Control Chart for Number of Defects per Unit

SESSION 10

Structure

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Session 7 of MSTL-002 (Industrial Statistics Lab).
- Unit 4 of MSTE-001 (Industrial Statistics-I).

10.1 INTRODUCTION

In Lab Session 9, you have learnt how to apply the c-chart for monitoring and controlling the number of defects. You have also noted that the c-chart is applied only when the **sample size is 1 or constant**, i.e., the number of units or items is the same in each sample. When the sample size varies for some reason, we use the u-chart for controlling the number of defects. The u-chart can also be used when the **sample size is constant**.

You have learnt how to construct the u-chart in Unit 4 of MSTE-001 (Industrial Statistics-I). In this lab session, we explain how to construct the u-chart for monitoring and controlling the number of defects for **varying sample sizes** with the help of MS Excel 2007.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for the u-chart;
- construct the u-chart; and
- interpret the u-chart.

10.2 PROBLEM DESCRIPTION

Let us consider the problem faced by a textile company, which observes defects in its woven fabric. As part of an overall quality improvement programme, the company decided to monitor the number of defects found in the woven fabric each day. The fabric produced each day is one metre

wide but varies in length. For this purpose, the length of woven fabric and the total number of defects inspected on each day were recorded for 35 days. The data is given in Table 1.

Table 1: Number of defects in fabric

Day	Length of Fabric (in metre)	Number of Defects	Day	Length of Fabric (in metre)	Number of Defects
1	50	7	19	35	8
2	40	3	20	50	4
3	45	1	21	33	2
4	35	3	22	43	5
5	45	6	23	38	12
6	30	2	24	38	1
7	40	4	25	48	5
8	35	5	26	43	3
9	35	7	27	43	6
10	45	3	28	33	5
11	40	12	29	48	7
12	40	2	30	53	2
13	30	4	31	48	4
14	45	8	32	38	5
15	50	3	33	43	4
16	45	7	34	38	3
17	35	6	35	53	6
18	40	3			

The quality control inspector of the company needs to construct a control chart to check whether the process is under statistical control or not and also compute the revised control limits, if necessary.

Therefore, the problem for this session is to construct the control chart for number of defects per unit, i.e., the u-chart for the data given in Table 1.

10.3 PROCEDURE FOR THE CONSTRUCTION OF u-CHART

You have learnt how to construct the u-chart in Unit 4 of MSTE-001. You know that the control limits are the function of sample size (n_i) in the u-chart. So the control limits vary with sample size. We calculate the control limits separately for each sample as explained in Lab Session 7. Here we list the formulae and procedure for computing control limits.

The u-chart plays the same role for the number of defects as the p-chart for the number of defectives with variable sample size discussed in Session 7. Therefore, we can use two approaches for constructing the u-chart in the same way as the p-chart with variable sample size.

First Approach

According to the first approach, we calculate the control limits for each sample separately as follows:

Step 1: If $c_1, c_2, ..., c_k$ represent the number of defects in the 1st, 2nd, …, kth samples of size $n_1, n_2, ..., n_k$, respectively, the number of defects per unit for each sample is given by

$$u_i = \frac{c_i}{n_i}; \quad (i = 1, 2, ..., k) \qquad \ldots(1)$$

Step 2: The average number of defects per unit is given by

$$\bar{u} = \frac{\sum_{I=1}^{k} c_i}{\sum_{I=1}^{k} n_i} \qquad \ldots(2)$$

Step 3: The centre line remains constant for each sample because it is not a function of sample size and is given as

$$\text{Centre line}(CL) = \bar{u} \qquad \ldots(3)$$

Step 4: The upper and lower control limits for each sample are:

- ✓ Upper control limit $(UCL_i) = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$ ...(4)
- ✓ Lower control limit $(LCL_i) = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$ ...(5)

Step 5: Interpretation of the u-chart.

Second Approach

According to the second approach, the control limits are calculated using the average sample size. This approach is used only when the variations among the sample sizes are not large and future sample sizes are not expected to differ significantly from the average sample size. Using this approach, we get constant control limits just as in c-chart.

