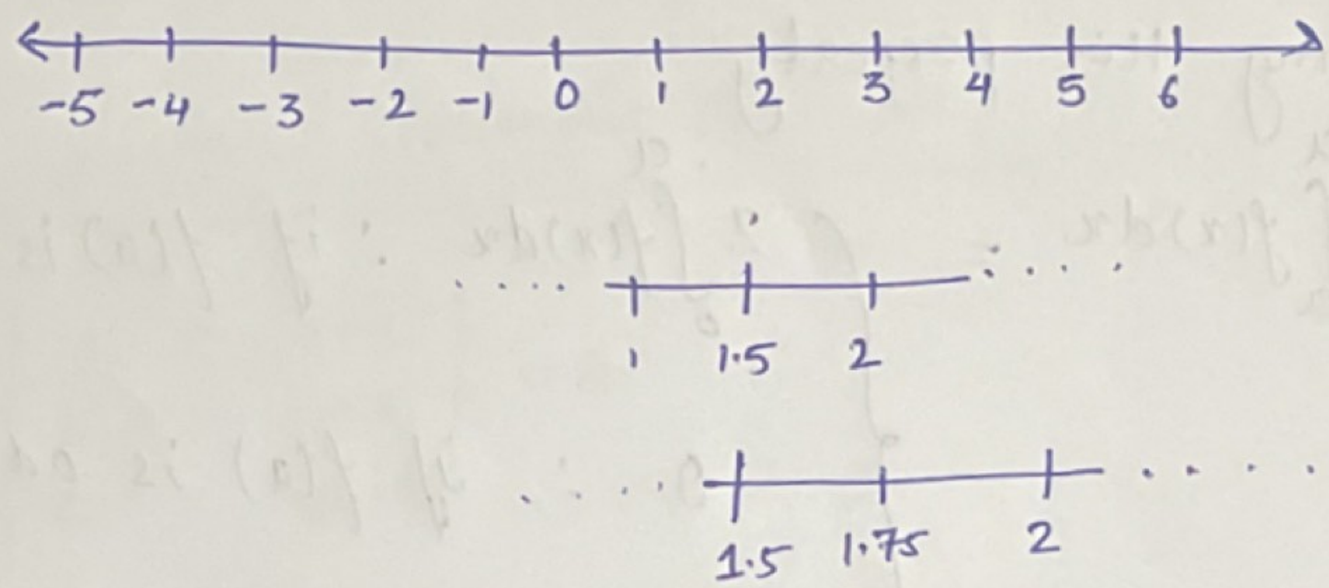
Question No 1 State whether the following statements are True or False. Give reason in support of your answer:

(a) Between any two different rational numbers there is another rational number.

According to this property, there is always a rational numbers (a subset of real numbers) between two rational number.



Between two rational numbers there are infinitly many rational numbers.

$\boxed{\text{Given Statement is TRUE}}$

(b) $$\int_{-1000}^{+1000} (x^{1001} + x^{2001} + x^{3001})\, dx = 0$$

Let $$I = \int_{-1000}^{+1000} (x^{1001} + x^{2001} + x^{3001})\, dx$$

Say $f(x) = x^{1001} + x^{2001} + x^{3001}$

$$f(-x) = (-x)^{1001} + (-x)^{2001} + (-x)^{3001}$$

$$= -x^{1001} - x^{2001} - x^{3001}$$

$$= -1\left(x^{1001} + x^{2001} + x^{3001}\right)$$

$$\boxed{f(x) = f(-x)}$$

i.e $f(x)$ is a odd function

using this property

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_{0}^{a} f(x)\,dx & : \text{if } f(a) \text{ is even} \\ 0 & : \text{if } f(a) \text{ is odd.} \end{cases}$$

Clearly $I = \int_{-1000}^{+1000} \left(x^{1001} + x^{2001} + x^{3001}\right) dx = 0$

[$\because f(x)$ is odd]

$\boxed{\text{Given Statement is TRUE}}$

(c) 210 is 51st term of the sequence 10, 15, 20, 25, ...

Sol^n Given sequence

10, 15, 20, 25, ...

