# Regulator Design, Observer Design & Feedback Design of a Mechanical System

## RA 602 - Control Engineering for Robotics (Dept. CICPS)


**Team Members**

AKASH VISHWAKARMA - 224102101

SATYAM MISHRA - 224102108

ANKIT SINGH - 224102104

BANOTH VISHNU VARDHAN - 224102106

BASU HELA - 224101013



Under the Supervision of

**Dr. Parijat Bhowmick**

**Assistant Professor, Dept. of EEE**

**Indian Institute of Technology, Guwahati (Assam)**

# 1. Objective:

By considering the mechanical system -

a) Perform Mathematical Modelling and find state space of the given mechanical system.
b) Design Observer gain Matrix for pole placement, the desired closed loop poles for the full order observer to be s = -15, s = -16 s = -25 s = -16
c) Design Feedback gain Matrix for pole placement, the desired closed loop poles for the full order observer to be $s = -2+j2\sqrt{3}$, $s = -2 - j2\sqrt{3}$, $s = -10$, $s = -10$
d) Obtain the response of the system to an initial condition**.** Initial conditions are $y_1(0) = 0.1$, $y_2(0) = 0$, $e_1(0) = 0.1$, $e_2(0) = 0.05$ where $e_1$ and $e_2$ are defined by $e_1 = y_1 - y'_1$ $e_2 = y_2 - y'_2$
e) Design Regulator in such a way that response due to Disturbance is zero.

# 2. Software:

MATLAB R2022a

# 3. Mechanical System:

**Specifications:** m1=1 kg, m2=2 kg, k=36 N/m, and b=0.6 N-s/m.

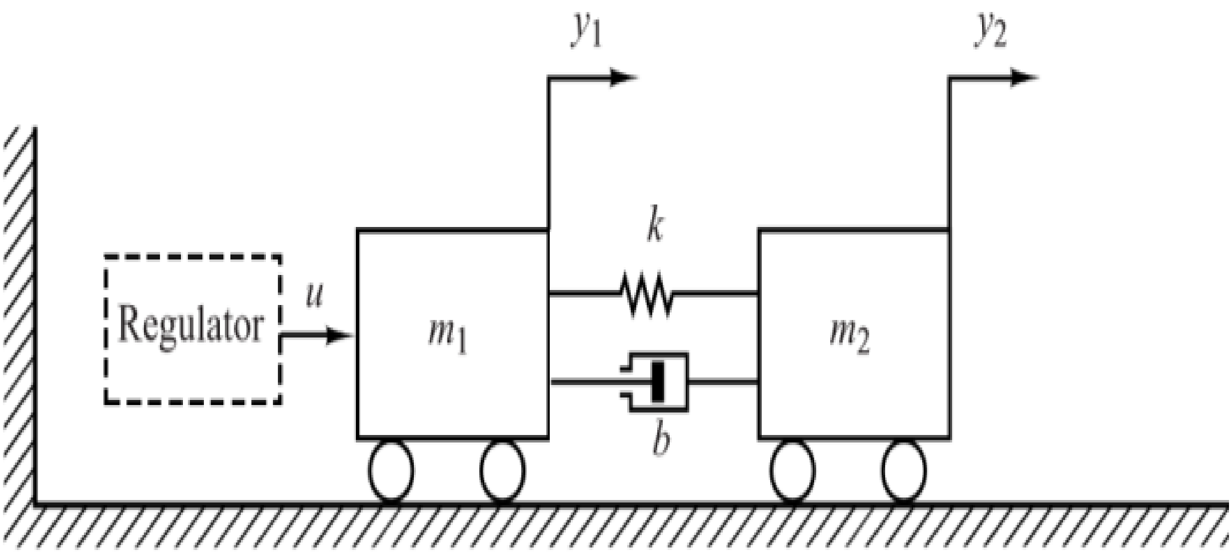



**Figure 1:** Mechanical System

# 4. Project work:

## a) Mathematical Modelling and find state space of a mechanical system:

Differential equation of the above system is-

$$m_1 \frac{d^2 y_1}{dt^2} + b\frac{d(y_1 - y_2)}{dt} + k(y_1 - y_2) = u$$

$$\frac{d^2 y_1}{dt^2} = -36y_1 - 0.6\frac{dy_1}{dt} + 36y_2 + 0.6\frac{dy_2}{dt} + u \qquad (1)$$

