possible to design the calibration so that they can be considered as calibration parameters. This is currently under investigation as an extension of this work. In our current model we assume that weather conditions are known. This is a reasonable assumption since we use monthly average temperatures that are much less variable across years than daily temperatures.

Table 2Ranked list of model parameters by normative model and energyplus model.

Rank	Normative model	Energyplus model
1	Intercept c	Intercept c
2	Indoor temperature	Indoor temperature
3	Infiltration rate	Infiltration rate
4	Discharge coefficient	Discharge coefficient
5	Appliance power density	Appliance power density
6	Window <i>U</i> -value	Lighting power density
7	Heating distribution loss factor	Glass emissivity
8	Lighting power density	Boiler nominal efficiency
9	Envelope heat capacity	Insulation conductivity
10	Occupant heat gain	Glass solar transmittance

4.2. Parameter screening

Before proceeding to calibrate the model, we need to reduce the number of calibration parameters. Indeed, exploring the parameter space in 19 dimensions is computationally prohibitive even with a light-weight normative energy model that takes less than 1 s per run. We therefore use a parameter screening technique, known as the Morris method [40], to rank parameters by their relative effect on the energy consumption of the building. The Morris method first discretizes the parameter space: it divides each parameter interval into a chosen number of levels that correspond to a pre-selected number of quantiles of the corresponding parameter. This forms a grid of values in the parameter space. After starting from an initial fixed point in that grid, the move to the next step is done by changing one parameter value at a time while the other parameter values stay the same; there is no diagonal move, only moves along axes. Eventually, this allows moves in all directions. At the end of each step, we obtain a number: the elementary effect equal to the change in the model outcome as the result of the change in one input value. At the end of the entire procedure, we obtain distributions of elementary effects for all parameters. This technique is quite useful as it allows us to select a smaller and more manageable set of calibration parameters more objectively than using our own judgment.

We used Simlab version 2.2 [41] to execute the Morris method. Table 2 shows the list of model parameters ranked by their importance to heating energy consumption of our building. We selected the top four parameters to calibrate our model. It is a reasonable number given that we have 3 years of monthly gas consumption data (36 observations). As the third step, we compute model outputs given prior distributions of the four parameters. We use Latin Hypercube Sampling technique to ensure that the parameter space is sufficiently covered. These model outputs, along with prior distributions of calibration parameters and observations (monitored gas consumption data) are given as input to the Bayesian calibration model.

4.3. Model calibration

The Kennedy and O'Hagan [23] formulation of Bayesian calibration requires three sets of data as input: (1) monthly gas consumption values as observations y(x), (2) computer outputs from exploring the space of calibration parameters $\eta(x,\theta)$, and (3) the prior probability density functions of calibration parameters $p(\theta)$. Given these, the model outputs $\eta(x,\theta)$ and the bias term $\delta(x)$ are both modeled as Gaussian processes [42]. A Gaussian process is a generalization of a multivariate normal vector to the case where the index set is infinite. The energy model output under specific known conditions and for a chosen set of calibration parameters, despite being deterministic, is assumed to follow a normal distribution. Jointly, with several outputs under different sets of known conditions, they form a multivariate normal vector with a specific

covariance structure. With such distributional assumptions one can expect to make probabilistic statements about energy model outputs at unknown sets of conditions and parameter values. Indeed, for very similar conditions and calibration parameters, the energy simulation outputs will be highly correlated, and the Gaussian process framework accommodates such joint variations. Ideally, one wants to evaluate the energy model outputs over a very dense set of conditions and parameter values. Since this is infeasible, one uses design of experiments to explore the parameter space as much as possible with a manageable computational burden. The Gaussian process formulation itself necessitates the use of hyper-parameters describing the aforementioned covariance structure. In general, the prior distributions on the hyper-parameters control which terms on the right hand side of Eq. (1) are dominant. Given a good energy model and reliable observations, one would expect the calibrated model to explain most of the variation in the observations with a relatively smaller bias and even smaller observation errors, unless the data says otherwise.

