



McCOMBS SCHOOL OF BUSINESS

Salem Center for Policy

Causality

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Cause-and-effect

Potential outcomes and Counterfactuals

RCTs and Obs Studies

Causality and policy analysis?



These two ideas seem completely unrelated.

But, they are and stealthily show up EVERYWHERE!













The Federal Unemployment Bonus Holds the Recovery Back

The Help Wanted signs came down in my town only when Arkansas opted out of the \$300 supplement.

Covid-19 Rekindles Debate Over License Requirements for Many Jobs

Hair styling and medical fields are among the occupations where state rules can bar entry; Biden has pushed for changes

Beefed-Up Sanctions Could Limit the Damage in Afghanistan

The Taliban's control of the government will significantly increase their wealth and influence.

Medicare Advantage Shows the Path Forward

The savings spill over in [Harvard's Katherine Baicker, Michael Chernew and Jacob Robbins](#) have showed that as penetration of Medicare Advantage increases in different counties, hospital costs and length-of-stays decline not only for seniors enrolled in Medicare managed care plans, but also for beneficiaries still on the traditional program. For every 10% increase in the uptake of Medicare Advantage, inpatient spending among fee-for-service Medicare seniors falls by 5% to 10%. Similar findings are also observed among commercially-insured people under 65 in regions with rapid diffusion of Medicare Advantage.

How Often Should You Shower? Celebrities Ignite a Ferocious Debate

Hollywood types including Jake Gyllenhaal, Mila Kunis, Ashton Kutcher and Dax Shepard take a lax approach to hygiene, stoking a contentious uproar on how often one should bathe. It mirrors a similar discord in the medical community, and among everyday people.

Cause-and-effect \iff policy impacts



These two paradigms in the title are one and the same! One is a general framework, and one is specific to the policy arena.

Cause is a statement of something being manipulated or changed
Effect is a measure of the change in an outcome of interest

Cause-and-effect \iff policy impacts



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Effect is a measure of the change in an outcome of interest

- “Cause” is the same as a policy introduction or change
- “Effect” is the unique, independent measurement of how the cause modulated some other part of our system

Cause-and-effect: policy decisions as cause



We see **causes** all of the time

→ The federal government increases the minimum wage

Cause-and-effect: policy decisions as cause



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- The percentage of vaccinated individuals exceeds 50% of population

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- Sweetgreen decreases its Kale caesar salad price by \$2
- States implement stay-at-home orders during the pandemic

Cause-and-effect: varying features (treatments) as cause



Causes might also just be variation in a population

→ Are dog-lovers nicer people?

Cause-and-effect: varying features (treatments) as cause



Causes might also just be variation in a population

- Are dog-lovers nicer people?
- Does race affect hiring decisions?

Cause-and-effect: varying features (treatments) as cause



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Cause-and-effect: varying features (treatments) as cause



Causes might also just be variation in a population

- Are dog-lovers nicer people?
- Does race affect hiring decisions?
- Does age affect COVID-19 mortality?
- Is there a "gender-gap" in salary?

Cause-and-effect \iff policy impacts



The **difficult** question is, what about the **effects**?



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- What outcomes do we look at?
- How do we measure them?
- Are there other variables that might affect the outcomes *and* the causes?



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- What outcomes do we look at? (**lower class unemployment rate, income, ...**)
- How do we measure them? (**government data, surveys, ...**)
- Are there other variables that might affect the outcomes *and* the causes? (**current economic conditions, differences among states, ...**)



The **difficult** question is, what about the **effects**?

- What outcomes do we look at? (**revenue, count of kale caesars sold, number of daily lunch visitors, ...**)
- How do we measure them? (**financial data, ...**)
- Are there other variables that might affect the outcomes *and* the causes? (**time of year (seasonality), temperature, weather, length of daily wait time, ...**)

Racial discrimination in hiring?



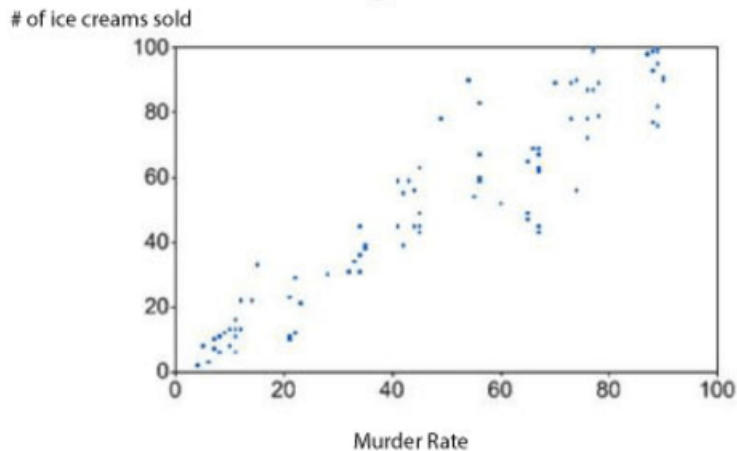
The **difficult** question is, what about the **effects**?