$$m_2 \frac{d^2 y_2}{dt^2} + b\frac{d(y_2 - y_1)}{dt} + k(y_2 - y_1) = 0$$

$$\frac{d^2 y_2}{dt^2} = 18y_1 + 0.3\frac{dy_1}{dt} - 18y_2 - 0.3\frac{dy_2}{dt} \qquad (2)$$

Now, I am taking four states because we have 4 variables.
Assuming,

$$\dot{y}_1 = \frac{dy_1}{dt} \text{ and } \ddot{y}_1 = \frac{d^2 y_1}{dt^2}$$

$$y_1 = x_1$$

$$\dot{y}_1 = \dot{x}_1, \ddot{y}_1 = \dot{x}_2, \dot{y}_2 = \dot{x}_3, \ddot{y}_2 = \dot{x}_4$$

By using above equations (1), (2) and above assumptions we can write the state equations. State equations are following-

$$\dot{x}_1 = x_2 \qquad (3)$$

$$\dot{x}_2 = -36\,x_1 - 0.6\,x_2 + 36\,x_3 + 0.6\,x_4 + u \qquad (4)$$

$$\dot{x}_3 = x_4 \qquad (5)$$

$$\dot{x}_4 = 18\,x_1 + 0.3\,x_2 - 18\,x_3 - 0.3\,x_4 \qquad (6)$$

By using above four equation we can write state matrices-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -36 & -0.6 & 36 & 0.6 \\ 0 & 0 & 0 & 1 \\ 18 & 0.3 & -18 & -0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} [u]$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [u]$$

$$[\dot{x}] = [A][x] + [B][u]$$

$$[y] = [C][x] + [D][u]$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}, \; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \; y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$[\mathbf{A}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -36 & -0.6 & 36 & 0.6 \\ 0 & 0 & 0 & 1 \\ 18 & 0.3 & -18 & -0.3 \end{bmatrix} \quad [\boldsymbol{B}] = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$[\mathbf{C}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad [\boldsymbol{D}] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

## b) Designing of Observer gain Matrix ($K_e$):

According to the desired closed loop poles s = -15, s = -16 s = -25 s = -16 observer gain matrix can be determine by-

- First write the characteristic equation by using close loop poles.
- Then find the determinant of matrix $s[I] - ([A] - [K_e][C])$
- By comparing above two matrix we will get each elements of the matrix $K_e$ known as observer gain matrix.
- MATLAB code for getting observer gain matrix is –

```
        %for writing state matrix
A=[0 1 0 0; -36 -0.6 36 0.6; 0 0 0 1; 18 0.3 -18 -0.3];
B=[0; 1; 0; 0];
C=[1 0 0 0; 0 0 1 0];
D=[0; 0;];
        %for designing observer gain matrix
poles1 =[-15 -16 -25 -16];
Ke=place(A',C',poles1)'
```

Finally observer gain matrix is -

$$[K_e] = \begin{bmatrix} 30.43 & 1.34 \\ 186.69 & 71.38 \\ 0.68 & 40.67 \\ 33.09 & 369.74 \end{bmatrix}$$

## c) Designing of Feedback gain Matrix:

According to the desired closed loop poles s = -2+j2√ 3, s = −2 − j2 √ 3, s = −10, s = −10 feedback gain matrix can be determine by-

