

Quantum Annealing and Computation: A Brief Documentary Note*

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Abstract

Major breakthrough in quantum computation has recently been achieved using quantum annealing to develop analog quantum computers instead of gate based computers. After a short introduction to quantum computation, we retrace very briefly the history of these developments and discuss the Indian researches in this connection and provide some interesting documents (in the Figs.) obtained from a chosen set of high impact papers (and also some recent news etc. blogs appearing in the Internet). This note is also designed to supplement an earlier note by Bose (Science and Culture, **79**, pp. 337-378, 2013).

1 Introduction

Quantum computers are actively being sought for the last couple of decades. Basic hope being that quantum mechanics promises several features to help faster computations if quantum features are properly implemented in the hardware architecture of such computers. Traditional architecture of classical computers is logical gate-based ones. The linear superposition processing of the wave functions in quantum mechanics helps simultaneous (probabilistic) processing of the binary bits or qubits (of information). Quantum mechanics also promises major advantages of parallel operation of these gates in appropriate architectures. The problem of decoherence has not allowed so far any gate-based quantum computer which is able to handle more than a couple of qubits. Even in classical computers, in order to solve computationally hard problems, like the traveling salesman problem, one artificially generates stochastic algorithms, like the simulated annealing techniques (in the so-called ‘Boltzmann Machine’) for practical and efficient searches. The essential problem in such searches is that the system gets locked in some local minimum separated from the other deeper minimum by (cost function or free energy) barriers. Noting that quantum tunneling feature across such barriers can help [1] searching for the minimum (optimal solution), quantum annealing techniques have been developed recently [1-29] (see Figs. 1-28). In the last couple of years, such techniques have been efficiently implemented in the computer architectures and such quantum annealing computers have already arrived in the market (see Fig. 1) and major successes are being demonstrated [18, 20, 21, 22, 23, 24, 25, 26, 28]. These exciting developments are also being captured in several recent notes (e.g., [30]), reviews (e.g., [25, 29]) and books (e.g., [31, 32]).

2 A brief history

Ray et al. [1] and Thirumalai et al. [2] suggested (in 1989; see Figs. 4 and 5) that quantum tunneling from local trap states across the (narrow) free energy barriers of macroscopic height (of order N ; coming from the requirement of the flips of finite fraction of all N spins) in a Sherrington-Kirpatrick spin glass (in transverse field) can help evolving such a complex system towards its (degenerate) ground state(s). Although the idea was criticized heavily in the subsequent literature, essentially on the ground that the incoherent phase overlaps of the tunneling states (waves) will localize the system and will not allow evolution towards the ground state(s). Later, theoretical investigations ([5, 7, 9, 10]) and the experimental demonstrations ([8]) lead to a very promising development resulting in the quantum annealing technique [12, 13, 14, 15, 23] with the hardware implementation by D-Wave system [18]. Successful checking and applications [20, 24, 26, 28] led to the emergence of a new era in quantum computing (see e.g., the comments by Bose [30], Fig. 27).

*Dedicated to Prof. Bikas K. Chakrabarti on the occasion of his 60-th birthyear

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3 A short story of the development

In May this year (2013), there have been several news posts and blogs in popular science journals as well as newspapers informing about the successful tests of a quantum computer with about 100 qubits (order of magnitude higher than those available otherwise), based on quantum annealing technique and marketed by the D-Wave Systems Inc. Interestingly, the NASA group of Consortium had already placed an order to them for a 512 qubit quantum annealing computer. Fig. 1 shows the BBC news blog on the purchase deal with D-Wave Inc. by the NASA-Google consortium. In an attempt to explain in a popular way how quantum tunneling can help such analog computers to get out of the “local” solutions and anneal down quantum mechanically to the “global” or “optimal” solutions with proper tuning of the tunneling term, a Scientific American news blog appeared last May (see Fig. 2). It explains in brief the working principle involved, using the Wikipedia entry on quantum annealing (partly reproduced in Fig. 3).

The rugged nature of the (free) energy landscape (energy versus spin configurations) of a classical (Ising) spin glass does not allow searches for the global energy minima by simple rolling down the landscape (say, using an energy dissipative dynamics). Essentially the system gets trapped in the local minima, separated from the global minima often by macroscopically high (free) energy barriers. Ray et al. [1] first pointed out that if such barriers are narrow, quantum mechanical tunneling (as in a transverse Ising spin glass model) can help such searches (see Fig. 4). As mentioned already, this indication was criticized heavily in the following literatures. However, some crucial features of such tunneling effects were immediately checked, with positive results in some solid-state samples discovered by Wu et al. (see Fig. 7). The possibility of tuning the quantum tunneling term to achieve the minimization of a multi-variable optimization problem was pointed out by Finnila et al. [5] (see Fig. 8). However, the use of this annealing (of the tunneling field) in the well-characterized ground state search problems of (frustrated) spin systems were convincingly demonstrated, using numerical techniques, by Kadwaki and Nishimori [7] (see also Fig. 9) and the reported success in this paper made a major impact on the following developments. Soon, Brook et al. [8] extended their earlier experimental investigations (see Fig. 7) and with suitable tuning of the tunneling field, observed clear advantages of the quantum annealing in the search for ground state(s) for such samples (see Fig. 10). This experimental demonstration of the clear advantages of quantum annealing established the field. Soon major investigations by several groups from all over the world started. Farhi et al. [9] (see Fig. 11) indicated that such an adiabatic evolution (zero temperature annealing) algorithm may help solving the computationally (NP-) hard problems. The estimate of the growth of errors in the optimal search, with the decrease in the annealing time, was made first by Santoro et al. [10] (see Fig. 12) and the extensions (and some clarifications) of the experimental investigations by Ancona-Torres et al. (see Fig. 13) enthused the community to explore further in much more meaningful way. Soon (during 2006-2008), some important reviews on quantum annealing techniques were written (see Figs. 14, 15 and 16): Santoro and Tosatti [13] and Morita and Nishimori [15] reviewed the mathematical structures of the quantum annealing techniques, while Das and Chakrabarti [14] reviewed the physical structure of quantum annealing and discussed its possible implications for analog quantum computations.¹

In 2011, D-wave system announced [18] the arrival of “World’s first commercially available quantum computer, operating on a 128 qubit chip-set using quantum annealing” (see Figs. 17, 18). As indicated already, this news created huge enthusiasms as well as a lot of criticisms in the community of scientists. However, soon some leading research groups came forward to test the performances of these machines with remarkably positive results (see e.g., Figs. 19, 20, 24 and 26). There were parallel investigations on the possible performances of the quantum annealing technique in the context of various computationally hard problems (see e.g., Figs. 21, 22 and 23).²

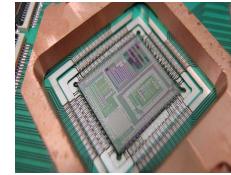
¹For extensions to non-adiabatic quantum annealing see M. Ohzeki, and H. Nishimori, J. Comp. and Theor. Nanoscience **8**, 963(2011) and D. Aharonov et al., SIAM J. Comp., **37**, 166 (2007), <http://arxiv.org/abs/quant-ph/0405098>, for showing complexity equivalence between adiabatic quantum computation and circuit computation.