$a = 10$ so, $a_n = a + (n-1)d$

$d = 5$

$n = 51$ $a_n = 10 + (51-1)5$

$a_n = 10 + (50 \times 5)$

$a_n = 260$

i.e 51st term in given sequence is 260.

$\boxed{\text{Given statement is false}}$

(d) $\begin{vmatrix} a & x & b+c \\ b & x & c+a \\ c & x & a+b \end{vmatrix} = 0$

$I = \begin{vmatrix} a & x & b+c \\ b & x & c+a \\ c & x & a+b \end{vmatrix}$ $C_1 \rightarrow C_1 + C_2 + C_3$

$I = \begin{vmatrix} a+b+c+x & x & b+c \\ a+b+c+x & x & c+a \\ a+b+c+x & x & a+b \end{vmatrix}$

$I = (a+b+c+x)(x) \begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$.

Here, two columns (C_1 & C_2) are identical so value of the determinant vanishes.

So, $\begin{vmatrix} a & x & b+c \\ b & x & c+a \\ c & x & a+b \end{vmatrix} = 0$

$\boxed{\text{Given statement is True.}}$

(e) The range of the data shown in the following frequency distribution is 350.

Classes	200-250	250-300	300-350	350-400	400-450	450-500	500-550
Frequency	0	7	3	8	4	0	0

We know that Range = $x_m - x_0$

$= 450 - 200 = 200$

Here x_m = the upper class boundary of highest class

x_0 = the lower class boundary of lower class

In cases (450-500) and (500-550) the frequencies are zero. So we will not count it for Range.

$\boxed{\text{Given Statement is False.}}$

Question No 2 (a) A carpenter was hired to build 192 window frames. The first day he made five frames and each day thereafter he made two more frames than he made the day before. How many days he will take to finish his job?

Solution:- Total no. of frames to be made $= 192$

1st day he made five frames $(a_1) = 5$

2nd day $(a_2) = 7$

this will form arithmetic progression with $a = 2$

let the carpenter finish his job in n days

then $S_n = 192$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$192 = \frac{n}{2}[(2 \times 5) + (n-1)2]$$

$$192 \times 2 = 2n^2 + 8n$$

$$n^2 + 4n - 192 = 0$$

$$n(n-12) + 16(n-12) = 0$$

$$\boxed{n = 12}$$

Question 2 (b) Set having values

$\frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49} \ldots$ is countable.

$= \{ (\frac{1}{2})^2, (\frac{1}{3})^2, (\frac{1}{4})^2, (\frac{1}{5})^2 \ldots \}$

Hence we may write the set as

$A = \{ x : x = \frac{1}{n^2}, n \in N \text{ and } 1 < n \}$

(c) How many words each of three vowels and two consonants can be formed from the letters of the words INVOLUTE?

Sol^n

INVOLUTE → Vowels I, O, U, E

INVOLUTE → Consonants N, V, L, T

	#	# to be chosen	# of ways to choose
Vowels	4	3	$^4C_3$
Consonants	4	2	$^4C_2$

No. of ways selecting 3 vowels and 2 consonants

$= {}^4C_3 \times {}^4C_2$

$= \frac{4!}{3!\,1!} \times \frac{4!}{2!\,2!} = 24.$

Now, we have to arrange

No. of arrangements of 5 letter is

given by $= {}^5P_5$

$$= \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5!$$

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$= 120$$

Thus, Total no. of words

= No. of ways of selecting × No. of arrangements

$$= 24 \times 120 = 2880$$

Question 3 (a) Express 700. 1400. 2100. 2800. 3500. 42000 in terms of factorial

$$= 100^6 [7 \cdot 14 \cdot 21 \cdot 28 \cdot 35 \cdot 40]$$

$$= 7^6 \times 100^6 [1 \cdot 2 \cdot 3 \cdot 4 \cdot 5]$$

$$= (7 \times 100)^6 \times 5! = (700)^6 (5!)$$

(b) How many different signals are possible with 5 blue, 4 red, 3 white and 2 green flags by using all at a time in a queue?

