Appendix A

Transforming to a Rotating Frame

A very useful tool, and one which was frequently used in the preceding publication, is transforming the Hamiltonian of the system to an arbitrarily specified rotating frame, which is tailored to have properties which make it easier to study than in, say, the Schrödinger picture. The general procedure is outlined below.

This is in general carried out as follows. Assuming that we have the Hamiltonian $\hat{H}$ appropriate for the system in the Schrödinger picture, we clearly have the following wave equation for $|\Psi\rangle$, the wave-function in the Schrödinger picture:

$$i\hbar\frac{d|\Psi\rangle}{dt} = \hat{H}|\Psi\rangle. \tag{A.1}$$

We make a judicious choice of unitary operator,

$$\hat{U} = e^{-i\hat{A}t/\hbar}, \tag{A.2}$$

where $\hat{A}$ must clearly be self-adjoint. Note that this is not the most general kind of unitary operator, as it possesses the simplest kind of time dependence. The general procedure does not fundamentally differ in the case of more complicated time dependences however, and most useful rotating frames can be defined in these terms.

We use this unitary operator to transform the Schrödinger wave-function;

$$|\tilde{\Psi}\rangle = \hat{U}^\dagger|\Psi\rangle. \tag{A.3}$$

We now find the corresponding transformed Hamiltonian $\tilde{H}$ for the transformed wave-function $|\tilde{\Psi}\rangle$ by studying the Schrödinger equation for $|\tilde{\Psi}\rangle = \hat{U}^\dagger|\Psi\rangle$.

$$\begin{aligned} i\hbar\frac{d\hat{U}^\dagger|\Psi\rangle}{dt} &= i\hbar\hat{U}^\dagger\frac{d|\Psi\rangle}{dt} + i\hbar\frac{d\hat{U}^\dagger}{dt}|\Psi\rangle \\ &= \hat{U}^\dagger\hat{H}|\Psi\rangle - \hat{A}\hat{U}^\dagger|\Psi\rangle \\ &= \left(\hat{U}^\dagger\hat{H}\hat{U} - \hat{A}\right)\hat{U}^\dagger|\Psi\rangle, \end{aligned} \tag{A.4}$$

and therefore the equation

$$\tilde{H} = \hat{U}^\dagger\hat{H}\hat{U} - \hat{A}. \tag{A.5}$$

describes the general form of the transformed Hamiltonian operator $\tilde{H}$, where the transformation is defined by a unitary operator of the form given in Eq. (A.2).