Contest (1)

.bashrc

2 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g
    -std=c++14 \
  -fsanitize=undefined,address'
```

template.cpp

13 lines

```
#include <bits/stdc++.h>
using namespace std;
#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
#define pb push_back
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;
int main() {
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);
}
```

Data structures (2)

OrderedSet.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null_type.
Time: $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>,
    rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge
      t2 into t
}
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

e62fac, 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] +=
        dif;
  }
  ll query(int pos) { // sum of values in [0, pos
      )
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos
        -1];
    return res;
  }
  int lower_bound(ll sum) {// min pos st sum of
      [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if
        empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= sz(s) && s[pos + pw-1] <
          sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

FenwickTree2D.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
Time: $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" 157f07, 22 lines

```
struct FT2 {
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].
        push_back(y);
  }
  void init() {
    for (vi& v : ys) sort(all(v)), ft.
        emplace_back(sz(v));
  }
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[
        x].begin()); }
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }
};
```

Treap.h

Description: cutting and moving array. everything is [l, r] 0 based indexing.
Usage: `Treap<int> tr(arr);`
Time: $\mathcal{O}(\log N)$

419fcb, 166 lines

```
struct node {
  int prior, val, min1, lazy, size;
  bool rev;
  node *l, *r;
};
typedef node* pnode;

template<class T = int>
class Treap {
public:
  pnode root;
  pnode getnode(T val) {
    pnode t = new node;
    t->l = t->r = NULL;
    t->prior = rand(); t->size = 1; t->rev =
        false;
    t->lazy = 0; t->min1 = t->val = val;
    return t;
  }
  inline int sz(pnode t) { return t ? t->size :
      0;}
  // t may denote same node as l or r, so take
      care of that.
  void combine(pnode &t,pnode l,pnode r) {
    if(!l or !r) return void(t = (l ? l : r));
    t->size = sz(l) + sz(r); t->min1 = min(l->
        min1, r->min1);
```

```
  }
  void operation(pnode t) {
    if(!t) return;
    // reset t;
    t->size = 1; t->min1 = t->val;
    push(t->l); push(t->r);
    // combine
    combine(t, t->l, t); combine(t, t, t->r);
  }
  void push(pnode t) {
    if(!t) return;
    if(t->rev) {
      swap(t->r, t->l);
      if(t->r) t->r->rev = not t->r->rev;
      if(t->l) t->l->rev = not t->l->rev;
      t->rev = false;
    }
    if(t->lazy) {
      t->val += t->lazy;
      t->min1 += t->lazy;
      if(t->r) t->r->lazy += t->lazy;
      if(t->l) t->l->lazy += t->lazy;
      t->lazy = 0;
    }
  }
  // l = [0, pos], r = rest
  void split(pnode t,pnode &l,pnode &r,int pos,
     int add=0) {
    push(t);
    if(!t) return void(l=r=NULL);
    int curr_pos = add + sz(t->l);
    if(pos >= curr_pos) {
      split(t->r,t->r, r, pos, curr_pos + 1);
      l = t;
    } else {
      split(t->l, l, t->l, pos, add);
      r = t;
    }
    operation(t);
  }
  void merge(pnode &t,pnode l,pnode r) {
    push(l); push(r);
    if(!l or !r) return void(t = (l ? l : r));
    if(l->prior > r->prior) {
      merge(l->r, l->r, r);
      t = l;
    } else {
      merge(r->l, l, r->l);
      t = r;
    }
    operation(t);
  }
  void heapify(pnode t) {
    if(!t) return ;
     pnode max = t;
     if (t->l != NULL && t->l->prior > max->
         prior)
         max = t->l;
     if (t->r != NULL && t->r->prior > max->
         prior)
         max = t->r;
     if (max != t) {
         swap (t->prior, max->prior);
         heapify (max);
     }
  }
  // O(n) treap build given array is increasing
  pnode build(T *arr,int n) {
    if(n==0) return NULL;
    int mid = n/2;
    pnode t = getnode(arr[mid]);
    t->l = build(arr, mid);
    t->r = build(arr + mid + 1, n - mid - 1);
    heapify(t); operation(t);
    return t;
  }
  Treap(vector<T> &arr) {
    root = NULL;
    for(int i=0;i<arr.size();i++) {
      T c = arr[i];
      merge(root, root, getnode(c));
    }
  }
  void add(int l,int r,T d) {
    if(l>r) return;
    pnode L, mid, R;
    split(root, L, mid, l-1); split(mid, mid, R,
       r-l);
    if(mid) {
      mid->lazy += d;
    }
    merge(L, L, mid); merge(root, L, R);
  }
  void reverse(int l,int r) {
    if(l>r) return;
    pnode L, mid, R;
    split(root, L, mid, l-1); split(mid, mid, R,
       r-l);
    if(mid) {
      mid->rev = not mid->rev;
    }
    merge(R, mid, R); merge(root, L, R);
  }
  void revolve(int l,int r,int cnt) {
    if(cnt<=0 or l>r) return;
    int len = r - l + 1;
    // cnt = len => no rotation;
    cnt %= len;
    if(cnt == 0) return;
    // pick cnt elements from the end  // => (len
         - cnt) from front
    int mid = l + (len - cnt) - 1; pnode L, Range
       , R;
    split(root, L, Range, l-1); split(Range,
       Range, R, r - l);
    pnode first, second;
    split(Range, first, second, (len-cnt-1));
    merge(Range, second, first);
    merge(L, L, Range); merge(root, L, R);
  }
  void insert(int after,T val) {
    pnode L, R; split(root, L, R, after);
    merge(L, L, getnode(val)); merge(root, L, R);
  }
  void del(int pos) {
    pnode L, mid, R;
    split(root, L, mid, pos-1); split(mid, mid, R
       , 0);
    if(mid) {
      delete mid;
    }
    merge(root, L, R);
  }
  T range_min(int l,int r) {
    pnode L, mid, R;
    split(root, L, mid, l-1); split(mid, mid, R,
       r-l);
    push(mid); T ans = mid->min1;
    merge(L, L, mid); merge(root, L, R);
    return ans;
  }
  void inorder(pnode curr) {
    push(curr); if(!curr) return;
    inorder(curr->l); cerr<<curr->val<<" ";
       inorder(curr->r);
  }
  int query(int pos) {
    pnode l, mid, r;
    split(root, l, mid, pos-1); split(mid, mid, r
       , 0);
```

```
    int ans = mid->val;
    merge(l, l, mid); merge(root, l, r);
    return ans;
  }
};
```

SQRT.h

Description: Square Root Decomposition
Time: Amul Knows

976251, 65 lines

