Sensitivity analysis techniques for building thermal simulation programs

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Abstract

Three sensitivity analysis techniques, differential sensitivity analysis (DSA), Monte Carlo analysis (MCA), and stochastic sensitivity analysis (SSA), are appraised using three detailed finite difference simulation programs, ESP, HTB2, and SERI-RES. The applicability of the methods to simpler programs is considered. Domestic-scale, passive solar buildings are used as vehicles for testing the methods. The total sensitivities, in both hourly and daily average predictions, due to the uncertainties in over 70 input parameters, are compared for DSA and MCA. The sensitivities of the predictions to changes in a reduced set of inputs are compared for DSA and SSA. It was found that in this case SSA had drawbacks. It is suggested that, at present, DSA is used to obtain the sensitivities of predictions to individual input parameter uncertainties and that MCA is used to obtain the total sensitivities in the predictions. With further work, it may be possible to extract individual sensitivities from MCA, which would make this the preferred technique.

Introduction

In this paper three sensitivity analysis techniques are compared, differential sensitivity analysis (DSA), Monte Carlo analysis (MCA) and stochastic sensitivity analysis (SSA). These were tested using three detailed finite difference thermal simulation programs, ESP [1], HTB2 [2] and SERI-RES [3, 4]. The work was part of the Applicability Study 1 research program, sponsored by the UK Department of Energy to assess the reliability and resolution of simulation programs for the design of passive solar buildings in the UK [5, 6], and has previously been discussed elsewhere [7].

Two types of sensitivity can be evaluated:

(i) individual sensitivities, which describe the influence on predictions of variations in each individual input;

(ii) total sensitivities, due to the uncertainties in all the input data.

A knowledge of the influence that the variations in the individual inputs has on the outputs has numerous practical benefits:

(a) To identify the inputs to which the outputs are particularly sensitive and those to which they are insensitive. It is therefore possible to identify the parameters which must be chosen with care, so that the accuracy of program predictions is not compromised, and the parameters for which accurate specification is unnecessary.

(b) To identify inputs to which the programs are sensitive, but for which adequate data is not yet available, so field experiments can be suggested to produce more accurate values.

(c) To identify the features of a building to which a particular output, e.g., energy consumption, is particularly sensitive. This can guide the designer towards an improved design and the fabricator towards improved quality control in critical areas.

(d) To identify parameters which should be removed from the control of the program user because they cannot (except perhaps by very skilled users) be assigned sufficiently accurate values.

The total uncertainty in outputs due to all the input uncertainties enables the following to be assessed:

(e) The resolution of the programs (i.e., the maximum accuracy) which can be expected in absolute predictions. This information is important for empirical validation studies, in which predictions are compared with measured data, since it allows sound judgement to be made about the validity of the programs.

(f) The probability distribution of the results and hence a knowledge of, for example, the likelihood

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that the energy use will not exceed a particular value.

(g) The significance of uncertainties due to computational inputs (such as time-step and node placement [8]), modelling assumptions (e.g., number of building zones) and different algorithms (e.g., for window conduction) by providing a precision benchmark.

These uses indicate that sensitivity analysis encompasses both parametric studies, in which input parameters are deliberately and systematically changed (by large amounts) to determine the influence on program predictions, and error analysis, in which the consequences on predictions of (small) errors, or uncertainties, in input data are assessed.

This paper concentrates on error analysis and, in particular, the errors in the predictions which occur because the geometrical and thermophysical properties which describe a building are not precisely known at the design stage. However, the conclusions drawn about the three methods may be equally valid for a wider range of program input data or when viewed from the perspective of parametric studies.

Differential sensitivity analysis

Theory

Differential sensitivity analysis (DSA) is widely used because it enables the sensitivity of the program outputs to input parameter changes to be explored directly. By making suitable assumptions, it also permits the total uncertainty in a chosen output, due to changes in many inputs, to be found. DSA involves varying just one input for each simulation whilst the remaining inputs stay fixed at their most likely 'base-case' values (Fig. 1). The changes in a predicted parameter (p) are therefore a direct measure of the effect of the change made in the single input parameter (i). Repeated simulations, varying a different input parameter each time, enable the individual effects (Δp_i) of all the input changes to be determined:

$$\Delta p_i = p_i - p_B \tag{1}$$

where p_i = value predicted using modified value of input i, and p_B = value predicted using base-case inputs. The predicted values, for which uncertainties are quoted, may be instantaneous (hourly) values (such as air temperature or peak energy use) or time-averaged values (such as pre-heat time or annual energy use).

In common with the other two techniques (MCA, SSA), the DSA method does not impose a restriction on the form of the input data uncertainty, but in the absence of information to the contrary, it is often assumed that each input is distributed normally about the modal, or base-case, value. This assumption was made in this study (Fig. 1). Since there may be a non-linear relationship between any input change and the consequential change in the output, an average value for the sensitivity, over the likely range of input parameter change, is usually the most relevant measure. Thus input changes Δi (of say 2.33 standard deviations, s) are appropriate since there is only about a 1% probability that the input value could, by chance, lie outside these bounds. There is therefore only about a 1% probability that errors or uncertainties in the input i could lead to changes greater than Δp_i in the predicted value.

The values of $\Delta p_i/\Delta i$ are an *estimate* of the first-order differential sensitivities of a particular output p, with respect to a particular input, i, and have been termed influence coefficients by Spitler *et al.* [9]. However, in ref. 9 it was proposed to make arbitrary (small) changes (di) to each input to find the effect (dp_i) on a prediction and then extrapolate to obtain the actual effect of (larger) input parameter changes. This is dangerous if the system (thermal program) behaves in a highly non-linear way. It is safer to estimate the effect of smaller changes to i by interpolation. For example:

$$dp_i = di\,\frac{\Delta p_i}{\Delta i} \tag{2}$$

In the case of a perfectly linear system this equation is exactly true and the Spitler *et al.* approach, and the approach proposed here, are identical.

Provided all input parameters are varied by the same amount (say 2.33s), their combined, or total, influence on the predicted parameter (Δp_{tot}) may be estimated from the quadrature sum of the influences due to each of the (I) inputs:

$$\Delta p_{\text{tot}} = \left(\sum_{i=1}^{I} \Delta p_i^{\,2}\right)^{1/2} \tag{3}$$

This method was suggested by workers at the Solar Energy Research Institute (SERI) within their validation program [10, 11] and has since been used successfully by others in empirical validation studies [12, 13]. Assuming that the uncertainty in the predicted parameter is normally distributed, Δp_{tot} gives the total uncertainty expressed as 2.33s, i.e., the same point on the distribution curve as that chosen for each input uncertainty (Fig. 1).

The value of Δp_{tot} is only strictly correct if the sensitivity to each individual input is independent of the value of the other inputs (i.e., the thermal

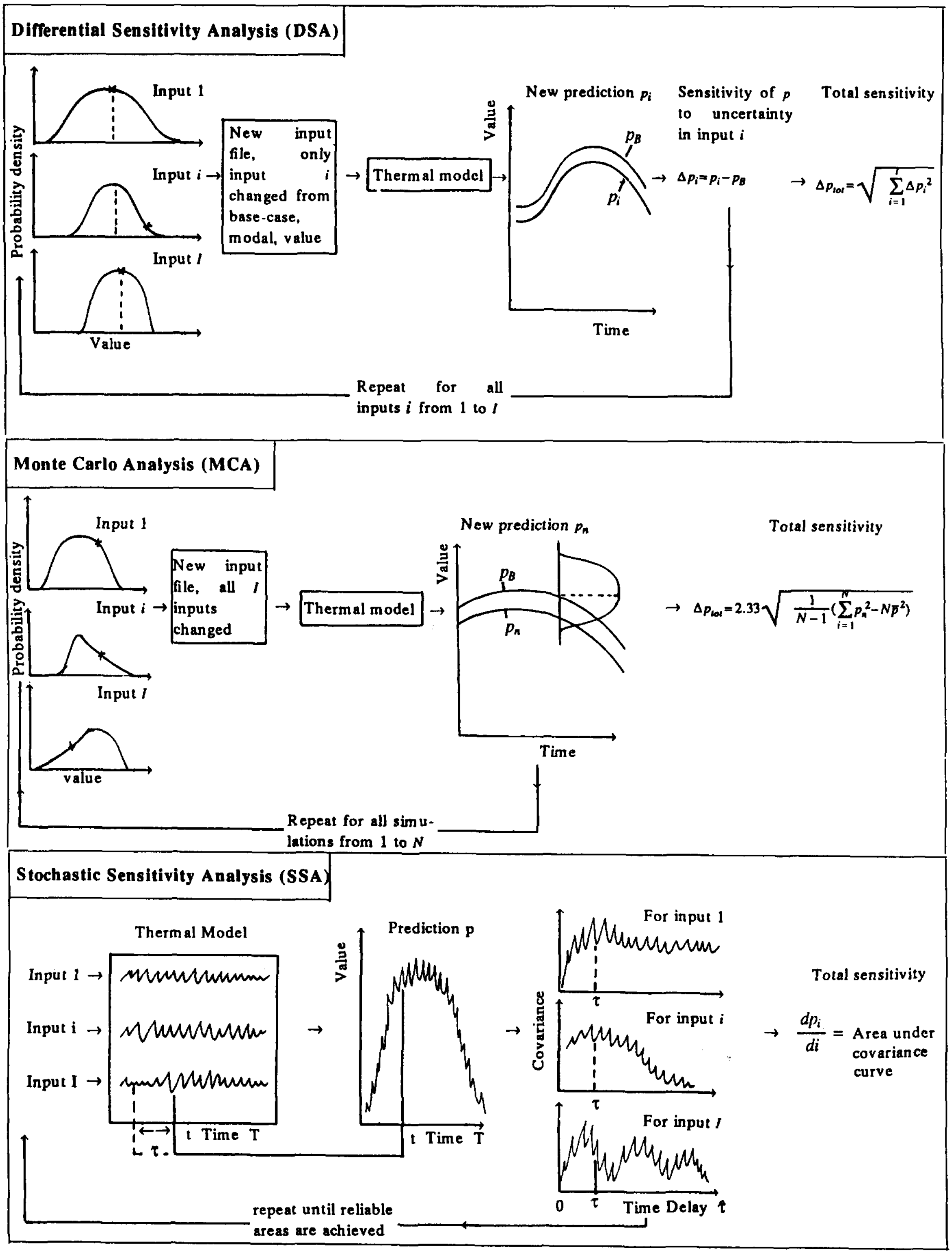


Fig. 1. Diagrammatic comparison of the three sensitivity analysis techniques.

program behaves as a superposable system). For most systems (buildings and imposed weather and operating conditions) this is not strictly true. (Indeed, if it were true, then predicting the thermal performance of buildings would be a relatively trivial task and there would be no need for complex programs.) Nevertheless, for small changes in the input data the assumption may be reasonable. One purpose of this study is to assess the errors which arise from making such an assumption.

The DSA method itself does not place any restrictions on the parameters which are predicted or the inputs to which their sensitivity is being sought. The inputs could be values which are usually fixed (such as the thermal conductivity of a material, the ground reflectivity or the thermostat set-point), or values which may vary with time (such as casual gains, solar radiation or surface convection coefficients). However, in the case of inputs which vary with time, any perturbations must be applied to every (hourly) occurrence of that parameter. Furthermore, since the errors (e.g., in radiation measurements) are frequently stated as a percentage (e.g., 3%), the absolute value of Δi is likely to vary (hour-by-hour) as the parameter assumes different base-case values.

Implementation

The process of selecting an input, perturbing it, creating the new input file, performing the simulation, collecting the results, and then calculating the individual and total uncertainties is repetitive. It was therefore automated, using a Fortran program (Fig. 2). This program takes the information required to conduct the analysis from a 'perturbation' input file, which contains a list of all the inputs to be perturbed, their base-case values, and the amount Δi by which they should change. The program firstly conducts a simulation with each input at its base-case value and stores all the predicted values (p_B). Then, one simulation is conducted for each input listed in the perturbation file. For each of these the base-case value of each input to be perturbed is overwritten with the new value, the thermal simulation is repeated, the new set of predictions is saved, and the base-case predictions are subtracted from the corresponding new predictions to find the individual sensitivities (Δp_i).