- What outcomes do we look at? (**whether or not a job applicant receives a callback**)
- How do we measure them? (**follows directly from above ...**)
- Are there other variables that might affect the outcomes *and* the causes? (**other resume characteristics, average GPA, brand of university,...**)

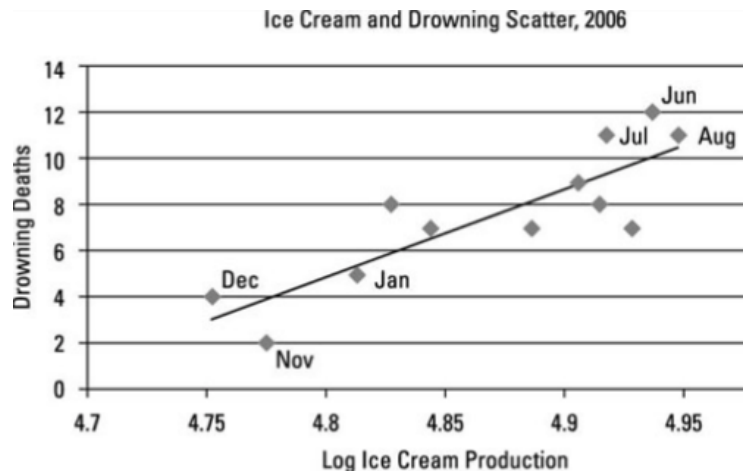
Ice cream and NYC murder rate



Figure 1



Ice cream and drownings



To sum up cause-and-effect



- Challenges are related to both the **system of study** and **ability to gather the right data**.
- With data in hand, you can start to formulate hypotheses and test them.
- There might be lurking variables driving an underlying relationship (**ice cream**). Only an expert (you!) can identify those and take them into account.

Let's formalize these ideas with some basic notation



- units of study are indexed by i (states, stores, customers, people)



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$$Y_i(z_i)$$



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- with this notation, we have the building blocks to talk about policy **effects**!

Example: COVID-19 lockdowns



How did state lockdowns at the beginning of 2020 affect the spread of the virus?

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What are the **potential outcomes**?

Example: COVID-19 lockdowns



Let's set up the structure of this problem.

Let i denote a state, so

$$i \in \{\text{New York, California, Florida, Texas, South Dakota, ...}\}$$

First, we define what z_i is:

$$z_i = \begin{cases} 0 & \text{no lockdown} \\ 1 & \text{lockdown} \end{cases}$$

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Second, we define our outcome: Y_i : let's choose the cases per capita (in state i) after lockdown or no lockdown.



A brief aside:

Defining exactly what the treatment z_i is very hard! It could be a combination of many available data.

- masking
- bar and restaurant closures
- school closures
- curfews
- limits to exercise
- retail store closures



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- bar and restaurant closures
- school closures
- curfews
- limits to exercise
- retail store closures

Q: How would you define z_i ?



i (state)	z_i (lockdown)	$Y_i(0)$	$Y_i(1)$
New York			
Florida			
California			
Texas			
South Dakota			
Illinois			
\vdots	\vdots	\vdots	\vdots



i (state)	z_i (lockdown)	$Y_i(0)$	$Y_i(1)$
New York	1		
Florida	0		
California	1		
Texas	0		
South Dakota	0		
Illinois	1		
\vdots	\vdots	\vdots	\vdots

Organizing our data: The Science Table (CDC, cases/100k)



i (state)	z_i (lockdown)	$Y_i(0)$	$Y_i(1)$
New York	1		.0034
Florida	0	.007	
California	1		.0014
Texas	0	.004	
South Dakota	0	.0028	
Illinois	1		.002
⋮	⋮	⋮	⋮



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What is the ideal scenario?



We are able to know both of the potential outcomes for each state!



Defining a causal effect for NY



We can define the **causal effect** of the “lockdown treatment” as the difference between the two potential outcomes.

$$\tau_{NY} = Y_{NY} \left(\text{Image of busy New York City street} \right) - Y_{NY} \left(\text{Image of quiet New York City street} \right)$$

or written more generally:

$$\tau_i = Y_i(1) - Y_i(0)$$

The fundamental problem of causal inference



We only observe one of the two potential outcomes for New York and all other states. In general, we always only observe one of two potential outcomes for our units of study.

- **economics of COVID policy**: a state either locks down or doesn't
- **drug trials**: an individual either receives the medicine or the placebo
- **gender wage gap**: a person is either male or female

The unknown outcomes are called the **missing potential outcomes** or **counterfactuals**. This is what makes causality a nontrivial task ... it is a **missing data problem**.

Is all hope lost?



Is all hope lost?



Definitely not! The potential outcomes will **always** be used as a starting point. Depending on the data and question to be answered, there are several approaches:

- Randomization and the sample average treatment effect
- Observational data – before-and-after and DiD approaches
- Fancier (probabilistic) models to address confounding. Regression, etc. (the “Prediction” part of class).

The average causal effect across the sample



This is called the **sample average treatment effect**. In stats language, it is called an **estimand**. Let's suppose we have N units in our data.

$$\begin{aligned}\mathbf{SATE} &= \frac{1}{N} \sum_{i=1}^N \tau_i \\ &= \frac{1}{N} \sum_{i=1}^N \{Y_i(1) - Y_i(0)\}\end{aligned}$$

We still don't know how to calculate this because of the fundamental problem of causal inference.

However, here's an idea ...

Estimator of the **SATE**



We have the **observed** outcome and treatment. Let's call them:

$$Y_{\text{obs}} = (Y_1, \dots, Y_N)$$

$$Z_{\text{obs}} = (Z_1, \dots, Z_N)$$

Let's define our **estimator** of the **SATE** as the simple **difference-in-means (DiM)** between the treated and control units.

$$\widehat{\text{SATE}} = \frac{1}{\sum_i \mathbb{1}(Z_i = 1)} \sum_i \mathbb{1}(Z_i = 1) Y_i - \frac{1}{\sum_i \mathbb{1}(Z_i = 0)} \sum_i \mathbb{1}(Z_i = 0) Y_i$$

Q: When can this be reasonably interpreted as the average **causal effect**, when can it not?



