Absolutely separating quantum maps and channels

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Abstract. Absolutely separable states ϱ remain separable under arbitrary unitary transformations $U \varrho U^\dagger$. By example of a three qubit system we show that in multipartite scenario neither full separability implies bipartite absolute separability nor the reverse statement holds. The main goal of the paper is to analyze quantum maps resulting in absolutely separable output states. Such absolutely separating maps affect the states in a way, when no Hamiltonian dynamics can make them entangled afterwards. We study general properties of absolutely separating maps and channels with respect to bipartitions and multipartitions and show that absolutely separating maps are not necessarily entanglement breaking. We examine stability of absolutely separating maps under tensor product and show that $\Phi^{\otimes N}$ is absolutely separating for any N if and only if Φ is the tracing map. Particular results are obtained for families of local unital multiqubit channels, global generalized Pauli channels, and combination of identity, transposition, and tracing maps acting on states of arbitrary dimension. We also study the interplay between local and global noise components in absolutely separating bipartite depolarizing maps and discuss the input states with high resistance to absolute separability.

1. Introduction

The phenomenon of quantum entanglement is used in a variety of quantum information applications [1, 2]. The distinction between entangled and separable states has an operational meaning in terms of local operations and classical communication, which cannot create entanglement from a separable quantum state [3]. Natural methods of entanglement creation include interaction between subsystems, measurement in the basis of entangled states, entanglement swapping [4, 5, 6], and dissipative dynamics towards an entangled ground state [7, 8]. On the other hand, dynamics of any quantum system is open due to inevitable interaction between the system and its environment. The general transformation of the system density operator for time t is given by a dynamical map Φ_t, which is completely positive and trace preserving (CPT) provided the initial state of the system and environment is factorized [9]. CPT maps are called

quantum channels [10]. Dissipative and decoherent quantum channels describe noises acting on a system state. Properties of quantum channels with respect to their action on entanglement are reviewed in the papers [11, 12, 13].

Suppose a quantum channel Φ such that its output $\varrho_{\text{out}} = \Phi[\varrho]$ is separable for some initial system state ϱ. It may happen either due to entanglement annihilation of the initially entangled state ϱ [14, 15], or due to the fact that the initial state ϱ was separable and Φ preserved its separability. Though the state ϱ_{out} is inapplicable for entanglement-based quantum protocols, there is often a possibility to make it entangled by applying appropriate control operations, e.g. by activating the interaction Hamiltonian H among constituting parts of the system for a period τ. It results in a unitary transformation $\varrho_{\text{out}} \rightarrow U\varrho_{\text{out}}U^\dagger$, where $U = \exp(-iH\tau/\hbar)$, $\hbar$ is the Planck constant. Thus, if a quantum system in question is controlled artificially, one can construct an interaction such that the state $U\varrho_{\text{out}}U^\dagger$ may become entangled. It always takes place for pure output states $\varrho_{\text{out}} = |\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|$, however, such an approach may fail for mixed states. These are *absolutely separable* states that remain separable under action of *any* unitary operator U [16, 17]. Properties of absolutely separable states are reviewed in the papers [18, 19, 20, 21]. Even if the dynamical map Φ is such that $\Phi[\varrho]$ is absolutely separable for a given initial state ϱ, one may try and possibly find a different input state ϱ' such that $\Phi[\varrho']$ is not absolutely separable, and the system entanglement could be recovered by a proper unitary transformation. It may happen, however, that whatever initial state ϱ is used, the output $\Phi[\varrho]$ is always absolutely separable. Thus, a dynamical map Φ may exhibit an *absolutely separating* property, which means that its output is always absolutely separable and cannot be transformed into an entangled state by any Hamiltonian dynamics. The only deterministic way to create entanglement in a system acted upon by the absolutely separating channel Φ is to use a nonunitary CPT dynamics afterwards, e.g. a Markovian dissipative process $\widetilde{\Phi}_t = e^{t\mathcal{L}}$ with the only fixed point ϱ_∞, which is entangled. From experimental viewpoint it means that absolutely separating noises should be treated in a completely different way in order to maintain entanglement.

The goal of this paper is to characterize absolutely separating maps Φ, explore their general properties, and illustrate particular properties for specific families of quantum channels.

The paper is organized as follows.

In section 2, we review properties of absolutely separable states and known criteria for their characterization. We establish an upper bound on purity of absolutely separable states. Also, we pay attention to the difference between absolute separability with respect to a bipartition and that with respect to a multipartition. We show the relation between various types of absolute separability and conventional separability in tripartite systems. In section 3, we review general properties of absolutely separating maps and provide sufficient and (separately) necessary conditions of absolutely separating property. Section 4 is devoted to the analysis of N-tensor-stable absolutely separating maps, i.e. maps Φ such that $\Phi^{\otimes N}$ is absolutely separating with respect to any valid

bipartition. In section 5, we consider specific families of quantum maps, namely, local depolarization of qubits (section 5.1), local unital maps on qubits (section 5.2), generalized Pauli diagonal channels constant on axes [22] (section 5.3). In section 5.4, we consider a combination of tracing map, transposition, and identity transformation acting on a system of arbitrary dimension. Such maps represent a two-parametric family comprising a global depolarization channel and the Werner-Holevo channel [23] as partial cases. In section 5.5, we deal with the recently introduced three-parametric family of bipartite depolarizing maps [24], which describe a combined physical action of local and global depolarizing noises on a system of arbitrary dimension. In section 6, we discuss the obtained results and focus attention on initial states ϱ such that $\Phi_t[\varrho]$ remains not absolutely separable for the maximal time t. In section 7, brief conclusions are given.

2. Absolutely separable states

Associating a quantum system with the Hilbert space $\mathcal{H}$, a quantum state is identified with the density operator ϱ acting on $\mathcal{H}$ (Hermitian positive semidefinite operator with unit trace). By $\mathcal{S}(\mathcal{H})$ denote the set of quantum states. We will consider finite dimensional spaces $\mathcal{H}_d$, where the subscript d denotes $\dim\mathcal{H}$.

2.1. Bipartite states

A quantum state $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$, $m, n \geqslant 2$ is called separable with respect to the particular partition $\mathcal{H}_{mn} = \mathcal{H}_m^A \otimes \mathcal{H}_n^B$ on subsystems A and B if ϱ adopts the convex sum resolution $\varrho = \sum_k p_k \varrho_k^A \otimes \varrho_k^B$, $p_k \geqslant 0$, $\sum_k p_k \geqslant 0$ [25]. We will use a concise notation $\mathcal{S}(\mathcal{H}_m^A|\mathcal{H}_n^B)$ for the set of such separable states. Usually, subsystems A and B denote different particles or modes [26], depending on experimentally accessible degrees of freedom. If the state $\varrho \notin \mathcal{S}(\mathcal{H}_m^A|\mathcal{H}_n^B)$, then ϱ is called entangled with respect to $\mathcal{H}_m^A|\mathcal{H}_n^B$.

In contrast, a quantum state $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$ is called *absolutely separable* with respect to partition $m|n$ if ϱ remains separable with respect to *any* partition $\mathcal{H}_{mn} = \mathcal{H}_m^A \otimes \mathcal{H}_n^B$ without regard to the choice of A and B [18, 19, 20]. Denoting by $\mathcal{A}(m|n)$ the set of absolutely separable states, we conclude that $\mathcal{A}(m|n) = \cap_{A,B}\mathcal{S}(\mathcal{H}_m^A|\mathcal{H}_n^B)$. The physical meaning of absolutely separable states is that they remain separable without respect to the particular choice of relevant degrees of freedom. In terms of the fixed partition $\mathcal{H}_m^A|\mathcal{H}_n^B$, the state $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$ is absolutely separable with respect to $m|n$ if and only if $U\varrho U^\dagger \in \mathcal{S}(\mathcal{H}_m^A|\mathcal{H}_n^B)$ for any unitary operator U.

Let us notice that specification of partition is important. For instance, 12-dimensional Hilbert space allows different partitions $2|6$ and $3|4$. Moreover, one can always imbed the density operator $\varrho \in \mathcal{S}(\mathcal{H}_{d_1})$ into a higher-dimensional space $\mathcal{S}(\mathcal{H}_{d_2})$, $d_2 > d_1$ and consider separability with respect to other partitions.

Absolutely separable states cannot be transformed into entangled ones by unitary operations. In quantum computation circuits, the application of unitary entangling gates (like controlled-NOT or $\sqrt{\text{SWAP}}$) to absolutely separable states is useless from the

viewpoint of creating entanglement. Thus, a quantum dynamics transforming absolutely separable states into entangled ones must be non-unitary. Example of a dynamical map Φ, which always results in an entangled output, is a Markovian process $\Phi_t = e^{t\mathcal{L}}$ with the only fixed point $\varrho_\infty \notin \mathcal{S}(\mathcal{H}_m|\mathcal{H}_n)$; the output state $\Phi_t[\varrho]$ is always entangled for some time $t > t_*$ even if the input state ϱ is absolutely separable.

In analogy with the absolutely separable states, absolutely classical spin states were introduced recently [27]. The paper [27] partially answers the question: what are the states of a spin-j particle that remain classical no matter what unitary evolution is applied to them? These states are characterized in terms of a maximum distance from the maximally mixed spin-j state such that any state closer to the fully mixed state is guaranteed to be classical.

2.2. Criteria of absolute separability with respect to bipartition

Note that two states ϱ and $V\varrho V^\dagger$, where V is unitary, are both either absolutely separable or not. In other words, they exhibit the same properties with respect to absolute separability. Let V diagonalize $V\varrho V^\dagger$, i.e. $V\varrho V^\dagger = \mathrm{diag}(\lambda_1, \ldots, \lambda_{mn})$, where $\lambda_1, \ldots, \lambda_{mn}$ are eigenvalues of ϱ. It means that the property of absolute separability is defined by the state spectrum only.

A necessary condition of separability is positivity under partial transpose (PPT) [28, 29]: $\varrho \in \mathcal{S}(\mathcal{H}_m^A|\mathcal{H}_n^B) \implies \varrho^{\Gamma_B} = \sum_{i,j=1}^n I^A \otimes |j\rangle^B\langle i| \, \varrho \, I^A \otimes |j\rangle^B\langle i| \geqslant 0$, where I is the indentity operator, $\{|i\rangle\}_{i=1}^n$ is an orthonormal basis in $\mathcal{H}_n^B$, and positivity of Hermitian operator O means $\langle\varphi|O|\varphi\rangle \geqslant 0$ for all $|\varphi\rangle$. In analogy with absolutely separable states one can introduce *absolutely PPT* states with respect to partitioning $m|n$, namely, $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$ is absolutely PPT with respect to $m|n$ if $\varrho^{\Gamma_B} \geqslant 0$ for all decompositions $\mathcal{H} = \mathcal{H}_m^A \otimes \mathcal{H}_n^B$ [30, 18]. Equivalently, $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$ is absolutely PPT with respect to $m|n$ if $(U\varrho U^\dagger)^{\Gamma_B} \geqslant 0$ for all unitary operators U. The set of absolutely PPT states with respect to $m|n$ denote $\mathcal{A}_{\mathrm{PPT}}(m|n)$. It is clear that $\mathcal{A}(m|n) \subset \mathcal{A}_{\mathrm{PPT}}(m|n)$ for all m, n.

The set $\mathcal{A}_{\mathrm{PPT}}(m|n)$ is fully characterized in [18], where necessary and sufficient conditions on the spectrum of ϱ are found under which ϱ is absolutely PPT. These conditions become particularly simple in the case $m = 2$: the state $\varrho \in \mathcal{S}(\mathcal{H}_{2n})$ is absolutely PPT if and only if its eigenvalues $\lambda_1, \ldots, \lambda_{2n}$ (in decreasing order $\lambda_1 \geqslant \ldots \geqslant \lambda_{2n}$) satisfy the following inequality:

$$\lambda_1 \leqslant \lambda_{2n-1} + 2\sqrt{\lambda_{2n}\lambda_{2n-2}}. \tag{1}$$

Due to the fact that separability is equivalent to PPT for partitions $2|2$ and $2|3$ [29], $\mathcal{A}(2|2) = \mathcal{A}_{\mathrm{PPT}}(2|2)$ and $\mathcal{A}(2|3) = \mathcal{A}_{\mathrm{PPT}}(2|3)$. Moreover, the recent study [19] shows that $\mathcal{A}(2|n) = \mathcal{A}_{\mathrm{PPT}}(2|n)$ for all $n = 2, 3, 4, \ldots$. Thus, equation (1) is a necessary and sufficient criterion for absolute separability of the state $\varrho \in \mathcal{S}(\mathcal{H}_{2n})$ with respect to partition $2|n$.

For general m, n there exists a sufficient condition of absolute separability based on the fact that the states ϱ with sufficiently low purity $\mathrm{tr}[\varrho^2]$ are separable [30, 31, 32, 33].

Suppose the state $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$ satisfies the requirement

$$\text{tr}[\varrho^2] = \sum_{k=1}^{mn} \lambda_k^2 \leqslant \frac{1}{mn-1}, \tag{2}$$

then $\varrho \in \mathcal{S}(\mathcal{H}_m|\mathcal{H}_n)$. Since unitary rotations $\varrho \to U\varrho U^\dagger$ do not change the Frobenius norm, the states inside the separable ball (2) are all absolutely separable, i.e. (2)$\Longrightarrow$ $\varrho \in \mathcal{A}(m|n)$.