```
const int N = 1e5 + 13, Q = 1e5 + 13, B = 500;
int S[N/B + 13][B + 13], len[N/B + 13], prv[N],
    nxt[N], st[N/B + 13], en[N/B + 13], A[N];
map<int,set<int>> pos; int n, q;

void add_link(int p,int val) {
    nxt[p] = val; prv[val] = p;
    if(p < 1 or p > n) return;
    int b = p / B;
    for(int i = st[b]; i <= en[b]; i++) {
        S[b][i - st[b] + 1] = nxt[i];
    }
    sort(S[b] + 1, S[b] + len[b] + 1);
}
// set A_x = y
void point_update(int x,int y) {
    // update the original link
    add_link(prv[x], nxt[x]); pos[A[x]].erase(x);
    // insert new links
    A[x] = y; pos[A[x]].insert(x);
    int pr = 0, nx = n + 1;
    if(*pos[A[x]].begin() != x) pr = *prev(pos[A[
        x]].find(x));
    if(*pos[A[x]].rbegin() != x) nx = *next(pos[A
        [x]].find(x));
    add_link(pr, x); add_link(x, nx);
}

int query_block(int s,int e,int k) {
    int ans = 0;
    for(int i = s; i <= e; i++)
        ans += ((S[i] + len[i] + 1) - upper_bound
            (S[i] + 1, S[i] + len[i] + 1, k));
    return ans;
}

int query_elements(int s,int e,int k) {
    int ans = 0;
    for(int i = s; i <= e; i++)
        ans += (nxt[i] > k);
    return ans;
}

int range_query(int l,int r) {
    int lb = l / B, rb = r / B;
    if(lb == rb) return query_elements(l, r, r);
    return query_elements(l, en[lb], r)
        + query_block(lb + 1, rb - 1, r)
        + query_elements(st[rb], r, r);
}
for(int i = 1; i <= n; i++) {
    nxt[i] = n + 1;
    if(!pos[A[i]].empty()) {
        prv[i] = *pos[A[i]].rbegin();
        nxt[prv[i]] = i;
    }
    pos[A[i]].insert(i);
}
for(int i = 1; i <= n; i++) {
    int b = i / B;
    if(!len[b])
        st[b] = i;
    en[b] = i;
    len[b]++;
    S[b][len[b]] = nxt[i];
}
for(int i = 0; i <= n/B; i++) {
    sort(S[i] + 1, S[i] + len[i] + 1);
}
```

LazyDynamicSegTree.h

Description: Segment Tree based on large [L, R] range (includes range updates)
Time: $\mathcal{O}(\log(R-L))$ in addition and deletion

391dcb, 31 lines

```
using T=ll; using U=ll;   // exclusive right
    bounds
T t_id; U u_id; // t_id: total (normal), u_id:
    lazy (default)
T op(T a, T b){ return a+b; }
void join(U &a, U b){ a+=b; }
void apply(T &t, U u, int x){ t+=x*u; }
T part(T t, int r, int p){ return t/r*p; }
struct DynamicSegmentTree {
  struct Node { int l, r, lc, rc; T t; U u;
    Node(int l, int r):l(l),r(r),lc(-1),rc(-1),t(
        t_id),u(u_id){}
  };
  vector<Node> tree;
  DynamicSegmentTree(int N) { tree.push_back({0,N
      }); }
  void push(Node &n, U u){ apply(n.t, u, n.r-n.l)
      ; join(n.u,u); }
  void push(Node &n){push(tree[n.lc],n.u);push(
      tree[n.rc],n.u);n.u=u_id;}
  T query(int l, int r, int i = 0) { auto &n =
      tree[i];
    if(r <= n.l || n.r <= l) return t_id;
    if(l <= n.l && n.r <= r) return n.t;
    if(n.lc < 0) return part(n.t, n.r-n.l, min(n.
        r,r)-max(n.l,l));
    return push(n), op(query(l,r,n.lc),query(l,r,
        n.rc));
  }
  void update(int l, int r, U u, int i = 0) {
      auto &n = tree[i];
    if(r <= n.l || n.r <= l) return;
    if(l <= n.l && n.r <= r) return push(n,u);
    if(n.lc < 0) { int m = (n.l + n.r) / 2;
      n.lc = tree.size();       n.rc = n.lc+1;
      tree.push_back({tree[i].l, m}); tree.
          push_back({m, tree[i].r});
    }
    push(tree[i]); update(l,r,u,tree[i].lc);
        update(l,r,u,tree[i].rc);
    tree[i].t = op(tree[tree[i].lc].t, tree[tree[
        i].rc].t);
  }
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k
      < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b
      )
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
```

```
    if (x->k == y->k) x->p = x->m > y->m ? inf :
        -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y
        = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p
        )
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

Graph (3)

Dinic.h

Description: Complexity: (1) O(V^2E): General (2) O(Flow$E$): General (3) O($E\sqrt{V}$): when sum of edge capacities is O(n), we can treat edge with weight x as x edges with weight 1. (4) O($EV\log(Flow)$): Dinics with scaling

a1ff6e, 61 lines

```
const int INF = 1e9 + 13;
template<class T = long long>
class Dinic {
  // {to: to, rev: reverse_edge_id, c: cap, oc:
      original cap}
  struct Edge {
    int to, rev;
    T c, oc;
    T flow() { return max(oc - c, (T)0); } // if
        you need flows
  };
  int N;
  vector<int> lvl, ptr, q; vector<vector<Edge>>
      adj;
public:
  vector<vector<T>> Flow;
  Dinic(int n) {
    N = n; Flow.assign(n, vector<T>(n, (T)0));
    lvl.resize(n); adj.resize(n); ptr.resize(n);
        q.resize(n);
  }
  // automatically adds a reversed edge
  void addEdge(int a, int b, T c, T rcap = 0) {
    adj[a].push_back({b, sz(adj[b]), c, c});
    adj[b].push_back({a, sz(adj[a]) - 1, rcap,
        rcap});
  }
  T dfs(int v, int t, T f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < sz(adj[v]); i++) {
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (T p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;
        }
    }
    return 0;
  }
  T calc(int s, int t) {
    T flow = 0; q[0] = s;
    // bfs part, setting the lvl here
    for(int L = 0; L < 31; L++) do { // 'int L
        =30' maybe faster for random data
      lvl = ptr = vector<int>(sz(q));
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {
        int v = q[qi++];
        for (Edge e : adj[v])
          if (!lvl[e.to] && e.c >> (30 - L))
            q[qe++] = e.to, lvl[e.to] = lvl[v] +
                1;
      }
      // dfs part, setting ptr and checking for a
          path.
      while (T p = dfs(s, t, INF)) flow += p;
    } while (lvl[t]);
    return flow;
  }
  bool leftOfMinCut(int a) { return lvl[a] != 0;
      }
  void buildFlow() {
    for(int i=0;i<N;i++) {
      for(auto e : adj[i]) {
        int j = e.to;
        Flow[i][j] = e.flow();
      }
    }
  }
};
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}};
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i};
  rep(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with
        prio. queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();
      rep(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
}
```