²Of course, the story is not a closed one and many critical aspects of the development are also being addressed by the scientists. See for example:

B. Altshuler et al., Proc. Nat. Acad. Sc., **107**, 12446 (2010), <http://www.pnas.org/content/107/28/12446.full> (on why it may fail for general disorder);

C. R. Laumann et al., Phys. Rev. Lett. **109**, 030502 (2012), <http://arxiv.org/abs/1202.3646> (showing both unexpected success and failure modes of quantum annealing);

E. Farhi et al., Phys. Rev. A **86**, 052334 (2012), <http://arxiv.org/abs/1208.3757> (discussing how Quantum Annealing can fail in some generic satisfiability problems);



A \$15m computer that uses "quantum physics" effects to boost its speed is to be installed at a Nasa facility. It will be shared by Google, Nasa, and other scientists, providing access to a machine said to be up to 3,600 times faster than conventional computers. Unlike standard machines, the D-Wave Two processor appears to make use of an effect called quantum tunnelling. This allows it to reach solutions to certain types of mathematical problems in fractions of a second. Effectively, it can try all possible solutions at the same time and then select the best. Google wants to use the facility at Nasa's Ames Research Center in California to find out how quantum computing might advance techniques of machine learning and artificial intelligence, including voice recognition. University researchers will also get 20% of the time on the machine via the Universities Space Research Agency (USRA). Nasa will likely use the commercially available machine for scheduling problems and planning. Canadian company D-Wave Systems, which makes the machine, has drawn scepticism over the years from quantum computing experts around the world. Until research outlined earlier this year, some even suggested its machines showed no evidence of using specifically quantum effects. Quantum computing is based around exploiting the strange behaviour of matter at quantum scales. Most work on this type of computing has focused on building quantum logic gates similar to the gate devices at the basis of conventional computing. But physicists have repeatedly found that the problem with a gate-based approach is keeping the quantum bits, or qubits (the basic units of quantum information), in their quantum state. "You get drop out... decoherence, where the qubits lapse into being simple 1s and 0s instead of the entangled quantum states you need. Errors creep in," says Prof Alan Woodward of Surrey University. One gate opens... instead, D-Wave Systems has been focused on building machines that exploit a technique called quantum annealing - a way of distilling the optimal mathematical solutions from all the possibilities. Annealing is made possible by an effect in physics known as quantum tunnelling, which can endow each qubit with an awareness of every other one. "The gate model... is the single worst thing that ever happened to quantum computing", Geordie Rose, chief technology officer for D-Wave, told BBC Radio 4's Material World programme. "And when we look back 20 years from now, at the history of this field, we'll wonder why anyone ever thought that was a good idea." Dr Rose's approach entails a completely different way of posing your question, and it only works for certain questions. But according to a paper presented this week (the result of benchmarking tests required by Nasa and Google), it is very fast indeed at finding the optimal solution to a problem that potentially has many different combinations of answers. In one case it took less than half a second to do something that took conventional software 30 minutes. A classic example of one of these "combinatorial optimisation" problems is that of the travelling sales rep, who needs to visit several cities in one day, and wants to know the shortest path that connects them all together in order to minimise their mileage. The D-Wave Two chip can compare all the possible itineraries at once, rather than having to work through each in turn. Reportedly costing up to \$15m, housed in a garden shed-sized box that cools the chip to near absolute zero, it should be installed at Nasa and available for research by autumn 2013. US giant Lockheed Martin earlier this year upgraded its own D-Wave machine to the 512 qubit D-Wave Two.

Figure 1: BBC news blog on the purchase deal of the latest 512 qubit quantum annealer of D-Wave Inc. by the NASA-Google consortium (website: <http://www.bbc.co.uk/news/science-environment-22554494>).

News Blog

SCIENTIFIC AMERICAN™

Is It Quantum Computing or Not?

By Alan Woodward | May 17, 2013

This week I had a fascinating discussion on BBC Radio 4 with Dr Geordie Rose, the CTO of DWave, triggered by the news that NASA and Google are investing in DWave's "quantum computer". The idea is to set up a facility that is used by both NASA and Google but also allows academics to book time on the system to try out new ideas. Our radio conversation brought out an important issue that has dogged this subject for several years: when is a quantum computer not a quantum computer?

I began by explaining the theory behind quantum computing and why they hold the promise of significantly faster processing. In essence, it relies upon the fact that whilst conventional "bits" can be 0 or 1, quantum bits (so called qubits) can be both 0 and 1 at the same time (known as superposition). If you can process values that expand exponentially with the number of qubits you entangle. As with conventional programming, these qubits are passed through various logic gates to achieve the desired results. Hence, this is known as the "gate theory" of quantum computing.

Many academics, and increasingly large corporations such as IBM and Microsoft, have spent years working on the algorithms, error correction and a variety of techniques for creating qubits, ranging from photons to ion traps to braided anyons. To date, we have found it extremely difficult to maintain these qubits in superposition and to ensure they are truly entangled. "Decoherence", where the qubits drop out of superposition and become just a 0 or a 1, is the bane of all quantum computer engineers.

This decoherence problem has spurred many to look for methods that are naturally immune from the effect. DWave were one such group. They have based their processor on an effect called **quantum annealing**, also sometimes referred to as adiabatic quantum computing, which was first discussed in 2000 as a possible means of conducting certain calculations.

The quantum annealing process is, as the name suggests, a quantum level effect. At the scale of a qubit, you can use the effect to determine the lowest "energy" state of a system. Hence, if you can describe a problem in terms of a function that has a "cost" or "energy" versus some other parameter, you can find the configuration that represents the optimal state. So, for example, think of the classic travelling salesman problem where one tries to find the shortest path when travelling between a number of cities. If you did this using simple trial and error on a conventional computer it would take longer than the age of the universe by the time you were up to 30 cities. Using quantum annealing you can define the problem as an optimisation task which means you can programme a DWave system to calculate it.

An obvious question is how much faster is quantum annealing than conventional computers? Based upon solving specific problems, that question was addressed in a paper just published, where academics compared conventional computers with a DWave system when solving optimisation problems which were known to be computationally hard. The DWave system is reported as being many thousands of times faster in some cases.

Thus, we have a system that can do useful computations based on quantum effects. It may not be a quantum computer as some purists might define it, but it does have one huge advantage: it exists and is available to do meaningful work. For all the theory, quantum computers based upon gate theory are still very experimental and can muster only a handful of qubits. Gate based quantum computing will come eventually; the money being invested and the screaming need for the technology as we head towards the end of Moore's law mean that it's a question of when not if. But, on the journey, which is currently of uncertain length, we should not be blind to opportunities on the way. It may prove to be a detour, but many interesting developments have arisen in computing by those who spotted just such an opportunity.

So, is DWave's system a quantum computer? I think that's the wrong question. Better to ask if the DWave system can help with some computations that were previously impractical, in which case the answer is yes.

Reference: Quantum Annealing: Wikipedia
Images: DWave Systems, Inc., Arnab Das, WhiteTimberwolf, Saurabh.harsh

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Figure 2: A Scientific American news blog on the NASA-Google investment in D-Wave quantum (annealing) computer. It explains briefly the working principle involved, using the Wikipedia entry on quantum annealing (Fig. 3; website: <http://blogs.scientificamerican.com/guest-blog/2013/05/17/is-it-quantum-computing-or-not/>).