Suppose $\varrho \in \mathcal{A}_{\text{PPT}}(m|n)$, then decreasingly ordered eigenvalues $\lambda_1, \ldots, \lambda_{mn}$ of ϱ satisfy ([20], theorem 8.1)

$$\lambda_1 \leqslant \lambda_{mn-2} + \lambda_{mn-1} + \lambda_{mn}. \tag{3}$$

Since $\mathcal{A}(m|n) \subset \mathcal{A}_{\text{PPT}}(m|n)$, equation (3) represents a readily computable necessary condition of absolute separability with respect to bipartition $m|n$. The physical meaning of equation (3) is that the absolutely separable state cannot be close enough to any pure state, because for pure states $\lambda_1^\downarrow = 1$ and $\lambda_2^\downarrow = \ldots = \lambda_{mn}^\downarrow = 0$, which violates the requirement (3).

Moreover, a factorized state $\varrho_1 \otimes \varrho_2$ with $\varrho_1 \in \mathcal{S}(\mathcal{H}_m)$ and $\varrho_2 \in \mathcal{S}(\mathcal{H}_n)$, $m, n \geqslant 2$, cannot be absolutely separable with respect to partition $m|n$ if either ϱ_1 or ϱ_2 belongs to a boundary of the state space. In fact, a boundary density operator $\varrho_1 \in \partial \mathcal{S}(\mathcal{H}_m)$ has at least one zero eigenvalue, which implies at least $n \geqslant 2$ zero eigenvalues of the operator $\varrho_1 \otimes \varrho_2$. Consequently, $\lambda_{(m-1)n} = \ldots = \lambda_{mn} = 0$ and equation (3) cannot be satisfied.

Condition (3) imposes a limitation on the purity of absolutely separable states. Next proposition provides a quantitative description of the maximal ball, where all absolutely separable states are located.

Proposition 1. *An absolutely separable state $\varrho \in \mathcal{A}(m|n)$ necessarily satisfies the inequality*

$$1 + \sqrt{\frac{k\,\text{tr}[\varrho^2] - 1}{k-1}} \leqslant 3k\sqrt{\frac{\text{tr}[\varrho^2]}{mn+8}} \quad \textit{if} \quad \frac{1}{k} \leqslant \text{tr}[\varrho^2] \leqslant \frac{1}{k-1}, \quad k = 2, 3, \ldots, mn \tag{4}$$

and its simpler implication

$$\text{tr}[\varrho^2] \leqslant \frac{9}{mn+8}. \tag{5}$$

Proof. Let $\text{tr}[\varrho^2] = \mu$. It is not hard to see that in general $(\lambda_{mn-2} + \lambda_{mn-1} + \lambda_{mn})^2 \leqslant 3(\lambda_{mn-2}^2 + \lambda_{mn-1}^2 + \lambda_{mn}^2)$ and $\lambda_{mn-2}^2 + \lambda_{mn-1}^2 + \lambda_{mn}^2 \leqslant \frac{3}{mn-1}\sum_{i=2}^{mn} \lambda_i^2 = 3\frac{\mu - \lambda_1^2}{mn-1}$. Consequently, if

$$\lambda_1^2 > 9\frac{\mu - \lambda_1^2}{mn-1}, \tag{6}$$

then $\lambda_1 > \lambda_{mn-2} + \lambda_{mn-1} + \lambda_{mn}$ and the necessary condition for absolute separability (3) is not fulfilled.
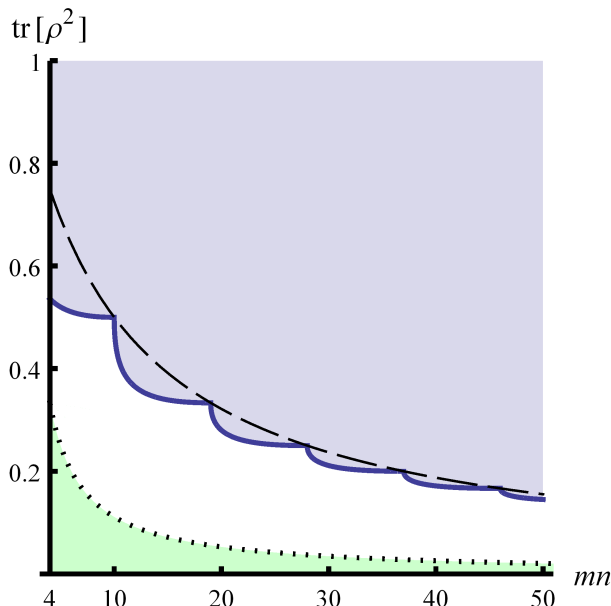
Figure 1. Purity $\text{tr}[\varrho^2]$ of states $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$ vs. dimension mn: below dotted line $\varrho \in \mathcal{A}(m|n)$ due to equation (2); above solid line $\varrho \notin \mathcal{A}(m|n)$ due to equation (4). Dashed line corresponds to the boundary established in equation (5).

Suppose the purity μ is known, then the maximal eigenvalue λ_1 has a lower bound, which can be found by the method of Lagrange multipliers with constraints $\sum_{i=1}^{mn} \lambda_i = 1$ and $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_{mn} \geqslant 0$. The eigenvalue λ_1 is minimal if $\lambda_1 = \ldots = \lambda_{k-1}$ and $\lambda_{k+1} = \lambda_{k+2} = \ldots = 0$ for some $1 < k \leqslant mn$. Then $\lambda_k = 1 - (k-1)\lambda_1$ and $\mu = (k-1)\lambda_1^2 + [1 - (k-1)\lambda_1]^2$. If $\frac{1}{k} \leqslant \mu \leqslant \frac{1}{k-1}$, then the minimal largest eigenvalue reads

$$\min \lambda_1 = \frac{1}{k} \left(1 + \sqrt{\frac{k\mu - 1}{k - 1}} \right). \tag{7}$$

Substituting $\min \lambda_1$ for λ_1 in equation (6), we obtain a converse to inequality (4). This converse relation specifies the region of purities $\mu \in (\mu_0, 1]$, where the state $\varrho \notin \mathcal{A}(m|n)$. Thus, equation (4) is necessary for absolute separability. Formula (5) follows from equation (4) and describes a hyperbola, which passes through all breaking points of μ_0 as a function of mn, see figure 1. $\qquad \square$

Proposition 1 shows, in particular, that two qubit states with $\text{tr}[\varrho^2] > (\sqrt{3} - 1)^2 \approx 0.536$ cannot be absolutely separable states with respect to partition $2|2$. A state $\varrho \in \mathcal{S}(\mathcal{H}_d)$ is not absolutely separable with respect to any partition $m|n$ ($d = mn \geqslant 4$, $m, n \geqslant 2$) if $\text{tr}[\varrho^2] > \frac{9}{d+8}$.

2.3. Absolute separability with respect to multipartition

An N-partite quantum state $\varrho \in \mathcal{S}(\mathcal{H}_{n_1 \ldots n_N})$, $n_k \geqslant 2$ is called fully separable with respect to the partition $\mathcal{H}_{n_1 \ldots n_N} = \mathcal{H}_{n_1}^{A_1} \otimes \ldots \otimes \mathcal{H}_{n_N}^{A_N}$ on subsystems $A_1, \ldots, A_N$ if ϱ

adopts the convex sum resolution $\varrho = \sum_k p_k \varrho_k^{A_1} \otimes \ldots \otimes \varrho_k^{A_N}$, $p_k \geqslant 0$, $\sum_k p_k = 1$. The set of fully separable states is denoted by $\mathcal{S}(\mathcal{H}_{n_1}^{A_1}|\ldots|\mathcal{H}_{n_N}^{A_N})$. The criterion of full separability is known, for instance, for 3-qubit Greenberger-Horne-Zeilinger (GHZ) diagonal states [34, 35].

We will call a state $\varrho \in \mathcal{S}(\mathcal{H}_{n_1\ldots n_N})$ *absolutely separable* with respect to *multipartition* $n_1|\ldots|n_N$ if ϱ remains separable with respect to *any* multipartition $\mathcal{H}_{n_1|\ldots|n_N} = \mathcal{H}_{n_1}^{A_1} \otimes \ldots \otimes \mathcal{H}_{n_N}^{A_N}$ or, equivalently, $U\varrho U^\dagger \in \mathcal{S}(\mathcal{H}_{n_1}^{A_1}|\ldots|\mathcal{H}_{n_N}^{A_N})$ for any unitary operator U and fixed multipartition $A_1|\ldots|A_N$. We will use notation $\mathcal{A}(n_1|\ldots|n_N)$ for the set of states, which are absolutely separable with respect to *multipartition* $n_1|\ldots|n_N$.

A sufficient condition of absolute separability with respect to multipartition follows from consideration of separability balls [33]. Consider an N-qubit state $\varrho \in \mathcal{S}(\mathcal{H}_{2^N})$ such that

$$\mathrm{tr}[\varrho^2] \leqslant \frac{1}{2^N}\left(1 + \frac{54}{17} 3^{-N}\right), \tag{8}$$

then $\varrho \in \mathcal{A}(\underbrace{2|\ldots|2}_{N \text{ times}})$ [33].

To illustrate the relation between different types of separability under bipartitions and multipartitions let us consider a three-qubit case.

Example 1. *The inclusion $\mathcal{A}(2|2|2) \subset \mathcal{S}(\mathcal{H}_2|\mathcal{H}_2|\mathcal{H}_2) \subset \mathcal{S}(\mathcal{H}_2|\mathcal{H}_4) \subset PPT(\mathcal{H}_2|\mathcal{H}_4) \subset \mathcal{S}(\mathcal{H}_8)$ is trivial. Also, $\mathcal{A}(2|2|2) \subset \mathcal{A}(2|4) = \mathcal{A}_{\mathrm{PPT}}(2|4) \subset \mathcal{S}(\mathcal{H}_2|\mathcal{H}_4)$. The relation to be clarified is that between $\mathcal{A}(2|4)$ and $\mathcal{S}(\mathcal{H}_2|\mathcal{H}_2|\mathcal{H}_2)$.*

Firstly, we notice that the pure state $|\psi_1\rangle\langle\psi_1| \otimes |\psi_2\rangle\langle\psi_2| \otimes |\psi_3\rangle\langle\psi_3|$ is fully separable but not absolutely separable with respect to partition $2|4$ as there exists a unitary transformation U, which transforms it into a maximally entangled state $|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|$, where $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. Thus, $\mathcal{S}(\mathcal{H}_2|\mathcal{H}_2|\mathcal{H}_2) \not\subset \mathcal{A}(2|4)$.

Secondly, consider a state $\varrho \in \mathcal{A}(2|4) = \mathcal{A}_{\mathrm{PPT}}(2|4)$, then its spectrum $\lambda_1, \ldots, \lambda_8$ in decreasing order satisfies equation (1) for $n = 4$. Maximizing the state purity $\sum_{k=1}^{8} \lambda_k^2$ under conditions $\lambda_1 \geqslant \ldots \geqslant \lambda_8 \geqslant 0$, $\sum_{k=1}^{8} \lambda_k = 1$, and inequality (1), we get $\lambda_1 = \lambda_2 = \frac{11}{48}$, $\lambda_3 = \frac{23}{144}$, $\lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = \frac{11}{144}$. Any 3-qubit state ϱ with such a spectrum is absolutely separable with respect to partition $2|4$. Consider a particular state

$$\varrho = \sum_{k=1}^{8} \lambda_k |\mathrm{GHZ}_k\rangle\langle\mathrm{GHZ}_k|, \tag{9}$$

where the binary representation of $k - 1 = 4k_1 + 2k_2 + k_3$, $k_i = 0, 1$, defines GHZ-like states

$$|\mathrm{GHZ}_k\rangle = \frac{1}{\sqrt{2}}((-1)^{k-1}|k_1\rangle \otimes |k_2\rangle \otimes |k_3\rangle + |\bar{k}_1\rangle \otimes |\bar{k}_2\rangle \otimes |\bar{k}_3\rangle \tag{10}$$

with $\bar{0} = 1$ and $\bar{1} = 0$. The state (9) is GHZ diagonal, so we apply to it the necessary and sufficient condition of full separability ([35], theorem 5.2), which shows that (9) is not fully separable. Thus, $\mathcal{A}(2|4) \not\subset \mathcal{S}(\mathcal{H}_2|\mathcal{H}_2|\mathcal{H}_2)$.

Finally, to summarize the results of this example, we depict the Venn diagram of separable and absolutely separable 3 qubit states in figure 2.
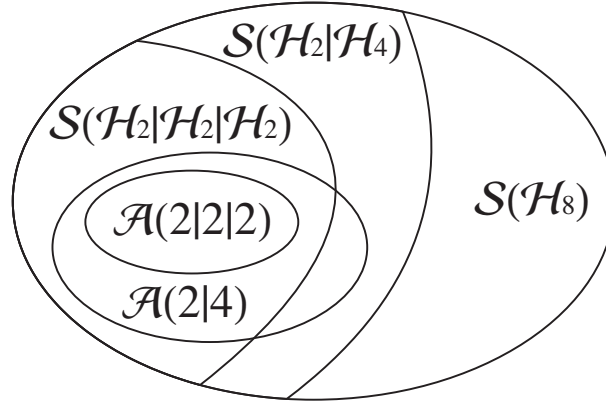
Figure 2. The relation between separability classes of three qubit states: $\mathcal{S}(\mathcal{H}_8)$ is the set of density operators, $\mathcal{S}(\mathcal{H}_2|\mathcal{H}_4)$ is the set of states separable with respect to a fixed bipartition $\mathcal{H}_2|\mathcal{H}_4$, $\mathcal{S}(\mathcal{H}_2|\mathcal{H}_2|\mathcal{H}_2)$ is the set of fully separable states with respect to a multipartition $\mathcal{H}_2|\mathcal{H}_2|\mathcal{H}_2$, $\mathcal{A}(2|4)$ is a set of absolutely separable states with respect to partition $2|4$, and $\mathcal{A}(2|2|2)$ is a set of absolutely separable states with respect to partition $2|2|2$. Convex figures correspond to convex sets.

