Diffusion and Auction on Graphs

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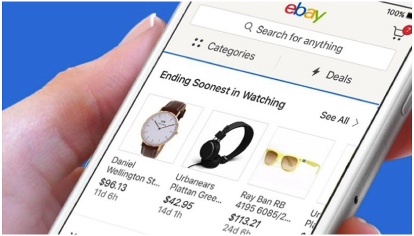
August 16, 2019

What happened in traditional auctions?

Suppose you are buying something through an eBay auction, would you inform others of the sale?



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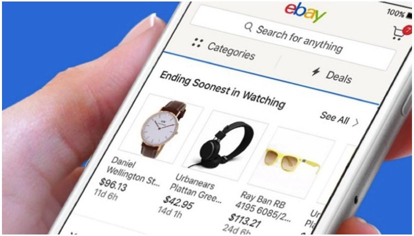


NO!

Traditional Promotions

- Promotions in shopping centres
- Keywords based ads via search engines such as Google

However...

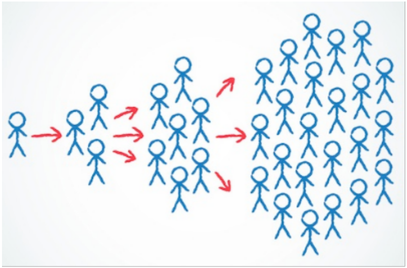
Challenge

- The return of these promotions is unpredictable.
- The seller may LOSE from the promotions.

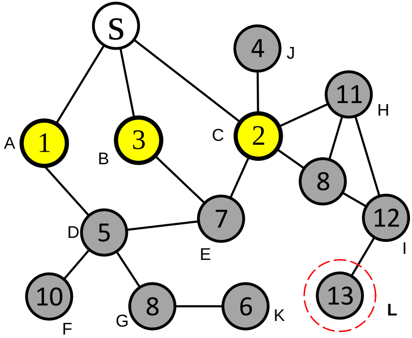
Tackle the Challenge

Build promotion inside the market mechanism such that

- the promotion will never bring negative utility/revenue to the seller.
- all buyers who are aware of the sale are incentivized to diffuse the sale information to all her neighbours.



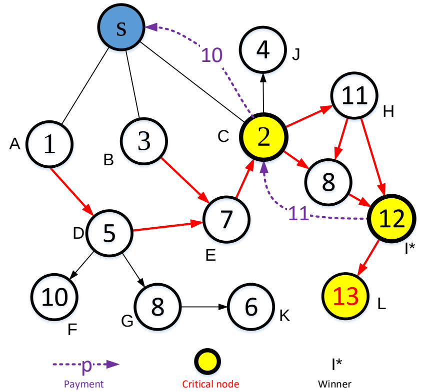
Auction on graphs



The Basic Model

- One item
- Private valuations
- Limited communication
- Initially, only seller's neighbors are aware of the auction

Information Diffusion Mechanism [Li et al., AAAI 2017]



Allocation policy

$$\pi_i^{idm}(\mathbf{a}') = \begin{cases} 1 & \text{if } i \in C_m \setminus \{m\} \text{ and } v_i' = v^*_{-d_{i+1}}, \\ 1 & \text{if } i = m, \\ 0 & \text{otherwise.} \end{cases}$$

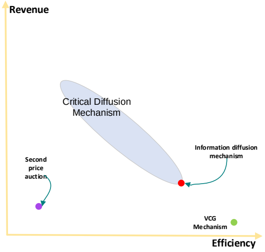
Payment policy

$$p_i^{idm}(\mathbf{a}') = \begin{cases} v^*_{-d_i} - v^*_{-d_{i+1}} & \text{if } i \in C_w \setminus \{w\}, \\ v^*_{-d_i} & \text{if } i = w, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem

IDM is IC, IR, BB and the seller's revenue is no less than that given in the VCG mechanism with/without information diffusion.

Our contributions



In unweighted graphs:

- A class of diffusion mechanisms.
- The lower bound.

