CS323 : Project Report

VEHICLE ROUTING PROBLEM (VRP)

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INTRODUCTION

The **vehicle routing problem** (**VRP**) is a combinatorial optimization and integer programming problem which questions "What is the optimal set of routes for a vehicle to traverse in order to deliver to a given set of customers”? In this problem we are provided with a list of customer coordinates and we set our depot at origin. Our goal is to find what is the shortest possible route that starts at the depot and visits each customer exactly once and returns back to the depot.

VRP has drawn enormous interest from many researchers during the last decades because of its vital role in planning of distribution systems and logistics in many sectors such as garbage collection, mail delivery, snow ploughing and task sequencing.

VRP falls under the class of local search problems. In this project three local search techniques i.e. hill climbing with random walk, hill climbing with random restart and simulated annealing are applied in order to get the solution and comparison among these techniques is done. Complexity of a VRP is very high as for a given 50 customer coordinates there are 50! (>10^64) routes possible for a vehicle to visit each customer. Such a large number of possible solutions is what makes this problem challenging and difficult to solve.

PROBLEM DEFINITION

VRP can be modelled as an undirected weighted graph, such that customers are the graph's vertices, paths are the graph's edges, and a path's distance is the edge's weight. It is a minimization problem starting and finishing at a specified vertex i.e. depot after having visited each other vertex i.e. customers exactly once.
In VRP data is needed in the form of the coordinates of the customers. Initial state is defined as an arbitrary solution i.e. a complete route and goal state is the optimal route. In this project Cost function evaluates complete distance travelled by a vehicle in a single possible route (solution) and there is one another function left i.e. neighbor function which gives all the possible neighbors of a current solution by swapping two customers position in the current route. local search technique is used to reach the goal state. So we start with an initial solution and will optimize our cost function in order to get the optimal solution.

BACKGROUND SURVEY

VRP is first appeared in a paper by George Dantzig and John Ramser in 1959,[1] in which the first algorithmic approach was written and was applied to petrol deliveries.In 1964, Clarke and Wright improved on Dantzig and Ramser's approach using an effective greedy approach called the savings algorithm. Taillard (1993) and Rochat and Taillard (1995) have applied Tabu Search (TS) to many VRP variants, where the best known results to benchmark VRPs were obtained. Various authors have reported similar results, obtained using TS, or Simulated Annealing (SA) (Baker & Ayechew 2003). However, it has been reported by Renaud et al. (1996) that such heuristics require considerable computing times and several parameter settings.

DISCUSSIONS

Using a simple greedy approach or hill climbing search algorithm to solve this problem will mostly end up in stucking at local minima rather than reaching global minima since when the state space has local minima, any search that moves only in greedy direction can not be complete. Using stochastic version of hill climbing i.e. hill climbing with random walk and random restart will help in getting out of these local minimas and will eventually end up in providing a better solution (close to global minima) then the greedy search in less amount of time and if luck favours then we may end up at global minima in very less computation and if luck does not favour then there may be the chances that we do not reach global minima even after large number of computation . That is why hill climbing with random walk and random restart is asymptotically complete. But such a solution is not something that we can rely upon but if given a hard time constraint then this solution can be applied as it definitely gives the best computed solution during the time constraint. For a given data set of 9 customer below images shows the optimal path found by hill climbing, hill climbing with random restart and hill climbing with random walk

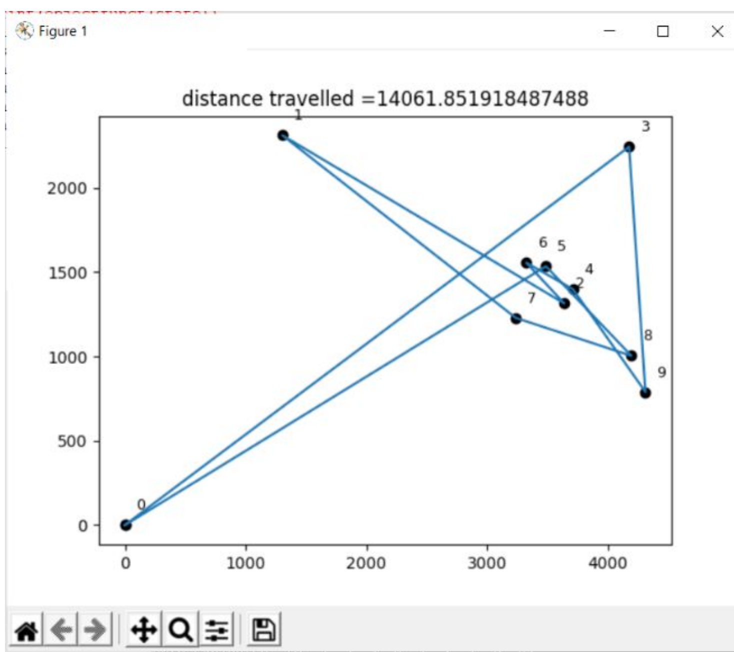

Fig. a) Optimal path from using greedy approach (stuck at local minima)