Note that the state ϱ in (9) is separable for *any* bipartition $2|4$ and entangled with respect to multipartition $\mathcal{H}_2|\mathcal{H}_2|\mathcal{H}_2$. In particular, ϱ is separable with respect to bipartitions $\mathcal{H}_2^A|\mathcal{H}_4^{BC}$, $\mathcal{H}_2^B|\mathcal{H}_4^{AC}$, and $\mathcal{H}_2^C|\mathcal{H}_4^{AB}$, but entangled with respect to tripartition $\mathcal{H}_2^A|\mathcal{H}_2^B|\mathcal{H}_2^C$. The states with such a property were previously constructed via unextendable product bases [36, 37]. Note, however, that even if a 3 qubit state ξ is separable with respect to the specific partitions $A|BC$, $B|AC$, and $C|AB$, it does not imply that ξ is absolutely separable with respect to partition $2|4$, because $U\xi U^\dagger$ is separable with respect to $A|BC$ only for permutation matrices $U(A \leftrightarrow B)$, $U(B \leftrightarrow C)$, and $U(A \leftrightarrow C)$, but not general unitary operators U.

3. Absolutely separating maps and channels

In quantum information theory, positive linear maps $\Phi : \mathcal{S}(\mathcal{H}) \mapsto \mathcal{S}(\mathcal{H})$ represent a useful mathematical tool in characterization of bipartite entanglement [29], multipartite entanglement [38, 39], characterization of Markovianity in open system dynamics [40, 41], etc. A quantum channel is given by a CPT map Φ such that $\Phi \otimes \mathrm{Id}_k$ is a positive map for all identity transformations $\mathrm{Id}_k : \mathcal{S}(\mathcal{H}_k) \mapsto \mathcal{S}(\mathcal{H}_k)$. Thus, entanglement-related properties are easier to explore for positive maps [13] but deterministic physical evolutions are given by quantum channels. It means that the set of absolutely separating channels is the intersection of CPT maps with the set of positive absolutely separating maps introduced below.

We recall that a linear map $\Phi : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_m|\mathcal{H}_n)$ is called positive entanglement annihilating with respect to partition $\mathcal{H}_m|\mathcal{H}_n$, concisely, PEA$(\mathcal{H}_m|\mathcal{H}_n)$. For multipartite composite systems, $\Phi : \mathcal{S}(\mathcal{H}_{n_1...n_N}) \mapsto \mathcal{S}(\mathcal{H}_{n_1}|...|\mathcal{H}_{n_N})$ is positive entanglement annihilating, PEA$(\mathcal{H}_{n_1}|...|\mathcal{H}_{n_N})$. The map $\Phi : \mathcal{S}(\mathcal{H}_m) \mapsto \mathcal{S}(\mathcal{H}_m)$ is

called entanglement breaking (EB) if $\Phi \otimes \mathrm{Id}_n$ is positive entanglement annihilating for all n [42, 43, 44, 45, 46]. Note that an EB map is automatically completely positive, which means that any EB map is a quantum channel (CPT map).

In this paper, we focus on positive absolutely separating maps $\Phi : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{A}(m|n)$, whose output is always absolutely separable for valid input quantum states. We will denote such maps by $\mathrm{PAS}(m|n)$. Clearly, $\mathrm{PAS}(m|n) \subset \mathrm{PEA}(\mathcal{H}_m|\mathcal{H}_n)$. Absolutely separating channels with respect to partition $m|n$ are the maps $\Phi \in \mathrm{CPT} \cap \mathrm{PAS}(m|n)$. Note that the concept of absolutely separating map can be applied not only to linear positive maps but also to non-linear physical maps originating in measurement procedures, see e.g. [47]. In this paper, however, we restrict to linear maps only.

Let us notice that the application of any positive map $\Phi : \mathcal{S}(\mathcal{H}_n) \mapsto \mathcal{S}(\mathcal{H}_n)$ to a part of composite system cannot result in an absolutely separating map.

Proposition 2. *The map $\Phi \otimes \mathrm{Id}_n$ is not absolutely separating with respect to partition $m|n$ for any positive map $\Phi : \mathcal{S}(\mathcal{H}_m) \mapsto \mathcal{S}(\mathcal{H}_m)$, $n \geqslant 2$.*

Proof. Consider the input state $\varrho_{\mathrm{in}} = \varrho_1 \otimes |\psi\rangle\langle\psi|$, then the output state is $\varrho_{\mathrm{out}} = \Phi[\varrho_1] \otimes |\psi\rangle\langle\psi|$. Spectrum of ϱ_{out} does not satisfy the necessary condition of absolute separability, equation (3), so $\Phi \otimes \mathrm{Id}_n$ is not absolutely separating. $\square$

The physical meaning of proposition 2 is that there exists no local action on a part of quantum system, which would make all outcome quantum states absolutely separable. This is in contrast with separability property since entanglement breaking channels disentangle the part they act on from other subsystems. Proposition 2 means that one-sided quantum noises $\Phi \otimes \mathrm{Id}$ can always be compensated by a proper choice of input state ϱ and unitary operations U in such a way that the outcome state $U(\Phi \otimes \mathrm{Id}[\varrho])U^\dagger$ becomes entangled.

It was emphasized already that the absolutely separable state can be transformed into an entangled one only by non-unitary maps. However, not every non-unitary map is adequate for entanglement restoration. For instance, unital quantum channels cannot result in entangled output for absolutely separable input.

Proposition 3. *Suppose Φ_1 is absolutely separating channel with respect to some (multi)partition and Φ_2 is a unital channel, i.e. $\Phi_2[I] = I$. Then the concatenation $\Phi_2 \circ \Phi_1$ is also absolutely separating with respect to the same partition.*

Proof. From absolute separability of Φ_1 it follows that $\varrho = \Phi_1[\varrho_{\mathrm{in}}]$ is absolutely separable for any input ϱ_{in}. Since the channel Φ_2 is unital, $\Phi_2[\varrho] \prec \varrho$ for any density operator ϱ [48], i.e. the ordered spectrum of $\Phi_2[\varrho]$ is majorized by the ordered spectrum of ϱ, with ϱ being absolutely separable in our case. Thus, the spectrum of the state $\Phi_2 \circ \Phi_1[\varrho_{\mathrm{in}}]$ is majorized by the spectrum of the absolutely separable state and according to Lemma 2.2 in [20] this implies absolute separability of $\Phi_2 \circ \Phi_1[\varrho_{\mathrm{in}}]$. $\square$

There exist such physical maps $\Phi : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$ that are not sensitive to unitary rotations of input states and translate that property to the output states. We will call

the map $\Phi : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$ covariant if

$$\Phi[U \varrho U^\dagger] = U\Phi[\varrho]U^\dagger \tag{11}$$

for all $U \in \mathrm{SU(d)}$. The example of covariant map is the depolarizing channel $\mathcal{D}_q : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$ acting as follows:

$$\mathcal{D}_q[X] = qX + (1-q)\mathrm{tr}[X]\frac{1}{d}I_d, \tag{12}$$

which is completely positive if $q \in [-1/(d^2-1), 1]$.

Proposition 4. *A covariant map $\Phi : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ is absolutely separating with respect to partition $m|n$ if and only if it is entanglement annihilating with respect to partition $\mathcal{H}_m|\mathcal{H}_n$.*

Proof. Suppose Φ is covariant and entanglement annihilating. Since Φ is entanglement annihilating, then the left hand side of equation (11) is separable for all U with respect to partition $\mathcal{H}_m|\mathcal{H}_n$. Due to covariance property it means that $U\Phi[\varrho]U^\dagger \in \mathcal{S}(\mathcal{H}_m|\mathcal{H}_n)$ for all unitary U, i.e. Φ is PAS($m|n$).

Suppose Φ is covariant and absolutely separating with respect to partition $m|n$. Consider pure states $\varrho = |\psi\rangle\langle\psi| \in \mathcal{S}(\mathcal{H}_{mn})$. Since Φ is absolutely separating, the right hand side of equation (11) is separable with respect to a fixed partition $\mathcal{H}_m|\mathcal{H}_n$ for all U. By covariance this implies $\Phi[U|\psi\rangle\langle\psi|U^\dagger] \in \mathcal{S}(\mathcal{H}_m|\mathcal{H}_n)$ for all unitary U, i.e. $\Phi[|\varphi\rangle\langle\varphi|] \in \mathcal{S}(\mathcal{H}_m|\mathcal{H}_n)$ for all pure states $|\varphi\rangle$. Since the set of input states $\mathcal{S}(\mathcal{H}_{mn})$ is convex, it implies that $\Phi[\varrho_{\mathrm{in}}] \in \mathcal{S}(\mathcal{H}_m|\mathcal{H}_n)$ for all input states ϱ_{in}, i.e. Φ is entanglement annihilating with respect to partition $\mathcal{H}_m|\mathcal{H}_n$. $\square$

Example 2. *The depolarizing channel $\mathcal{D}_q : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ is known to be PEA($\mathcal{H}_m|\mathcal{H}_n$) if $q \leqslant \frac{2}{mn+2}$ [12, 24]. Therefore, $\mathcal{D}_q$ is absolutely separating with respect to partition $m|n$ if $q \leqslant \frac{2}{mn+2}$ because $\mathcal{D}_q$ is covariant.*

The following results show the behaviour of absolutely separating maps under tensor product.

Proposition 5. *Suppose $\Phi_1 : \mathcal{S}(\mathcal{H}_{m_1 n_1}) \mapsto \mathcal{S}(\mathcal{H}_{m_1 n_1})$ and $\Phi_2 : \mathcal{S}(\mathcal{H}_{m_2 n_2}) \mapsto \mathcal{S}(\mathcal{H}_{m_2 n_2})$ are such positive maps that $\Phi = \Phi_1 \otimes \Phi_2$ is absolutely separating with respect to partition $m_1 m_2 | n_1 n_2$. Then Φ_1 is PAS($m_1|n_1$) and Φ_2 is PAS($m_2|n_2$).*

Proof. Let $\varrho_{\mathrm{in}} = \varrho_1 \otimes \varrho_2$, where $\varrho_1 \in \mathcal{S}(\mathcal{H}_{m_1 n_1})$ and $\varrho_2 \in \mathcal{S}(\mathcal{H}_{m_2 n_2})$, then

$$U\Phi(\varrho_1 \otimes \varrho_2)U^\dagger = U\Phi_1(\varrho_1) \otimes \Phi_2(\varrho_2)U^\dagger \tag{13}$$

is separable with respect to a specific bipartition $\mathcal{H}_{m_1 m_2}^{AB}|\mathcal{H}_{n_1 n_2}^{CD}$ for any unitary operator U. So the state (13) can be written as

$$U\Phi_1(\varrho_1) \otimes \Phi_2(\varrho_2)U^\dagger = \sum_k p_k \varrho_k^{AB} \otimes \varrho_k^{CD}. \tag{14}$$

Tracing out subsystem BD we get

$$\text{tr}_{BD}\left(\sum_k p_k \varrho_k^{AB} \otimes \varrho_k^{CD}\right) = \sum_k p_k \varrho_k^A \otimes \varrho_k^C, \tag{15}$$

which is separable with respect to bipartition $A|C$. Suppose $U = U_1 \otimes U_2$ in (13), then we obtain that $U_1 \Phi_1(\varrho_1) U_1^\dagger$ is separable with respect to bipartition $A|C$ for all U_1, which means that Φ_1 is PAS$(m_1|n_1)$. By the same line of reasoning, Φ_2 is PAS$(m_2|n_2)$. $\square$

However, even if two maps $\Phi_1 \in$ PAS$(m_1|n_1)$ and $\Phi_2 \in$ PAS$(m_2|n_2)$, the map $\Phi_1 \otimes \Phi_2$ can still be not absolutely separable with respect to partition $m_1 m_2 | n_1 n_2$, which is illustrated by the following example.

Example 3. *Consider a four qubit map $\Phi : \mathcal{S}(\mathcal{H}_{16}) \mapsto \mathcal{S}(\mathcal{H}_{16})$ of the form $\Phi = \mathcal{D}_q \otimes \mathcal{D}_q$, where $\mathcal{D}_q : \mathcal{S}(\mathcal{H}_4) \mapsto \mathcal{S}(\mathcal{H}_4)$ is a two qubit global depolarizing channel given by equation (12). Let $q = \frac{1}{3}$ then $\mathcal{D}_{1/3}$ is absolutely separating with respect to partition $2|2$ by example 2. Despite the fact that both parts of the tensor product $\mathcal{D}_{1/3} \otimes \mathcal{D}_{1/3}$ are absolutely separating with respect to $2|2$, Φ is not absolutely separating with respect to $4|4$. In fact, let $U = \begin{pmatrix} I_7 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & I_7 \end{pmatrix}$ be a 16×16 unitary matrix in the conventional four-qubit basis, $\varrho = (|\psi\rangle\langle\psi|)^{\otimes 2}$, $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, then $U\Phi[\varrho]U^\dagger$ is entangled with respect to partition $\mathcal{H}_4|\mathcal{H}_4$ because the the partially transposed output density matrix $(U\Phi[\varrho]U^\dagger)^\Gamma$ has negative eigenvalue $\lambda < -0.0235$. Thus, $\Phi = \mathcal{D}_{1/3} \otimes \mathcal{D}_{1/3}$ is not absolutely separating with respect to partition $4|4$ even though each $\mathcal{D}_{1/3}$ is absolutely separating with respect to partition $2|2$.*

The practical criterion to detect absolutely separating channels follows from the consideration of norms. Let us recall that for a given linear map Φ and real numbers $1 \le p, q \le \infty$, the induced Schatten superoperator norm [49, 50, 51] of Φ is defined by formula

$$\|\Phi\|_{q \to p} := \sup_X \{\|\Phi[X]\|_p : \|X\|_q = 1\}, \tag{16}$$

where $\|\cdot\|_p$ and $\|\cdot\|_q$ are the Schatten p- and q-norms, i.e. $\|A\|_p = \left[\text{tr}\left((A^\dagger A)^{\frac{p}{2}}\right)\right]^{\frac{1}{p}}$. Physically, in the case $q = 1$ and $p = 2$ equation (16) provides the maximal output purity $(\|\Phi\|_{1 \to 2})^2 = \max_{\varrho \in \mathcal{S}(\mathcal{H})} \text{tr}[(\Phi[\varrho])^2]$.