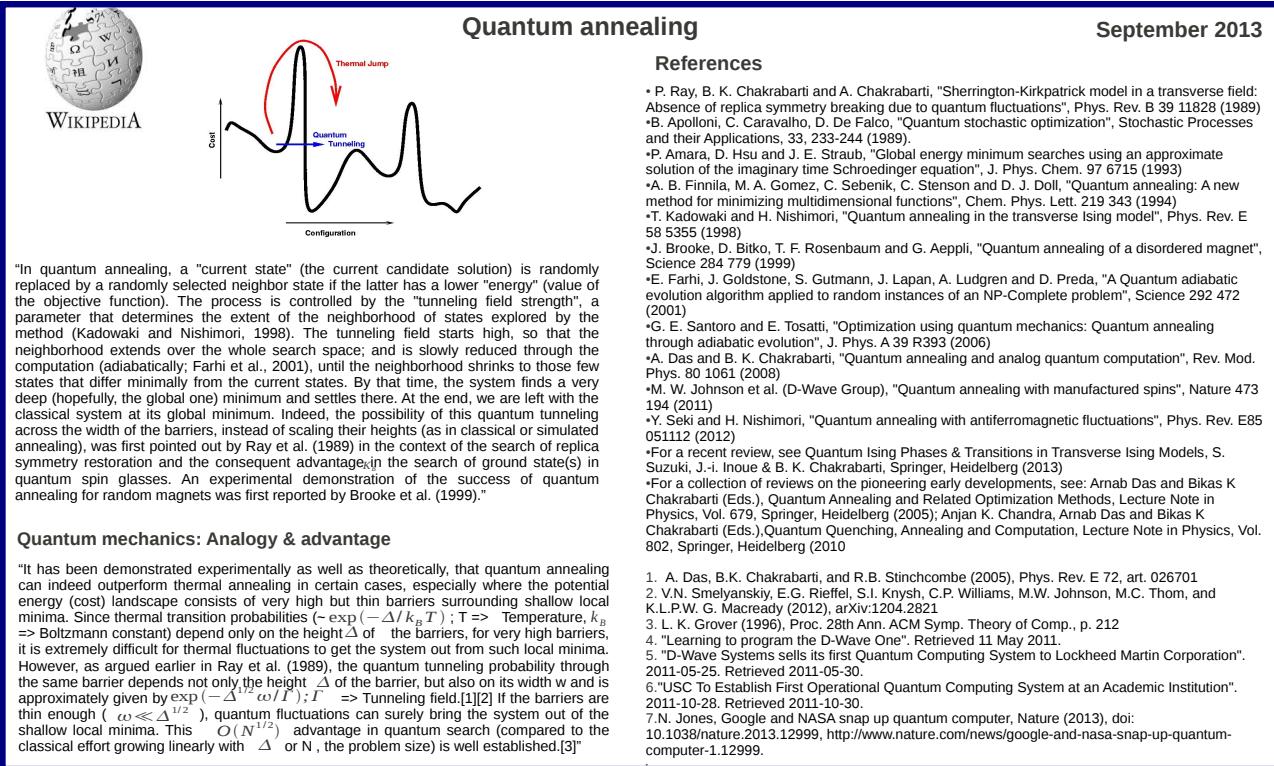


Figure 3: Part of the entry on quantum annealing in Wikipedia (as in September 2013; website: <http://en.wikipedia.org/wiki/Quantum-annealing>).

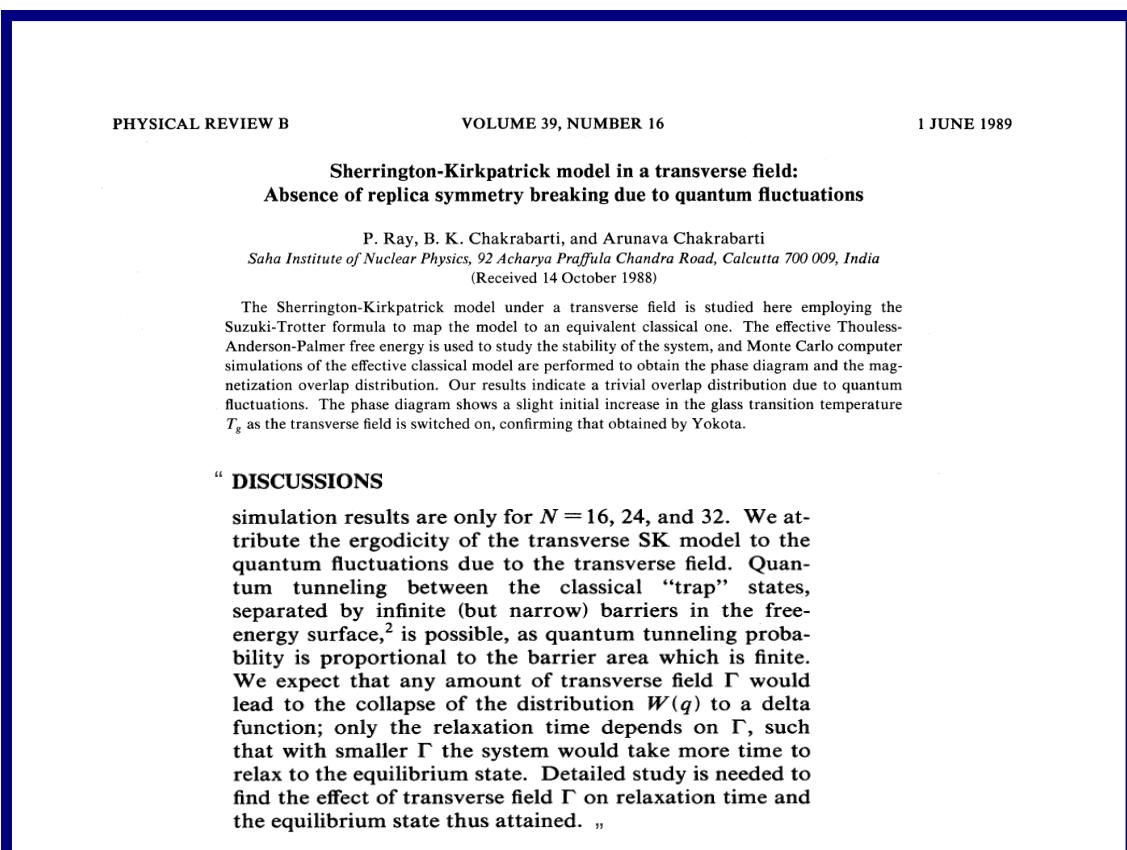


Figure 4: Title, abstract and some excerpts from the first published paper arguing that quantum tunneling across the free energy barriers in Sherrington-Kirkpatrick spin glass model can lead to efficient search possibilities for its ground state(s). It may be noted that computationally hard problems can often be mapped into such long-range spin glass models and the advantage of quantum tunneling in such quantum spin glass models has led ultimately to the development of the quantum annealer/computer discussed here.

Infinite-range Ising spin glass in a transverse field

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Abstract. The infinite-range Ising spin glass in the presence of a transverse field is considered. Using the generalised Trotter formula and replica methods, the free energy for the system with quenched random bond interactions is evaluated using the static approximation. It is shown that, when the strength of the transverse field equals the largest eigenvalue of the random bond interaction matrix, the spin glass transition is destroyed. We also show that a replica-symmetric spin glass phase is *stable* in a certain region of the temperature-field phase diagram. Finally we present suggestive arguments which indicate that, when the time dependence of the dynamic susceptibility is included, then the stability of the replica-symmetric phase is enhanced.

“Conclusions”

A plausible physical reason for the stability of the replica-symmetric solution can be given in terms of the pure state picture that is suggested by the Parisi solution to the SK model. The ordered phase of the SK model is thought to consist of many pure states, all with the same free energy per spin. The pure states are separated by barrier and the timescale for crossing the barrier scales as $\tau \sim \exp(N^{0.25})$ and hence becomes infinite as $N \rightarrow \infty$ [23]. The transverse field essentially induces tunnelling between the pure states and we suggest that the diverging barriers are somehow renormalised to finite values as r becomes large enough, i.e. when $\Gamma/J \sim O(1)$. Under these circumstances, the systems coherently tunnels between what were originally ‘pure’ states and this leads to the overlap distribution given by (4.1). This hypothesis can be verified by quantum Monte Carlo simulations.”

Figure 5: Title, abstract and some excerpts from [2], arguing (in almost the same language as in ref. [1]; see Fig. 4) for the possible advantage of quantum tunneling between well characterised (classically) localized states in the same long-range transverse Ising spin glass model.

