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Question No1 State whether the following statements are True or false. Give reason in support of your answer:

(a) Statistical quality control (SQC) is a technique of process control only.

Answer $\boxed{\text{Statement is FALSE}}$

SQC is defined as the technique of applying Statiscal method based on the theory of probability and sampling to establish quality standard and to maintain it in the most economical manner.

SQC is a technique of both process control and product control.

(b) Twenty pieces of different length of cloth contained 2, 4, 1, 3, 5, 4, 2, 7, 3, 5, 2, 2, 4, 5, 6 4, 2, 1, 2, 4 defects respectively. To check the process is under control with respect to the number of defects, we should use

np-chart

Solution:— The Statement is FALSE

Since the number of defects is given, we may use either c-chart or u-chart. Since the sample size varies, we use the u-chart.

(c) If the probabilities are not associated with the occurences of different state of nature, then the situation is known as decision making under risk.

Solution:— The Statement is FALSE

If the probabilities are not associated with the occurences of different states of nature, then the situation is known as Decision Making Under Uncertainity.

(d) In single sampling plan, if we increase acceptance number then the OC curve will be steeper.

Solution:— The statement is FALSE

Changing the acceptance number does

not drastically change the slope of the OC-curve. As the acceptance number is decreased the OC-curve shifts to the left. Usually OC-curves become steeper when sample size increases.

(c) A system has four components connected in parallel configuration with reliability 0.2, 0.4, 0.5, 0.8. To improve the reliability of the system most, we have to replace the component which reliability is 0.2.

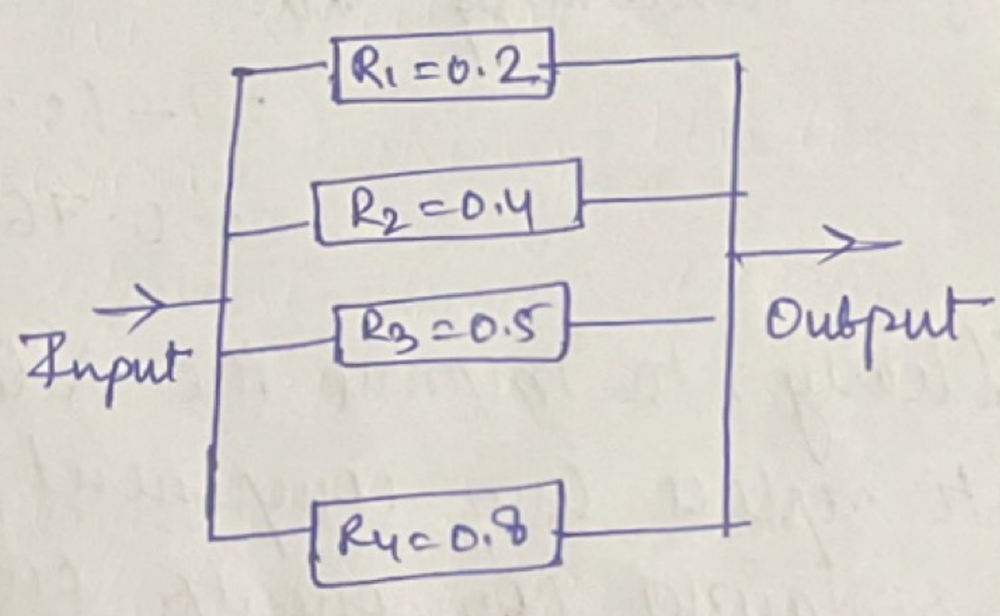
Solution :- $\boxed{\text{The Statement is FALSE}}$

Given that $R_1 = 0.2$

$R_2 = 0.4$

$R_3 = 0.5$

$R_4 = 0.8$



Initial Reliability $R = 1-(1-R_1)(1-R_2)(1-R_3)(1-R_4)$

$= 1-(0.8)(0.6)(0.5)(0.2)$

$= 0.952$

After Removing $R_1 = 0.8$

$$R = 1 - (1-R_2)(1-R_3)(1-R_4)$$

$$= 0.94$$

After Removing $R_2 = 0.4$ then

$$R = 1 - (1-R_1)(1-R_3)(1-R_4)$$

$$= 1 - (1-0.8)(0.5)(0.2)$$

$$= 0.92$$

After Removing $R_3 = 0.5$ then

$$R = 1 - (1-R_1)(1-R_2)(1-R_4)$$

$$= 1 - (0.8)(0.6)(0.2)$$

$$= 0.904$$

After Removing $R_4 = 1 - (1-R_1)(1-R_2)(1-R_3)$

$$= 1 - (0.8)(0.6)(0.5)$$

$$= 0.76$$

Clearly, to improve the reliability there is no need to replace any component. Because after replacing components one by one the reliability of system decreases.

Question No 2 Twenty samples each of size 10 were inspected. The number of defectives detected in each of them is given below:
0, 1, 0, 3, 9, 2, 0, 7, 0, 1, 1, 0, 0, 3, 1, 0, 0, 2, 1, 0. Find the control limits for the number of defectives and establish quality standards for the future. Plot the graph and interpret.

Solution Since the number of defects is given we can use p-chart or np-chart. But it is convenient to use the np-chart.

Now, we will calculate the center and control limits for the np-chart. We are given that