Proposition 6. *A positive linear map $\Phi : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ is absolutely separating with respect to partition $m|n$ if*

$$(\|\Phi\|_{1 \to 2})^2 \le \frac{1}{mn - 1}. \tag{17}$$

Proof. If (17) holds, then the state $\Phi[\varrho]$ satisfies equation (2) and belongs to the separability ball, i.e. $\Phi[\varrho]$ is absolutely separable with respect to partition $m|n$ for all $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$. $\square$

Proposition 7. *A positive linear map* $\Phi : \mathcal{S}(\mathcal{H}_{2^N}) \mapsto \mathcal{S}(\mathcal{H}_{2^N})$ *is absolutely separating with respect to partition* $\underbrace{2|\ldots|2}_{N \ times}$ *if*

$$(\|\Phi\|_{1\to 2})^2 \leqslant \frac{1}{2^N} \left(1 + \frac{54}{17} 3^{-N} \right). \tag{18}$$

Proof. If (18) holds, then the N-qubit state $\Phi[\varrho]$ satisfies equation (8) and belongs to the full separability ball, i.e. $\Phi[\varrho]$ is absolutely separable with respect to partition $2|\ldots|2$ for all $\varrho \in \mathcal{S}(\mathcal{H}_{2^N})$. $\qquad\square$

A necessary condition for the map $\Phi : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ to be absolutely separating with respect to partition $m|n$ follows from equation (3) which must be satisfied by all output states $\Phi[\varrho]$. If a map has a local structure, $\Phi = \Phi_1 \otimes \Phi_2$, then the output state $\Phi[\varrho_1 \otimes \varrho_2] = \Phi_1[\varrho_1] \otimes \Phi_2[\varrho_2]$ is factorized for factorized input states $\varrho_1 \otimes \varrho_2$.

Proposition 8. *A local map* $\Phi_1 \otimes \Phi_2$ *with* $\Phi_1 : \mathcal{S}(\mathcal{H}_m) \mapsto \mathcal{S}(\mathcal{H}_m)$ *and* $\Phi_2 : \mathcal{S}(\mathcal{H}_n) \mapsto \mathcal{S}(\mathcal{H}_n)$ *is not absolutely separating with respect to partition* $m|n$ *if the image (range) of* Φ_1 *or* Φ_2 *contains a boundary point of* $\mathcal{S}(\mathcal{H}_m)$ *or* $\mathcal{S}(\mathcal{H}_n)$, *respectively.*

Proof. Suppose the image of Φ_1 contains a boundary point of $\mathcal{S}(\mathcal{H}_m)$, i.e. there exists a state ϱ_1 such that $\Phi_1[\varrho_1] \in \partial\mathcal{S}(\mathcal{H}_m)$, then $\Phi_1[\varrho_1] \otimes \Phi_2[\varrho_2]$ is not absolutely separating with respect to partition $m|n$, see the discussion after equation (3). Analogous proof takes place if the image of Φ_2 contains a boundary point of $\mathcal{S}(\mathcal{H}_n)$. $\qquad\square$

Example 4. *Suppose* $\Phi_1 : \mathcal{S}(\mathcal{H}_m) \mapsto \mathcal{S}(\mathcal{H}_m)$ *is an amplitude damping channel [2] and* $\Phi_2 : \mathcal{S}(\mathcal{H}_n) \mapsto \mathcal{S}(\mathcal{H}_n)$ *is an arbitrary channel, then* $\Phi_1 \otimes \Phi_2$ *is not absolutely separating with respect to* $m|n$ *by proposition 8, because* Φ_1 *has a fixed point, which is a pure state.*

Similarly, if the maximal output purity of a positive map is large enough, then it cannot be absolutely separating.

Proposition 9. *A positive linear map* $\Phi : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ *is not absolutely separating with respect to partition* $m|n$ *if*

$$(\|\Phi\|_{1\to 2})^2 > \frac{9}{mn + 8}. \tag{19}$$

Proof. Inequality (19) implies that there exists a state $\varrho \in \mathcal{S}(\mathcal{H}_{mn})$ such that the output state $\Phi[\varrho]$ violates inequality (5), i.e. $\Phi[\varrho]$ is not absolutely separable with respect to partition $m|n$ and the map Φ is not absolutely separating. $\qquad\square$

4. Tensor-stable absolutely separating maps

Suppose a map $\Phi : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$. We will refer to Φ as N-tensor-stable absolutely separating if $\Phi^{\otimes N}$ is absolutely separating with respect to any valid partitions $m|n$, $d^N = mn$, $m, n \geqslant 2$. If Φ is N-tensor-stable absolutely separating for all $N = 1, 2, \ldots$,

then Φ is called tensor-stable absolutely separating. These definitions are inspired by the paper [52], where the stability of positive maps under tensor product was studied.

In what follows we show that all N-tensor-stable absolutely separating maps $\Phi : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$ are close to the tracing map $\mathrm{Tr}[\varrho] = \mathrm{tr}[\varrho]\frac{I_d}{d}$. To quantify such a closeness, one can use either the maximal output purity $(\|\Phi\|_{1\to 2})^2$ or the minimal output entropy [10]:

$$h(\Phi) = \min_{\varrho \in \mathcal{S}(\mathcal{H}_d)} \mathrm{tr}\Big[- \Phi[\varrho] \log \Phi[\varrho] \Big], \tag{20}$$

where log stands for the natural logarithm. Note that $\frac{1}{d} \leqslant (\|\Phi\|_{1\to 2})^2 \leqslant 1$ and $0 \leqslant h(\Phi) \leqslant \log d$. Since $(\|\Phi\|_{1\to 2})^2 = \frac{1}{d}$ and $h(\Phi) = \log d$ if and only if $\Phi = \mathrm{Tr}$, the differences $(\|\Phi\|_{1\to 2})^2 - \frac{1}{d}$ and $\log d - h(\Phi)$ can be interpreted as the measure of closeness between maps Φ and Tr.

Proposition 10. *A map* $\Phi : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$ *is not* N-*tensor-stable absolutely separating if*

$$N > \frac{8}{d(\|\Phi\|_{1\to 2})^2 - 1} + 1 \tag{21}$$

or

$$N > 8 \left(\frac{\log d + 1}{\log d - h(\Phi)} \right)^2 + 1. \tag{22}$$

Proof. Suppose the map $\Phi^{\otimes N}$ and the input state $\varrho^{\otimes N}$, then $\Phi^{\otimes N}[\varrho^{\otimes N}] = (\Phi[\varrho])^{\otimes N}$. Let decreasingly ordered eigenvalues of $\Phi[\varrho]$ be $\lambda_1, \ldots, \lambda_d$, then the decreasingly ordered eigenvalues $\Lambda_1, \ldots, \Lambda_{d^N}$ of $(\Phi[\varrho])^{\otimes N}$ satisfy the following relations:

$$\lambda_1^N = \Lambda_1, \qquad \lambda_1 \lambda_d^{N-1} \geqslant \Lambda_{d^N-2}, \qquad \lambda_1 \lambda_d^{N-1} \geqslant \Lambda_{d^N-1}, \qquad \lambda_1 \lambda_d^{N-1} \geqslant \lambda_d^N = \Lambda_{d^N}. \tag{23}$$

If $\Lambda_1 > \Lambda_{d^N-2} + \Lambda_{d^N-1} + \Lambda_{d^N}$, then $(\Phi[\varrho])^{\otimes N}$ is not absolutely separable with respect to any partition in view of equation (3) and Φ is not N-tensor-stable absolutely separating. On the other hand, inequality $\Lambda_1 > \Lambda_{d^N-2} + \Lambda_{d^N-1} + \Lambda_{d^N}$ follows from the inequalities $\lambda_1^{N-1} > 3\lambda_d^{N-1}$ and $(d\lambda_1)^{N-1} > 3$ because $\frac{1}{d} \geqslant \lambda_d$.

Let ϱ be a state, which maximizes the purity of $\Phi[\varrho]$, then $(\|\Phi\|_{1\to 2})^2 = \sum_{i=1}^d \lambda_i^2$ and $\lambda_1^2 \geqslant \frac{1}{d}(\|\Phi\|_{1\to 2})^2$. Consequently, $(d\lambda_1)^2 \geqslant d(\|\Phi\|_{1\to 2})^2$ and the inequality

$$d(\|\Phi\|_{1\to 2})^2 > 1 + \frac{8}{N-1} > \sqrt[N-1]{9} \tag{24}$$

implies $(d\lambda_1)^{N-1} > 3$. Finally, the first inequality in equation (24) is equivalent to inequality (21) and provides a sufficient condition for the map Φ not to be N-tensor-stable absolutely separable.

Let ϱ be a state, which minimizes the entropy of $\Phi[\varrho]$, then $h(\Phi) = -\sum_{i=1}^d \lambda_i \log \lambda_i$. Denote $T = \|\Phi[\varrho] - \frac{1}{d}I\|_1 = \sum_{i=1}^d |\lambda_i - \frac{1}{d}| < 2$, then $\frac{T}{2} = \sum_{i: \lambda_i \geqslant \frac{1}{d}} (\lambda_i - \frac{1}{d}) \leqslant d(\lambda_1 - \frac{1}{d})$ and $\lambda_1 \geqslant \frac{1}{d}\left(1 + \frac{T}{2}\right)$. Using results of the paper [53], we obtain

$$\log d - h(\Phi) \leqslant T \log d + \min\left(-T \log T, \frac{1}{e} \right) \leqslant T \log d + \sqrt{2T} \leqslant \sqrt{2T}(\log d + 1). \tag{25}$$

Therefore,

$$(d\lambda_1)^{N-1} \geqslant \left(1 + \frac{T}{2}\right)^{N-1} \geqslant 1 + \frac{N-1}{2}T \geqslant 1 + \frac{N-1}{4}\left(\frac{\log d - h(\Phi)}{\log d + 1}\right)^2. \tag{26}$$

If inequality (22) is fulfilled, then the right hand side of equation (26) is greater than 3, which implies $(d\lambda_1)^{N-1} > 3$ and Φ is not N-tensor-stable absolutely separating. $\qquad\square$

If $\Phi \neq \mathrm{Tr}$, then there exists N such that $\Phi^{\otimes N}$ is not absolutely separating. On the contrary, if $\Phi = \mathrm{Tr}$, then $\Phi^{\otimes N}$ is absolutely separating with respect to any partition for all N because $\Phi^{\otimes N}[\widetilde{\varrho}] = \frac{1}{d^N}I_{d^N}$ for all $\widetilde{\varrho}$.

Corollary 1. *A map $\Phi : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$ is tensor-stable absolutely separating if and only if $\Phi = \mathrm{Tr}$.*

Physical interpretation of this result can be also based on the fact that $\varrho^{\otimes N}$ allows Schumacher compression [54], namely, $\varrho^{\otimes N} \approx P \oplus 0 \approx |\psi\rangle\langle\psi| \otimes P$, where P is a projector onto the typical subspace of dimension $\mathrm{e}^{S(\varrho)N}$ and $|\psi\rangle \in \mathcal{H}_{\mathrm{e}^{[\log d - S(\varrho)]N}}$. If $S(\varrho) \neq \log d$, then for sufficiently large number N of identical mixed states ϱ the dimension $\mathrm{e}^{[\log d - S(\varrho)]N}$ exceeds 4, so $|\psi\rangle$ can be transformed into an entangled state $U|\psi\rangle$ by the action of a proper unitary operator U.

5. Specific absolutely separating maps and channels

In this section we focus on particular physical evolutions and transformations, which either describe specific dynamical maps or represent interesting examples of linear state transformations. We characterize the region of parameters, where the map is absolutely separating and find states robust to the loss of property to be not absolutely separable.

5.1. Local depolarizing qubit maps and channels

Let us analyze a map of the form $\mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}$, where

$$\mathcal{D}_q[X] = qX + (1-q)\mathrm{tr}[X]\frac{1}{2}I. \tag{27}$$

Map $\mathcal{D}_q$ is positive for $q \in [-1, 1]$ and completely positive if $q \in [-\frac{1}{3}, 1]$. As absolutely separating maps are the subset of entanglement annihilating maps, it is worth to mention that entanglement-annihilating properties of the map $\mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}$ and their generalizations (acting in higher dimensions) are studied in the papers [12, 13, 24].

Since depolarizing maps are not sensitive to local changes of basis states, we consider a pure input state $|\psi\rangle\langle\psi|$, where $|\psi\rangle$ always adopts the Schmidt decomposition $|\psi\rangle = \sqrt{p}|00\rangle + \sqrt{1-p}|11\rangle$ in the proper local bases. We denote $\varrho_{\mathrm{out}} = \mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}[|\psi\rangle\langle\psi|]$. Using proposition 6, we conclude that $\mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}$ is absolutely separating with respect to partition $2|2$ if $\mathrm{tr}[\varrho_{\mathrm{out}}^2] \leqslant \frac{1}{3}$ for all $p \in [0, 1]$, which reduces to

$$q_1^2 + q_2^2 + q_1^2 q_2^2 \leqslant \frac{1}{3}. \tag{28}$$

Note that equation (28) provides only sufficient condition for absolutely separating maps $\mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}$. The area of parameters q_1, q_2 satisfying equation (28) is depicted in figure 3.