QUANTUM STOCHASTIC OPTIMIZATION

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We propose a combinatorial optimization procedure based on the physical idea of using the quantum tunnel effect to allow the search of global minima of a function of many Boolean variables to escape from poor local minima. More specifically, the function V to be minimized is viewed as the potential energy term in a Schrödinger Hamiltonian H for a quantum spin 1/2 system, the kinetic energy term being the generator of a random walk tailored to the neighborhood structure associated with V . The distorted random walk associated with (a suitable approximation of) the ground state eigenfunction of H defines then our approximate optimization strategy. A numerical application to the graph partitioning problem is presented.

combinatorial optimization * global minima * random walk * Schrödinger Hamiltonian * potential energy * graph partitioning

Figure 6: Title and abstract of ref. [3] indicating the formulation of a quantum stochastic optimization trick.

From Classical to Quantum Glass

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(Received 11 March 1991)

We study the effects of a transverse magnetic field on the dynamics of the randomly diluted, dipolar coupled, Ising magnet $\text{LiHo}_{0.16}\text{Y}_{0.83}\text{F}_4$. The transverse field mixes the eigenfunctions of the ground-state Ising doublet with the otherwise inaccessible excited-state levels. We observe a rapid decrease in the characteristic relaxation times, large changes in the spectral form of the relaxation, and a depression of the spin-glass transition temperature with the introduction of quantum fluctuations.

"Various authors [2-4] have considered spin glasses in transverse fields, with predictions ranging from the destruction of the spin-glass state [3] to an enhancement in the transition temperature [4] with the introduction of quantum fluctuations. We report here static and dynamic measurements on a physical realization of the Ising spin glass in a transverse magnetic field."

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Figure 7: Title, abstract and some excerpts from the first paper (from Univ. Chicago and Bell Labs. groups) reporting on the experimental realization of a sample described precisely by a transverse Ising spin glass model, with tunable transverse field, and observations in agreement with the results of Ray et al. (1989).

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18 March 1994

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CHEMICAL PHYSICS LETTERS

Quantum annealing: a new method for minimizing multidimensional functions

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Received 29 November 1993

Abstract

Quantum annealing is a new method for finding extrema of multidimensional functions. Based on an extension of classical, simulated annealing, this approach appears robust with respect to avoiding local minima. Further, unlike some of its predecessors, it does not require an approximation to a wavefunction. We apply the technique to the problem of finding the lowest energy configurations of Lennard-Jones clusters of up to 19 particles (roughly 10^5 local minima). This early success suggests that this method may complement the widely implemented technique of simulated annealing.

Figure 8: Title and abstract of the first published paper demonstrating the ground state cluster search for a Lennard-Jones system with 'quantum annealing' [in the title]. Of course, the first claim for 'A numerical implementation of quantum annealing' in a single particle Hamiltonian for minimizing a real function of Boolean variables was published by B. Apolloni, C. Carvalho and D. De Falco in a conference (held in July 1988) proceedings, 'Stochastic Processes, Physics & Geometry', Eds. S. Albeverio et al., World Scientific, Singapore, 1990, pp. 97-111].

Quantum annealing in the transverse Ising model

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(Received 30 April 1998)

We introduce quantum fluctuations into the simulated annealing process of optimization problems, aiming at faster convergence to the optimal state. Quantum fluctuations cause transitions between states and thus play the same role as thermal fluctuations in the conventional approach. The idea is tested by the transverse Ising model, in which the transverse field is a function of time similar to the temperature in the conventional method. The goal is to find the ground state of the diagonal part of the Hamiltonian with high accuracy as quickly as possible. We have solved the time-dependent Schrödinger equation numerically for small size systems with various exchange interactions. Comparison with the results of the corresponding classical (thermal) method reveals that the quantum annealing leads to the ground state with much larger probability in almost all cases if we use the same annealing schedule. [S1063-651X(98)02910-9]

Figure 9: Title and abstract of a quantum annealing paper demonstrating clear advantages of quantum annealing in well characterized computationally hard problems of Ising models with frustrating interactions. The demonstrations of clear advantages in some well studied spin models made major impact on the subsequent developments.



Quantum Annealing of a Disordered Magnet
J. Brooke et al.
Science **284**, 779 (1999);
 DOI: 10.1126/science.284.5415.779

Quantum Annealing of a Disordered Magnet

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Traditional simulated annealing uses thermal fluctuations for convergence in optimization problems. Quantum tunneling provides a different mechanism for moving between states, with the potential for reduced time scales. Thermal and quantum annealing are compared in a model disordered magnet, where the effects of quantum mechanics can be tuned by varying an applied magnetic field. The results indicate that quantum annealing hastens convergence to the optimum state.

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Figure 10: Title and abstract of the first experimental demonstration of the advantages of quantum annealing in extracting ground state of disordered magnets. This experimental demonstration had put the quantum annealing trick on firm physical ground.

A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem

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Joshua Lapan,³ Andrew Lundgren,³ Daniel Predu³

A quantum system will stay near its instantaneous ground state if the Hamiltonian that governs its evolution varies slowly enough. This quantum adiabatic behavior is the basis of a new class of algorithms for quantum computing. We tested one such algorithm by applying it to randomly generated hard instances of an NP-complete problem. For the small examples that we could simulate, the quantum adiabatic algorithm worked well, providing evidence that quantum computers (if large ones can be built) may be able to outperform ordinary computers on hard sets of instances of NP-complete problems.

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Figure 11: Title and abstract of zero temperature quantum annealing algorithm for NP-hard problems.

Theory of Quantum Annealing of an Ising Spin Glass

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Roberto Car⁵

Probing the lowest energy configuration of a complex system by quantum annealing was recently found to be more effective than its classical, thermal counterpart. By comparing classical and quantum Monte Carlo annealing protocols on the two-dimensional random Ising model (a prototype spin glass), we confirm the superiority of quantum annealing relative to classical annealing. We also propose a theory of quantum annealing based on a cascade of Landau-Zener tunneling events. For both classical and quantum annealing, the residual energy after annealing is inversely proportional to a power of the logarithm of the annealing time, but the quantum case has a larger power that makes it faster.

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Figure 12: Title and abstract of a paper on application of quantum annealing in estimating the remaining fraction of undesired solutions in some optimization searches in Ising spin-glasses.

Quantum and Classical Glass Transitions in $\text{LiHo}_x\text{Y}_{1-x}\text{F}_4$ C. Ancona-Torres,¹ D. M. Silevitch,¹ G. Aeppli,² and T. F. Rosenbaum^{1,*}¹The James Franck Institute and Department of Physics, The University of Chicago, Chicago, Illinois 60637, USA²London Centre for Nanotechnology and Department of Physics and Astronomy, UCL, London, WC1E 6BT, United Kingdom
(Received 14 January 2008; revised manuscript received 2 April 2008; published 30 July 2008)

When performed in the proper low-field, low-frequency limits, measurements of the dynamics and the nonlinear susceptibility in the model Ising magnet in a transverse field $\text{LiHo}_x\text{Y}_{1-x}\text{F}_4$ prove the existence of a spin-glass transition for $x = 0.167$ and 0.198 . The classical behavior tracks for the two concentrations, but the behavior in the quantum regime at large transverse fields differs because of the competing effects of quantum entanglement and random fields.

References (Partial):

"Research on spin glasses [1] has not only led to deep insights into disordered materials and the glassy state but has generated novel approaches to problems ranging from computer architecture through protein folding to economics. The rugged free energy landscape characteristic of such systems defies usual equilibrium analyses, with pronounced nonlinear responses and history dependence. At low temperatures, and in cases where barriers to relaxation are tall and narrow, quantum mechanics can enhance the ability to traverse the free energy surface [2]. The $\text{LiHo}_x\text{Y}_{1-x}\text{F}_4$ family of materials represents the simplest quantum spin model, the Ising magnet in a transverse field, and it has been an especially useful system to probe the interplay of disorder, glassiness, random magnetic fields, and quantum entanglement [3–10]."

