Absolute Destruction of Nonlocal Resources

Absolute Destruction of Nonlocal Resources

Tamal Guha,[1,*] Bihalan Bhattacharya,[2] Debarshi Das,[3,†] Some Sankar Bhattacharya,[1,‡] Amit Mukherjee,[4,§] Arup Roy,[1,¶] Kaushiki Mukherjee,[5,**] Nirman Ganguly,[6,††] and A. S. Majumdar[2,‡‡]

[1]*Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B. T. Road, Kolkata 700108, India.*
[2]*S. N. Bose National Centre for Basic Sciences, Block JD, Sector III, Salt Lake, Kolkata 700 098, India*
[3]*Center for Astroparticle Physics and Space Science (CAPSS), Bose Institute, Block EN, Sector V, Salt Lake, Kolkata 700 091, India*
[4]*Optics and Quantum Information Group, The Institute of Mathematical Sciences, C. I. T. Campus, Taramani, Chennai 600113, India.*
[5]*Department of Mathematics, Government Girls General Degree College, Ekbalpore, Kolkata, India.*
[6]*Department of Mathematics, Heritage Institute of Technology, Kolkata -107, India.*

Nonlocal quantum states have significant applications as resources in various quantum informational tasks. Environmental interactions may transform a nonlocal state into a local one with or without steerability. This transformed state may again retrieve its steerability or nonlocality under global unitary interaction. However, in certain cases the nonlocal resource is *absolutely* destroyed, i.e., the transformed state *cannot* repossess its steerability or Bell-nonlocality under arbitrary global unitary action. In the present study, modelling environmental interactions through four particular quantum channels, we investigate how the usefulness of a two qubit nonlocal state as resource can be *absolutely* destroyed. Here the Bell-locality and unsteerability of the transformed two qubit state are studied with respect to a particular local realist inequality and a particular steering inequality, respectively.

I. INTRODUCTION

Nonlocality [1] is one of the key quantum features that enables distinguishing quantum mechanics (QM) from classical mechanics. Quantum nonlocality is generally interpreted as the failure to describe quantum mechanical (QM) correlations arising due to local measurements on space-like separated sub-systems of a composite system by local realist models. Bell-CHSH (Bell-Clauser-Horne-Shimony-Holt) inequalities [2, 3] are used to reveal such incompatibility between QM and local-realism. A state which satisfies the Bell-CHSH inequalities cannot be guaranteed as local, as there may exist other local-realist inequalities that it violates. On the other hand, QM violation of any Bell-CHSH inequality is a signature of quantum nonlocality. However, the complete set of Bell-CHSH inequalities is the necessary and sufficient criteria for local-realism in the $2 \times 2 \times 2$ experimental scenario (2 parties, 2 measurement settings per party, 2 outcomes per mea surement setting). A state is termed as local only if the correlations arising due to performing local measurements on it admit a local hidden variable (LHV) model [4].

The pioneering study by Einstein, Podolsky and Rosen (EPR) [5] demonstrating the incompleteness of the QM description of 'reality' motivated Schrodinger to propose the concept of 'quantum steering' [6]. EPR steering arises in the scenario where local quantum measurements on one part of a bipartite system prepare different ensembles on the other part. This scenario demonstrates EPR steering if these ensembles cannot be explained by a local hidden state (LHS) model [7, 8]. This kind of interpretation of steering has induced great interest in foundational research in recent times as evidenced by a wide range of studies [9–19]. Reid first proposed a criteria for testing EPR-steering in continuous-variable systems based on position-momentum uncertainty relation [20], which was experimentally tested by Ou et al. [21]. Cavalcanti et al. constructed experimental EPR-stee ring criteria based on the assumption of the existence of LHS model [22]. This general construction is applicable to discrete as well as continuous-variable observables and Reid's criterion appears as a special case of this general formulation.

Entanglement [23] marks the primary departure of QM from classical physics, without which Bell-nonlocality or steerability are not possible. The correlations admitting a LHV model can be simulated by states that are not entangled, i. e., by separable states. The inequivalence of entanglement and nonlocality is exemplified by showing the

* g.tamal91@gmail.com
† debarshidas@jcbose.ac.in
‡ somesankar@gmail.com
§ amitisiphys@gmail.com
¶ arup145.roy@gmail.com
** kaushiki˙mukherjee@rediffmail.com
†† nirmanganguly@gmail.com
‡‡ archan@bose.res.in

existence of certain entangled states producing QM correlations that admit a LHV model [24]. Bell-nonlocal states form a strict subset of steerable states which also form a strict subset of entangled states [7, 25].

Apart from being important candidates in foundational studies of QM, quantum nonlocal, steerable and entangled states serve as resource in various quantum tasks, for example, teleportation [26], randomness certification [27–29], cryptography [30] and so on. Motivated by this fact a number of studies have been performed towards revealing 'hidden' nonlocality from quantum states that failed to demonstrate nonlocality under the standard Bell scenario [1]. Local filtering operation is one such procedure [31, 32], which can be broadly classified into two categories (a) performing single local measurement [32] and (b) subjecting the state to suitable sequence of local measurements [31]. Similarly, the issue of revealing of 'hidden' quantum steerability by using local filters has also been studied [25, 33].

In practical scenarios, a state is subjected to ubiquitous environmental interaction, and hence, may lose its entanglement or nonlocal character partially or completely. Thus, it is of considerable interest to study the behaviour of entangled states as well as nonlocal states under local noise. The issue related to entanglement breaking channels which transform an entangled state into a separable one, has acquired a lot of significance as witnessed by a number of studies [34, 35]. On the other hand, studies on nonlocality breaking maps, which transform a nonlocal state into a local one, have also probed the role of environment in destructing quantum resources [36].

A separable state can be transformed into an entangled one when subjected to a global unitary action acting on the composite system. However, it has been shown that there are separable states, dubbed as absolutely separable states, which cannot be transformed into an entangled one under any global unitary interaction [37–40]. Recently, the effect of global unitary interactions on the nonlocality of a state has been probed, with the focus on the Bell-CHSH inequality for two qubit systems. A state initially satisfying the Bell-CHSH inequality can violate it after a global unitary interaction. On the other hand, it has been demonstrated that there are states which preserve their Bell-CHSH local character under arbitrary global unitary action. These states are termed as absolutely Bell-CHSH local states [41, 42]. The question of transforming a separable, but not absolutely separable, state into an absolutely separable one under environmental interac tions is important in practical situations, and has been investigated recently [43]. In the context of absolutely Bell-CHSH local states, the issue of transforming an absolutely Bell-CHSH local state into a nonlocal one has also been presented in a recent study [44]. In a similar spirit, the effect of global unitary interactions on states demonstrating EPR steering has also been studied [45] in the context of steering inequalities derived by Cavalcanti et al. [22]. In particular, the issue of non-violation of the steering inequality with three measurement settings per party [22] by any two qubit system under arbitrary global unitary action has been studied in detail. For our convenience, we will denote the states which preserve their non-violation of the steering inequality with three measurement settings per party derived by Cavalcanti et al. [22] under arbitrary global unitary action as "absolutely 3-settings unsteerable states".