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- The **treatment** and **control** groups that you're computing the "means" over are otherwise equal in all other attributes!



- The difference-in-means (DiM) estimator is useful and interpretable for randomized experiments. **Why?**
- The **treatment** and **control** groups that you're computing the "means" over are otherwise equal in all other attributes!
- This eliminates the confounding issue!
- In short, randomizing the **treatment** (cause) is the **gold standard** for understanding causality.

So we have a point estimate of the causal effect ...



But, what about uncertainty?

So we have a point estimate of the causal effect ...



But, what about uncertainty?

- Let's use the known process of randomization to our advantage
- It is common to characterize uncertainty under a particular **null hypothesis**.
- Think of the **null** as the not interesting or exciting causal conclusion. **"There is no causal effect"**.

$$H_0 : Y_i(0) = Y_i(1) \quad \text{for all } i$$

(Then, we use our actual data to probe this statement.)

What is the advantage of assuming H_0 ?



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We now know all potential outcomes!

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\vdots	\vdots	\vdots	\vdots

But where does uncertainty come in?



- Let's go back to our original DiM estimate
- This is given by a single number denoting the difference in average outcome of treated and control units

$$Y_{\text{obs}} = (Y_1, \dots, Y_N)$$

$$Z_{\text{obs}} = (Z_1, \dots, Z_N)$$

$$\begin{aligned} \text{DiM} &= \frac{1}{\sum_i \mathbb{1}(Z_i = 1)} \sum_i \mathbb{1}(Z_i = 1) Y_i - \frac{1}{\sum_i \mathbb{1}(Z_i = 0)} \sum_i \mathbb{1}(Z_i = 0) Y_i \\ &= \text{"function of } Y \text{ and } Z\text{"} \end{aligned}$$

So, let's write out the estimate explicitly as DiM(Y, Z).

Randomization from the experiment is the uncertainty!



- If I have a randomized experiment, I know exactly how the analyst **allocated** units to **treatment** and **control**

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- For example, in the simplest case, the randomization could be done by flipping a fair coin for each unit i . If heads, unit i is in control, if tails, unit i is in treatment.

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- If I have a randomized experiment, I know exactly how the analyst **allocated** units to **treatment** and **control**
- For example, in the simplest case, the randomization could be done by flipping a fair coin for each unit i . If heads, unit i is in control, if tails, unit i is in treatment.
- The point is that I have an experimental design, $P(Z)$, from which I can generate many **alternative treatments**!
- We write an alternative treatment as $Z' \sim P(Z)$

Importantly, under the H_0 , I can now generate a bunch of alternative values of the DiM statistic DiM(Y, Z')

Randomization from the experiment is the uncertainty!



Q: Why do we obtain new values of the difference-in-means (DiM) for alternative draws of the treatment/control allocation, Z' ?

Randomization from the experiment is the uncertainty!

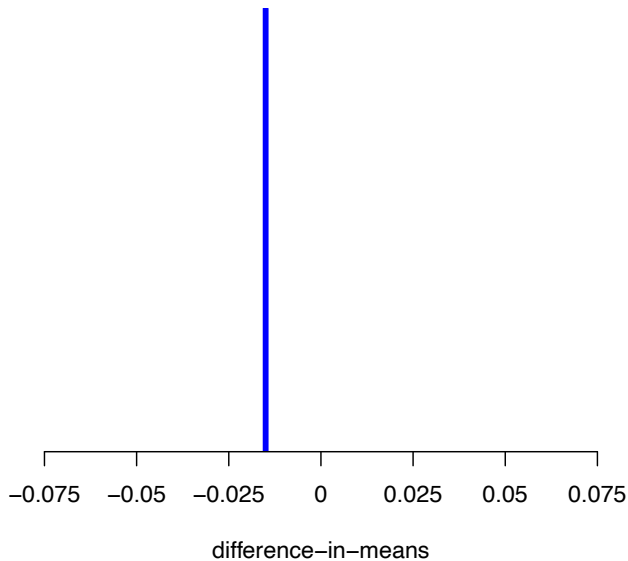


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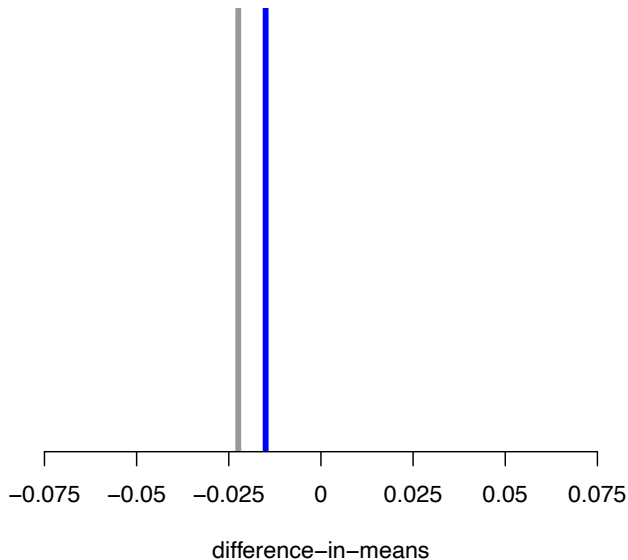
$$\text{DiM}(Y, Z) = \text{average } Y \text{ for treated} - \text{average } Y \text{ for control}$$

For a new Z' , the groups of treated units and control units are different! So, the averages will be different and thus $\text{DiM}(Y, Z')$ will be different.