This Gaussian process formulation enables us to compute likelihoods of observations given model parameters $p(y|\theta)$. Through Bayes theorem, we can update the prior distributions $p(\theta)$, and obtain posterior distributions $p(\theta|y)$ of the model parameters given the observations:

$$p(\theta|y) \propto p(y|\theta) \times p(\theta)$$
 (6)

We employ Markov Chain Monte Carlo (MCMC), specifically the Metropolis–Hastings algorithm, to draw from the joint multivariate posterior distribution. As a result, we collect a sample of size n, $\{\theta^{(0)},\ldots,\theta^{(n)}\}$ which approximates the theoretical posterior probability density functions (pdfs) of calibration parameters. To get a visual depiction of these pdfs, we bin these draws for each individual calibration parameter. The resulting plots are empirical histograms, and can be compared to prior distributions in order to assess the influence of observations on the calibration exercise.

Fig. 2 shows the posterior distributions of the four calibration parameters against the prior distributions assigned to them, from which we can infer the following: (a) posterior distribution of intercept c is towards the lower bound within the range -3.80 to (-2.09), and the mode of the distribution is at -3.6. In physical terms, it means that the proportion of open windows is quite strongly at the lower limit of the prior estimates, (between -3.80 and -3.0and most likely at -3.6). (b) The shape of the uncertainty distribution of indoor temperature remains the same after calibration, but it shifts to the lower end by 1 °C. The observations tell us that the mean indoor temperature is distributed around 21 °C and not 22 °C as we originally estimated. (c) Upon reviewing relevant literature, we estimated that the air tightness of our building is most likely to correspond to the average value attributed to UK buildings $(0.5 \, h^{-1})$, and similarly assigned the limits of uncertainty to be within the lower and upper values of infiltration rates reported for office buildings in the UK. The results from the calibration show that the infiltration rate of our building is likely to be much higher $(1.0 \, h^{-1})$, and the spread of uncertainty about it is much smaller. In other words, infiltration heat losses in our building are strongly higher than our prior estimates. (d) The posterior pdf of the discharge coefficient does not change significantly from the prior estimates. This is likely because we were able to give a very good prior estimate of this value. The discharge coefficient is relevant only when windows are open, and the proportion of window area open during the heating season is relatively small (we are calibrating the model with gas energy consumption, and hence primarily for the heating period of the building).

Thus, the calibration exercise enables us to learn from observations and derive better estimates of parameter values by changing the prior uncertainty distributions. The resulting posterior pdfs not only challenge our expertise and establish understanding of

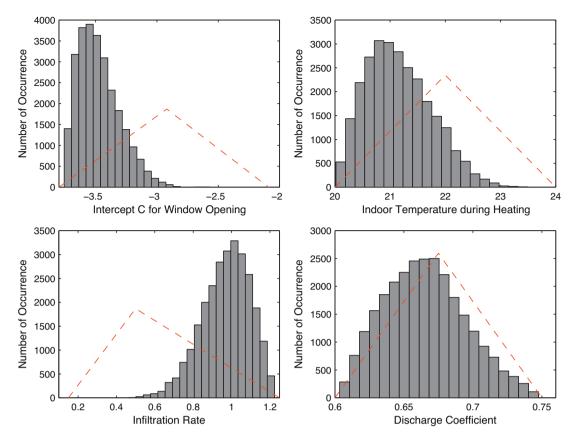


Fig. 2. Posterior distributions of calibration parameters with normative model: posterior - histogram; prior - line.

parameter values, but allow us to improve the predictive power of the energy model for the particular building under study. Fig. 3 shows that predictions from the calibrated normative model match actual energy consumption far better than the uncalibrated model displayed in Fig. 1. At this point we should also re-emphasize that posterior pdfs of individual parameters are derived from a joint multivariate distribution, and are hence correlated. This means that a specific value of one parameter coincides with a certain value of other parameters. Hence, posterior pdfs of all four calibration parameters should be applied in conjunction when exercising the model for retrofit analysis.