Against the above backdrop the motivation underpinning the present study is to investigate under what kind of environmental interactions a two qubit nonlocal state can be transformed into an absolutely Bell-CHSH local state or an absolutely 3-settings unsteerable state. The significant result revealed by the present study indicates that a nonlocal state subjected to certain environmental interactions cannot be used as a nonlocal (or steerable) resource even after applying global unitary interactions. Hence, the nonlocal states must be protected from such environmental interactions in order to use them as resources for various quantum tasks. A previous study [36] on nonlocality breaking maps shows the transformation of a nonlocal state into a local one, which *can* again retrieve its nonlocality under global unitary actions. The point to be stressed here is that on the other hand, in the present study the transformed local states (with respect to Bell-CHSH inequality) or the transformed unsteerable states (with respect to the steering inequality with three me asurement settings per party derived by Cavalcanti et al. [22]) *cannot* retrieve their nonlocality or steerability under any global unitary action. Here lies the importance of the present study.

The plan of the present paper is as follows. In Section II some preliminary notions required for the present study have been briefly discussed. The single interaction and the sequential interaction protocol used for this study has been presented in Section III. Section IV contains all the results of the present study which is followed by the concluding section.

II. PRELIMINARIES

Let us start with some preliminary ideas required for the present study.

A. Bell-CHSH locality

A bipartite state is said to be Bell-CHSH local if and only if (iff) the correlations obtained by performing local measurements on the two subsystems of the composite state do not violate the Bell-CHSH inequality. The necessary and sufficient criteria for QM violation of the CHSH inequality by arbitrary bipartite qubit states has been established in [46].

An arbitrary qubit state can be expressed in terms of the Hilbert-Schmidt basis as

$$\rho = \frac{1}{4}(\mathbb{I}\otimes\mathbb{I} + \vec{r}.\vec{\sigma}\otimes\mathbb{I} + \mathbb{I}\otimes\vec{s}.\vec{\sigma} + \sum_{i,j=1}^{3} t_{ij}\sigma_i\otimes\sigma_j). \tag{1}$$

Here $\mathbb{I}$ is the identity operator acting on $\mathbb{C}^2$, σ_is are the three Pauli matrices and $\vec{r},\vec{s}$ are vectors in $\mathbb{R}^3$ with norm less than unity. $\vec{r}.\vec{\sigma} = \sum_{i=1}^{3} r_i\sigma_i$. $\vec{s}.\vec{\sigma} = \sum_{i=1}^{3} s_i\sigma_i$. The condition $Tr(\rho^2) \leq 1$ imples

$$\sum_{i=1}^{3}\left(r_i^2 + s_i^2\right) + \sum_{i,j=1}^{3} t_{ij}^2 \leq 3, \tag{2}$$

where the equality is achieved for the pure states. In addition, for being a valid density matrix, ρ has to be positive semidefinite.

Let us consider the matrix $V = TT^t$, where T is the correlation matrix of the state (1) with coeffiecient $t_{ij} = Tr(\rho\sigma_i\otimes\sigma_j)$. u_1, u_2 are two greatest eigenvalues of V. Let us consider the quantity given by,

$$M(\rho) = u_1 + u_2. \tag{3}$$

The state given by Eq.(1) violates the Bell-CHSH inequality iff $M(\rho) > 1$. Hence, the state (1) is Bell-CHSH local iff $M(\rho) \leq 1$.

B. Absolutely Bell-CHSH locality

The concept of absolutely Bell-CHSH local states has recently been introduced in [41]. A Bell-CHSH local quantum state is said to be absolutely Bell-CHSH local if the state remains Bell-CHSH local under the action of any global unitary operation. For a given spectrum, the maximal Bell-CHSH violation is attained at the respective Bell-diagonal state [47]. Therefore, if a Bell-diagonal state is Bell-CHSH local, it is absolutely Bell-CHSH local. Hence, if a_1, a_2, a_3 are the three largest eigenvalues of the given two qubit state ρ taking in descending order, then the state ρ is absolutely Bell-CHSH local *iff* [42]

$$A(\rho) = (2a_1 + 2a_2 - 1)^2 + (2a_1 + 2a_3 - 1)^2 \leq 1. \tag{4}$$

C. Absolutely 3-settings unsteerability

Cavalcanti et al. have provided a series of steering inequalities to certify whether a bipartite state is steerable when each of the two parties are allowed to perform n measurements on his or her part [22]. In particular for $n = 3$, the inequality is given by,

$$F^3 = \frac{1}{\sqrt{3}}\Big|\sum_{i=1}^{3}\langle A_i\otimes B_i\rangle\Big| \leq 1, \tag{5}$$

where, $A_i = \hat{u}_i.\vec{\sigma}$, $B_i = \hat{v}_i.\vec{\sigma}$, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is a vector composed of Pauli matrices, $\hat{u}_i \in \mathbb{R}^3$ are unit vectors, $\hat{v}_i \in \mathbb{R}^3$ are orthonormal vectors. $\langle A_i\otimes B_i\rangle = \mathrm{Tr}(\rho A_i\otimes B_i)$ with $\rho \in \mathcal{B}(\mathcal{H}_A\otimes\mathcal{H}_B)$ is the bipartite quantum system shared between the two parties.

The states, which preserve their non-violation of the steering inequality (5) under arbitrary global unitary action, are called "absolutely 3-settings unsteerable states". A given two qubit state ρ is absolutely 3-settings unsteerable *iff* [45]

$$B(\rho) = 3\ \mathrm{Tr}(\rho^2) - 2(x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4) \leq 1, \tag{6}$$

where x_i are the eigenvalues of the two qubit state ρ.

D. Quantum channels

In practical scenarios it is very hard to isolate a qubit from its environment. Environmental interactions can be represented by different quantum channels. Quantum channels are completely positive trace preserving (CPTP) maps acting on the space of density matrices [48]. Every quantum channel admits the operator sum representation. Let ε be a quantum channel, then its action on the state ρ can be expressed as

$$\varepsilon(\rho) = \sum_{i=0}^{d} K_i \rho K_i^{\dagger}, \tag{7}$$

where K_i's are Krauss operators for the corresponding channel with $\sum_{i=0}^{d} K_i^{\dagger} K_i = \mathbb{I}$ ($\mathbb{I}$ is the identity operator). In the present study we restrict ourselves to four quantum channels, viz. 1) Phase-flip channel , 2) Bit-flip channel, 3) Depolarizing channel, and 4) Phase damping channel.

1. *Phase-flip channel*

The action of the phase-flip channel on the state ρ can be expressed as [48]

$$\varepsilon(\rho) = \sum_{i=0}^{1} K_i \rho K_i^{\dagger}. \tag{8}$$

The Krauss operators for the phase-flip channel are given by

$$K_0 = \sqrt{1 - \frac{p}{2}}\mathbb{I}, K_1 = \sqrt{\frac{p}{2}}\sigma_z,$$

where p is the channel strength with $0 \leq p \leq 1$. It can be checked that $\sum_{i=0}^{1} K_i^{\dagger} K_i = \mathbb{I}$.