Step 1: The average sample size is given by

$$\bar{n} = \frac{n_1 + n_2 + ... + n_k}{k} = \frac{1}{k}\sum_{i=1}^{k} n_i \qquad \ldots(6)$$

Step 2: The average number of defects per unit are given by

$$\bar{u} = \frac{\sum_{i=1}^{n} c_i}{k\bar{n}} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} n_i} \qquad \ldots(7)$$

Step 3: The centre line, upper and lower control limits are:

- ✓ Centre line$(CL) = \bar{u}$ ...(8)
- ✓ Upper control limit $(UCL) = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$ ...(9)
- ✓ Lower control limit $(LCL) = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$ ...(10)

Step 4: Interpretation of the u-chart.

10.4 STEPS INVOLVED IN THE CONSTRUCTION OF u-CHART IN EXCEL 2007

We now give the steps for constructing the u-chart in Excel 2007 for the data given in Sec.10.2.

Step 1: We enter the given data in the MS Excel spreadsheet as shown in Fig.10.1.

	A	B	C
1			
2	Days	Length of Fabric	Number of Defects
3	1	50	7
4	2	40	3
5	3	45	1
6	4	35	3
7	5	45	6
8	6	30	2
9	7	40	4
10	8	35	5
11	9	35	7
12	10	45	3
13	11	40	12
14	12	40	2
15	13	30	4
16	14	45	8
17	15	50	3
18	16	45	7
19	17	35	6
20	18	40	3
21	19	35	8

u Chart

Fig. 10.1: Partial screenshot of the spreadsheet for the given data.

Step 2: We compute the number of defects per metre fabric for each sample in Column D. We type "=C3/B3" in Cell D3 and drag down Cell D3 up to Cell D37 to obtain the number of defects per metre fabric for all 35 samples as shown in Fig. 10.2.

D3 =C3/B3

	A	B	C	D
1				
2	Days	Length of Fabric	Number of Defects	u
3	1	50	7	0.140
4	2	40	3	
5	3	45	1	
6	4	35	3	
7	5	45	6	

DRAG IT DOWN

	A	B	C	D
1				
2	Days	Length of Fabric	Number of Defects	u
3	1	50	7	0.140
4	2	40	3	0.075
5	3	45	1	0.022
6	4	35	3	0.086
7	5	45	6	0.133
8	6	30	2	0.067
9	7	40	4	0.100
10	8	35	5	0.143
11	9	35	7	0.200
12	10	45	3	0.067
13	11	40	12	0.300

Fig. 10.2

Step 3: We calculate the total length of fabric and total number of defects by using "=Sum(B3:B37)" and "=Sum(C3:C37)" function in Cells B38 and C38, respectively. The output is shown in Fig. 10.3.

	A	B	C
38	Total	1450	168

Used **"=Sum(B3:B37)"** function Used **"=Sum(C3:C37)"** function

Fig. 10.3

Step 4: We determine the average number of defects per unit ($\bar{u}$) by typing "=C38/B38" in Cell B39 as shown in Fig. 10.4.

B39 f_x =C38/B38

	A	B	C
39	$\bar{u}$	0.116	
40			

Fig. 10.4

Step 5: We calculate the values of the centre line, upper and lower control limits from equations (3) to (5) in Columns E, F and G, respectively, by typing

The formula with dollar sign ($) is used for an absolute reference.

i) "=$B$39" in Cell E3 to find the centre line as shown in Fig. 10.5a.

ii) "=E3+3*Sqrt(E3/B3)" in Cell F3 to calculate the upper control limit as shown in Fig. 10.5b.

iii) "=E3-3*Sqrt(E3/B3)" in Cell G3 to compute the lower control limit as shown in Fig. 10.5c.

(a)

E3 f_x =$B$39

	E	F	G	H
1		Control Limits		
2	Centre Line	UCL	LCL	
3	0.116			
4				

(b)

F3 f_x =E3+3*SQRT(E3/B3)

	F	G	H	I
1	Control Limits			
2	UCL	LCL		
3	0.260			
4				

(c)

G3 f_x =E3-3*SQRT(E3/B3)

	G	H	I	J
1				
2	LCL			
3	-0.029			
4				

Fig. 10.5

Step 6: We select Cells E3:G3 and drag them down up to Row 37 to get the control limits corresponding to each sample. The output is shown in Fig. 10.6.

