McCOMBS SCHOOL OF BUSINESS
Salem Center for Policy

Causality

David Puelz

September 9, 2021

Outline

Cause-and-effect

Potential outcomes and Counterfactuals

RCTs and Obs Studies

Causality and policy analysis?

These two ideas seem completely unrelated.

But, they are and steathly show up EVERYWHERE!

The Federal Unemployment Bonus Holds the Recovery Back

The Help Wanted signs came down in my town only when Arkansas opted out of the $300 supplement.

Covid-19 Rekindles Debate Over License Requirements for Many Jobs

Hair styling and medical fields are among the occupations where state rules can bar entry; Biden has pushed for changes

Beefed-Up Sanctions Could Limit the Damage in Afghanistan

The Taliban's control of the government will significantly increase their wealth and influence.

Medicare Advantage Shows the Path Forward

The savings spill over in

Harvard's Katherine Baicker, Michael Chernew and Jacob Robbins have showed that as penetration of Medicare Advantage increases in different counties, hospital costs and length-of-stays decline not only for seniors enrolled in Medicare managed care plans, but also for beneficiaries still on the traditional program. For every 10% increase in the uptake of Medicare Advantage, inpatient spending among fee-for-service Medicare seniors falls by 5% to 10%. Similar findings are also observed among commercially-insured people under 65 in regions with rapid diffusion of Medicare Advantage.

How Often Should You Shower? Celebrities Ignite a Ferocious Debate

Hollywood types including Jake Gyllenhaal, Mila Kunis, Ashton Kutcher and Dax Shepard take a lax approach to hygiene, stoking a contentious uproar on how often one should bathe. It mirrors a similar discord in the medical community, and among everyday people.

Cause-and-effect $\iff$ policy impacts

These two paradigms in the title are one and the same! One is a general framework, and one is specific to the policy arena.

Cause is a statement of something being manipulated or changed
Effect is a measure of the change in an outcome of interest

Cause-and-effect $\iff$ policy impacts

These two paradigms in the title are one and the same! One is a general framework, and one is specific to the policy arena.

Cause is a statement of something being manipulated or changed
Effect is a measure of the change in an outcome of interest

- "Cause" is the same as a policy introduction or change
- "Effect" is the unique, independent measurement of how the cause modulated some other part of our system

Cause-and-effect: policy decisions as cause

We see causes all of the time

$\rightarrow$ The federal government increases the minimum wage

Cause-and-effect: policy decisions as cause

We see causes all of the time

→ The federal government increases the minimum wage
→ The FAA mandates face coverings on planes

Cause-and-effect: policy decisions as cause

We see causes all of the time

$\rightarrow$ The federal government increases the minimum wage
$\rightarrow$ The FAA mandates face coverings on planes
$\rightarrow$ The percentage of vaccinated individuals exceeds 50% of population

Cause-and-effect: policy decisions as cause

We see causes all of the time

→ The federal government increases the minimum wage
→ The FAA mandates face coverings on planes
→ The percentage of vaccinated individuals exceeds 50% of population
→ It's windy on an Austin afternoon

Cause-and-effect: policy decisions as cause

We see causes all of the time

$\rightarrow$ The federal government increases the minimum wage
$\rightarrow$ The FAA mandates face coverings on planes
$\rightarrow$ The percentage of vaccinated individuals exceeds 50% of population
$\rightarrow$ It's windy on an Austin afternoon
$\rightarrow$ Sweetgreen decreases its Kale caesar salad price by $2

Cause-and-effect: policy decisions as cause

We see causes all of the time

→ The federal government increases the minimum wage
→ The FAA mandates face coverings on planes
→ The percentage of vaccinated individuals exceeds 50% of population
→ It's windy on an Austin afternoon
→ Sweetgreen decreases its Kale caesar salad price by $2
→ States implement stay-at-home orders during the pandemic

Causes might also just be variation in a population

→ Are dog-lovers nicer people?

Cause-and-effect: varying features (treatments) as cause

Causes might also just be variation in a population

$\rightarrow$ Are dog-lovers nicer people?
$\rightarrow$ Does race affect hiring decisions?

Cause-and-effect: varying features (treatments) as cause

Causes might also just be variation in a population

→ Are dog-lovers nicer people?
→ Does race affect hiring decisions?
→ Does age affect COVID-19 mortality?

Cause-and-effect: varying features (treatments) as cause

Causes might also just be variation in a population

→ Are dog-lovers nicer people?
→ Does race affect hiring decisions?
→ Does age affect COVID-19 mortality?
→ Is there a "gender-gap" in salary?

Cause-and-effect $\iff$ policy impacts

The **difficult** question is, what about the effects?

Cause-and-effect $\iff$ policy impacts

The **difficult** question is, what about the effects?

- What outcomes do we look at?
- How do we measure them?
- Are there other variables that might affect the outcomes *and* the causes?

Increasing the minimum wage

The **difficult** question is, what about the effects?

- What outcomes do we look at? (lower class unemployment rate, income, ...)
- How do we measure them? (government data, surveys, ...)
- Are there other variables that might affect the outcomes *and* the causes? (current economic conditions, differences among states, ...)

Sweetgreen salad price increase

The **difficult** question is, what about the effects?

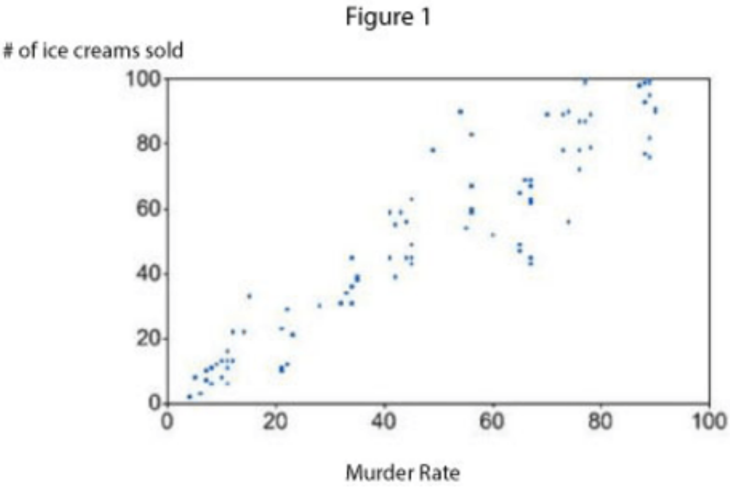
- What outcomes do we look at? (revenue, count of kale caesars sold, number of daily lunch visitors, ...)
- How do we measure them? (financial data, ...)
- Are there other variables that might affect the outcomes *and* the causes? (time of year (seasonality), temperature, weather, length of daily wait time, ...)

Racial discrimination in hiring?

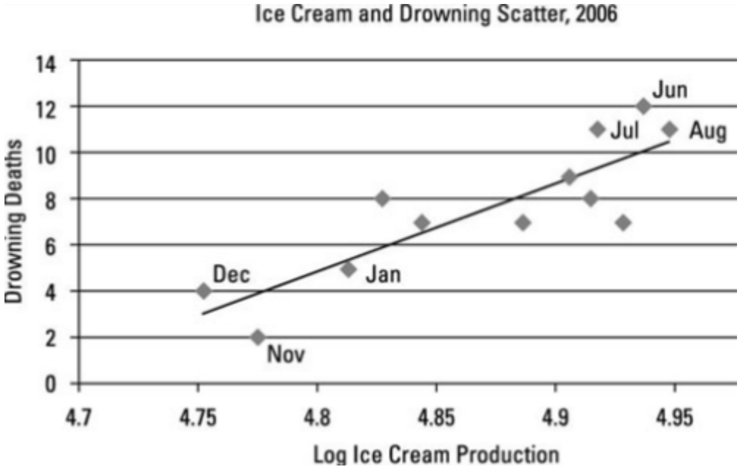
The **difficult** question is, what about the effects?