- First write the characteristic equation by using close loop poles.
- Then find the determinant of matrix $s[I] - ([A] - [B][K])$
- By comparing above two matrix we will get each elements of the matrix K known as feedback gain matrix.
- MATLAB code for getting feedback gain matrix is –

```
        %for writing state matrix
A=[0 1 0 0; -36 -0.6 36 0.6; 0 0 0 1; 18 0.3 -18 -0.3];
B=[0; 1; 0; 0];
C=[1 0 0 0; 0 0 1 0];
D=[0; 0;];
        %for designing Feedback gain matrix
poles2=[-2+2*sqrt(3)*j; -2-2*sqrt(3)*j; -10; -10];
K=acker(A,B,poles2)
```

Finally feedback gain matrix is -

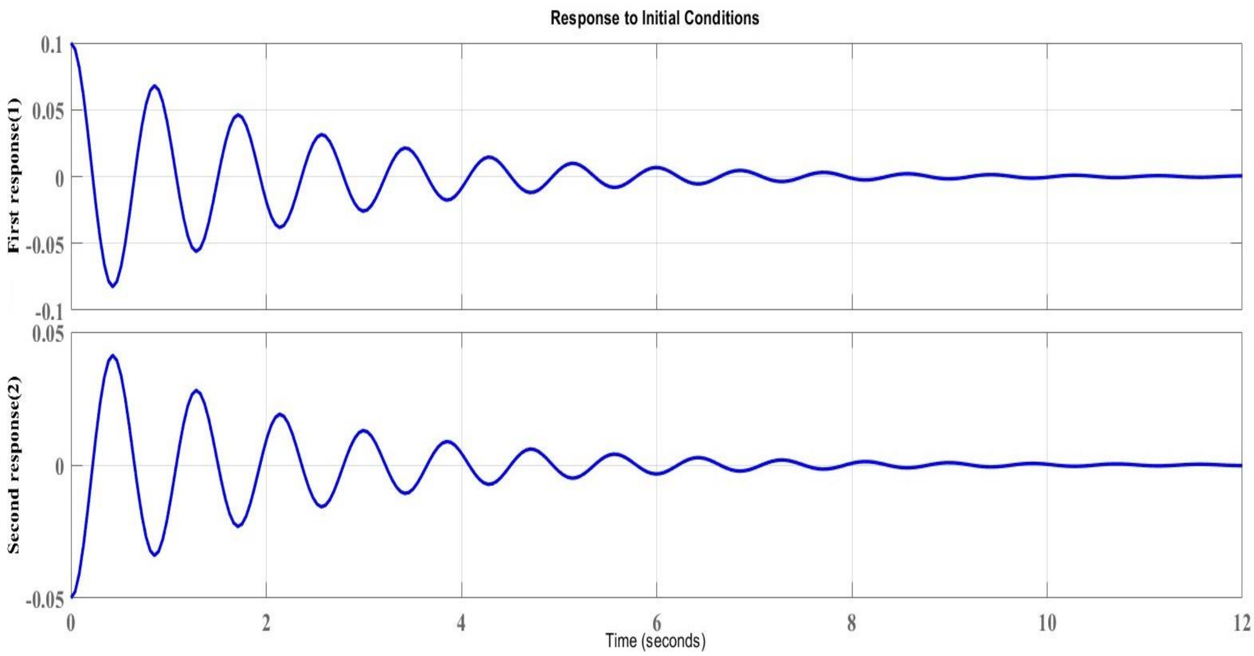
$$[\boldsymbol{K}] = [130.44 \quad 23.10 \quad -41.56 \quad 15.41]$$

