INDEX

S.No.	Question No.	Page No.	Total Marks	Marks Obtained	T. Sign
1.	Question No 1	1-4	10		
2.	Question No 2	5-6	13		
3.	Question No 3	7-8	7		
4.	Question No 4	9-11	10		
5.	Question No 5	12-14	10		
6.	Question No 6	15-17	15		
7.	Question No 7	18-20	15		
8.	Question No 8	21-29	20		

SUBJECT NAME : STATISTICAL INFERENCE

SUBJECT CODE : MST-004

Question No 1 State whether the following statements are True or False. Give reason in support of your answer:

(a) If the probability of non rejection of H_0 when H_1 is true is 0.4 then power of the test will be 0.6

Solution $\boxed{\text{The statement is TRUE}}$

The decision relating to non-rejecting of null hypothesis H_0 when it is false (i.e H_1 is true) is called Type-II error. The prob. of committing Type-II error is generally denoted by β and is given by

$$\beta = P[\text{Non rejection of } H_0 \text{ when } H_1 \text{ is true}]$$

$$\beta = 0.4$$

Power of the test $= 1 - \beta = 1 - 0.4 = 0.6$

(b) If T_1 and T_2 are two estimators of the parameter θ such that $Var(T_1) = 1/n$ and $Var(T_2) = n$ then T_1 is more efficient than T_2.

Solution:- $\boxed{\text{The Statement is TRUE}}$

We know that if T_1 and T_2 are two estimators of the parameter θ then T_1 is said to be

more efficient than T_2 for all sample sizes if $Var(T_1) < Var(T_2) \ \forall n$ — ①

As it is given that

$$Var(T_1) = \frac{1}{n} \text{ and } Var(T_2) = n$$

$$\forall n \in N \quad \frac{1}{n} < n$$

$\Rightarrow Var(T_1) < Var(T_2)$ which satisfy the condition ①

i.e T_1 is more efficient than T_2.

(c) A 95% confidence interval is smaller than 99% confidence interval.

Solution $\boxed{\text{The statement is TRUE}}$

A 99% confidence interval will be wider than a 95% confidence interval because we need to include more alternative values within the interval to be more certain than the actual population value falls within the interval.

(d) If the level of significance is the same, the area of the rejection region in a two-tailed test is less than in a one-tailed test.

Solution: The Statement is TRUE

Example:- For a two tailed test with a sample size of 18, the critical values are 4 and 14 at significance level of 5% (the actual size of rejection region is 3.08%.)

For a one tailed test at the same nominal significance level, the critical value is 5 (Actual size of rejection region is 4.81%)

Clearly, Area of rejection region in a two tailed test is less than in a one-tailed test.

(e) Non-parametric tests are more powerful than the parametric tests.

Solution: The Statement is FALSE

when the assumptions of parametric tests are met, parametric tests are more powerful than non-parametric tests.

When compared to the t-test the wilcoxon test has around 95% power. It indicates that if the null hypothesis is incorrect, the wilcoxon test takes 5% more observations in the sample than the t-test to have the same power.

For example, the wilcoxon test needs 20 observations, but the t-test necessitates 19 observations to attain the same power.

Question No2 If a finite population has four elements : 6, 1, 3 2

(a) How many different sample of size $n=2$ can be selected from this population if you sample without replacement?

Solution: No. of samples from the population

$$^4C_2 = \frac{4!}{2!\,2!} = \frac{4\times3\times2!}{2\times1\times2!} = 6.$$

(b) list of all possible sample of sizes $n=2$

Possible samples = {(6,1) (6,3) (6,2) (1,3) (1,2) (3,2)}

(c) Compute the sample mean for each of the samples given in part b.

Sample No	Sample Obs	Sample mean ($\bar{x}$)
1	(6,1)	3.5
2	(6,3)	4.5
3	(6,2)	4
4	(1,3)	2
5	(1,2)	1.5
6	(3,2)	2.5

(d) Find the sampling distribution of $\bar{x}$ and draw the histogram.