Once all I simulations have been completed, there are I files containing the Δp_i values. These are then accessed to add the corresponding Δp_i values in quadrature in order to obtain Δp_{tot}.

One important feature of the computer implementation of DSA was that much of it was not program-specific and so it did not demand access to the coding of the thermal program itself. The method can therefore be applied equally well to programs for which only object code is available

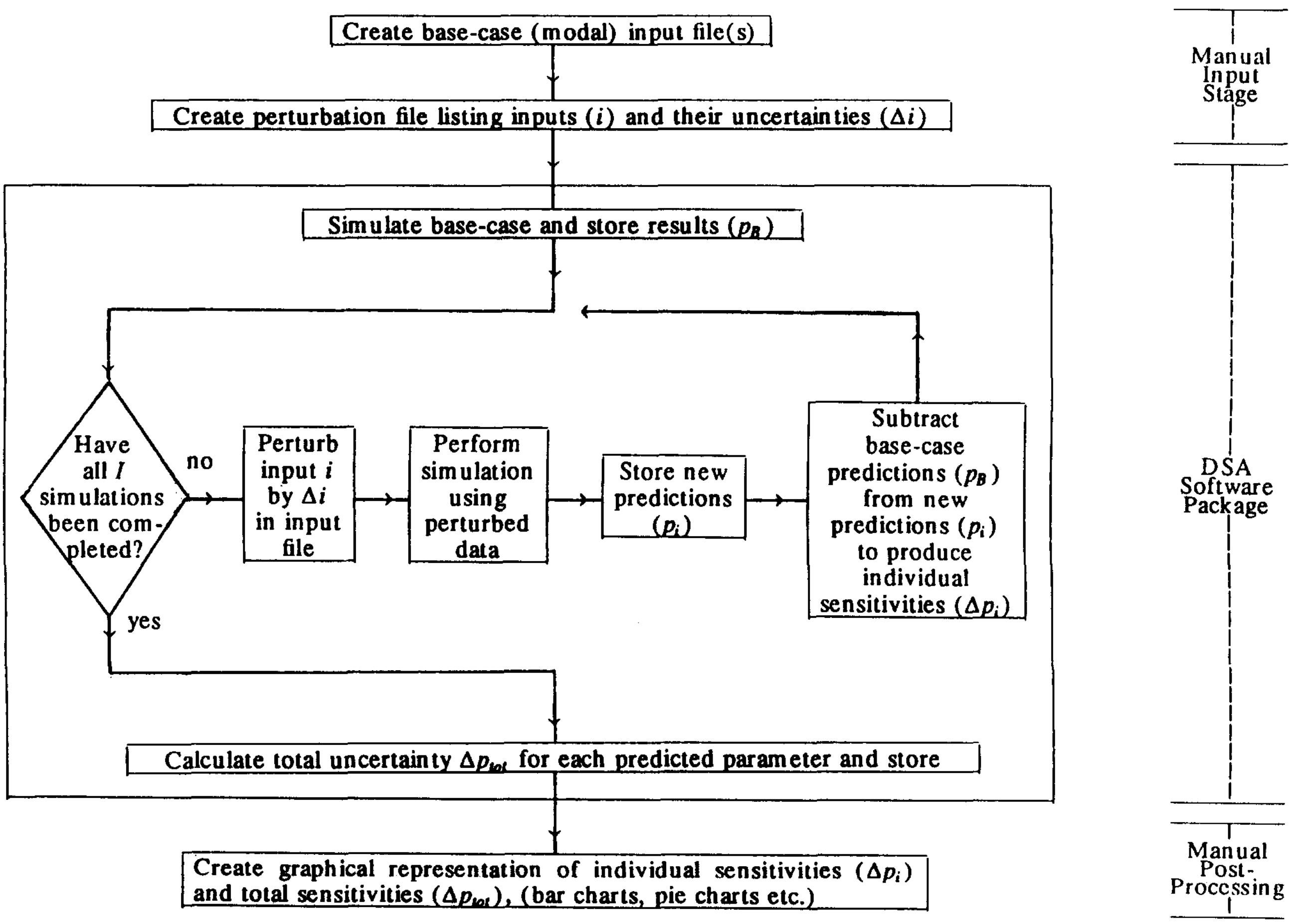


Fig. 2. Implementation of the differential sensitivity analysis procedure.

as it can to those for which source code is available. It can also be applied to simplified dynamic programs, such as those based on the admittance procedure [14], steady-state programs, such as BREDEM [15], electrical analogue models [16] or hand-calculation techniques, such as the UK Chartered Institution of Building Services Engineers (CIBSE) design-day and degree-day methods [17].

Monte Carlo analysis

Theory

The use of MCA in the thermal modelling field has been suggested by workers at the SERI [10] and the Los Alamos National Laboratory [18] and it is widely used to analyse systems in other fields of scientific endeavour. In MCA all the uncertain inputs to a program must be assigned a definite probability distribution. For each simulation one value is selected at random for each input based on its probability of occurrence. For inputs which are normally distributed, values near the modal value are more likely to be selected than extreme values. The predictions produced by this unique set of input values are saved and the process is repeated many times, using a different and unique set of inputs on each occasion. Upon completion many values of each predicted parameter will have been obtained and these will have a particular distribution. Provided there are a large number of inputs, irrespective of their individual distributions, the values predicted for a particular parameter (p) are likely to be normally distributed (Fig. 1). Thus, the total uncertainty in the predictions may be expressed by the standard deviation (s):

$$s=\left[\frac{1}{N-1}\left(\sum_{n=1}^{N}p_n^2-N\bar{p}^2\right)\right]^{1/2} \qquad (4)$$

where n = simulation number, N = total number of simulations, and $\bar{p}$ = mean value of output parameter p.

An estimate of s can be obtained after any number of simulations and the accuracy of this estimate can be determined using the χ^2-distribution to calculate a confidence interval around s (e.g., see ref. 19). The accuracy of s depends only on the number of simulations undertaken (N), and not, as for DSA, on the number of uncertain input parameters (I). It can be seen (Fig. 3) that, irrespective of the number of inputs and outputs to the program, only marginal improvements in accuracy are obtained after 60–80 simulations.

Since all the inputs are perturbed simultaneously the method fully accounts for any interactions be-

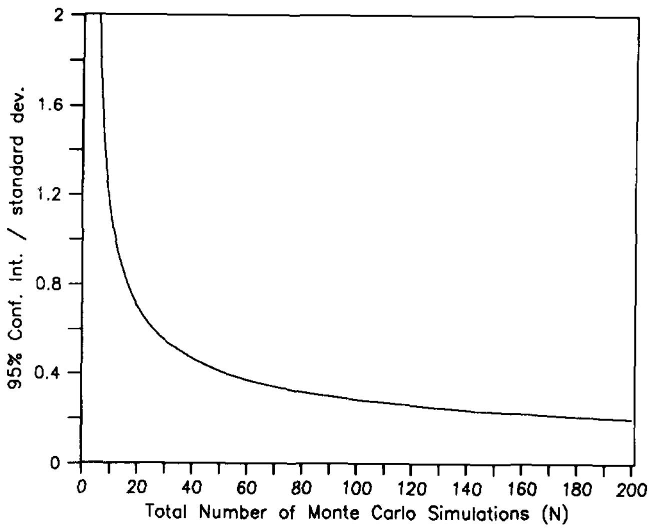


Fig. 3. Relationship between normalized confidence interval and number of MC simulations.

tween the inputs and, in particularly, any synergistic effects. Thus, unlike the DSA method, it is not necessary to assume that the effects of the inputs are superposable. Furthermore any non-linearities in the input/output relationships are fully accounted for. The obvious disadvantage is that, because the inputs are varied simultaneously, the sensitivities of the predictions to the individual input parameter changes are not divulged.

In this study the Δp_{tot} values calculated by DSA were represented by 2.33s so similar values were produced for MCA:

$$\Delta p_{tot}=2.33\left[\frac{1}{N-1}\left(\sum_{n=1}^{N}p_n^2-N\bar{p}^2\right)\right]^{1/2} \qquad (5)$$

Implementation

The MCA procedure was automated in a similar way to the DSA procedure (Fig. 4) and the same perturbation file, which lists the uncertain inputs and their degree of uncertainty, was used to control the procedure. Two algorithms for generating the normally distributed random numbers (r_n), necessary as the basis for input value selection, were tested. One, taken from ref. 20, used the equation:

$$r_n=S\sin\left(\frac{\pi}{2}r_1\right)\left(2\ln\frac{1}{r_2}\right)^{1/2} \qquad (6)$$

where r_1, r_2 = uniform random numbers with $0\leqslant r_1$, $r_2\leqslant 1$, and S = random sign.

This produced values which approximated very closely to a normal distribution with a mean of zero and a standard deviation of one (Fig. 5). The dispersion from an exact normal distribution was no greater than that produced by the algorithm embed-

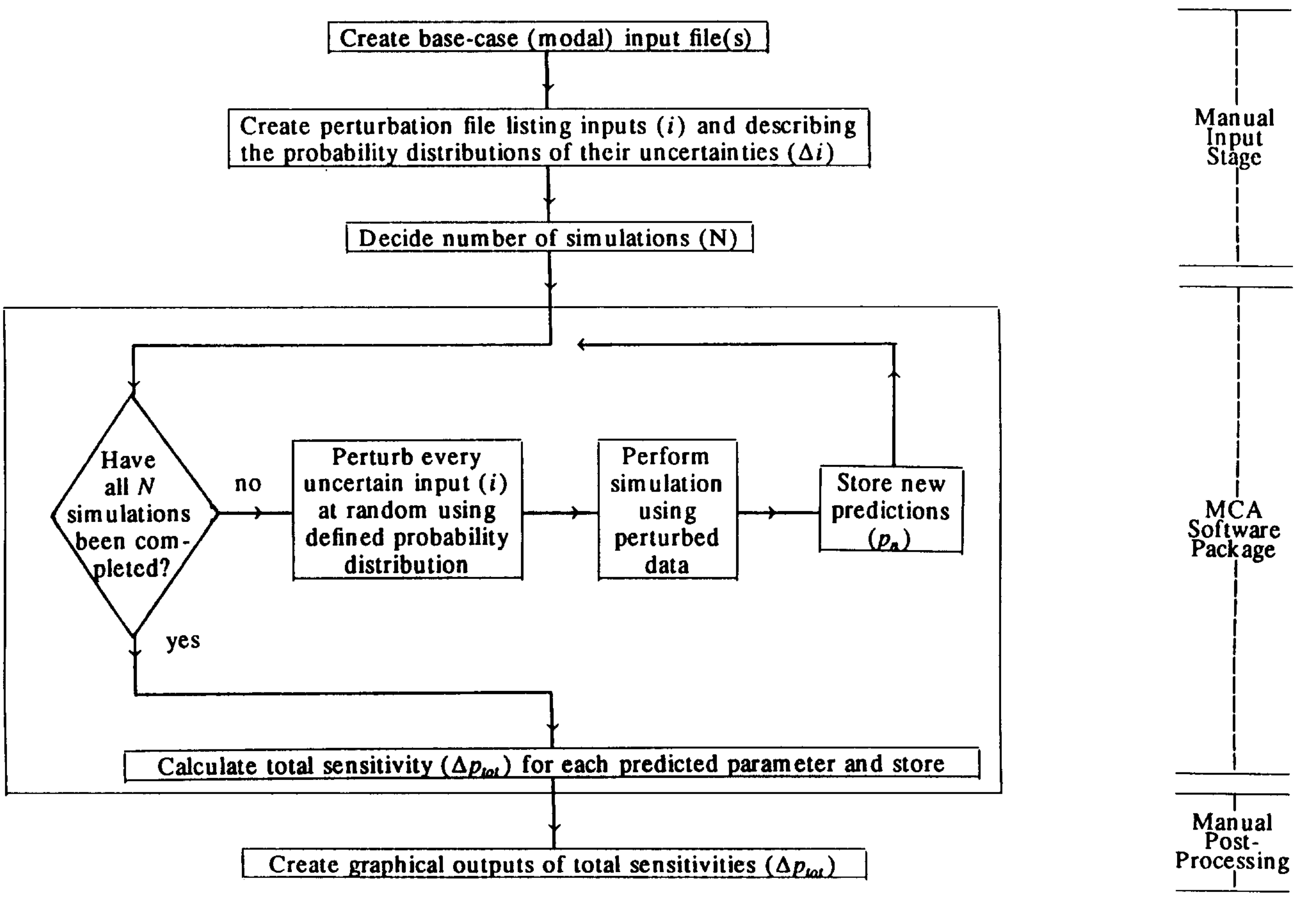


Fig. 4. Implementation of the Monte Carlo analysis procedure.

ded in NAG subroutine g05ddf [21]. With either algorithm, it is important that the uniform random number generator is set to begin at a random point so that a different sequence of normalized variates is produced in successive Monte Carlo analyses (Fig. 5).