Proposition 11. *Two-qubit local depolarizing map $\mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}$ is absolutely separating with respect to partition 2|2 if and only if*

$$q_1(1 + |q_2|) \leqslant \sqrt{1 - q_1^2}\,(1 - |q_2|) \quad \text{if } q_1 \geqslant q_2, \tag{29}$$

$$q_2(1 + |q_1|) \leqslant \sqrt{1 - q_2^2}\,(1 - |q_1|) \quad \text{if } q_1 \leqslant q_2. \tag{30}$$

Proof. We use equation (1) with $n = 2$ and apply it to all possible output states $\varrho_{\text{out}} = \mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}[|\psi\rangle\langle\psi|]$ with $|\psi\rangle = \sqrt{p}|00\rangle + \sqrt{1-p}|11\rangle$. It is not hard to see that the Schmidt decomposition parameter $p = 0$ or 1 for eigenvalues $\lambda_1, \ldots, \lambda_4$ saturating inequality (1). If $p = 0, 1$, then equation (1) reduces to equations (29)–(30). □

The area of parameters q_1, q_2 satisfying equations (29)–(30) is shown in figure 3. The fact that $p = 0, 1$ in derivation of equations (29)–(30) means that, in the case of local depolarizing noises, the *factorized* states exhibit the most resistance to absolute separability when affected by local depolarizing noises.

If $q_1 = q_2 = q$, then the sufficient condition (28) provides $\mathcal{D}_q \otimes \mathcal{D}_q \in \text{PAS}(2|2)$ if $|q| \leqslant \sqrt{\frac{2}{\sqrt{3}} - 1} \approx 0.3933$, whereas the exact conditions (29)–(30) provide $\mathcal{D}_q \otimes \mathcal{D}_q \in \text{PAS}(2|2)$ if $|q| \leqslant q_* \approx 0.3966$, with q_* being a solution of equation $2q_*^3 - 2q_*^2 + 3q_* - 1 = 0$.

The boundary points of both equation (28) and equations (29)–(30) are $q_1 = \pm\frac{1}{\sqrt{5}}, q_2 = \pm\frac{1}{3}$ and $q_1 = \pm\frac{1}{3}, q_2 = \pm\frac{1}{\sqrt{5}}$. Let us recall that $\mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}$ is entanglement breaking if and only if $|q_1|, |q_2| \leqslant \frac{1}{3}$. Thus, the two qubit map $\mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}$ can be entanglement breaking but not absolutely separating and vice versa. Thus, $\text{PAS}(2|2) \not\subset \text{EB}$ and $\text{EB} \not\subset \text{PAS}(2|2)$. This is related with the fact that factorized states remain separable under the action of local depolarizing channels, but they are the most robust states with respect to preserving the property not to be absolutely separable.

Moreover, $\text{PAS}(2|2) \not\subset \text{CPT}$. In fact, $\mathcal{D}_{q_1} \otimes \mathcal{D}_{q_2}$ is completely positive if and only if $q_1, q_2 \in [-\frac{1}{3}, 1]$, whereas the map $\mathcal{D}_0 \otimes \mathcal{D}_{-1/\sqrt{2}}$ is positive and absolutely separating.

One more interesting feature is related with the fact that $\mathcal{D}_0 \otimes \mathcal{D}_{q_2}$ is not absolutely separating if $q_2 > \frac{1}{\sqrt{2}}$. Physically, even though one of the qubits is totally depolarized in the state $\mathcal{D}_0 \otimes \mathcal{D}_{q_2}[\varrho] = \frac{1}{2}I \otimes \mathcal{D}_{q_2}[\text{tr}_A[\varrho]]$, there exists a unitary operator U (Hamilton dynamics) and a two qubit state ϱ such that $U(\mathcal{D}_0 \otimes \mathcal{D}_{q_2}[\varrho])U^\dagger$ is entangled with respect to $\mathcal{H}_2^A|\mathcal{H}_2^B$ if $q_2 > \frac{1}{\sqrt{2}}$. To overcome absolute separability of the outcome, the initial state ϱ should meet the requirement $q_2^2(\lambda_1 - \lambda_2)^2 > [1 + q_2(2\lambda_1 - 1)][1 + q_2(2\lambda_2 - 1)]$, where λ_1, λ_2 are eigenvalues of the reduced density operator $\text{tr}_A\varrho$. If the state ϱ satisfies this inequality, then one can choose $U = |\psi_1\psi_1\rangle\langle\psi_1\psi_1| + |\psi_2\psi_2\rangle\langle\psi_2\psi_2| + \frac{1}{\sqrt{2}}e^{i\pi/4}(|\psi_1\psi_2\rangle\langle\psi_2\psi_1| + |\psi_2\psi_1\rangle\langle\psi_2\psi_1|) + \frac{1}{\sqrt{2}}e^{-i\pi/4}(|\psi_1\psi_2\rangle\langle\psi_2\psi_1| + |\psi_2\psi_1\rangle\langle\psi_1\psi_2|)$, where $|\psi_1\rangle, |\psi_2\rangle$ are eigenvectors of the reduced density operator $\text{tr}_A\varrho$.
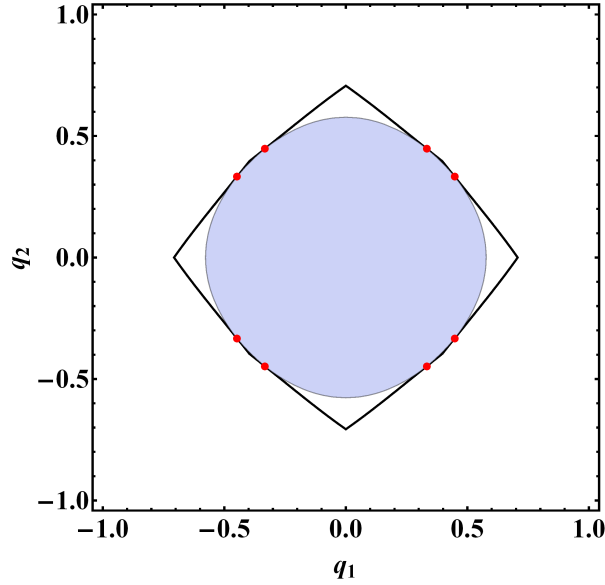
Figure 3. Local depolarizing two-qubit map $\Phi_{q_1} \otimes \Phi_{q_2}$ is absolutely separating with respect to partition $2|2$ for parameters (q_1, q_2) inside the solid line region, equations (29)–(30). The shaded area corresponds to sufficient condition (28). Points of contact between two figures are marked by dots.

Proposition 12. *An N-qubit local depolarizing channel $\mathcal{D}_{q_1} \otimes \ldots \otimes \mathcal{D}_{q_N}$ is absolutely separating with respect to multipartition* $\underbrace{2|\ldots|2}_{N \ times}$ *if*

$$\prod_{k=1}^{N}(1 + q_k^2) \leqslant 1 + \frac{54}{17}3^{-N}. \tag{31}$$

Proof. The channel $\mathcal{D}_{q_1} \otimes \ldots \otimes \mathcal{D}_{q_N}$ satisfies multiplicativity condition of the maximum output purity [55, 56], therefore $(\| \bigotimes_{k=1}^{N} \mathcal{D}_{q_k} \|_{1\to2})^2 = \prod_{k=1}^{N}(\|\mathcal{D}_{q_k}\|_{1\to2})^2 = 2^{-N}\prod_{k=1}^{N}(1 + q_k^2)$. Using proposition 7, we obtain equation (31) guaranteeing the desired absolutely separating property of $\mathcal{D}_{q_1} \otimes \ldots \otimes \mathcal{D}_{q_N}$. $\square$

Example 5. *A local depolarizing channel $\mathcal{D}_q^{\otimes N}$ acting on $N \geqslant 3$ qubits is absolutely separating with respect to multipartition* $\underbrace{2|\ldots|2}_{N \ times}$ *if $q \leqslant \frac{21\sqrt{2}}{17\sqrt{N \cdot 3^N}}$.*

Suppose each qubit experiences the same depolarizing noise, then one can find a condition under which the resulting channel is not absolutely separating with respect to any bipartition.

Proposition 13. *An N-qubit local uniform depolarizing channel $\mathcal{D}_q^{\otimes N}$ is not absolutely separating with respect to any partition $2^k|2^{N-k}$ if*

$$\sqrt{\frac{1 + |q|}{1 - |q|}} > \frac{3 + |q|}{1 + |q|}, \tag{32}$$

or $|q| > \frac{1}{N}$.

Proof. Consider a factorized input state $(|\psi\rangle\langle\psi|)^{\otimes N}$, then decreasingly ordered eigenvalues of $\mathcal{D}_q^{\otimes N}[(|\psi\rangle\langle\psi|)^{\otimes N}]$ are $\lambda_1 = (1+|q|)^N/2^N$, $\lambda_{2^N-2} = \lambda_{2^N-1} = (1-|q|)^{N-1}(1+|q|)/2^N$, and $\lambda_{2^N} = (1-|q|)^N/2^N$. If equation (32) is satisfied, then the necessary condition of absolute separability (3) is violated and $\mathcal{D}_q^{\otimes N}$ is not absolutely separating with respect to any partition $2^k|2^{N-k}$. Condition $|q| > \frac{1}{N}$ implies equation (32) so it serves as a simpler criterion of the absence of absolutely separating property. $\square$

5.2. Local unital qubit maps and channels

In this subsection we consider unital qubit maps $\Upsilon : \mathcal{S}(\mathcal{H}_2) \mapsto \mathcal{S}(\mathcal{H}_2)$, i.e. linear maps preserving maximally mixed state, $\Upsilon[I] = I$. By a proper choice of input and output bases the action of a general unital qubit map reads [57]

$$\Upsilon[X] = \frac{1}{2} \sum_{j=0}^{3} \lambda_j \mathrm{tr}[\sigma_j X] \sigma_j, \tag{33}$$

where $\sigma_0 = I$ and $\{\sigma_i\}_{i=1}^3$ is a conventional set of Pauli operators. In what follows we consider trace preserving maps (33) with $\lambda_0 = 1$.

Consider a local unital map acting on two qubits, $\Upsilon \otimes \Upsilon'$. General properties of such maps are reviewed in [15, 39].

Proposition 14. *The local unital two-qubit map $\Upsilon \otimes \Upsilon'$ is absolutely separating with respect to partition $2|2$ if*

$$\left(1 + \max(\lambda_1^2, \lambda_2^2, \lambda_3^2)\right)\left(1 + \max(\lambda_1'^2, \lambda_2'^2, \lambda_3'^2)\right) \leqslant \frac{4}{3}. \tag{34}$$

Proof. The output purity $\mathrm{tr}\left[(\Upsilon \otimes \Upsilon'[\varrho])^2\right]$ is a convex function of ϱ and achieves its maximum $(\|\Upsilon \otimes \Upsilon'\|_{1\to 2})^2$ at pure states $\varrho = |\psi\rangle\langle\psi|$. The Schmidt decomposition of any pure two-qubit state $|\psi\rangle$ is $|\psi\rangle = \sqrt{p}|\phi \otimes \chi\rangle + \sqrt{1-p}|\phi_\perp \otimes \chi_\perp\rangle$, where $0 \leqslant p \leqslant 1$, $\{|\phi\rangle, |\phi_\perp\rangle\}$ and $\{|\chi\rangle, |\chi_\perp\rangle\}$ are two orthonormal bases. We use the following parametrization by the angles $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$:

$$|\phi\rangle = \begin{pmatrix} \cos(\theta/2)\exp(-\mathrm{i}\phi/2) \\ \sin(\theta/2)\exp(\mathrm{i}\phi/2) \end{pmatrix}, \quad |\phi_\perp\rangle = \begin{pmatrix} -\sin(\theta/2)\exp(-\mathrm{i}\phi/2) \\ \cos(\theta/2)\exp(\mathrm{i}\phi/2) \end{pmatrix}. \tag{35}$$

The basis $\{|\chi\rangle, |\chi_\perp\rangle\}$ is obtained from above formulas by replacing $|\phi\rangle \to |\chi\rangle, |\phi_\perp\rangle \to |\chi_\perp\rangle, \theta \to \theta', \phi \to \phi'$. Thus, any pure input state $\varrho = |\psi\rangle\langle\psi|$ of two qubits can be parameterized by 5 parameters: $p, \theta, \phi, \theta', \phi'$. The pair $\{p, 1-p\}$ is the spectrum of reduced single-qubit density operator.