Soln 5 blue, 4 red, 3 white, 2 green

Total no. of flags = $5+4+3+2 = 14$

$\therefore$ required no. of signals $= \dfrac{14!}{5! \times 4! \times 3! \times 2!}$

$$= \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 2 \times 1}$$

$$= 2522520$$

(c) If in a hall there are 10 randomly selected students then how many numbers of ways are there such that all of them have different birthday. Assume that all of have their birthday in non-leap years.

Solution:- In a hall there are 10 randomly selected students all of them have their birthday in non leap years

$\Rightarrow$ Day in a year $= 365$

No. of ways are there such that all of them have different birthday

⇒ we need to select 10 birthday out of 365 and these 10 birthday can be arranged in 10! ways

$$= {}^{365}C_{10} \times 10! \quad \text{or} \quad {}^{365}P_{10}$$

$$= \frac{365!}{355!}$$

$$= \frac{365 \times 364 \times 363 \times 362 \times 361 \times 360 \times 359 \times 358 \times 357 \times 356 \times \cancel{355!}}{\cancel{355!}}$$

No. of ways are there such that all of them have different birthday $= \frac{365!}{355!}$

# of ways are there such that none of them have different birthday.

⇒ 1 birthday has to be selected out of 365, Hence 365 days

⇒ # of ways are there such that none of them have different birthday = 365.

Question NO4 Discuss the continuity and differentiability of the function at $x = 2/3$

$$f(x) = \begin{cases} |x - 2/3|, & x \neq 2/3 \\ 0, & x = 2/3 \end{cases}$$

Solution

Checking for continuity

Here $\lim_{x \to \frac{2}{3}} f(x)$, where $f(x) = \begin{cases} |x - 2/3|, & x \neq 2/3 \\ 0, & x = 2/3 \end{cases}$

$$\text{LHL at } \lim_{x \to \frac{2}{3}^-} = \lim_{x \to \frac{2}{3}^-} \left| x - \frac{2}{3} \right|$$

Putting $x = \frac{2}{3} - h$

where h is +ive real and very small.

$$L.H.L = \lim_{h \to 0^-} \left| \frac{2}{3} - h - \frac{2}{3} \right|$$

$$= \lim_{h \to 0^-} |-h| = \lim_{h \to 0^-} h = 0 \quad \text{—(1)}$$

Putting $x = \frac{2}{3} + h$ as $x \to \frac{2}{3}^+ \Rightarrow h \to 0^+$

$$R.H.L = \lim_{h \to 0^+} \left| \frac{2}{3} + h - \frac{2}{3} \right|$$

$$= \lim_{h \to 0^+} |h| = \lim_{h \to 0^+} h = 0 \quad \text{—(2)}$$

from eqn ① and ②

L.H.L = R.H.L

$\lim_{x \to \frac{2}{3}} f(x)$ exist and equal to $x \to \frac{2}{3}$

Also, $f\left(\frac{2}{3}\right) = \left|\frac{2}{3} - \frac{2}{3}\right| = 0$

$\therefore$ L.H.L = R.H.L = $f(2/3)$

$\Rightarrow$ $\boxed{f(x) \text{ is continow at } x = \frac{2}{3}}$

Checking differentiability

$$\text{L.H.D at } x = 2/3 = \lim_{x \to \frac{2}{3}^-} \frac{f(x) - f(2/3)}{x - 2/3}$$

$$= \lim_{h \to 0^-} \frac{f(2/3 - h) - f(2/3)}{2/3 - h - 2/3}$$

$$= \lim_{h \to 0^-} \frac{2/3 - (2/3 - h) - 0}{-h}$$

$$= \lim_{h \to 0^-} \frac{h}{-h} = -1$$

$$\text{R.H.D at } x = 2/3 = \lim_{x \to \frac{2}{3}^+} \frac{f(x) - f(2/3)}{x - 2/3}$$

$$= \lim_{h \to 0^+} \frac{f(2/3 + h) - f(2/3)}{2/3 + h - 2/3}$$

$$= \lim_{h \to 0^+} \frac{2/3 + 2/3 - h - 0}{h}$$

$$= \lim_{h \to 0^+} \frac{h}{h} = 1$$

$L.H.D_{x=2/3} \neq R.H.D_{x=3}$

Hence $f(x)$ is lontinous but <u>not differentiable</u> at $x = 2/3$

<u>Question No 5</u> Evaluate the following example.

