



McCOMBS SCHOOL OF BUSINESS

**Salem Center for Policy**

# Networks

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Networks and data from networks

Summarizing networks

Generating networks – association rules



**Network data** is everywhere. In most cases network relationships are assumed to not exist, but they are actually important and meaningful.

Examples:

- Spatial networks, countries and trade networks
- City streets / NYC subway
- Social media: Meta (Facebook), Instragram, Linkedin
- Spotify
- Classrooms / households



Graphical models provide a language for networks:

- 0/1 connections between people/sites/covariates

Think of graphs like a binary version of correlation

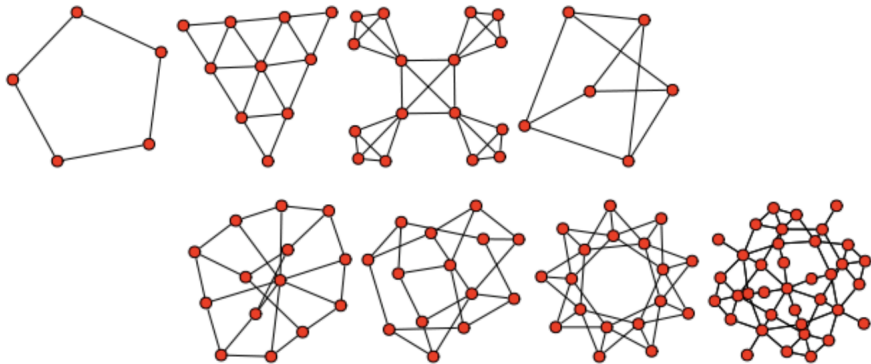
Graph Structure (how to describe a network?):

- Summarization: nodes and edges, direction
- Measuring connectivity and betweenness

Association and networks

- Market Basket Analysis

# What comprises a network?



The network has **nodes** (vertices), such as a website or worker, and **edges** are the (directed or undirected) links between nodes.

# Network data is connected



A network consists of variables and connections between them. A connection is discrete: it's either there or it's not.

Data living on a network:

- Word usage in text and language (what words follow?)
- Organization charts and employment (who's boss?)
- Business credit, supply, and competition networks
- Genes
- Everything on the internet!

Sometimes the network is given, other times we just get a glimpse of traffic on the network.



Start with a given network: you see all connections.

We'll reduce dimension + summarize important properties.  
In particular, we'll focus on measures of network connectivity.

Each node has connectivity statistics

**Degree:** How many other nodes are you connected to?

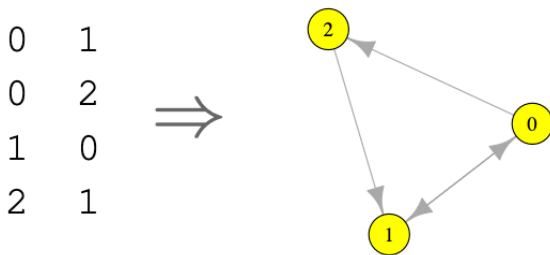
**Betweenness:** How many node-to-node paths go through you?

You can also make a lot of cool illustrations for graphs. These tend to be more pretty than informative, but that doesn't mean they aren't useful.

[igraph](#) is a great toolbox for visualizing and summarizing graphs. It has front-ends for R and Python.

Unlike most R packages, `igraph` is well documented. Type `help(igraph)` to get started.

For most applications, you'll read graphs from an [edgelist](#):





## Example: Marriage and power

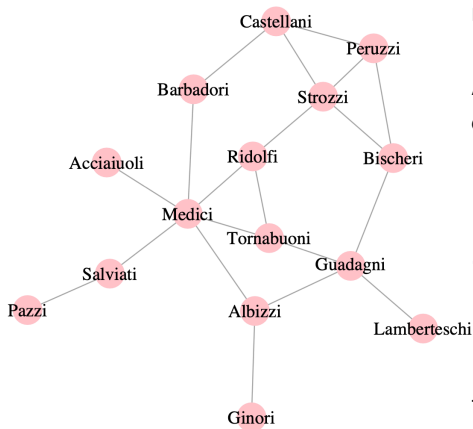


Early Renaissance Florence was ruled by an oligarchy of powerful families.

By the 15th century, the Medicis emerged supreme, & Medici Bank became the largest in Europe.

Political ties were established via marriage. [How did Medici win?](#)

# Marriage in Florence: 1250-1450



Network links can be used to measure “social capital”

A node's **degree** is its number of edges

```
> sort(degree(marriage))  
Ginori ... Strozzi Medici  
1          4          6
```

The Medicis are connected!

## Betweenness – a deeper measure of network structure



An alternative to degree, **betweenness** measures the proportion of shortest paths containing a given node

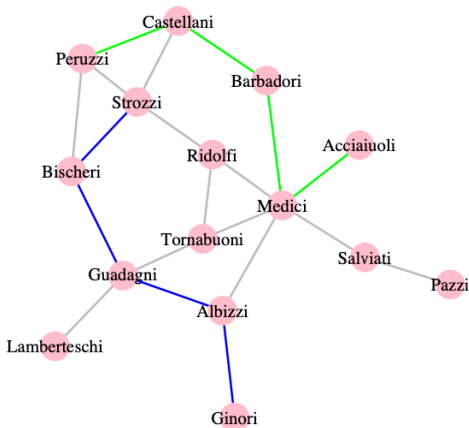
Shortest path: fewest steps from  $i$  to  $j$  (direction matters)

Say  $s_k(i, j)$  is the proportion of shortest paths from  $i$  to  $j$  containing node  $k$ .

$$\text{betweenness}(k) = \sum_{i, j: i \neq j, k \notin \{i, j\}} s_k(i, j)$$

**Intuitively**, this measure how much influence a node has over connections between others!

# Betweenness versus degree



Medicis have the highest degree, but only by a factor of 1.5 over the Strozzi.

```
> sort(betweenness(marriage))  
Ginori ... Strozzi Medici  
0.0      9.3    47.5
```

But their **betweenness** is 5 times higher!