MinCostMaxFlow.h

Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: Approximately $\mathcal{O}\left(E^2\right)$

fe85cc, 81 lines

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;
typedef vector<ll> VL;

struct MCMF {
  int N;
  vector<vi> ed, red;
  vector<VL> cap, flow, cost;
  vi seen;
```

```
  VL dist, pi;
  vector<pii> par;

  MCMF(int N) :
    N(N), ed(N), red(N), cap(N, VL(N)), flow(cap)
        , cost(cap),
    seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost)
      {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
    ed[from].push_back(to);
    red[to].push_back(from);
  }

  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});

    auto relax = [&](int i, ll cap, ll cost, int
        dir) {
      ll val = di - pi[i] + cost;
      if (cap && val < dist[i]) {
        dist[i] = val;
        par[i] = {s, dir};
        if (its[i] == q.end()) its[i] = q.push({-
            dist[i], i});
        else q.modify(its[i], {-dist[i], i});
      }
    };

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (int i : ed[s]) if (!seen[i])
        relax(i, cap[s][i] - flow[s][i], cost[s][
            i], 1);
      for (int i : red[s]) if (!seen[i])
        relax(i, flow[i][s], -cost[i][s], 0);
    }
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (int p,r,x = t; tie(p,r) = par[x], x !=
          s; x = p)
        fl = min(fl, r ? cap[p][x] - flow[p][x] :
            flow[x][p]);
      totflow += fl;
      for (int p,r,x = t; tie(p,r) = par[x], x !=
          s; x = p)
        if (r) flow[p][x] += fl;
        else flow[x][p] -= fl;
    }
    rep(i,0,N) rep(j,0,N) totcost += cost[i][j] *
        flow[i][j];
    return {totflow, totcost};
  }

  // If some costs can be negative, call this
      before maxflow:
  void setpi(int s) { // (otherwise, leave this
      out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)
        for (int to : ed[i]) if (cap[i][to])
          if ((v = pi[i] + cost[i][to]) < pi[to])
            pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

hopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `VI btoa(m, -1); hopcroftKarp(g, btoa);`
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

fd54bf, 43 lines

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa,
    vi& A, vi& B) {
  if (A[a] != L) return 0;
  A[a] = -1;
  for (int b : g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g,
        btoa, A, B))
      return btoa[b] = a, 1;
  }
  return 0;
}

int hopcroftKarp(vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);
    fill(all(B), 0);
    cur.clear();
    for (int a : btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a)
        ;
    for (int lay = 1;; lay++) {
      bool islast = 0;
      next.clear();
      for (int a : cur) for (int b : g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        }
        else if (btoa[b] != a && !B[b]) {
          B[b] = lay;
          next.push_back(btoa[b]);
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      for (int a : next) A[a] = lay;
      cur.swap(next);
    }
    rep(a,0,sz(g))
      res += dfs(a, 0, g, btoa, A, B);
  }
  return sz(btoa) - (int)count(all(btoa), -1);
}
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"hopcroftKarp.h" bfb654, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = hopcroftKarp(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] =
      false;
```

```
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e]
        != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost.
Time: $\mathcal{O}\left(N^2M\right)$

1e0fe9, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j
            ];
        if (cur < dist[j]) dist[j] = cur, pre[j]
            = j0;
        if (dist[j] < delta) delta = dist[j], j1
            = j;
      }
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -=
            delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a\|\|b)\&\&(!a\|\|c)\&\&(d\|\|!b)\&\&...$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).
Usage: `TwoSat ts(number of boolean variables);`
`ts.either(0, ~3); // Var 0 is true or var 3 is false`
`ts.setValue(2); // Var 2 is true`
`ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true`
`ts.solve(); // Returns true iff it is solvable`
`ts.values[0..N-1] holds the assigned values to the vars`
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

5f9706, 56 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  }
  void setValue(int x) { either(x, x); }

  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1])
        return 0;
    return 1;
  }
};
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
Time: $\mathcal{O}(V+E)$

780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges,
     int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just
      cycles
```

```
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end =
        sz(gr[x]);
    if (it == end){ ret.push_back(x); s.pop_back
        (); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
    }}
  for (int x : D) if (x < 0 || sz(ret) != nedges
      +1) return {};
  return {ret.rbegin(), ret.rend()};
}
```

CondensationGraph.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.

Usage: `scc(graph, [&](VI& v) { ... })` visits all components
in reverse topological order. comp[i] holds the component
index of a node (a component only has edges to components with
lower index). ncomps will contain the number of components.

Time: $\mathcal{O}(E+V)$

f5e4c5, 53 lines

```
// 0 based indexing
void condense(vector<vi> adj,vector<vi> &adj_scc,
              vector<vi> &comp,vi &root_of,int n) {
  vector<vi> rev_adj(n);
  rep(u,0,n) {
    for(auto v : adj[u]) {
      rev_adj[v].push_back(u);
    }
  }
  vector<bool> vis(n, false); vi order, component
    , root_nodes;
  function<void(int)> dfs1 = [&](int x) {
    vis[x] = true;
    for(auto nx : adj[x]) {
      if(!vis[nx]) {
        dfs1(nx);
      }
    }
    order.push_back(x);
  };
  rep(i, 0, n) { if(!vis[i]) dfs1(i); }
  vis.clear(); vis.assign(n, false);
  // order is now kind of topologically sorted
  reverse(order.begin(), order.end());
  function<void(int)> dfs2 = [&](int x) {
    vis[x] = true;
    component.push_back(x);
    for(auto u : rev_adj[x]) {
      if(!vis[u]) {
        dfs2(u);
      }
    }
  };
  comp.clear(); comp.resize(n);
  root_of.clear(); root_of.resize(n);
  for(auto v : order) {
    if(!vis[v]) {
      dfs2(v);
      int root = component.front();
      for(auto u : component) root_of[u] = root;
      root_nodes.push_back(root);
      comp[root] = component;
      component.clear();
    }
  }
  adj_scc.clear(); adj_scc.resize(n);
  rep(u, 0, n) {
    for(auto v : adj[u]) {
      if(root_of[u] != root_of[v]) {
        adj_scc[root_of[u]].push_back(root_of[v])
          ;
      }
    }
  }
}
```

BridgeTree.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

5e0207, 31 lines

