

Diffusion and Auction on Graphs

¹Bin Li, ¹Dong Hao, ²Dengji Zhao, and ³Makoto Yokoo

¹University of Electronic Science and Technology of China

²ShanghaiTech University

³Kyushu University



The Scenario

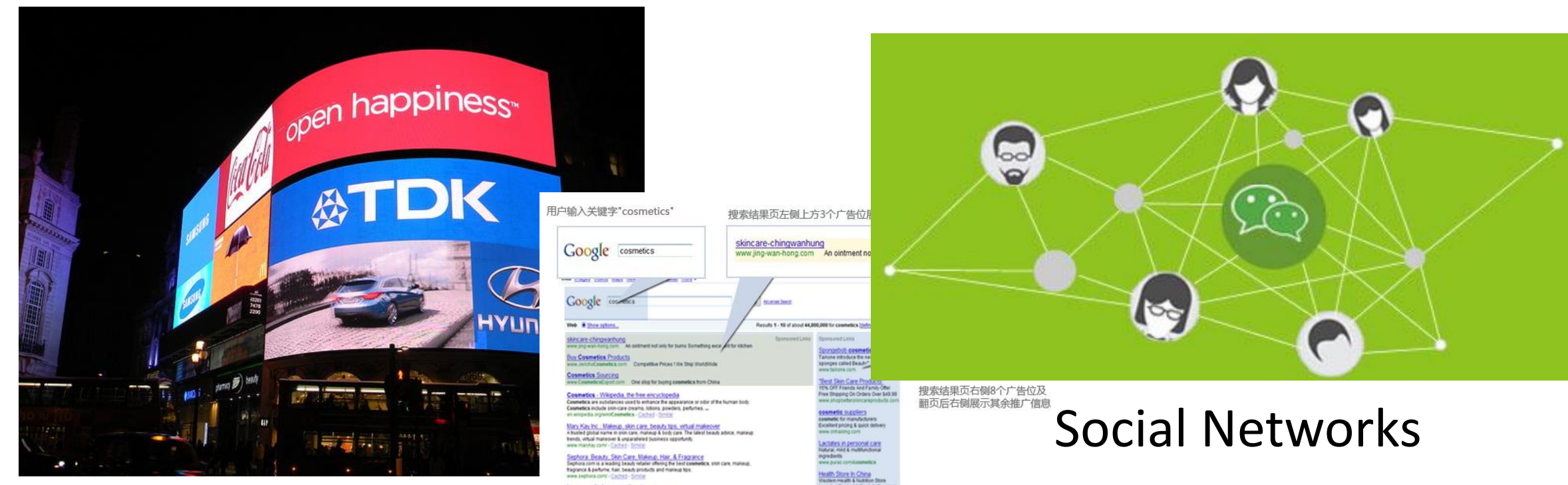
SETTING: One seller obsesses one item for sale. However, she can only reach her neighbors. How could she further improve the revenue?

Basic Diffusion Auction Model on Graphs

Traditional Sale Promotion



- Each one can only communicate with her neighbors.
- Each one has privately known valuations.
- Initially, only seller's neighbors know the auction



Challenges

- The return of these promotions are unpredictable.
- The seller may **LOSE** from the promotions.

Our solutions

Build promotion inside the market mechanism such that:

- the promotion will **never bring negative utility/revenue** to the seller.
- all buyers who are aware of the sale are **incentivized to diffuse the sale information** to all her neighbors.

Auction on unweighted graphs: Critical Diffusion Mechanism

Definition: Predefine an edge selection function $\alpha_i(j)$ for each diffusion critical node j of bidder i which satisfies conditions (1)-(3).