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Figure 13: Title, abstract and some excerpts from a paper extending and clarifying the method used in ref. [8] (see fig. 10) for quantum glasses.

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL
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TOPICAL REVIEW

Optimization using quantum mechanics: quantum annealing through adiabatic evolution

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Abstract We review here some recent work in the field of quantum annealing, alias adiabatic quantum computation. The idea of quantum annealing is to perform optimization by a quantum adiabatic evolution which tracks the ground state of a suitable time-dependent Hamiltonian, where ‘‘h’’ is slowly switched off. We illustrate several applications of quantum annealing strategies, starting from textbook toy-models—double-well potentials and other one-dimensional examples, with and without disorder. These examples display in a clear way the crucial differences between classical and quantum annealing. We then discuss applications of quantum annealing to challenging hard optimization problems, such as the random Ising model, the travelling salesman problem and Boolean satisfiability problems. The techniques used to implement quantum annealing are either deterministic Schrödinger’s evolutions, for the toy models, or pathintegral MonteCarlo and Green’s functionMonteCarlo approaches, for the hard optimization problems. The crucial role played by disorder and the associated non-trivial Landau-Zener tunnelling phenomena is discussed and emphasized.

"The idea of quantum annealing (QA) is an elegant and fascinating alternative to classical thermal simulated annealing (CA); it consists in helping the system escape the local minima using quantum mechanics—by tunnelling through the barriers rather than thermally overcoming them—with an artificial and appropriate source of quantum fluctuations (the counterpart of the temperature) initially present and slowly (adiabatically) switched off. To the best of our knowledge, this idea was first explicitly formulated, and tested in numerical simulations, in the early 1990s [9–11]. In the Ising spin glass context—more precisely, for the infinite range Sherrington-Kirkpatrick model [12]—the idea that the addition of a transverse field quantum term $- \Gamma \sum_i \sigma_i^z$ (σ_i^z being Pauli matrices at site i) might help the system in tunnelling through the infinitely high classical barriers separating the infinitely many metastable states was indeed put forward even earlier, in [13, 14]. More recently, experimental evidence in the disordered Ising ferromagnet $\text{LiHo}_{0.44}\text{Y}_{0.56}\text{F}_4$ in a transverse magnetic field showed that the QA strategy is not only feasible but presumably winning in certain cases [15, 16]."

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Figure 14: Title, abstract and some excerpts for the first review on adiabatic quantum computation/annealing.

Colloquium: Quantum annealing and analog quantum computation

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(Published 5 September 2008)

The recent success in quantum annealing, i.e., optimization of the cost or energy functions of complex systems utilizing quantum fluctuations is reviewed here. The concept is introduced in successive steps through studying the mapping of such computationally hard problems to classical spin-glass problems, quantum spin-glass problems arising with the introduction of quantum fluctuations, and the annealing behavior of the systems as these fluctuations are reduced slowly to zero. This provides a general framework for realizing analog quantum computation.

“SUMMARY AND DISCUSSIONS”

Unlike gate-based quantum computers see, e.g., Ekert and Jozsa, 1996; Nielsen and Chuang, 2000; Galindo and Martin-Delgado, 2002, annealing of a physical system toward the optimal state encoded in the ground state of the final Hamiltonian in the classical limit naturally achieves analog quantum computation. As discussed here, utilization of quantum-mechanical tunneling through classically localized states in annealing of glasses has opened up this new paradigm for analog quantum computation of hard optimization problems through adiabatic reduction of quantum fluctuations. We reviewed the recent success in annealing, or optimizing, the cost functions of complex systems, utilizing quantum fluctuations rather than thermal fluctuations (see Santoro and Tosatti, 2006, for a more technical review). As mentioned, following the early indication by Ray et al. 1989 and the pioneering demonstrations, theoretically by Amara et al. 1993, Finnila et al. 1994, Kadokawa and Nishimori 1998, Farhi, Goldstone, Gutmann, et al. 2001, and Santoro et al. 2002, and experimentally by Brooke et al. 1999, the quantum annealing technique has now emerged as a successful technique for optimization of complex cost functions. The literature exploring its success and also its limitations is also considerably developed at present.”

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Figure 15: Title, abstract and some excerpts from a review on quantum annealing and quantum computation.

Mathematical foundation of quantum annealingSatoshi Morita^{1,a)} and Hidetoshi Nishimori²¹*International School for Advanced Studies (SISSA), Via Beirut 2-4, I-34014 Trieste, Italy*²*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan*

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Quantum annealing is a generic name of quantum algorithms that use quantummechanical fluctuations to search for the solution of an optimization problem. It shares the basic idea with quantum adiabatic evolution studied actively in quantum computation. The present paper reviews the mathematical and theoretical foundations of quantum annealing. In particular, theorems are presented for convergence conditions of quantum annealing to the target optimal state after an infinite-time evolution following the Schrödinger or stochastic Monte Carlo dynamics. It is proved that the same asymptotic behavior of the control parameter guarantees convergence for both the Schrödinger dynamics and the stochastic dynamics in spite of the essential difference of these two types of dynamics. Also described are the prescriptions to reduce errors in the final approximate solution obtained after a long but finite dynamical evolution of quantum annealing. It is shown there that we can reduce errors significantly by an ingenious choice of annealing schedule time dependence of the control parameter without compromising computational complexity qualitatively. A review is given on the derivation of the convergence condition for classical simulated annealing from the view point of quantum adiabaticity using a classical-quantum mapping.

“Let us now turn our attention to QA see Refs. [6–11] In SA, we make use of thermal classical fluctuations to let the system hop from state to state over intermediate energy barriers to search for the desired lowest-energy state. Why then not try quantum-mechanical fluctuations quantum tunneling for state transitions if such may lead to better performance? In QA we introduce artificial degrees of freedom of quantum nature, noncommutative operators, which induce quantum fluctuations. We then ingeniously control the strength of these quantum fluctuations so that the system finally reaches the ground state, just like SA in which we slowly reduce the temperature.”

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Figure 16: Title, abstract and some excerpts from a review on quantum annealing, discussing rigorous mathematical bounds for errors and convergence times in different optimization cases.



WIKIPEDIA

D-Wave Systems

September 2013

"D-Wave Systems, Inc. is a quantum computing company, based in Burnaby, British Columbia. On May 11, 2011, D-Wave System announced D-Wave One, labeled "the world's first commercially available quantum computer," operating on a 128 qubit chip-set[1] using **quantum annealing** [2][3][4][5] to solve optimization problems. In May 2013 it was announced that a collaboration between NASA, Google and the Universities Space Research Association (USRA) launched a Quantum Artificial Intelligence Lab using a 512 qubit D-Wave Two that would be used for research into machine learning, among other fields of study.[6]"

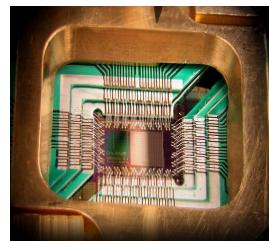
History

D-Wave was founded by Haig Farris (former chair of board), Geordie Rose (CTO and former CEO), Bob Wiens (former CFO), and Alexandre Zagoskin (former VP Research and Chief Scientist). Farris taught an entrepreneurship course at UBC (University of British Columbia), where Rose obtained his Ph.D. and Zagoskin was a postdoctoral fellow. The company name refers to their first qubit designs, which used d-wave superconductors.

