Modern Complexity Theory

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1.2

a) The number of possible arrangements where we toss a coin and get 101 consecutively are as follows.

① Case I

$\boxed{101\_\,\_}$

In this case we obtain 101 once in the beginning, thus total cases $= 2\times2 - 1 = 4-1 = 3$

The case we subtract is $\boxed{10101}$.

② Case II

We obtain 101 ~~occurs~~ consecutively in the form $\boxed{\_\,101\,\_}$ and the total number of such cases is $2\times2 = 4$

③ Case III

We obtain 101 once at the end and no where else. Thus total number of such $\boxed{\_\,\_\,101}$ cases $= 2\times2 - 1 = 4-1 = 3$

Case IV

We obtain consecutive 101 twice.

There is only 1 such case $\boxed{10101}$.

Thus total cases where we obtain 101 consecutively

at least once $= ① + ② + ③ + ④$

$= 3 + 4 + 3 + 1$

$= 11$ — ⑤

Total possibilities $= 2^5$ — ⑥

Thus, required probability $= \dfrac{\text{total favorable cases}}{\text{total cases}}$

$= \dfrac{11}{2^5} = \boxed{0.34375}$

b) Here, number of cases where at least three people are mutual friends is equivalent to number of graphs with 5 nodes and at least one cycle of length 3.

Let that set of graphs be $G_{5,3}$ then $|G_{5,3}|$ is the reqd. number of favourable cases.

Now, we know that a graph is called bipartite if it has only even cycles (no odd cycles) whenever there is a cycle.

Let B_5 denote the set of bipartite graphs having 5 nodes.

Now, $|G_{5,3}| + |G_{5,5}| = 2^{10} - 376 = \cancel{648}\ 648$

where $|G_{5,5}|$ is the total number of graphs with 5 nodes and atleast one cycle of length 5.

Thus, $|G_{5,3}| = 648 - |G_{5,5}| = 648 - 12$

$= 636$

Thus, total number of favorable cases $= 636$

Thus, required probability $= \dfrac{\text{no of favorable cases}}{\text{no. of cases}}$

$= \dfrac{636}{1024}$

Thus, the reqd. probability is $\boxed{0.62109375}$

1.1

a) We know that number of functions from A to B i.e., $f: A \to B$ is $|B|^{|A|}$.

Thus, number of functions from $\{0,1\}^n$ to $\{0,1\}^k$ i.e., $f: \{0,1\}^n \longrightarrow \{0,1\}^k$

shall be $|\{0,1\}^k|^{|\{0,1\}^n|} = |2^k|^{2^n} = 2^{k2^n}$.

Thus required number of functions is 2^{k2^n}.

If $k=2$ then number of possible functions shall be

$$2^{2.2^n} = \boxed{2^{2^{n+1}}}.$$

b) Number of possible functions such that $f: \{0,1\}^n \longrightarrow \{0,1\}^k$ is symmetric if for any reordering σ of $\{1, \ldots, n\}$ and any $f(x) = f(\sigma(x))$ is $\boxed{(2^k)^{n+1}}$

This is because $|\{0,1\}^k| = 2^k$ and number of sets of inputs with independent outputs is $n+1$,

$$|\{S_1, S_2, S_3, \ldots, S_n, S_0\}| = n+1,$$

where S_i = set of inputs with i zeroes.