MTP 290: PRACTICE PROBLEMS

(1) Write MATLAB script to implement the Euler's method to solve the initial value problems(IVPs) given in problem 1 to problem 3.

(2) Consider the IVP
$$y' = \frac{y \ln y}{x}, \; y(2) = e.$$
Use Euler's method with h = 0.1 to obtain the approximation to y(3).

(3) Consider the IVP
$$y' = y - x, \; y(0) = \frac{1}{2}.$$
Use Euler's method with h = 0.1 and h = 0.05 to obtain the approximation to y(1). Given that the exact solution to the IVP is
$$y(x) = x + 1 - \frac{e^x}{2},$$
compare the true errors in the two approximations to y(1).

(4) Consider the IVP
$$y' = 2xy^2, \; y(0) = 0.5.$$
Use modified Euler's method with h = 0.1 to obtain the approximation to y(1). Write down the MATLAB code for the same.

(5) Write MATLAB script to implement the Runge-Kutta (RK) methods of order 2 and order 4 to solve the below given IVPs.

(6) Redo problem no. 3 for Runge-Kutta method of order 2.

(7) Consider the IVP
$$y' = \frac{y}{x} - \left(\frac{y}{x}\right)^2, \; x \in [1, 2], \; y(1) = 1.$$
Use Runge-Kutta method of order 2 with h = 0.1 to obtain the approximation to y(2).

(8) Redo problem no. 3 for Runge-Kutta method of order 4.

(9) Consider the IVP
$$y' = xe^{3x} - 2y, \; x \in [0, 1], \; y(0) = 0.$$
Use Runge-Kutta method of order 4 method with h = 0.5 to obtain the approximation to y(1).

(10) Solve the following IVPs using the Modified Euler's method and Runge-Kutta method of order four:

a. $y' = 1 + (x - y)^2$, $2 \leq x \leq 3$, $y(2) = 1$, with step size $h = 0.5$, actual solution $y(x) = x + \frac{1}{1-x}$.

b. $y' = 1 + y/x$, $1 \leq x \leq 2$, $y(1) = 2$, with step size $h = 0.25$, actual solution $y(x) = x \ln x + 2x$.

c. $y' = \cos 2x + \sin 3x$, $0 \leq x \leq 1$, $y(0) = 1$, with step size $h = 0.25$, actual solution $y(x) = \frac{1}{2} \sin 2x - \frac{1}{3} \cos 3x + \frac{4}{3}$.