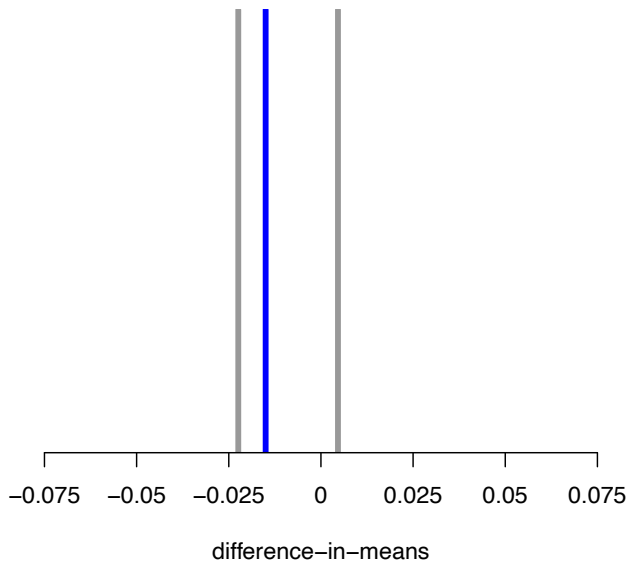
Observed DiM and its alternative values



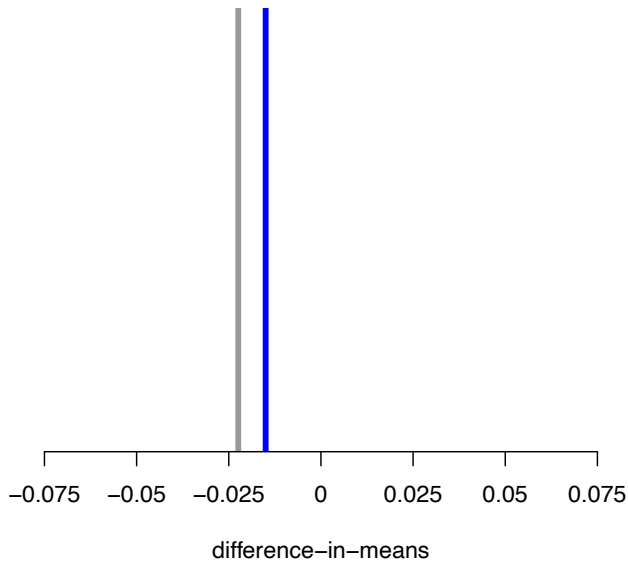
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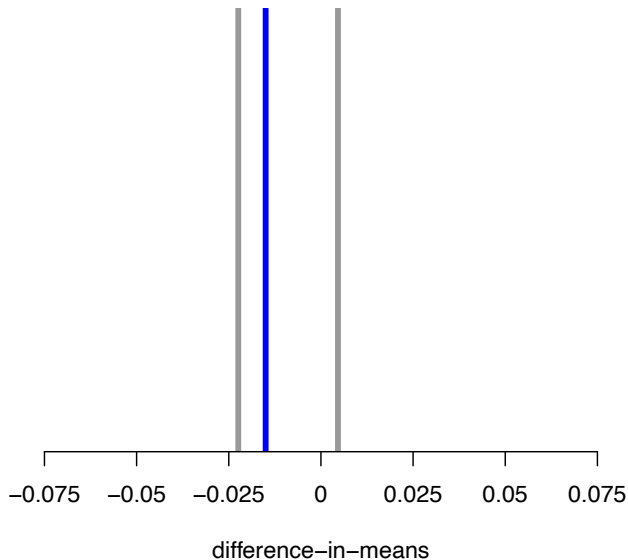
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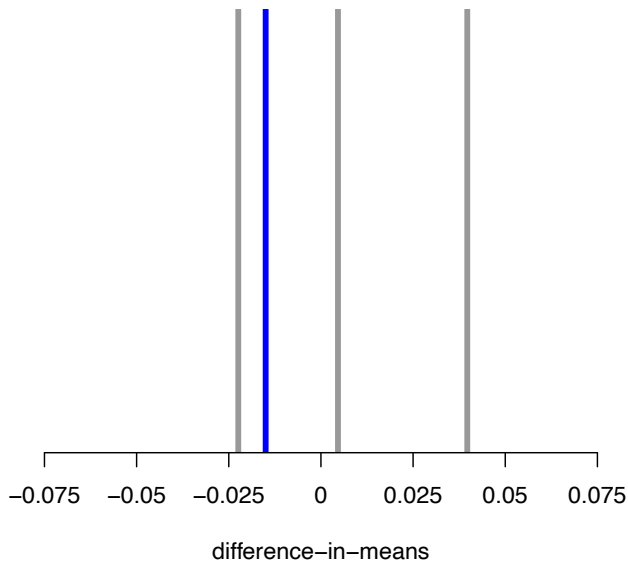
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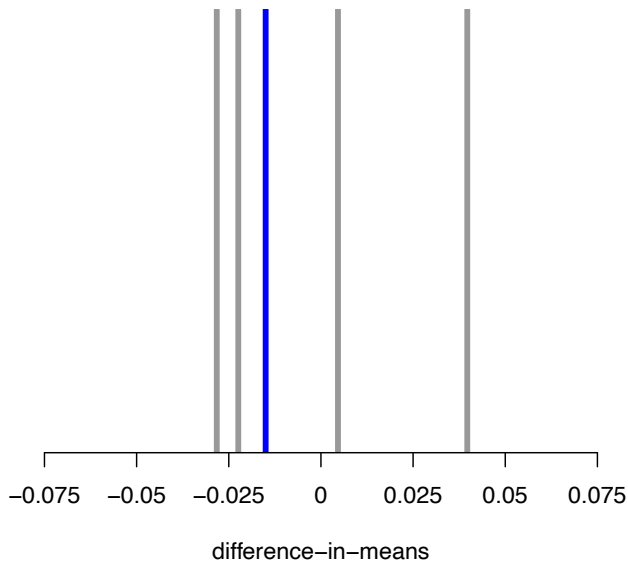
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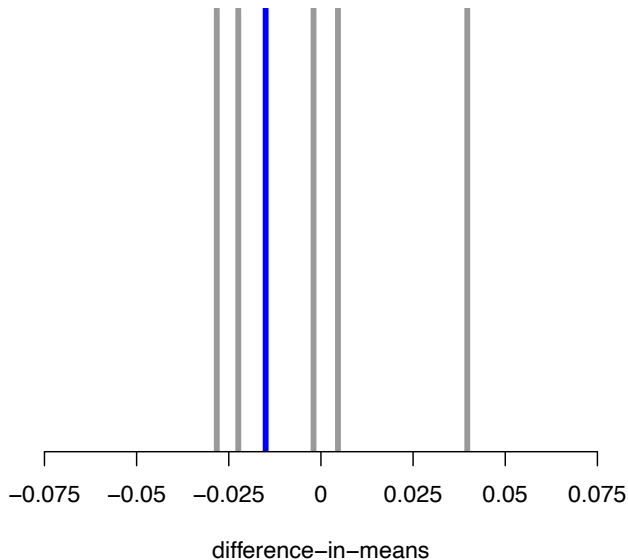
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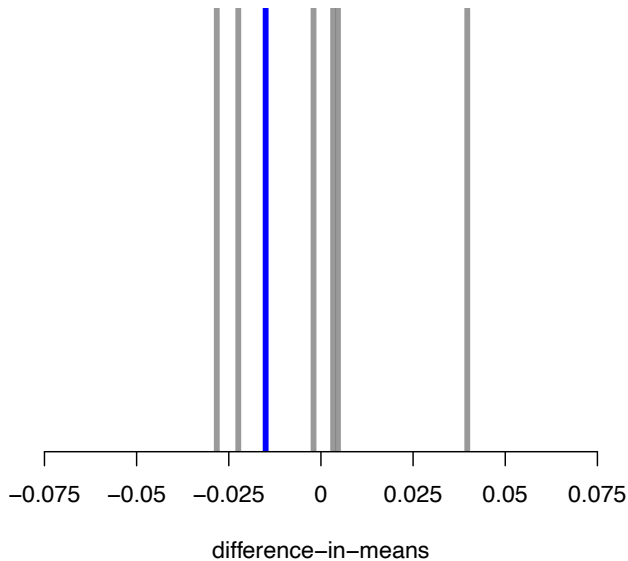