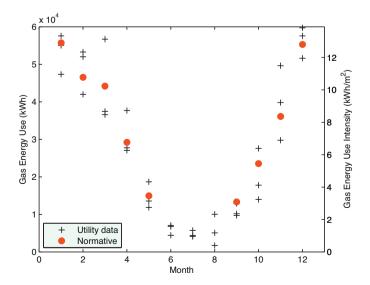


Fig. 3. Monitored gas consumption against expected values from calibrated model.

5. Calibration of the transient energy model

In section 2 we proposed that evaluating retrofit technologies for overall energy savings does not require the most advanced simulation models and a normative model with added calibration can be in fact better, since it derives probabilistic outcomes that quantify the uncertainty in predictions. In order to verify our proposition, we need to compare the outputs from the calibrated normative model (described in the previous section) and the calibrated transient model of the same building. These comparisons test if the calibrated (and hence improved) normative model is as good as the transient model in terms of accuracy of predictions and for evaluating energy saving from competing retrofit technologies. Thus, we calibrate a transient energy model of our building following the process outlined in the previous section. We use energyplus as the transient model, since it is validated and widely used in both research and industry.

The initial model in energyplus takes significantly more time to build than the normative model, mainly due to the level of detail required in modeling (roughly 10 times more than the time required by the normative model to represent main elements of the building). The energyplus model takes approximately 3 min to compute annual heating consumption (on an hourly basis), whereas the normative model takes less than a second for the same (using monthly averages). Therefore, Bayesian calibration of the energyplus model can be computationally expensive. The number of model runs required for parameter screening alone is 150, taking approximately 8 h.

Table 2 shows the uncertain parameters, ranked in the order of dominance using the parameter screening technique. The five most dominant parameters in the energyplus model are the same as those ranked as most dominant parameters in the normative energy model. Further below, the ranking of parameters in

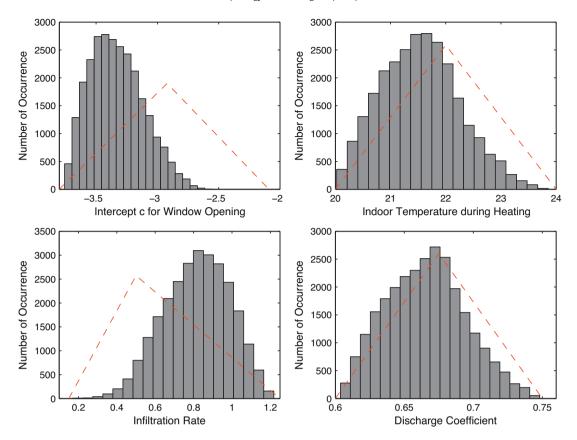


Fig. 4. Posterior distributions of calibration parameters with energyplus: posterior - histogram; prior - line.

the two model differs, which is expected since they compute heat and mass flows in a building at very different resolutions. Since the purpose is to compare the outcomes of the calibrated normative and transient model, we selected the same parameters to calibrate the energyplus model. Fig. 4 shows the posterior pdfs of the four calibration parameters; they are very similar to the posterior pdfs of parameters derived from the calibrated normative model.

6. Comparison between normative and transient energy model

We use standard metrics such as the index of agreement d and coefficient of variation of the root mean square error CV(RMSE) for comparing the outputs from the calibrated model with observed values of energy consumption.

$$d = 1 - \frac{\sum_{i=1}^{n} (O_i - P_i)^2}{\sum_{i=1}^{n} (|P_i - \overline{O}| + |O_i - \overline{O}|)^2}$$
 (7)

$$CV(RMSE) = \frac{\sqrt{\sum_{i=1}^{n} (O_i - P_i)^2 / n}}{\overline{O}}$$
 (8)

where O_i is observed value and P_i is the model prediction in month i. The observed average value of gas energy consumption over the year is denoted by \overline{O} . ASHRAE Guideline 14 [43] stipulates tolerance limits of calibrated simulation models in terms of CV(RMSE); if CV(RMSE) is less than 15% models are declared to be calibrated (see Table 5.2 in [43]). The index of agreement d is also widely used to assess the efficiency of models in comparison to measured data [44,45]. The range of d lies between 0 and 1, with higher values signifying good fit between the model and data. Based on these tests, if the calibrated model is deemed satisfactory, it can be exercised for computing energy-saving potential of different retrofit options.