The map $\Upsilon \otimes \Upsilon'$ transforms $|\psi\rangle\langle\psi|$ into the operator

$$\varrho_{\mathrm{out}}(\lambda_1, \lambda_2, \lambda_3, \lambda_1', \lambda_2', \lambda_3', p, \theta, \phi, \theta', \phi')$$
$$= \frac{1}{4}\Big\{ I \otimes I + (\mathbf{n} \cdot \boldsymbol{\sigma}) \otimes (\mathbf{n}' \cdot \boldsymbol{\sigma}') + (2p-1)[(\mathbf{n} \cdot \boldsymbol{\sigma}) \otimes I + I \otimes (\mathbf{n}' \cdot \boldsymbol{\sigma}')]$$
$$+ 2\sqrt{p(1-p)}[(\mathbf{k} \cdot \boldsymbol{\sigma}) \otimes (\mathbf{k}' \cdot \boldsymbol{\sigma}') + (\mathbf{l} \cdot \boldsymbol{\sigma}) \otimes (\mathbf{l}' \cdot \boldsymbol{\sigma}')]\Big\}, \tag{36}$$

where $(\mathbf{n} \cdot \boldsymbol{\sigma}) = n_1 \sigma_1 + n_2 \sigma_2 + n_3 \sigma_3$ and vectors $\mathbf{n}, \mathbf{k}, \mathbf{l} \in \mathbb{R}^3$ are expressed through parameters of map Υ by formulas

$$\mathbf{n} = (\lambda_1 \cos \phi \sin \theta, \lambda_2 \sin \phi \sin \theta, \lambda_3 \cos \theta), \tag{37}$$

$$\mathbf{k} = (-\lambda_1 \cos \phi \cos \theta, -\lambda_2 \sin \phi \cos \theta, \lambda_3 \sin \theta), \tag{38}$$

$$\mathbf{l} = (\lambda_1 \sin \phi, -\lambda_2 \cos \phi, 0). \tag{39}$$

The vectors $\mathbf{n}', \mathbf{k}', \mathbf{l}'$ are obtained from $\mathbf{n}, \mathbf{k}, \mathbf{l}$, respectively, by replacing $\lambda \to \lambda', \theta \to \theta', \phi \to \phi'$. Maximizing the output purity $\mathrm{tr}[\varrho_{\mathrm{out}}^2]$ over $p \in [0, 1]$, we get

$$\max_{p \in [0,1]} \mathrm{tr}\left[\left(\varrho_{\mathrm{out}}(\lambda_1, \lambda_2, \lambda_3, \lambda_1', \lambda_2', \lambda_3', p, \theta, \phi, \theta', \phi')\right)^2\right]$$
$$= \frac{1}{4}\left(1 + \lambda_3^2 \cos^2 \theta + (\lambda_1^2 \cos^2 \phi + \lambda_2^2 \sin^2 \phi) \sin^2 \theta\right)$$
$$\times \left(1 + \lambda_3'^2 \cos^2 \theta' + (\lambda_1'^2 \cos^2 \phi' + \lambda_2'^2 \sin^2 \phi') \sin^2 \theta'\right), \tag{40}$$

which is achieved at factorized states ($p = 0$ or $p = 1$). Maximizing equation (40) over angles $\theta, \phi, \theta', \phi'$, we get

$$(\|\Upsilon \otimes \Upsilon'\|_{1 \to 2})^2 = \frac{1}{4}\left(1 + \max(\lambda_1^2, \lambda_2^2, \lambda_3^2)\right)\left(1 + \max(\lambda_1'^2, \lambda_2'^2, \lambda_3'^2)\right). \tag{41}$$

By proposition 6, $\Upsilon \otimes \Upsilon'$ is absolutely separating with respect to partition $2|2$ if $(\|\Upsilon \otimes \Upsilon'\|_{1 \to 2})^2 \leqslant \frac{1}{3}$, which implies equation (41). $\square$

As in the case of local depolarizing maps, pure factorized states are the most resistant to absolute separability under action of $\Upsilon \otimes \Upsilon'$. If the map $\Upsilon \otimes \Upsilon'$ were completely positive, one could use the multiplicativity condition for calculation of the maximal output purity [58]. However, in our case the map $\Upsilon \otimes \Upsilon'$ is not necessarily completely positive.

The map $\Upsilon \otimes \Upsilon$ is absolutely separating with respect to partition $2|2$ if $\max(\lambda_1^2, \lambda_2^2, \lambda_3^2) \leqslant \frac{2}{\sqrt{3}} - 1$. The area of parameters $\lambda_1, \lambda_2, \lambda_3$ satisfying this inequality is depicted in figure 4. Clearly, $\Upsilon \otimes \Upsilon$ may be absolutely separating even if it is not completely positive.

Proposition 15. *An N-qubit local unital channel $\Upsilon^{(1)} \otimes \ldots \otimes \Upsilon^{(N)}$ is absolutely separating with respect to multipartition $\underbrace{2|\ldots|2}_{N \ \text{times}}$ if*

$$\prod_{k=1}^{N} \left\{1 + [\max(|\lambda_1^{(k)}|, |\lambda_2^{(k)}|, |\lambda_3^{(k)}|)]^2\right\} \leqslant 1 + \frac{54}{17} 3^{-N}. \tag{42}$$

Proof. The proof follows from the multiplicativity of the maximum output purity [58] and proposition 7. $\square$
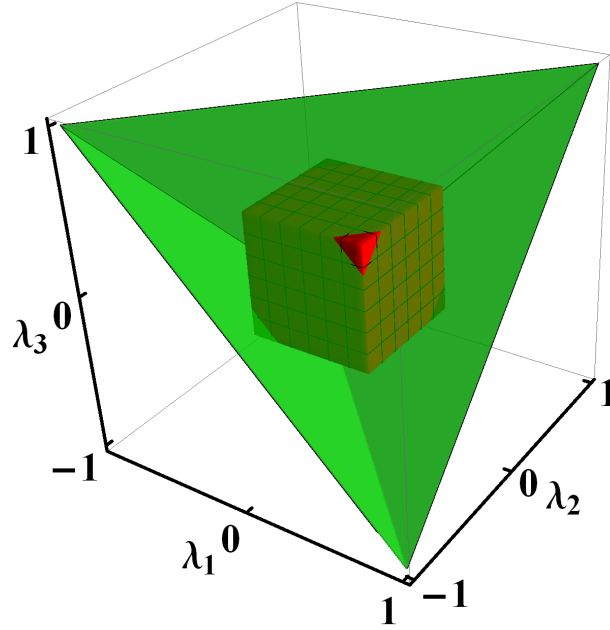
Figure 4. Parameters $\lambda_1, \lambda_2, \lambda_3$ of the Pauli map Υ, where $\Upsilon \otimes \Upsilon$ is completely positive (green tetrahedron) and absolutely separating with respect to partition $2|2$ by proposition 14 (red cube).

5.3. Generalized Pauli channels

The maps considered in previous subsections were local. Let us consider a particular family of non-local maps called generalized Pauli channels or Pauli diagonal channels constant on axes [22]. Suppose an mn-dimensional Hilbert space $\mathcal{H}_{mn}$ and a collection $\mathcal{B}_J = \{|\psi_k^J\rangle\}_{k=1}^{mn}$ of orthonormal bases in $\mathcal{H}_{mn}$. For simplicity denote $d = mn$ and define the operators

$$W_J = \sum_{k=1}^{d} \omega^k |\psi_k^J\rangle\langle\psi_k^J|, \qquad J = 1, 2, \ldots, d+1, \tag{43}$$

where $\omega = e^{i2\pi/d}$. If d is a power of a prime number, then there exist $d+1$ mutually unbiased bases [59]. The corresponding d^2-1 unitary operators $\{W_J^m\}_{m=1,\ldots,d-1,J=1,\ldots,d+1}$ satisfy the orthogonality condition $\mathrm{tr}[(W_J^j)^\dagger W_K^k] = d\delta_{JK}\delta_{jk}$ and, hence, form an orthonormal basis for the subspace of traceless matrices.

A generalized Pauli channel Φ acts on $\varrho \in \mathcal{S}(\mathcal{H}_d)$ as follows:

$$\Phi[\varrho] = \frac{(d-1)s+1}{d}\varrho + \frac{1}{d}\sum_{J=1}^{d+1}\sum_{j=1}^{d-1} t_J W_J^j \varrho (W_J^j)^\dagger. \tag{44}$$

Conditions

$$s + \sum_{J=1}^{d+1} t_J = 1, \quad t_J \geqslant 0, \quad s \geqslant -\frac{1}{d-1} \tag{45}$$

on parameters $s, t_1, \ldots, t_{d+1}$ ensure that Φ is trace preserving and completely positive (Φ is a quantum channel). To analyse absolutely separating properties we use Theorem 27 in [22]: the maximal output purity of Φ is achieved with an axis state, i.e. there exist n and J such that $(\|\Phi\|_{1\to 2})^2 = \mathrm{tr}\left[(\Phi[|\psi_n^J\rangle\langle\psi_n^J|])^2\right]$. On the other side, action of the generalized Pauli channel on an axis state $|\psi_n^J\rangle\langle\psi_n^J|$ reads

$$\Phi[|\psi_n^J\rangle\langle\psi_n^J|] = (1 - s - t_J)\frac{1}{d}I + (s + t_J)|\psi_n^J\rangle\langle\psi_n^J|, \tag{46}$$

whose purity equals $[1 + (d-1)(s+t_J)^2]/d$. Thus, if the obtained purity is less or equal to $(d-1)^{-1}$ for all J, then by proposition 6 Φ is absolutely separating with respect to $m|n$. To conclude, a generalized Pauli channel $\Phi : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ is absolutely separating with respect to partition $m|n$ if $|s + t_J| \leqslant (mn - 1)^{-1}$ for all $J = 1, \ldots, mn + 1$.

5.4. Combination of tracing, identity, and transposition maps

Let us consider a two-parametric family of positive maps $\Phi_{\alpha\beta} : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$ representing linear combinations of tracing map, identity transformation, and transposition $\top$ in a fixed orthonormal basis:

$$\Phi_{\alpha\beta}[X] = \frac{1}{d + \alpha + \beta}\left(\mathrm{tr}[X]\, I + \alpha X + \beta X^\top\right) \tag{47}$$

with real parameters α and β satisfying inequalities $1 + \alpha \geqslant 0$, $1 + \beta \geqslant 0$, and $1 + \alpha + \beta \geqslant 0$ (guaranteeing $\Phi_{\alpha\beta}$ is positive). Note that $\Phi_{\alpha\beta}$ is trace preserving. Equation (47) reduces to the depolarizing map if $\beta = 0$ and to the Werner-Holevo channel [23] if $\alpha = 0$ and $\beta = -1$. A direct calculation of the Choi-Jamiołkowski operator [60, 61] shows that $\Phi_{\alpha\beta}$ is completely positive if $\alpha \geqslant -\frac{1}{d}$ and $-(1 + d\alpha) \leqslant \beta \leqslant 1$.

Suppose $\mathcal{H}_d = \mathcal{H}_m \otimes \mathcal{H}_n$, then we can explore the absolute separability of the output $\Phi_{\alpha\beta}[\varrho]$ with respect to partition $m|n$.

Proposition 16. $\Phi_{\alpha\beta} : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ *is absolutely separating with respect to partition* $m|n$ *if*

$$-1 \leqslant \alpha + \beta \leqslant \frac{mn}{mn - 2} \tag{48}$$

and

$$(\alpha - \beta)^2 + \frac{mn - 3}{mn - 1}\left(\alpha + \beta - \frac{2}{mn - 3}\right)^2 \leqslant \frac{2(mn - 2)}{mn - 3}. \tag{49}$$

Proof. Since $\mathrm{tr}[\varrho] = \mathrm{tr}[\varrho^\top] = 1$ for a density matrix ϱ and $\mathrm{tr}[\varrho^2] = \mathrm{tr}[(\varrho^\top)^2]$, the output purity of the map $\Phi_{\alpha\beta}$ reads $(d + \alpha + \beta)^{-2}\{d + 2(\alpha + \beta) + 2\alpha\beta\,\mathrm{tr}[\varrho\varrho^\top] + (\alpha^2 + \beta^2)\mathrm{tr}[\varrho^2]\}$. If $\alpha\beta \geqslant 0$, then the output purity is maximal when $\mathrm{tr}[\varrho\varrho^\top] = 1$ and $\mathrm{tr}[\varrho^2] = 1$. Substituting the obtained value of the maximum output purity in equation (2), we get equation (48). If $\alpha\beta < 0$, then the output purity is maximal when $\mathrm{tr}[\varrho\varrho^\top] = 0$ and $\mathrm{tr}[\varrho^2] = 1$. If this is the case, equation (2) results in equation (49). Combining two criteria, we see that if both conditions (48) and (49) are fulfilled, then $\Phi_{\alpha\beta}$ is absolutely separating with respect to partition $m|n$ by proposition 6. $\qquad\square$

The region of parameters α, β satisfying equations (48)–(49) is the intersection of a stripe and an ellipse depicted in figure 4.

According to proposition 16 the Werner-Holevo channel $\Phi_{0,-1} : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ is absolutely separating with respect to partition $m|n$ for all $m, n = 2, 3, \ldots$.

If $\beta = 0$, then $\Phi_{\alpha,0} : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ is absolutely separating with respect to partition $m|n$ when $-1 \leqslant \alpha \leqslant \frac{mn}{mn-2}$, which corresponds to the global depolarizing map $\mathcal{D}_q$ with $|q| \leqslant \frac{1}{mn-1}$.

If $m = 2$, one can use a necessary and sufficient condition (1) of absolute separability with respect to partition $2|n$ and apply it to the map (47).

Proposition 17. $\Phi_{\alpha\beta} : \mathcal{S}(\mathcal{H}_{2n}) \mapsto \mathcal{S}(\mathcal{H}_{2n})$ *is absolutely separating with respect to partition* $2|n$ *if and only if (i)* $\alpha, \beta \geqslant 0$ *and* $\alpha + \beta \leqslant 2$; *(ii)* $\alpha \geqslant 0$ *and* $\alpha^2 - 4 \leqslant 4\beta < 0$; *(iii)* $\beta \geqslant 0$ *and* $\beta^2 - 4 \leqslant 4\alpha < 0$; *(iv)* $\alpha, \beta < 0$ *and* $\alpha + \beta \geqslant -1$.