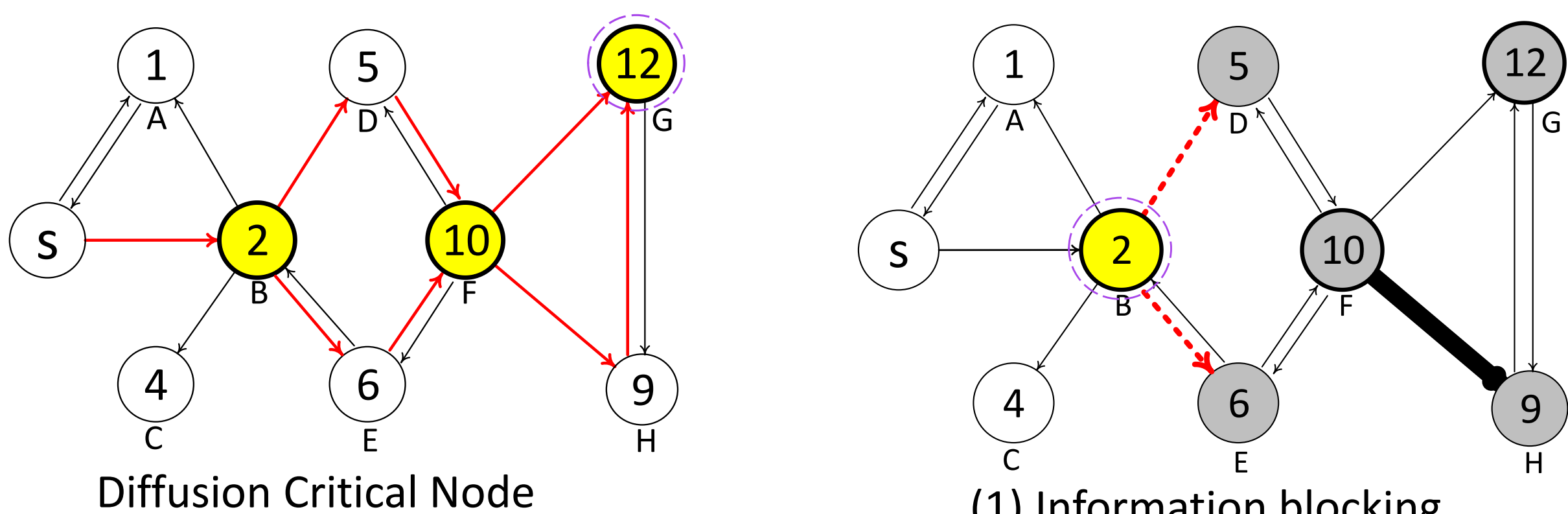
Critical Diffusion Mechanism

Allocation policy:

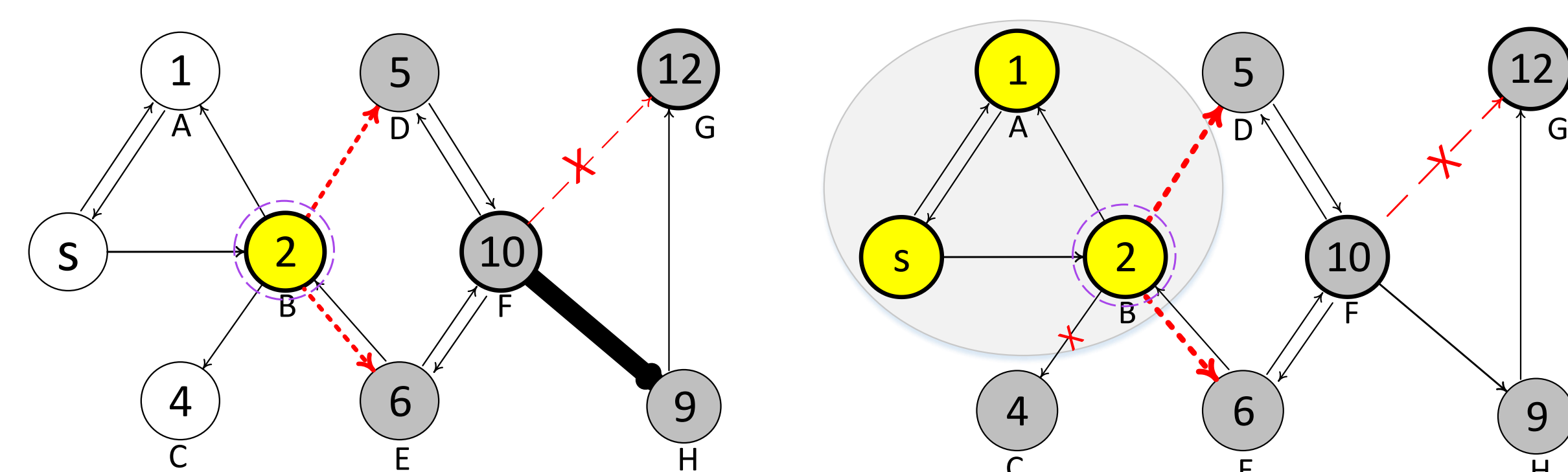
- Identify the highest bidder m .
- For m 's critical diffusion node j :
 - Removing $\alpha_m(j)$.
 - Allocate the item to j if she becomes the highest bidder.
 - Otherwise, set $j=j+1$.