D-Wave operated as an offshoot from UBC[citation needed], while maintaining ties with the Department of Physics and Astronomy. It funded academic research in quantum computing, thus building a collaborative network of research scientists. The company collaborated with several universities and institutions, including UBC[citation needed], IPHT Jena[citation needed], Université de Sherbrooke[citation needed], University of Toronto[citation needed], University of Twente[citation needed], Chalmers University of Technology[citation needed], University of Erlangen[citation needed], and Jet Propulsion Laboratory[citation needed]. These researchers worked with D-Wave scientists and engineers. Some of D-Wave's peer-reviewed technical publications come from this period. Some publications have D-Wave employees as authors, while others include employees of their partners as well or only. As of 2005, these partnerships were no longer listed on D-Wave's website.[18][19]

D-Wave operated from various locations in Vancouver, Canada, and laboratory spaces at UBC before moving to its current location in the neighboring suburb of Burnaby."

Photograph of a chip constructed by D-Wave Systems Inc., designed to operate as a 128-qubit superconducting adiabatic quantum optimization processor, mounted in a sample holder.



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LETTER

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Quantum annealing with manufactured spins

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Many interesting but practically intractable problems can be reduced to that of finding the ground state of a system of interacting spins; however, finding such a ground state remains computationally difficult¹. It is believed that the ground state of some naturally occurring spin systems can be effectively attained through a process called quantum annealing [2,3]. If it could be harnessed, quantum annealing might improve on known methods for solving certain types of problem [4,5]. However, physical investigation of quantum annealing has been largely confined to microscopic spins in condensed-matter systems [6–12]. Here we use quantum annealing to find the ground state of an artificial Ising spin system comprising an array of eight superconducting flux quantum bits with programmable spin–spin couplings. We observe a clear signature of quantum annealing, distinguishable from classical thermal annealing through the temperature dependence of the time at which the system dynamics freezes. Our implementation can be configured *in situ* to realize a wide variety of different spin networks, each of which can be monitored as it moves towards a low-energy configuration [13,14].

This programmable artificial spin network bridges the gap between the theoretical study of ideal isolated spin networks and the experimental investigation of bulk magnetic samples. Moreover, with an increased number of spins, such a system may provide a practical physical means to implement a quantum algorithm, possibly allowing more-effective approaches to solving certain classes of hard combinatorial optimization problems.

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Figure 18: Title and abstract of the first paper by D-Wave group giving the basic architecture of their quantum annealing precessor.

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Finding low-energy conformations of lattice protein models by quantum annealing

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Lattice protein folding models are a cornerstone of computational biophysics. Although these models are a coarse grained representation, they provide useful insight into the energy landscape of natural proteins. Finding low-energy threedimensional structures is an intractable problem even in the simplest model, the Hydrophobic-Polar (HP) model. Description of protein-like properties are more accurately described by generalized models, such as the one proposed by Miyazawa and Jernigan (MJ), which explicitly take into account the unique interactions among all 20 amino acids. There is theoretical and experimental evidence of the advantage of solving classical optimization problems using quantum annealing over its classical analogue (simulated annealing). In this report, we present a benchmark implementation of quantum annealing for lattice protein folding problems (six different experiments up to 81 superconducting quantum bits). This first implementation of a biophysical problem paves the way towards studying optimization problems in biophysics and statistical mechanics using quantum devices.

"Harnessing quantum-mechanical effects to speed up the solving of classical optimization problems is at the heart of quantum annealing algorithms (QA) [11–15]. There is theoretical [11,12,16–18] and experimental [19] evidence of the advantage of solving classical optimization problems using QA [11–14] over its classical analogue (simulated annealing [20]). In QA, quantum mechanical tunneling allows for more efficient exploration of difficult potential energy landscapes such as that of classical spin-glass problems. In our implementation of lattice folding, quantum fluctuations (tunneling) occurs between states representing different model protein conformations or folds."

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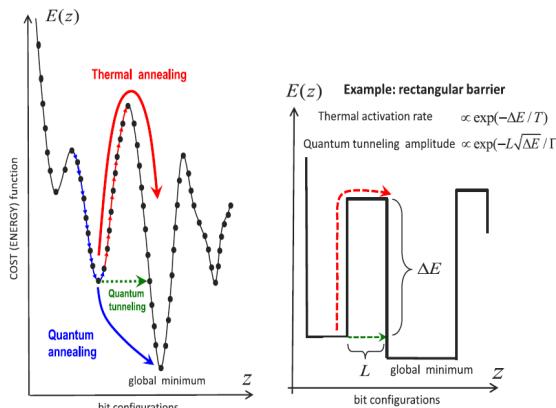
Figure 19: Title, abstract and excerpts from the first major paper supporting the claim of D-Wave quantum computer used in searching the low energy conformations of lattice protein model, reported by the Harvard University group.

A Near-Term Quantum Computing Approach for Hard Computational Problems in Space Exploration

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Advances in quantum hardware mean that empirical testing of one particular family of quantum algorithms, Quantum Annealing algorithms, may be possible in the near term. Theoretical studies and classical simulations suggest that Quantum Annealing [9–12] can provide dramatic improvements, both in the algorithmic runtime and quality of the solutions, to many instances of hard optimization problems where state-of-the-art classical approaches fail.

“ Fig. 2(b) illustrates a simple example [19] in which Quantum Annealing can perform better than Simulated (Thermal) Annealing. The tunneling rate through a rectangular potential barrier of height ΔE and length L is $\propto \exp(-L\sqrt{\Delta E}/\Gamma)$, where Γ is a quantum annealing constant analogous to the temperature in SA. Unlike the classical thermal activation rate for SA to cross the barrier, $\exp(-\Delta E/T)$, quantum tunneling depends not only on the barrier height but also on its width. We emphasize that the quantum tunneling rate exponent grows only as $\sqrt{\Delta E}$, whereas the classical activation rate exponent displays a faster linear growth with ΔE . Therefore, QA will have a tendency to beat SA in landscapes dominated by *high narrow barriers*, with $L \ll \sqrt{\Delta E}$ for typical values [18].

Fig. 2

An example of quantum annealing with a rectangular barrier of width L and height ΔE [19]. The tunneling rate through the barrier is $\propto \exp(-L\sqrt{\Delta E}/\Gamma)$, where $\Gamma = \Gamma(t)$ is a QA constant that is gradually reduced to zero throughout the algorithm. By comparison, the rate of thermal over-the-barrier activation in SA is $\propto \exp(-\Delta E/T)$, where $T = T(t)$ is the annealing temperature that is gradually reduced to zero by the end of SA. ”

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Figure 20: Title, abstract and some excerpts of a paper (from NASA Ames Research centre, Jet Propulsion Lab, CALTECH and University of Southern California) explaining the basic principle of quantum annealing, D-Wave computers and the possibilities of searching solutions of hard computational problems in space science and technology (website: <http://arxiv.org/abs/1204.2821>).

Invited Paper

Quantum Computing vs. Coherent Computing

NEW
GENERATION
COMPUTING
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"An Ising model also provides a prototype framework for studying various magnetic orders in frustrated spin lattice and random spin glasses.[6, 7] Hence, an Ising machine that can find a ground state of Eq. (1) efficiently has been extensively searched in both classical and quantum domains [8]. Quantum annealing is proposed to solve Ising models by utilizing quantum uncertainty, more specifically quantum mechanical tunneling across a potential energy landscape (PEL) [9-14.] Experimental realization of quantum annealing employed either a sample of real magnetic crystal[15, 16] or molecular NMR technique [17]."

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Figure 21: Title and some excerpts of a paper (from National Institute of Informatics, Tokyo and Stanford University, California) comparing different quantum algorithms (including quantum annealing) in hardware platforms.