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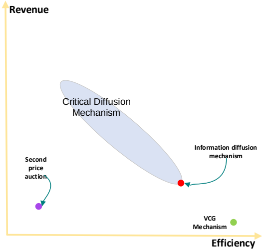
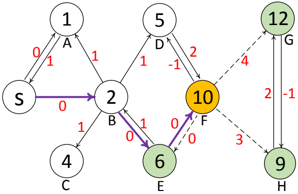




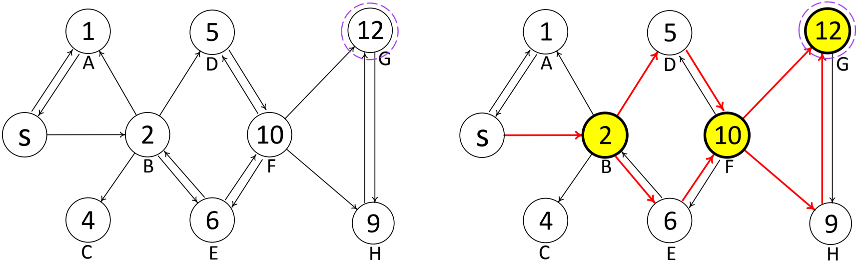
In unweighted graphs:

- A class of diffusion mechanisms.
- The lower bound.

In weighted graphs:

- Weighted diffusion mechanism.

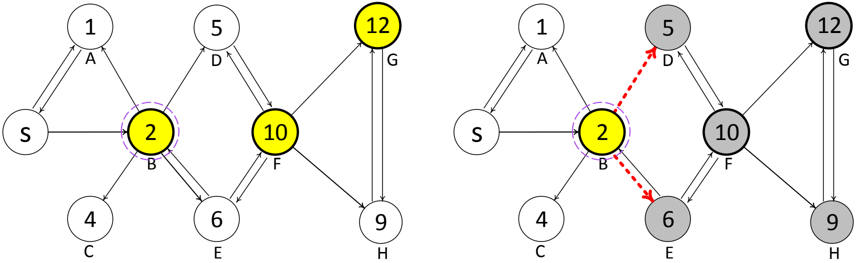
Critical Diffusion Nodes



Critical Diffusion Nodes

- They are cut nodes.
- They are ordered (critical diffusion sequence).

Edge selection function $\alpha_i(j)$



properties of $\alpha_i(j)$

- information blocking (produce a graph cut)
- node independence
- diffusion monotonicity

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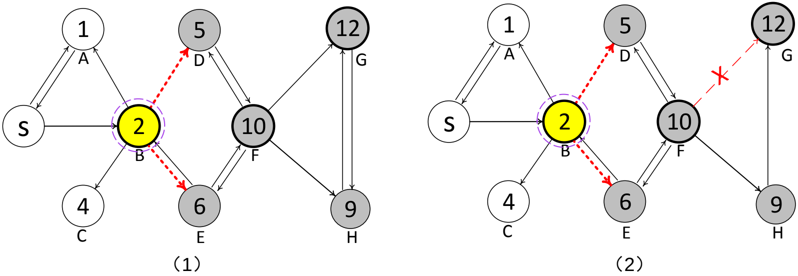

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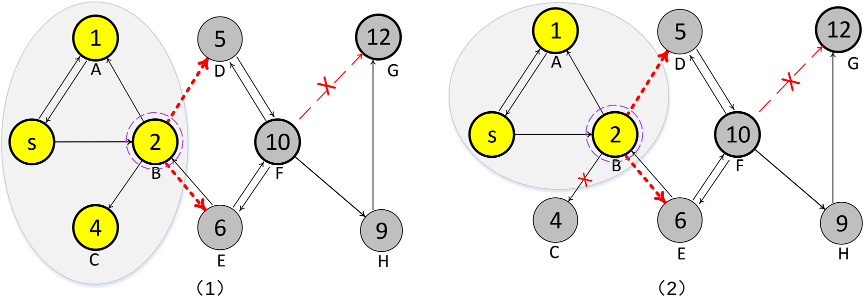