Payment policy:

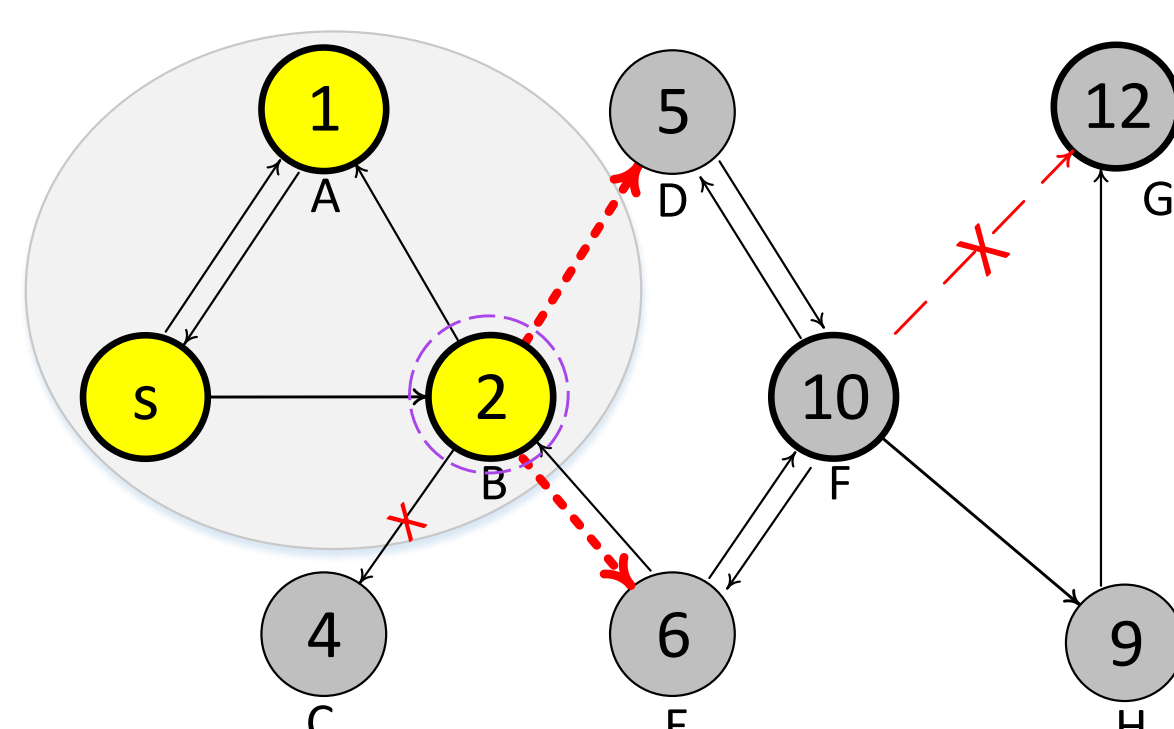
- Suppose w wins the item.
- w pays the highest bid without her participation.
- For $j \in \{1, 2, \dots, w-1\}$,
$$x_j = W^*(t'_j) - W^*(t'_{-\alpha_m(j)})$$
- Others pay zero.



(1) Information blocking



(2) Node independence



(3) Diffusion monotonicity

Theorem 1: The critical diffusion mechanism defined above is individually rational and incentive-compatible. More importantly, it dominates the Vickrey auction both in the seller's revenue and the allocation efficiency.