The number of items inspected in each sample (n) = 10

The number of samples (k) = 20

The total number of defectives is

$$\sum_{i=1}^{n} d_i = 0+1+0+3+9+2+0+7+0+1+1+0+0+3+1+0+0+2+1+0 = 31$$

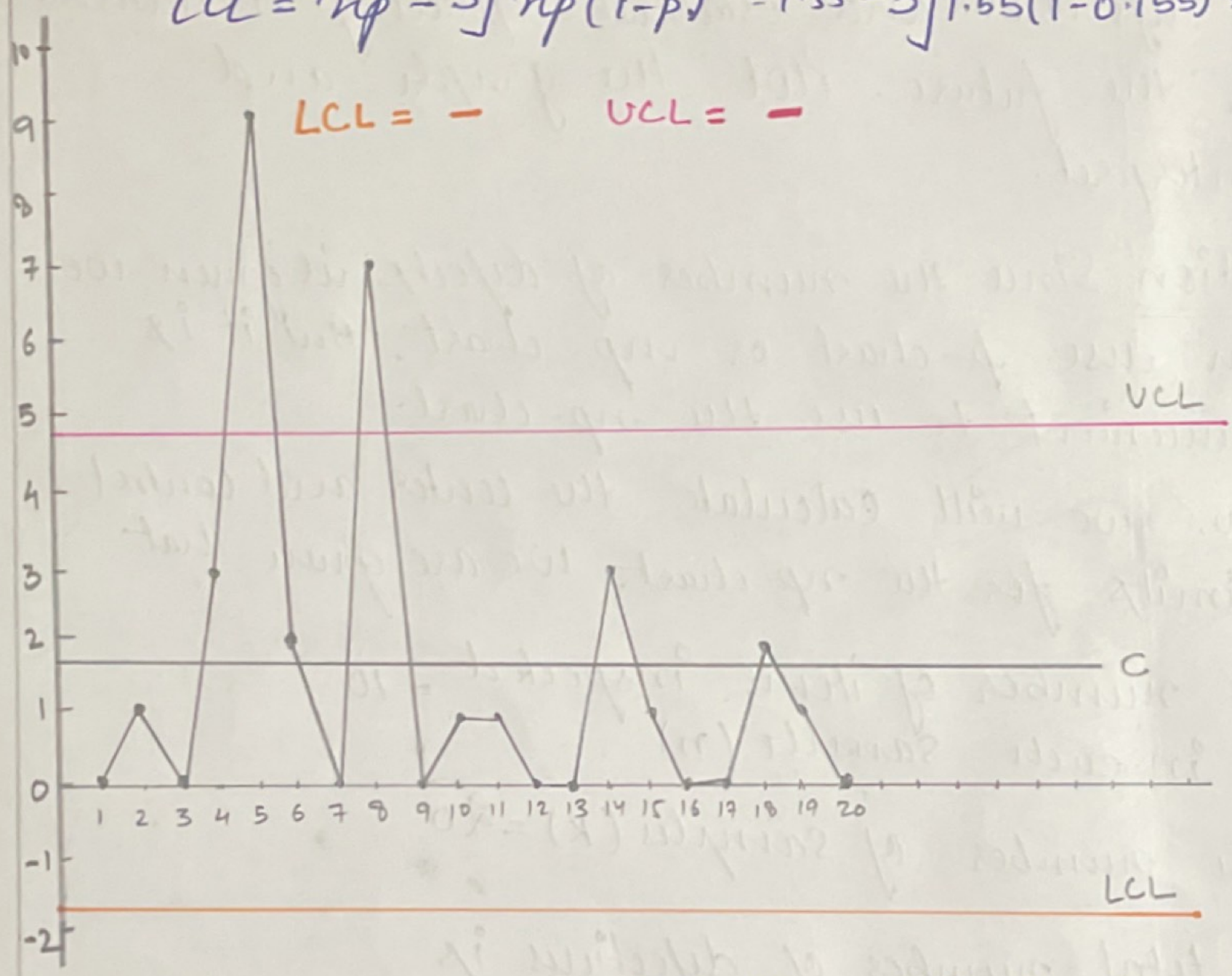
We calculate $\bar{p}$: $\bar{p} = \frac{\sum_{i=1}^{n} d_i}{nk} = \frac{31}{10 \times 20} = 0.155$

Now, we will calculate the combine line and control limits of np-chart as follows:

$$CL = n\bar{p} = 10 \times 0.155 = 1.55$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 1.55 + 3\sqrt{1.55(1-0.155)} = 4.88$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 1.55 - 3\sqrt{1.55(1-0.155)} = -1.88$$



Interpretation:- Sample No 5, 8 lies outside the CL.

∴ Process is out of control.

To bring process under control, it is necessary to investigate. We eliminate out of control samples 5, 8 and calculate the revised center line, upper and lower control limits for np-chart using the remaining samples.

We first calculate the new $\bar{p}$:

$$\bar{p}_{new} = \frac{\sum_{i=1}^{k} di - \sum_{j=1}^{d} dj}{n(k-d)}$$

$$= \frac{31-(9+7)}{10(20-2)} = \frac{31-16}{10\times 18}$$

$$\bar{p}_{new} = 0.083$$

Now, we will calculate the center line and central limits of np-chart as follows:

$$CL = n\bar{p}_{new} = 10\times 0.1083 = 0.83$$

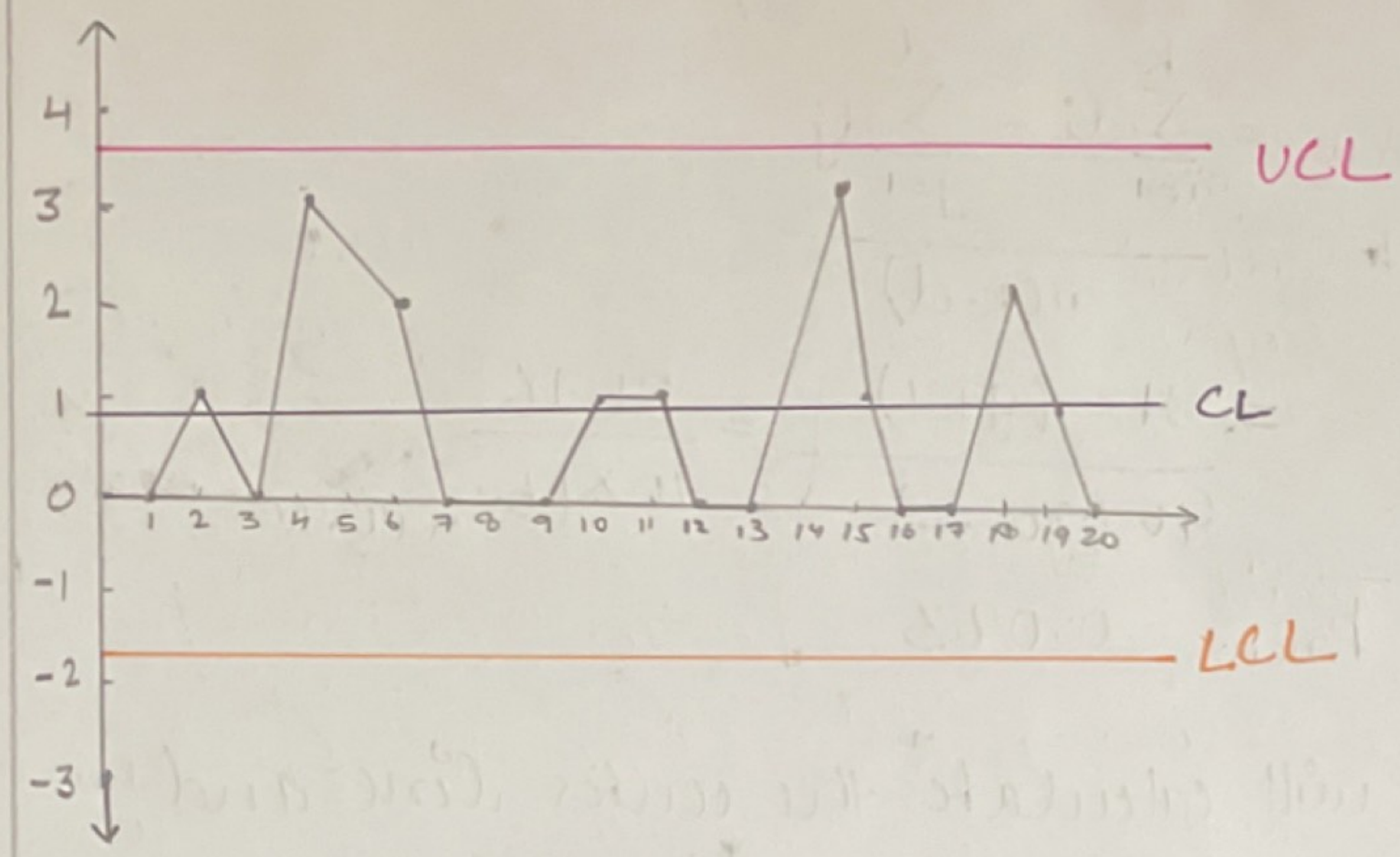
$$UCL = n\bar{p}_{new} + 3\sqrt{n\bar{p}_{new}(1-\bar{p}_{new})} = 0.83+3\sqrt{0.83(1-0.083)}$$

$$= 3.44$$

$$LCL = n\bar{p}_{new} - 3\sqrt{n\bar{p}_{new}(1-\bar{p}_{new})}$$

$$= 0.83 - 3\sqrt{0.83(1-0.083}$$

$$= -1.79$$



The control chart for defective items reveals that the process is under statistical control since no point lies outside the control limits of the np chart. We may conclude that the process is under statistical control with respect to the no. of defective items.

Question No 3 A manufacturer of men's jeans purchases zippers in lots of 500. The jeans manufacturer uses single-sample acceptance sampling with a same size of 10 to determine whether to accept the lot. The manufacturer uses $c=2$ as the acceptance number. Suppose 3% nonconforming zippers are acceptable to the manufacturer and 8% nonconforming zippers are not acceptable.

(i) Construct an OC curve.

Solution It is given that

$N = 500$ Incoming quality =

$n = 10$ $p = 0.04$ (4%)

$c = 2$

For constructing OC curve, we have to calculate the probabilities of accepting the lot corresponding to different quality levels.

If x represents no. of defective zippers in the sample, the manufacturer accepts the lot if $X \leq C$ i.e $X \leq 2$