2. *Bit-flip channel*

The action of the bit-flip channel on the state ρ can be expressed as [48]

$$\varepsilon(\rho) = \sum_{i=0}^{1} K_i \rho K_i^{\dagger}. \tag{9}$$

The Krauss operators for the bit-flip channel are given by

$$K_0 = \sqrt{1 - \frac{p}{2}}\mathbb{I}, K_1 = \sqrt{\frac{p}{2}}\sigma_x,$$

where p is the channel strength with $0 \leq p \leq 1$.

3. *Depolarizing channel*

The action of the depolarizing channel on the state ρ can be expressed as [48]

$$\varepsilon(\rho) = \sum_{i=0}^{3} K_i \rho K_i^{\dagger}. \tag{10}$$

The Krauss operators for the depolarizing channel are given by

$$K_0 = \sqrt{1 - p}\mathbb{I}, K_1 = \sqrt{\frac{p}{3}}\sigma_x, K_2 = \sqrt{\frac{p}{3}}\sigma_y, K_3 = \sqrt{\frac{p}{3}}\sigma_z,$$

where p is the channel strength with $0 \leq p \leq 1$.

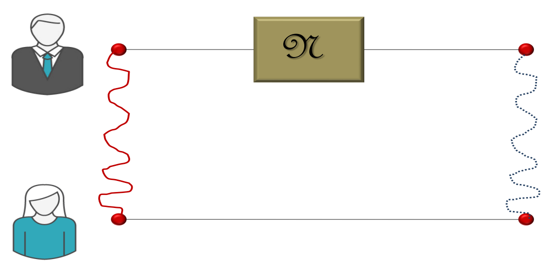

FIG. 1: Single Interaction

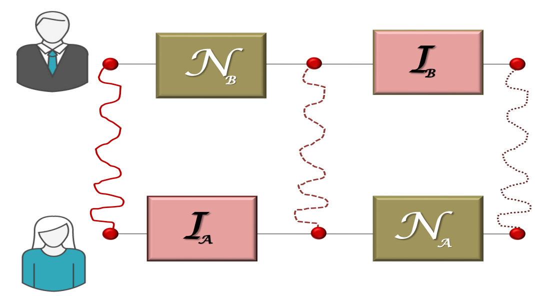

FIG. 2: Sequential Interaction.

4. Phase damping channel

The action of the phase damping channel on the state ρ can be expressed as [48]

$$\varepsilon(\rho) = \sum_{i=0}^{2} K_i \rho K_i^{\dagger}. \tag{11}$$

The Krauss operators for Phase damping channel are given by

$$K_0 = \begin{bmatrix} \sqrt{1-p} & 0 \\ 0 & \sqrt{1-p} \end{bmatrix},\ K_1 = \begin{bmatrix} \sqrt{p} & 0 \\ 0 & 0 \end{bmatrix},\ K_2 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{bmatrix}.$$

Here p is the channel strength with $0 \leq p \leq 1$.

III. PROTOCOL

In order to study the effect of environmental interactions on nonlocal states, we have considered two protocols as follows:

A. Single interaction

Let Alice and Bob hold a bipartite qubit nonlocal quantum state. Bob passes his qubit through a quantum channel of strength p. They finally obtain a family of states which is a function of p. We calculate the range of the channel strengths for which the state becomes Bell-CHSH local, or absolutely Bell-CHSH local, or absolutely 3-settings unsteerable, or state having an LHV model. Figure 1 depicts the above mentioned protocol.

B. Sequential Interaction

Let Alice and Bob hold a bipartite qubit nonlocal quantum state. Bob passes his qubit through a quantum channel of strength p. Then, Alice also applies the same quantum channel with strength p on her qubit and they finally obtain a family of states which is a function of p. In this case also we calculate the range of the channel strengths for which the state becomes Bell-CHSH local, or absolutely Bell-CHSH local, absolutely 3-settings unsteerable, or state having an LHV model. Figure 2 depicts the above mentioned protocol.

IV. RESULTS

In this section we are going to present the results obtained by following the aforementioned two protocols.

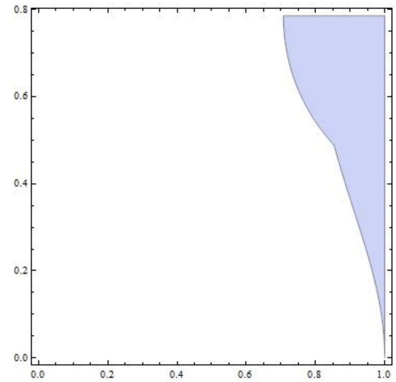

FIG. 3: The horizontal axis represents λ and the vertical axis represents θ. The shaded region depicts the nonlocal region of the state (12)

A. Nonlocal state $\rightarrow$ Bell-CHSH local state, absolutely Bell-CHSH local state and absolutely 3-settings unsteerable state

Let us consider that the following two parameter family of states is shared among two spatially separated parties (say, Alice and Bob),

$$\rho_i(\lambda,\theta) = \begin{bmatrix} \frac{1-\lambda}{2} & 0 & 0 & 0 \\ 0 & \lambda\sin^2\theta & \frac{\lambda}{2}\sin 2\theta & 0 \\ 0 & \frac{\lambda}{2}\sin 2\theta & \lambda\cos^2\theta & 0 \\ 0 & 0 & 0 & \frac{1-\lambda}{2} \end{bmatrix}. \tag{12}$$

Figure 3 depicts the nonlocal region for the family of states given by Eq.(12). let us choose two initial states from the above two parameter family of states, one with $\lambda = 0.95$, $\theta = 0.6$ and another with $\lambda = 0.80$ and $\theta = 0.6$, such that the two initial states are nonlocal. Let us consider that initial states are subjected to the aforementioned single and sequential interactions of different quantum channels with channel strength p. Let R_1, R_2 and R_3 denote the ranges of p for which the resulting states are Bell-CHSH local, absolutely Bell-CHSH local, and absolutely 3-settings unsteerable, respectively.