To test the method, the sets of predictions produced by each Monte Carlo simulation were stored and post-processed to generate the error estimates. However, if the distribution of predicted values is not needed, the calculations necessary to calculate s (sums of p_n and p_n^2) could be conducted after each simulation thereby avoiding the storage of the N result sets.

The implementation of the method was not program-specific and it did not demand access to the program source code. As for DSA, the method does not place any limitation on the output parameters which are examined or on the inputs to which uncertainty is attached, neither does it place limitations on the complexity or simplicity of the program being used.

Stochastic sensitivity analysis

Theory

Stochastic sensitivity analysis (SSA), like DSA, seeks to generate the sensitivity of predictions to the individual parameter uncertainties. In contrast to DSA, being a comparatively new technique, little use has been made of SSA in the building simulation field. In SSA all the uncertain input parameters are varied simultaneously as the simulation progresses, typically at every time-step (Fig. 1). This is in contrast to the other two techniques, in which the uncertain parameters are varied before the simulation commences and are then held constant for the duration of the simulation. SSA is mathematically and computationally far more complex than the other two methods so only a brief description is given here. A more detailed theoretical treatment can be found in Appendix 1 and ref. 22.

To understand the method, consider initially a simple linear and time-invariant system which has an input (i), which is held constant at all times (t) during the simulation, and which generates a pre-

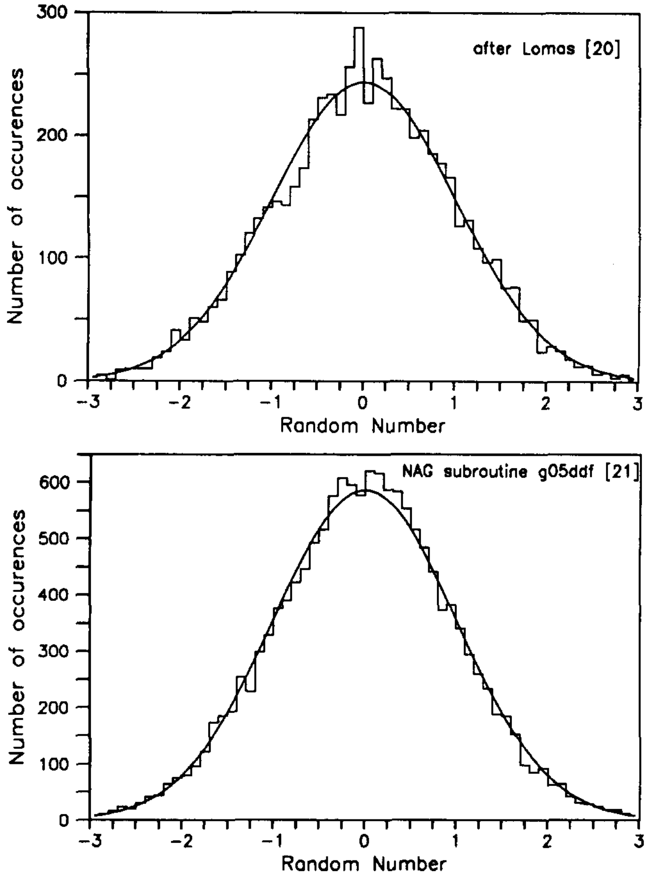


Fig. 5. Comparison of algorithms for producing normalized random numbers.

diction (p) which is constant. Other inputs and outputs are being ignored for the time being. The input could be, for example, the window area and the prediction could be the air temperature in a building. If the input (i) is subject to a brief impulsive change at time t of one unit (in practice of minimum duration one time-step) the predicted value will also change, although the change will be delayed and/or smeared out over a period of time after t due to the time-lags associated with the building. The shape of the prediction generated by the impulse change is in fact the impulse response curve $h(\tau)$ relating the prediction to the input. If the system is assumed to be linear, the effect on the prediction at any time τ after a non-unit change in the input i can be found by simply factoring up or down depending on whether the change is greater or less than unity. By extending this line of thinking it can be seen that $h(\tau)$ can be used, in the discrete form of the convolution equation, to establish the change in a prediction at time t of the impulses which occurred at the previous $(t-\tau)$ time steps:

$$p_t = \sum_{\tau=0}^{\hat{\tau}} h(\tau) i_{t-\tau} \Delta\tau \tag{7}$$

where p_t = predicted value at time t, $i_{t-\tau}$ = input value at time $t-\tau$, $h(\tau)$ = impulse response for time delay τ, $\Delta\tau$ = increment in time delay, and $\hat{\tau}$ = length of impulse response of the system. In the thermal modelling field this formulation is used as the basis for the response factor method for calculating the influence of heat flux through building fabric.

The stochastic sensitivity of the prediction due to input parameter changes is defined as the area under the impulse response curve (Appendix 1):

$$\frac{dp_i}{di} = \sum_{\tau=0}^{\hat{\tau}} h(\tau)\Delta\tau \tag{8}$$

The SSA method hinges therefore on the calculation of $h(\tau)$, but it can be shown, from the Wiener–Hopf equation (e.g., see ref. 23), that if the changes to the input at each time step are completely random (i.e., white noise):

$$h(\tau)\Delta\tau = \frac{C_{ip}(\tau)}{C_{ii}(0)} \tag{9}$$

where $C_{ip}(\tau)$ = cross-covariance between input i and prediction p at time delay τ, and $C_{ii}(0)$ = auto-covariance of the input with itself for time delay zero, then:

$$C_{ii}(0) = \sum_{t=0}^{T} (\Delta i_t)^2 \tag{10}$$

If such white noise is applied to an input (e.g., the value of window area is randomly changed at each time step) then, provided the simulation is long enough, the average change during the simulation ($\overline{\Delta i}$) will be zero:

$$\overline{\Delta i} = \sum_{t=0}^{T} (i_t - i_B) = \sum_{t=0}^{T} (\Delta i_t) = 0 \tag{11}$$

where i_t = input value at time step t, i_B = base-case, unperturbed, input value, and T = duration of simulation.

Consequently, although the new prediction at each time (p_t) differs from the corresponding base-case value (p_B), the average change ($\overline{\Delta p}$), over the simulation period will also be zero:

$$\overline{\Delta p} = \sum_{t=0}^{T} (p_t - p_B) = \sum_{t=0}^{T} (\Delta p_t) = 0 \tag{12}$$

In this situation, the covariance function between the input i and the predicted parameter p at any time delay τ ($C_{ip}(\tau)$) is given by:

$$C_{ip}(\tau) = \sum_{t=0}^{T} (\Delta i_{t-\tau} \Delta p_t) \tag{13}$$

Discussion of the technique

With the SSA technique, any number of inputs may be varied simultaneously provided that the applied stochastic variations (white noise) are uncorrelated. The auto-covariance function for each input time series ($C_{ii}(0)$) and the cross-covariance function for each input/output pair ($C_{ip}(\tau)$) can then be generated simultaneously. Hence the pairwise impulse response function ($h(\tau)$) and the individual pairwise sensitivities may be calculated. Since only a single simulation may be required to generate all the sensitivities, the technique is potentially much faster, once implemented, than the DSA technique, when a large number of input/output pairs are to be used. However, the main attraction of the technique is that information about the time-delay response of the outputs, due to variations in the inputs, could be obtained, along with an estimate of the accuracy of the calculated sensitivities. Having said this, the DSA technique is easier to implement and is usually computationally faster.

The SSA technique produces a mean sensitivity for the duration of the simulation and not instantaneous values (for a chosen time step). Hence sensitivities of, e.g., peak air temperature or peak power output, cannot be obtained. At present the method cannot deal with discontinuities in the input data, for instance scheduled casual gains, plant operation, or ventilation rates (due to window openings), but this deficiency is being investigated. The method is also limited to programs which operate in discrete time steps, such as finite difference simulation programs.

For any one stochastic simulation the impulse response curves for each input/output pair can be very 'noisy'. This is because the instantaneous sensitivities vary with time (for example the sensitivities of the heat flux through glazing will vary depending on the temperature difference across it). Also, because the other inputs are being varied simultaneously there can be some 'leakage' of this noise across to the input of interest. Furthermore the impulse response curves will vary from one stochastic simulation to the next because of the random nature of the perturbation being made. To try and overcome these problems, a stochastic simulation may be repeated a number of times (say 50) to produce a reliable impulse response curve.

Implementation

The method was implemented in HTB2 (Fig. 6) because it permitted frequent stochastic variations to the inputs, i.e., at every 30-second time step. HTB2 is also a well-structured code and this permitted the stochastic software (to perturb the chosen inputs, and to capture and analyse the chosen input/output pairs) to be embedded with minimum difficulty.

By adopting a 'rolling barrel' technique, it was possible to minimize the amount of computer storage required, because it was necessary to store all the previous values of the chosen time series. However, the barrel must have the ability to store results for more than $\hat{\tau}$ time steps and, in buildings with time constants of a few hours, this could be hundreds of time steps. By calculating a number of 'running' summations 'on-line' (e.g., $\Sigma_{t=0}^{T}(\Delta i_{t-\tau}\Delta p_t)$), a reduced amount of post-processing time was required in order to calculate the individual auto-covariance and cross-covariance functions. From these, the pairwise discrete response functions were generated and the area under them calculated. The stochastic simulations were repeated 50 times to produce a mean value for the areas. This limit was dictated purely by the availability of human and computer time resources.

Comparison of DSA and MCA

Test conditions

The total uncertainties predicted by DSA and MCA were compared for ESP, HTB2 and SERI-RES, for a range of single-zoned buildings subject to different sets of daily weather data and various occupancy scenarios. Accurate and compatible input files had been developed for each program as part of the Applicability Study 1 project [5, 6], so there was minimal risk that inter-program differences between the predicted sensitivities could be due to discrepancies in the base-case input data. The results shown in this paper are typical, and were generated for a building representing the rectangular living space, on the ground floor, of a modern passive solar dwelling subject to cool, clear winter weather conditions [24].

The walls, partitions and floor were of well-insulated heavyweight construction and the only window consisted of 7.2 m^2 of south-facing single glazing (Fig. 7). To test the robustness of the sensitivity analysis methods, stressful operating conditions were chosen. The room was intermittently occupied with the lights on from 17:00 to 23:00 and a television operated from 18:00 to 19:00 and 20:00 to 22:00. Heating was supplied from 07:00 to 09:00 and 16:00 to 23:00 by an air-based system controlled by a thermostat set at 21 °C. At night (17:00 to 08:00) blinds were drawn across the window reducing the U-value from 5.6 W m^{-2} K^{-1} to 2.79 W m^{-2} K^{-1}. The 'system' modelled was therefore inherently non-stationary.