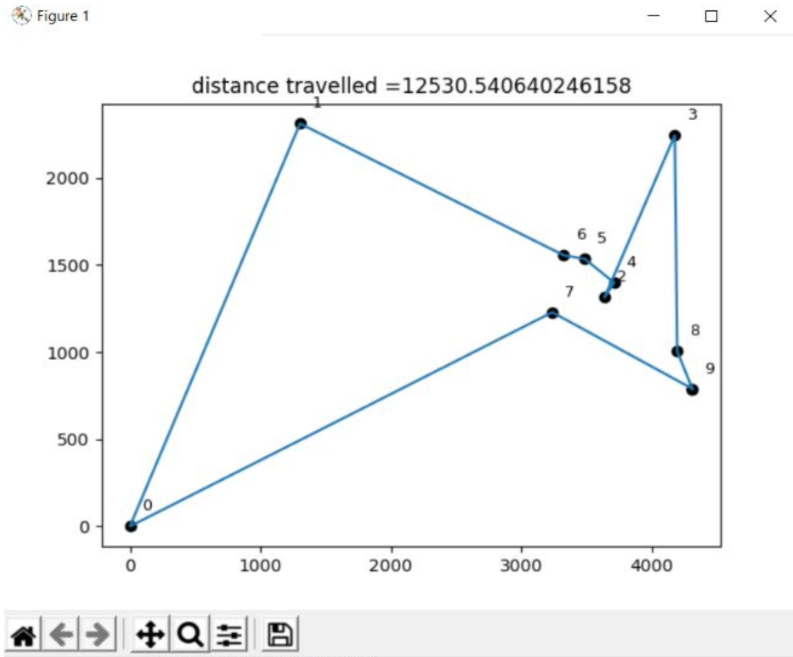

Fig. b) optimal path (close to global minima) using hill climbing with random restart (with 50 iteration)

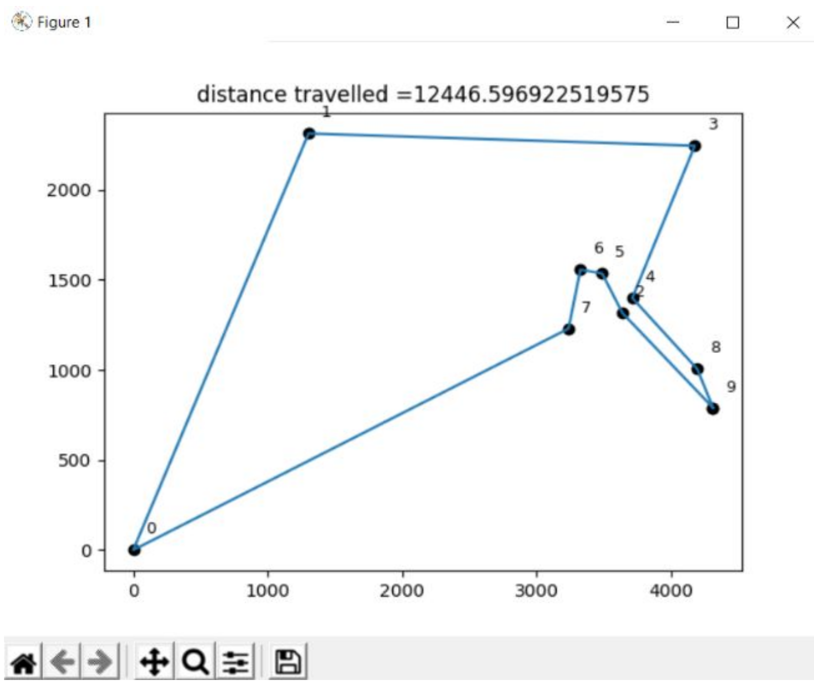

Fig. c) optimal path using hill climbing with random walk(with 50 iteration)

The above picture shows that with small number of customers given and using large number of iteration we can reach global minima with great probability using this stochastic hill climbing.But now the question comes are these stochastic hill climbing version perform efficiently in case of when data of like 50 or more customers are given.

Answer to this question is no, as for 50 customer data even using 1000 iteration stochastic hill climbing does not provide a solution which is close to optimal solution more generally. Below images show the output of stochastic hill climbing on 48 customers data set.