- What outcomes do we look at? (whether or not a job applicant receives a callback)
- How do we measure them? (follows directly from above ...)
- Are there other variables that might affect the outcomes *and* the causes? (other resume characteristics, average GPA, brand of university,...)

Ice cream and NYC murder rate



Ice cream and drownings



To sum up cause-and-effect

- Challenges are related to both the system of study and ability to gather the right data.
- With data in hand, you can start to formulate hypotheses and test them.
- There might be lurking variables driving an underlying relationship (ice cream). Only an expert (you!) can identify those and take them into account.

Let's formalize these ideas with some basic notation

Notation

- units of study are indexed by i (states, stores, customers, people)

Notation

- units of study are indexed by i (states, stores, customers, people)
- cause aka treatment aka policy for unit i is given by z_i

$$z_i = \begin{cases} 0 & \text{not treated (control)} \\ 1 & \text{treated} \end{cases}$$

Notation

- units of study are indexed by i (states, stores, customers, people)
- cause aka treatment aka policy for unit i is given by z_i

$$z_i = \begin{cases} 0 & \text{not treated (control)} \\ 1 & \text{treated} \end{cases}$$

- denote a outcome of interest for each i as Y_i, but let's write it as a function of the cause.

$$Y_i(z_i)$$

Notation

- units of study are indexed by i (states, stores, customers, people)
- cause aka treatment aka policy for unit i is given by z_i

$$z_i = \begin{cases} 0 & \text{not treated (control)} \\ 1 & \text{treated} \end{cases}$$

- denote a outcome of interest for each i as Y_i, but let's write it as a function of the cause.

$$Y_i(z_i) \leftarrow \textbf{potential outcome} \text{ for unit } i$$

Notation

- units of study are indexed by i (states, stores, customers, people)
- cause aka treatment aka policy for unit i is given by z_i

$$z_i = \begin{cases} 0 & \text{not treated (control)} \\ 1 & \text{treated} \end{cases}$$

- denote a outcome of interest for each i as Y_i, but let's write it as a function of the cause.

$$Y_i(z_i) \leftarrow \textbf{potential outcome} \text{ for unit } i$$

- with this notation, we have the building blocks to talk about policy effects!

Example: COVID-19 lockdowns

How did state lockdowns at the beginning of 2020 affect the spread of the virus?

Example: COVID-19 lockdowns

How did state lockdowns at the beginning of 2020 affect the spread of the virus?

What are the potential outcomes?

Example: COVID-19 lockdowns

Let's set up the structure of this problem.

Let i denote a state, so

$$i \in \{\text{New York, California, Florida, Texas, South Dakota, ...}\}$$

First, we define what z_i is:

$$z_i = \begin{cases} 0 & \text{no lockdown} \\ 1 & \text{lockdown} \end{cases}$$

Example: COVID-19 lockdowns

Let's set up the structure of this problem.

Let i denote a state, so

$$i \in \{\text{New York, California, Florida, Texas, South Dakota, ... }\}$$

First, we define what z_i is:

$$z_i = \begin{cases} 0 & \text{no lockdown} \\ 1 & \text{lockdown} \end{cases}$$

Second, we define our outcome: Y_i : let's choose the cases per capita (in state i) after lockdown or no lockdown.

Example: COVID-19 lockdowns

A brief aside:

Defining exactly what the treatment z_i is very hard! It could be a combination of many available data.

- masking
- bar and restaurant closures
- school closures
- curfews
- limits to exercise
- retail store closures

Example: COVID-19 lockdowns

A brief aside:

Defining exactly what the treatment z_i is very hard! It could be a combination of many available data.

- masking
- bar and restaurant closures
- school closures
- curfews
- limits to exercise
- retail store closures

Q: How would you define z_i?

i **(state)**	z_i **(lockdown)**	$Y_i(0)$	$Y_i(1)$
New York			
Florida			
California			
Texas			
South Dakota			
Illinois			
$\vdots$	$\vdots$	$\vdots$	$\vdots$

i **(state)**	z_i **(lockdown)**	$Y_i(0)$	$Y_i(1)$
New York	1		
Florida	0		
California	1		
Texas	0		
South Dakota	0		
Illinois	1		
⋮	⋮	⋮	⋮

Organizing our data: The Science Table (CDC, cases/100k)

i **(state)**	z_i **(lockdown)**	$Y_i(0)$	$Y_i(1)$
New York	1		.0034
Florida	0	.007	
California	1		.0014
Texas	0	.004	
South Dakota	0	.0028	
Illinois	1		.002
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Organizing our data: The Science Table (CDC, cases/100k)

i **(state)**	z_i **(lockdown)**	$Y_i(0)$	$Y_i(1)$
New York	1	??	.0034
Florida	0	.007	??
California	1	??	.0014
Texas	0	.004	??
South Dakota	0	.0028	??
Illinois	1	??	.002
$\vdots$	$\vdots$	$\vdots$	$\vdots$

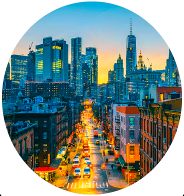
What is the ideal scenario?

We are able to know both of the potential outcomes for each state!

Y_{NY} () and Y_{NY} ()

Defining a causal effect for NY

We can define the **causal effect** of the "lockdown treatment" as the difference between the two potential outcomes.

$\tau_{\text{NY}} = Y_{\text{NY}}\Bigg($  $\Bigg) - Y_{\text{NY}}\Bigg($  $\Bigg)$

or written more generally:

$$\tau_i = Y_i(1) - Y_i(0)$$

The fundamental problem of causal inference

We only observe one of the two potential outcomes for New York and all other states. In general, we always only observe one of two potential outcomes for our units of study.

- economics of COVID policy: a state either locks down or doesn't
- drug trials: an individual either receives the medicine or the placebo
- gender wage gap: a person is either male or female

The unknown outcomes are called the **missing potential outcomes** or **counterfactuals**. This is what makes causality a nontrivial task ... it is a **missing data problem**.

Is all hope lost?

Is all hope lost?

Definitely not! The potential outcomes will **always** be used as a starting point. Depending on the data and question to be answered, there are several approaches:

- Randomization and the sample average treatment effect
- Observational data – before-and-after and DiD approaches
- Fancier (probabilistic) models to address confounding. Regression, etc. (the "Prediction" part of class).

The average causal effect across the sample

This is called the **sample average treatment effect**. In stats language, it is called an estimand. Let's suppose we have N units in our data.

$$
\begin{aligned}
\textbf{SATE} &= \frac{1}{N}\sum_{i=1}^{N} \tau_i \\
&= \frac{1}{N}\sum_{i=1}^{N} \{Y_i(1) - Y_i(0)\}
\end{aligned}
$$

We still don't know how to calculate this because of the fundamental problem of causal inference.

However, here's an idea ...

Estimator of the **SATE**

We have the **observed** outcome and treatment. Let's call them:

$$Y_{\text{obs}} = (Y_1, \ldots, Y_N)$$
$$Z_{\text{obs}} = (Z_1, \ldots, Z_N)$$

Let's define our estimator of the **SATE** as the simple difference-in-means (DiM) between the treated and control units.

$$\widehat{\textbf{SATE}} = \frac{1}{\sum_i \mathbb{1}(Z_i = 1)} \sum_i \mathbb{1}(Z_i = 1) Y_i - \frac{1}{\sum_i \mathbb{1}(Z_i = 0)} \sum_i \mathbb{1}(Z_i = 0) Y_i$$

Q: When can this be reasonably interpreted as the average causal effect, when can it not?

