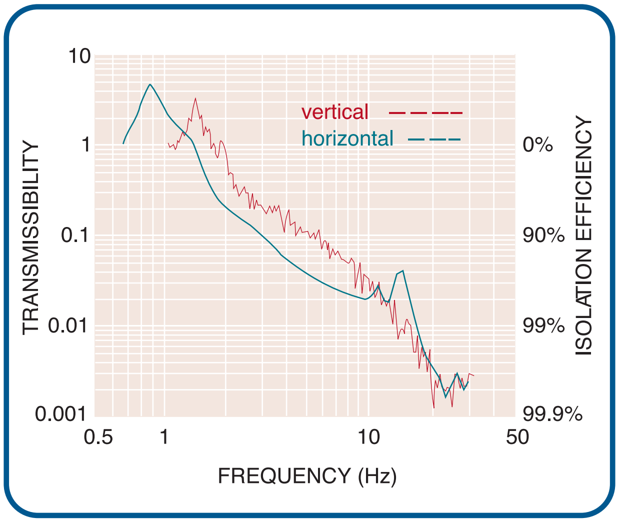
Resonance

In the example just outlined, the transmissibility of the spring is very dependent on the frequency of the forcing vibration. The idealized transmissibility of this system is given by

$$T = \sqrt{\frac{1+(2\wp f / f_n)^2}{(1-f^2/f_n^2)^2+(2\wp f/f_n)^2}}$$

where f is the frequency of the forcing function, f_n is the natural resonant frequency of the system, and $\wp$ is the damping ratio of actual damping to critical damping. Critical damping is the minimum amount of damping necessary to prevent resonant isolation following application of an impulsive force.

The following graph shows a plot of this idealized transmissibility as a function of frequency. Note the similarity to the compliance transfer function previously discussed in the table vibration text. Again, there are three distinct regions on the curve.



A typical transmissibility vs frequency curve for a system with one degree of freedom

At low or zero frequencies, the ball and spring move synchronously with the table and with the same amplitude; thus, the transmissibility is unity and the system behaves as though the spring were rigid, with no vibrational isolation.

As the frequency increases, a resonant condition is approached. The ball has mass and momentum when moving and cannot change direction instantaneously in response to the rapidly changing forcing vibration. The vibration of the ball starts to lag behind the forcing vibration, so they are no longer in phase. Eventually, this phase lag becomes exactly 90° and at this point, the system is vibrating at its natural or resonant frequency. At this resonant frequency, the system accumulates vibrational energy and increases in amplitude during the time that the forcing vibration is applied (i.e., the system acts as a vibrational amplifier). As defined by the equation for transmissibility, the amplitude of this resonant motion does not grow to infinity because of damping.

At frequencies much higher than the resonant frequency, the response of the ball is determined solely by the mass term, which is much larger than the stiffness term. In other words, the spring is relatively soft and the vibrational force travels slowly along it in the form of compression extension waves. This slow transmission effectively spreads out the oscillatory nature of the forcing vibration. Essentially, the ball experiences a time-averaged force due to the fast moving vibration, and unless the vibration involves a net displacement, this time-averaged force tends to zero with increasing vibrational frequency. As the transmissibility approaches zero, the position of the ball is not affected by the vibration in the large mass. This is a classic example of seismic mounting.

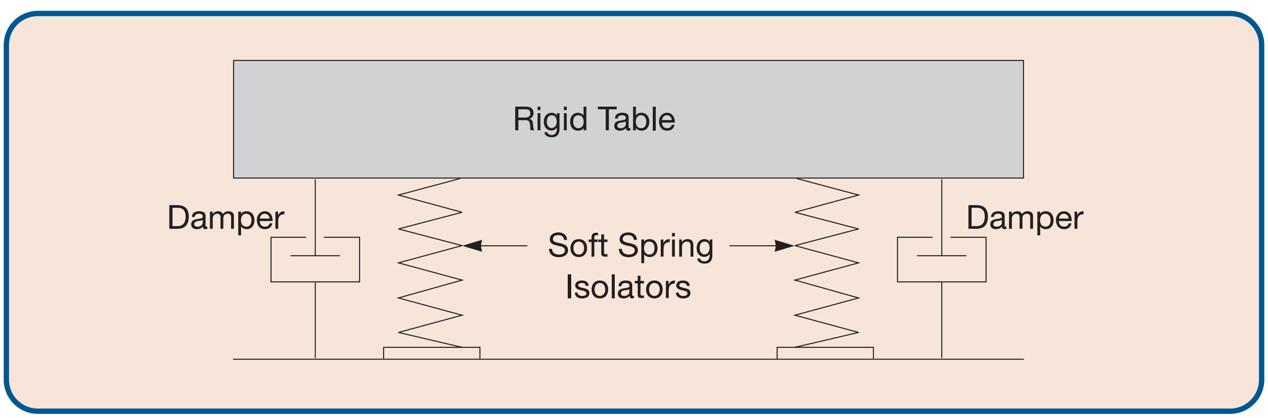
Damping

Damping refers to any process that causes an oscillation in a system to decay rapidly to zero amplitude. It is a very important phenomenon in vibration suppression or isolation. Damping causes the energy to be diverted from vibration to other sinks. Damping in a system is usually defined as the ratio of actual damping (C) to critical damping (CC). Critical damping is the minimum amount of damping in a system necessary to prevent resonant oscillation, following application of an impulsive force.

Damping is a resonant effect that primarily affects the transmissibility function at or near resonance. At resonance, the transmissibility becomes

$$T = \frac{1}{2\wp}$$

where $\wp$ is the damping ratio (C/CC).



Ideal seismic mounting of an optical table, consisting of weak spring supports with added damping