```
// 0 based indexing
int n, m, Tin;
vector<vii> adj, adjn;
vi vis, low;
vector<array<int, 3>> bridges;
Dsu<int> ds;

int dfs0(int x,int p=-1,int w=0) {
  vis[x] = 1; low[x] = Tin++;
  int crl = low[x];
  for(auto nx : adj[x]) {
    if(nx.ff == p) continue;
    else if (vis[nx.ff]) crl = min(crl, low[nx.ff
        ]);
    else crl = min(crl, dfs0(nx.ff, x, nx.ss));
  }
  if(crl == low[x] and p != -1) bridges.pb({x, p,
      w});
  else if (p != -1) ds.join(x, p);
  return crl;
}

void build_bridgetree() {
  // CLEAR global variables
  ds.build(n); // INITIALIZE DSU HERE
  rep(i,0,n) if(!vis[i]) dfs0(i);
  for(auto arr : bridges) {
    int u = ds.find(arr[0]), v = ds.find(arr[1]),
        w = arr[2];
    if(u != v) {
      adjn[v].pb({u, w}); adjn[u].pb({v, w});
    }
  }
}
```

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

Time: $\mathcal{O}(NM)$

e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N),
      loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second
      ];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0,
        i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d
        ]) != -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at =
        adj[at][cd])
```

```
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d
          ]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc
          [i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1;
          z++);
  }
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v
        ;) ++ret[i];
  return ret;
}
```

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X
    ={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

f7c0bc, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    for (auto& v : r) v.d = 0;
    for (auto& v : r) for (auto j : r) v.d += e[v
        .i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d
        > b.d; });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return;
      q.push_back(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.
          push_back({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(sz(qmax) -
            sz(q) + 1, 1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i];
              };
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear
              ();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vi maxClique() { init(V), expand(V); return
      qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(
      C)), old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  }
};
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.

Time: All operations take amortized $\mathcal{O}(\log N)$.

5909e2, 90 lines

```
struct Node { // Splay tree. Root's pp contains
    tree's parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if
        wanted)
  }
  void pushFlip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z
         = b ? y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
```

```
    swap(pp, y->pp);
  }
  void splay() {
    for (pushFlip(); p; ) {
      if (p->p) p->p->pushFlip();
      p->pushFlip(); pushFlip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this)
        ;
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    makeRoot(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u,
      v)
    Node *x = &node[u], *top = &node[v];
    makeRoot(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in
      the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void makeRoot(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
};
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a
    ->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g)
    {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b],
      new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]])
            ;
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] =
            -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end
            ]}});
      }
    }
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto& [u,t,comp] : cycs) { // restore sol
      (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

HLD.h

Description: Heavy Light Decomposition
Time: $\mathcal{O}(logN * TimetakenbyRangeQueryDS)$

3a0a27, 74 lines

```
// requires a segment tree with init function
class HLD {
    SegmentTrees sgt; vector<vi> adj;
    vi sz, par, head, sc, st, ed;
    int t, n;
public:
    HLD(vector<vector<int>> &adj1,int n1): sz(n1
        +1), par(n1+1),
```

```
    head(n1+1), sc(n1+1), st(n1+1), ed(n1+1) {
     n = n1; adj = adj1; t = 0;
    }
    void dfs_sz(int x,int p = 0) {
        sz[x] = 1; par[x] = p; head[x] = x;
        for(auto nx : adj[x]) {
            if(nx == p) continue;
            dfs_sz(nx, x);
            sz[x] += sz[nx];
            if(sz[nx] > sz[sc[x]]) sc[x] = nx;
        }
    }
    void dfs_hld(int x,int p = 0) {
        st[x] = t++;
        if(sc[x]) {
            head[sc[x]] = head[x];
            dfs_hld(sc[x], x);
        }
        for(auto nx : adj[x]) {
            if(nx == p or nx == sc[x]) continue;
            dfs_hld(nx, x);
        }
        ed[x] = t - 1;
    }
    void build(int base = 1) {
        dfs_sz(base);
        dfs_hld(base);
        sgt.init(t);
    }
    bool anc(int x,int y) {
        if(x == 0) return true; if(y == 0) return
             false;
        return (st[x] <= st[y] and ed[x] >= ed[y
            ]);
    }
    int lca(int x,int y) {
        if(anc(x, y)) return x; if(anc(y, x))
            return y;
        while(!anc(par[head[x]], y)) x = par[head
            [x]];
        while(!anc(par[head[y]], x)) y = par[head
            [y]];
        x = par[head[x]]; y = par[head[y]];
        // one will overshoot the lca and the
           other will reach lca.
        return anc(x, y) ? y : x;
    }
    void update_up(int x,int p,ll add) {
        while(head[x] != head[p]) {
            sgt.update(st[head[x]], st[x], add,
               0, t-1);
            x = par[head[x]];
        }
        sgt.update(st[p], st[x], add, 0, t - 1);
    }
    void range_update(int u,int v,T add) {
        int l =  lca(u, v);
        update_up(u, l, add); update_up(v, l, add
            );
        update_up(l, l, -add);
    }
    T query_up(int x,int p) {
        T ans = 0;
        while(head[x] != head[p]) {
            ans = min(ans, sgt.query(st[head[x]],
                 st[x], 0, t-1));
            x = par[head[x]];
        }
        ans = min(ans, sgt.query(st[p], st[x], 0,
             t - 1));
        return ans;
    }
    T range_min(int u,int v) {
        int l = lca(u, v);
        return min(query_up(u, l), query_up(v, l)
            );
    }
};
```

CentroidDecomposition.h

Time: $\mathcal{O}(N \log N + Q)$

ca0a79, 59 lines