Summary of DiM

- The difference-in-means (DiM) estimator is useful and interpretable for randomized experiments.

Summary of DiM

– The difference-in-means (DiM) estimator is useful and interpretable for randomized experiments. **Why?**

Summary of DiM

- The difference-in-means (DiM) estimator is useful and interpretable for randomized experiments. **Why?**
- The treatment and control groups that you're computing the "means" over are otherwise equal in all other attributes!

Summary of DiM

- The difference-in-means (DiM) estimator is useful and interpretable for randomized experiments. **Why?**
- The treatment and control groups that you're computing the "means" over are otherwise equal in all other attributes!
- This eliminates the confounding issue!
- In short, randomizing the treatment (cause) is the **gold standard** for understanding causality.

So we have a point estimate of the causal effect ...

But, what about uncertainty?

So we have a point estimate of the causal effect ...

But, what about uncertainty?

- Let's use the known process of randomization to our advantage
- It is common to characterize uncertainty under a particular **null hypothesis**.
- Think of the **null** as the not interesting or exciting causal conclusion. **"There is no causal effect"**.

$$\mathbf{H_0} : Y_i(0) = Y_i(1) \quad \text{for all } \ i$$

(Then, we use our actual data to probe this statement.)

What is the advantage of assuming $\mathbf{H_0}$?

What is the advantage of assuming $\mathbf{H_0}$?

We now know all potential outcomes!

What is the advantage of assuming $\mathbf{H_0}$?

We now know all potential outcomes!

i **(unit)**	z_i **(treatment)**	$Y_i(0)$	$Y_i(1)$
unit 1	1	??	.0034
unit 2	0	.007	??
unit 3	1	??	.0014
unit 4	0	.004	??
unit 5	0	.0028	??
unit 6	1	??	.002
$\vdots$	$\vdots$	$\vdots$	$\vdots$

What is the advantage of assuming $\mathbf{H_0}$?

$\mathbf{H_0} : Y_i(0) = Y_i(1) \quad \text{for all} \quad i$

i **(unit)**	z_i **(treatment)**	$Y_i(0)$	$Y_i(1)$
unit 1	1	**.0034**	.0034
unit 2	0	.007	**.007**
unit 3	1	**.0014**	.0014
unit 4	0	.004	**.004**
unit 5	0	.0028	**.0028**
unit 6	1	**.002**	.002
⋮	⋮	⋮	⋮

But where does uncertainty come in?

- Let's go back to our original DiM estimate
- This is given by a single number denoting the difference in average outcome of treated and control units

$$Y_{\text{obs}} = (Y_1, \ldots, Y_N)$$
$$Z_{\text{obs}} = (Z_1, \ldots, Z_N)$$

$$\mathbf{DiM} = \frac{1}{\sum_i \mathbb{1}(Z_i = 1)} \sum_i \mathbb{1}(Z_i = 1) Y_i - \frac{1}{\sum_i \mathbb{1}(Z_i = 0)} \sum_i \mathbb{1}(Z_i = 0) Y_i$$
$$= \text{“function of } Y \text{ and } Z\text{”}$$

So, let's write out the estimate explicitly as $\underline{\mathbf{DiM}(Y, Z)}$.

Randomization from the experiment is the uncertainty!

- If I have a randomized experiment, I know exactly how the analyst **allocated** units to treatment and control

Randomization from the experiment is the uncertainty!

- If I have a randomized experiment, I know exactly how the analyst **allocated** units to treatment and control
- For example, in the simplest case, the randomization could be done by flipping a fair coin for each unit i. If heads, unit i is in control, if tails, unit i is in treatment.

Randomization from the experiment is the uncertainty!

- If I have a randomized experiment, I know exactly how the analyst **allocated** units to treatment and control
- For example, in the simplest case, the randomization could be done by flipping a fair coin for each unit i. If heads, unit i is in control, if tails, unit i is in treatment.
- The point is that I have an experimental design, $P(Z)$, from which I can generate many **alternative treatments**!
- We write an alternative treatment as $Z' \sim P(Z)$

Importantly, under the H_0, I can now generate a bunch of alternative values of the DiM statistic .... $\underline{\mathrm{DiM}(Y, Z')}$

Randomization from the experiment is the uncertainty!

Q: Why do we obtain new values of the difference-in-means (DiM) for alternative draws of the treatment/control allocation, Z'?

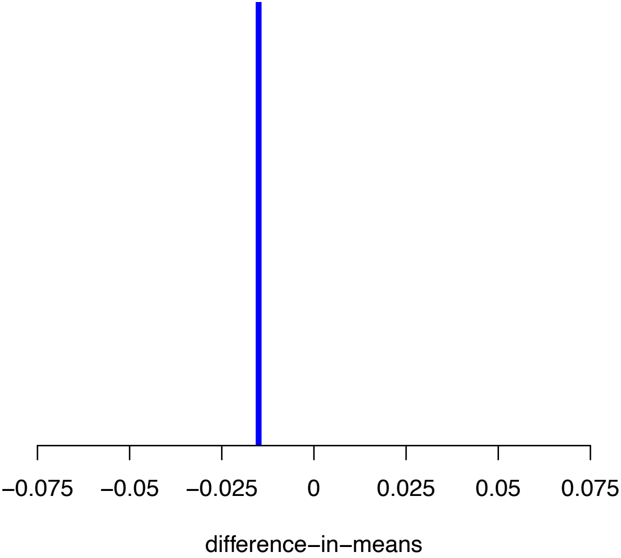
Randomization from the experiment is the uncertainty!

Q: Why do we obtain new values of the difference-in-means (DiM) for alternative draws of the treatment/control allocation, Z'?

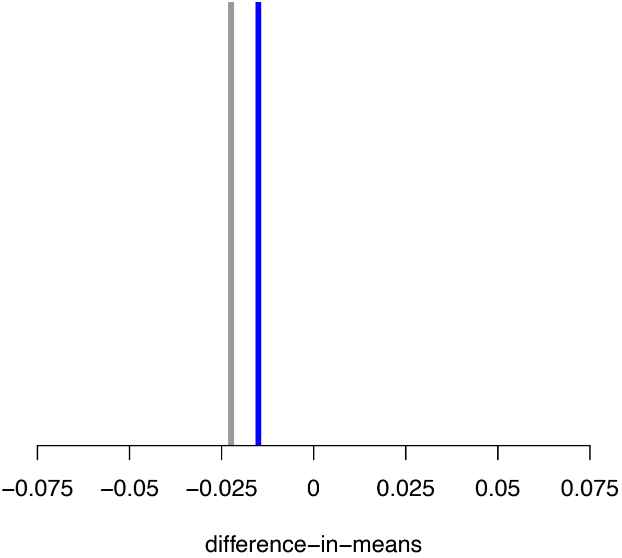
$$\text{DiM}(Y, Z) = \text{average } Y \text{ for treated} - \text{average } Y \text{ for control}$$

For a new Z', the groups of treated units and control units are different! So, the averages will be different and thus $\text{DiM}(Y, Z')$ will be different.