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Critical Diffusion Mechanism (CDM)

Critical Diffusion Mechanism

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predefine an edge selection function α_i(j);
initialize allocation π(t') = ∅ and
  payments {x_i(t') = 0}_{i∈V\{s}};
locate the highest bidder m, break tie
  arbitrarily;
compute m's critical diffusion sequence
  C*_m(t') and denote it by {1, 2, ··· , m};
for i ← 1 to m do
    compute α_m(i);
    if v_i = W*(t'_{−α_m(i)}) then
        set π(t') to be any trading path
          of i and x_i(t') = W*(t'_{−i});
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    else
        set x_i(t') =
          W*(t'_{−i}) − W*(t'_{−α_m(i)});
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Allocation policy:

1. identify the highest bidder m.
2. compute m's critical diffusion sequence, denoted by $\{1, 2, \cdots, m\}$.
3. allocate the item as follows:
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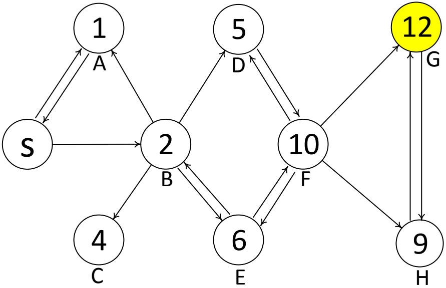
A running example



Allocation

1. the highest bidder is G.

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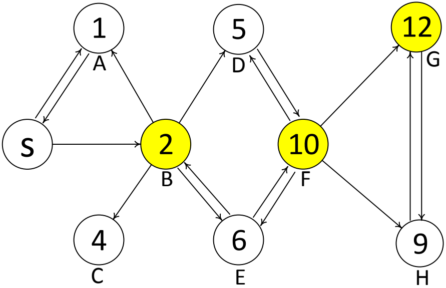


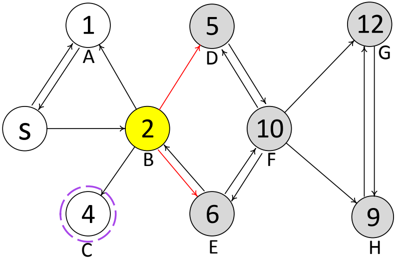
Allocation

1. the highest bidder is G.
2. her critical diffusion sequence is $\{B \to F \to G\}$.

A running example

Edge selection function

$\alpha_i(j)$ is defined as the minimum edge set between j and her neighbors, by removing which buyer $j+1$ cannot join in the auction.



Allocation

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3.1. $\alpha_G(B) = \{(B, D), (B, E)\}$ and C is the highest bidder after removing $\alpha_G(B)$.

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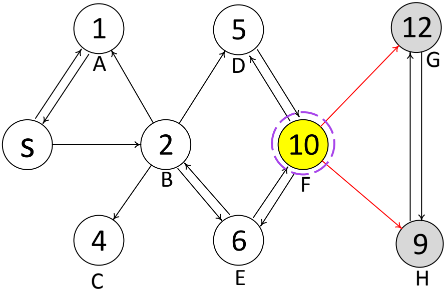

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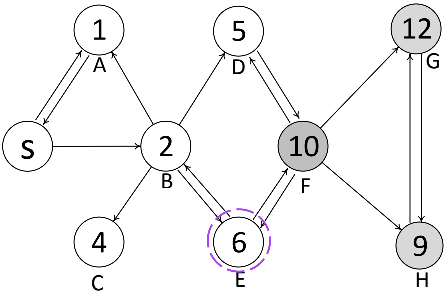
3.2. $\alpha_G(F) = \{(F, G), (F, H)\}$ and F is the highest bidder after removing $\alpha_G(F)$.

- Therefore, F wins the item.