```
const int N = 5e4 + 13, logN = 17;
vi adj[N], sub(N), par(N,-1), lvl(N), done(N),
    par_adj(N);
vector<vi> dist(N, vi(logN, 0)), anc(N, vi(logN,
    0));
int nn = 0, root;
void dfs_size(int x,int p) {
  nn++; sub[x] = 1;
  for(auto nx : adj[x]) if(!done[nx] and nx != p)
       {
      dfs_size(nx, x); sub[x] += sub[nx];
    }
}
int find_ct(int x,int p) {
  for(auto nx : adj[x]) if(!done[nx] and nx != p
     and sub[nx] > nn/2)
    return find_ct(nx, x);
  return x;
}
void dfs(int x,int p,int ct) {
  anc[x][lvl[ct]] = ct;
  for(auto nx : adj[x]) if(!done[nx] and nx != p)
       {
      dist[nx][lvl[ct]] = 1 + dist[x][lvl[ct]];
      dfs(nx, x, ct);
    }
}
// par_adj[ct] = adjacent vertex to parent of ct
   in OT in subtree of ct.
int decompose(int x,int p=-1) {
  nn = 0; dfs_size(x, x);
  int ct = find_ct(x, x);
  if(p) lvl[ct] = 1 + lvl[p];
  done[ct] = 1; par[ct] = p;
  dfs(ct, ct, ct);
  for(auto nx : adj[ct]) if(!done[nx]) {
      int nct = decompose(nx, ct);
      par_adj[nct] = nx;
    }
  return ct;
}
vector<vi> child_cntb(N), my(N);
rep(x,0,n) for(int y = x; y >= 0; y = par[y]) {
      my[y].pb(dist[x][lvl[y]]);
      if(par[y] >= 0)
        child_cntb[y].pb(dist[x][lvl[par[y]]]);
  }
rep(x,0,n) {
    sort(all(my[x])); sort(all(child_cntb[x]));
}
// number of nodes <= k in v.
auto cnt_k = [&](vi &v,int k) {
  int l = upper_bound(all(v), k) - v.begin();
  return l;
};
auto k_dists = [&](int x,int k) {
  int ans = cnt_k(my[x], k);
  int ch = x, q = x; x = par[x];
  while(x >= 0) {
    ans += (cnt_k(my[x], k - dist[q][lvl[x]]));
    ans -= (cnt_k(child_cntb[ch], k - dist[q][lvl
        [x]]));
    ch = x; x = par[x];
  }
  return ans;
};
```

AuxiliaryTrees.h

Description: Creates a auxiliary tree of k nodes.
Time: $\mathcal{O}(k)$

1b4c5b, 62 lines

```
using vvi = vector<vector<int>>
struct Tree {
  int n;
  vvi adj;
  vi pos, tour, depth, pos_end, max_depth, dp,
      max_up;
  Tree(int n) : n(n), adj(n), max_depth(n), dp(n)
      , max_up(n) {}
  void add_edge(int s, int t) {
    adj[s].pb(t); adj[t].pb(s);
  }
  vvi table;
  int argmin(int i, int j) { return depth[i] <
      depth[j] ? i : j; }
  void rootify(int r) {
    pos.resize(n); pos_end.resize(n);
    function<void (int,int,int)> dfs = [&](int u,
        int p, int d) {
      pos[u] = pos_end[u] = depth.size();
      tour.pb(u); depth.pb(d);
      for (int v: adj[u]) {
        if (v != p) {
          dfs(v, u, d+1);
          pos_end[u] = depth.size();
          tour.pb(u);
          depth.pb(d);
        }
      }
    }; dfs(r, r, 0);
    int logn = sizeof(int)*__CHAR_BIT__-1-
        __builtin_clz(tour.size()); // log2
    table.resize(logn+1, vi(tour.size()));
    iota(all(table[0]), 0);
    for (int h = 0; h < logn; ++h)
      for (int i = 0; i+(1<<h) < tour.size(); ++i
          )
        table[h+1][i] = argmin(table[h][i], table
            [h][i+(1<<h)]);
  }
  int lca(int u, int v) {
    int i = pos[u], j = pos[v]; if (i > j) swap(i
        , j);
    int h = sizeof(int)*__CHAR_BIT__-1-
        __builtin_clz(j-i); // = log2
    return i == j ? u : tour[argmin(table[h][i],
        table[h][j-(1<<h)])];
  }
  int getDepth(int u){
    return depth[pos[u]];
  }
  void aux_Tree(vi nodes, vvi & adj_aux, vi &
      start_times){
    // adj_aux stores the children
    for(int x : nodes) start_times.pb(pos[x]);
    sort(all(start_times));
    for(int i = 1; i < (int) nodes.size();  i++){
      start_times.pb(pos[lca(tour[start_times[i
          ]], tour[start_times[i - 1]])]);
    }
    sort(all(start_times));
    start_times.erase(unique(start_times.begin(),
        start_times.end()), start_times.end());
    adj_aux.resize(start_times.size());
    stack<int> st;
        // nodes now indexed according to
            start_times
    st.push(0);
    for(int i = 1; i < (int)start_times.size(); i
        ++){
      while(pos_end[tour[start_times[st.top()]]]
          < start_times[i]){
        st.pop();
      }
      adj_aux[st.top()].pb(i);
      st.push(i);
    }
  }
};
```

Blossom.h

Description: Blossom Algorithm

1b2a6f, 52 lines

```
vector<int> Blossom(vector<vector<int>>& graph) {
  int n = graph.size(), timer = -1;
  vector<int> mate(n, -1), label(n), parent(n),
              orig(n), aux(n, -1), q;
  auto lca = [&](int x, int y) {
    for (timer++; ; swap(x, y)) {
      if (x == -1) continue;
      if (aux[x] == timer) return x;
      aux[x] = timer;
      x = (mate[x] == -1 ? -1 : orig[parent[mate[
          x]]]);
    }
  };
  auto blossom = [&](int v, int w, int a) {
    while (orig[v] != a) {
      parent[v] = w; w = mate[v];
      if (label[w] == 1) label[w] = 0, q.
          push_back(w);
      orig[v] = orig[w] = a; v = parent[w];
    }
  };
  auto augment = [&](int v) {
    while (v != -1) {
      int pv = parent[v], nv = mate[pv];
      mate[v] = pv; mate[pv] = v; v = nv;
    }
  };
  auto bfs = [&](int root) {
    fill(label.begin(), label.end(), -1);
    iota(orig.begin(), orig.end(), 0);
    q.clear();
    label[root] = 0; q.push_back(root);
    for (int i = 0; i < (int)q.size(); ++i) {
      int v = q[i];
      for (auto x : graph[v]) {
        if (label[x] == -1) {
          label[x] = 1; parent[x] = v;
          if (mate[x] == -1)
            return augment(x), 1;
          label[mate[x]] = 0; q.push_back(mate[x
              ]);
        } else if (label[x] == 0 && orig[v] !=
            orig[x]) {
          int a = lca(orig[v], orig[x]);
          blossom(x, v, a); blossom(v, x, a);
        }
      }
    }
    return 0;
  };
  // Time halves if you start with (any) maximal
      matching.
  for (int i = 0; i < n; i++)
    if (mate[i] == -1)
      bfs(i);
  return mate;
}
```

Strings (4)

Hashing.h

Description: Various self-explanatory methods for string hashing. Use on Codeforces, which lacks 64-bit support and where solutions can be hacked.

<sys/time.h> eb5e9e, 36 lines

