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Code updating history of Jump Code Experiment

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Notes about the Matlab code used in the Jump Code Experiments.

BACKGROUND (APR 07, 2015)

All the codes are available in SVN 'https:\\github.com\bannislloyd\dawei-qip-matlab\trunk\Jump Code'

The flipping error channel and erasing error channel are

$$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-r} \end{pmatrix}, A_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{r} \end{pmatrix}, \tag{1}$$

Since $A_0 = [(1+\sqrt{1-r})/2]I + [(1-\sqrt{1-r})/2]Z$, we know in the Hadamard basis the qubit will be flipped with the probability

$$p_F = [(1-\sqrt{1-r})/2]^2, \tag{2}$$

and no error probability

$$p_0 = [(1+\sqrt{1-r})/2]^2. \tag{3}$$

And $A_1|+\rangle = \sqrt{r/2}|1\rangle, A_1|-\rangle = -\sqrt{r/2}|1\rangle$, so A_1 will erase all the information in the Hadamard basis with the probability $p_E = r/2$.

To make a unitary transformation, we need one ancilla. Assume the ancilla starts at $|0\rangle$, so we can use the following way to construct unitary. The channel Φ on these 2 qubits (ancilla A and system B) should give us

$$\Phi(\rho_{AB}) = |0\rangle\langle 0| \otimes A_0 \rho_B A_0^\dagger + |1\rangle\langle 1| \otimes A_1 \rho_B A_1^\dagger. \tag{4}$$

Φ is not unitary but we can use a unitary channel U with the aid of post selection to realize it. It means if we start from $|00\rangle$ and $|01\rangle$ (Note we have assumed the ancilla A is from $|0\rangle$), by applying U we will have

$$U|00\rangle = |00\rangle, U|01\rangle = \sqrt{1-r}|01\rangle + \sqrt{r}|11\rangle. \tag{5}$$

So the unitary can be chosen in this way

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-r} & 0 & \sqrt{r} \\ 0 & 0 & -1 & 0 \\ 0 & \sqrt{r} & 0 & -\sqrt{1-r} \end{pmatrix}. \tag{6}$$

In summary, we have three probabilities

$$\begin{aligned} p_0 &= [(1+\sqrt{1-r})/2]^2, \\ p_F &= [(1-\sqrt{1-r})/2]^2, \\ p_E &= r/2. \end{aligned} \tag{7}$$

CLASSICAL CASE WITHOUT CORRECTION (APR 07, 2015)

First consider the classical case. Without correction, one needs one bit with the two states $|+\rangle$ or $|-\rangle$. Suppose it is $|+\rangle$.

We can use 2 qubits (ancilla and system) starting from $|0+\rangle$. Applying U and post-select the ancilla. If we get $|0\rangle$, we know A_0 happens and with probability p_0 no error happens, with probability p_F $|+\rangle$ is flipped to $|-\rangle$. If we get $|1\rangle$, we know A_1 happens (the probability of this case is p_E) and we will get $|1\rangle$ on the system. We have half chance to recover it to the correct state $|+\rangle$. Therefore, the fidelity for this classical case will be

$$F^C = p_0 + \frac{1}{2}p_E = \frac{1}{2} + \frac{1}{2}\sqrt{1-r} \tag{8}$$

The matlab code is (for a given r)

```
% projective measurement
M0 = kron(ST0, I); M1 = kron(ST1, I);

rho_projective0 = M0*rho_channel*M0'; % if ST0 is detected on the ancilla
% equal to  rho_echo = Gz_echo(rho_channel,2), simulated by Gradient Echo
rho_sys0 = ptrace(rho_projective0, 1, [2 2]); % trace out ancilla
Fidelity0 = trace(System*rho_sys0);

rho_projective1 = M1*rho_channel*M1';  % if ST1 is detected on the ancilla
% equal to  rho_echo = Gz_echo(rho_channel,2), simulated by Gradient Echo
rho_sys1 = ptrace(rho_projective1, 1, [2 2]); % trace out ancilla
Fidelity1 = trace(System*rho_sys1);
```

The total fidelity is Fidelity0+Fidelity1 and equals to F^C.

CLASSICAL CASE WITH CORRECTION (APR 07, 2015)

Without correction, one needs two bits with the two states $|++\rangle$ or $|--\rangle$ to encode one bit information. Suppose it is $|++\rangle$.

We have to use 4 qubits (ancilla, system, system, ancilla) to simulate this case. Applying U and post-select the 2 ancillas.