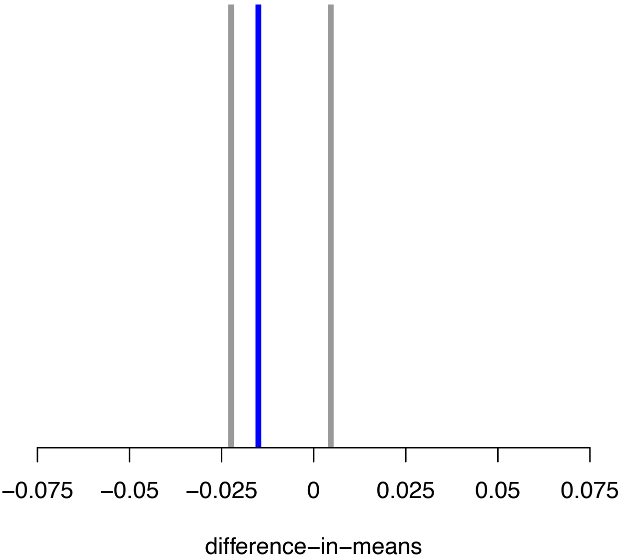
Observed DiM and its alternative values



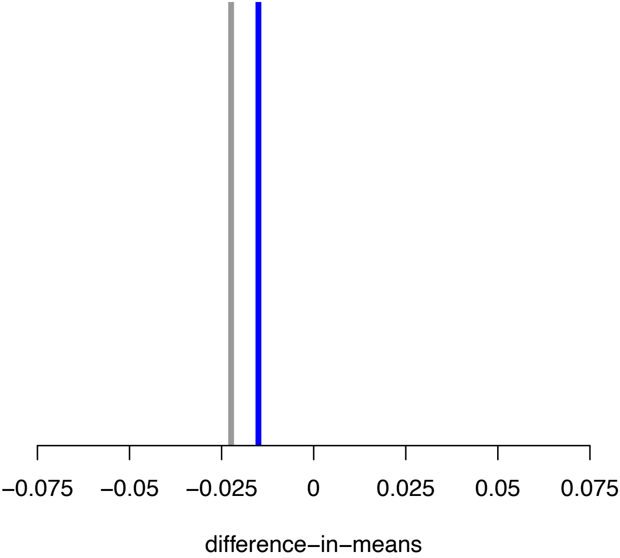
Observed DiM and its alternative values



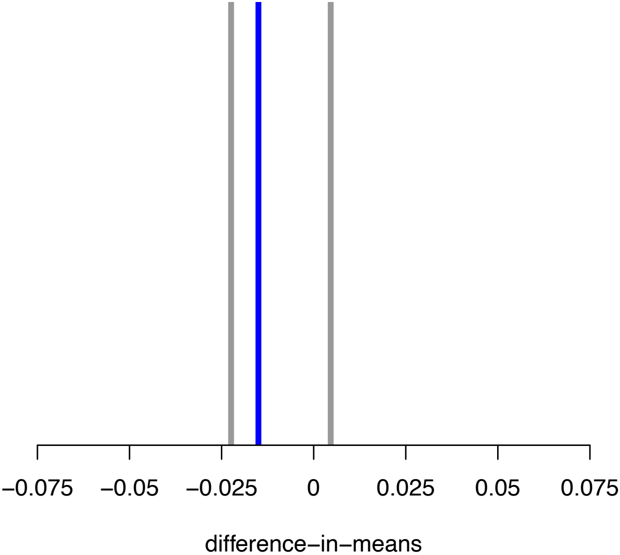
Observed DiM and its alternative values



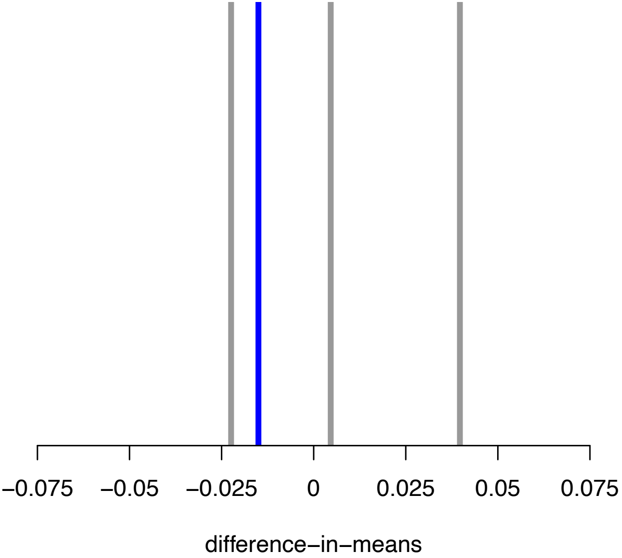
Observed DiM and its alternative values



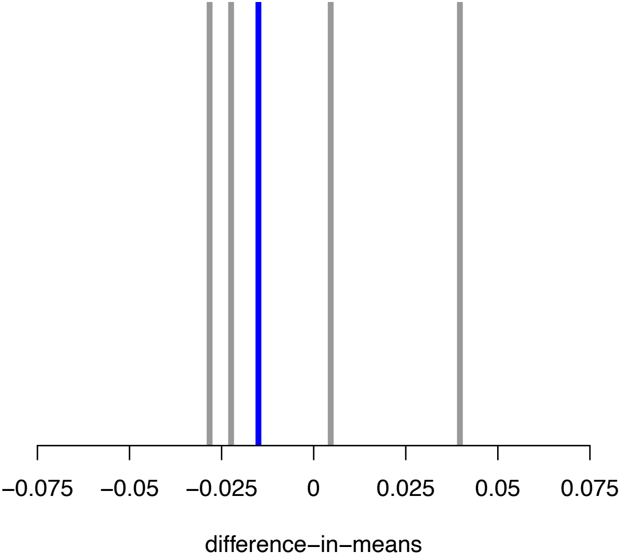
Observed DiM and its alternative values



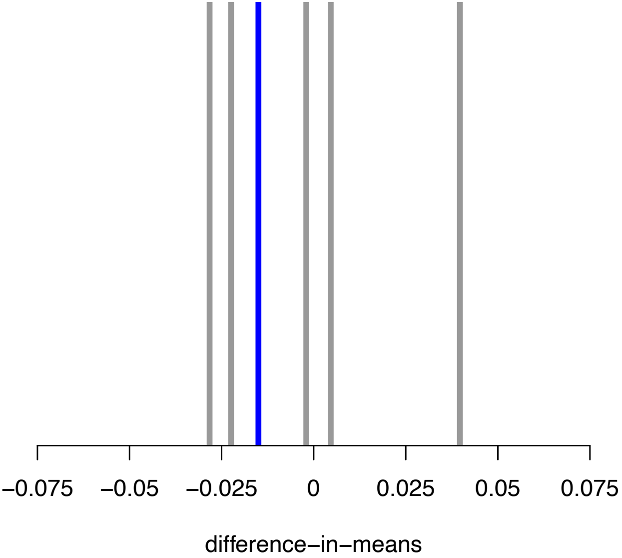
Observed DiM and its alternative values



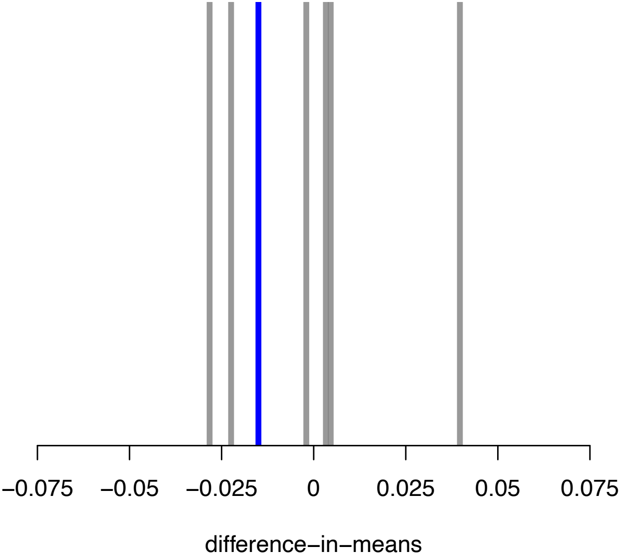
Observed DiM and its alternative values



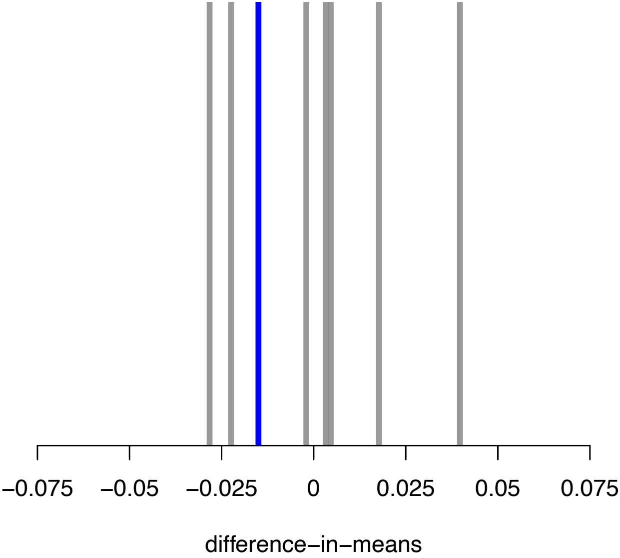
Observed DiM and its alternative values



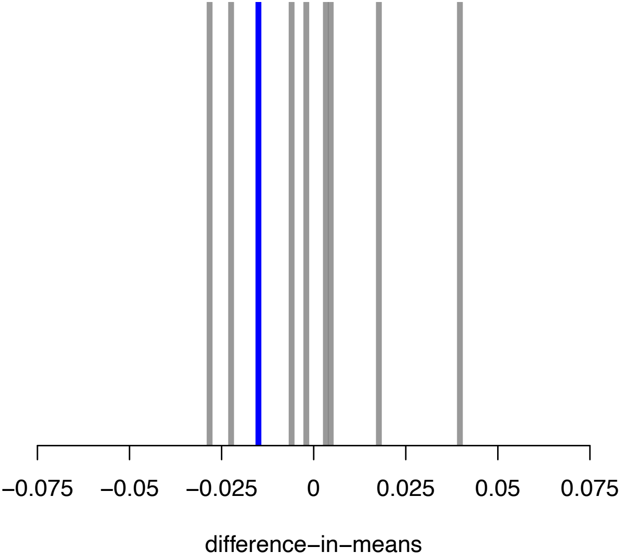
Observed DiM and its alternative values



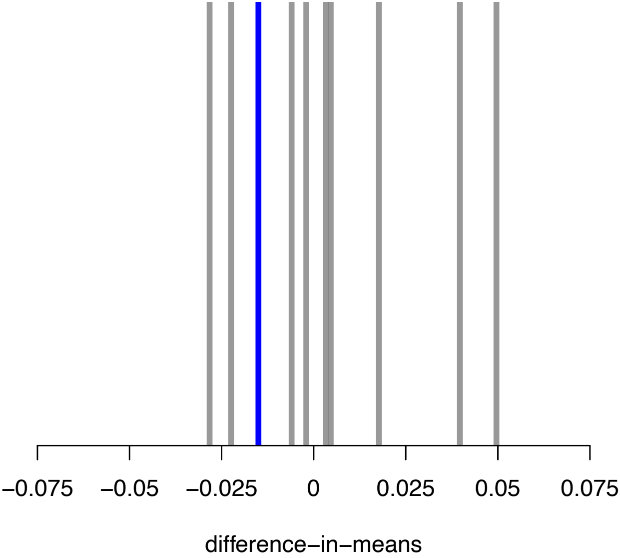
Observed DiM and its alternative values



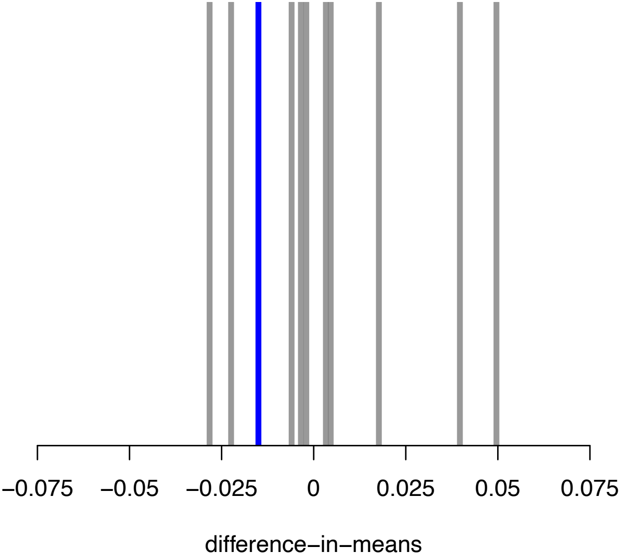
Observed DiM and its alternative values



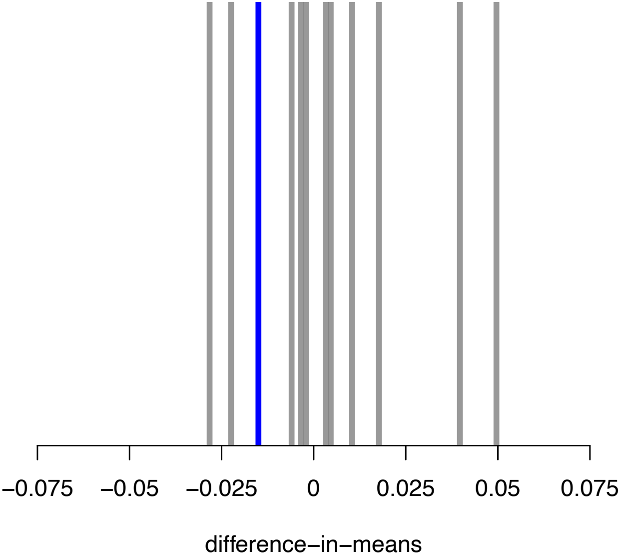
Observed DiM and its alternative values



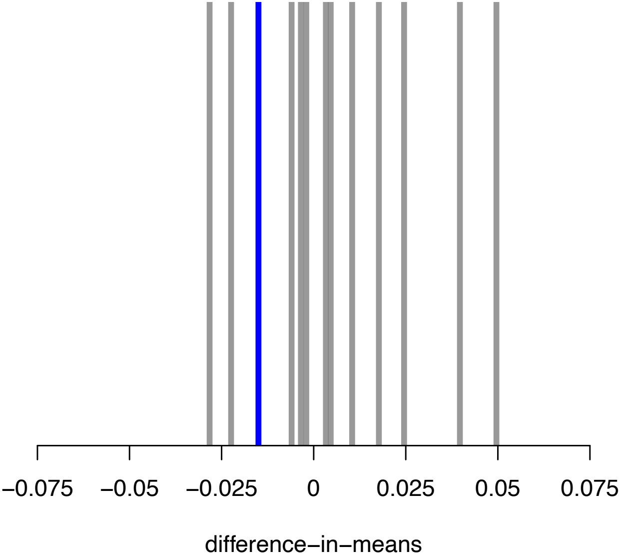
Observed DiM and its alternative values



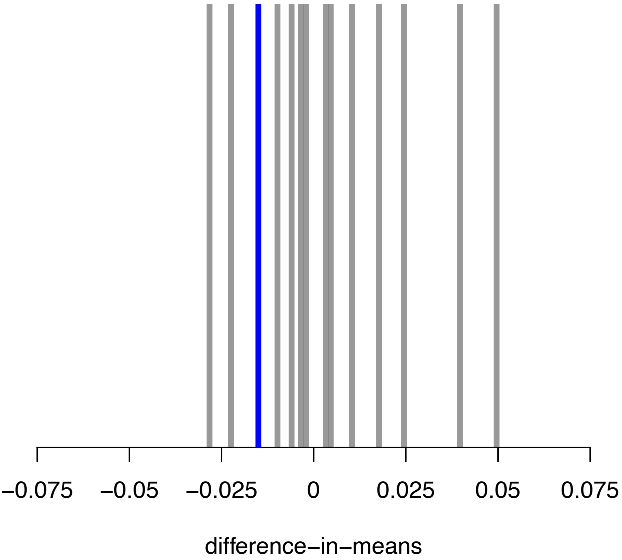
Observed DiM and its alternative values



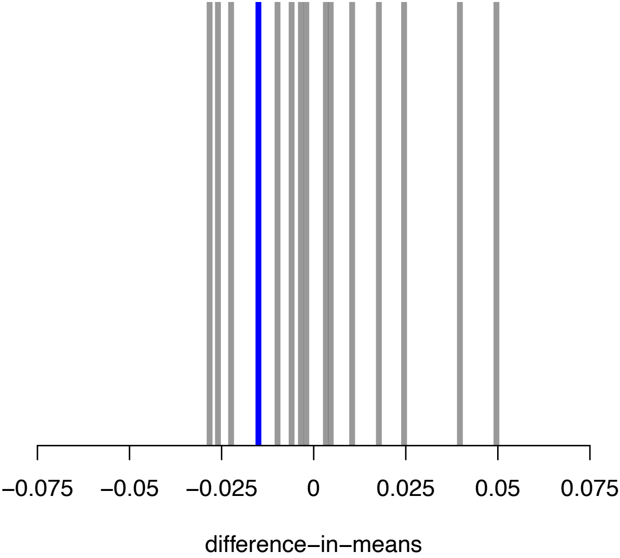
Observed DiM and its alternative values



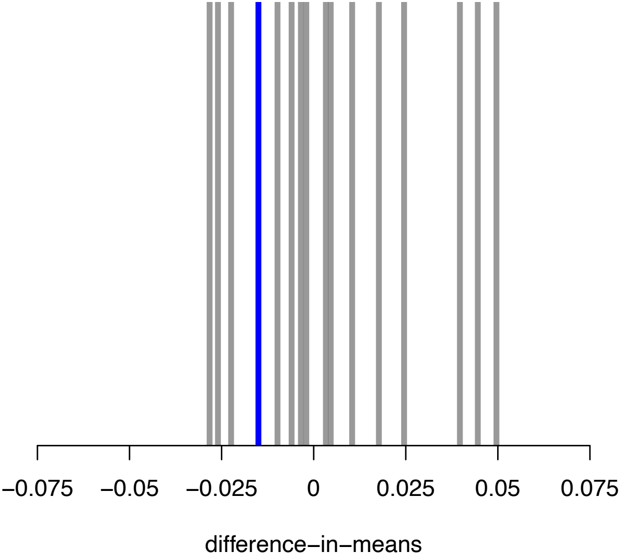
Observed DiM and its alternative values



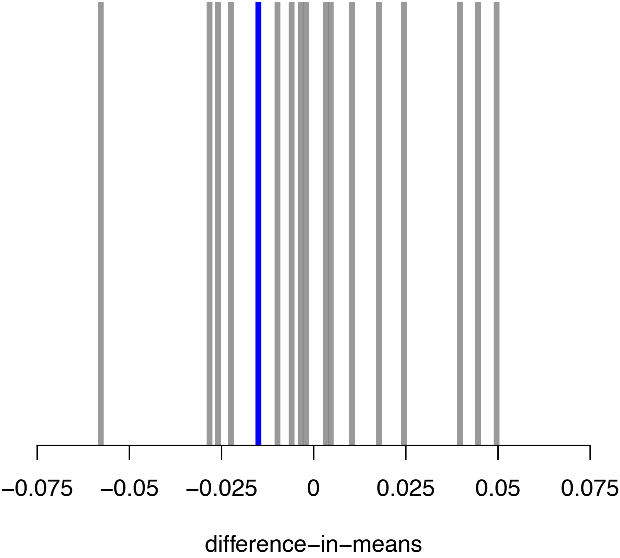
Observed DiM and its alternative values



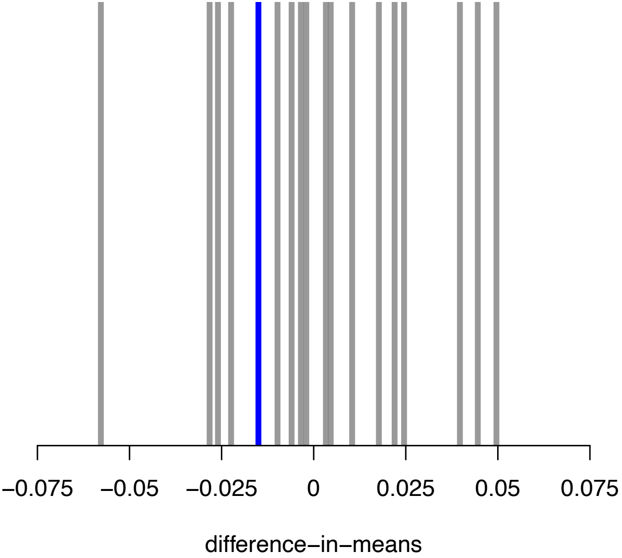
Observed DiM and its alternative values



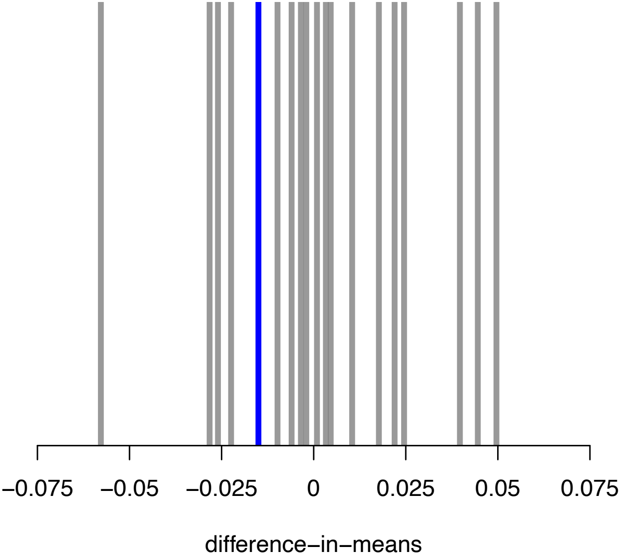
Observed DiM and its alternative values



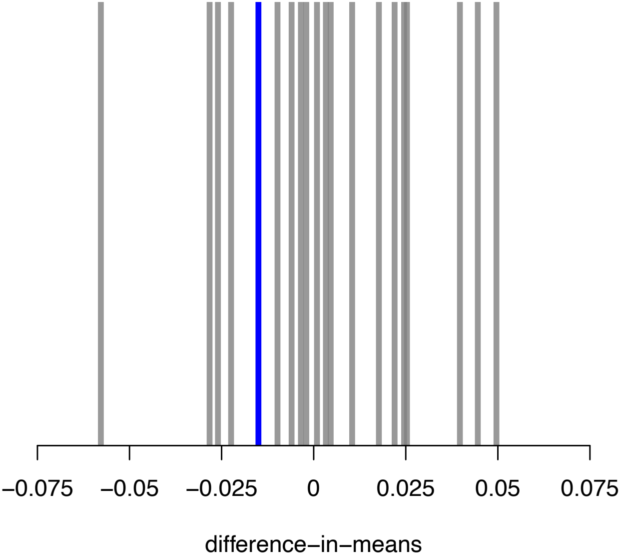
Observed DiM and its alternative values



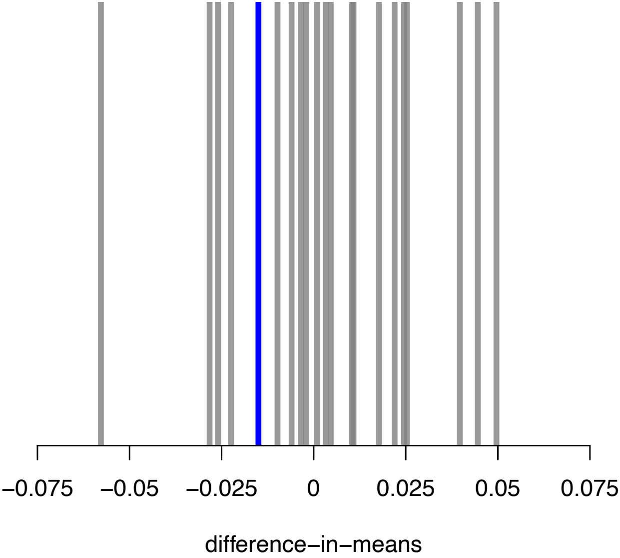
Observed DiM and its alternative values



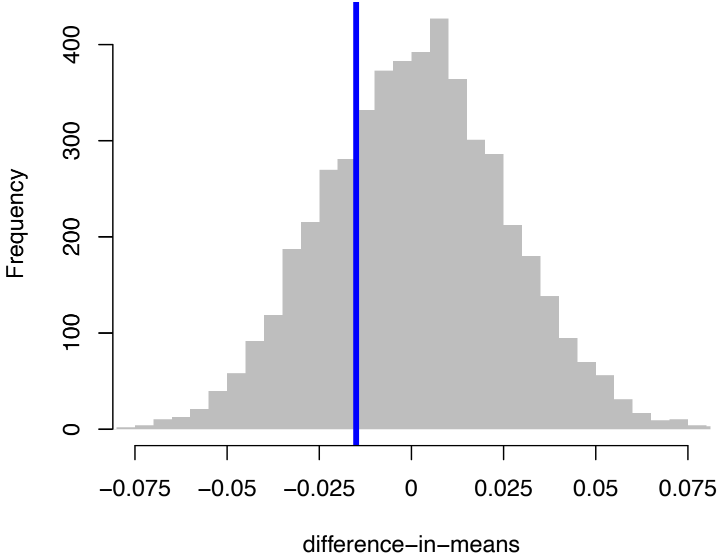
Observed DiM and its alternative values



Observed DiM and its alternative values