```
typedef uint64_t ull;
static int C; // initialized below

// Arithmetic mod two primes and 2^32
    simultaneously.
// "typedef uint64_t H;" instead if Thue-Morse
    does not apply.
template<int M, class B>
struct A {
  int x; B b; A(int x=0) : x(x), b(x) {}
  A(int x, B b) : x(x), b(b) {}
  A operator+(A o){int y = x+o.x; return{y - (y>=
      M)*M, b+o.b};}
  A operator-(A o){int y = x-o.x; return{y + (y<
      0)*M, b-o.b};}
  A operator*(A o) { return {(int)(1LL*x*o.x % M)
      , b*o.b}; }
  explicit operator ull() { return x ^ (ull) b <<
       21; }
};
typedef A<1000000007, A<1000000009, unsigned>> H;

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(SZ(str)+1), pw(
      ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};

int main() {
  timeval tp;
  gettimeofday(&tp, 0);
  C = (int)tp.tv_usec; // (less than modulo)
  assert((ull)(H(1)*2+1-3) == 0);
  // ...
}
```

Kmp.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

d4375c, 16 lines

```
vi pi(const string& s) {
  vi p(sz(s));
  rep(i,1,sz(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
  }
  return p;
}

vi match(const string& s, const string& pat) {
  vi p = pi(pat + '\0' + s), res;
  rep(i,sz(p)-sz(s),sz(p))
    if (p[i] == sz(pat)) res.push_back(i - 2 * sz
        (pat));
  return res;
}
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$

d07a42, 8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0,
        k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$

38db9f, 23 lines

```
struct SuffixArray {
  vi sa, lcp;
  SuffixArray(string& s, int lim=256) { // or
      basic_string<int>
    int n = sz(s) + 1, k = 0, a, b;
    vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n
        );
    sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j *
        2), lim = p) {
      p = j, iota(all(y), n - j);
      rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] -
          j;
      fill(all(ws), 0);
      rep(i,0,n) ws[x[i]]++;
      rep(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y
          [i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
        (y[a] == y[b] && y[a + j] == y[b + j]) ?
            p - 1 : p++;
    }
    rep(i,1,n) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]]
        = k)
      for (k && k--, j = sa[rank[i] - 1];
          s[i + k] == s[j + k]; k++);
  }
};
```

Z.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

ee09e2, 12 lines

```
vi Z(const string& S) {
  vi z(sz(S));
  int l = -1, r = -1;
```

```
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - 1]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z
        [i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

DynamicAhoCorasik.h

Description: Deletion happens by creating another aho corasik. Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

aa86f2, 84 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'}; // change this!
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1,
        nmatches = 0;
    Node(int v) { memset(next, v, sizeof(next));
        }
  };
  vector<Node> N;
  vi backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    for (char c : s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = sz(N); N.
          emplace_back(-1); }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) : N(1, -1) {
    rep(i,0,sz(pat)) insert(pat[i], i);
    N[0].back = sz(N);
    N.emplace_back(0);

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      rep(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[
            i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[
              ed].start])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  vi find(string word) {
    int n = 0;
    vi res; // ll count = 0;
    for (char c : word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vi> findAll(vector<string>& pat, string
      word) {
    vi r = find(word);
    vector<vi> res(sz(word));
    rep(i,0,sz(word)) {
      int ind = r[i];
      while (ind != -1) {
        res[i - sz(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};

vector<string> vc;
vc.push_back(s);
for(int i=0;i<LIM;i++) {
  if(ad[0][i].vs.size()>0) {
    for(auto x: ad[0][i].vs) {
      vc.push_back(x);
    }
    ad[0][i]=Aho();
  }
  else {
    for(auto x: vc) {
      ad[0][i].add(x);
    }
    ad[0][i].build_aho();
    break;
  }
}
```

Number theory (5)

LinearDiophantine.h

Description: Solving linear diophantine eqns ($ax + by = c$).

87b1ec, 74 lines

```
int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}

bool find_any_solution(int a, int b, int c, int &
    x0, int &y0, int &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}

void shift_solution(int & x, int & y, int a, int
    b, int cnt) {
```

```
    x += cnt * b;
    y -= cnt * a;
}

int find_all_solutions(int a, int b, int c, int
    minx, int maxx, int miny, int maxy) {
    int x, y, g;
    if (!find_any_solution(a, b, c, x, y, g))
        return 0;
    a /= g;
    b /= g;

    int sign_a = a > 0 ? +1 : -1;
    int sign_b = b > 0 ? +1 : -1;

    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx)
        shift_solution(x, y, a, b, sign_b);
    if (x > maxx)
        return 0;
    int lx1 = x;

    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx)
        shift_solution(x, y, a, b, -sign_b);
    int rx1 = x;

    shift_solution(x, y, a, b, -(miny - y) / a);
    if (y < miny)
        shift_solution(x, y, a, b, -sign_a);
    if (y > maxy)
        return 0;
    int lx2 = x;

    shift_solution(x, y, a, b, -(maxy - y) / a);
    if (y > maxy)
        shift_solution(x, y, a, b, sign_a);
    int rx2 = x;

    if (lx2 > rx2)
        swap(lx2, rx2);
    int lx = max(lx1, lx2);
    int rx = min(rx1, rx2);

    if (lx > rx)
        return 0;
    return (rx - lx) / abs(b) + 1;
}
```

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.