	E	F	G
1		Control Limits	
2	Centre Line	UCL	LCL
3	0.116	0.260	-0.029
4	0.116	0.277	-0.046
5	0.116	0.268	-0.036
6	0.116	0.288	-0.057
7	0.116	0.268	-0.036
8	0.116	0.302	-0.071
9	0.116	0.277	-0.046
10	0.116	0.288	-0.057
11	0.116	0.288	-0.057
12	0.116	0.268	-0.036
13	0.116	0.277	-0.046
14	0.116	0.277	-0.046
15	0.116	0.302	-0.071
16	0.116	0.268	-0.036
17	0.116	0.260	-0.029
18	0.116	0.268	-0.036
19	0.116	0.288	-0.057
20	0.116	0.277	-0.046
21	0.116	0.288	-0.057

u Chart

Fig. 10.6

If the values of lower control limits are negative, we set the lower control limits as zero because negative number of defects is not possible.

Step 7: The values of LCL are negative (Fig. 10.6), which is not possible and, therefore, not acceptable. We set it to the value 0 (zero) and denote the limit by LCL* as shown in Fig. 10.7 to plot the control chart.

	D	E	F	G	H
1			Control Limits		
2	u	Centre Line	UCL	LCL	LCL*
3	0.140	0.116	0.260	-0.029	0
4	0.075	0.116	0.277	-0.046	0
5	0.022	0.116	0.268	-0.036	0
6	0.086	0.116	0.288	-0.057	0
7	0.133	0.116	0.268	-0.036	0
8	0.067	0.116	0.302	-0.071	0
9	0.100	0.116	0.277	-0.046	0
10	0.143	0.116	0.288	-0.057	0
11	0.200	0.116	0.288	-0.057	0
12	0.067	0.116	0.268	-0.036	0
13	0.300	0.116	0.277	-0.046	0
14	0.050	0.116	0.277	-0.046	0
15	0.133	0.116	0.302	-0.071	0
16	0.178	0.116	0.268	-0.036	0
17	0.060	0.116	0.260	-0.029	0
18	0.156	0.116	0.268	-0.036	0
19	0.171	0.116	0.288	-0.057	0
20	0.075	0.116	0.277	-0.046	0
21	0.229	0.116	0.288	-0.057	0

u Chart

Fig. 10.7

Step 8: To plot the u-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1. It means that we

1. select Cells D3:F37 and H3:H37 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting u-chart is shown in Fig. 10.8.

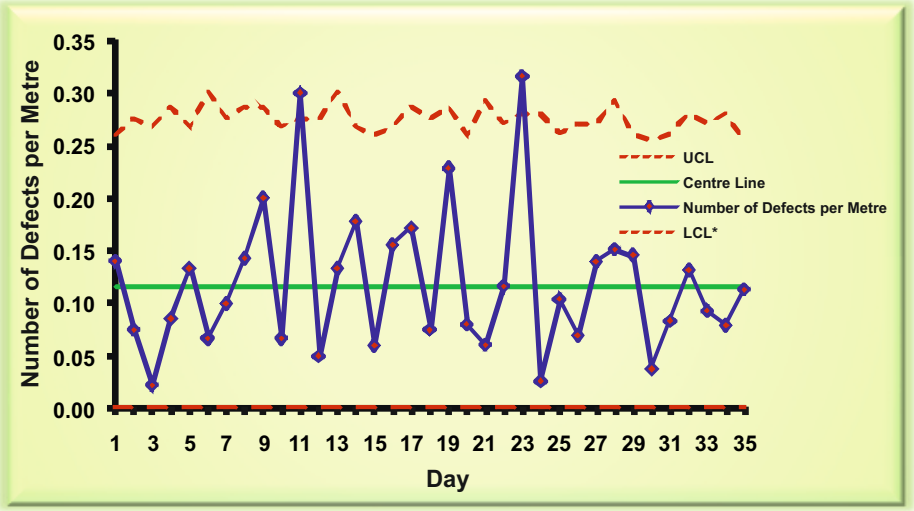

Fig. 10.8

Interpretation

The control chart for number of defects (shown in Fig. 10.8) indicates that the process is not under statistical control. Some assignable causes are present in the process. The samples corresponding to the 11th and 23rd days are outside the upper control limit. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. We now remove the out-of-control samples for the 11th and 23rd days and calculate the revised centre line and control limits for the u-chart using the remaining samples. These limits are known as the revised control limits and are discussed in Sec.10.5. But before studying Sec. 10.5, you should practice how to construct the u-chart using second approach.

Activity 1

✓ Compute the centre line and control limits in Excel 2007 using the second approach which you have learnt in Sec. 4.4 of MSTE-001.