Quantum Speedup by Quantum Annealing

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We study the glued-trees problem from A. M. Childs, R. Cleve, E. Deotto, E. Farhi, S. Gutmann, and D. Spielman, in *Proceedings of the 35th Annual ACM Symposium on Theory of Computing* (ACM, San Diego, CA, 2003), p. 59, in the adiabatic model of quantum computing and provide an annealing schedule to solve an oracular problem exponentially faster than classically possible. The Hamiltonians involved in the quantum annealing do not suffer from the so-called sign problem. Unlike the typical scenario, our schedule is efficient even though the minimum energy gap of the Hamiltonians is exponentially small in the problem size. We discuss generalizations based on initial-state randomization to avoid some slowdowns in adiabatic quantum computing due to small gaps.

Quantum annealing is a powerful heuristic to solve problems in optimization [1,2]. In quantum computing, the method consists of preparing a low-energy or ground state $|\psi\rangle$ of a quantum system such that, after a simple measurement, the optimal solution is obtained with large probability. $|\psi\rangle$ is prepared by following a particular annealing schedule, with a parametrized Hamiltonian path subject to initial and final conditions. A ground state of the initial Hamiltonian is then transformed to $|\psi\rangle$ by varying the parameter slowly. In contrast to more general quantum adiabatic state transformations, the Hamiltonians along the path in quantum annealing are termed *stoquastic* and do not suffer from the so-called *numerical sign problem* [3]: for a specified basis, the off-diagonal Hamiltonian-matrix entries are nonpositive [4]. This property is useful for classical simulations [2].

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Figure 22: Title, abstract and some excerpts of a paper discussing possible speed-ups in quantum annealing computers.

Parameter Tuning Patterns for Random Graph Coloring with Quantum Annealing

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Abstract

Quantum annealing is a combinatorial optimization technique inspired by quantum mechanics. Here we show that a spin model for the k -coloring of large dense random graphs can be field tuned so that its acceptance ratio diverges during Monte Carlo quantum annealing, until a ground state is reached. We also find that simulations exhibiting such a diverging acceptance ratio are generally more effective than those tuned to the more conventional pattern of a declining and/or stagnating acceptance ratio. This observation facilitates the discovery of solutions to several well-known benchmark k -coloring instances, some of which have been open for almost two decades.

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"Quantum annealing [1–6] is a combinatorial optimization technique that employs a quantum fluctuation parameter C for the purpose of escaping local minima. The parameter C is often a transverse magnetic field in the presence of a low temperature T [3]. Quantum annealing studies have been carried out on NP-hard [7] problems such as the traveling salesman problem [8] and the graph coloring problem [9,10]."

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Figure 23: Title, abstract and some excerpts from a paper discussing successes of quantum annealing algorithms for graph-coloring problems.

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Experimental signature of programmable quantum annealing

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Quantum annealing is a general strategy for solving difficult optimization problems with the aid of quantum adiabatic evolution. Both analytical and numerical evidence suggests that under idealized, closed system conditions, quantum annealing can outperform classical thermalization-based algorithms such as simulated annealing. Current engineered quantum annealing devices have a decoherence timescale which is orders of magnitude shorter than the adiabatic evolution time. Do they effectively perform classical thermalization when coupled to a decohering thermal environment? Here we present an experimental signature which is consistent with quantum annealing, and at the same time inconsistent with classical thermalization. Our experiment uses groups of eight superconducting flux qubits with programmable spin–spin couplings, embedded on a commercially available chip with 4100 functional qubits. This suggests that programmable quantum devices, scalable with current superconducting technology, implement quantum annealing with a surprising robustness against noise and imperfections.

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Figure 24: Title and abstract of a paper reporting on quantum signatures in D-Wave machines and on their ‘surprising robustness against noise and imperfections’.

Experimental Determination of Ramsey Numbers

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Ramsey theory is a highly active research area in mathematics that studies the emergence of order in large disordered structures. Ramsey numbers mark the threshold at which order first appears and are extremely difficult to calculate due to their explosive rate of growth. Recently, an algorithm that can be implemented using adiabatic quantum evolution has been proposed that calculates the two-color Ramsey numbers $R(m, n)$. Here we present results of an experimental implementation of this algorithm and show that it correctly determines the Ramsey numbers $R(3, 3)$ and $R(m, 2)$ for $4 \leq m \leq 8$. The $R(8, 2)$ computation used 84 qubits of which 28 were computational qubits. This computation is the largest experimental implementation of a scientifically meaningful adiabatic evolution algorithm that has been done to date.

Figure 25: Title and abstract from ref. [27] claiming a precise success (though very limited in scope) in a problem of party size calculation (with conflicting choices among the party members), using the D-wave computer.

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Quantum annealing with more than one hundred qubits

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Quantum technology is maturing to the point where quantum devices, such as quantum communication systems, quantum random number generators and quantum simulators, may be built with capabilities exceeding classical computers. A quantum annealer, in particular, solves hard optimisation problems by evolving a known initial conformation at non-zero temperature towards the ground state of a Hamiltonian encoding a given problem. Here, we present results from experiments on a 108 qubit D-Wave One device based on superconducting flux qubits. The strong correlations between the device and a simulated quantum annealer, in contrast with weak correlations between the device and classical annealing or classical spin dynamics, demonstrate that the device performs quantum annealing. We find additional evidence for quantum annealing in the form of small-gap avoided level crossings characterizing the hard problems. To assess the computational power of the device we compare it to optimised classical algorithms.

"Annealing a material by slow cooling is an ancient technique to improve the properties of glasses, metals and steel that has been used for more than seven millennia [1]. Mimicking this process in computer simulations is the idea behind simulated annealing as an optimisation method [2], which views the cost function of an optimisation problem as the energy of a physical system. Its configurations are sampled in a Monte Carlo simulation using the Metropolis algorithm [3], escaping from local minima by thermal fluctuations to find lower energy configurations. The goal is to find the global energy minimum (or at least a close approximation) by slowly lowering the temperature and thus obtain the solution to the optimisation problem. The phenomenon of quantum tunneling suggests that it can be more efficient to explore the state space quantum mechanically in a quantum annealer [4-6]. In simulated quantum annealing [7, 8], one makes use of this effect by adding quantum fluctuations, which are slowly reduced while keeping the temperature constant and positive ultimately ending up in a low energy configuration of the optimisation problem."

Unlike adiabatic quantum computing [16], which has a similar schedule but assumes fully coherent adiabatic ground state evolution at zero temperature, quantum annealing [4-6, 10] is a positive temperature method involving an open quantum system coupled to a thermal bath."

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Figure 26: Title, abstract and some excerpts from a paper by scientists from University of Southern California, University of California, ETH Zurich and Microsoft Research, reporting on the precise quantum nature of the performance of the D-Wave computer, compared with that of classical or conventional computers (website: <http://arxiv.org/abs/1304.4595>).

Breakthrough in Quantum Computation

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In disciplines ranging over the social, physical and biological sciences, one sometimes encounters processes or phenomena involving a search for the global minimum of an appropriately defined function over a set of candidate states or configurations. The search problem acquires complexity when the function (may be free energy or a cost function) has a highly rugged structure consisting of several local minima (valleys) separated by barriers (hills). In this case, the problem of locating the global minimum becomes non-trivial.

A well-known procedure for finding the global minimum, termed simulated annealing, allows a system to escape the traps set by the local minima , thus facilitating the system's journey to the global minimum. This is achieved via progressive changes in the temperature from high to low values. In 1989, P. Ray, B. K. Chakrabarti and A. Chakrabarti proposed [Physical Review B **39**, 11828 (1989), see also A. Das and B. K. Chakrabarti, Rev. Mod. Phys. **80**, 1061 (2008)] a new and more efficient method, now known as quantum annealing, to locate the global minimum in a rugged landscape utilizing the principle of quantum mechanical tunneling. In simulated annealing, the system utilizes thermal energy to cross a barrier separating neighbouring minima whereas quantum tunneling allows the system to tunnel through a barrier rather than cross over it. The seminal proposal of Ray et al. was taken up by other groups in the world including an experimental demonstration by Brooke et al. [Science 284, 779 (1999)] that quantum annealing is superior to simulated annealing in finding the lowest energy state, the ground state, of a disordered magnet.