Time: LIM=1e9 $\approx$ 1.5s

6b2912, 20 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM /
      2;
  vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log
      (LIM)*1.1));
  vector<pii> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i])
      {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve
        [j] = 1;
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p))
          block[i-L] = 1;
    rep(i,0,min(S, R - L))
      if (!block[i]) pr.push_back((L + i) * 2 +
          1);
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) ==
      3;
  ull A[] = {2, 325, 9375, 28178, 450775,
      9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" a33cf6, 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return modmul(x, x, n) +
      1; };
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))
        ) prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

CRT.h

Description: Chinese Remainder Theorem.

crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \mathrm{lcm}(m, n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
```

```
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p}$ $\forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i
    )
    for (int j = i; j < LIM; j += i) phi[j] -=
        phi[j] / i;
}
```

Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \;\; b = k \cdot (2mn), \;\; c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \bot n$, and either m or n even.
$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^{\times}$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.
$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorics (6)

Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g $(g.x = x)$.

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \;\; p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.

Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able). $B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),$$
$$c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\dots(x+n-1)$$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, \dots$

Algebra (7)

$$f(x) = f(a) + f'(a)\frac{x-a}{1!} + f''(a)\frac{x-a}{2!} + \dots$$

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i
        +1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin,
    double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$

"../number-theory/ModPow.h" 96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) %
        mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.

Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`

Time: $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) %
          mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] *
          tr[j]) % mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) %
      mod;
  return res;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.

Time: $\mathcal{O}\left(n^2 m\right)$

44c9ab, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m
      )
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" 08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto
      fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.

Time: $\mathcal{O}\left(n^2 m\right)$

fa2d7a, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int
    m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break
        ;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
```

```
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9\cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N\log N)$ with $N = |A| + |B|$ (~$1s$ for $N = 2^{22}$)

00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    }
}
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  rep(i,0,sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
}
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
Time: $\mathcal{O}(N\log N)$

"../number-theory/ModPow.h" ced03d, 33 lines

```
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)};
    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    }
}
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s), n = 1 << B;
  int inv = modpow(n, mod - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  rep(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
  ntt(out);
  return {out.begin(), out.begin() + s};
}
```

WalshHadamard.h

Description: $C_k = \sum_{i \otimes j = k} A_i B_j$
Usage: `Apply the transform, point multiply and invert`
Time: $\mathcal{O}(N\log N)$

922b72, 11 lines

```
void WalshHadamard(Poly &P, bool invert) {
  for (int len = 1; 2 * len <= sz(P); len <<= 1) {
    for (int i = 0; i < sz(P); i += 2 * len) {
      rep(j, 0, len) {
        auto u = P[i + j], v = P[i + len + j];
        P[i + j] = u + v, P[i + len + j] = u - v; // XOR
      }
    }
  }
  if (invert) for (auto &x : P) x /= sz(P);
}
```

OnlineFFT.h

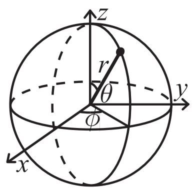
Description: Given $B_1, \ldots B_m$, compute $A_i = \sum_{j=1}^{i-1} A_j * B_{i-j}$
Usage: `1-indexed, pad B[i] = 0 for i > m`
Time: $\mathcal{O}\left(N\log^2 N\right)$

c0e86b, 18 lines

```
void online(const Poly &B, CD a1, int n, Poly &A) {
  const int m = SZ(B) - 1;
  A.assign(n + 1, 0); A[1] = a1;
```

```
  auto bst = B.begin(), ast = A.begin();
  REP(i, 1, n) {
    A[i + 1] += A[i] * B[1];
    if (i + 2 <= n) A[i + 2] += A[i] * B[2];
    for (int pw = 2; i % pw == 0 && pw + 1 <= m;
        pw <<= 1) {
      Poly blockA(ast + i - pw, ast + i);
      Poly blockB(bst + pw + 1, bst + min(pw * 2,
          m) + 1);
      Poly prod = conv(blockA, blockB);
      REP(j, 0, sz(prod)) {
        if (i + 1 + j <= n)
          A[i + 1 + j] += prod[j];
      }
    }
  }
}
```

Geometry (8)



$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\theta\sin\phi \qquad \theta = \operatorname{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r\cos\theta \qquad \phi = \operatorname{atan2}(y,x)$$

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

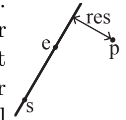
<bits/stdc++.h> e8d121, 44 lines

```
template <class T> int sgn(T x) { return (x > T
    (0)) - (x < T(0)); }
template <class T> struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x = 0, T y = 0) : x(x), y(y) {
      }
  bool operator<(P p) const { return tie(x, y) <
      tie(p.x, p.y); }
  bool operator==(P p) const { return tie(x, y)
      == tie(p.x, p.y); }
  P operator+(P p) const { return P(x + p.x, y +
      p.y); }
  P operator-(P p) const { return P(x - p.x, y -
      p.y); }
  P operator*(T d) const { return P(x * d, y * d)
      ; }
  P operator/(T d) const { return P(x / d, y / d)
      ; }
  T dot(P p) const { return x * p.x + y * p.y; }
  T cross(P p) const { return x * p.y - y * p.x;
      }
  T cross(P a, P b) const { return (a - *this).
      cross(b - *this); }
  T dist2() const { return x * x + y * y; }
  // abs() == dist()
  double dist() const { return sqrt((double)dist2
      ()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this / dist(); } //
      makes dist()=1
  P perp() const { return P(-y, x); }        //
      rotates +90 degrees
  P normal() const { return perp().unit(); }
  P translate(P v) { return P(x + v.x, y + v.y);
      }
  // scale an object by a certain ratio alpha
      around a
  // center c, we need to shorten or lengthen the
       vector
  // from c to every point by a factor alpha,
      while
  // conserving the direction
  P scale(P c, double factor) { return c + (*this
       - c) * factor; }
  // returns point rotated 'a' radians ccw around
       the origin
  P rotate(double a) const {
    return P(x * cos(a) - y * sin(a), x * sin(a)
        + y * cos(a));
  }
  friend ostream &operator<<(ostream &os, P p) {
    return os << "(" << p.x << "," << p.y << ")";
  }

  // Additional random shit
  bool isPerp(P p) { return P(x, y).dot(p) == 0;
      }
  double angle(P p) {
    double costheta = P(x, y).dot(p) / (*this).
        dist() / p.dist();
    return acos(fmax(-1.0, fmin(1.0, costheta)));
  }
  T orient(P b, P c) { return (b - *this).cross(c
       - *this); }
};
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

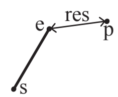


"Point.h" f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P&
    p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



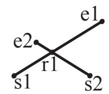
Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).
      dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

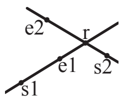
Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage:
```
vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
```
"Point.h", "OnSegment.h" 9d57f2, 13 lines
```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage:
```
auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```
"Point.h" a01f81, 8 lines
```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 9 lines
```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