Therefore the probability of accepting the lot is given by

$$P_a(p) = P[X \le c]$$

$$= P[X \le 2] = \sum_{x=0}^{2} [P(X=x)]$$

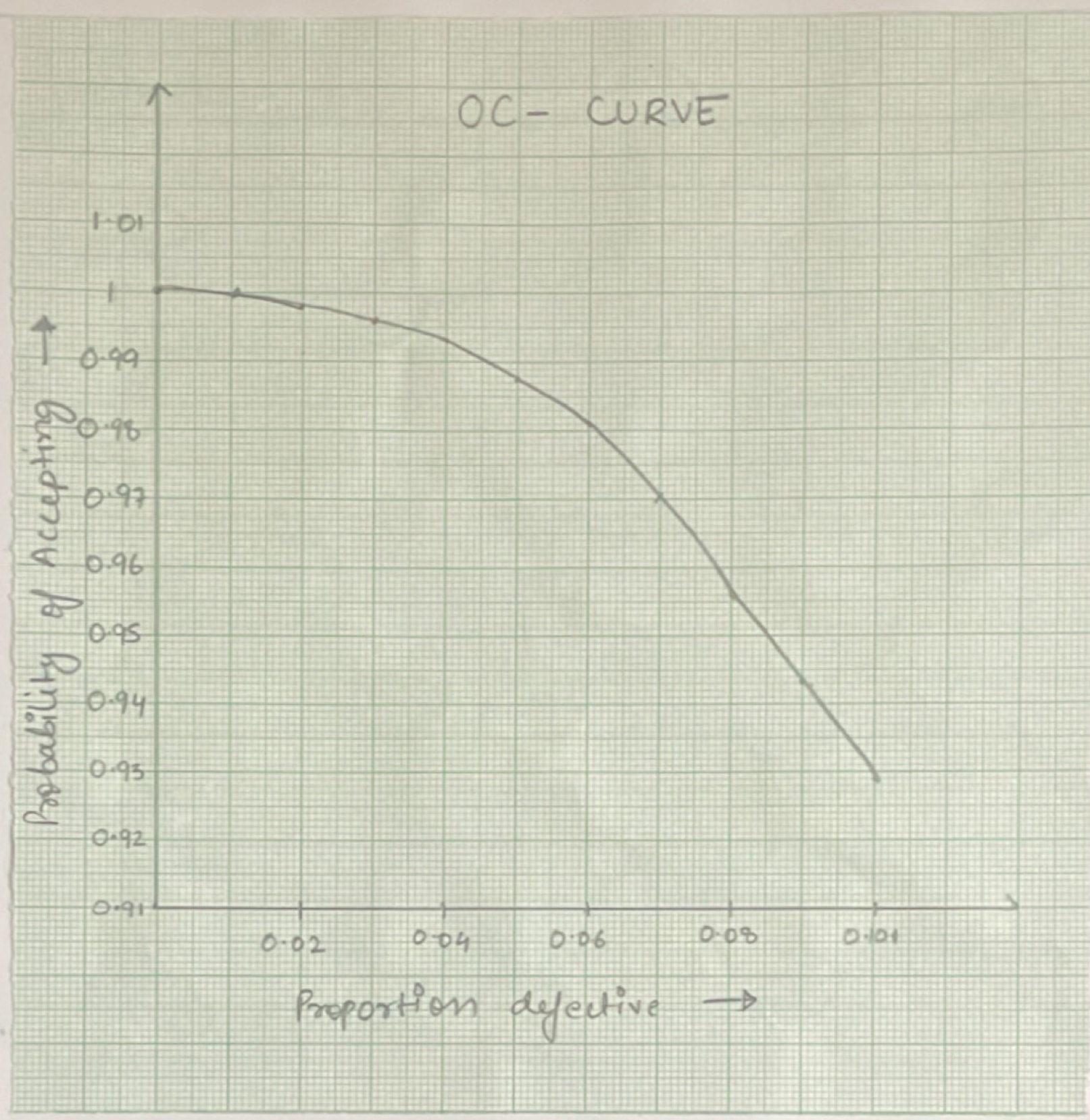
Using Binomial distribution

$$P_a(p) = P[X \le 2]$$

$$= \sum_{x=0}^{2} {}^nC_x \, p^x (1-p)^{n-x}$$

Incoming lot Quality	Probability of Accepting the lot
0	1
0.01	0.9998
0.02	0.9991
0.03	0.9972
0.04	0.9937
0.05	0.9884
0.06	0.9811
0.07	0.9716
0.08	0.9599
0.09	0.9499
0.10	0.9298

We now construct the OC-curve by taking the quality level (proportion defective) on the x-axis and prob. of accepting the lot on y axis



(ii) Average outing quality (AOQ), if the rejected lots are screened and all defectives zippers are replaced by non-defectives.

Average outing quality (AOQ) for single sampling plan is given by

$$\boxed{AOQ = \frac{p(N-n)\,P_a}{N}} \quad -(1)$$

$$P_a(p) = P[x \le c] = P[x \le 2] = \sum_{x=0}^{2} {}^nC_x p^x (1-p)^{n-x}$$

$\therefore p = 0.04$

$$= {}^{10}C_0 (0.04)^0 (1-0.04)^{10} + {}^{10}C_1 (0.04)(1-0.04)^9$$
$$+ {}^{10}C_2 (0.04)^2 (1-0.04)^8$$

$P_a(p) = 0.9937$ Put in eq^n ① – ②

$$AOQ = \frac{0.04(500-10)(0.9337)}{500}$$

$$\boxed{AOQ = 0.036}$$

(iii) Average Total Inspection (ATI)

$$ATI = n + (1 - P_a)(N-n)$$
$$ATI = 10 + (1-0.9937)(500-10)$$
$$ATI = 10 + [6.3 \times 10^{-3} \times 430]$$

$$ATI = 13.087$$

$$\boxed{ATI \simeq 13}$$

Question No4 An office supply ordered a lot of 400 printers. When a lot arrives the company inspector will randomly inspect 12 printers. If more than three printers in the sample are non-conforming, the lot will be rejected. If fewer than two printers are non-conforming, the lot will be accepted. Otherwise, a second sample of size 8 will be taken. Suppose the inspector finds two non-conforming printers in the first sample and two in the second sample. Also. AQL and LTPD are 0.05 and 0.10 respectively. Let incoming quality be 4%.

(i) What is the probability of accepting the lot at the first sample?

Solution :→

Given values

$N = 900$

$n_1 = 12$ $\quad n_2 = 8$

Assumption $C_1 = 1$ $\quad C_2 = 2$.

$p = 0.04\% \ (4\%)$

$AQL(p_1) = 0.05$ $\quad LTPD\ (p_2) = 0.10$

If X represents the no. of defective printers in the first sample of size 12, the lot is accepted on the first sample if $X \le C_1 = 1$

$$\therefore P_a(p) = P[X \le C_1] = P[X \le 1]$$

$$= \sum_{x=0}^{1} {}^{n}C_x \, p^x (1-p)^{n-x}$$

$$= {}^{12}C_0 (0.04)^0 (1-0.04)^{12} + {}^{12}C_1 (0.04)(1-0.04)^{11}$$

$$= 0.6127 + 0.3063$$

$$\boxed{P_a(p) = 0.9190}$$

(ii) What is the probability of accepting the lot at the second sample?

If Y represents the number of defective mobiles in the second sample the lot is accepted on the second sample if $C_1 < X + Y \le C_2$

i.e $1 < X + Y \le 2$, Here $n_2 = 8$, $C_2 = 2$.

It means that we will accept the lot if

$X = C_1 + 1 = 1 + 1 = 2$, $Y \le 2 - X = 2 - 2 = 0$

$$P_{a_2}(p) = P[X=2] \cdot P[Y \le 0]$$

$$= P[X=2] \cdot P[Y \le 0] \quad - (1)$$

Consider $P[X=2] = {}^{12}C_2 (0.04)^2 (1-0.04)^{10}$

$$= 0.0702 \quad - (a)$$

$$P[Y=0] = {}^{8}C_0 (0.04)^0 (1-0.04)^8$$

$$= 0.7213 \quad - (b)$$

Put these (a) & (b) in equation (1)

$$P_{a_2}(p) = 0.0702 \times 0.7213$$

$$\boxed{P_{a_2}(p) = 0.0506}$$

(iii) Find AOQ

$$AOQ = p\left[\left(1 - \frac{n_1}{N}\right)(P_{a_1} + P_{a_2}) - \frac{n_2}{N} P_{a_2}\right]$$

$$= 0.04\left[\left(1 - \frac{12}{400}\right)(0.9190 + 0.0506) - \frac{8}{400}(0.0506)\right]$$

$$= 0.04\left[\frac{388 \times 0.9696}{400} - \frac{0.4048}{400}\right]$$

$$\boxed{AOQ = 0.3758}$$

Find ATI :-

$$\boxed{ATI = n_1 P_{a_1} + (n_1 + n_2) P_{a_2} + N[1 - P_a]}$$

here $P_a = P_{a_1} + P_{a_2}$

$= (12 \times 0.9190) + (12 + 8) \times (0.0506)$

$+ 400 (1 - \{0.9190 + 0.0506\})$

$= 11.028 + 1.012 + 12.16$

$$\boxed{ATI = 24.2}$$

Question No 5 A two-person zero sum game having the following payoff matrix for player A.

