Theoretical Study on Baryon Acoustic Oscillations

Author: Yingqiu He
Supervisor: Xinjuan Yang

College of Science, Zhejiang University of Technology (ZJUT), Hangzhou, China

Introduction

Baryon Acoustic Oscillations (BAO) are a standard ruler for cosmological distance, originating from pressure waves in the early universe. They are important for understanding the dynamics of the cosmic expansion, particularly since the robust evidence of BAO was first documented in 2005 [1]. The objective of this project is to investigate the properties of BAO theoretically, using methods of linear perturbation theory and the Halo Model to deepen the understanding of the evolution of large-scale structures.

By applying the Flat Lambda Cold Dark Matter (ΛCDM) model, this study computes the linear matter power spectrum and two-point correlation function to identify oscillatory features indicative of the BAO scale in both real and redshift space. This is essential for quantifying acoustic waves imprints on galaxies distribution. The halo model is also used to probe the nonlinear regime of structure formation. It separates the power spectrum into contributions from individual halos and interactions between different halos, allowing us to know the behaviors of matter clustering at smaller scales and its effect on the BAO signal. Throughout the study we adopt a flat ΛCDM cosmology with $h = 0.7$, $\sigma_8 = 0.8110$, $\Omega_{\rm m} = 0.31$, $\Omega_\Lambda = 0.69$, $\Omega_{\rm c}/\Omega_{\rm m} = 0.2$, and $n_s = 0.99$.

Methodologies

Linear Matter Power Spectrum

To delineate the BAO feature in the matter density field $\delta_{\rm m}(k, z)$, we apply linear perturbation theory and derive the linear power spectrum as follows:

$$P(k, z) \equiv \langle \delta_{\rm m}(k, z)^2 \rangle = AT(k)^2 D(z)^2 k^{n_s} \qquad (1)$$

Here, k^{n_s} represents the primordial power spectrum, setting the initial conditions for matter fluctuations with n_s as the spectral index. $T(k)$ is the transfer function using the Eisenstein & Hu fitting formula [2]. $D(z)$ is the growth factor, which adjusts the initial conditions in both scale and time to reflect the dynamics of the expanding universe. A is a normalization factor using σ_8, which quantifies the density fluctuations within spheres of radius $8h^{-1}$Mpc. This normalization scales the amplitude of the power spectrum to match the observed density variance at that scale, using the top-hat window function.

Transitioning from the Fourier space to to the configuration space yields the two-point correlation function:

$$\xi(r) = \frac{1}{2\pi^2} \int_0^\infty P_{\rm lin}(k) \frac{\sin(kr)}{kr} k^2 dk \qquad (2)$$

Here, r is used in comoving coordinates. In addition, we consider the velocity induced redshift space distortion (RSD), a phenomenon influenced by Doppler effects. We derive the Kaiser formula from the fluctuation theory:

$$P_{\rm lin,RSD}(k, z) = (1 + f\mu^2)^2 P_{\rm lin}(k, z) \qquad (3)$$

where $f(z) = d \ln D/da$ is the growth rate, $a = (1+z)^{-1}$ is the scale factor, and μ is the cosine of angle between the line of sight and the wavevector of a given mode in Fourier space.

Halo Model

The dark matter halo model is used to evaluate the nonlinear power spectrum, incorporating contributions from the single halo term $P_{\rm 1h}$

plus the clustering term P_{2h} (see [3]):

$$P_{\mathrm{HM}}(k) = P_{\mathrm{2h}}(k) + P_{\mathrm{1h}}(k) \tag{4}$$

This model takes into account the abundance of halos, calculated using the Press-Schechter formalism, and their spatial distribution, described by the halo bias. It also considers the internal matter distribution within each halo, typically modelled by the NFW (Navarro-Frenk-White) profile. By integrating these elements, the model assesses whether BAO signatures can be completely obscured at scales affected by nonlinear structure.