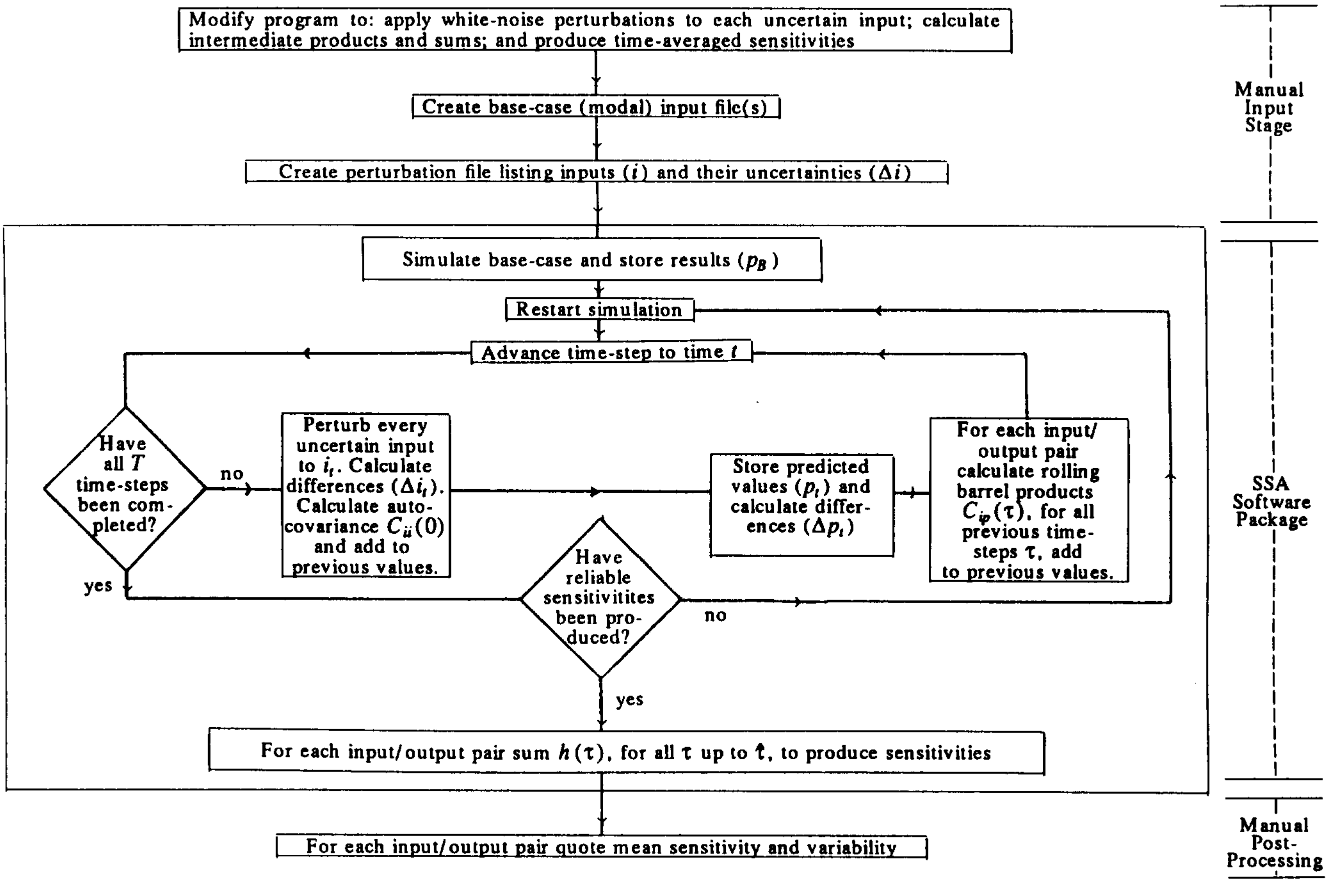


Fig. 6. Implementation of the stochastic sensitivity analysis procedure.

For all the simulations, three nodes were used in each layer of material and the day's weather data were repeated 19 times to pre-condition each program. The number of time steps per hour were 4 for ESP, 120 for HTB2 and 56 for SERI-RES.

The hourly values predicted for each program were: the power demand, the internal air temperature, the internal surface temperatures, the mean radiant temperature (MRT), and the dry resultant temperature (DRT). From these, the following daily values were calculated: the energy consumption; the peak power demand; and the maximum, minimum, mean and swing of the air temperature, mean radiant temperature and dry resultant temperature. The total uncertainty in both the hourly values and the daily performance indicators are presented.

Variability of input parameters

There are numerous inputs to detailed simulation programs and the uncertainty to be attributed to them depends on the reason for conducting the sensitivity analysis and the building being studied. Here, the situation in which a program is being used to predict the performance of proposed (as yet unbuilt) dwellings is considered. If 'standard' weather conditions and operating schedules are adopted, the (only) uncertainties which exist are in the input data which describe the building itself. These arise because the structure, once finished, will differ slightly from that which was modelled, even if it is built in line with the designer's specification. These 'designer' inputs encompass:

(a) the geometry of the building (height, length, breadth);

(b) the thermophysical properties of materials (conductivity, density, specific heat, emissivity and absorptivity);

(c) the transmission properties of glazing;

(d) the area and orientation of glazing;

(e) the radiant/convective split of heat input and temperature sensed.

Estimating the errors in each designer input is a difficult task, however, a great deal of progress was made at Leicester Polytechnic in the UK Science and Engineering Research Council (SERC)/Building Research Establishment (BRE) validation project [12]. The magnitude of the input data errors varies considerably, for example, the geometry of the room, the area of glazing, and the orientation are likely

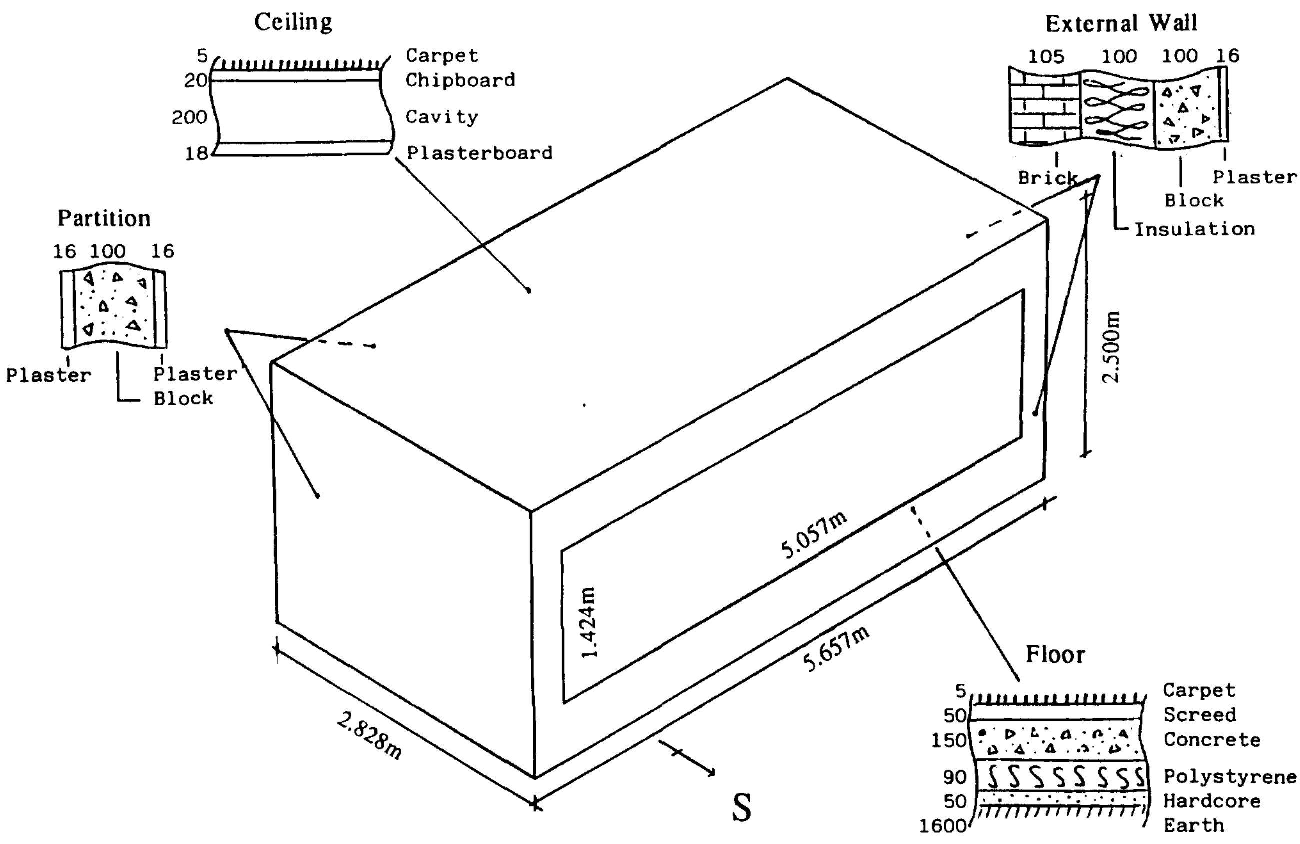


Fig. 7. Geometry and constructions for the comparison of DSA and MCA.

to be very close to those proposed by the designer, whereas the thermophysical properties of materials and the radiant/convective split of the heat from an emitter are quantities which are much more difficult to define. (They may also vary with time, for example as the moisture content of the materials changes.) In this work, the upper and lower bound values, such that there was only about 1% chance that the actual (on site) values could lie beyond them, were estimated from published literature. As mentioned above, the distribution of the errors was assumed to be roughly normal and the value midway between the upper and lower bounds was chosen as the base-case, or modal value, for each parameter (Tables 1 and 2).

In addition to these uncertain designer inputs, uncertainty was assigned to the casual heat gains and the thermostat set-point. These were assigned rather arbitrary values and were included simply to test that the DSA and MCA techniques could also cope with uncertainty in these 'driver' inputs.

Work on the general topic of data uncertainty is continuing [27] and a methodology for assigning appropriate uncertainties is being sought. In practice, careful attention should be given to the uncertainty in parameters to which the program is particularly sensitive but, for the remainder, any underestimates in the error on one parameter is likely to be balanced by overestimates for another. The values assigned in this work are unlikely to be in serious error and they are certainly adequate for testing sensitivity analysis techniques, which is the primary topic of this paper.

Linearity and superposability tests

The use of quadrature addition to generate the total uncertainty in predictions from DSA is based on the premise that the effects of the input uncertainties on the outputs are superposable over the range of input changes being considered. To test for superposability a single simulation can be conducted in which all the input parameters are perturbed simultaneously and the resulting predictions can be compared with those which are obtained by adding *linearly* (i.e., not in quadrature) the influences of each of the individual inputs. If they are essentially the same the system is superposable (but not necessarily linear).

It is possible to test whether the effects of an input parameter are linear, by firstly changing the input by $+\Delta i$ and then by $-\Delta i$. If the magnitudes

TABLE 1. Uncertainties in thermophysical properties of materials for DSA and MCA

Material	Property*	Minimum	Mode	Maximum	Source
Brick inner	λ	0.600	0.650	0.700	[25]
	ρ	1700	1850	2000	
	c	760	800	840	
	d	0.103	0.105	0.107	Estimated
Insulation	λ	0.036	0.0465	0.057	[12]
	ρ	9	11	13	
	c	770	805	840	
	d	0.090	0.100	0.110	Estimated
Concrete block	λ	0.310	0.490	0.670	[25]
	ρ	848	1124	1400	
	c	837	918.5	1000	
	d	0.097	0.100	0.103	Estimated
Plaster	λ	0.220	0.285	0.350	[25]
	ρ	600	775	950	
	c	840	920	1000	
	d	0.010	0.012	0.014	Estimated
Brick outer	λ	0.750	0.855	0.960	[25]
	ρ	1700	1850	2000	
	c	650	745	840	
	d	0.103	0.105	0.107	Estimated
Plasterboard	λ	0.140	0.180	0.220	[12]
	ρ	760	1030	1300	
	c	800	970	1140	
	d	0.011	0.012	0.013	Estimated
Plywood	λ	0.120	0.135	0.150	[12]
	ρ	505	545	585	
	c	855	1740	2625	
	d	0.008	0.010	0.012	Estimated
Carpet	λ	0.045	0.0725	0.100	[17]
	ρ	160	280	400	
	c	1000	1180	1360	
	d	0.004	0.005	0.006	Estimated
Screed	λ	0.340	0.410	0.480	[25]
	ρ	900	1200	1500	
	c	800	820	840	
	d	0.045	0.050	0.055	Estimated
Concrete slab	λ	1.300	1.500	1.700	[25]
	ρ	2000	2200	2400	
	c	800	840	880	
	d	0.135	0.150	0.165	Estimated
Hardcore	λ	1.790	1.830	1.870	Estimated
	ρ	1800	2200	2600	
	c	612	712	812	
	d	0.047	0.05	0.053	
Polystyrene	λ	0.025	0.0345	0.044	[25]
	ρ	11	22.5	34	
	c	1214	1342	1470	
	d	0.080	0.090	0.100	Estimated

(continued)

TABLE 1. (*continued*)

Material	Property*	Minimum	Mode	Maximum	Source
Chipboard	λ	0.100	0.140	0.180	[25]
	ρ	650	1000	1350	
	c	2093	2216.5	2340	
	d	0.018	0.020	0.022	Estimated
Earth	λ	1.100	1.300	1.500	[25]
	ρ	100	100	100	Fixed value
	c	500	500	500	Fixed value
	d	1.300	1.300	1.300	Fixed value
Air gap	R	0.15	0.18	0.21	[17]

*Units: λ (Wm^{-1} K^{-1}); ρ (kgm^{-3}); c (Jkg^{-1} K^{-1}); d (m); R (m^2 KW^{-1}).