Player A \ Player B	I	II	III	IV	V
I	2	4	3	8	4
II	5	6	3	7	8
III	6	7	9	8	7
IV	4	2	8	4	3

(i) Check whether saddle point exit or not.

Solution:-

		PLAYER B					ROW Minima	Max Value
		B_1	B_2	B_3	B_4	B_5		
PLAYER A	A_1	2	4	3	8	4	2	max $\{2,3,6,2\}$ $=6$
	A_2	5	6	3	7	8	3	
	A_3	6	7	9	8	7	6	
	A_4	4	2	8	4	3	2	
Column Maxima		6	7	9	8	8		
Minimax value.		min $\{6,7,9,8,8\} = 6$						

"Yes, there is a saddle point exit between player A and player B.

(ii) If saddle point does not exist then determine optimal strategies for both the manufactures and value of the game.

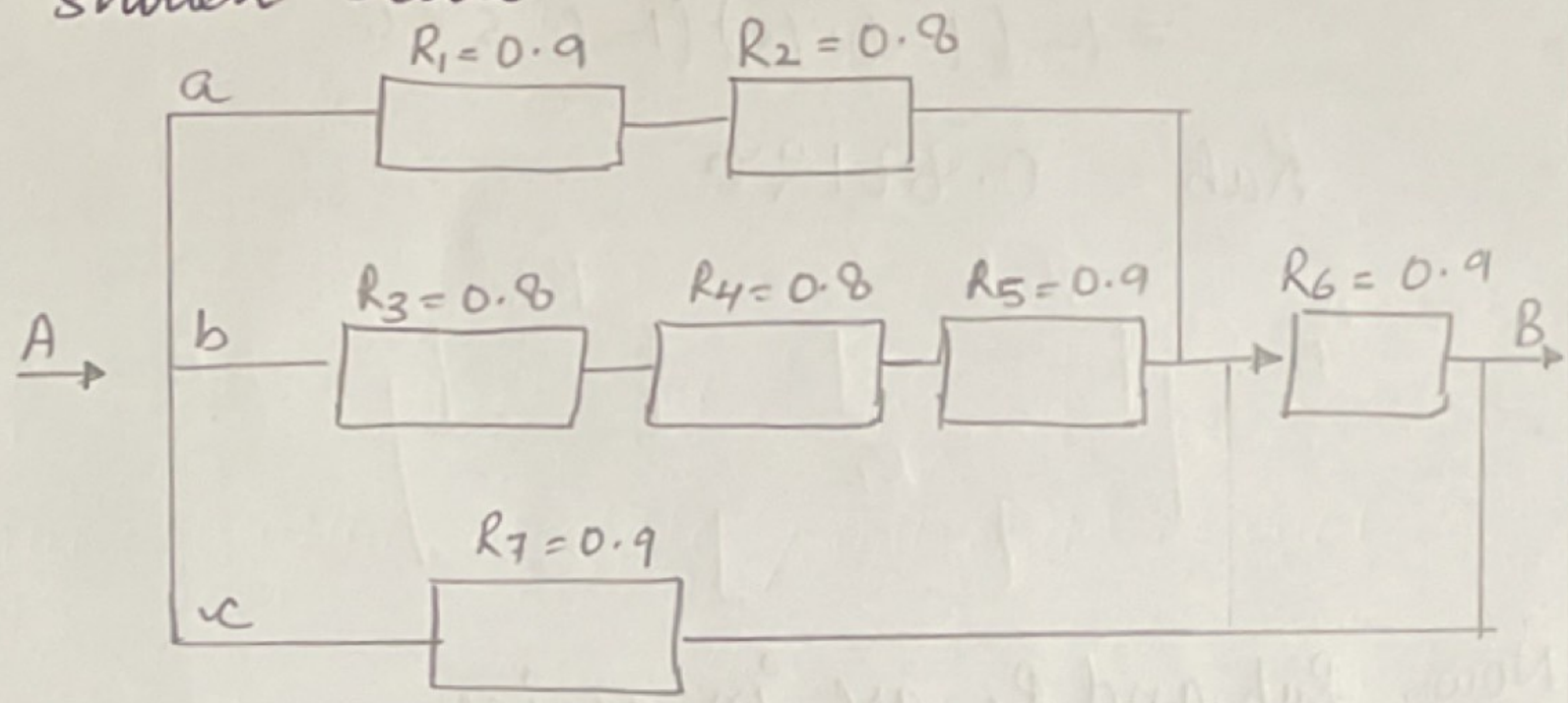
Select minimum from the maximum of columns. Column MiniMax = (6)

Select maximum from the minimum Row minimax = (6)

Here, Column MiniMax = Row MiniMax = 6

∴ this game has a saddle point and value of the game is 6.

Question No 6 A system has seven independent components and reliability block diagram of it shown below:



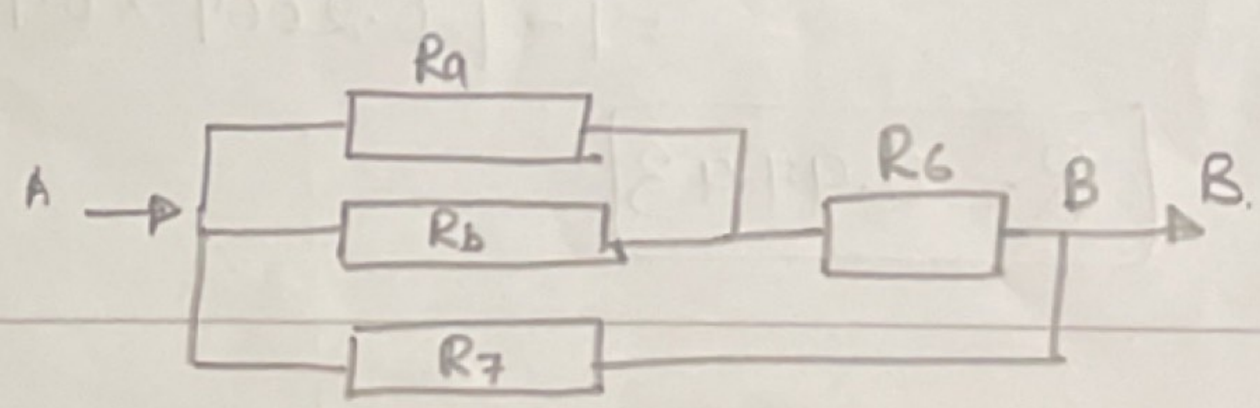
Find the reliability of the system

Solution In the above figure, Reliability block diagram R_1 and R_2 are in series

$$\therefore R_a = R_1 \times R_2$$
$$= 0.9 \times 0.8 = 0.72 \quad \text{—(1)}$$