1. *Phase-flip channel*

Let us consider that the state (12), with $\lambda = 0.95$ and $\theta = 0.6$, is subjected to the aforementioned single interaction of phase-flip channel with channel strength p. The resulting state is given by,

$$\rho_f^{PF1} = \begin{bmatrix} 0.025(1-p)+0.025p & 0 & 0 & 0 \\ 0 & 0.303(1-p)+0.303p & 0.443(1-p)-0.443p & 0 \\ 0 & 0.443(1-p)-0.443p & 0.647(1-p)+0.647p & 0 \\ 0 & 0 & 0 & 0.025(1-p)+0.025p \end{bmatrix}. \tag{13}$$

In Figure 4 we have plotted $M(\rho)$, $A(\rho)$ and $B(\rho)$ for the state ρ_f^{PF1} against different values of p. We note that for $0.2538 \leq p \leq 0.7461$, the state ρ_f^{PF1} is Bell-CHSH local. Hence $R_1 = [0.2538, 0.7461]$. For $0.3490 \leq p \leq 0.6510$, the state ρ_f^{PF1} is absolutely Bell-CHSH local, and hence $R_2 = [0.3490, 0.6510]$. Thus, $R_2 \subset R_1$. Further, for $0.2538 \leq p < 0.3490$ and for $0.6510 < p \leq 0.7461$ the the state ρ_f^{PF1} is Bell-CHSH local, but not absolutely Bell-CHSH local. We also note that there does not exist any channel strength p for which the resulting final state is absolutely 3-settings unsteerable. We also find that for $0.3490 \leq p < 0.4718$ and for $0.5282 < p \leq 0.6510$ the the state

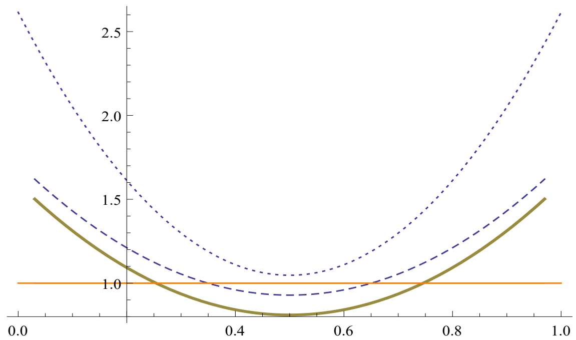

FIG. 4: The thick, dashed, and dotted curves represent $M(\rho)$ versus p, $A(\rho)$ versus p, and $B(\rho)$ versus p, respectively, for the resulting state after single interaction of phase-flip channel with channel strength p is applied on the state (12) with $\lambda = 0.95$ and $\theta = 0.6$.

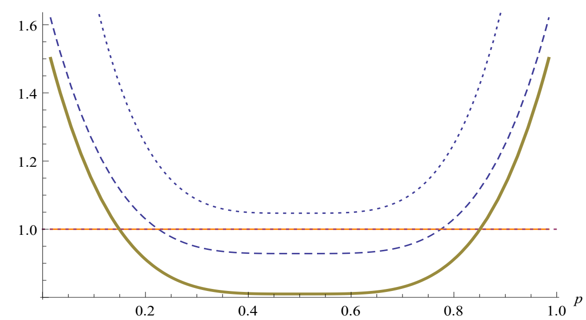

FIG. 5: The thick, dashed, and dotted curves represent $M(\rho)$ versus p, $A(\rho)$ versus p, and $B(\rho)$ versus p, respectively, for the resulting state after sequential interaction of phase-flip channel with channel strength p is applied on the state (12) with $\lambda = 0.95$ and $\theta = 0.6$.

ρ_f^{PF1} is absolutely Bell-CHSH local and entangled, which is checked through the positive partial transpose (PPT) condition [49, 50].

Now, consider that the state (12), with $\lambda = 0.95$ and $\theta = 0.6$, is subjected to the sequential interaction of phase-flip channel with channel strength p. In Figure 5 we have plotted $M(\rho)$, $A(\rho)$ and $B(\rho)$ for the resulting state against different values of p. From this figure it is clear that for $0.1492 \leq p \leq 0.8508$, the resulting state is Bell-CHSH local. Hence, $R_1 = [0.1492, 0.8508]$. For $0.2252 \leq p \leq 0.7747$, the resulting state is absolutely Bell-CHSH local. Hence, $R_2 = [0.2252, 0.7747]$. Thus, $R_2 \subset R_1$. For $0.1492 \leq p < 0.2252$ and for $0.7747 < p \leq 0.8508$ the resulting state is Bell-CHSH local, but not absolutely Bell-CHSH local.We note that there does not exist any channel strength p of phase-flip channel for which the resulting final state after sequential interaction is absolutely 3-settings unsteerable. It is also found that for $0.2252 \leq p < 0.3812$ and for $0.6188 < p \leq 0.7747$ the state is absolutely Bell-CHSH local as well as entangled, which is probed through the PPT criterion [49, 50].

Next, we consider the initial state (12) with $\lambda = 0.80$ and $\theta = 0.6$. Following similar analysis it is found that for the final state after single interaction of phase-flip channel with channel strength p, $R_1 = [0.0258, 0.9742]$, $R_2 = [0.0675, 0.9325]$ and $R_3 = [0.1743, 0.8257]$. On the other hand, for the final state after sequential interaction of phase-flip channel with channel strength p, $R_1 = [0.0131, 0.9869]$, $R_2 = [0.0350, 0.9650]$ and $R_3 = [0.0964, 0.9036]$.

2. *Bit-flip channel*

Let us consider that the state (12), with $\lambda = 0.95$ and $\theta = 0.6$, is subjected to the aforementioned single interaction of the bit-flip channel with channel strength p. In Figure 6 we have plotted $M(\rho)$, $A(\rho)$ and $B(\rho)$ for the resulting state against different values of p. We find that for $0.2445 \leq p \leq 0.7542$ the resulting state is Bell-CHSH local. Therefore, $R_1 = [0.2445, 0.7542]$. For $0.3532 \leq p \leq 0.6443$, the resulting state is absolutely Bell-CHSH local. Therefore, $R_2 = [0.3532, 0.6443]$. For $0.3807 \leq p \leq 0.6193$ the resulting state is absolutely 3-settings unsteerable. Therefore, $R_3 = [0.3807, 0.6193]$. We also note that $R_3 \subset R_2 \subset R_1$. Hence, within the range $0.2445 \leq p < 0.3532$ and $0.6443 < p \leq 0.7542$ the states are Bell-CHSH local but not absolutely Bell-CHSH local. We also note that for $0.5321 < p < 0.6443$ the states are absolutely Bell-CHSH local as well as entangled.

Now the state (12) with $\lambda = 0.95$ and $\theta = 0.6$ is subjected to the aforementioned sequential interaction of bit-flip channel with channel strength p. In Figure 7 we have plotted the $M(\rho)$, $A(\rho)$ and $B(\rho)$ for the resulting state against different values of p. From this figure it is clear that for $0.1378 \leq p \leq 0.8622$ the resulting state is Bell-CHSH local. Hence, $R_1 = [0.1378, 0.8622]$. For $0.1856 \leq p \leq 0.8144$ the resulting state is absolutely Bell-CHSH local. Hence, $R_2 = [0.1856, 0.8144]$. For $0.2256 \leq p \leq 0.7744$ the resulting state is absolutely 3-settings unsteerable. Therefore, $R_3 = [0.2256, 0.7744]$. Thus, $R_3 \subset R_2 \subset R_1$. We also note that for $0.1378 \leq p < 0.1856$ and $0.8144 < p \leq 0.8622$ the resulting state is Bell-CHSH local, but not absolutely Bell-CHSH local. Another important point is that for $0.1856 \leq p < 0.3733$ and $0.6266 < p \leq 0.8144$ the resulting state is absolutely Bell-CHSH local as well as entangled.