Proof. Since the state space is convex and the map $\Phi_{\alpha\beta}$ is linear, it is enough to check absolute separability of the output $\Phi_{\alpha\beta}[|\psi\rangle\langle\psi|]$ for pure states $|\psi\rangle \in \mathcal{H}_{2n}$ only. Transposition $(|\psi\rangle\langle\psi|)^\top = |\overline{\psi}\rangle\langle\overline{\psi}|$ is equivalent to complex conjugation $|\psi\rangle \mapsto |\overline{\psi}\rangle$ in a fixed basis. Eigenvalues of the operator $I + \alpha|\psi\rangle\langle\psi| + \beta|\overline{\psi}\rangle\langle\overline{\psi}|$ are 1 with degeneracy $2n-2$ and $1 + \frac{1}{2}(\alpha+\beta) \pm \frac{1}{2}\sqrt{(\alpha - \beta)^2 + 4\alpha\beta|\langle\psi|\overline{\psi}\rangle|^2}$. Since $0 \leqslant |\langle\psi|\overline{\psi}\rangle|^2 \leqslant 1$, the largest possible eigenvalue is $\lambda_1 = 1 + \frac{1}{2}(\alpha + \beta) + \frac{1}{2}\max(|\alpha + \beta|, |\alpha - \beta|)$, while the smallest eigenvalue is $\lambda_{2n} = 1 + \frac{1}{2}(\alpha + \beta) - \frac{1}{2}\max(|\alpha + \beta|, |\alpha - \beta|)$, with other eigenvalues being equal to 1. Substituting such a spectrum in equation (1), we get conditions (i)–(iv). $\square$

We depict the region of parameters α, β corresponding to $\Phi_{\alpha,\beta} \in \mathrm{PAS}(2|4)$ in figure 5.

Let us consider a necessary condition of the absolutely separating property of $\Phi_{\alpha\beta}$.

Proposition 18. *Suppose* $\Phi_{\alpha\beta} : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ *is absolutely separating with respect to partition* $m|n$, *then* $\max(|\alpha + \beta|, |\alpha - \beta|) \leqslant 2$.

Proof. If $\Phi_{\alpha\beta} \in \mathrm{PAS}(m|n)$, then equation (3) is to be satisfied for the spectrum of states $\Phi_{\alpha\beta}[\varrho]$. Let us recall that the largest and smallest eigenvalues of the operator $(mn + \alpha + \beta)\Phi_{\alpha\beta}[|\psi\rangle\langle\psi|]$ read $\lambda_1 = 1 + \frac{1}{2}(\alpha + \beta) + \frac{1}{2}\max(|\alpha + \beta|, |\alpha - \beta|)$ and $\lambda_{2n} = 1 + \frac{1}{2}(\alpha + \beta) - \frac{1}{2}\max(|\alpha + \beta|, |\alpha - \beta|)$, respectively. Eigenvalues $\lambda_2 = \ldots = \lambda_{mn-1} = 1$. Substituting $\lambda_1, \lambda_{mn-2}, \lambda_{mn-1}, \lambda_{mn}$ into equation (3), we get $\max(|\alpha + \beta|, |\alpha - \beta|) \leqslant 2$. $\square$

The obtained necessary condition does not depend on m and n and is universal for the maps $\Phi_{\alpha\beta}$.

Finally, by proposition 7, $\Phi_{\alpha\beta} : \mathcal{S}(\mathcal{H}_{2^N}) \mapsto \mathcal{S}(\mathcal{H}_{2^N})$ is absolutely separating with respect to N-partition $2|\ldots|2$ if

$$2^N + 2(\alpha + \beta) + |\alpha\beta| + \alpha\beta + \alpha^2 + \beta^2 \leqslant \frac{(2^N + \alpha + \beta)^2}{2^N}\left(1 + \frac{54}{17}3^{-N}\right). \quad (50)$$

As an example we illustrate the region of parameters α, β, where $\Phi_{\alpha\beta}$ is PAS(2|2|2), see figure 5.
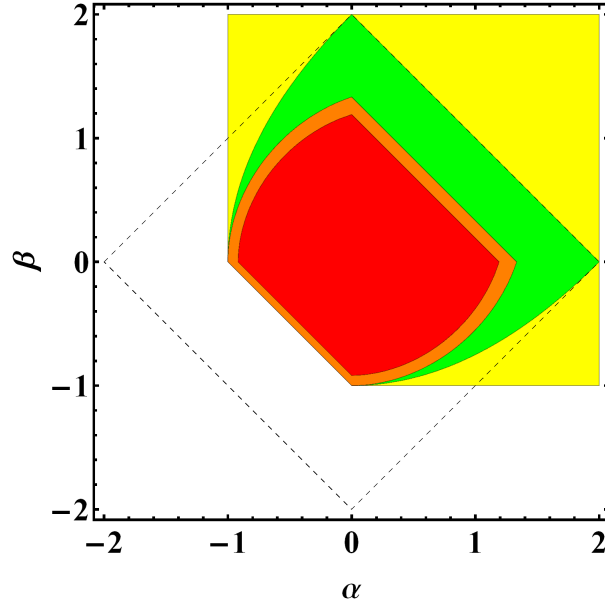
Figure 5. Nested structure of two-parametric maps $\Phi_{\alpha\beta}$: $\mathcal{S}(\mathcal{H}_8) \mapsto \mathcal{S}(\mathcal{H}_8)$. Shaded areas from outer to inner ones: $\Phi_{\alpha\beta}$ is positive, $\Phi_{\alpha\beta}$ is absolutely separating with respect to partition $2|4$ by necessary and sufficient criterion of proposition 17, sufficient condition of absolutely separating property with respect to partition $2|4$ by proposition 16, $\Phi_{\alpha\beta}$ is absolutely separating with respect to multipartition $2|2|2$ by equation (50). The dashed square represents a necessary condition of absolute separating property with respect to any bipartition $m|n$.

5.5. Bipartite depolarizing channel

Suppose a bipartite physical system whose parts are far apart from each other, then the interaction with individual environments leads to local noises, for instance, local depolarization considered in section 5.1. In contrast, if the system is compact enough to interact with the common environment as a whole, the global noise takes place. As an example, the global depolarization is a map $\Phi_{\alpha,0}$ considered in section 5.4. In general, two parts of a composite system AB can be separated in such a way that both global and local noises affect it. Combination of global and local depolarizing maps results in the map $\Phi : \mathcal{S}(\mathcal{H}_m^A \otimes \mathcal{H}_n^B) \mapsto \mathcal{S}(\mathcal{H}_m^A \otimes \mathcal{H}_n^B)$

$$\Phi_{\alpha\beta\gamma}[X] = \frac{I_{mn}\mathrm{tr}[X] + \alpha I_m \otimes \mathrm{tr}_A[X] + \beta \mathrm{tr}_B[X] \otimes I_n + \gamma X}{mn + \alpha m + \beta n + \gamma}, \qquad (51)$$

whose positivity and entanglement annihilating properties were explored in the paper [24].

The output purity reads

$$\mathrm{tr}\left[(\Phi_{\alpha\beta\gamma}[\varrho])^2\right] = (mn + \alpha m + \beta n + \gamma)^{-2}$$
$$\times \left\{ mn + 2(\alpha m + \beta n + \alpha\beta + \gamma) + \gamma^2 \mathrm{tr}[\varrho^2] + (\alpha^2 m + 2\alpha\gamma)\mathrm{tr}[\varrho_B^2] + (\beta^2 n + 2\beta\gamma)\mathrm{tr}[\varrho_A^2] \right\}, \qquad (52)$$

where $\varrho_A = \mathrm{tr}_B \varrho$ and $\varrho_B = \mathrm{tr}_A \varrho$. Also, we have taken into account the fact that $\mathrm{tr}[\varrho(I_m \otimes \varrho_B)] = \mathrm{tr}[\varrho_B^2]$ and $\mathrm{tr}[\varrho(\varrho_A \otimes I_n)] = \mathrm{tr}[\varrho_A^2]$.

Let us recall that $\Phi_{\alpha\beta\gamma}$ is absolutely separating with respect to partition $m|n$ if and only if $\Phi_{\alpha\beta\gamma}[|\psi\rangle\langle\psi|] \in \mathcal{A}(m|n)$ for all pure states $|\psi\rangle \in \mathcal{S}(\mathcal{H}_{mn})$. It means that we can restrict ourselves to the analysis of pure input states $\varrho = |\psi\rangle\langle\psi|$ satisfying $\mathrm{tr}[\varrho^2] = 1$. On the other hand, reduced density operators ϱ_A and ϱ_B have the same spectra if ϱ is pure, therefore $\mathrm{tr}[\varrho_A^2] = \mathrm{tr}[\varrho_B^2] = \mu \in [\frac{1}{\min(m,n)}, 1]$. Thus,

$$\mathrm{tr}\left[(\Phi_{\alpha\beta\gamma}[|\psi\rangle\langle\psi|])^2\right] = (mn + \alpha m + \beta n + \gamma)^{-2}$$
$$\times \left\{ mn + 2(\alpha m + \beta n + \alpha\beta + \gamma) + \gamma^2 + [\alpha^2 m + \beta^2 n + 2\gamma(\alpha + \beta)]\mu \right\}. \tag{53}$$

If $\alpha^2 m + \beta^2 n + 2\gamma(\alpha + \beta) > 0$, then expression (53) achieves its maximum at a factorized state $|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$, when $\mu = 1$. If $\alpha^2 m + \beta^2 n + 2\gamma(\alpha + \beta) < 0$, then expression (53) achieves its maximum at the maximally entangled state $|\psi\rangle = \frac{1}{\sqrt{\min(m,n)}} \sum_{i=1}^{\min(m,n)} |i\rangle \otimes |i\rangle$ with $\mu = \frac{1}{\min(m,n)}$.

Using the explicit form of the maximum output purity (53) and proposition 6, we get the following result.

Proposition 19. $\Phi_{\alpha\beta\gamma} : \mathcal{S}(\mathcal{H}_{mn}) \mapsto \mathcal{S}(\mathcal{H}_{mn})$ is $PAS(m|n)$ if $\alpha^2 m + \beta^2 n + 2\gamma(\alpha + \beta) \geqslant 0$ and

$$(\alpha + \beta + \gamma)^2 + \alpha^2(m - 1) + \beta^2(n - 1) - 1 \leqslant \frac{(\alpha m + \beta n + \gamma + 1)^2}{mn - 1}, \tag{54}$$

or $\alpha^2 m + \beta^2 n + 2\gamma(\alpha + \beta) \leqslant 0$ and

$$\gamma^2 + 2\alpha\beta - 1 + \frac{\alpha^2 m + \beta^2 n + 2\gamma(\alpha + \beta)}{\min(m, n)} \leqslant \frac{(\alpha m + \beta n + \gamma + 1)^2}{mn - 1}. \tag{55}$$

If $m = n$, then equation (55) reduces to $(n^2 - 1)|\gamma| \leqslant |\gamma + n(\alpha + \beta) + n^2|$.

Proposition 20. *Suppose* $\Phi_{\alpha\beta\gamma} \in PAS(m|n)$, *then the decreasingly ordered vectors* $\boldsymbol{\lambda}$ *of the form*

$$(1 + \alpha + \beta + \gamma, \underbrace{1 + \alpha, \dots,}_{\substack{m - 1 \\ times}} \underbrace{1 + \beta, \dots,}_{\substack{n - 1 \\ times}} \underbrace{1, \dots}_{\substack{(m - 1) \\ \times (n - 1) \\ times}})^{\downarrow}, \tag{56}$$

$$(1 + \frac{\alpha + \beta}{\min(m,n)} + \gamma, \underbrace{1 + \frac{\alpha + \beta}{\min(m,n)}, \dots,}_{\substack{[\min(m, n)]^2 - 1 \\ times}} \underbrace{1, \dots}_{\substack{mn \\ -[\min(m, n)]^2 \\ times}})^{\downarrow} \tag{57}$$

must satisfy $\lambda_1^{\downarrow} \leqslant \lambda_{mn-2}^{\downarrow} + \lambda_{mn-1}^{\downarrow} + \lambda_{mn}^{\downarrow}$ *and* $\lambda_{mn}^{\downarrow} \geqslant 0$.

Proof. Equations (56) and (57) are nothing else but the spectra of the operator $I_{mn}\mathrm{tr}[\varrho] + \alpha I_m \otimes \mathrm{tr}_A[\varrho] + \beta \mathrm{tr}_B[\varrho] \otimes I_n + \gamma\varrho$ for the factorized state $\varrho = |\varphi\rangle\langle\varphi| \otimes |\chi\rangle\langle\chi| \in \mathcal{S}(\mathcal{H}_m|\mathcal{H}_n)$ and the maximally entangled state $\varrho = \frac{1}{\min(m,n)} \sum_{i,j=1}^{\min(m,n)} |i\rangle\langle j| \otimes |i\rangle\langle j| \in \mathcal{S}(\mathcal{H}_{mn})$, respectively. Since $\Phi_{\alpha\beta\gamma}[\varrho] \in \mathcal{A}(m|n)$, the spectra of $\Phi_{\alpha\beta\gamma}[\varrho]$ must satisfy equation (3), and so do the spectra (56)–(57) in view of the relation (51). Requirement $\lambda_{mn}^{\downarrow} \geqslant 0$ is merely the positivity requirement for output density operators. $\square$
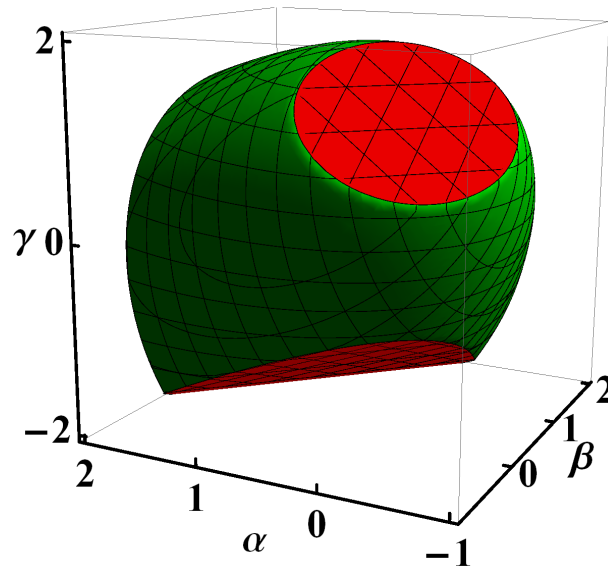
Figure 6. Region of parameters, where $\Phi_{\alpha\beta\gamma} : \mathcal{S}(\mathcal{H}_4) \mapsto \mathcal{S}(\mathcal{H}_4)$ is absolutely separating with respect to partition $2|2$. Plane sections (red) corresponds to maximally entangled input states, convex surface (green) corresponds to factorized input states.