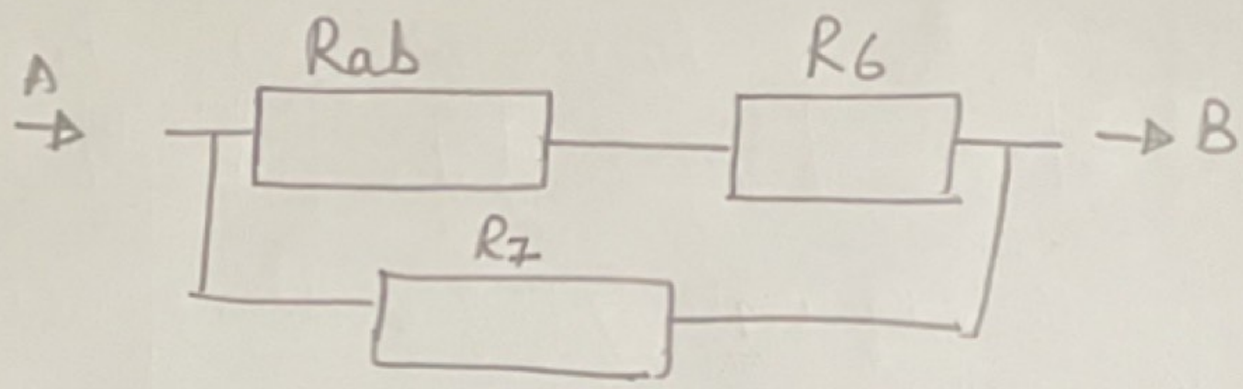
Also, R_3, R_4 & R_5 are in series configuration

$$\therefore R_b = R_3 \times R_4 \times R_5$$
$$= 0.8 \times 0.8 \times 0.9$$
$$R_b = 0.576 \quad \text{—(2)}$$



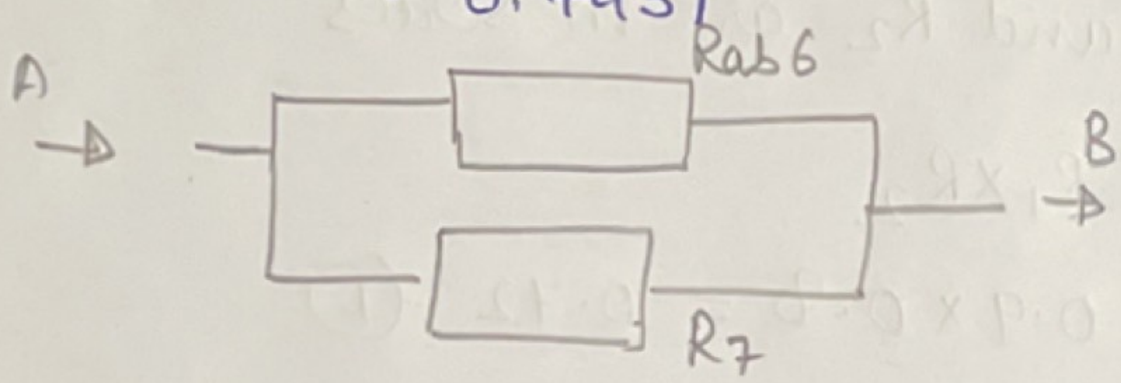
Now, R_a and R_b in parallel configuration.

$$R_{ab} = 1-(1-R_a)(1-R_b)$$
$$= 1-(1-0.72)(1-0.576$$
$$R_{ab} = 0.88128$$



Now, R_{ab} and R_6 are in series.

$$R_{ab6} = R_{ab} \times R_6$$
$$= 0.88128 \times 0.9$$
$$= 0.7931$$



Finally, R_{ab6} and R_c are in parallel configuration

$\therefore$ Reliability of the System $= 1-(1-R_{ab6})(1-R_c)$

$$= 1-[1-0.793][1-0.9]$$
$$= 1-[0.2069 \times 0.1]$$

$$\boxed{R = 0.9793}$$

Question No 7 the failure data for 40 electronic components is shown below:

Ops (hrs)	0-5	5-10	10-15	15-20	20-25	25-30
No. of fail	5	7	6	4	5	4
Ops (hrs)	30-35	35-40	40-45	45-50	≥50	
No. of fail	4	0	2	1	2	

Estimate the reliability, cumulative failure distribution, failure density and failure rate functions.

Solution

Time Δt (t)	No. of failures (f)	Cumulative failure (N_f)	No. of Survivors (N_s)	Reliability (R_t)	Cumulative Failure Distribution (F_t)	Failure density function $f(t)$	Hazard rate $\lambda(t)$
0	0	0	40	1	0	0.025	0.025
5	5	5	35	0.875	0.125	0.035	0.04
10	7	12	28	0.7	0.3	0.05	0.05
15	6	18	22	0.55	0.45	0.02	0.036
20	4	22	18	0.45	0.55	0.025	0.55
25	5	27	13	0.325	0.675	0.02	0.06
30	4	31	9	0.225	0.775	0.02	0.08
35	4	35	5	0.25	0.875	0	0
40	0	35	5	0.25	0.875	0.01	0.08
45	2	37	3	0.075	0.925	0.005	0.06
50	1	38	2	0.05	0.95	0.01	0.2
≥50	2	40	0	0	1	–	–

CALCULATIONS EXPLAINED.