## d) Response due to Initial condition of the system:

- The initial condition of the system is
  $y_1(0) = 0.1$, $y_2(0) = 0$,
  $e_1(0) = 0.1$, $e_2(0) = 0.05$
  where e1 and e2 are defined by
  $e_1 = y_1 - y'_1$ $e_2 = y_2 - y'_2$
  now remaining initial conditions are-
  $y'_1(0) = 0$, $y'_2(0) = -0.05$,
- MATLAB code for the Response due to initial condition

```
    %for writing state matrix
A=[0 1 0 0; -36 -0.6 36 0.6; 0 0 0 1; 18 0.3 -18 -0.3];
B=[0; 1; 0; 0];
C=[1 0 0 0; 0 0 1 0];
D=[0; 0;];
    %response of the system due to initial condition
G=ss(A,B,C,D);
X0=[0.1; 0; -0.05; 0];
initial(G,X0)
grid
```

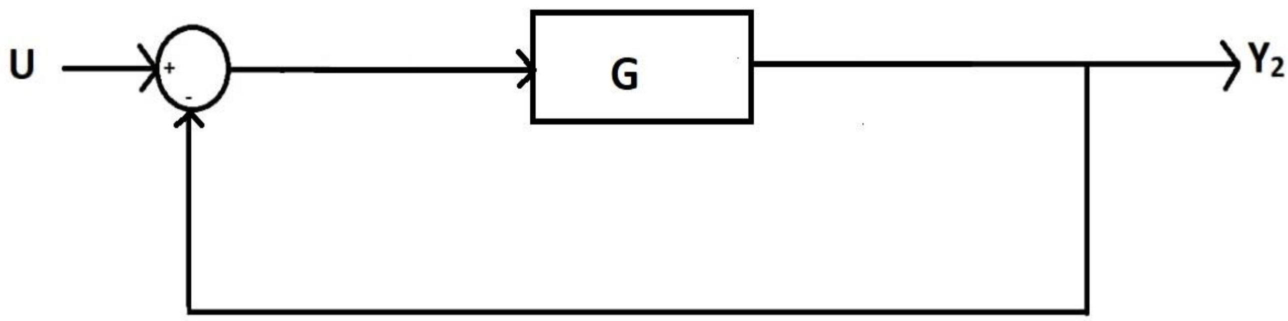
Response ($y_1$ and $y_2$) of the system due to initial condition is-



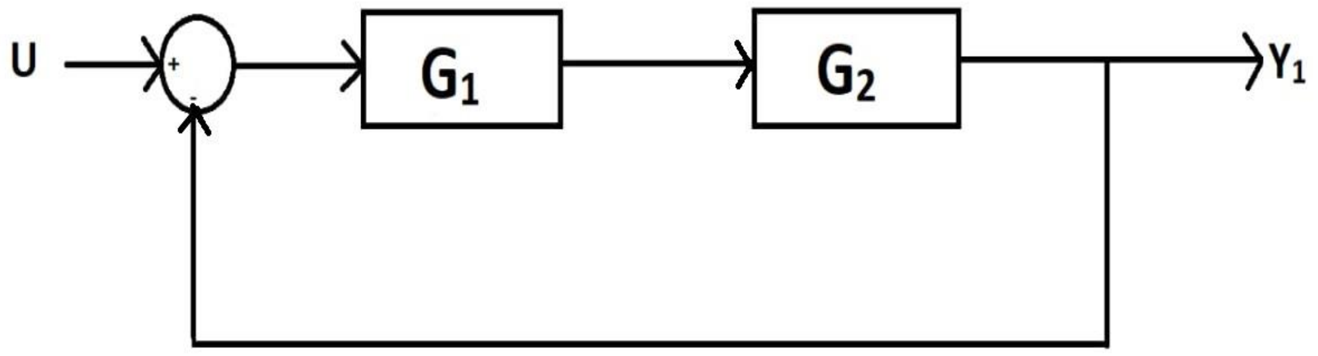

**e) Designing of a Regulator:**

If system experiencing any disturbance, then it's effect comes on output. For that we are designing a regulator in such a way that effect of the disturbance minimized .