Next, we consider the initial state (12) with $\lambda = 0.80$ and $\theta = 0.6$. Following similar analysis it is found that for the final state after single interaction of bit-flip channel with channel strength p, $R_1 = [0.0531, 0.9468]$, $R_2 =$

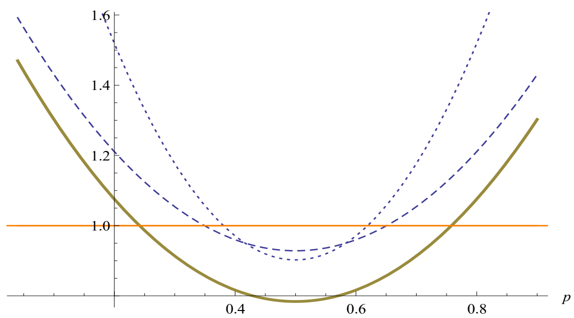

FIG. 6: The thick , dashed, and dotted curves represent $M(\rho)$ versus p, $A(\rho)$ versus p, and $B(\rho)$ versus p, respectively, for the resulting state after single interaction of bit-flip channel with channel strength p is applied on the state (12) with $\lambda = 0.95$ and $\theta = 0.6$.

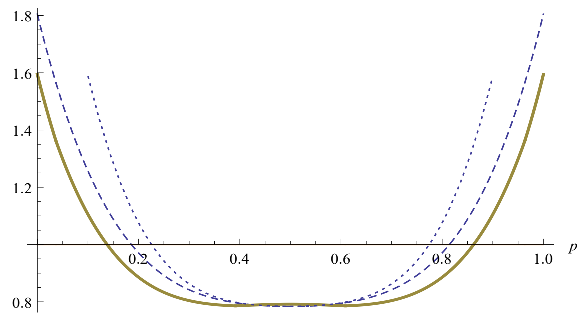

FIG. 7: The thick, dashed, dotted curves represent $M(\rho)$ versus p, $A(\rho)$ versus p and $B(\rho)$ versus p, respectively, for the resulting state after sequential interaction of bit-flip channel with channel strength p is applied on the state (12) with $\lambda = 0.95$ and $\theta = 0.6$.

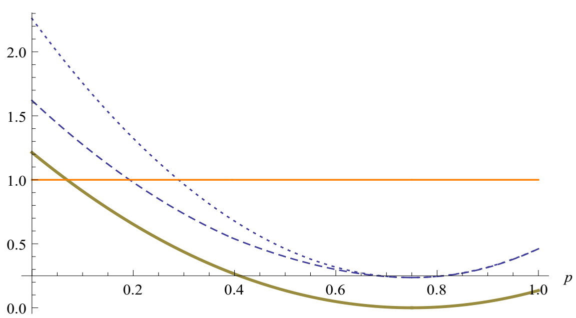

FIG. 8: The thick, dashed, and dotted curves represent $M(\rho)$ versus p, $A(\rho)$ versus p and $B(\rho)$ versus p, respectively, for the resulting state after single interaction of depolarizing channel with channel strength p is applied on the state (12) with $\lambda = 0.95$ and $\theta = 0.6$.

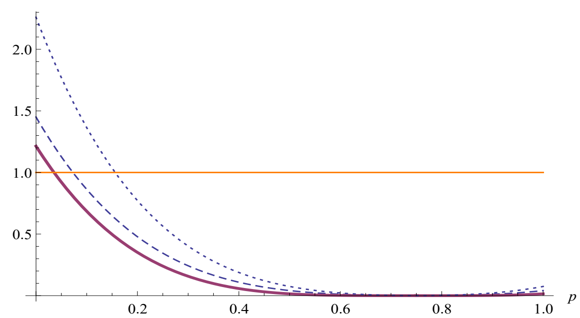

FIG. 9: The thick, dashed, and dotted curves represent $M(\rho)$ versus p, $A(\rho)$ versus p and $B(\rho)$ versus p, respectively, for the resulting state after sequential interaction of depolarizing channel with channel strength p is applied on the state (12) with $\lambda = 0.95$ and $\theta = 0.6$.

$[0.1250, 0.8750]$ and $R_3 = [0.2000, 0.8000]$. On the other hand, for the final state after sequential interaction of bit-flip channel with channel strength p, $R_1 = [0.0273, 0.9727]$, $R_2 = [0.0654, 0.9345]$ and $R_3 = [0.1093, 0.8907]$.

3. Depolarizing Channel

Let us consider that the state (12) with $\lambda = 0.95$ and $\theta = 0.6$ is subjected to the aforementioned single interaction of the depolarizing channel with channel strength p. In Figure 8 we have plotted the $M(\rho)$, $A(\rho)$ and $B(\rho)$ for the resulting state against different values of p. We notice that for $p \geq 0.0685$ the resulting state is Bell-CHSH local. Hence, $R_1 = [0.0685, 1]$. For $p \geq 0.1928$ the resulting state is absolutely Bell-CHSH local. Therefore, $R_2 = [0.1928, 1]$. Hence, for $0.0685 \leq p < 0.1928$ the resulting state is Bell-CHSH local, but not absolutely Bell-CHSH local. We also note that for $p \geq 0.2893$ the resulting state is absolutely 3-settings unsteerable. Hence, $R_3 = [0.2893, 1]$. thus, in this case too $R_3 \subset R_2 \subset R_1$. We also find that for $0.1928 \leq p < 0.4471$ the resulting state is absolutely Bell-CHSH local as well as entangled, probed through the PPT criterion [49, 50].

Next, let us consider that the state (12) with $\lambda = 0.95$ and $\theta = 0.6$ is subjected to the sequential interaction of depolarizing channel with channel strength p. In Figure 9 we have plotted the $M(\rho)$, $A(\rho)$ and $B(\rho)$ for the resulting

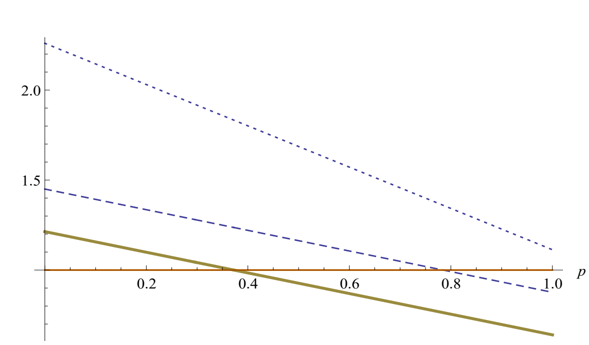

FIG. 10: The thick, dashed, and dotted curves represent $M(\rho)$ versus p, $A(\rho)$ versus p and $B(\rho)$ versus p, respectively, for the resulting state after single interaction of the phase damping channel with channel strength p is applied on the state (12) with $\lambda = 0.95$ and $\theta = 0.6$.