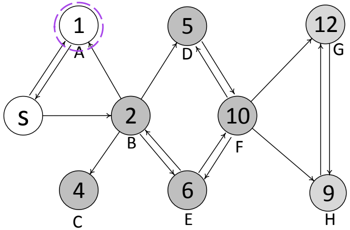
A running example

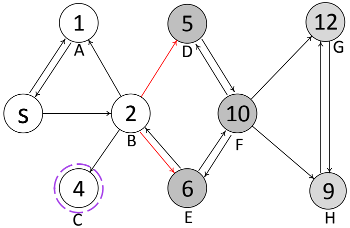


Payment of F

- winner F pays the highest bid without her participation, i.e., $v_E = 6$.

A running example

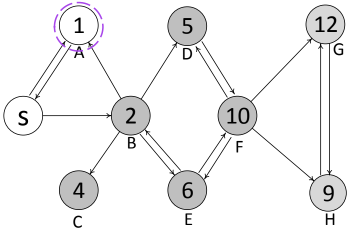


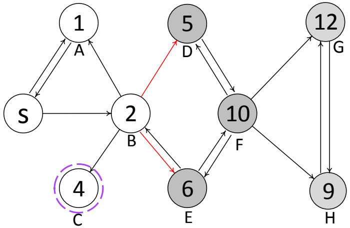


Payment of B

- the highest bid without B's participation is $v_A = 1$.

A running example





Payment of B

- the highest bid without B's participation is $v_A = 1$.
- the highest bid after deleting $\alpha_G(B)$ is $v_C = 4$.

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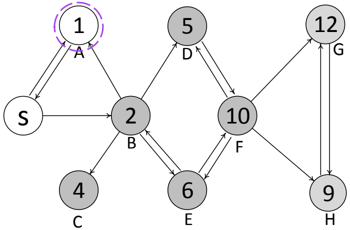

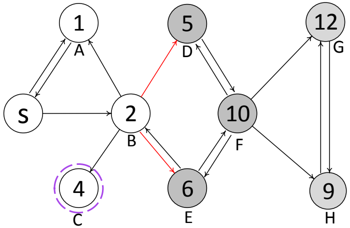

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- the highest bid without B's participation is $v_A = 1$.
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- therefore B's payment is $1 - 4 = -3$, i.e., the seller pays 3 to B.

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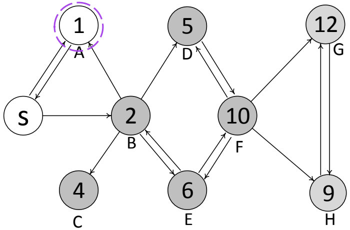

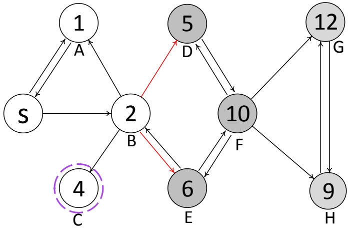

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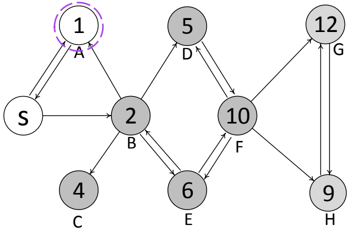

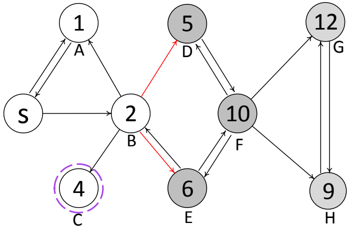

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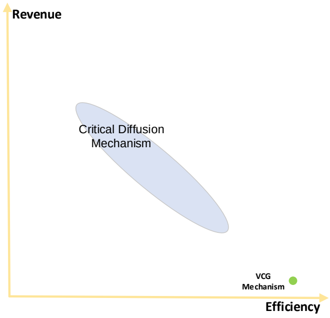
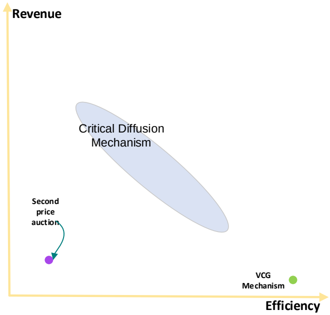
- therefore B's payment is $1 - 4 = -3$, i.e., the seller pays 3 to B.
- others pay zero.
- the seller's revenue is $-3 + 6 = 3 > 1$—the revenue obtained in the second price auction.