Table 3 shows the values of *d* and CV(RMSE) of the model predictions before and after calibration. Based on these values, the calibrated normative model predicts as accurately as the calibrated energyplus model and better than the uncalibrated energyplus model. The CV(RMSE) of the calibrated models is 17%, which is higher than the 15% required by ASHRAE Guideline 14. However, it should be noted that the guideline evaluates the accuracy of deterministically calibrated models. Even with lower agreements with measured data, calibrated models based on a Bayesian approach (such as the one we apply) still outweigh deterministic models because they can quantify uncertainties in model predictions.

We also assess the parity of predictions between the two calibrated models by using them to evaluate three energy conservation measures (ECMs) for our building. We consider wall insulation upgrade, window replacement, and improving airtightness. Retrofits in buildings are typically assessed on the basis of cost-benefit ratios and simple payback time [20]. We compute simple payback time (SPT), which is defined as investment costs divided by annual saving costs. Table 4 lists the parameter values specified for these three improvements, and Table 5 lists the capital costs roughly estimated for them. Prior estimates of uncertainties in parameter values of the ECMs are derived from literature

Table 3Validation measures for uncalibrated and calibrated models.

Type of model	Index of agreement	CV(RMSE)
Before calibration		
Normative model	0.76	0.34
Energyplus model	0.88	0.30
After calibration		
Normative model	0.97	0.17
Energyplus model	0.97	0.17

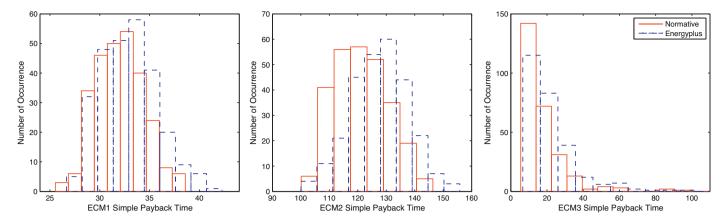


Fig. 5. SPT predictions of three options: normative - solid line; energyplus - dotted line.

Table 4 Specification of three ECMs for retrofit analysis.

Parameters	Base	Min	Max
ECM 1: Insulation addition <i>U</i> -value (W/m ² K)	0.30	0.27	0.33
ECM 2: Window replacement U-value (W/m² K) Solar transmittance Emissivity	1.41 0.65 0.05	1.27 0.63 0.04	1.55 0.67 0.06
ECM 3: Airtightness Infiltration rate reduction (%)	11	1	31

[24,46], and their cost is based on values and standard deviation suggested in [47]. For the calculation of annual saving costs, we use gas price as 2.4 p/kWh [48]. Fig. 5 shows the SPT distributions for the three options generated by the calibrated normative model (solid line) and energyplus model (dotted line). Overall the SPT distributions predicted by the normative model coincide well with those predicted by the energyplus model.

Finally, we also evaluate the two calibrated models to see if they derive consistent results for supporting decisions. The final evaluation of the calibrated models is based on three measures associated with plausibale decision-making scenarios. The first measure uses the expected value of SPT, E(SPT), in the conventional practice. The second measure evaluates guaranteed savings (95-quantile of SPT distributions), $q_{95}(SPT)$, as required by ESCOs [20]. The third measure is based on a saving curve score, developed for actuarial pricing of retrofit projects [16]. The score is defined as the mean savings divided by risk, and the magnitude of risk is computed as the standard deviation of mean savings estimate. Since we use the payback time inverse to mean savings, we use 1/E(SPT) instead of mean savings and $1/(E(SPT) \times \sigma(SPT))$ as the measure. Table 6 shows the ranking of the three ECMs for the three different performance metrics. It shows that the normative and the energyplus model resulted in the same ranking of the three ECMs. This result demonstrates that lower resolution of the normative model does not bias decisions in the retrofit analysis process, and the normative model can adequately support retrofit analysis. Furthermore, it is noted that the ranking of the options differs depending on

Table 5Cost estimates of the three ECMs in 1000£.