(1) Cumulative failure (Colum 3)

Entries in this column simply represent the cumulative frequencies of the frequencies. If $N_f(t)$ denotes the number of components that have failed by time t. then the first 3 entries of the column are calculated as follows:

First entry $N_f(0) = 0$

Second entry $N_f(5) = 0 + 5 = 5$

third entry $N_f(10) = 5 + 7 = 12$ . . . .

(2) Number of Survivor (N_s)

$N_s(t)$ denotes the no. of components that are performing their intended function at time t then

$$\boxed{N_s(t) = \text{Total failure} - N_f^{\,i^{th}}}$$

where N_f = Cumulative failure frequencies of the ith time.

(3) Reliability (R_t) - Column 5

If $R(t)$ denotes the reliability at time t, then

$$\boxed{R(t) = \frac{N_s(t)}{N_0}}$$

Where $N_s(t)$ = Number of Survivors

N_0 = total no. of failures.

(4) Cumulative failure Distribution $F(t)$ - Column 6.

If $F(t)$ denotes cumulative failure distribution at time (t)

$$\boxed{F(t) = \frac{N_f(t)}{N_0}}$$

Where $N_f(t)$ = Cumulative failures

N_0 = total no. of failures.

(5) Failure Density function $f(t)$ - Column 7.

$f(t)$ denotes the value of failure density function at time t

$$\boxed{f(t) = \frac{N_f(t+\Delta t) - N_f(t)}{N_0 \times \Delta t}}$$

Where $\Delta t = 5$

(6) Hazard Rate ($\lambda(t)$).

Hazard Rate at different points in time.
$\lambda(t)$ denotes the value of failure rate at time t.

$$\boxed{\lambda(t) = \frac{N_f(t+\Delta t) - N_f(t)}{N_s(t) \cdot \Delta t}}$$

Question No 8 At a call center, callers have to wait till an operator is ready to take their call. To monitor this process 5 calls were recorded every hour for the 8-hour working day. The data below shows the waiting time in seconds.

Time	Sample Number				
	1	2	3	4	5
9.am	8	9	15	4	11
10	7	10	7	6	8
11	11	12	10	9	10
12	12	8	6	9	12
1 p.m	11	10	6	14	11
2	7	7	10	4	11
3	10	7	4	10	10
4	8	11	11	11	7

(i) Use the data to construct control charts for mean and variability and comment about the process. If process is out of control, the calculate the revised control limits.

Solution :→

Time	X_1	X_2	X_3	X_4	X_5	$\bar{X}$	R
9a.m	8	9	15	4	11	9.4	11
10am	7	10	7	6	8	7.6	4
11am	11	12	10	9	10	10.4	3
12am	12	8	6	9	12	9.4	6
1 p.m	11	10	6	14	11	10.4	8
2.PM	7	7	10	4	11	7.8	7
3PM	10	7	4	10	10	8.2	6
4PM	8	11	11	11	7	9.6	4
						$\Sigma\bar{x}=72.8$	49

Given $n=5$, $A_2=0.577$

To calculate the center line and control limits, we first calculate the values $\bar{\bar{x}}$ & $\bar{R}$

$$\bar{\bar{x}} = \frac{\Sigma\bar{x}}{K} = \frac{72.3}{8} = 9.1$$

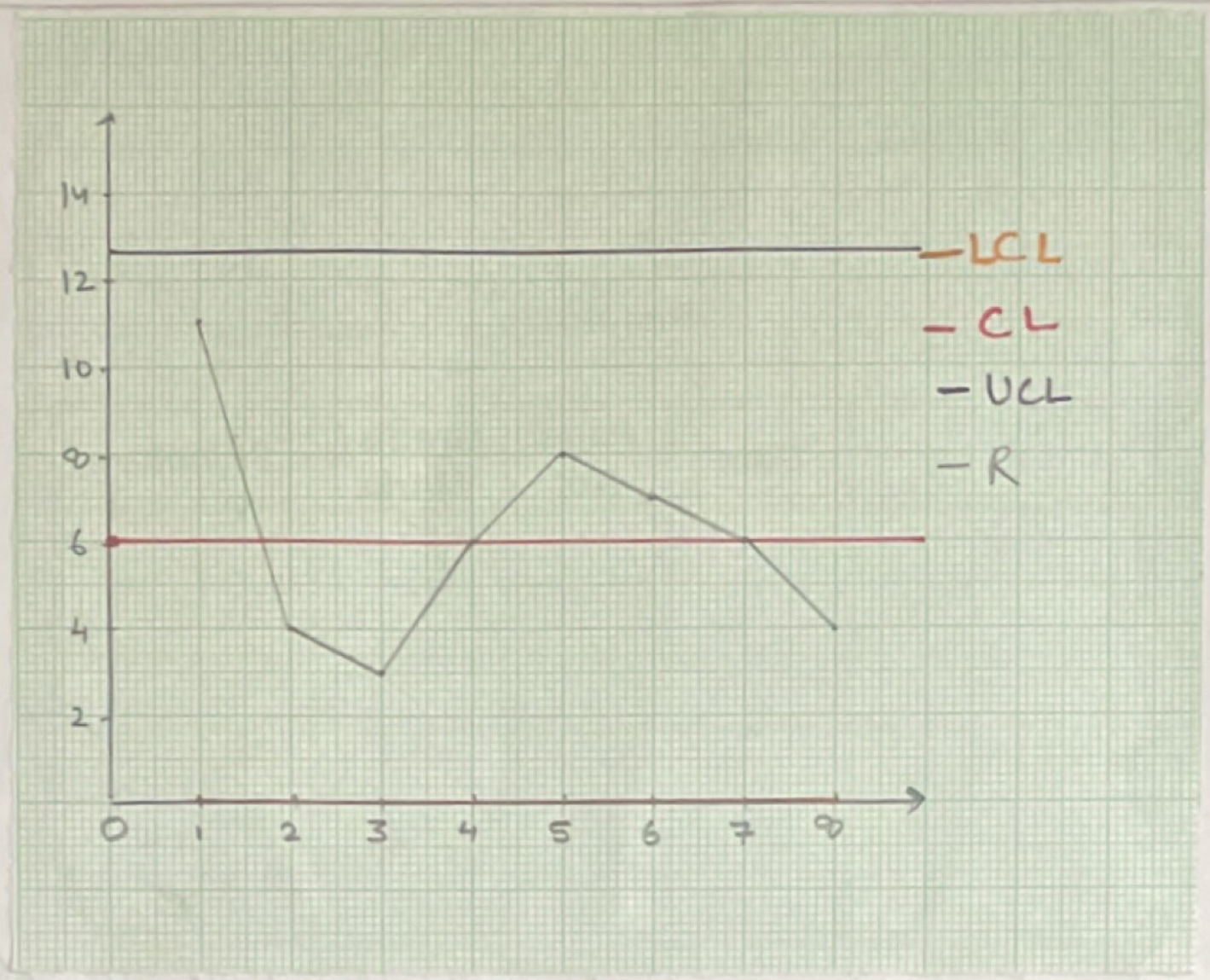
$$\bar{R} = \frac{\Sigma R}{K} = \frac{49}{8} = 6.125$$