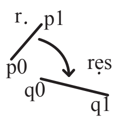
OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" c597e8, 3 lines
```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 03a306, 6 lines
```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage:
```
vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i
```
0f0602, 35 lines
```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
```

```
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CirclePolygonIntersection.h

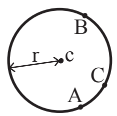
Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

"../../content/geometry/Point.h" a1ee63, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

circumcircle.h

Description:
The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

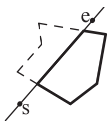
Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h" 9706dc, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i
      ++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



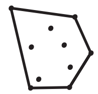
Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" f2b7d4, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s,
    P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.
        back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).
          second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.



Time: $\mathcal{O}(n \log n)$

"Point.h" 310954, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts
      )))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p
          ) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h
      [0] == h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i
          ], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1]
          - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict
    = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.
      back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0],
      l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n
    ]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i -
    1 + n) < 0
template <class P> int extrVertex(vector<P>& poly
    , P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
```

```
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ?
        hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly)
    {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2)
          % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz
        (poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; }
      );
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}}
      ;
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.
        upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}
          });
    S.insert(p);
  }
  return ret.second;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x <
    b.x; }
bool on_y(const P& a, const P& b) { return a.y <
    b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in
      it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; //
      bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared
      distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal
          ...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x :
          on_y);
      // divide by taking half the array for each
           child (not
      // best performance with many duplicates in
          the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() +
          half});
      second = new Node({vp.begin() + half, vp.
          end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({
      all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the
          point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(),
          node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance
        (p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f
        , s);

    // search closest side first, other side if
        needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its
      squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords
    are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any
    other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the
    circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.
      cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new
      Quad{0}}}};
  H = r->o; r->r()->r() = r;
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1
       ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b
      ->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}
pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s
        [1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c :
         b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())
      ) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)
             ));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (
    valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) {
        \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev())
        ;
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H
        (LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts
      .end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e
      ->o;
#define ADD { Q c = e; do { c->mark = 1; pts.
    push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c
      != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark)
      ADD;
  return pts;
}
```

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L&
    trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b
      ]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y
      (y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y,
       z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y,
       z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d
      ); }
```

```
  P operator/(T d) const { return P(x/d, y/d, z/d
      ); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z
      ; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y
        - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2
      ()); }
  //Azimuthal angle (longitude) to x-axis in
      interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in
      interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*
      y),z); }
  P unit() const { return *this/(T)dist(); } //
      makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw
      around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u =
        axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*
        s;
  }
};
```

Various (9)

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).

Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`

Time: $\mathcal{O}(\log(b-a))$

9155b4, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

Convolution.h

Description: Getting different convolutions

Time: $\mathcal{O}(n2^n)$

c8b662, 41 lines

```
// Zeta/SOS, N*2^N
rep(i,0,M)
  for(int mask = (1<<M) - 1; mask >= 0; mask--)
    if((mask>>i)&1)
      F[mask] += F[mask ^ (1 << i)];
// Rev mask loop and invert bit condition for
    superset sum

// Base from SOS
for(int i = M - 1; i >= 0; i--)
    for(int mask = (1 << M) - 1; mask >= 0; mask
        --)
      if((mask >> i)&1)
        F[mask] -= F[mask ^ (1 << i)];
// Rev mask loop and invert condition for base
    from Sum over superset

// Mobius, F[s] = SUM(-1^{s/s'} * F[s']), N*2^N
// F[1011] = F[1011] - F[0011] - F[1001] - F
    [1010] + F[1000] ...
rep(i,0,M) rep(mask, 0, 1<<M) if((mask>>i)&1)
      F[mask] -= F[mask ^ (1 << i)];

// sos(mu(f(x))) = f(x) = mu(sos(f(x)))

// fog[s] = SUM(f[s']*g[s/s']), N^2 * 2^N
// Make fhat[][] = {0} and ghat[][] = {0}
rep(mask,0,1<<N) {
    fhat[__builtin_popcount(mask)][mask] = f[mask
        ];
    ghat[__builtin_popcount(mask)][mask] = g[mask
        ];
}
// Apply zeta transform on fhat[][] and ghat[][]
rep(i,0,N+1) rep(j,0,N) rep(mask,0,1<<N) if((mask
    >>j)&1) {
  fhat[i][mask] += fhat[i][mask ^ (1 << j)];
  ghat[i][mask] += ghat[i][mask ^ (1 << j)];
}
// Do the convolution and store into h[][] = {0}
rep(mask,0,(1<<N)) rep(i,0,N+1) rep(j,0,i+1)
          h[i][mask] += fhat[j][mask] * ghat[i
              - j][mask];
// Apply inverse SOS dp on h[][]
rep(i,0,N+1) rep(j,0,N) rep(mask,0,1<<N) if((mask
    >>j)&1)
  h[i][mask] -= h[i][mask ^ (1 << j)];

rep(mask,0,1<<N) fog[mask] = h[__builtin_popcount
    (mask)][mask];
```

PolyModPoly.h

Description: Poly Mod Poly

c40f13, 33 lines

```
#define rsz resize
poly RSZ(poly p, int x) { p.rsz(x); return p; }
poly rev(poly p) { reverse(all(p)); return p; }
poly inv(poly A, int n) { // Q-(1/Q-A)/(-Q^{-2})
  poly B{1/A[0]};
  while (sz(B) < n) {
    int x = 2*sz(B);
    B = RSZ(2*B-conv(RSZ(A,x),conv(B,B)),x); } //
        fft
  return RSZ(B,n);
}
pair<poly,poly> divi(const poly& f, const poly& g
    ) {
  if (sz(f) < sz(g)) return {{},f};
  auto q = mul(inv(rev(g),sz(f)-sz(g)+1),rev(f));
  q = rev(RSZ(q,sz(f)-sz(g)+1));
  auto r = RSZ(f-mul(q,g),sz(g)-1); return {q,r};
}
typedef vector<mi> vmi; // mi = modular int
struct MultipointEval {
  poly stor[1<<18];
  void prep(vmi v, int ind = 1) { // v -> places
      to evaluate at
    if (sz(v) == 1) { stor[ind] = {-v[0],1};
        return; }
    int m = sz(v)/2;
    prep(vmi(begin(v),begin(v)+m),2*ind);
    prep(vmi(m+all(v)),2*ind+1);
    stor[ind] = conv(stor[2*ind],stor[2*ind+1]);
  }
  vmi res;
  void eval(vmi v, int ind = 1) {
    v = divi(v,stor[ind]).s;
    if (sz(stor[ind]) == 2) { res.pb(v[0]);
        return; }
    eval(v,2*ind); eval(v,2*ind+1);
  }
};
```