TABLE 2. Uncertainties in other input data for DSA and MCA

Parameter	Minimum	Mode	Maximum	Remarks
Zone volume (m^3)	39.81	39.99	40.18	±10 mm on x, y, z
Window area (m^2)	7.160	7.200	7.234	±5 mm in x, y
Window orientation (°)	175	180	185	Est. error in site layout
Ground reflectivity (−)	0.15	0.225	0.30	[12]
Ground temperature (°C)	5.28	7.28	9.28	Estimated
Glazing U-value (Wm^{-2} K^{-1})	5.2	5.6	6.0	Without blind – est.
Glazing U-value (Wm^{-2} K^{-1})	2.59	2.79	2.99	With blind – est.
Extinction coefficient (mm^{-1})	0.0301	0.0343	0.0385	±2.5% in transmittance
Internal floor absorptivity (−)	0.65	0.725	0.80	Estimated
External absorptivity (−)	0.55	0.725	0.90	[17]
Internal emissivity (−)	0.85	0.90	0.95	[12]
External emissivity (−)	0.85	0.90	0.95	[12]
Heating conv. fraction[a] (−)	0.95	0.975	1.00	Estimated
Thermostat air fraction[b] (−)	0.95	0.975	1.00	Estimated
Thermostat setpoint (°C)	20	21	22	Estimated
Casual gains:				
occupants sensible heat (W)	69	90	111	[26]
occupants latent heat (W)	46	60	74	[26]
occupants convective fraction[a] (−)	0.75	0.80	0.85	Estimated
lighting (W)	190.8	212.0	233.2	±10%
lighting convective fraction[a] (−)	0.15	0.20	0.25	Estimated
small power (W)[c]	121.5	135	148.5	±10%
small power (W)[c]	142.2	158	173.8	±10%
small power conv. fraction[a] (−)	0.75	0.80	0.85	Estimated

[a]Remaining fraction radiant.
[b]Remaining fraction mean radiant temperature.
[c]Two different output levels scheduled.

of the positive and negative changes in Δp_i are the same, then the system is linear.

It is possible to get an insight into both aspects at the same time by undertaking the superposability test with all the input parameters simultaneously taking one extreme value (either $+\Delta i$ or $-\Delta i$) and then repeating the test with all the inputs simultaneously taking the opposite value. This approach was adopted in this study for ESP and SERI-RES and hourly predictions of energy use and air temperature. The *actual* effect of simultaneously changing all the inputs was obtained by perturbing all the inputs (by either $+2.33s$ or $-2.33s$) so that they would all contribute towards shifting the prediction of interest (hourly energy use or air temperature) in a positive direction. This 'extreme' result was then compared with that *calculated* by adding linearly the individual input parameter sensitivities Δp_i. In this case all the Δp_i values would be positive (even though some of the Δi values would be negative and some positive). It can be seen (Fig. 8) that the actual and calculated curves produced for both energy use and air temperature are virtually identical at all time.

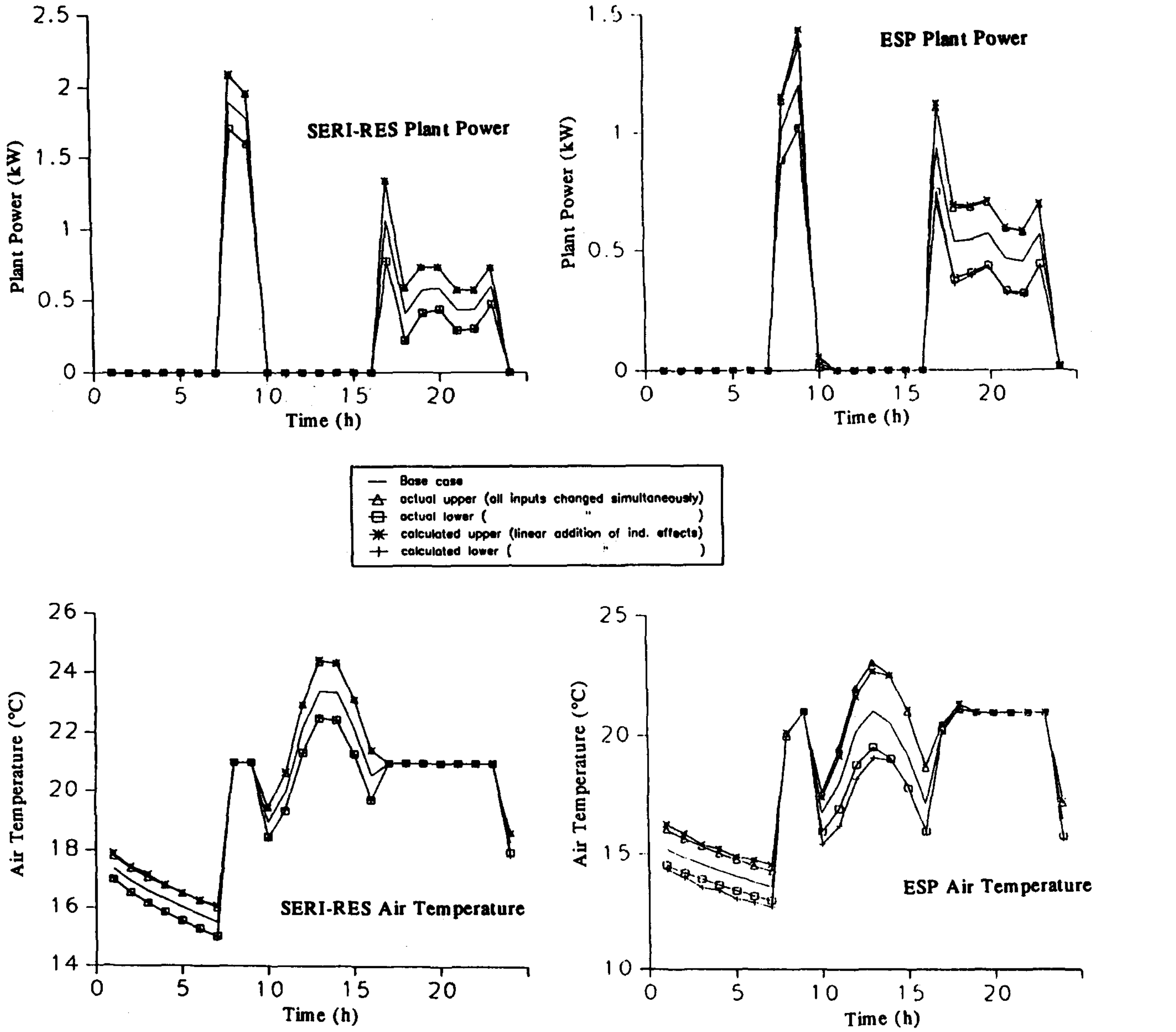


Fig. 8. Linearity and superposability test.

The process was then repeated, but with all inputs being shifted in the opposite direction so that they each contributed a negative change $-\Delta p_i$ to the prediction. Again the actual and calculated hourly values were very similar (Fig. 8), only the ESP air temperature curves differ slightly suggesting that the system is, to a limited extent, non-superposable.

Since the upper pair of curves is displaced from the base-case result by an equal amount to that which the lower pair of curves is displaced (Fig. 8), this study suggests that overall (when all the input changes are considered simultaneously), the programs behave substantially like a linear system. However, it could be that non-linearities for one input parameter are being compensated for by equal and opposite non-linearities in another. So, the issue of linearity was also studied for each individual input parameter (see below). Others [9, 12, 28] have also indicated a high degree of linearity over typical (small) ranges of input parameter uncertainty.

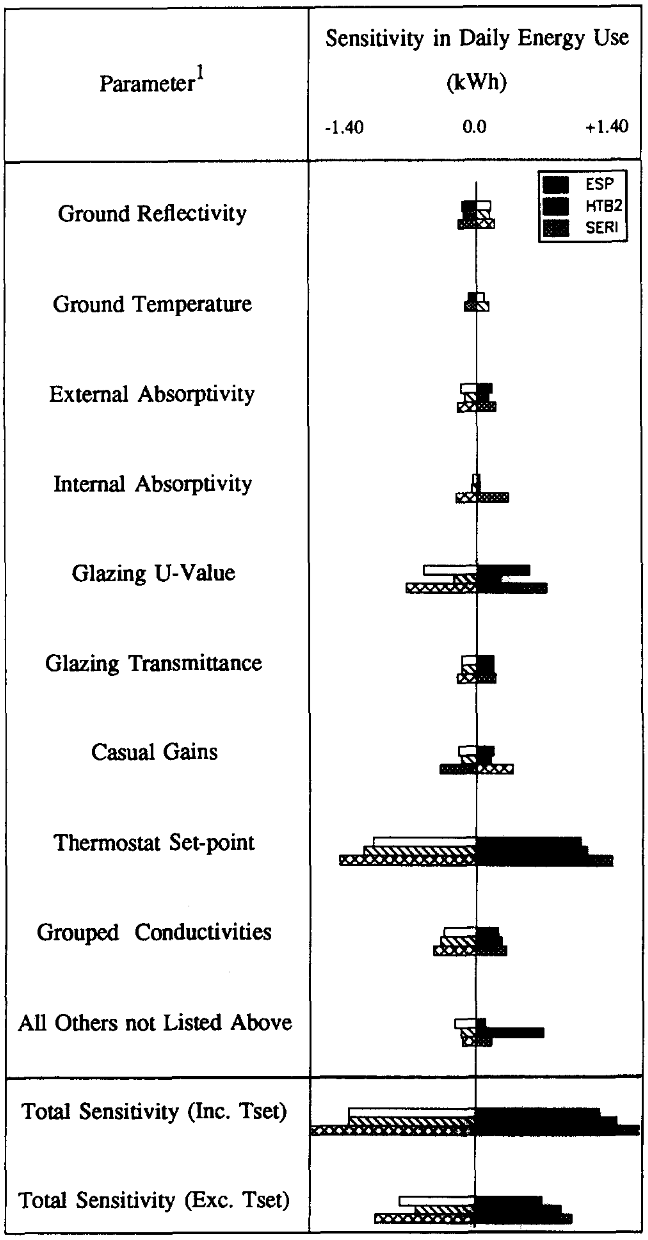
Individual sensitivities produced by DSA

The sensitivity of all the daily performance indicators to errors in each of the inputs (in Tables 1 and 2) were studied for all three programs. In this paper only the sensitivity of the daily energy use predictions due to the individual input uncertainties is presented, however, these results are typical and the methods of presentation, as bar or pie charts, are equally applicable to other predicted parameters.

The bar charts permit a comparison of the effects of different inputs as well as the results for different programs (Fig. 9). The results are presented such that a black or tightly hatched bar represents the effect of an increase in the input parameter value ($+2.33s$) and an open or lightly hatched bar represents the effect of decreasing the input parameter value ($-2.33s$). The pie charts give an overview of the contribution of the individual uncertainties to the total uncertainty for each program. The total area of the pie chart reflects the magnitude of the total uncertainty in the prediction given by quadrature addition (eqn. (3)) and the area (angle) of the sector reflects the relative contribution of each input parameter (Fig. 10).