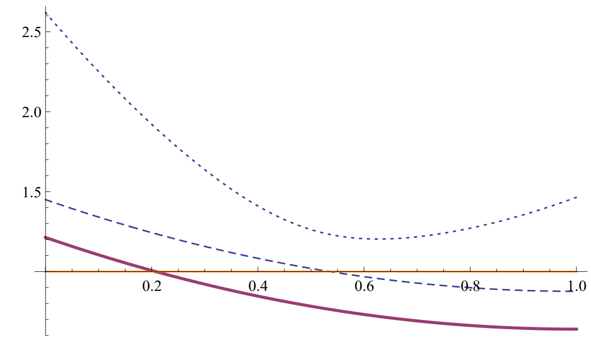

FIG. 11: The thick, dashed, dotted curves represent $M(\rho)$ versus p, $A(\rho)$ versus p and $B(\rho)$ versus p, respectively, for the resulting state after sequential interaction of the phase damping channel with channel strength p is applied on the state (12) with $\lambda = 0.95$ and $\theta = 0.6$.

state against different values of p. We notice that for $p \geq 0.0354$ the resulting state is Bell-CHSH local, which implies that $R_1 = [0.0354, 1]$. We also find that $R_2 = [0.0727, 1]$ as for $p \geq 0.0727$ the resulting state is absolutely Bell-CHSH local. Hence, for $0.0354 \leq p < 0.0727$ the resulting state is Bell-CHSH local, but not absolutely Bell-CHSH local. We note that for $p \geq 0.1560$ the resulting state is absolutely 3-settings unsteerable. Hence, $R_3 = [0.1560, 1]$. Thus, $R_3 \subset R_2 \subset R_1$ again. We have further checked that for $0.0727 \leq p < 0.2570$ the resulting state is absolutely Bell-CHSH local as well as entangled, probed with the help of the PPT criterion [49, 50].

Next we consider the initial state (12) with $\lambda = 0.80$ and $\theta = 0.6$. Following similar analysis it is found that for the final state after single interaction of the depolarizing channel with channel strength p, $R_1 = [0.0692, 1]$, $R_2 = [0.1928, 1]$ and $R_3 = [0.2893, 1]$. On the other hand, for the final state after sequential interaction of the depolarizing channel with channel strength p, $R_1 = [0.0196, 1]$, $R_2 = [0.0481, 1]$ and $R_3 = [0.0922, 1]$.

4. Phase Damping Channel

Let us consider that the state (12) with $\lambda = 0.95$ and $\theta = 0.6$ is subjected to the aforementioned single interaction of the phase damping channel with channel strength p. In Figure 10 we have plotted the $M(\rho)$, $A(\rho)$ and $B(\rho)$ for the resulting state against different values of p. It is observed that for $p \geq 0.3723$ the resulting state is Bell-CHSH local, which implies that $R_1 = [0.3723, 1]$. We have also found that $R_2 = [0.7846, 1]$ as for $p \geq 0.7846$ the resulting state is absolutely Bell-CHSH local. Hence, for $0.3723 \leq p < 0.7846$ the resulting state is Bell-CHSH local, but not absolutely Bell-CHSH local. Hence, in this case $R_2 \subset R_1$. We have checked that there is no channel strength which takes the initial state to absolute 3-settings unsteerable state. We have also checked that for the channel strength $p > 0.9826$ the states are absolutely Bell-CHSH local as well as entangled.

Next, let us consider that the state (12) with $\lambda = 0.95$ and $\theta = 0.6$ is subjected to the sequential interaction of the phase damping channel with channel strength p. In Figure 11 we have plotted the $M(\rho)$, $A(\rho)$ and $B(\rho)$ for the resulting state against different values of p. It is observed that for $p \geq 0.2077$ the resulting state is Bell-CHSH local, which implies that $R_1 = [0.2077, 1]$. We have also found that $R_2 = [0.5359, 1]$ as for $p \geq 0.5359$ the resulting state is absolutely Bell-CHSH local. Hence, for $0.2077 \leq p < 0.5359$ the resulting state is Bell-CHSH local, but not absolutely Bell-CHSH local. Hence, in this case too $R_2 \subset R_1$. We have found that there is no channel strength which takes the initial state to an absolute 3-settings unsteerable state. We have also checked that for the channel strength $p > 0.8679$ the states are absolutely Bell-CHSH local as well as entangled.

Next, we consider the initial state (12) with $\lambda = 0.80$ and $\theta = 0.6$. Following similar analysis it is found that for the final state after single interaction of the phase damping channel with channel strength p, $R_1 = [0.0516, 1]$, $R_2 = [0.1350, 1]$ and $R_3 = [0.3485, 1]$. On the other hand, for the final state after sequential interaction of phase damping channel with channel strength p, $R_1 = [0.0261, 1]$, $R_2 = [0.0699, 1]$ and $R_3 = [0.1928, 1]$.

B. Nonlocal state → abosultely Bell-CHSH local states as well as absolutely 3-settings unsteerable states having LHV models

Let us consider the following two parameter family of bipartite qubit quantum states initially shared between two spatially separated parties (say, Alice and Bob),

$$\rho_i(q,s) = q\Big(s|\phi^+\rangle\langle\phi^+| + (1-s)\frac{\mathbb{I}}{4}\Big) + (1-q)\Big(\frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|\Big), \tag{14}$$

where, $|0\rangle$ and $|1\rangle$ are the eigenstates of the operator σ_z with eigenvalues $+1$ and -1 respectively, $|\phi^+\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)$, $\mathbb{I}$ is the identity operator, $0 \leq q \leq 1$ and $0 \leq s \leq 1$.

1. *Phase damping channel*

Let us choose $q = 0.96$, $s = 0.74$ such that the initial state given by Eq.(14) is nonlocal. In case of the phase damping channel with channel strength p, if this state undergoes single interaction as described earlier, then for $p = p_1 = 0.65$, the state becomes

$$\rho_f = \frac{1}{2}\sigma + \frac{1}{2}\Big(\frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|\Big), \tag{15}$$

where σ is the two qubit isotropic state given by $\sigma = \frac{1}{2}\Big(|\phi^+\rangle\langle\phi^+| + \frac{\mathbb{I}}{4}\Big)$, $|\phi^+\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)$. It has been shown that the correlation produced by the state ρ_f given by Eq.(15) has a local hidden variable-local hidden state (LHV-LHS) model [51]. Hence, the correlation produced by the state ρ_f has a LHV model, since the states having a LHV-LHS model form a subset of the states having a LHV model. It can easily be checked that the state ρ_f is absolutely Bell-CHSH local according to the condition given by (4). The state ρ_f is absolutely 3-settings unsteerable also according to the condition given by (6).

Again, in case of the phase damping channel with channel strength p, if the state $\rho_i(q,s)$ given by Eq.(14) (with $q = 0.96$, $s = 0.74$) undergoes sequantial interaction as described earlier, then for $p = p_2 = 0.41$ the state becomes ρ_f given by Eq.(15).