Retrofit options	Base	Min	Max
ECM 1: Insulation addition	16	15	17
ECM 2: Window replacement	204	194	214
ECM 3: Airtightness	10.5	10	11

Table 6Ranking of three ECM options for the three measures.

ECM	Measure 1	Measure 2	Measure 3
Normative			
ECM 1	2	1	1
ECM 2	3	3	3
ECM 3	1	2	2
Energyplus			
ECM 1	2	1	1
ECM 2	3	3	3
ECM 3	1	2	2

decision-makers' willingness to take risks. For measures 2 and 3, in which decision-makers are risk-conscious, ECM 1 is preferred instead of ECM 3 even though ECM 3 has a higher expected value. In contrast, measure 1 is more optimistic, and based on the most likely performance. Thus, ECM 3 is preferred when measure 1 is used. The changes in the ranking indicate the importance of information about uncertainty in the retrofit decision-making process. In conclusion, the normative model can adequately serve as a tool to support decision-makings with uncertainties taken into account according to decision-makers' intentions.

7. Conclusions and further work

Despite the increasing need to improve the energy efficiency of the existing building stock, the current methods are not capable to support retrofit decision-makings at large scale with adequate risk management. In order to tackle the limitation of the current methods, we have presented a scalable, probabilistic methodology based on Bayesian calibration of normative energy models. Normative energy models enable modeling a large portfolio of buildings while greatly reducing modeling burdens. Normative models are furthermore calibrated under a Bayesian approach such that the resulting calibrated models quantify uncertainties in the model. In addition, Bayesian calibration models can incorporate additional uncertainties coming from retrofit interventions to generate probabilistic predictions of retrofit performance. Probabilistic outputs can be straightforwardly translated to quantify risks of underperformance associated with retrofit interventions. The case study demonstrates that this methodology can correctly evalute energy retrofit options and support risk-conscious decision-making by explicitly inspecting risks associated with each retrofit option.

In order to improve the applicability of the proposed methodology into standard practice, we need to resolve the following issues:

 More case studies are necessary to confirm the preliminary findings across various buildings (e.g. building function, building

- design, system design). Furthermore, as an illustrative example, the case study applied only envelope-related ECMs, but the proposed analysis framework can, in principle, evaluate any ECMs if uncertainty in model parameters associated with each ECM is provided. More case studies with model improvements will allow other ECMs to be included in the analysis with confidence.
- The current Bayesian calibration module is based on the sophisticated statistical formulation, following Kennedy and O'Hagan's framework. This formulation can cause computational burdens in the calibration process for the large-scale analysis, and its applicability is limited to the cases in which the source of measured data is at one building level. Hence, it is required to develop a simplified, computationally efficient method for Bayesian calibration that can be extended to calibrate a model with various sources of sparse monitored data (e.g. containing a mix of building specific as well as portfolio-aggregated consumption data).
- The study proposes a calibration and retrofit analysis methodology that can potentially be used without deep modeling and calibration expertise. In the current stage, however, the proposed method still relies heavily on experts' judgments especially in the choice of calibration parameters, quantification of their prior distribution, and quantification of uncertainties in other parameters. This will remain so until a repository for standard estimates of parameter uncertainty for variety of building cases becomes available. The development of such a comprehensive database requires an extensive effort and will rely on collaboration within the research community. Only through such effort, model calibration for retrofit analysis will become consistent and transparent, and ultimately automated.
- This paper provides the analysis framework that is ready to incorporate all sources of uncertainty for ECM predictions. Nonetheless, the sources of uncertainty in the case study have been limited to physical properties, equipment performance, and investment costs. The case study ignores other uncertainties such as system degradation over the lifetime and detailed economic factors. In order to correctly evalute the volatility of ECMs, we need to further quantify the full spectrum of uncertainties related to performance and financial risks of energy retrofit projects and ECM selection. For dynamic parameters that may change over time, their uncertainty may also differ accordingly over time horizon. Hence, uncertainty in these parameters should be carefully quantified to capture time-series variation in uncertainty. To capture the stochastic nature of uncertainty in these parameters, it may be required to propagate the uncertainty through stochastic models embedded in the building energy model.

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