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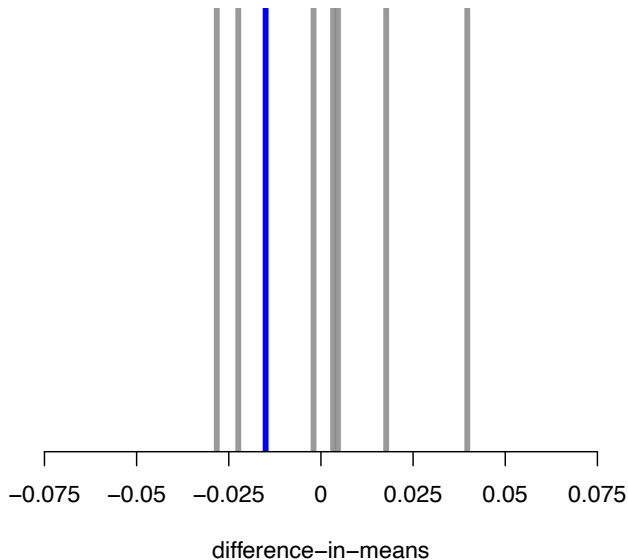
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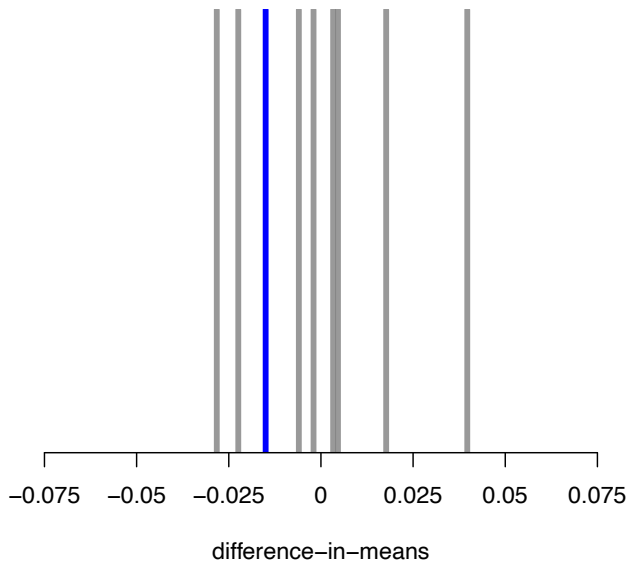
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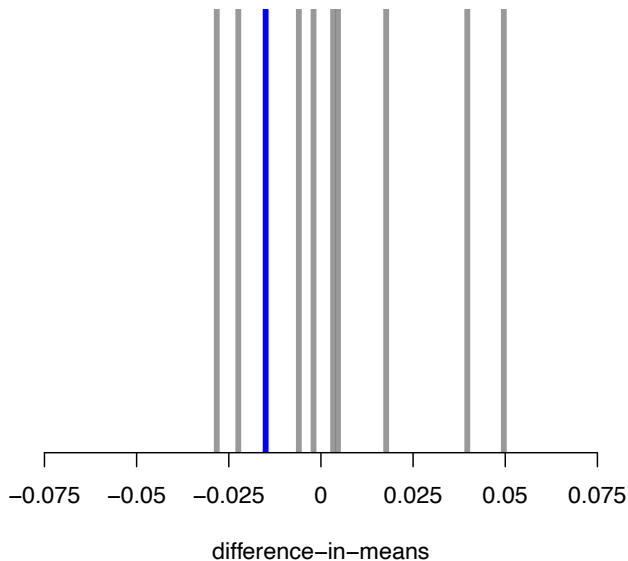
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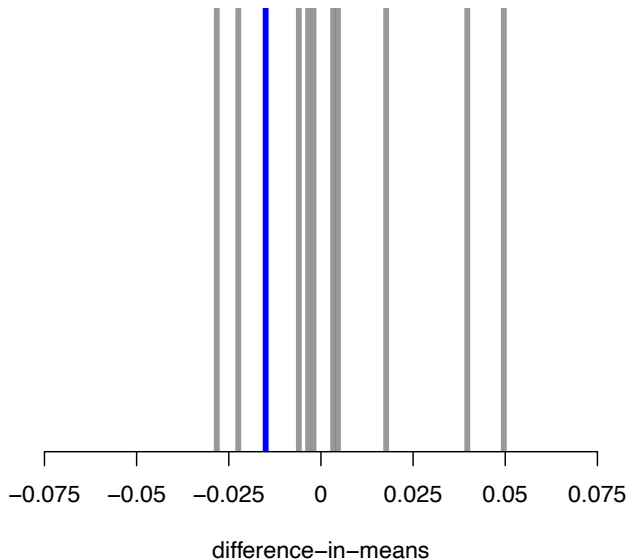
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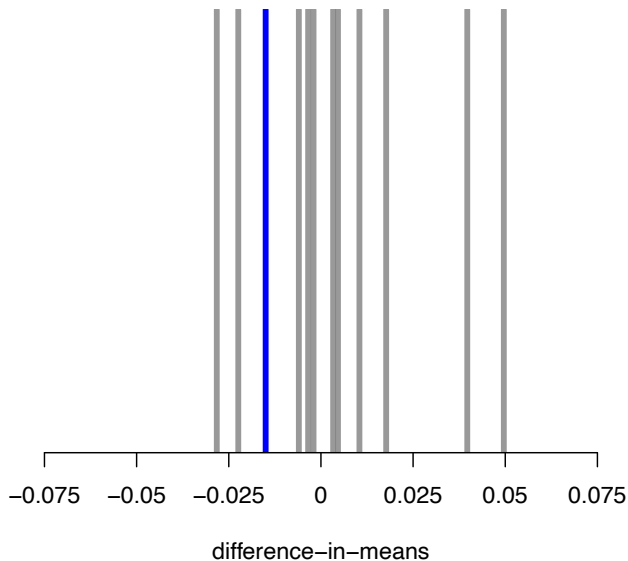
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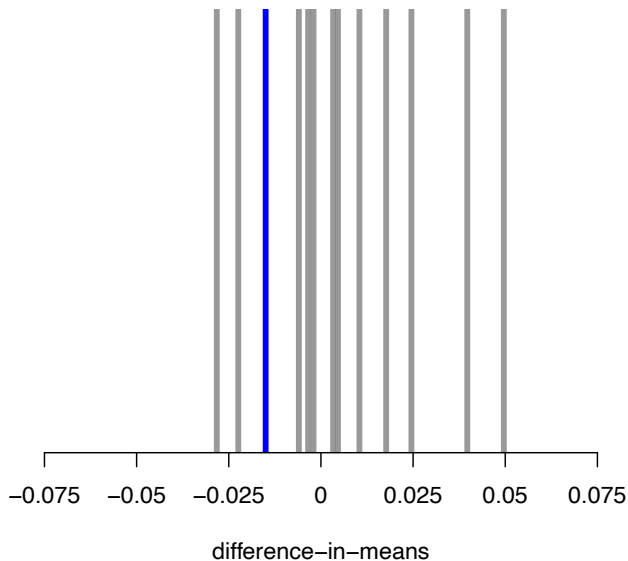
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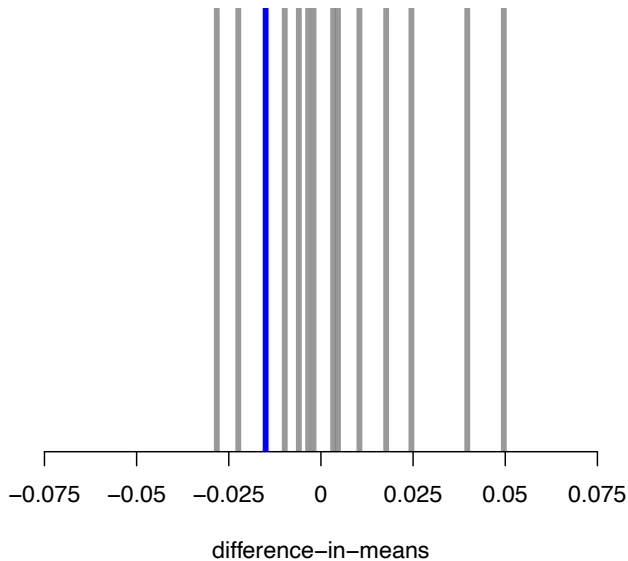