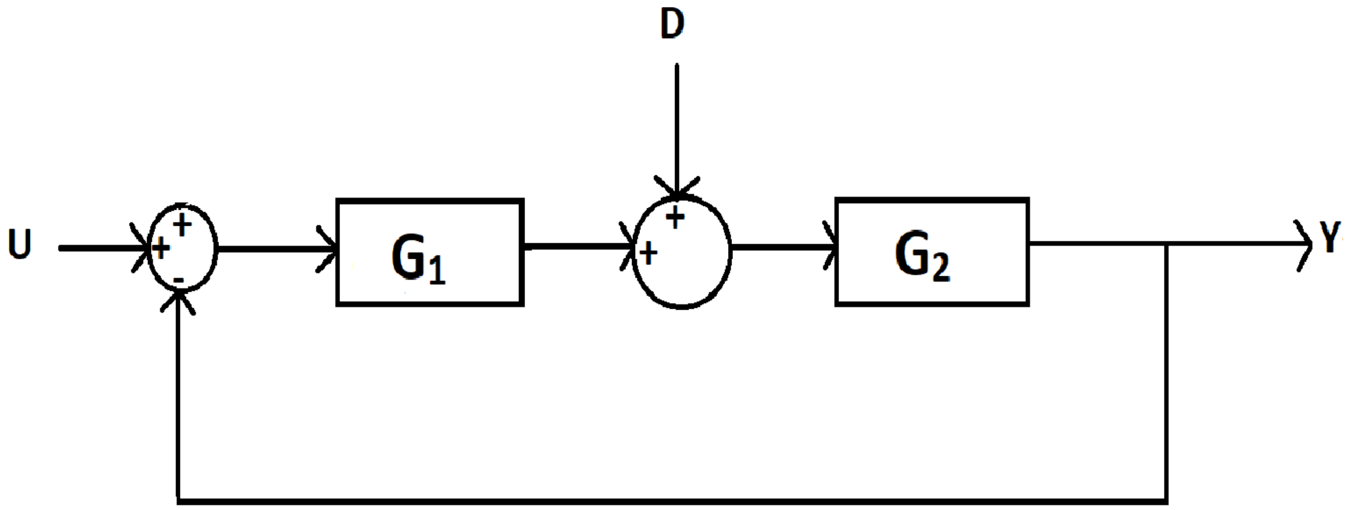
- First find transfer function from the state space.



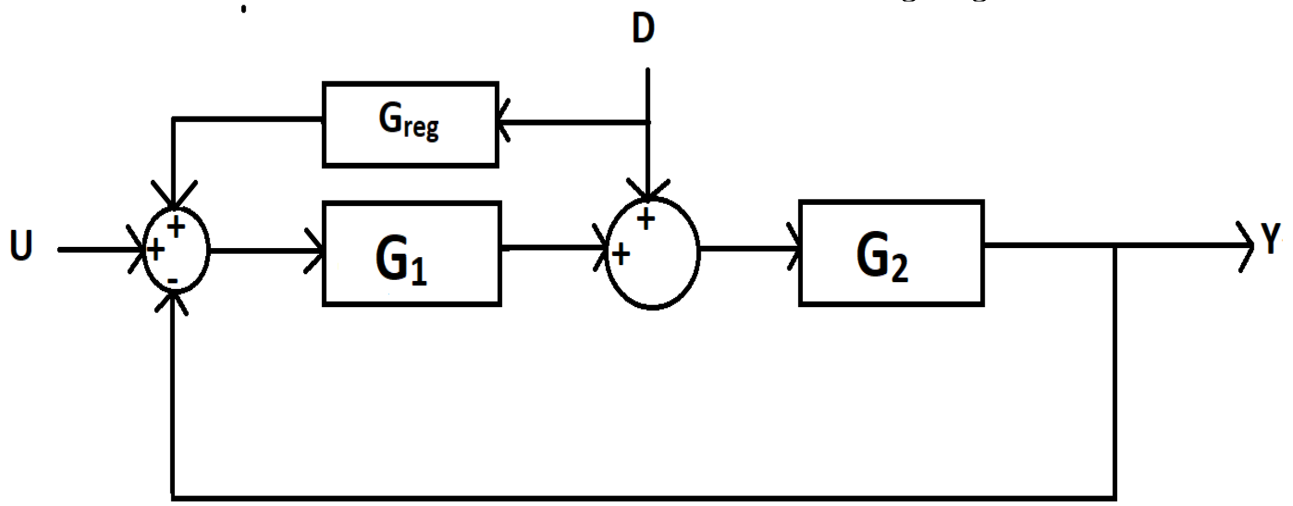
- Find it open loop transfer function ($G = G_1.G_2$)