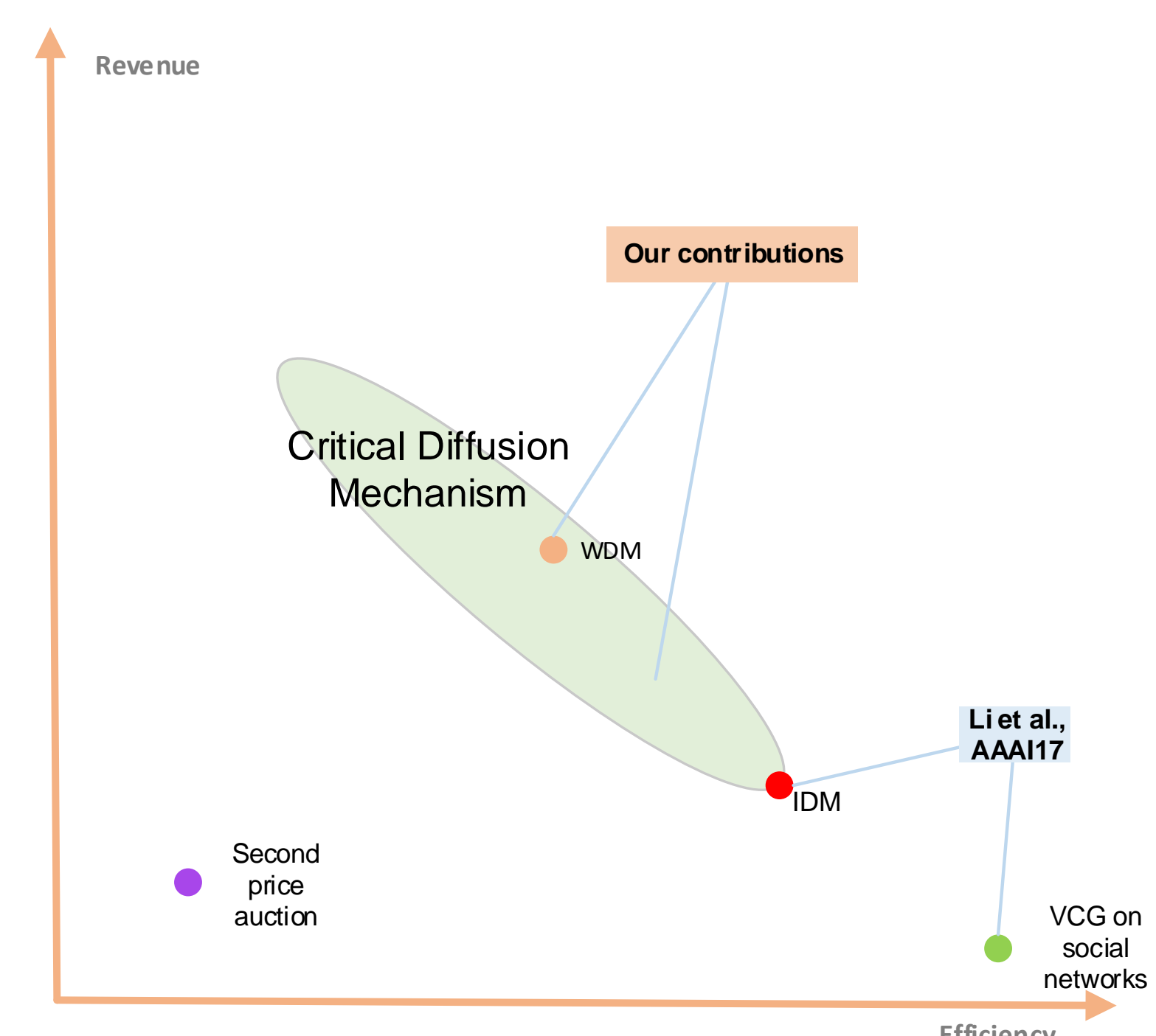
Proposition: In all critical diffusion mechanisms, the information diffusion mechanism proposed in [Li et al., AAAI 17] is the one with the lowest revenue.

Auction on weighted graphs: Weighted Diffusion Mechanism

The Big Picture

- ① Identify the efficient allocation.
- ② Determine the winner along the efficient allocation
- ③ Find the winner's critical opponents.
- ④ Compute the winner's critical value.

Theorem 2: The weighted diffusion mechanism is individually rational and incentive-compatible, and dominates the Vickrey auction.



Key References:

[1] Bin Li, Dong Hao, Dengji Zhao, Makoto Yokoo. Diffusion and Auction on Graphs. IJCAI 2019.

[2] Bin Li, Dong Hao, Dengji Zhao, and Tao Zhou. Mechanism design in social networks. AAAI 2017.

[3] Bin Li, Dong Hao, Dengji Zhao, Tao Zhou. Customer Sharing in Economic Networks with Costs. IJCAI 2018.

[4] Dengji Zhao, Bin Li, Junping Xu, Dong Hao, Nick Jennings. Selling Multiple Items via Social Networks. AAMAS 2018.