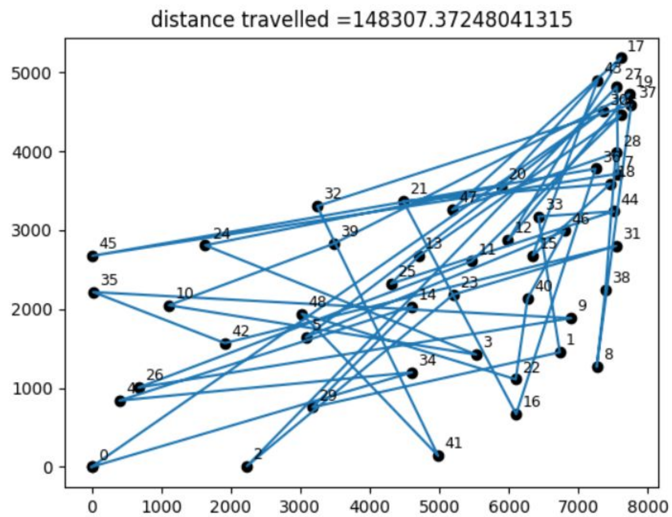

Fig. d) solution find by hill climbing with random walk (1000 iterations)

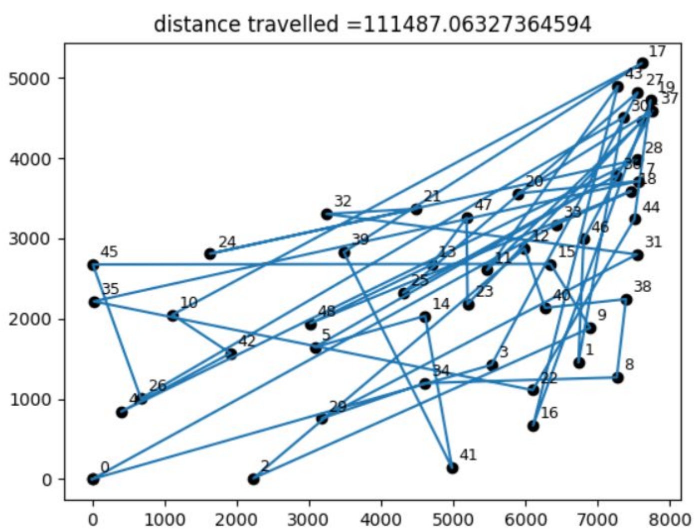

Fig. e) solution find by hill climbing with random restart (1000 iterations)

Optimal distance travelled for the above shown data set of 48 costumers is in between 35k to 40k. But the solutions found by these stochastic hill climbing techniques are greater than 100k which is not even close to an optimal solution. Hence stochastic hill climbing technique tends to fail in case of large data given which shows the complexity of this problem and shows that we need to do more improvement in order to construct a reliable engineer based solution.

Solution to this issue is given by Simulated annealing (concept derived from metal annealing) search in which whenever we stuck at a local minima then a shake is provided in order to get out of it and reach global minima. The shake is given by formula $e^{(h(A)-h(A'))/T}$ where h(a) is current solution cost , h(a') is adjacent neighbor solution cost and T is temperature which is given some initial value that computed according to several parameter setting and based on the problem itself. The solution for the same data of 48 customers given by Simulated Annealing is shown below image.

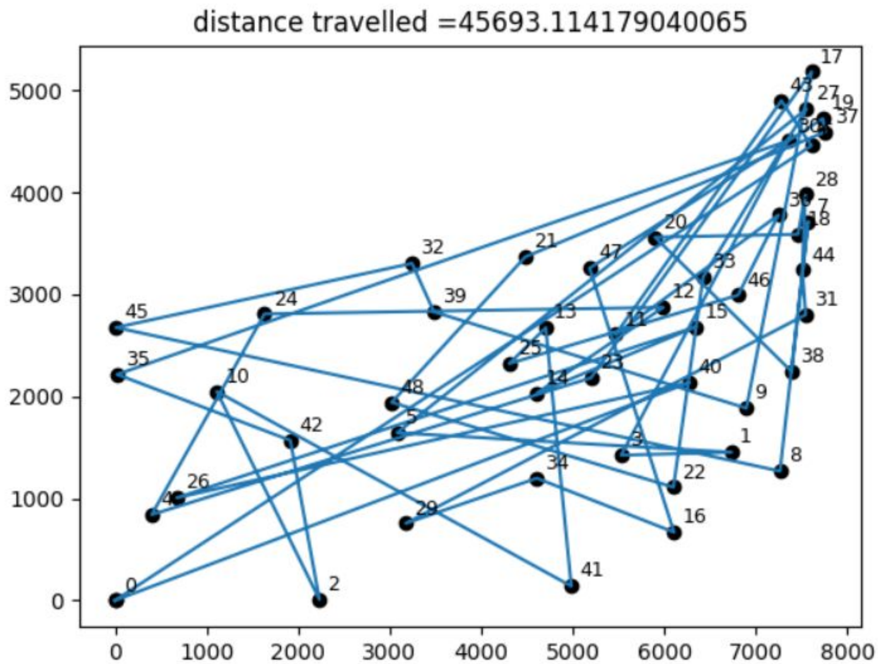

Fig. f) solution found by Simulated annealing (temp= 10 and alpha =0.995)