Hence, here we have presented the transformation of a nonlocal state into an absolutely Bell-CHSH local state (as well as absolutely 3-settings unsteerable state) with an LHV model under single and sequential interaction of the phase damping channel. It is clear that $p_2 < p_1$, which implies that the state $\rho_i(q,s)$ (with $q = 0.96$, $s = 0.74$) can be transformed into an absolutely Bell-CHSH local state (as well as an absolutely 3-settings unsteerable state) having a LHV model under sequential interaction of the phase damping channel with smaller channel strength compared to that under single interaction.

2. *Depolarizing channel*

In this case let us choose $q = 0.34$, $s = 0.97$ such that the initial state given by Eq.(14) is nonlocal. In case of the depolarizing channel with channel strength p, if this state undergoes single interaction as described earlier, then for $p = p_1 = 0.18$ the state becomes ρ_f given by Eq.(15). If the state $\rho_i(q,s)$ given by Eq.(14) (with $q = 0.34$, $s = 0.97$) undergoes sequential interaction of the depolarizing channel as described earlier, then for $p = p_2 = 0.10$ the state becomes ρ_f given by Eq.(15).

Hence, here we have presented the transformation of a nonlocal state into an absolutely Bell-CHSH local state (as well as absolutely 3-settings unsteerable state) having a LHV model under single and sequential interaction of the depolarizing channel. Here also we notice that $p_2 < p_1$, which implies that the state $\rho_i(q,s)$ (with $q = 0.34$, $s = 0.97$) can be transformed into an absolutely Bell-CHSH local state (as well as an absolutely 3-settings unsteerable state) having an LHV model under sequential interaction of the depolarizing channel with smaller channel strength compared to that under single interaction.

V. CONCLUSIONS

Nonlocal as well as steerable quantum states are of potential interest due to their applications as resources in various quantum informational tasks. In practical scenarios, the nonlocal resources are subjected to interaction with the environment, which may transform them into local states with or without steerability. Studies related to nonlocality breaking maps [36] demonstrate this fact by showing the transformation of nonlocal states into local states, which *can* again retrieve their nonlocality under global unitary actions. However, some environmental interactions may transform a nonlocal state into a local one which *cannot* retrieve its nonlocality under any global unitary interactions, or into an unsteerable one which *cannot* retrieve its steerability under any global unitary interactions. Motivated by this fact, we have studied the transformation of nonlocal states into absolutely Bell-CHSH local states [41, 42] and absolutely 3-settings unsteerable states [45] under environmental interactions. Moreover, since non-violation of the Bell-CHSH inequality does not imply that the states have LHV model, we have further investigated conversion of different initial states under noisy channels into states having LHV models.

A comparative analysis of the different protocols used here and the different channels employed leads one to certain relevant observations. Our results confirm the expectation that sequential interactions destroy nonlocal resources in larger regions of the channel parameters than single interactions. This is evident from the fact that the regions R_1, R_2 and R_3 are larger in case of sequential interactions compared to that in case of single interactions for both the initial states and for all the quantum channels considered here. On the other hand, from the present results it is not possible to claim whether any particular channel is more efficient in resource destruction compared to the others. For example, in the case of the first initial state considered (Eq.(12) with $\lambda = 0.95$, $\theta = 0.6$) the parameter range of the depolarizing channel is the largest in the context of nonlocal resource destruction. However, in case of the second initial state considered (Eq.(12) with $\lambda = 0.80$, $\theta = 0.6$) the parameter range of the phase-flip channel is the largest. Hence, the role of different quantum channels in destroying nonlocal resources depends on the initial states.

We have discussed a number of possible ways by which nonlocal resources can be *absolutely* destroyed, i.e., their usefulness as nonlocal resources can never be retrieved under arbitrary unitary interactions. The present study prescribes some specific interactions from which the nonlocal bipartite qubit states must be protected in order to use them as resources in different quantum tasks. The environmental interactions have been modelled through four quantum channels, *viz.* phase-flip channel, bit-flip channel, depolarizing channel and phase damping channel. Before concluding, it may be noted that we have restricted ourselves to bipartite qubit systems. Further useful results may be obtained by generalizations of this study to higher dimensional or multipartite systems. Finally, though in the present study we have considered some specific channels to understand how nonlocal resources can be absolutely destroyed, a full characterization under arbitrary quantum channels is worth probing.

ACKNOWLEGEMENT

We would like to gratefully acknowledge fruitful discussions with Prof. Guruprasad Kar. BB acknowledges support from the Department of Science and Technology (DST), Government of India. DD acknowledges the financial support from University Grants Commission (UGC), Government of India. ASM acknowledges Project no. DST/ICPS/Qust/2018/98 of Department of Science and Technology, Government of India.

[1] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, *Bell nonlocality*, Rev. Mod. Phys. **86**, 419 (2014).
[2] J. S. Bell, *On the Einstein-Podolsky-Rosen paradox*, Physics **1**, 195 (1965).
[3] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Proposed Experiment to Test Local Hidden-Variable Theories*, Phys. Rev. Lett. **23**, 880 (1969).
[4] A. Fine, *Hidden Variables, Joint Probability, and the Bell Inequalities*, Phys. Rev. Lett. **48**, 291 (1982).
[5] A. Einstein, B. Podolsky, and N. Rosen, *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*, Phys. Rev. **47**, 777 (1935).
[6] E. Schrodinger, Proc. Cambridge Philos. Soc. **31**, 553 (1935); **32**, 446 (1936).
[7] H. M. Wiseman, S. J. Jones, and A. C. Doherty, *Steering, Entanglement, Nonlocality, and the Einstein-Podolsky-Rosen Paradox*, Phys. Rev. Lett. **98**, 140402 (2007).
[8] S. J. Jones, H. M. Wiseman, and A. C. Doherty, *Entanglement, Einstein-Podolsky-Rosen correlations, Bell nonlocality, and steering*, Phys. Rev. A. **76**, 052116 (2007).
[9] B. Wittmann, S. Ramelow, F. Steinlechner, N. K. Langford, N. Brunner, H. Wiseman, R. Ursin, and A. Zeilinger, *Loophole-free Einstein-Podolsky-Rosen experiment via quantum steering*, New.J. Phys. **14**, 053030 (2012).
[10] A. Milne, S. Jevtic, D. Jennings, H. Wiseman, and T. Rudolph, *Quantum steering ellipsoids, extremal physical states and monogamy*, New. J. Phys. **16**, 083017 (2014).
[11] S. Jevtic, M. Pusey, D. Jennings, and T. Rudolph, *Quantum Steering Ellipsoids*, Phys. Rev. Lett. **113**, 020402 (2014).
[12] S. Jevtic, M. J. W. Hall, M. R. Anderson, M. Zwierz, and H. M. Wiseman, *Einstein-Podolsky-Rosen steering and the steering ellipsoid*, JOSA B, Vol. 32, Issue 4, pp. A40-A49 (2015).

[13] J. Bowles, T. Vertesi, M. T. Quintino, and N. Brunner, *One-way Einstein-Podolsky-Rosen Steering*, Phys. Rev. Lett. **112**, 200402 (2014).