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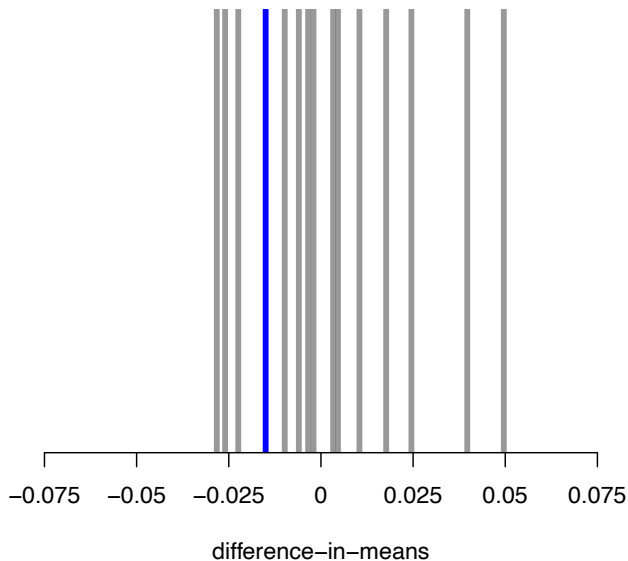
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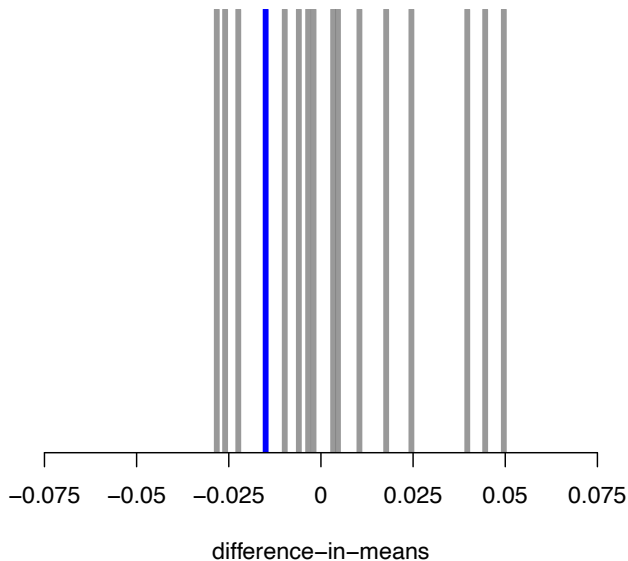
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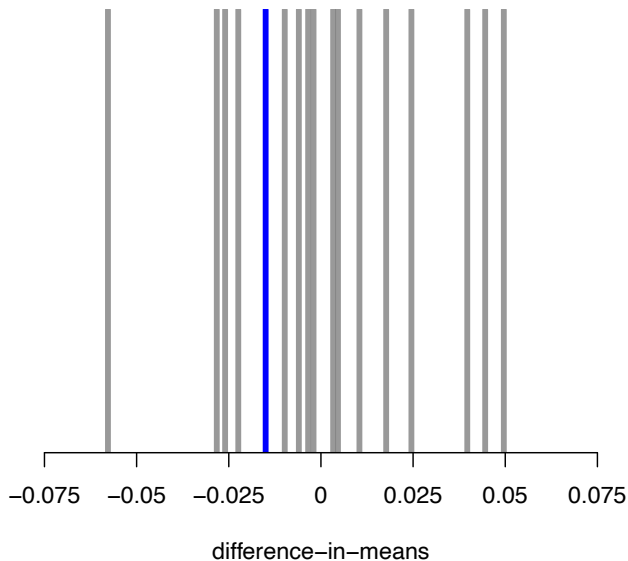
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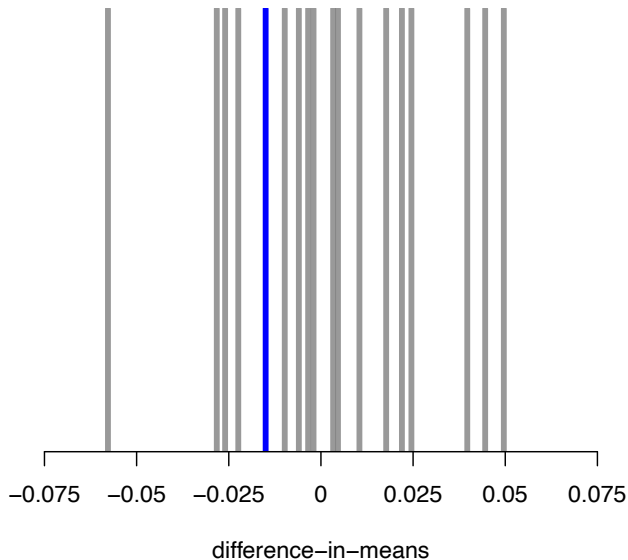
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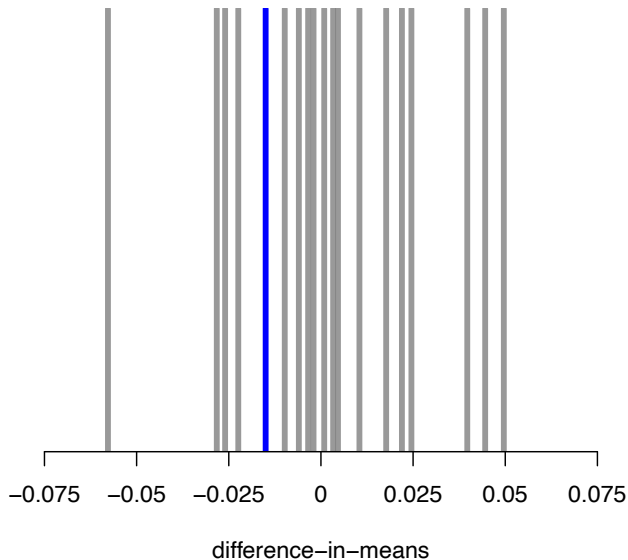
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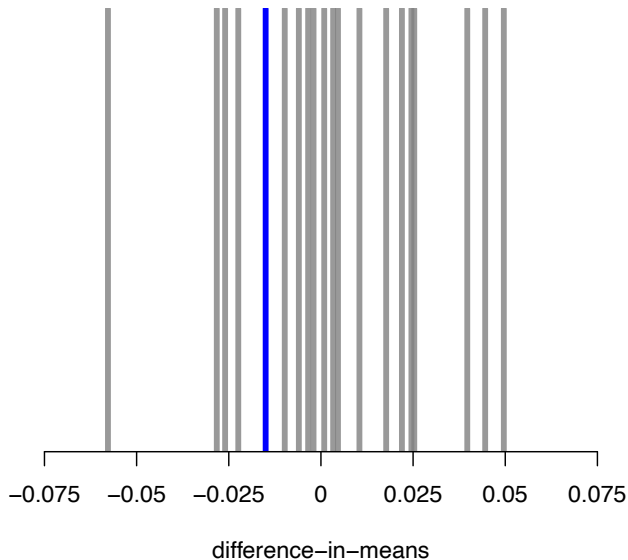