✓ Plot the u-chart and compare the result with the output obtained in Sec. 10.4. If any point goes out of control, compute the revised control limits in the same manner as discussed in Sec. 10.5.

10.5 REVISED u-CHART

We plot the u-chart for the revised control limits using the remaining samples. We repeat the procedure until all points come within UCL and LCL. The main steps for computing the revised control limits are as follows:

First Approach

Step 1: For the revised limits for u-chart, we calculate the new $\bar{u}$ as follows:

$$\bar{u}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{\sum_{i=1}^{k} n_i - \sum_{j=1}^{d} n_j} \quad \ldots(10)$$

where d – number of discarded samples, and

$\sum_{j=1}^{d} c_j$ – total number of defects within the discarded sample.

Step 2: Then we reconstruct the centre line and control limits for the u-chart by replacing $\bar{u}$ by $\bar{u}_{new}$ as follows:

✓ Centre line (CL) = $\bar{u}_{new}$ ...(11)

✓ Upper control limit (UCL_i) = $\bar{u}_{new} + 3\sqrt{\frac{\bar{u}_{new}}{n_i}}$ ...(12)

✓ Lower control limit (LCL_i) = $\bar{u}_{new} - 3\sqrt{\frac{\bar{u}_{new}}{n_i}}$ ...(13)

Step 3: Interpretation of the u-chart.

Second Approach

Step 1: For the revised limits for u-chart, the value of new $\bar{u}$ will be same as equation (10).

Step 2: Then we reconstruct the centre line and control limits for the u-chart by replacing $\bar{u}$ by $\bar{u}_{new}$ as follows:

✓ Centre line (CL) = $\bar{u}_{new}$ ...(14)

✓ Upper control limit (UCL) = $\bar{u}_{new} + 3\sqrt{\frac{\bar{u}_{new}}{\bar{n}}}$ ...(15)

✓ Lower control limit (LCL) = $\bar{u}_{new} - 3\sqrt{\frac{\bar{u}_{new}}{\bar{n}}}$ ...(16)

Step 3: Interpretation of the u-chart.

Steps in Excel

The revised centre line and control limits for the u-chart in Excel 2007 using the remaining samples are as follows:

Step 1: We observe from Fig. 10.8 that the samples corresponding to the 11th and 23rd days lie outside the upper control limit. So we highlight the samples which lie outside the control limits, i.e., the 11th and 23rd samples, with light orange colour as shown in Fig. 10.9.

	A	B	C	D	E	F	G	H
1						Control Limits		
2	Days	Length of Fabric	Number of Defects	u	Centre Line	UCL	LCL	LCL*
3	1	50	7	0.140	0.116	0.260	-0.029	0
4	2	40	3	0.075	0.116	0.277	-0.046	0
5	3	45	1	0.022	0.116	0.268	-0.036	0
6	4	35	3	0.086	0.116	0.288	-0.057	0
7	5	45	6	0.133	0.116	0.268	-0.036	0
8	6	30	2	0.067	0.116	0.302	-0.071	0
9	7	40	4	0.100	0.116	0.277	-0.046	0
10	8	35	5	0.143	0.116	0.288	-0.057	0
11	9	35	7	0.200	0.116	0.288	-0.057	0
12	10	45	3	0.067	0.116	0.268	-0.036	0
13	11	40	12	0.300	0.116	0.277	-0.046	0
14	12	40	2	0.050	0.116	0.277	-0.046	0
15	13	30	4	0.133	0.116	0.302	-0.071	0
16	14	45	8	0.178	0.116	0.268	-0.036	0
⋮								
23	21	33	2	0.061	0.116	0.294	-0.062	0
24	22	43	5	0.116	0.116	0.272	-0.040	0
25	23	38	12	0.316	0.116	0.282	-0.050	0
26	24	38	1	0.026	0.116	0.282	-0.050	0
27	25	48	5	0.104	0.116	0.263	-0.032	0

Fig. 10.9

Step 2: For the revised limits of u-chart, we calculate the new $\bar{u}$ by typing "=(C38-C13-C25)/(B38-B13-B25)" in Cell B40 as shown in Fig. 10.10. We also put d = 2 in Cell B41.

B40 f_x =(C38-C13-C25)/(B38-B13-B25)

	A	B	C	D
38	Total	1450	168	
39	$\bar{u}$	0.116		
40	$\bar{u}_{new}$	0.105		
41	d	2		

Fig. 10.10

Step 3: We reconstruct the centre line and control limits by replacing $\bar{u}$ by $\bar{u}_{new}$. We also replace the negative LCL by zero (0) and denote the limit by LCL* as shown in Fig. 10.11.