Results

The dimensionless matter power spectrum analyzed here is defined as:

$$\Delta^2(k,z) = \frac{k^3 P(k,z)}{2\pi^2} \tag{5}$$

as shown in Figure 1.

At large scales ($k < 0.1h\,\mathrm{Mpc}^{-1}$), the power spectrum aligns with the Harrison-Zel'dovich spectrum, which predicts a scale-invariant distribution of density perturbations from the early universe. In this regime, the clustering of halos is significant. The baryonic matter and cold dark matter gravitational influence the formation of large-scale structures similarly. When $k \sim 1h\,\mathrm{Mpc}^{-1}$, the spectrum reveals a series of acoustic oscillations consistent with the transfer function, with a dimensionless power spectrum value around ~ 1. Here, the coupling between baryons and photons during the epoch of recombination is a key feature. This scale also marks the transition from linear to nonlinear matter density perturbations, indicating that BAO features are not prominent at smaller scales. At smaller scales ($k > 1h\,\mathrm{Mpc}^{-1}$), the baryon-photon coupling is no longer significant since temperature is low. The dominant influence at these scales shifts to the 1-halo term, reflecting observations of structures that are smaller than the typical scale of dark halos. The dominance of the 1-halo term suggests that the signal is largely due to the matter distribution within individual halos, rather than to clustering between halos.

As shown in Figure 2, varying the cosmological parameter of matter density, Ω_{m}, affects the growth of structure via gravitational clustering. An increase in Ω_{m} enhances the gravitational clustering, which in turn increases the amplitude of the power spectrum across all scales, particularly evident before $0.1h\,\mathrm{Mpc}^{-1}$. This indicates stronger density perturbations. Conversely, a decrease in Ω_{m} not only reduces the amplitude of the power spectrum but also slightly shifts the position of its peak towards smaller scales, indicating a weaker gravitational clustering at these scales. Thus, the overall shape and the specific scaling characteristics of the power spectrum at the epoch of matter-radiation equality are dependent on Ω_{m}.

In Figure 3, the correlation function shows a peak around $r \approx 100h^{-1}$Mpc, marking a scale of significant spatial correlation associated with the BAO. This peak reflects distances where galaxies have a probability of being separated due to sound waves propagating in the early universe. In addition, the amplitude of the correlation function is larger in redshift space than in real space, illustrating the impact of peculiar velocities on the observed distribution of matter. These velocities result in an apparent increase in clustering along the line of sight in redshift measurements, thereby amplifying the correlation function's amplitude in redshift space compared to real space.

Discussion and Conclusion

This study analyses the influence of the BAO on the power spectrum and correlation functions within the flat ΛCDM model. The results show how variations in the cosmological matter density parameter shape the scale features of the BAO and take account for redshift space distortions in modifying observed matter clustering. We identify oscillatory scale features at $k = 0.1h\,\mathrm{Mpc}^{-1}$ and $r = 100h^{-1}\,\mathrm{Mpc}$, which serve as important cosmic distance markers. The application of the dark matter halo model reveals that the BAO features are obscured at small scales due to nonlinear effects. While the power spectrum at large scales $k < 1h\,\mathrm{Mpc}^{-1}$ remains similar to the linear spectrum, at $k > 1h\,\mathrm{Mpc}^{-1}$,