It can be seen (Fig. 9) that positive and negative changes to the input parameters affect the predicted value by a similar amount (but in opposite directions). This is further evidence that the programs are operating as almost linear systems over the range of input parameter uncertainties being investigated.

The programs display similar sensitivities to each of the inputs and they rank them a similar order (thermostat set-point, glazing U-value and grouped conductivities being, in descending order, the most



[1] see Table 1 for modal values and uncertainties

Fig. 9. Bar chart representation of the DSA results for daily energy use.

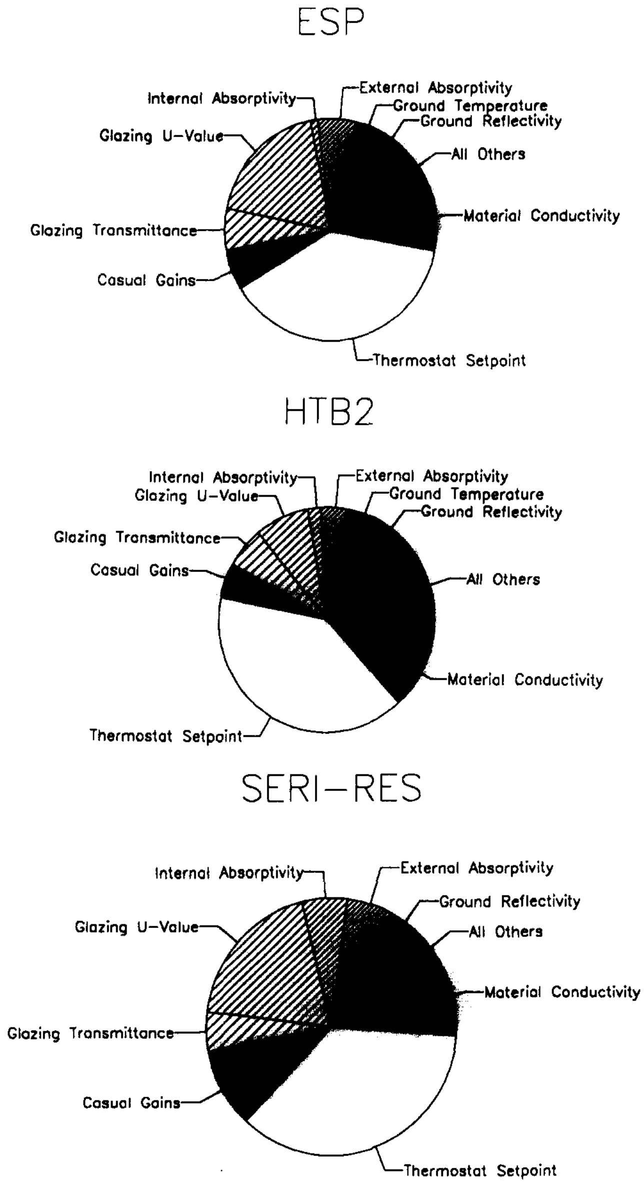


Fig. 10. Pie chart representation of the DSA results for daily energy use.

influential input parameters, Fig. 9). The similarity in the sensitivity of the programs is of course to be expected since they all purport to model, in a similar level of detail, the same building.

Total uncertainties predicted by MCA

An initial study of 200 Monte Carlo simulations was conducted with ESP and SERI-RES to study the distribution of the predicted values of daily energy use, peak power and peak air temperature. From these (Fig. 11) it seemed appropriate to assume (as the central-limit theorem suggests) that the predictions were normally distributed and therefore that the dispersion could be represented by either the standard deviation s or (to permit direct comparison with the DSA results) $2.33s$ (eqn. (5)).

MCA was undertaken for all the uncertain input parameters (Tables 1 and 2), except for thermostat set-point. This was done to prevent the large uncertainty generated by this parameter from dominating the process. (When MCA is being used to quantify uncertainty in program predictions, rather than, as here, to evaluate the method itself, all uncertain parameters must be included.) The effect of the number of Monte Carlo simulations on the estimates of s and $2.33s$ (and their 95% confidence intervals) are shown for daily energy use and all three programs in Fig. 12. It can be seen that after about 30 simulations the curves become steady, and that the s and $2.33s$ values calculated from 50 simulations are within 5% of the values from 100 simulations. Furthermore, the confidence intervals reduce very little after 50 simulations. These results apply to all three programs and similar results were obtained for the other parameters studied, peak air temperature and peak power.

When considering the hourly results, it is clear that non-normal distributions will arise as a result of the non-stationary nature of the system. For example, heating plant power output cannot be less than zero, so distributions will be skewed when the heating system power output is very small. (It is these isolated occurrences which tend to pervert the tendency of daily average results (such as energy use) to assume a normal distribution.) Care must therefore be taken when placing error estimates on hourly results. The use of s (or $2.33s$) at all times may be inappropriate. Another approach is to define the upper and lower bound values exceeded by a chosen number of Monte Carlo simulations. For example, the value exceeded by 15 simulations from a 100-simulation analysis would give the bound exceeded 15% of the time, a similar bound could be defined for the opposite end of the distribution. Were this distribution normal, these would reflect approximately the $\pm s$ values. To find the upper and lower bounds equivalent to $\pm 2.33s$ (1% and 99%) would require a minimum of 100 simulations, but probably more for accurate definition.

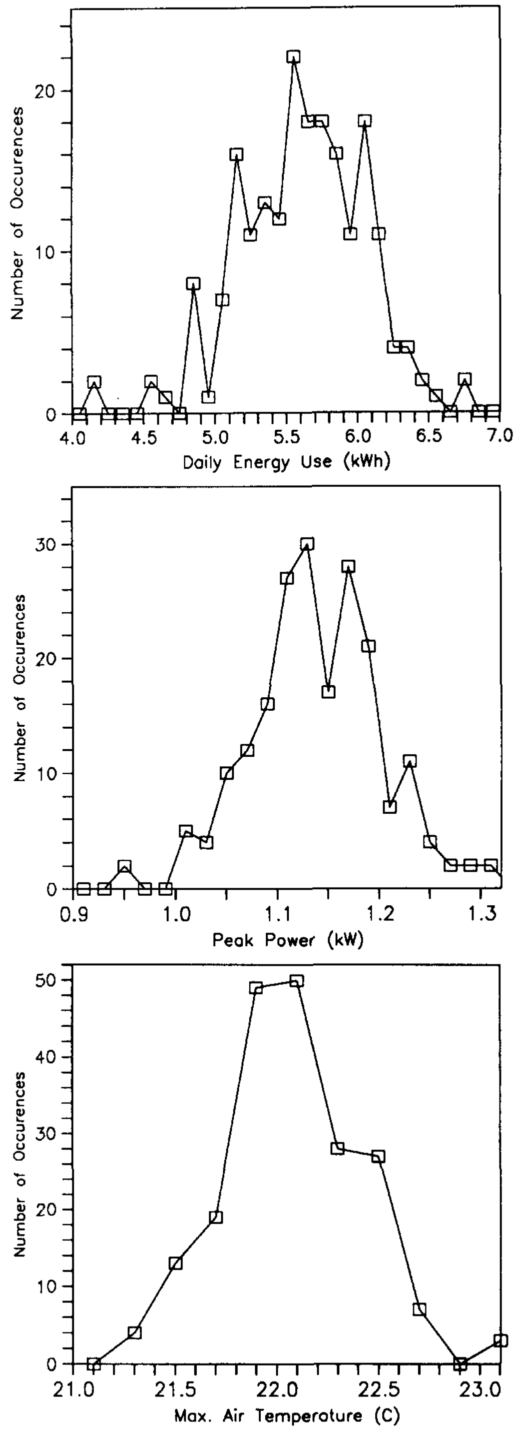


Fig. 11. Distribution of MCA results (ESP).

Comparison of total uncertainties

The total uncertainties in the hourly predictions of plant power output and internal air temperature

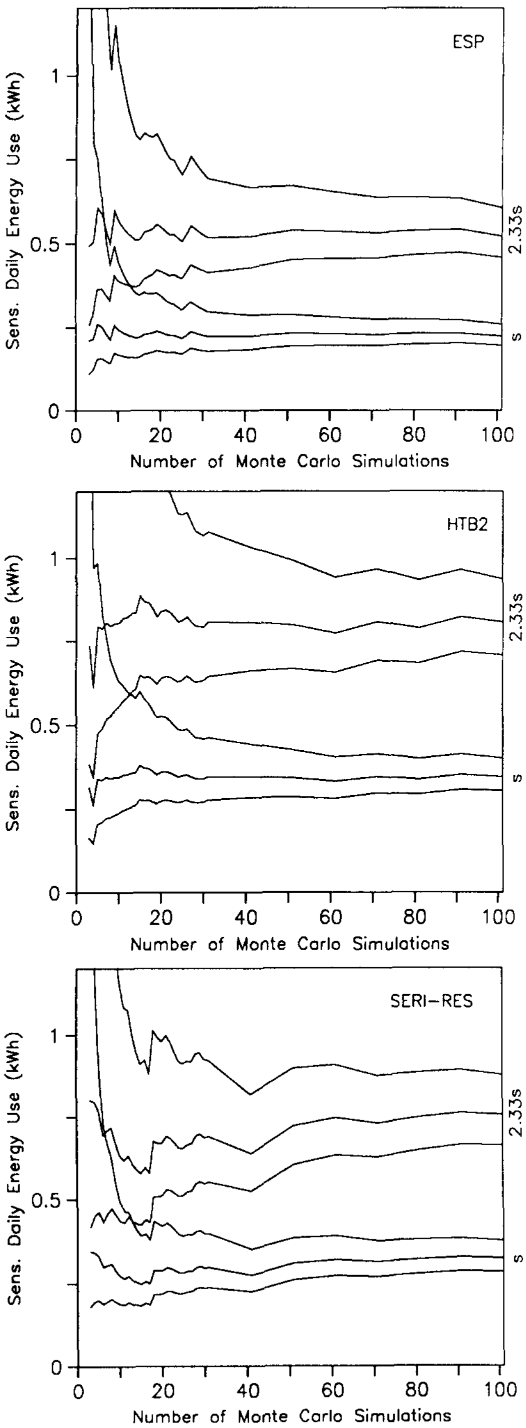


Fig. 12. MCA sensitivity in daily energy use.

due to the uncertainties in all the input parameters (except thermostat set-point) are compared in Fig. 13. It is clear that, despite the restrictions imposed by the theoretical basis of DSA, the results for any program are very similar at all times to the MCA results. This similarity extends to virtually all the daily performance metrics studied (Table 3). Because the building was intermittently heated to a fixed set-point, the uncertainty in the temperature metrics was small (less than 1 °C). The sensitivities predicted by the two methods were invariably within ± 0.2 °C of each other (e.g., minimum dry resultant temperature, for ESP was ± 0.6 °C by DSA and ± 0.4 °C by MCA). There were only three isolated instances where the two methods differed by over ± 0.2 °C (Table 3). The two methods also predicted reasonably similar estimates for peak power output. (For example, the biggest difference occurred for ESP and was ± 0.1 kW (DSA) compared to ± 0.08 kW (MCA).) The large difference between the uncertainties predicted by HTB2 for the daily energy use requires further investigation.

These similarities typify the results obtained for other buildings, occupancy schedules and weather data, and they suggest that there is little practical difference between the DSA and MCA results for these buildings. However, where precise estimates of total uncertainty only are required, MCA may be preferred since it generates the distribution of the results and the theoretical basis does not restrict it to linear superposable systems. When the number of uncertain inputs to a program is very large (over 50), MCA will generate results more rapidly than DSA (Table 4). The converse is true when the number of inputs is small.

An explanation for the large differences in the absolute predictions made by the three programs (e.g., base-case daily energy use, Table 3) is a major aim of the Applicability Study 1 project. Considerable progress has been made [6] and the work is continuing.

Comparison of DSA and SSA

Test conditions

It was intended that the stochastic sensitivities would be obtained for a continuously heated well-insulated dwelling. However, since the heating system could switch on and off during the simulation (in response to solar and internal gains), this would make the system non-stationary, thus violating a key restriction imposed by the SSA method. This early observation severely restricts the potential scope of the method. For example, buildings which are heated, or cooled to a set-point or those with other scheduled parameters (e.g., casual gains or curtain operation) could not be assessed. Further theoretical work may enable the technique to extend to such non-stationary systems.