	I	J	K	L
1	Revised Control Limits			
2	Centre Line	UCL	LCL	LCL*
3	0.105	0.242	-0.032	0
4	0.105	0.259	-0.049	0
5	0.105	0.250	-0.040	0
6	0.105	0.269	-0.059	0
7	0.105	0.250	-0.040	0
8	0.105	0.282	-0.072	0
9	0.105	0.259	-0.049	0
10	0.105	0.269	-0.059	0
11	0.105	0.269	-0.059	0
12	0.105	0.250	-0.040	0
13	0.105	0.259	-0.049	0
14	0.105	0.259	-0.049	0

u Chart

Fig. 10.11

Step 4: Finally, we plot the defects per metre of the fabric, centre line, UCL and LCL* as discussed in Step 8 of Sec. 10.4. We also format the chart as explained in Sec.1.4 of Lab Session 1. The resulting chart is shown in Fig. 10.12.

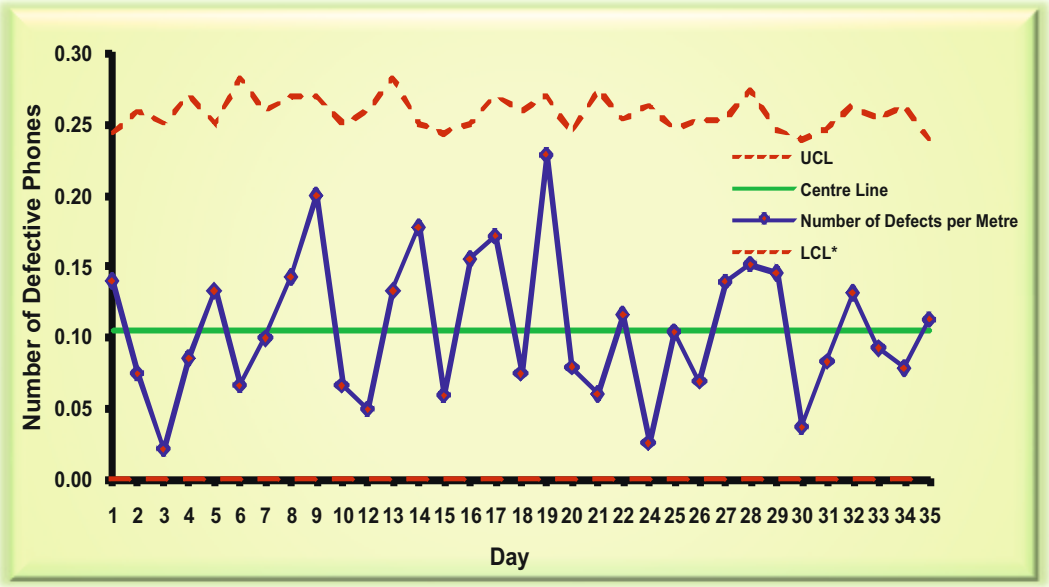

Fig. 10.12

Interpretation

The u-chart shown in Fig. 10.12 reveals that no point lies outside the control limits. So we may conclude that the process is under statistical control with respect to defects per metre fabric.

You should now apply the method on other problems for practice.

Activity 2

Construct the control charts for the number of defects per unit with the help of MS Excel 2007 and interpret the results for

A1) Example 4 given in Unit 4 of MSTE-001.

A2) Exercise E6 given in Unit 4 of MSTE-001.

Match the results with the manual calculation done in Unit 4 of MSTE-001.

Continuous Assessment 10

Suppose an almirah manufacturing company wants to judge the quality of almirah produced. The number of scratch marks on an almirah was recorded for this purpose. The data of 30 samples are given as follows:

Table 2: Number of scratch marks on almirah

Sample No.	1	2	3	4	5	6	7	8	9	10
Scratch Marks	7	3	1	3	6	2	4	5	7	3
Sample No.	11	12	13	14	15	16	17	18	19	20
Scratch Marks	6	3	14	7	2	5	9	4	7	3
Sample No.	21	22	23	24	25	26	27	28	29	30
Scratch Marks	2	7	6	8	4	10	5	4	6	7

Which chart will be used in this case? Construct the suitable control chart. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 10.4 and 10.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 10" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.