(i) $\int (10x^9 + 40x^4 + 3)\sqrt{x^{10} + 8x^5 + 3x + 5}\, dx$

Let $x^{10} + 8x^5 + 3x + 5 = t$

$(10x^9 + 40x^4 + 3)\,dx = dt$

$$I = \int \sqrt{t}\, dt$$

$$= \frac{t^{3/2}}{3/2} + k = \frac{2}{3}(10x^9 + 40x^4 + 3x + 5)^{3/2} + k$$

where K is constant of Integeration.

(ii) $\int \frac{1}{(x-5)(x^2+4)} dx$

we can write Integerand as

$$\frac{1}{(x-5)(x^2+4)} = \frac{A}{(x-5)} + \frac{Bx+C}{(x^2+4)}$$

$$1 = A(x^2+4) + (Bx+C)(x-5)$$

Putting $x = +5$

$$1 = A(25+4) + (5B+C)(5-5)$$

$$\boxed{A = \frac{1}{29}}$$

Putting $x = 0$

$$1 = \frac{1}{29}(0+4) + (B\times 0 + C)(0-5)$$

$$1 = \frac{4}{29} + (-5)C \qquad \boxed{C = \frac{-5}{29}}$$

Putting $x = 1$

$$1 = \frac{1}{29}(1+4) + (B\times 1 + \left(\frac{-5}{29}\right)(1-5)$$

$$\boxed{B = -\frac{1}{29}}$$

Now,

$$\int \frac{1}{(x-5)(x^2+4)} = \underbrace{\int \frac{1}{29} \times \frac{1}{(x-5)} dx}_{I} + \underbrace{\int -\frac{1}{29} \times \frac{x}{x^2+4} dx}_{II} + \underbrace{\int \frac{-5}{29} \times \frac{1}{x^2+4} dx}_{III}$$

From I $= \frac{1}{29}\int \frac{1}{x-5} dx = \frac{1}{29} \log|x-5|$ — eqn (1)

From III $\frac{-5}{29}\int \frac{1}{x^2+4}dx = \frac{-5}{29} \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) = \frac{-5}{58}\tan^{-1}\left(\frac{x}{2}\right)$ — eqn ②

From II $-\frac{1}{29}\int \frac{x}{x^2+4}dx = -\frac{1}{29\times 2}\int \frac{2x}{x^2+4}dx$

Let $x^2+4 = t$

$2x\,dx = dt$

$\Rightarrow -\frac{1}{58}\int \frac{dt}{t} = \frac{-1}{58}\log|t|$ — eqn ③

$= -\frac{1}{58}|x^2+4|$

From eqn ①, ②, ③

$$\boxed{I = \frac{1}{29}\log|x-5| - \frac{1}{58}\log|x^2+4| - \frac{5}{58}\tan^{-1}\left(\frac{x}{2}\right) + C}$$

QUESTION NO 6 Find value of x, y, z given that

$5x + y + z = 36$

$x + y + z = 16$

$10x + 2y + 2z = 72$

You are bound to use the matrix techniques to solve.

Soln System of eqn can be written as

$$\begin{bmatrix} 5 & 1 & 1 \\ 1 & 1 & 1 \\ 10 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 36 \\ 16 \\ 72 \end{bmatrix} \quad \text{or } AX = B$$

$|A| = \begin{vmatrix} 5 & 1 & 1 \\ 1 & 1 & 1 \\ 10 & 2 & 2 \end{vmatrix}$ Expanding along R_1

$= 5(2-2) - 1(2-10) + 1(2-10) = 0 + 8 - 8 = 0$

$\therefore A^{-1}$ does not exist

$\Rightarrow$ System has either no solution or infinite many solution.