In 2011, a Canadian company, D-wave Systems, introduced the first commercial quantum annealer to the market under the tag D-Wave One. The system was an impressive achievement in quantum computation with a 128 qubit (quantum bit) processor chipset. In 2011 itself, the Lockheed Martin Corporation, USA purchased a D-Wave One system for its operations. In May 2013, a consortium of Google, NASA AMES and the non-profit Universities Space Research Association has purchased a quantum computer from the D-Wave Systems with 512 qubits. The Academia are also not far behind in the use of the quantum computer based on quantum annealing. The power of the quantum computer has already been demonstrated in finding the ground state of an engineered spin system [Johnson et al., Nature 473, 194 (2011)] and a study by a group of Harvard University scientists on finding low-energy conformations of lattice protein models by quantum annealing [Perdomo-Ortiz et al., Scientific Reports 2: 571 (2012)]. This has been followed by other important studies in the last few months presaging a significant surge in research activity on quantum annealing based quantum computation [see e.g., Boixo, et al, Nature Comm. 4, 3067 (2013), and arXiv:1304.4595 from Universities of California and ETH, Zurich].

Physicists have been trying since the last few decades to come up with practical realizations of quantum computers utilizing quantum mechanical principles. The usual route in this endeavour has been to utilize quantum correlations in the form of a quantity called entanglement and logic gate operations via specific quantum mechanical transformations. The quantum computers which could be constructed following this route are few-qubit systems. The quantum annealer, on the other hand, adopts a different strategy, namely, quantum tunneling for the purpose of computation and is now operational with the number of qubits of the order of a few hundreds. The annealer is, however, designed to address only a specific class of problems but lays open the possibility of exploring unconventional routes towards quantum computation.

Research activity on quantum computation is of significant importance considering the fact that quantum computers are expected to be more efficient and versatile than classical computers. Prof. Bikas K. Chakrabarti of the Saha Institute of Nuclear Physics, Calcutta (the idea of quantum annealing based computation was first mooted by his group) has the following to say on the origin and prospects of quantum annealing based computation: "The essential reason for the NP hardness of computational problems seems to be the inability to get out of a local minimum past a barrier (of height N in time less than exponential in N , using classical mechanics). Noting that even an "infinitely high" (but narrow) delta function like barrier is penetrable (in finite time; generally perhaps in polynomial in N time!) due to quantum tunneling, it was fun for us (in 1989) to suggest possible success in such computational problems using quantum tunneling mechanics, instead of classical mechanics. It was shocking to see the criticisms in the subsequent literature claiming that the decoherence among the phases of the tunneling waves from such random barriers will lead to localization and total failure of any such idea! It is great fun again to see in the last decade that such an idea is indeed working and quantum annealing computers, based on it are already in the market. Distinguished universities and research centres are now reporting major successes of such quantum computers in solving well characterized computational problems and also often acknowledging our original idea!" .

Figure 27: A very recent news note by Bose [30] conveying the excitement.

Further proof for controversial quantum computer

Is the world's only commercial quantum computer really a quantum device, or a just regular computer in disguise? Controversy has long swirled around the computer produced by D-Wave, a company based near Vancouver, Canada. Now a paper published on the arXiv preprint server takes a step forward in showing that it really does operate on a quantum level. D-Wave's computer is a special type of quantum device: its quantum bits (or qubits) seek out a low-energy state that represents the answer to a given problem. Unlike a universal computer, this kind of computer, called an annealer, cannot answer any question thrown at it. Instead, it can only answer 'discrete optimization' problems. This is a type of problem where a set of criteria are all fighting to be simultaneously met, and there is one best solution that meets the most of them — one example being the simulation of protein folding, in which the system seeks a state of minimal free energy. The hope is that a quantum annealer should be able to solve these problems much more quickly than a classical one. The company's current top-line computer has 512 qubits. In some ways, this is miles ahead of work in universal quantum computers, where academics struggle to get just a handful of qubits to operate usefully. But even D-Wave admits that it doesn't know exactly how its computer works, and critics have complained that it might not be quantum at all. Instead, it could be using classical physics to crunch calculations. In 2011, a group led by scientists working with D-Wave published a paper in Nature with evidence that their 8-qubit system was working on a quantum level: it responded to temperature changes as expected for a quantum device. Now, a group of independent scientists follows that up by showing that the 128-qubit version of the D-Wave computer (or at least the 108 functioning qubits in the specific computer that they analysed) also seems to be behaving quantumly. Simulations of quantum versus classical annealers show that a classical one has a fairly uniform probability of solving a problem correctly; a quantum device should instead have a low probability of success at solving hard problems, and a high probability of success solving easy ones. This is what they see with the D-Wave computer. Scott Aaronson, a theoretical computer scientist at the Massachusetts Institute of Technology in Cambridge who has historically been sceptical of D-Wave's claims, says that he is fairly convinced by the data, but that there are plenty of important questions remaining — including whether the current or future versions of the D-Wave computer will actually be any faster than classical machines. The new paper, Aaronson notes, shows that a quantum annealer is actually expected to be slower than a classical one in many circumstances. "It may be that they really have built a quantum annealing device, which is academically very interesting, but that it provides no [speed] advantage. That may be the case," says Aaronson. The paper's authors include several researchers from the University of Southern California in Los Angeles, which has a deal to use and experiment with the D-Wave computer recently purchased by aerospace company Lockheed Martin. The co-author contacted by this reporter declined to comment on the work until it appears in a peer-reviewed publication. As of March, that group now has a 512-qubit version of the D-Wave to play with, which could start to show a speed advantage over classical annealers.

Figure 28: Nature news blog regarding the recent reporting by scientists confirming the quantum nature of computation in D-Wave computers (website: <http://blogs.nature.com/news/2013/04/further-proof-for-controversial-quantum-computer.html>).

4 Summary and conclusions

Approximate solutions of computationally hard problems were obtained more easily in Boltzmann-like machines which employ stochastic searches and (classical or thermal) annealing rather than employing sequential search methods. It was noted that for NP-hard problems, the effective cost function landscape (in the solution state or configuration space) becomes extremely rugged as discussed in the text. Even with classical annealing tricks the probability of escape from a local minimum to another lower one separated by a barrier of height of order N (the problem size) decreases as $\exp(-N)$, implying that the time to arrive at the solution is not bounded by any polynomial in N . Noting that the quantum tunneling probability across such a barrier decreases with width of the barrier (becoming finite in the delta function barrier limit), Ray et al. [1] proposed in 1989 that quantum tunneling might help solving NP-hard problems in polynomial (in N) time! Subsequently, the researches by Finnila et al. [5], Kadowaki and Nishimori [7], Brooke et al. [8] and Ferhi et al. [9] led to the robust development of quantum annealing technique, indicating clearly the possible development of analog quantum computers with such tricks (cf. Santoro and Tosatti [13], Das and Chakrabarti [14]). With the major breakthrough achieved by D-Wave computers [18] (with indications [28] of search time $\sim \exp(N^\alpha)$; $\alpha < 1$), a new era in quantum computing has started: See e.g., [19, 22, 23, 25, 26] etc. as a few chosen examples only of the rapidly growing publications which (as shown in Figs. 19, 20, 21, 24, 26) indicate also the role of the original papers in this remarkable development. We note that the initial contributions by our Indian colleagues had indeed been pioneering, though the follow-up researches and contributions have been rather slow.

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