S.No.	Sample mean ($\bar{x}$)	Freq. (y)	Prob.
1	3.5	1	1/6
2	4.5	1	1/6
3	4	1	1/6
4	2	1	1/6
5	1.5	1	1/6
6	2.5	1	1/6

(e) Compute standard Error.

S.No.	$\bar{x}$	$\bar{x}^2$
1	3.5	12.25
2	4.5	20.25
3	4	16
4	2	4
5	1.5	2.2
6	2.5	6.25

$\Sigma \bar{x} = 18$

$\Sigma \bar{x}^2 = 61$

Question No 3 A study was conducted to compare the mean numbers of police emergency calls per 8-hour shift in two districts of a large city. Samples of 100 8 hour shifts were randomly selected from the police records for each of two regions and the number of emergency calls was recorded for each shift. The sample statistics are listed here:

	Region	
	1	2
Sample size	100	100
Sample mean	2.4	3.1
Sample variance	1.44	2.64

Find a 90% Confidence Interval for the difference in the mean numbers of police emergency calls per shift b/w two district of the city. Interpret the Interval.

Solution Here we are given that

Sample 1 :- $n_1 = 100$, $\bar{x}_1 = 2.4$, $s_1 = 1.44$

Sample 2 :- $n_2 = 100$, $\bar{x}_2 = 3.1$, $s_2 = 2.64$

Since population variances are known,

therefore we use $(1-\alpha)100\%$ CI limits for the difference of population mean when population variances are unknown which are given by

$$(\bar{x}-\bar{y}) \pm Z_{0.005}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

For 90% Confidence Interval

$$1-\alpha = 0.90 \Rightarrow \alpha = 0.10 \Rightarrow \alpha/2 = 0.05$$

Also, $Z_{\alpha/2} = 1.64$

So, 90% CI of the difference of two population averages given by

$$(3.1-2.4) \pm 1.64\sqrt{\frac{(1.44)^2}{100}+\frac{(2.64)^2}{100}}$$

$$= 0.7 \pm 1.64\sqrt{\frac{2.073+6.969}{100}}$$

$$= 0.7 \pm 1.64 \times 0.9007$$

$$= 0.2069 \text{ to } 1.1931$$

Hence required 90% confidence interval of $(\mu_1-\mu_2)$ is $[0.2069, 1.1931]$

Question No 4 A bond proposal for school construction will be submitted to the voters at the next municipal election. A major portion of the money derived from this bond issue will be used to build schools in a rapidly developing section of the city and the remainder will be used to renovate and update school buildings in the rest of the city. To asses the viability of the bond proposal, a random sample of $n_1 = 50$ residents in the developing section and $n_2 = 100$ residents from the other parts of the city were asked whether they plan to vote for the proposal. The results are tabulated below:

	Developing Section	Rest of the city.
Sample size	50	100
No. favouring proposal.	38	65

(a) Estimate the difference in the true proportions favoring the bond proposal with a 99% confidence Interval.

<u>Solution</u> Here we have given

$n_1 = 50$ $\quad$ $n_2 = 100$

$X_1 = 38$ $\quad$ $X_2 = 65$

<u>for sample 1</u> $p_1 = \frac{X_1}{n_1} = \frac{38}{50} = 0.76$

$q_1 = 1 - p_1 = 1 - 0.76 = 0.24.$

<u>for sample 2</u>

$p_2 = \frac{X_2}{n_2} = \frac{65}{100} = 0.65$

$q_2 = 1 - p_2 = 1 - 0.65 = 0.35$

We know that $(1-\alpha)100\%$ CI limits for the difference in proportions is given by

$$(p_1 - p_2) \pm Z_{\alpha/2}\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