Example 6. *Let $m = n = 2$. Parameters α, β, γ satisfying both propositions 19 and 20 are depicted in figure 6. Note that these sufficient and (separately) necessary conditions do coincide for parameters α, β, γ in the vicinity of the upper plane section in figure 6, with the maximally entangled state being the most resistant to absolute separability. Lower plane section in figure 6 corresponds to positivity condition $\lambda_{mn}^{\downarrow} \geqslant 0$.*

Example 7. *Let $m = n = 3$. Parameters α, β, γ satisfying proposition 19 are depicted by a shaded body in figure 7. Plane sections correspond to maximally entangled states (red), and convex surface (green) corresponds to factorized input states. A polyhedron in figure 7 corresponds to proposition 20. The upper and lower faces of that polyhedron correspond to maximally entangled initial states, and all other faces correspond to factorized input states.*

6. Discussion of state robustness

Let us now summarize observations of the state resistance to absolute separability.

Suppose a dynamical process Φ_t described by a local depolarizing or a unital N-qubit channel, $\bigotimes_{k=1}^{N} \mathcal{D}_{q_k}$ and $\bigotimes_{k=1}^{N} \Upsilon^{(k)}$, with monotonically decreasing parameters $q_k(t)$ or $\lambda_i^{(k)}(t)$, $q_k(0) = \lambda_i^{(k)}(0) = 1$. Then the analysis of sections 5.1 and 5.2 shows that a properly chosen factorized pure initial state $\varrho = \bigotimes_{k=1}^{N} |\psi_k\rangle\langle\psi_k|$ affected by the dynamical map Φ_t remains not absolutely separable for the longer time t as compared to initially entangled states. The matter is that factorized states exhibit a less decrease of purity in this case as compared to entangled states whose purity decreases faster due to the destruction of correlations.
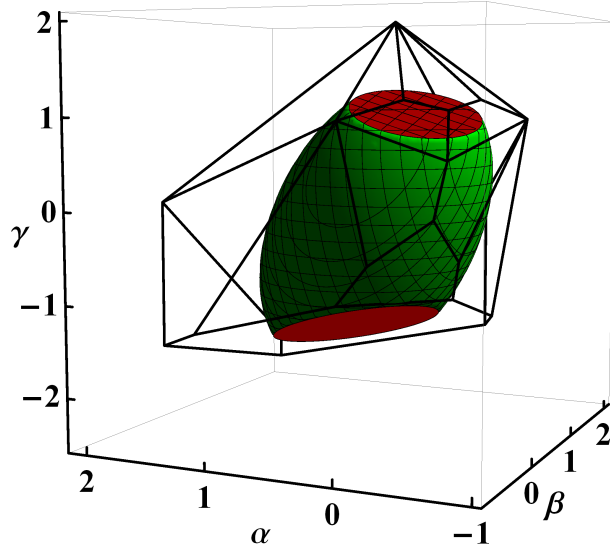
Figure 7. The map $\Phi_{\alpha\beta\gamma} : \mathcal{S}(\mathcal{H}_9) \mapsto \mathcal{S}(\mathcal{H}_9)$ is absolutely separating with respect to partition $3|3$ by proposition 19 for parameters α, β, γ inside the colored region (sufficient condition). Parameters α, β, γ must belong to the polyhedron for $\Phi_{\alpha\beta\gamma} : \mathcal{S}(\mathcal{H}_9) \mapsto \mathcal{S}(\mathcal{H}_9)$ to be absolutely separating with respect to partition $3|3$ (necessary condition).

State robustness is irrelevant to the initial degree of state entanglement in the case of evolution under a linear combination of global tracing, identity, and transposition maps. The only fact that matters is the initial state purity ($\varrho = |\psi\rangle\langle\psi|$) and the overlap with the transposed state ($|\langle\psi|\bar{\psi}\rangle|^2$).

In the case of combined local and global noises (section 5.5), robust states can be either entangled (for dominating global noise) or factorized (for dominating local noise). In fact, domination of the global depolarizing noise corresponds to large values of $\gamma > 0$ and small values of $\alpha, \beta \leqslant 0$, when $\alpha^2 m + \beta^2 n + 2\gamma(\alpha + \beta) \leqslant 0$ and the maximally entangled states exhibits higher output purity than factorized states. On the other hand, for local depolarizing channels $\Phi_{q_1} \otimes \Phi_{q_2}$ we have $\alpha = \frac{nq_2}{1-q_2}$, $\beta = \frac{mq_1}{1-q_1}$, and $\gamma = \frac{mnq_1q_2}{(1-q_1)(1-q_2)}$; a direct calculation yields $\alpha^2 m + \beta^2 n + 2\gamma(\alpha + \beta) > 0$ if $\Phi_{q_1} \otimes \Phi_{q_2}$ is positive, i.e. factorized initial states result in a larger output purity when local noises dominate.

7. Conclusions

In this paper, we have revised the notion of absolutely separable states with respect to bipartitions and multipartitions. In particular, we have found an interesting example of the three-qubit state, which is absolutely separable with respect to partition $2|4$, and consequently is separable with respect to any bipartition $\mathcal{H}_8 = \mathcal{H}_2 \otimes \mathcal{H}_4$, yet is not separable with respect to tripartition $\mathcal{H}_2 \otimes \mathcal{H}_2 \otimes \mathcal{H}_2$.

We have introduced the class of absolutely separating maps and explored their basic properties. This class is a subset of entanglement annihilating maps. Even in the case of local maps, a set of absolutely separating channels is not a subset of entanglement breaking channels. In general, a map can be positive and absolutely separating even if it is not completely positive. We have shown that one-sided channels cannot be absolutely separating, i.e. entanglement of the output state can always be recovered by a proper choice of the input state and the unitary operation applied afterwards. Even if the maps Φ_1 and Φ_2 are absolutely separating with respect to partitions $m_1|n_1$ and $m_2|n_2$, the tensor product $\Phi_1 \otimes \Phi_2$ can still be not absolutely separating with respect to partition $m_1 m_2|n_1 n_2$. Global depolarizing maps are absolutely separating if and only if they are entanglement annihilating.

We have also analyzed N-tensor-stable absolutely separating maps Φ, whose tensor power $\Phi^{\otimes N}$ is absolutely separating with respect to any valid partition. The greater N, the closer an N-tensor-stable absolutely separating map $\Phi : \mathcal{S}(\mathcal{H}_d) \mapsto \mathcal{S}(\mathcal{H}_d)$ to the tracing map $\text{Tr}[\varrho] = \text{tr}[\varrho]\frac{1}{d}I_d$. In fact, the tracing map is the only map that is N-tensor-stable absolutely separating for all N.

Particular characterization of absolutely separating property is fulfilled for specific families of local and global maps. We have fully determined parameters of two-qubit local depolarizing absolutely separating maps PAS(2|2) and provided sufficient conditions for local Pauli maps. The factorized pure states are shown to be the most robust to the loss of property being not absolutely separable under the action of local noises. Global noises are studied by examples of generalized Pauli channels and combination of tracing map, transposition, and identity transformation. Finally, the combination of local and global noises is studied by an example of so-called bipartite depolarizing maps. Robust states are shown to be either entangled or factorized depending on the prevailing noise component: global or local.

Acknowledgments

References

[1] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865
[2] Nielsen M A and Chuang I L 2000, *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[3] Heinosaari T and Ziman M 2012, *The Mathematical Language of Quantum Theory* (Cambridge: Cambridge University Press)

[4] Yurke B and Stoler D 1992 *Phys. Rev. A* **46** 2229

[5] Żukowski M, Zeilinger A, Horne M A and Ekert A K 1993 *Phys. Rev. Lett.* **71** 4287

[6] Pan J-W, Bouwmeester D, Weinfurter H and Zeilinger A 1998 *Phys. Rev. Lett.* **80** 3891

[7] Reiter F, Kastoryano M J and Sørensen A S 2012 *New J. Phys.* **14** 053022

[8] Lin Y, Gaebler J P, Reiter F, Tan T R, Bowler R, Sørensen A S, Leibfried D and Wineland D J 2013 *Nature* **504** 415

[9] Breuer H-P and Petruccione F 2002, *The Theory of Open Quantum Systems* (New York: Oxford University Press)

[10] Holevo A S 2012, *Quantum Systems, Channels, Information* (Berlin: Walter de Gruyter)

[11] Aolita L, de Melo F and Davidovich L 2015 *Rep. Prog. Phys.* **78** 042001

[12] Filippov S N 2014 *J. Russ. Laser Res.* **35** 484

[13] Filippov S N and Ziman M 2013 *Phys. Rev. A* **88** 032316

[14] Moravčíková L and Ziman M 2010 *J. Phys. A: Math. Theor.* **43** 275306

[15] Filippov S N, Rybár T and Ziman M 2012 *Phys. Rev. A* **85** 012303

[16] Kuś M and Zyczkowski K 2001 *Phys. Rev. A* **63** 032307

[17] Verstraete F, Audenaert K and De Moor B 2001 *Phys. Rev. A* **64** 012316

[18] Hildebrand R 2007 *Phys. Rev. A* **76** 052325

[19] Johnston N 2013 *Phys. Rev. A* **88** 062330

[20] Jivulescu M A, Lupa N, Nechita I and Reeb D 2015 *Lin. Algebra Appl.* **469** 276

[21] Arunachalam S, Johnston N and Russo V 2015 *Quantum Information & Computation* **15** 0694

[22] Nathanson M and Ruskai M B 2007 *J. Phys A: Math. Theor.* **40** 8171

[23] Werner R F and Holevo A S 2002 *J. Math. Phys.* **43** 4353

[24] Lami L and Huber M 2016 *J. Math. Phys.* **57** 092201

[25] Werner R F 1989 *Phys. Rev. A* **40** 4277

[26] Amosov G G and Filippov S N 2017 *Quantum Inf. Process.* **16** 2, doi:10.1007/s11128-016-1467-9

[27] Bohnet-Waldraff F, Giraud O and Braun D 2017 *Phys. Rev. A* **95** 012318

[28] Peres A 1996 *Phys. Rev. Lett.* **77** 1413

[29] Horodecki M, Horodecki P, Horodecki R 1996 *Phys. Lett. A* **223** 1

[30] Zyczkowski K, Horodecki P, Sanpera A and Lewenstein M 1998 *Phys. Rev. A* **58** 883

[31] Gurvits L and Barnum H 2002 *Phys. Rev A* **66** 062311

[32] Gurvits L and Barnum H 2005 *Phys. Rev A* **72** 032322

[33] Hildebrand R 2007 *Phys. Rev. A* **75** 062330

[34] Chen X-Y, Jiang L-Z, Yu P and Tian M 2015 *Quantum Inf. Proc.* **14** 2463

[35] Han K H and Kye S-H 2017 *J. Phys. A: Math. Theor.* **50** 145303

[36] Bennett C H, DiVincenzo D P, Mor T, Shor P W, Smolin J A and Terhal B M 1999 *Phys. Rev. Lett.* **82** 5385

[37] DiVincenzo D P, Mor T, Shor P W, Smolin J A and Terhal B M 2003 *Commun. Math. Phys.* **238** 379

[38] Filippov S N, Melnikov A A and Ziman M 2013 *Phys. Rev. A* **88** 062328

[39] Filippov S N and Magadov K Yu 2017 *J. Phys. A: Math. Theor.* **50** 055301

[40] Rivas Á, Huelga S F and Plenio M B 2010 *Phys. Rev. Lett.* **105** 050403

[41] Benatti F, Chruściński D and Filippov S 2017 *Phys. Rev. A* **95** 012112

[42] Holevo A S 1998 *Russ. Math. Surveys* **53** 1295

[43] King C 2002 *J. Math. Phys.* **43** 1247

[44] Shor P W 2002 *J. Math. Phys.* **43** 4334

[45] Ruskai M B 2003 *Rev. Math. Phys.* **15** 643

[46] Horodecki M, Shor P W and Ruskai M B 2003 *Rev. Math. Phys.* **15** 629

[47] Luchnikov I A and Filippov S N 2017 *Phys. Rev. A* **95** 022113

[48] Uhlmann A 1977 *Commun. Math. Phys.* **54** 21

[49] King C and Ruskai M B 2004 *Quantum Inf. Comput.* **4** 500

[50] King C, Nathanson M and Ruskai M B 2005 *Lin. Algebra Appl.* **404** 367

[51] Watrous J 2005 *Quantum Inf. Comput.* **5** 58
[52] Müller-Hermes A, Reeb D and Wolf M M 2016 *J. Math. Phys.* **57** 015202
[53] Fannes M 1973 *Commun. Math. Phys.* **31** 291
[54] Schumacher B 1995 *Phys. Rev. A* **51** 2738
[55] Amosov G G and Holevo A S 2003 *Theor. Prob. Appl.* **47** 123
[56] King C 2003 *IEEE Trans. Inf. Theory* **49** 221
[57] Ruskai M B, Szarek S and Werner E 2002 *Linear Algebra Appl.* **347** 159
[58] King C 2002 *J. Math. Phys.* **43** 4641
[59] Wootters W K and Fields B D 1989 *Annals of Physics* **191** 363
[60] Choi M-D 1975 *Lin. Algebra Appl.* **10** 285
[61] Jamiołkowski A 1972 *Rep. Math. Phys.* **3** 275