Case 1: If we get $|00\rangle$, we know A_0A_0 happens. We have p_0^2 probability for no error and p_F^2 for flip error on both qubits. The latter one cannot be corrected. Besides, we have $2p_0p_F$ probability that only one flip error happens. So we can project the system to $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$ basis. If we get $|++\rangle$ or $|--\rangle$, do nothing. If we get $|+-\rangle$ or $|-+\rangle$, discard the 2nd qubit and use the 1st qubit state as the real one. It means we have p_0p_F probability to succeed, and p_Fp_0 probability to fail. In total, for Case 1, the succeeding probability is $p_0^2 + p_0p_F$.

Case 2: If we get $|01\rangle$, we know A_0A_1 happens. Discard the 2nd qubit and use the 1st qubit state. The succeeding probability is thus p_0p_E.

Case 3: If we get $|10\rangle$, we know A_1A_0 happens. Discard the 1st qubit and use the 2nd qubit state. The succeeding probability is thus p_Ep_0.

Case 4: If we get $|11\rangle$, we know A_1A_1 happens. The final result would be $|11\rangle$ on the system and we have half chance to get the correct state $|++\rangle$. The probability is thus $\frac{1}{2}p_E^2$.

The total fidelity with error correction in the classical case will be

$$F^C_{code} = p_0^2 + p_0p_F + p_0p_E + p_Ep_0 + \frac{1}{2}p_E^2. \tag{9}$$

Comparing the classical case with and without error correction, we will have the following figure. And we know for any r, the error correction performs better than the original case.

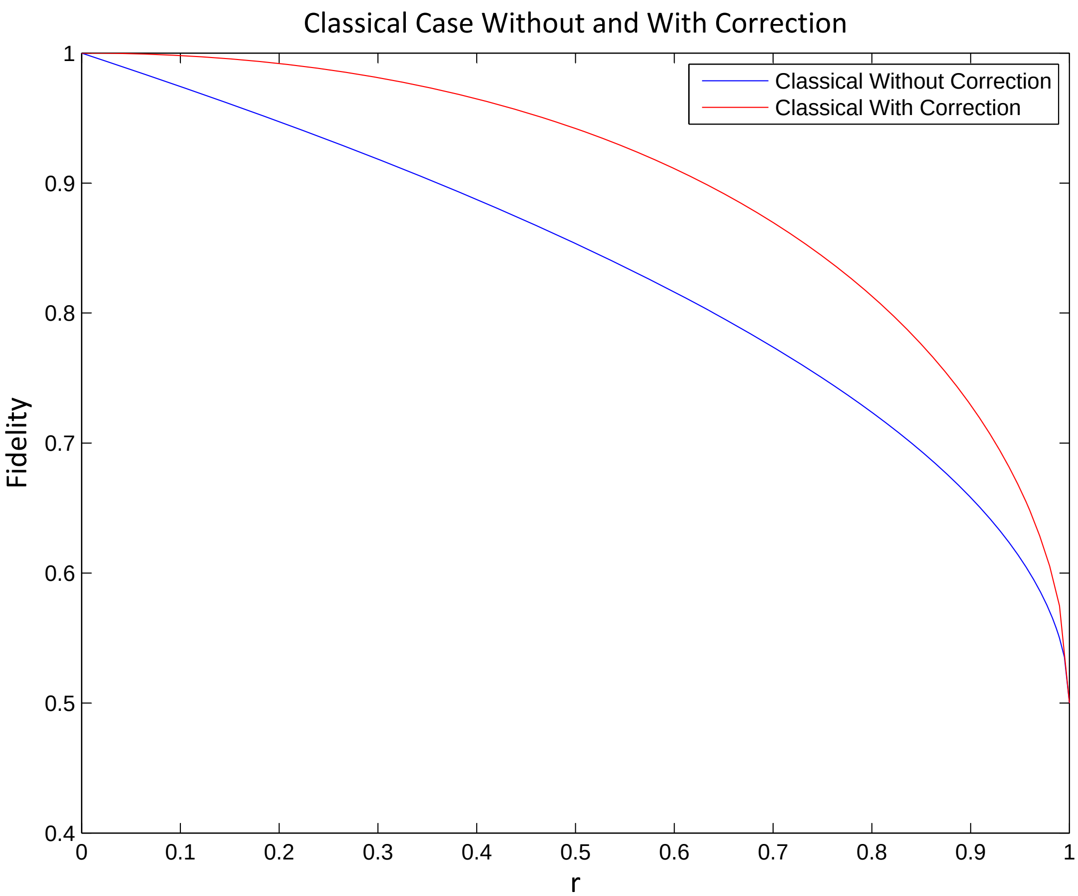

FIG. 1. Classical case with and without error correction.