$\therefore$ 99% confidence limit for the difference in proportions of in favor are:

$$(p_1 - p_2) \pm Z_{0.005}\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

$$= (0.76 - 0.65) \pm 2.58\sqrt{\frac{0.76 \times 0.24}{50} + \frac{0.65 \times 0.35}{100}}$$

$$= 0.11 \pm 2.58\sqrt{0.0036 + 0.0022}$$

$$= 0.11 \pm 2.58\sqrt{0.00587}$$

$$= 0.11 \pm 0.1977$$

$$[-0.0877, 0.3077]$$

(b) If both samples were pooled into one sample size $n = 150$ with 103 in favor of the proposal, provide a point estimate of the proportion of city residents who will vote for the bond proposal.

$$p = \frac{\text{favor proposal}}{\text{total proposal}}$$

$$= \frac{103}{150} = 0.686$$

$$= 68\%$$

Question No 5 The following data relate to the number of items produced per shift by two workers for a number of days.

Worker A - 19 22 24 27 24 18 20 19 25

Worker B - 26 37 40 35 30 40 26 30 35 45

Can it be inferred that worker A is more stable worker compared to B by testing the variation in the item produced by them at 5% level of significance.

Solution

Let σ_2^2 = variances of worker A

σ_1^2 = variances of worker B

Stating hypothesis

$H_0 : \sigma_1^2 \leq \sigma_2^2$

$H_1 : \sigma_1^2 > \sigma_2^2$ [worker B is more stable than worker A]

Test statistic given by

$$\boxed{F = \frac{S_1^2}{S_2^2} \sim F_{(n_1-1,\, n_2-1)}}$$

where $s_1^2 = \frac{1}{n-1} \sum (x_1 - \bar{x})^2$ and $s_2^2 = \frac{1}{n_2 - 1} \sum (y_i - \bar{y})^2$

A (x)	$x - \bar{x}$ $= x - 22$	$(x-\bar{x})^2$	B (y)	$(y-\bar{y})$ $= y - 34$	$(y-\bar{y})^2$	
19	-3	9	26	-8	64	
22	0	0	37	3	9	
24	2	4	40	6	36	
27	5	25	35	1	1	
24	2	4	30	-4	16	
18	-4	16	30	-4	16	
20	-2	4	40	6	36	
19	-3	9	26	-8	64	
25	3	9	30	-4	16	
			35	1	1	
			45	11	121	
198	0	80	374	0	380	Total

We have

$$\bar{x} = \frac{1}{n_1} \sum x = \frac{1}{9} \times 198 = 22$$

$$\bar{y} = \frac{1}{n_2} \sum x = \frac{1}{11} \times 374 = 34$$

Thus $s_1^2 = \frac{1}{8} \times 80 = 10$

$$s_2^2 = \frac{1}{10} \times 380 = 38$$

Putting the value of s_1^2 and s_2^2 in eq^n (4)

$$F = \frac{10}{38} = 0.263$$

table value

$$F_{(n_1-1),(n_2-1),\alpha} = F_{(10,8),0.01} = 5.81$$

Since calculated value of test statistic is less than the critical value, that means calculated value of test statistic lies in rejection region. so we do not reject the null hypothesis.

Thus, we conclude that sample work provide us sufficient evidence against the claim so worker B is not more stable (or constant) worker compared to A.

Question No 6 If magnitude of earthquakes recorded in a region of a country follows a distribution with parameter μ whose pdf is given below:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2} ; \infty < x, \mu < \infty$$

then show that the estimators of the parameter μ using maximum likelihood and method of moments are same.

Solution

USING MAXIMUM LIKELIHOOD

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n taken from normal population $N(\mu)$ whose p.d.f is given by

$$f(x, \mu) = \frac{1}{\sqrt{2\pi}} e^{-1/2 (x-\mu)^2}, -\infty < x, \mu < \infty$$

$\therefore$ The likelihood function for parameters μ can be obtained as

$$L(\mu) = L = f(x_1 . \mu) \cdot f(x_2 \cdot \mu) \cdots f(x_n \cdot \mu_n)$$

$$L = \left(\frac{1}{\sqrt{2\pi}}\right)^{n/2} e^{-1/2 \sum_{i=1}^{n} (x_i - \mu)^2}$$

taking log on both sides

$$\log L = \frac{n}{2}\log 1 - \frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2$$

differentiating eqn with respect to μ

$$\frac{\partial}{\partial u}(\log L) = 0 - 0 - \frac{1}{2}\sum_{i=1}^{n} 2(x_i-\mu)(-1)$$

for finding ML estimate of μ,

we put $\frac{\partial}{\partial a}(\log L) = 0$

$$\frac{1}{2}\sum_{i=1}^{n} 2(x_i-\mu) = 0$$

$$\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\boxed{\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}} \quad -(1)$$

Thus, the ML estimate ~~of~~ for μ is the observed sample mean $\bar{x}$.