Observed DiM and its alternative values



Fisher randomization test (1935)

$\mathbf{H_0}: Y_i(0) = Y_i(1) \quad \text{for all } i$

The procedure:

1. $\text{DiM}_{\text{obs}} = \text{DiM}(Y_{\text{obs}}, Z_{\text{obs}})$.
2. Draw $Z' \sim P(Z')$,

store $\text{DiM}_r = \text{DiM}(Y', Z') \overset{H_0}{=} \text{DiM}(Y_{\text{obs}}, Z')$.

3. p-value $= \mathbb{E}\left[\mathbb{1}\{\text{DiM}_r \geq \text{DiM}_{\text{obs}}\}\right]$.

This beautifully simple procedure tells you whether or not your causal effect is "significant" (under the null hypothesis of no effect)!

Summary of DiM and FRT

- First, we defined potential outcomes, $Y_i(0)$ and $Y_i(1)$
- Second, we wrote down an estimand (SATE) and estimator (DiM) of the average causal effect
- Third, we decided that a point estimate isn't enough, we need uncertainty!
- Fourth, we discussed the Fisher randomization test (FRT) as a simple and powerful procedure for characterizing uncertainty using the experimental design, $P(Z)$.

From randomized experiments to observational studies

The goal is always the same, estimate the causal effect of some cause/treatment/intervention.

→ **Randomized experimental data**: The treatment allocation is randomized among study units. This makes treatment and control group comparable and estimating causal effects straightforward.

From randomized experiments to observational studies

The goal is always the same, estimate the causal effect of some cause/treatment/intervention.

→ **Randomized experimental data**: The treatment allocation is randomized among study units. This makes treatment and control group comparable and estimating causal effects straightforward.

→ **Observational study data**: The treatment allocation is *not* randomized, but simply observed.

∗ "I observe states lockdown and not lockdown"

∗ "I observe minimum wage increases in some places and not others"