Properties of Critical Diffusion Mechanism



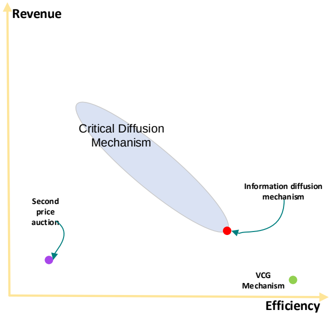
- Individually rational: no buyer will receive a negative utility to join the mechanism.
- Incentive-compatible: report true valuation and diffuse the sale information to all neighbours is a dominate strategy.

Properties of Critical Diffusion Mechanism



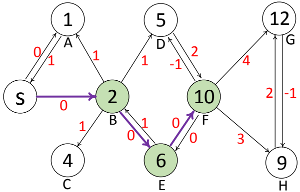
- The allocation efficiency and the seller's revenue are $\geq$ that given in the second price auction.

Properties of Critical Diffusion Mechanism



- The information diffusion mechanism [Li et al., AAAI17] is with the lowest revenue in this class.

Weighted Diffusion Mechanism



Interpretation of the weights

- Commissions
- Frights
- CPU times
- ...

Difficulties

- Which path should be selected
- How to evaluate nodes' efforts
- How to guarantee the revenue

Main results

Weighted Diffusion Mechanism (WDM)

WDM is IR, IC and BB. More importantly, both the seller's revenue and the allocation efficiency are no less than that given in the second price auction.

Allocation Rule

initialize $\pi(t') = \emptyset$;
compute $\pi^*(t')$, break tie arbitrarily;
denote $\pi^*(t')$ by
$L_m^*(t') = \{1^*, 2^*, \cdots, q^* = m\}$;
for $i \leftarrow 1^*$ **to** q^* **do**
    compute γ_i;
    if *i is allocated the item in* $\pi^*(t'_{-\gamma_i})$ **then**
        set $\pi(t') = L_i^*(t')$;
        break;

Payment Rule

initialize $\{x_i(t') = 0\}_{i \in V \setminus \{s\}}$ and $B_g^*(t') = 0$;
let $L_g^*(t')$ be the allocation achieved in Alg. 1;
denote $\tilde{w}(i, j) = \sum_{L_j^*(t'_{-\gamma_i}) \setminus \{j\}} w(l, l+1)$;
for $i \in L_g^*(t') \setminus \{g\}$ **do**
    compute γ_i;
    set $x_i(t') = W^*(t'_{-i}) - W^*(t'_{-\gamma_i})$;
    if *i is a secondary node* **then**
        update
        $B_g^*(t') = \max\{B_g^*(t'), v'_i - \tilde{w}(i, i) + \tilde{w}(i, g)\}$;
update $B_g^*(t') = \max\{B_g^*(t'), W^*(t'_{-g}) + \tilde{w}(g, g)\}$;
set $x_g(t') = B_g^*(t')$;

Key Refs

- Bin Li, Dong Hao, Dengji Zhao, Makoto Yokoo. Diffusion and Auction on Graphs. IJCAI 2019.
- Bin Li, Dong Hao, Dengji Zhao, Tao Zhou. Customer Sharing in Economic Networks with Costs. IJCAI 2018.
- Dengji Zhao, Bin Li, Junping Xu, Dong Hao, Nick Jennings. Selling Multiple Items via Social Networks. AAMAS 2018.
- Bin Li, Dong Hao, Dengji Zhao, and Tao Zhou. Mechanism Design in Social Networks. AAAI 2017.

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Thanks for your attention!