USING METHODS OF MOMENT

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n taken from ~~normal~~ distribution $N(\mu)$ whose p.d.f is given.

We know,

$\mu_1 = \mu$ ~~and $\mu_2 = \sigma^2$~~

and the corresponding sample moment are

$$M_1' = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$$

Therefore by the method of moments. we equate population moments with corresponding sample moment.

$$\mu_1' = M_1'$$

$$\boxed{\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}} \quad \text{— (2)}$$

From eqn (1) and (2) we can say that the estimator using maximum likelihood & method of moments are same.

Question No 7 A company plans to promote a new product by using one of three adv. campaigns. To investigate the extent of product recognition from these three campaigns, 15 market areas were selected and five were randomly assigned to each advertising plan. At the end of the ad campaigns, 400 adults were selected. Use an appropiate nonparametric method to answer this question at 5% level of significance.

Campaign		
1	2	3
0.33	0.28	0.21
0.29	0.41	0.30
0.21	0.34	0.26
0.32	0.39	0.33
0.25	0.27	0.31

Question No 8 A psychology class performed an experiment to determine whether a recall score in which instructions to form image of 25 words were given differs from an intial recall score for which no imaginery instructions were given. Twenty students participated in the experiment with the results listed in the table.

Student	With Imagery	Without Imagery	Student	With Imagery	Without Imagery
1	20	5	11	17	8
2	24	9	12	20	16
3	20	5	13	20	10
4	18	9	14	16	12
5	22	6	15	24	7
6	19	11	16	22	9
7	20	8	17	25	21
8	19	11	18	21	14
9	17	7	19	19	12
10	21	9	20	23	13

(a) What two testing procedures can be used to test for differences in the distribution of recall scores with and without imagery? What assumptions are required for the parametric procedure? Do these data satisfy these assumptions?

Solution (a)

Two testing procedures are

1. Parametric → Paired t-test
2. Non-Parametric → Wilcoxon Matched Paired Signed rank test.

Assumptions of parametric procedure

(i) The population of differences follows normal distribution

(ii) Samples are not independent

(iii) Size of both the samples are equal.

(iv) Population variances are unknown but not necessarily equal.

~~Yes~~ No, data does not satisfy the assumption. As it is not mention the data is normal or not.

(b) Use both the parametric and non-parametric tests for differences in the distribution of recall scores under these two condition.

Solution:- PARAMETRIC TEST

PAIRED t-TEST

- If μ_1 and μ_2 denotes the mean of with imagery and without imagery, so our claim is $\mu_1 \neq \mu_2$ and its complement $\mu_1 = \mu_2$. Therefore,
- Stating Hypothesis

  Null Hypothesis (H_0): $\mu_1 = \mu_2$

  Alternate Hypothesis (H_1): $\mu_1 \neq \mu_2$

Since alternative hypothesis is both tailed so the test is both tail test.

- For testing the null hypothesis, the test statistic is given by

$$\boxed{t = \frac{\bar{D}}{S_D/\sqrt{n}} \sim t_{(n-1)} \text{ under } H_0}$$

where $\bar{D}$ and S_D are mean & standard deviation.