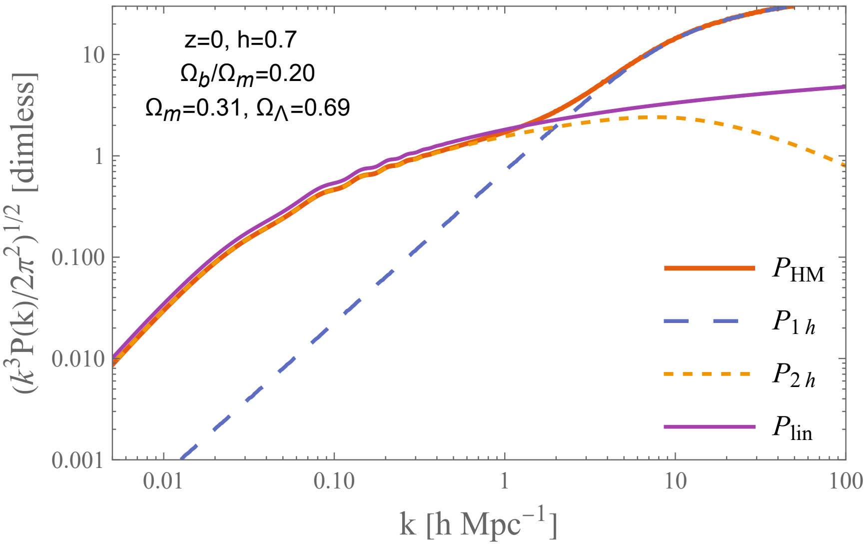

Figure 1: *The matter power spectra at $z = 0$, as predicted by the linear density perturbation (purple line) and the halo model (orange line). Both the 1-halo and 2-halo terms are also shown for clarity.*

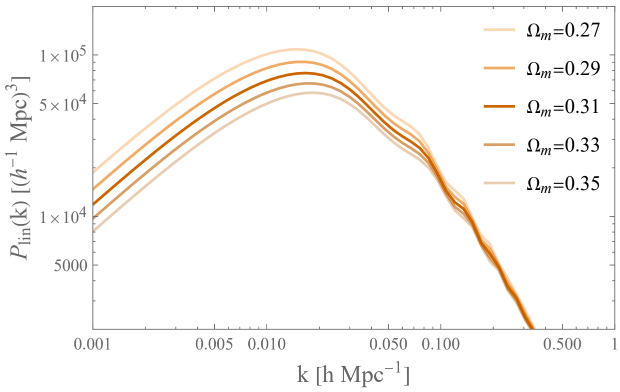

Figure 2: *Matter linear power spectrum at redshift $z = 0$ varying $\Omega_{\rm m}$ around the fiducial value, keeping $\Omega_{\rm m} + \Omega_\Lambda = 1$.*

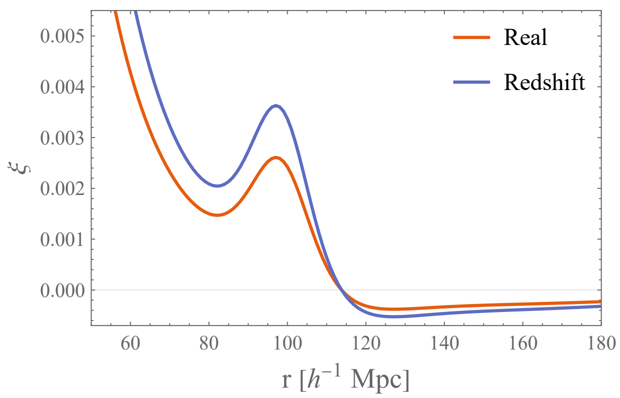

Figure 3: *The two-point correlation function in both real and redshift space.*

nonlinear dynamics dominate, significantly altering the spectrum.

This thesis concludes that the improvement of the BAO model will benefit significantly from the adoption of more advanced theoretical approaches, including modified gravity, sophisticated perturbation theories, and the study of nonlinear cosmic evolution. These improvements will lead to more accurate measurements of cosmological distance. In addition, new and upcoming high-precision surveys such as DESI (Dark Energy Spectroscopic Instrument), LSST (Vera C. Rubin Observatory), Eulid, and CSST (Chinese Space Station Telescope), will provide rich data that will further refine our models. The integration of additional methods, such as weak lensing and the 21cm line, will deepen our understanding of dark energy and the expansion rate of the Universe.

References

[1] D. J. Eisenstein *et al.* [SDSS], Astrophys. J. **633**, 560-574 (2005).

[2] D. J. Eisenstein and W. Hu, ApJ **496**, 605-614 (1998).

[3] A. Cooray and R. Sheth, Phys. Rept. **372**, 1-129 (2002).