

MTP 290-Practice Problems
Numerical Integration

1. Evaluate the integral $\int_0^4 (x^2 + \cos x) dx$ by using midpoint formula and composite midpoint rule with $n=5$.
2. Use Trapezoidal rule with $n = 8$ to estimate

$$\int_1^5 \sqrt{1+x^2} dx.$$

3. The following points were found empirically.

| | | | | | | |
|---|-----|-----|-----|-----|-----|-----|
| x | 2.1 | 2.4 | 2.7 | 3.0 | 3.3 | 3.6 |
| y | 3.2 | 2.7 | 2.9 | 3.5 | 4.1 | 5.2 |

Use composite Trapezoidal rule to evaluate $\int_{2.1}^{3.6} y dx$.

4. Approximate the integral of $f(x)=x^3$ on the interval $[1,2]$ by using composite trapezoidal method
 - (a) with four sub intervals,
 - (b) with eight sub intervals,
(Which approximation is much closer to the correct answer)
 - (c) Compute the true error in both the cases.
5. Use Simpson rule to evaluate
 - (a) $\int_0^{\pi/3} \cos^2 x dx$.
 - (b) Use your answer to part (a) to deduce an approximate value of integral $\int_0^{\pi/3} \sin^2 x dx$.
6. Use Simpson's rule with 5 ordinates to find an approximate value for the integral

$$\int_4^6 \frac{1}{3-\sqrt{x}} dx.$$

7. Approximate the integral of $f(x)=x^3$ on the interval $[1,2]$ by using composite Simpson's method
 - (a) with four sub intervals,
 - (b) with eight sub intervals,
(Which approximation is much closer to the correct answer)
 - (c) Compute the true error in both the cases.
8. Using Trapezoidal Rule and Simpson's rule with $n=4$ to approximate the value of the following integral and compute the true errors and approximation errors

$$\int_0^2 e^{x^2} dx.$$

9. Evaluate the following integral by using one point Gauss quadrature and compute the true error.

$$\int_0^{\pi/2} x \sin x \, dx$$

10. Redo Problem 5 by using two point Gauss quadrature formula.