Calculation for $\bar{D}$ and S_D

Student	With Imagery (X)	Without Imagery (Y)	D=(X-Y)	D^2
1	20	5	15	225
2	24	9	15	225
3	20	5	15	225
4	18	9	9	81
5	22	6	16	256
6	19	11	8	64
7	20	8	12	144
8	19	11	8	64
9	17	7	10	100
10	21	9	12	144
11	17	8	9	81
12	20	16	4	16
13	20	10	10	100
14	26	12	4	16
15	24	7	17	289
16	22	9	13	169
17	25	21	4	16
18	21	14	7	49
19	19	12	7	49
20	23	13	10	100

$\Sigma D = 205$ $\Sigma D^2 = 2413$

From above calculations, we have

$$\bar{D} = \frac{1}{n}\sum D = \frac{1}{20} \times 205 = 10.25$$

$$\boxed{S_D^2 = \frac{1}{n-1}\left\{\sum D^2 - \frac{(\sum D)^2}{n}\right\}}$$

$$= \frac{1}{20-1}\left\{2413 - \frac{(205)^2}{20}\right\}$$

$$= \frac{1}{19}\left\{2413 - 2101.25\right\}$$

$$S_D^2 = 16.407$$

$$S_D = \sqrt{16.407} = 4.0506.$$

Substituting these values

$$t = \frac{10.25}{4.05/\sqrt{20}} = \frac{10.25}{4.05/4.472} = \frac{10.95}{0.905}$$

$$t = 12.09$$

The critical value of test statistic t for two tailed test corresponding $(n-1) = 19$ df at 5% level of significance

$$t_{(n-1)\alpha} = t_{(20-1),0.05} = 2.093$$

Since calculated value of test statistic $t(=12.09)$ is more than the critical value $t_c(=2.093)$, that means t lies in non-rejection region. So we accept the null hypothesis and reject alternative hypothesis.

Thus, we conclude the sample passes to provide us sufficient evidence against the claim. There is no differences in the distribution of recall scores with and without imagery.

NON-PARAMETRIC TEST

It is case of before and after study so we can go for paired t-test or paired sign test or Wilcoxon matched-pair signed-rank. But t-test requires assumptions of normality which is not given. Also, measurement of observation is available in internal scale, so we will go for Wilcoxon matched-pair signed-rank test

* Stating the null hypothesis

If μ_1 and μ_2 denotes the mean of with Imagery or without imagery

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2 .$$

Since alternative hypothesis is two tailed so the test is two tailed test.

Student	with imagery (X)	without imagery (Y)	Diff (X-Y)	Abs. diff |d|	Rank of |d|	Signed Rank
1	20	5	15	15	8	8
2	24	9	15	15	8	8
3	20	5	15	15	8	8
4	18	9	9	9	4	4
5	22	6	16	16	9	9
6	19	11	8	8	3	3
7	20	8	12	12	6	6
8	19	11	8	8	3	3
9	17	7	10	10	5	5
10	21	9	12	12	6	6
11	17	8	9	9	4	4
12	20	16	4	4	1	1
13	20	10	10	10	5	5
14	16	12	4	4	1	1
15	24	7	17	17	10	10
16	22	9	13	13	7	7
17	25	21	4	4	1	1
18	21	14	7	7	2	2
19	19	12	7	7	2	2
20	23	13	10	10	5	5

From the above calculations we have

T^+ = Sum of positive ranks = 20

n = no. of non-zero d_i's = 20

So the value of test statistic $(T) = T^+ = 20$

The critical (tabulated) value of test-statistic for two-tailed test corresponding $n = 20$ at 5% level of significance is 53

Since calculated value of test statistic $t (= 20)$ is less than critical value $(= 53)$ so we reject the null hypothesis and support the alternative hypothesis.

Thus, we conclude that samples fail to provide us sufficient evidence against the claim, so we assume there is differences in the distribution of recall scores with and without imagery.

(c) Compare the results of the tests in part b. Are the conclusion for same? if not why not?

Solution:-

Withe parametric test :→

we Accept the null hypothesis

whereas with non-parametric test

we are rejecting the null hypothesis.

conclusion for bothe test are different. the reason being the assumption we took for parametric test does n't satisfy. ~~[illegible]~~ Since it is not mentioned the data is Normally distributed. whereas for non parametric test data satisfy all the assumptions taken.