Let A_{ij} denotes the cofactor of $(i,j)^{th}$ element of matrix A

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 1(2-2) = 0$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 10 & 2 \end{vmatrix} = (-1)(2-10) = 8$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 1 \\ 10 & 2 \end{vmatrix} = (1)(2-10) = -8$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = (-1)(2-2) = 0$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 1 \\ 10 & 2 \end{vmatrix} = (1)(10-10) = 0$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 1 \\ 10 & 2 \end{vmatrix} = (-1)(10-10) = 0$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = (1)(1-1) = 0$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} = (-1)(5-1) = -4$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} = (1)(5-1) = 4$$

$$\text{adj}A = \begin{bmatrix} 0 & 8 & -8 \\ 0 & 0 & 0 \\ 0 & -4 & 4 \end{bmatrix}^{T} = \begin{bmatrix} 0 & 0 & 0 \\ 8 & 0 & -4 \\ -8 & 0 & 4 \end{bmatrix}$$

$$(\text{adj}A)B = \begin{bmatrix} 0 & 0 & 0 \\ 8 & 0 & -4 \\ -8 & 0 & 4 \end{bmatrix}\begin{bmatrix} 36 \\ 16 \\ 72 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \text{ (null matrix)}$$

Hence system has infinite many solution and given by putting either $x=k$ or $y=k$ or $z=k$ in any two equations given by ①, ② and ③. Let's put $z=k$ in eq^n ① and ②. we get, where k is any real number

$$5x+y+k=36$$

$$5x+y=36-k \quad \text{—(i)}$$

$$x+y=16-k \quad \text{—(ii)}$$

$$4x=40$$

$$\boxed{x=10}$$

put in eq^n (ii)

$$y=36-k-50$$

$$\boxed{y=-24-k}$$

$$5\times10-24-k+z=36$$

$$z=36-26+z$$

$$\boxed{z=10+k}$$

where k is any real no.

Question 7 (a) Write flow charts of Cramer rule and matrix method.

* Cramer's Rule to solve $AX = B$

Find Δ

If $\Delta \neq 0$ then system have unique solution and given by $x = \frac{\Delta_1}{\Delta}$, $y = \frac{\Delta_2}{\Delta}$, $z = \frac{\Delta_3}{\Delta}$

$\Delta = 0$

If $\Delta = 0$ but atleast one of $\Delta_1, \Delta_2, \Delta_3 \neq 0$ then system has no solution

If $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ then system has infinite solution & obtained by putting x or y or z equal to K, where K is any real number

Matrix Method to Solve the Systems $AX = B$

Find $|A|$

If $|A| \neq 0$, then system has unique solution and given by $X = A^{-1}B$.

$|A| = 0$

If $(\text{adj}A)B \neq 0$ then system has no solution

If $(\text{adj}A)B = 0$ then system has infinite solutions and obtained by putting x, y or z equal to k, where k is any real number

Question 7 (b) Write whether the following data are discrete or continous. Give reason in support of your answer.

(i) Number of children in a family in a colony of 100 families.

Discrete Variable

(ii) Number of pages in each of the 50 books having some mistake.

Discrete Variable

(iii) Height of students of IGNOU who enrolled in 2021

Continous Variable

(iv) Waiting time of metro when a person reaches metro station.

Continous variable

(v) Monthly income of the family.

Discrete variable.

Question 8(a) Write any 10 principles of data visualisation.

Solution Some principles of data visualisation are :-

1. Determine your audience. What questions will they need answer?
2. Choose the right kind of chart to depict the type of information you have.
3. Keep it simple. Remove any non-essential information.
4. Use patterns (of chart types, colors or other design element) to identify similar types of information.
5. Use proportion carefully so that differences in design size fairly represent difference in value.
6. Form follows function. Focus on how your audience need to use data.
7. Choose colors carefully to draw attention
8. Seek balance in your visual elements, including texture, color, shape and space.
9. Provide necessary context of data.
10. Neat and clean diagram.

Question 8 (b) What is the relation of unit on y axis with unit on x-axis in histogram.

Solution:-

Y axis: The Y axis shows the number of times that the values accured within the intervals set by the x-axis

X axis: the X axis are internal that show the scale of values which the measurement fall under.