A common question in data mining: What do one person's choices say about another's?

As Amazon says: “people who buy this book also bought...”

These types of tasks are referred to as **collaborative filtering**: using **shared choices** to predict preferences.



It's a big field, with many tools:

- logistic regression of each product on to all other choices
- principal components analysis: Underlying taste factors.

But as an easy start, there are good fast algorithms for discovering low dimensional **association rules**. Foreshadow, we will build networks with these association rules.



Consider two binary variables:  $x_a$  &  $x_b$ .

If  $x_b = 1$  more often when  $x_a = 1$ , then we say  $x_a \implies x_b$  is an **association rule**.

Example: When you buy **chips**, you need **beer** to wash them down

Suppose that beer is purchased 10% of the time in general, but 50% of the time when the consumer grabs chips.

- The **support** for **beer** is 10%
- The **confidence** for **beer** is 50%
- The **lift** is 5



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Example: When you buy **chips**, you need **beer** to wash them down

Suppose that beer is purchased 10% of the time in general, but 50% of the time when the consumer grabs chips.

- The **support** for **beer** is 10% (total probability)
- The **confidence** for **beer** is 50% (conditional probability)
- The **lift** is 5 (ratio)





Market basket analysis – use purchase coincidence to build association rules

Our example basket: LHS (chips)  $\implies$  RHS (beer), where LHS is called the **antecedent**, and the RHS is called the **consequent**.

Every event has support: the proportion of times it occurred. This leads to two measures of association rule strength:

- confidence:  $\frac{\text{supp}(\text{LHS and RHS})}{\text{supp}(\text{LHS})}$  ... probability of RHS given LHS
- lift:  $\frac{\text{supp}(\text{LHS and RHS})}{\text{supp}(\text{LHS})\text{supp}(\text{RHS})}$  ... increase in probability of RHS given LHS

Interpretation? Remember Bayes rule!  $P(\text{RHS} \mid \text{LHS}) = P(\text{RHS}, \text{LHS})/P(\text{LHS})$



Generally, association rules with high **lift** are most useful because they tell you something you don't already know.

Low **support** does not preclude high **confidence** or high **lift**

- chips  $\implies$  beer is high support, but low lift if everybody always buys beer!
- caviar  $\implies$  vodka is low support, but high lift if people only buy vodka for their caviar parties

There's no deep theory around these rules, we often just scan the data for interesting relationships and go from there.

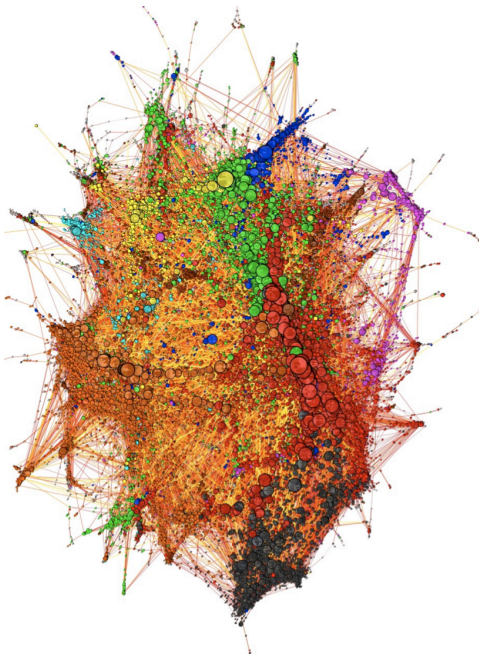


To find confidence and lift, just count the # of times **RHS** and **LHS** happen, and how often they happen together.

$$\text{supp}(\text{event}) = \frac{\# \text{ of times event occurs}}{\# \text{ of observations}}$$

However, counting all combinations can take a long time! The `apriori` function in R is very useful for this task.

# Streaming playlists and music preferences



Let's consider a large collection of playlists (like a shopping cart!)

metal, rock, pop, jazz, electronica, hip-hop  
reggae/ska, classical, folk/country/world.

This “network” shows artists sized by play count, with lines (edges) for shared users.

# Association rules for musical taste



lhs	rhs	support	confidence	lift
t.i.	=> kanye west	0.0104	0.5672	8.8544
pink floyd,				
the doors	=> led zeppelin	0.0106	0.5387	6.8020
beyonce	=> rihanna	0.0139	0.4686	10.8810
morrissey	=> the smiths	0.0112	0.4655	8.8961
megadeth	=> iron maiden	0.0132	0.4307	7.2677
jimi hendrix	=> the doors	0.0120	0.3062	5.3170
nelly furtado	=> madonna	0.0100	0.2750	5.0374
bright eyes	=> the shins	0.0102	0.2698	5.4623
elliott smith	=> modest mouse	0.0109	0.2679	5.1732
britney spears	=> lady gaga	0.0120	0.2612	7.7292
ramones	=> the clash	0.0104	0.2586	5.9052
franz ferdinand	=> kaiser chiefs	0.0132	0.2224	7.1153

Example: Given a new user that listens to a lot of Morrissey, we're 46% positive that they'll also like the Smiths. This is 9 times higher than if we didn't know about Morrissey.



Graphs can be a useful way to summarize all sorts of data. We can define a network using any measure of connectivity.

For example, an association network:

- Say there's an edge between LHS and RHS if **support** and **confidence** are greater than some thresholds
- If we just look at any shared membership in a playlist, we get our monster graph from the beginning

Let's check out `playlists.R` to see this in action.