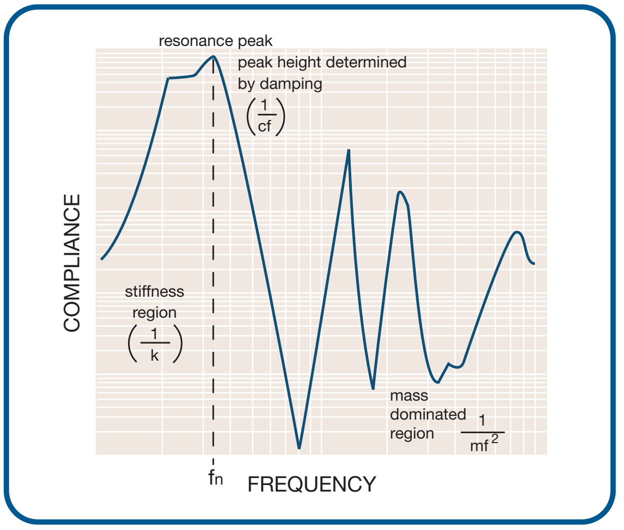
If we restate the previous equation in words then

$$compliance = \frac{1}{\sqrt{(stiffness - mass\ effects)^2 + (resonance\ effects)^2}}$$

A plot of compliance versus frequency, in accordance with this formula, shows that the compliance of a rigid body can be separated into three parts: stiffness, resonance effects, and mass effects.



Compliance versus frequency for a system with one degree of freedom

Low Frequency Compliance

At zero and low frequencies, the stiffness term dominates the compliance equation. When a low-frequency forcing vibration is applied to the unattached end of our bar, it bends in response. The amount of deflection is determined by the stiffness of the bar, which ultimately depends on its shape, the tensile modulus of elasticity (Young's modulus) of the bar material, and the method of mounting and/or constraining the bar.

Any solid body has a fixed equilibrium, and when forces are applied, it can be deformed from this equilibrium shape. The potential energy of the body rises, and this is manifested as resistive forces, which act to restore the equilibrium shape.

When the bar is deflected, the restoring forces return it to its equilibrium (linear) position. However, the momentum of the bar causes it to overshoot this position. Restoring forces now act in the opposite direction to return the bar to its equilibrium position, and again, momentum causes an overshoot beyond linearity. This oscillation of the bar is an example of a simple harmonic oscillator, and in the absence of damping, this oscillation would persist forever.

When a body is vibrating at its resonant frequency, the energy in the vibration alternates between potential and kinetic. At the maximum deflection or displacement, the velocity is zero, the acceleration and potential energy are at their maximum values, and the kinetic energy is zero. At equilibrium, or zero displacement, the opposite is true.

Compliance at Resonance

When the forcing vibration is at the resonant frequency, each maximum in the velocity of the forcing vibration coincides with a maximum in the acceleration of the excited vibration. This adds to the acceleration of the bar, which thereby accumulates vibrational energy and actually amplifies the forcing vibration. This is in accordance with the previous equation for compliance, which can be solved to show

$$\frac{x}{F} = \frac{1/k}{\sqrt{(1 - f^2/f_n^{\,2})^2 + (2\wp f/f_n)^2}}$$

where f is the frequency of the forcing function, f_n is the natural resonant frequency of the system, $\wp$ is the damping ratio, and k is the stiffness.

From this equation, it can be seen that when the frequency of the forcing function is close to the resonant frequency, the compliance is determined solely by the damping term and can be quite large.

High Frequency Compliance

At higher frequencies, the compliance is totally dominated by the mass (inertia) of the table. From the previous equations, the compliance at high frequencies is given by

$$compliance \frac{x}{F} \approx \frac{1}{mf^2}$$

where m is the effective mass and f is the frequency of the forcing vibration.

In real systems, the mass term in the general equation for compliance can become quite complex. In some very simple vibrational systems, such as our vibrating bar, it is fairly easy to evaluate the mass involved in the vibration. However, in the case of a structure such as an optical table with several types of bending and flexural resonant modes of vibration, different points on the table are undergoing different amplitudes of vibration. At nodal points, there is no vibrational amplitude at all for that specific vibrational node. The effective mass involved in the vibration is therefore a complicated function best determined by computer programs.

Compliance of a Real Table

The concept of an ideal rigid body is useful when considering optical table performance. This theoretical structure does not resonate and therefore has no compliance peaks. When plotted on a log:log scale, an ideal rigid body has a compliance proportional to $1/f^2$ and is represented by a straight line with a slope of -2. It represents the ultimate design goal when manufacturing optical tables: the closer the actual curve fits the straight line, the better the dynamic stiffness.

A compliance curve of a real table can now be examined. The figure on the next page shows a typical compliance curve measured at the corner of a Thorlabs tabletop.

Several aspects of this curve merit special comment. The initial portion of the plot (i.e., before the first table resonance) is determined primarily by the table supports, not the table itself.