finding CL, UCL, LCL to construct $\bar{x}$-chart

$$CL = \bar{\bar{x}} = 9.1$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 9.1 + (0.577 \times 6.125)$$
$$= 12.6341$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 9.1 - (0.577 \times 6.125) = 5.5658$$



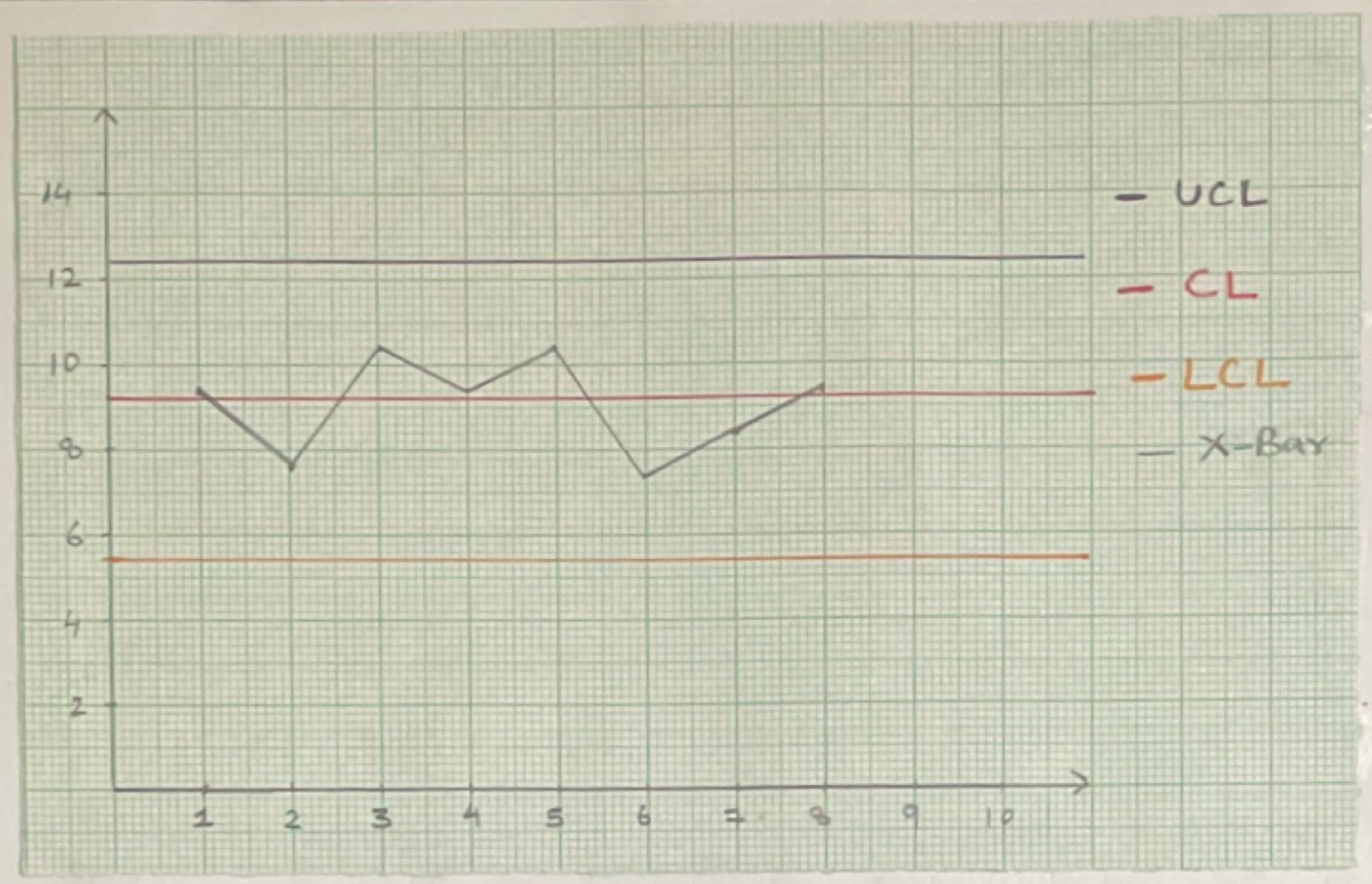
Since no point lies outside the control limits of R-chart. It indicates that the process variability is under statistical control.

(ii) If the specification limit as the 8 ± 2, the find the process capability. Does it appear that the process is capable of meeting the specification requirements?

<u>Solution</u> for $n=5$, $d_2 = 2.326$

i.e $USL = 8+2 = 10$

$LSL = 8-2 = 6$



Interpretation :- This $\bar{x}$- chart indicates that all points lie within the control limits. Hence we say that process variability is under statistical control.

R- Chart for Variability

Now, we will construct R-chart for variability

For $n = 5$ $A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.114$.

Control limits for R-chart

$$CL = \bar{R} = \frac{\Sigma R}{K} = \frac{49}{8} = 6.125$$

$$UCL = D_4 \bar{R} = 2.114 \times 6.125 = 12.948$$

$$LCL = D_3 \bar{R} = 0 \times 6.125 = 0$$

for $n=4$ $d_2 = 2.059$

Process capability $PC = \frac{6\bar{R}}{d_2} = \frac{6 \times 6.125}{2.326}$

$$\boxed{PC = 15.7996}$$

Tolerance $T = USL - LSL$

$= 10 - 6$

$T = 4$

Since process capability is greater than tolerance, that means the process is not capable of achieving the requirements specified in the specification.