MTP 290 - Problem Set 4

(1) Write a MATLAB script to solve the linear system $Ax = b$, where A is an invertible diagonal matrix. Taking $A = \text{diag}(1, 2, 3)$ and $b = [1, 1, 1]^T$, solve for x.

(2) Write MATLAB code to implement the forward substitution method to solve the linear system $Ax = b$, where A is a non-singular lower triangular matrix. Use it to solve for x if A and b are given as follows:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0.5 & 1 \end{pmatrix}$$

and $b = [1, 2, 1]^T$.

(3) Write a MATLAB script for implementing the backward substitution method to solve the system $Ax = b$, where A is a non-singular upper triangular matrix. Use this code to solve for x if A and b are as follows:

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -3 \\ 0 & 0 & -6.5 \end{pmatrix}$$

and $b = [1, 7, 6.5]^T$.

(4) Implement Gauss elimination method for solving a system of linear equations $Ax = b$, where A is a non-singular matrix.

(5) Use Gauss elimination method to solve

$$\begin{aligned} 4x_1 + x_2 - x_3 &= -2 \\ 5x_1 + x_2 + 2x_3 &= 4 \\ 6x_1 + x_2 + x_3 &= 6. \end{aligned}$$

(6) Consider

$$A = \begin{pmatrix} 10 & -7 & 0 \\ -3 & 2.099 & 6 \\ 5 & -1 & 5 \end{pmatrix}.$$

Find the determinant of A using Gauss elimination method.

(7) Use of Gauss elimination method on the coefficient matrix

$$A = \begin{pmatrix} 25 & c & 1 \\ 64 & a & 1 \\ 144 & b & 1 \end{pmatrix}.$$

reduces it to

$$B = \begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{pmatrix}.$$

What is the determinant of A?

(8) The following data is given for the velocity of the rocket as a function of time. To find the velocity at $t = 21$s, you are asked to use a quadratic polynomial, $v(t) = at^2 + bt + c$ to approximate the velocity profile.

t(s)	0	14	15	20	30	35
$v(t)$m/s	0	227.04	362.78	517.35	602.97	901.67

The correct set of equations that will find a, b and c are

$$\begin{aligned} 225a + 15b + c &= 362.78 \\ 400a + 20b + c &= 517.35 \\ 900a + 30b + c &= 602.97. \end{aligned}$$

Find the velocity at $t = 21$s.

(9) Implement the Gauss elimination method with partial pivoting to solve a system of linear equations $Ax = b$, where A is a non-singular matrix.

(10) Solve the following linear system by Gauss elimination method and Gauss elimination method with partial pivoting.

$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 2z &= 5 \\ 3x + 4y + 4z &= 12. \end{aligned}$$

(11) Apply the modified solver implemented in Problem 9 to solve the following system. Further check the difference between the computed solution x and the result of MATLAB built in solver $A\backslash b$.

$$\begin{aligned} x_1 + x_2 + x_4 &= 2 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ 4x_1 - x_2 - 2x_3 + 2x_4 &= 0 \\ 3x_1 - x_2 - x_3 + x_4 &= -3. \end{aligned}$$

(12) Implement Gauss Jordan method to solve a system of linear equations $Ax = b$, where A is a non-singular matrix.

(13) Redo the problem 5 and 10 using Gauss-Jordan method.