Hence the above image shows that how simulated annealing came out of this local minimas and provides a solution that costs around 45k which is close to that of optimal solution and much better than stochastic hill climbing techniques. In this project we choose the value of temperature and alpha just by using a hit and trial method due to which we get stuck at one of the local minima and not able to reach global minima.

***All the images used in this discussion part are taken from the output of the code itself not from any source**

ALGORITHMS

Various algorithms are developed in order to solve the VRP and simulated annealing is also one of the efficient algorithms to solve this problem if temperature and its tenure are defined precisely. After simulated annealing many different advanced algorithms like G*enetic Algorithm and Ant Colony Optimization Algorithm* are developed which are more randomized ,efficient then Simulated annealing and definitely provide an optimal solution.(even apply to large data set practical cases)

There are other various simple algorithms also like tabu search, heuristic approach and stochastics hill climbing (we see above already) that developed before simulated annealing and that will definitely find a solution but stuck in local minima in case of large dataset. Thus, In practical cases we can apply these above mentioned algorithm as all of the above algorithms will definitely return a solution that may or may not be optimal one.

Algorithm Complexity

Determining the optimal solution for VRP is NP-hard as for N number of cities the number of possible solutions are **N!**. Hence, time complexity of solving this problem using a naive approach i.e traversing each solution is **O(n!)** which is very big. Hence the state space size for VRP is (N!). In VRP if we found a neighbor solution by swapping two customers then there are **n(n-1)/2** possible neighbors to a current state (branching factor).

The time complexity of stochastic hill climbing algorithms (hill climbing with random restart and random walk) and simulated annealing algorithm that are applied in this project to solve VRP is **O(m*n(n-1)/2).**
where,
1. m -> no. of iteration or random restart given (in case of stochastic hill climbing) and number of iteration done in the specified temperature tenure (in case of simulated annealing)
2. n(n-1)/2 are the number of neighbor that are explored in each step

Space complexity of these algorithms is **O(n) i.e list containing n no. of cities.**

SUMMARY AND CONCLUSION

- VRP is a NP- hard problem in which state space size is N! for n number of customers to visit in a route.
- Local search method (stochastic hill climbing and simulated annealing) are applied in order to get the solution And Using greedy approach to solve VRP result in stucking at local minima
- For 9 customers dataset :

Algorithm	No. of iterations (m)	Distance travelled	Run time
Hill climbing with random restart	50	12,530	0.1s
Hill climbing with random restart	100	12,440	0.13s
Hill climbing with random walk	50	12,466	0.1s
Hill climbing with random walk	100	12207	0.15s

From the above table we can say that for small no. of customers dataset given stochastics hill climbing most probably find optimal solution and with increase in number of iteration the chances of getting optimal solution increases.

- For 48 customers dataset:

Algorithm	No. of iterations (m)	Distance travelled	Run time
Hill climbing with random restart	1000	148307	3.3s
Hill climbing with random walk	1000	111487	3.4s
Simulated annealing	50	45693	1.7s

From the above table we can conclude that for a complex given dataset simulated annealing is a much more efficient algorithm than stochastic hill climbing.

- For real life cases stochastics hill climbing algorithm is not one that we can rely on but will definitely give a solution given a hard time constraints.
- On the other hand simulated annealing is a much feasible solution to VRP and can be applied on real life problems to get optimum solution.
- Estimating temperature and its tenure is one of the key features that determines the efficiency of the simulated annealing and is one of the difficulties in implementing simulated annealing.
- Advanced Algorithms like genetic algorithm and ant colony optimization algorithm are the most efficient algorithms that guarantee to give optimal solutions for VRP.

References

1. https://en.wikipedia.org/wiki/Vehicle_routing_problem
2. **https://www.researchgate.net/publication/313005083_Vehicle_routing_problem_Models_and_solutions**
3. **http://www.psychicorigami.com/2007/06/28/tackling-the-travelling-salesman-problem-simmulated-annealing/**
4. **http://www.psychicorigami.com/2007/05/12/tackling-the-travelling-salesman-problem-hill-climbing/**
5. **Data set : https://people.sc.fsu.edu/~jburkardt/datasets/tsp/tsp.html**