We can't control what we observe, but we can still try to use this variation to answer interesting causal questions!

Observational studies

Let's use the minimum wage increase in NJ as our running example.

- 1992, NJ increased minimum wage from \$4.25 to \$5.05 per hour
- Causal question: Did this increase reduce or increase employment?

Observational studies

Let's use the minimum wage increase in NJ as our running example.

- 1992, NJ increased minimum wage from \$4.25 to \$5.05 per hour
- Causal question: Did this increase reduce or increase employment?
- There are a variety of ways to attempt an answer:
  - Compare outcome in NJ to a neighboring state, PA: cross-sectional comparison

Observational studies

Let's use the minimum wage increase in NJ as our running example.

- 1992, NJ increased minimum wage from \$4.25 to \$5.05 per hour

- Causal question: Did this increase reduce or increase employment?

- There are a variety of ways to attempt an answer:

  * Compare outcome in NJ to a neighboring state, PA: cross-sectional comparison
  * Compare groups of restaurants in NJ to *similar* groups of restaurants in PA: subclassification / statistical control

Observational studies

Let's use the minimum wage increase in NJ as our running example.

- 1992, NJ increased minimum wage from \$4.25 to \$5.05 per hour

- Causal question: Did this increase reduce or increase employment?

- There are a variety of ways to attempt an answer:

  * Compare outcome in NJ to a neighboring state, PA: cross-sectional comparison
  * Compare groups of restaurants in NJ to *similar* groups of restaurants in PA: subclassification / statistical control
  * Compare the same group *before* and *after* the intervention: panel comparison / before-after design

Observational studies

Let's use the minimum wage increase in NJ as our running example.

- 1992, NJ increased minimum wage from \$4.25 to \$5.05 per hour