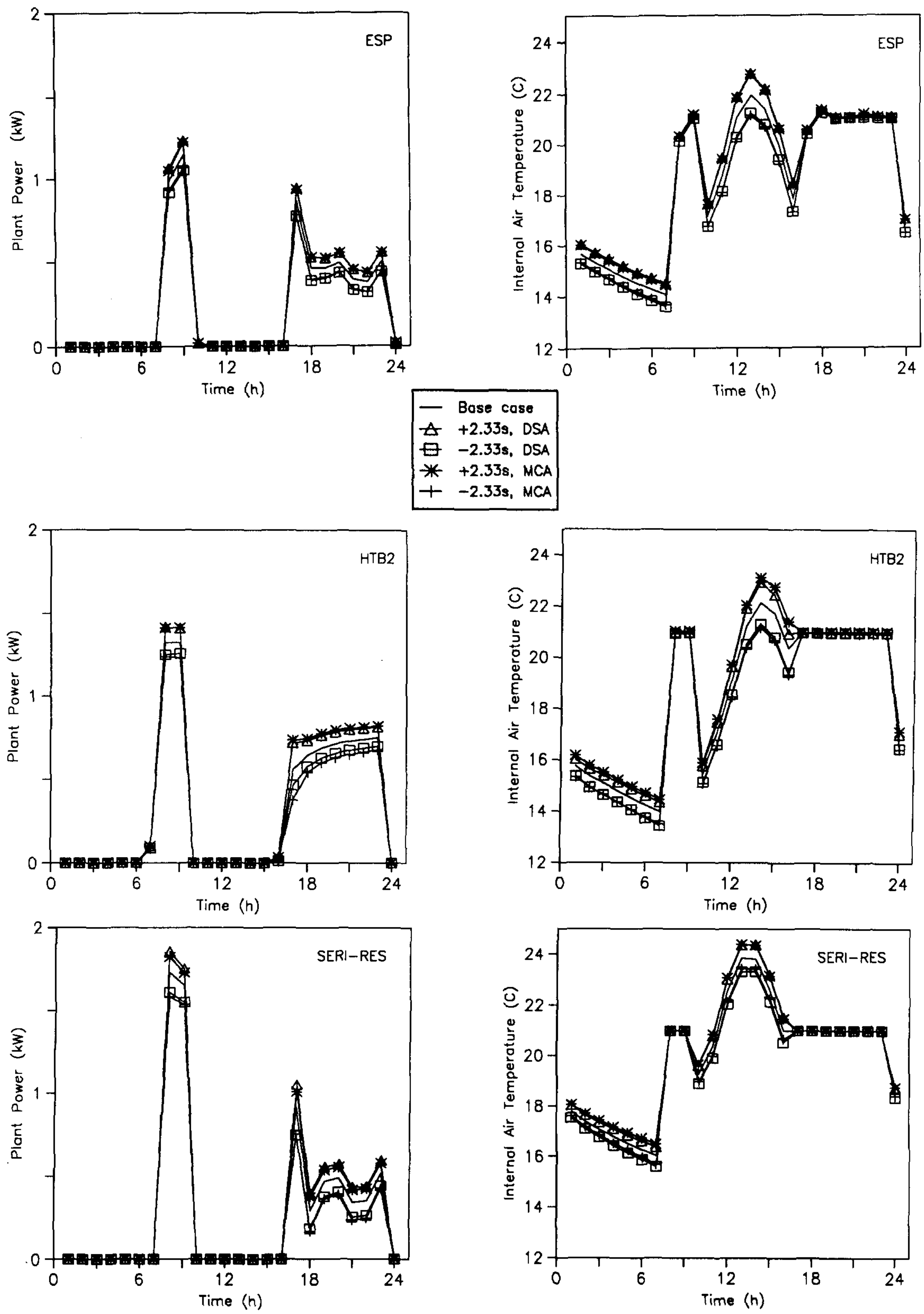


Fig. 13. Comparison of total sensitivities in hourly predictions of plant power and internal air temperature for DSA and MCA.

Results are presented here for a single-zoned dwelling which was very similar to that used to compare the DSA and MCA techniques. The only differences are that there were no occupants or other casual gains, no heating or cooling system and the window was double glazed ($U = 2.9$ Wm^{-2}

TABLE 3. Comparison of total uncertainties in daily performance metrics for DSA and MCA

Daily performance metric	Method	ESP Base	ESP Uncert.	HTB2 Base	HTB2 Uncert.	SERI-RES Base	SERI-RES Uncert.
Energy use (kWh)	DSA MCA	5.71	±0.58 (10.2%) ±0.52 (9.1%)	7.58	±0.45 (5.9%) ±0.81 (10.7%)	6.76	±0.77 (11.4%) ±0.76 (11.2%)
Peak power (kW)	DSA MCA	1.14	±0.10 (8.9%) ±0.08 (7.0%)	1.32	±0.08 (6.1%) ±0.09 (6.8%)	1.73	±0.11 (6.5%) ±0.12 (7.1%)
Mean air (°C)	DSA MCA	18.6	±0.3 ±0.3	18.6	±0.2 ±0.3	20.0	±0.4 ±0.2
Max. air (°C)	DSA MCA	21.9	±0.7 ±0.8	22.2	±0.7 ±1.0	23.8	±0.5 ±0.5
Min. air (°C)	DSA MCA	14.1	±0.5 ±0.4	14.0	±0.4 ±0.5	16.0	±0.4 ±0.4
Swing air (°C)	DSA MCA	7.9	±0.7 ±0.8	8.2	±0.7 ±0.9	7.8	±0.5 ±0.6
Mean MRT (°C)	DSA MCA	19.9	±0.5 ±0.4	17.9	±0.3 ±0.5	18.8	±0.4 ±0.2
Max. MRT (°C)	DSA MCA	24.4	±0.8 ±0.7	22.4	±0.6 ±1.0	23.8	±0.6 ±0.5
Min. MRT (°C)	DSA MCA	16.6	±0.5 ±0.4	14.5	±0.5 ±0.5	16.4	±0.4 ±0.4
Swing MRT (°C)	DSA MCA	7.7	±0.6 ±0.8	7.9	±0.7 ±0.9	7.4	±0.5 ±0.6
Mean DRT (°C)	DSA MCA	19.2	±0.4 ±0.3	18.2	±0.3 ±0.4	20.0	±0.4 ±0.2
Max. DRT (°C)	DSA MCA	23.1	±0.8 ±0.8	22.3	±0.6 ±1.0	23.8	±0.5 ±0.5
Min. DRT (°C)	DSA MCA	15.3	±0.6 ±0.4	14.3	±0.4 ±0.5	16.0	±0.4 ±0.4
Swing DRT (°C)	DSA MCA	7.8	±0.6 ±0.8	0.8	±0.7 ±0.9	7.8	±0.5 ±0.6

TABLE 4. Comparison of computer run times*

Program	Run time for one simulation (h)	DSA Run time[a] (h)	MCA Run time for 50 simulations[b] (h)	SSA Run time for 50 simulations[c] (h)
ESP	0.08	4.5	4.0	–
HTB2	1.12	62.7	56.0	~128
SERI-RES	0.09	4.0	4.5	–

*Values are for a 20-day period for a single-zone building on a SUN 3/60 workstation.
[a]Although 80 uncertain parameters were considered, the number of simulations required for DSA varied from 44 (SERI-RES) to 56 (ESP, HTB2), since not all inputs are used by all the programs. Furthermore, the density and the specific heat of the materials were varied together.
[b]Optimum number.
[c]Chosen number to generate response curves.

K^{-1}) rather than single glazed with no night-time blinds being simulated. To add a semblance of realism, the weather data used was for a hot sunny day [24] and the parameters predicted were the instantaneous values of air, mean radiant and window surface temperatures. The study thus approximates to an investigation of the likelihood of overheating.

Variability of input parameters

Although the method varies each input, by applying 'white noise', limits were placed on the upper and lower values which each input parameter should attain. At this stage, little consideration was given as to the precise relevance of the magnitudes adopted since the bounds do not refer to points on any assumed uncertainty distribution. As the method is in its infancy, only seven input parameters were varied (Table 5). Each input was varied at every time step, rather than every hour, so, for this building using HTB2, there were 120 variations per hour.

Comparison of individual uncertainties

The SSA software embedded in HTB2 was tested by making a single impulse change to external air temperature and recording the resultant discrete response function. This was compared with the curve generated by extracting the covariance functions, and hence the discrete response function, when the external air temperature was varied stochastically; a reasonable match both of form and magnitude was obtained.

For the inputs chosen (Table 5), the daily simulation was repeated 50 times to produce the discrete response functions (some of which are shown in Fig. 14) and hence the stochastic sensitivities and their variability (some of which are shown in Table 5). Only one hour of time delay is shown on the response function plots but the summation to generate the area under these curves, i.e., the sensitivity, was performed over the full length of time delay that had been set up. Both computer memory and speed can be limiting factors of the SSA technique but more information about the system is obtained than with either MCA or DSA. The SSA results shown in this paper were all obtained from a Prime 750 computer, which was found to be about eight times faster than the SUN 3/60 used for the DSA and MCA comparison. The computational times shown for SSA in Table 4 are estimates for a SUN 3/60 based on this run-time ratio.

The discrete response functions for the zone air temperature versus ground reflectivity, density of concrete block and the zone volume are shown in Fig. 14. These curves are typical and can be seen to be 'noisy', and hence the stochastic sensitivities for the chosen parameters, some of which are given in Table 5, are uncertain. For the most precisely determined, the external surface absorptivity, the sensitivity has a standard error (± 1.87) which is about 40% of the actual (SSA derived) sensitivity of 4.79 °C per unit change in absorptivity. For most inputs the standard errors quoted are more than $\pm 100\%$ of the sensitivity. It is thus not surprising that the stochastically and differentially derived sensitivities agree to within the limits of a standard '*t*-test'. However, no estimate of the uncertainties in the DSA were calculated, so no comparison of accuracy can be made between the techniques.

The SSA technique permits more than one input parameter to be varied at a time, but the magnitude of the resultant discrete response function may be small when compared with the size of the noise still present on the function. This can be seen in the middle plot shown in Fig. 14. The size of the noise on the resultant response functions can be reduced through performing further simulations, but, at this stage of the analysis, it can be inferred that the chosen output is insensitive to the particular input.

Discussion of results

A synopsis of the relative advantages and disadvantages of the three techniques is given in Table 6. The most novel and innovative of the three techniques, SSA, has a number of advantages and disadvantages, compared with the other two techniques.

(i) It is more difficult to implement than the other techniques, and is also far more demanding on computer resource.

TABLE 5. Input parameters and time-averaged sensitivities for DSA and SSA

Parameter	Input			Sensitivity of air temperature*	
	Min.	Mode	Max.	SSA	DSA
Ground reflectivity (−)	0.10	0.20	0.30	4.31 ± 2.42	9.73
Window area (m^2)	6.85	7.20	7.60	2.41 ± 3.76	1.56
Window orientation (°)	160	180	200	0.0462 ± 0.2	0.0361
Internal wall absorptivity (−)	0.4	0.5	0.6	-2.07 ± 1.86	0.233
Concrete block density (kg m^{-3})	1000	1400	1800	$3.01 \times 10^{-3} \pm 9.76 \times 10^{-2}$	-1.5×10^{-4}
Zone volume (m^3)	39.8	40.0	40.2	0.769 ± 24.81	-0.123
External wall absorptivity (−)	0.7	0.8	0.9	4.79 ± 1.87	3.31

* °C change in air temperature per unit change in input.