[14] D. A. Evans, and H. M. Wiseman, *Optimal measurements for tests of Einstein-Podolsky-Rosen steering with no detection loophole using two-qubit Werner states*, Phys. Rev. A. **90**, 012114 (2014).

[15] P. Chowdhury, T. Pramanik, A. S. Majumdar and G. S. Agarwal,??. *Einstein-Podolsky-Rosen steering using quantum correlations in non-Gaussian states*, Phys. Rev. A **89**, 012104 (2014).

[16] T. Pramanik, M. Kaplan and A. S. Majumdar, *Fine-grained EPR steering inequalitites*, Phys. Rev. A **90**, 050305 (R) (2014).

[17] P. Chowdhury, T. Pramanik, A. S. Majumdar, *Stronger steerability criterion for more uncertain continuous variable systems*, Phys. Rev. A **92**, 042317 (2015).

[18] D. Das, S. Datta, C. Jebaratnam, and A. S. Majumdar, *Cost of Einstein-Podolsky-Rosen steering in the context of extremal boxes*, Phys. Rev. A **97**, 022110 (2018).

[19] S. Sasmal, D. Das, S. Mal, and A. S. Majumdar, *Steering a single system sequentially by multiple observers*, arXiv:1712.10227 [quant-ph] (2017).

[20] M. D. Reid, *Demonstration of the Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification*, Phys. Rev. A. **40**, 913 (1989).

[21] Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng, *Realization of the Einstein-Podolsky-Rosen paradox for continuous variables*, Phys. Rev. Lett. **68**, 3663 (1992).

[22] E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid, *Experimental criteria for steering and the Einstein-Podolsky-Rosen paradox*, Phys. Rev. A. **80**, 032112 (2009).

[23] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, *Quantum entanglement*, Rev. Mod. Phys **81**, 865 (2009).

[24] R. F. Werner, *Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model*, Phys. Rev. A **40**, 4277 (1989).

[25] M. T. Quintino, T. Vertesi, D. Cavalcanti, R. Augusiak, M. Demianowicz, A. Acin, and N. Brunner, *Inequivalence of entanglement, steering, and Bell nonlocality for general measurements*, Phys. Rev. A **92**, 032107 (2015).

[26] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels*, Phys. Rev. Lett. **70**, 1895 (1993).

[27] S. Pironio, A. Acin, S. Massar, A. Boyer de la Giroday, D. N. Matsukevich, P. Maunz, S. Olmschenk, D. Hayes, L. Luo, T. A. Manning, and C. Monroe, *Random numbers certified by Bell's theorem*, Nature **464**; 1021 (2010).

[28] R. Colbeck, and R. Renner, *Free randomness can be amplified*, Nat. Phys. **8**, 450 (2012).

[29] A. Chaturvedi and M. Banik, *Measurement-device-independent randomness from local entangled states*, EPL **112**, 30003 (2015).

[30] C.H.Bennett, and G. Brassard, in Proceedings of the IEEE International Conference on Computers, Systems, and Signal Processing, Bangalore, p. 175 (1984).

[31] S. Popescu, *Bell's Inequalities and Density Matrices: Revealing "Hidden" Nonlocality*, Phys. Rev. Lett. **74**, 2619 (1995).

[32] N.Gisin, *Hidden quantum nonlocality revealed by local filters*, Phys. Lett. A. **210**, 151 (1996).

[33] T. Pramanik, Y.-W. Cho, S.-W. Han, S.-Y. Lee, Y.-S. Kim, and S. Moon, *Revealing of hidden quantum steerability using local filtering operations*, arXiv:1707.08285 (2017).

[34] P.W. Shor, *Additivity of the classical capacity of entanglement-breaking quantum channels*, JMP **43**, 4334 (2002).

[35] M. Horodecki, P. W. Shor, and M. B. Ruskai, *Entanglement Breaking Channels*, Rev. Math. Phys. **15**, 629 (2003).

[36] R. Pal, and S. Ghosh, *Non-locality breaking qubit channels: the case for CHSH inequality*, J. Phys. A: Mathematical and Theoretical **48** 155302 (2015).

[37] K. Zyczkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, *Volume of the set of separable states*, Phys. Rev. A **58**, 883 (1998).

[38] F. Verstraete, K. Audenaert, and B. De Moor, *Maximally entangled mixed states of two qubits*, Phys. Rev. A **64**, 012316 (2001).

[39] N. Johnston, *Separability from spectrum for qubit-qudit states*, Phys. Rev. A **88**, 062330 (2013).

[40] N. Ganguly, J. Chatterjee and A. S. Majumdar, *Witness of mixed separable states useful for entanglement creation*, Phys. Rev. A **89**, 052304 (2014).

[41] A. Roy, S. S. Bhattacharya, A. Mukherjee, and N. Ganguly, *Characterization of nonlocal resources under global unitary action*, Quantum Stud.: Math. Found. **5**, 315 (2018).

[42] N. Ganguly, A. Mukherjee, A. Roy, S. S. Bhattacharya, and B. Paul, *Bell-CHSH Violation Under Global Unitary Operations: Necessary and Sufficient conditions*, arXiv:1611.05586 [quant-ph] (2016).

[43] S. N. Filippov, K. Y. Magadov, and M. A. Jivulescu, *Absolutely separating quantum maps and channels*, New J. Phys. **19**, 083010 (2017).

[44] K. Mukherjee, B. Paul, D. Sarkar, A. Mukherjee, S. S. Bhattacharya, A. Roy, and N. Ganguly, *Generation of Nonlocality*, arXiv:1702.07782 [quant-ph] (2017).

[45] S. S. Bhattacharya, A. Mukherjee, A Roy, B. Paul, K. Mukherjee, I. Chakrabarty, C. Jebaratnam, and N. Ganguly, *Absolute non-violation of a three-setting steering inequality by two-qubit states*, Quantum Inf Process **17**, 3 (2018).

[46] R. Horordecki, P. Horordecki, M. Horodecki, *Violating Bell inequality by mixed spin-$\frac{1}{2}$ states: necessary and sufficient condition*, Phys. Lett. A **200**, 340 (1995).

[47] F. Verstraete, and M. M. Wolf, *Entanglement versus Bell Violations and Their Behavior under Local Filtering Operations*, Phys. Rev. Lett. **89**, 170401 (2002).

[48] M. A. Nielsen, and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, The Edinburgh Building, Cambridge CB2 8RU, UK).

[49] A. Peres, *Separability Criterion for Density Matrices*, Phys. Rev. Lett. **77**, 1413 (1996).

[50] M. Horodecki, P. Horodecki, and R. Horodecki, *Separability of mixed states: necessary and sufficient conditions*, Phys. Lett. A **223**, 1 (1996).

[51] J. Bowles, F. Hirsch, M. T. Quintino, and N. Brunner, *Sufficient criterion for guaranteeing that a two-qubit state is unsteerable*, Phys. Rev. A **93**, 022121 (2016).