- Causal question: Did this increase reduce or increase employment?
- There are a variety of ways to attempt an answer:
  - Compare outcome in NJ to a neighboring state, PA: cross-sectional comparison
  - Compare groups of restaurants in NJ to *similar* groups of restaurants in PA: subclassification / statistical control
  - Compare the same group *before* and *after* the intervention: panel comparison / before-after design
  - Compare before-after difference in NJ to before-after difference in PA: difference-in-differences

Cross-sectional comparison

This one is simple.

Define $\overline{Y}^{\text{NJ}}$ and $\overline{Y}^{\text{PA}}$ as the average percentage of full-time employees across fast-food restaurants in New Jersey and Pennsylvania, respectively.

$$\text{causal effect} = \overline{Y}^{\text{NJ}}\text{(treatment)} - \overline{Y}^{\text{PA}}\text{(control)}$$

Q: What are some issues with this?

Cross-sectional comparison

This one is simple.

Define $\overline{Y}^{\text{NJ}}$ and $\overline{Y}^{\text{PA}}$ as the average percentage of full-time employees across fast-food restaurants in New Jersey and Pennsylvania, respectively.

$$\text{causal effect} = \overline{Y}^{\text{NJ}}\text{(treatment)} - \overline{Y}^{\text{PA}}\text{(control)}$$

Q: What are some issues with this?

confounding! NJ and PA might be different in other ways, they have their own economies, political issues, populations, etc.

Statistical control (matching)

This one tries to address the confounding.

Pick covariates X in the data that might be confounding. Compute the causal effect only on treatment and control units with **identical** values of X.

$$\text{causal effect} = \overline{Y}^{\text{NJ}}_{X=\text{Taco Bell}} - \overline{Y}^{\text{PA}}_{X=\text{Taco Bell}}$$

It is called matching because you match on like values of covariates, i.e., compare apples-to-apples!