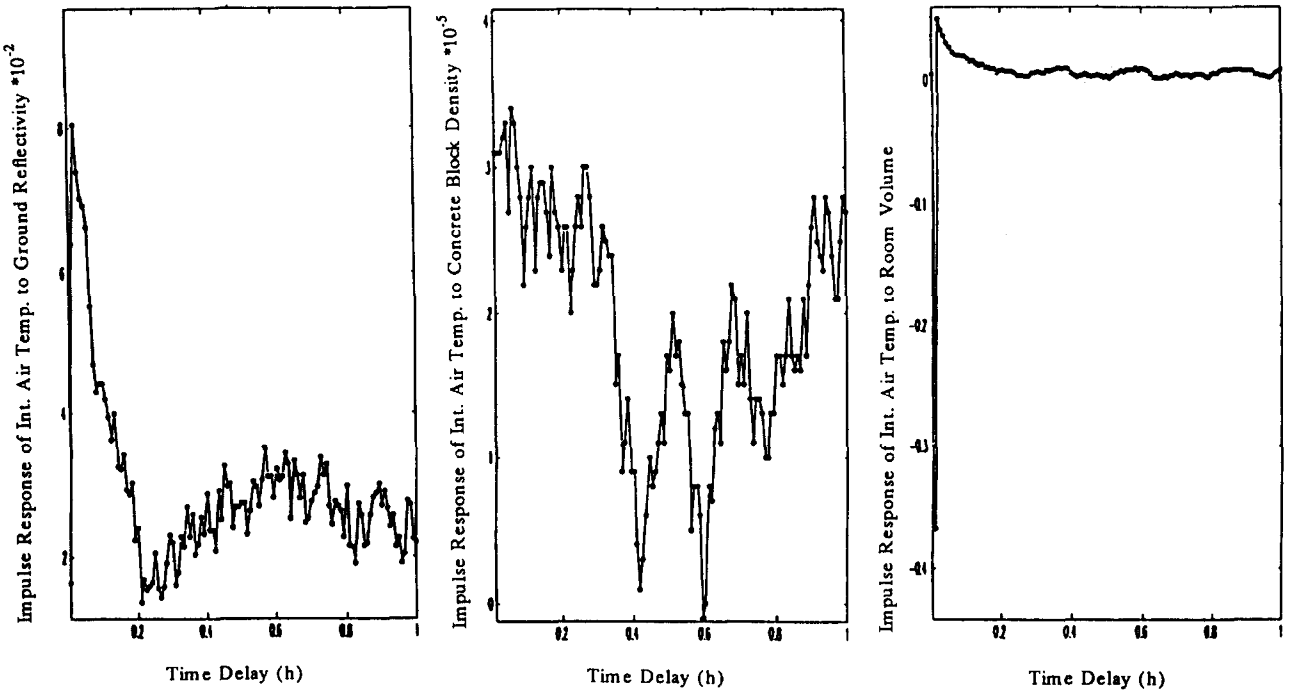


Fig. 14. Typical SSA results.

TABLE 6. Advantages and disadvantages of the three techniques

Technique	Advantages	Disadvantages
DSA	No code modifications necessary (black box approach), any program, easy to implement, any parameter, simple post-processing, individual and total uncertainties	Necessary to assume that input parameters are linear and superposable in their effect to calculate total uncertainties
MCA	No code modifications necessary (black box approach), any program, easy to implement, any parameter, simple post-processing, no assumptions necessary about linearity and superposability, theoretically more rigorous than DSA	At present only total uncertainties can be obtained, not individual uncertainties
SSA	Individual and total uncertainties, accuracy estimates of uncertainties, time-delay domain information, potential for second-order terms	Code modifications necessary, complex processing, computationally demanding, only programs which use discrete time steps, at present cannot be used with discontinuous input data and is limited to linear and superposable systems, time-averaged sensitivities only

(ii) The method, as implemented in this study, demanded access to the programs in order to implement the stochastic routines. Therefore, it cannot be implemented, in this form, when only object code is available or within programs which do not discretize time. Nevertheless, in other work, the technique has succeeded in extracting the thermal transmittance (U-value) and combined convective and radiative heat transfer coefficient values from field data. For such analyses, access to the source code is no longer seen as such a limitation because only the meterological parameters are perturbed.

(iii) The need to use long time-series data sets, together with the heavy computational demands, negates the possible run-time benefits over DSA when only first-order differential sensitivities are of interest. The advantages of SSA are that it produces an estimate of the accuracy of the sensitivity values and the underlying relationship between inputs and outputs. In particular, the response functions are obtained and can be analysed and interpreted for physical significance.

(iv) The SSA form used here, like DSA, is only applicable to linear and superposable systems; however, a non-linear generalization to this work is being developed. Unfortunately, the SSA technique is also restricted to quasi-time-invariant systems, thus buildings which are heated, cooled or which have scheduled occupancy, etc, cannot easily be studied.

(v) The method generates averaged sensitivities for the simulation period so the sensitivity of peak or daily average values cannot be extracted.

(vi) There was reasonable agreement between the SSA and DSA sensitivities, allowing for the large standard errors on the stochastic sensitivities and the non-existence of accuracy measurements on the differential sensitivities.

(vii) Considering the difficulty in calculating the first-order differential sensitivities, it is likely that the second-order sensitivities will be more difficult to calculate.

The other two methods, MCA and DSA, proved to be particularly useful. Both techniques were sufficiently robust that they could be applied to inherently non-stationary systems. They were relatively easy to implement and neither required code modifications *per se*. (The only minor difficulty was due to the large output files produced by some of the programs (particularly ESP), and these had to be trimmed to avoid computer memory problems.) Much of the sensitivity analysis software was common to both methods and was not program-specific.

With regard to computing time, there was little difference between DSA and MCA for calculating total uncertainties, the dominating factor was the number of simulations required. Typically, 50 simulations were necessary to produce reliable MCA results, which for the single-zone building (and a 19-day pre-conditioning period) took about four hours on a SUN 3/60 workstation for ESP and SERI-RES, and about 60 hours for HTB2. For DSA, the number of simulations necessary depends on the number of input parameters. In this study about 80 inputs were varied, although not all of them were used by every program (Table 4).

The three programs appeared to perform as essentially linear and superposable systems over the range of input uncertainties studied, thus the theoretical limitations of the DSA and SSA theory in this regard may be unimportant. However, the generality of this observation should be tested by studying other buildings. The DSA and the MCA total uncertainties were very similar, however DSA has the advantage that the sensitivity to changes in the individual inputs can be obtained. Therefore, for the type of sensitivity study conducted in this paper, the DSA technique has obvious advantages.

In a wider context, the MCA technique would have definite advantages over DSA when:

(i) the distribution of predicted values is needed;

(ii) a number of the key input parameter uncertainties are not normally distributed;

(iii) the inputs vary by large amounts so that the assumption that the system is linear and superposable becomes invalid;

(iv) very accurate estimates of total uncertainties are needed.

(These situations could occur when the variation of energy use in an estate of houses was being studied or when a probabilistic approach to the analysis of energy consumption was being undertaken.)

Bearing in mind the advantages and disadvantages of the three methods (summarized in Table 6), the choice of sensitivity analysis technique probably depends on the system under consideration and precisely what information is needed. However, the authors believe that a generalized, non-program-specific sensitivity analysis package, which would address most needs, could be based on MCA. It may also be possible to enhance the technique to extract, without excessive computing demands, the individual input/output sensitivities. This approach may also offer the opportunity to extract second-order differential sensitivities as well as the first-order sensitivities. Such a tool would be of great value to those involved with commuter simulation and is a goal worth pursuing.

Conclusions

(1) SSA is complex to implement, is computationally demanding and its theoretical basis restricts its sphere of applicability. However, it produces both an estimate of the accuracy of the sensitivities and information on the time-delayed response of the system.

(2) Both MCA and DSA can be applied to the widest range of thermal programs from manual steady-state techniques through to dynamic simulations. SSA is restricted to dynamic finite-difference programs.

(3) Because DSA produces both total and individual sensitivity information, it is the preferred technique for programs which can be assumed to operate as roughly linear and superposable systems.

(4) MCA is no more complex to implement than DSA and requires similar computational effort. Whilst it only generates total sensitivities, it does not demand that the system is linear and superposable. It is suggested that the technique could be extended to permit the extraction of individual sensitivities.

(5) A list of uncertainties which are appropriate to the input parameters used when simulating domestic-scale buildings has been given. Over this range of uncertainties, ESP, HTB2 and SERI-RES behaved as essentially linear and superposable systems. They produced very similar individual and total sensitivity results with both the MCA and DSA methods.

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Nomenclature

$C_{ip}(\tau)$	cross-covariance between input i and prediction p at time delay τ
$C_{ii}(0)$	auto-covariance of the input with itself for time delay zero
di	small change in input i
dp_i	small difference in prediction p due to small change in input i
$h(\tau)$	impulse response for time delay τ
i	a particular input
i_B	base-case, unperturbed, input value
i_t	input value at time t
$i_{t-\tau}$	input value at time $t-\tau$
I	total number of inputs
n	a particular simulation
N	total number of simulations
p	a particular output parameter
$\bar{p}$	mean value of output parameter p
p_B	value predicted using base-case inputs
p_i	value predicted using modified value of input i
p_n	value predicted by simulation n
p_t	value predicted at time t
r_n	normally distributed random number
r_1, r_2	uniform random numbers with $0 \leqslant r_1, r_2 \leqslant 1$
s	standard deviation
S	random sign
t	an instant in time
T	total duration of simulation
τ	time delay
$\hat{\tau}$	length of impulse response of the system
Δi	variation of input i
Δk	time-step length
$\overline{\Delta i}$	average change in input during simulation
Δp_i	effect of uncertainties in input i on output parameter p
$\overline{\Delta p}$	average change in output parameter during simulation
Δp_{tot}	combined effect of all input uncertainties on output parameter p
$\Delta \tau$	increment in time delay

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Appendix 1 – Theoretical basis of stochastic sensitivity analysis

If we assume that the system can be represented by a simple linear uni-variate time-invariant model, then with i_t and p_t representing the input and output time series of the system, one may write:

$$p_t = \int_{-\infty}^{+\infty} h(\tau) i_{t-\tau} d\tau$$

This may be recognized as the first-order convolution equation. Further, if we assume that the system is causal, i.e., $h(\tau)=0$ for $\tau<0$, and that the system has a finite memory of length $\hat{\tau}$, then:

$$p_t = \int_0^{\hat{\tau}} h(\tau) i_{t-\tau} d\tau$$

This is the continuous form of the convolution equation. If the time-series data is not continuous, but discrete, then this equation becomes:

$$p_t = \sum_{k=0}^{\hat{\tau}} h(k) i_{t-k} \Delta k$$

where Δk is of unit magnitude and represents the time-step used to discretize the model, and so both t and k must be integer multiples of Δk.

If the data are stationary stochastic sequences, then, by a series of mathematical steps we can transform the discrete representation of the system given above into the discrete form of the Wiener–Hopf equation [23]:

$$C_{ip}(\tau) = \sum_{k=0}^{\hat{\tau}} h(k) C_{ii}(\tau - k) \Delta k$$

This equation, in this case of a white-noise input time series, can be reduced to:

$$h(\tau)\Delta k = \frac{C_{ip}(\tau)}{C_{ii}(0)}$$

because the auto-covariance of a white-noise time series is a delta function.

We may also solve this equation for the response function values $h(\tau)$ by applying a recursive relationship which is not limited to the white-noise case, thereby extending the Wiener–Hopf white-noise identification and analysis technique.

$$h(\tau)\Delta k = \frac{C_{ip}(\tau) - \sum_{i=0}^{\hat{\tau}} h(i)C_{ii}(\tau - i)\Delta k}{C_{ii}(0)}$$

The sensitivity function of a linear, time-invariant, uni-variate system, will be equal to the derivative [29]:

$$S(i, p) = \frac{dp_i}{di}$$

where i and p are the input and output time series respectively. Hence, differentiation of the model we have chosen to represent the system will provide us with a measure of the sensitivity of the system. Differentiating the discrete equation we have chosen to apply to the system gives us (applying Leibnitz's rule (e.g., see ref. 30) in the continuous case),

$$\frac{dp_i}{di} = \sum_{k=0}^{\hat{\tau}} h(k)\Delta k$$

or in the continuous case,

$$\frac{dp_i}{di} = \int_{\tau=0}^{\hat{\tau}} h(\tau)d\tau$$

i.e., the area under the response function. Hence the definition of the stochastic sensitivity between a pair of values is defined as the area under the pairwise response function.