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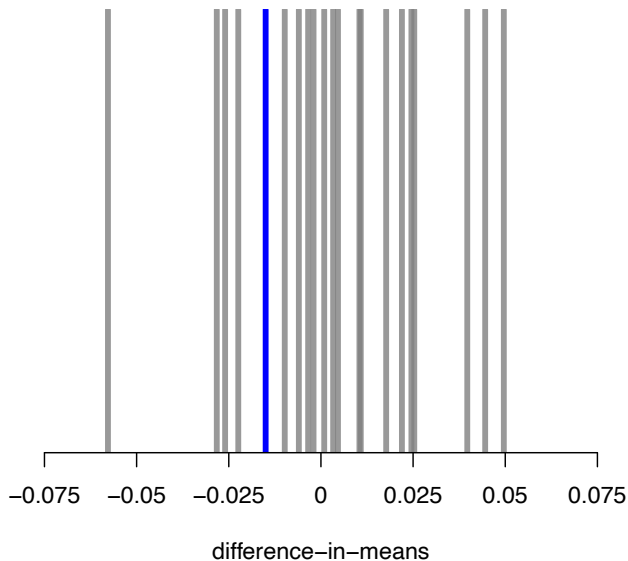
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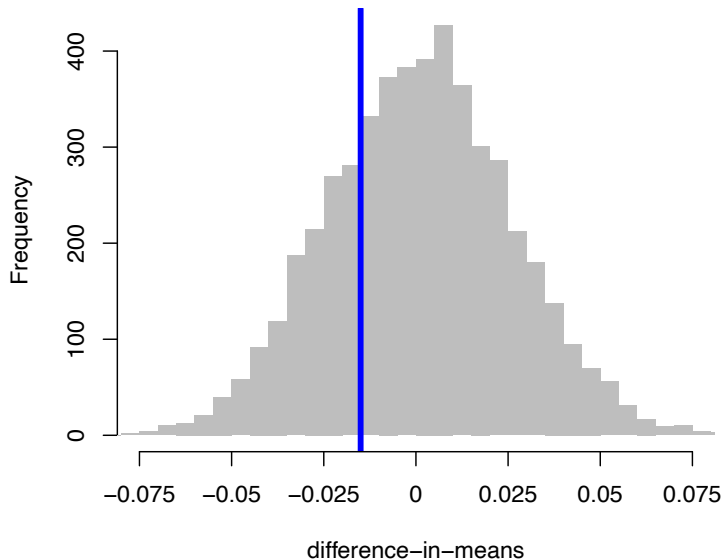
Observed DiM and its alternative values



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Observed DiM and its alternative values



Fisher randomization test (1935)



$$H_0 : Y_i(0) = Y_i(1) \quad \text{for all } i$$

The procedure:

1. $\text{DiM}_{\text{obs}} = \text{DiM}(Y_{\text{obs}}, Z_{\text{obs}})$.
2. Draw $Z' \sim P(Z')$,
store $\text{DiM}_r = \text{DiM}(Y', Z') \stackrel{H_0}{=} \text{DiM}(Y_{\text{obs}}, Z')$.
3. $\text{p-value} = \mathbb{E}[\mathbb{1}\{\text{DiM}_r \geq \text{DiM}_{\text{obs}}\}]$.

This beautifully simple procedure tells you whether or not your causal effect is “significant” (under the null hypothesis of no effect)!



- First, we defined potential outcomes, $Y_i(0)$ and $Y_i(1)$
- Second, we wrote down an **estimand** (SATE) and **estimator** (DiM) of the average causal effect
- Third, we decided that a point estimate isn't enough, we need **uncertainty**!
- Fourth, we discussed the Fisher randomization test (FRT) as a simple and powerful procedure for characterizing uncertainty using the experimental design, $P(Z)$.