Q: What other X's could be used for matching?

Statistical control (matching)

This one tries to address the confounding.

Pick covariates X in the data that might be confounding. Compute the causal effect only on treatment and control units with **identical** values of X.

$$\text{causal effect} = \overline{Y}^{\text{NJ}}_{X=\text{Taco Bell}} - \overline{Y}^{\text{PA}}_{X=\text{Taco Bell}}$$

It is called matching because you match on like values of covariates, i.e., compare apples-to-apples!

Q: What other X's could be used for matching?

geographic location, closeness to NJ-PA border, urban or rural area

Panel comparison

What if we had our data measured *over time*. This is called panel or longitudinal data.

Panel comparison

What if we had our data measured *over time*. This is called panel or longitudinal data.

$$Y_{i,\text{before}}^{\text{NJ}} \longrightarrow \textbf{min wage hike} \longrightarrow Y_{i,\text{after}}^{\text{NJ}}$$

$$Y_{i,\text{before}}^{\text{PA}} \longrightarrow \textbf{min wage hike} \longrightarrow Y_{i,\text{after}}^{\text{PA}}$$

where i is a fast-food restaurant.

$$\text{causal effect} = \overline{Y}_{\text{after}}^{\text{NJ}} - \overline{Y}_{\text{before}}^{\text{NJ}}$$

Q: What are some issues with this?

Panel comparison

What if we had our data measured *over time*. This is called panel or longitudinal data.

$$Y^{\text{NJ}}_{i,\text{before}} \longrightarrow \textbf{min wage hike} \longrightarrow Y^{\text{NJ}}_{i,\text{after}}$$

$$Y^{\text{PA}}_{i,\text{before}} \longrightarrow \textbf{min wage hike} \longrightarrow Y^{\text{PA}}_{i,\text{after}}$$

where i is a fast-food restaurant.

$$\text{causal effect} = \overline{Y}^{\text{NJ}}_{\text{after}} - \overline{Y}^{\text{NJ}}_{\text{before}}$$

Q: What are some issues with this?

actually, confounding again! time is potentially an issue (remember our ice cream example).

Difference-in-differences

This one tries to remove time-trend confounding by using panel data from both the treated state (NJ) and the control state (PA).

There are four outcomes of interest now (see previous slide):
$\overline{Y}^{NJ}_{\text{before}}, \overline{Y}^{NJ}_{\text{after}}, \overline{Y}^{PA}_{\text{before}}, \overline{Y}^{PA}_{\text{after}} \leftarrow$ average full-time employment rates

$$\text{causal effect (DiD)} = (\overline{Y}^{NJ}_{\text{after}} - \overline{Y}^{NJ}_{\text{before}}) - (\overline{Y}^{PA}_{\text{after}} - \overline{Y}^{PA}_{\text{before}})$$

Why does this work?

Difference-in-differences

This one tries to remove time-trend confounding by using panel data from both the treated state (NJ) and the control state (PA).

There are four outcomes of interest now (see previous slide):
$\overline{Y}^{\text{NJ}}_{\text{before}}, \overline{Y}^{\text{NJ}}_{\text{after}}, \overline{Y}^{\text{PA}}_{\text{before}}, \overline{Y}^{\text{PA}}_{\text{after}} \leftarrow$ average full-time employment rates

$$\text{causal effect (DiD)} = (\overline{Y}^{\text{NJ}}_{\text{after}} - \overline{Y}^{\text{NJ}}_{\text{before}}) - (\overline{Y}^{\text{PA}}_{\text{after}} - \overline{Y}^{\text{PA}}_{\text{before}})$$

Why does this work?

- answer 1: picture
- answer 2: some math

Difference-in-differences (math explanation)

4*N potential outcomes, 2 for each state.

$$\{Y_i^{\text{NJ}}(0), Y_i^{\text{NJ}}(1), Y_i^{\text{PA}}(0), Y_i^{\text{PA}}(1)\} \quad \text{for all } i = 1, ..., N$$

Difference-in-differences (math explanation)

$4*N$ potential outcomes, 2 for each state.

$$\{Y_i^{\text{NJ}}(0), Y_i^{\text{NJ}}(1), Y_i^{\text{PA}}(0), Y_i^{\text{PA}}(1)\} \quad \text{for all } i = 1, ..., N$$

$$\text{causal effect (DiD)} = (\overline{Y}^{\text{NJ}}_{\text{after}} - \overline{Y}^{\text{NJ}}_{\text{before}}) - (\overline{Y}^{\text{PA}}_{\text{after}} - \overline{Y}^{\text{PA}}_{\text{before}})$$

$$\begin{aligned}
&= \big(\mathbb{E}[Y_i^{\text{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{before}]\big) \\
&\quad - \big(\mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{before}]\big) + \mathbf{0} \\
&= \big(\mathbb{E}[Y_i^{\text{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{before}]\big) \\
&\quad - \big(\mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{before}]\big) \\
&\quad + \big(\mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{after}]\big) \\
&= \big(\mathbb{E}[Y_i^{\text{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{after}]\big) \\
&\quad + \big(\mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{before}]\big) \\
&\quad - \big(\mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{before}]\big)
\end{aligned}$$

Difference-in-differences (math explanation)

$$
\begin{aligned}
\text{causal effect (DiD)} = {} & \left(\mathbb{E}[Y_i^{\mathrm{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{after}]\right) \\
& + \left(\mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{before}]\right) \\
& - \left(\mathbb{E}[Y_i^{\mathrm{PA}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{PA}}(0) \mid \text{before}]\right)
\end{aligned}
$$

$$
\begin{aligned}
\textbf{first part} &= \mathbb{E}[Y_i^{\mathrm{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{after}] \\
&= \text{sample average treatment effect (of treated)}
\end{aligned}
$$

Difference-in-differences (math explanation)

$$
\begin{aligned}
\text{causal effect (DiD)} = {} & \left(\mathbb{E}[Y_i^{\text{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{after}]\right) \\
& + \left(\mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{before}]\right) \\
& - \left(\mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{before}]\right)
\end{aligned}
$$

$$
\begin{aligned}
\textbf{first part} &= \mathbb{E}[Y_i^{\text{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{after}] \\
&= \text{sample average treatment effect (of treated)}
\end{aligned}
$$

$$
\begin{aligned}
\textbf{second part} &= \left(\mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{NJ}}(0) \mid \text{before}]\right) \\
&\quad - \left(\mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\text{PA}}(0) \mid \text{before}]\right) \\
&= \text{slope of NJ line} - \text{slope of PA line}
\end{aligned}
$$

Difference-in-differences (math explanation)

$$
\begin{aligned}
\text{causal effect (DiD)} = &\left(\mathbb{E}[Y_i^{\mathrm{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{after}]\right) \\
&+ \left(\mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{before}]\right) \\
&- \left(\mathbb{E}[Y_i^{\mathrm{PA}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{PA}}(0) \mid \text{before}]\right)
\end{aligned}
$$

$$
\begin{aligned}
\textbf{first part} &= \mathbb{E}[Y_i^{\mathrm{NJ}}(1) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{after}] \\
&= \text{sample average treatment effect (of treated)}
\end{aligned}
$$

$$
\begin{aligned}
\textbf{second part} = &\left(\mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{NJ}}(0) \mid \text{before}]\right) \\
&- \left(\mathbb{E}[Y_i^{\mathrm{PA}}(0) \mid \text{after}] - \mathbb{E}[Y_i^{\mathrm{PA}}(0) \mid \text{before}]\right) \\
= &\ \text{slope of NJ line} - \text{slope of PA line}
\end{aligned}
$$

parallel trend assumption: **second part** $= 0$