- Create disturbance in forward path




- Find response from Disturbance
- Find condition to minimize the effect of disturbance and design regulator.



Response due to Disturbance when input is zero-

$$Y(s) = D\frac{G_{reg}G_1G_2 + G_2}{1 + G_1G_2}$$

For zero response the condition is –

$$G_{reg} = -\frac{1}{G_1}$$

- MATLAB code for finding Transfer Functions using state space-

```
    %for writing state matrix
A=[0 1 0 0; -36 -0.6 36 0.6; 0 0 0 1; 18 0.3 -18 -0.3];
B=[0; 1; 0; 0];
C=[1 0 0 0; 0 0 1 0];
D=[0; 0;];
    %to write close loop transfer function
syms s
m1=s*eye(4)-A;
m2=C*inv(m1)*B;
T1=m2(1,1)
T2=m2(2,1)
    %to write open loop transfer function
[n1,d1]=numden(T1);  [n2,d2]=numden(T2);
t1=n1/(d1-n1)
t2=n2/(d2-n)
```

Closed loop Transfer Functions of the system from first and second response –

$$T_1(s) = \frac{10s^2 + 3s + 180}{s^2(10s^2\ +\ 9s\ +\ 540)}$$

$$T_2(s) = \frac{3(s + 60)}{s^2(10s^2\ +\ 9s\ +\ 540)}$$

Open loop Transfer Functions of the system from first and second response –

$$t_1(s) = \frac{10s^2 + 3s + 180}{10s^4\ +\ 9s^3 +\ 530s^2 - 3s - 180)}$$

$$t_2(s) = \frac{3(s + 60)}{10s^4\ +\ 9s^3 +\ 540s^2 - 3s - 180)}$$

**Case 1:** with respect to response 1

writing the value of $G_1$ and $G_2$ by taking reference of above block diagram

$$G_1(s) = \frac{1}{(s+0.581)} \text{ and } G_2(s) = \frac{s^2+0.3s+18}{(s^2+0.9s+53.5)(s-0.581)}$$

From here the regulator should be-

$$G_{reg}(s) = -\frac{1}{G_1(s)} = -(s+0.581)$$

**Case 2:** with respect to response 2

writing the value of $G_1$ and $G_2$ by taking reference of above block diagram

$$G_1(s) = \frac{1}{(s+0.58)} \text{ and } G_2(s) = \frac{0.3s+18}{(s^2+0.9s+54.23)(s-0.58)}$$

From here the regulator should be

$$G_{reg}(s) = -\frac{1}{G_1(s)} = -(s+0.58)$$

For minimizing effect of disturbance on response we designed Regulator. For minimizing disturbance effect in first response, we designed regulator which was discusses in case 1. For minimizing disturbance effect in second response we designed regulator which was discusses in case 2. It is difficult to design single regulator which minimize disturbance effect for both outputs.

## Results:

Mathematical modelling with state space equations.
Designed observer gain matrix($K_e$) using pole placement approach using MATLAB.
Designed feedback gain matrix(K) using pole placement approach using MATLAB.
Got response due to initial conditions in the system using MATLAB.
Got open loop and close loop transfer function using MATLAB.
Designed regulator to minimize disturbance effect on output.