Fuck Logic ICPC Team Notebook

1 Intro

1.1 Template

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b);
    ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
        cin.tie(0)->sync_with_stdio(0);
        cin.exceptions(cin.failbit);
}
```

2 Data Structures

2.1 Matrix

```
template<class T, int N> struct Matrix {
        typedef Matrix M;
        array<array<T, N>, N> d{};
        M operator*(const M& m) const {
                M a;
                rep(i,0,N) rep(j,0,N)
                        rep(k,0,N) a.d[i][j]
                            += d[i][k]*m.d[k
                            ][j];
                return a;
        }
        vector<T> operator*(const vector<T>&
            vec) const {
                vector<T> ret(N);
                rep(i,0,N) rep(j,0,N) ret[i]
                    += d[i][j] * vec[j];
                return ret;
        }
        M operator^(ll p) const {
                assert(p >= 0);
                M a, b(*this);
                rep(i,0,N) a.d[i][i] = 1;
                while (p) {
                        if (p&1) a = a*b;
                        b = b*b;
                        p >>= 1;
                }
                return a;
        }
};
```

3 Dynamic Programming

3.1 Template

```
#include <bits/stdc++.h>
```

4 Graphs

4.1 Template

```
#include <bits/stdc++.h>
```

5 Strings

5.1 Template

```
#include <bits/stdc++.h>
```

6 Mathematics

6.1 Template

```
#include <bits/stdc++.h>
```

7 Geometry

7.1 Template

```
#include <bits/stdc++.h>
```

8 Miscellaneous

8.1 Template

```
#include <bits/stdc++.h>
```

9 Math Extra

9.1 Seldom used combinatorics

- Fibonacci in $O(\log(N))$ with memoization is:

$$f(0) = f(1) = 1$$

$$f(2k) = f(k)^2 + f(k-1)^2$$

$$f(2k+1) = f(k) \times (f(k) + 2 * f(k-1))$$

- Wilson's Theorem Extension:

  $B = b_1 b_2 \dots b_m (\mod n) = \pm 1$, all $b_i \leq n$ such that $\gcd(b_i, n) = 1$.

  If $n \leq 4$ or $n = (\text{odd prime})^k$ or $n = 2(\text{odd prime})^k$) then $B = -1$ for any k.

  Else $B = 1$.

- Stirling numbers of the second kind, denoted by $S(n,k) =$ number of ways to split n numbers into k non-empty sets.

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = kS(n-1,k) + S(n-1,k-1)$$

  $S(n-d+1, k-d+1) = S(n,k)$ where if indexes i,j belong to the same set, then $|i-j| \geq d$.

- Burnside's Lemma: $|\text{classes}| = \frac{1}{|G|}\sum_{\forall\ g\in G}(K^{C(g)})$, where:
  G = different permutations possible,
  $C(g)$ = number of cycles on the permutation g,
  and K = Number of states for each element
- Different ways to paint a necklace with N beads and K colors:

$$G = (1,2,\dots N),(2,3,\dots N,1),\dots (N,1,\dots N-1)$$

$g_i = (i, i+1, \dots i+N)$, (taking mod N to get it right) $i = 1 \dots N$ with i per step, that is, $i \to 2i \to 3i \dots$

Cycles in g_i all have size $n/\gcd(i,n)$, so

$$C(g_i) = \gcd(i,n)$$

$$\text{ans} = \frac{1}{N}\sum_{i=1\dots N}(K^{\gcd(i,n)})$$

$$\text{ans} = \frac{1}{N}\sum_{\forall\ d|N}(\phi(\frac{N}{d})K^d)$$

9.2 Combinatorial formulas

$$\sum_{k=0}^{n} k^2 = n(n+1)(2n+1)/6$$

$$\sum_{k=0}^{n} k^3 = n^2(n+1)^2/4$$

$$\sum_{k=0}^{n} k^4 = (6n^5+15n^4+10n^3-n)/30$$

$$\sum_{k=0}^{n} k^5 = (2n^6+6n^5+5n^4-n^2)/12$$

$$\sum_{k=0}^{n} x^k = (x^{n+1}-1)/(x-1)$$

$$\sum_{k=0}^{n} kx^k = (x-(n+1)x^{n+1}+nx^{n+2})/(x-1)^2$$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

$$\binom{n}{k} = \frac{n}{n-k}\binom{n-1}{k}$$

$$\binom{n}{k} = \frac{n-k+1}{k}\binom{n}{k-1}$$

$$\binom{n+1}{k} = \frac{n+1}{n-k+1}\binom{n}{k}$$

$$\binom{n}{k+1} = \frac{n-k}{k+1}\binom{n}{k}$$

$$\sum_{k=1}^{n} k\tbinom{n}{k} = n2^{n-1}$$

$$\sum_{k=1}^{n} k^2\tbinom{n}{k} = (n+n^2)2^{n-2}$$

$$\binom{m+n}{r} = \sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k}$$

$$\binom{n}{k} = \prod_{i=1}^{k}\frac{n-k+i}{i}$$

9.3 Number theory identities

Lucas' Theorem: For non-negative integers m and n and a prime p,

$$\binom{m}{n} \equiv \prod_{i=0}^{k}\binom{m_i}{n_i} \pmod p,$$

where

$$m = m_k p^k + m_{k-1}p^{k-1} + \cdots + m_1 p + m_0$$

is the base p representation of m, and similarly for n.

9.4 Stirling Numbers of the second kind

Number of ways to partition a set of n numbers into k non-empty subsets.

$$\left\{ {n \atop k} \right\} = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{(k-j)}\binom{k}{j}j^n$$

Recurrence relation:

$$\left\{ {0 \atop 0} \right\} = 1$$

$$\left\{ {n \atop 0} \right\} = \left\{ {0 \atop n} \right\} = 1$$

$$\left\{ {n+1 \atop k} \right\} = k\left\{ {n \atop k} \right\} + \left\{ {n \atop k-1} \right\}$$

9.5 Burnside's Lemma

Let G be a finite group that acts on a set X. For each g in G let X^g denote the set of elements in X that are fixed by g, which means $X^g = \{x \in X | g(x) = x\}$. Burnside's lemma assers the following formula for the number of orbits, denoted $|X/G|$:

$$|X/G| = \frac{1}{|G|}\sum_{g\in G}|X^g|$$

9.6 Numerical integration

RK4: to integrate $\dot{y} = f(t,y)$ with $y_0 = y(t_0)$, compute

$$k_1 = f(t_n, y_n)$$
$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$$
$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$$
$$k_4 = f(t_n + h, y_n + hk_3)$$
$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$