# Control Chart for Number of Defectives

SESSION 8

Structure



## 8.1 INTRODUCTION

**Prerequisite**

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Session 6 of MSTL-002 (Industrial Statistics Lab).
- Unit 3 of MSTE-001 (Industrial Statistics-I).

In Lab Sessions 6 and 7, you have studied the p-chart for fixed and varying sample size, respectively. In p-chart, we calculated the fraction defective for each sample and plotted the fraction defective against the sample number.

In this lab session, we count the number of defective items in the samples instead of calculating the fraction defective. We plot them against the sample numbers. This chart is known as the np-chart. Using np-chart, we may better understand the actual number of defective items than the fraction of defective. The np-chart is used only for equal sample size because we cannot compare the number of defectives for variable sample size as discussed in Sec. 3.5 of Unit 3 of MSTE-001 (Industrial Statistics-I). For example, if we get one defective item in a sample of size 10 and one defective item in a sample of size 100, the first sample shows a much higher rate of defectives, i.e., 10% in comparison with the second sample.

The np-chart is also known as the d-chart.

### Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for number of defectives;
- construct the control chart for number of defectives; and
- interpret the control chart for number of defectives.

## 8.2 PROBLEM DESCRIPTION

To monitor the manufacturing of mobile phones, a quality controller randomly inspects 45 mobile phones at the end of the production each day and notes the number of defective mobile phones. The number of defective mobile phones found each day is recorded in Table 1.

Table 1: Number of defective mobile phones

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of Defective Phones | 13 | 12 | 13 | 18 | 7 | 14 | 12 | 10 | 16 | 16 |
| Sample No. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Number of Defective Phones | 12 | 18 | 10 | 12 | 14 | 10 | 17 | 8 | 13 | 12 |
| Sample No. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Number of Defective Phones | 16 | 13 | 10 | 17 | 14 | 18 | 10 | 12 | 12 | 13 |

The quality control inspector of this company needs to construct the control chart for number of defective mobile phones to check whether the process is under statistical control or not.

Therefore, the problem for this session is to construct the control chart for number of defective mobile phones, i.e., the np-chart for the data given in Table 1 and also compute the revised control limits, if necessary.

## 8.3 PROCEDURE FOR THE CONSTRUCTION OF np-CHART

In Sec. 3.5 of Unit 3 of MSTE-001, you have grasped the various formulae and procedure for the construction of np-chart. Therefore, in this lab session, we briefly discuss the procedure of computing the centre line, upper and lower control limits. The main steps involved in the construction of the np-chart are as follows:

**Step 1:** We draw k samples of the same size n and $d_1, d_2, ..., d_k$ are the numbers of defectives in 1st, 2nd, …, kth sample, respectively. The average fraction defective of k samples is given by

$$\bar{p} = \frac{\text{Sum of defective items}}{\text{Total number of items inspected}} = \frac{1}{nk}\sum_{i=1}^{k} d_i \qquad \ldots(1)$$

**Step 2:** The centre line and control limits for the np-chart are given by

- ✓ Centre line (CL) $= n\bar{p}$ …(2)
- ✓ Upper control limit (UCL) $= n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ …(3)
- ✓ Lower control limit (LCL) $= n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$ …(4)

**Step 3:** After setting the centre line and control limits, we construct the np-chart by taking sample number on the X-axis and number of defectives in the sample on the Y-axis.

**Step 4:** Interpretation of the np-chart.

## 8.4 STEPS INVOLVED IN THE CONSTRUCTION OF np-CHART IN EXCEL 2007

You have already learnt about the manual computation of np-chart in Sec. 3.5 of Unit 3 of MSTE-001. The steps for plotting the np-chart in Excel 2007 are given below:

**Step 1:** We enter the given data in MS Excel spreadsheet as shown in Fig. 8.1.

| | A | B |
|---|---|---|
| 1 | Sample No. | Number of Defective Phones |
| 2 | 1 | 13 |
| 3 | 2 | 12 |
| 4 | 3 | 13 |
| 5 | 4 | 18 |
| 6 | 5 | 7 |
| 7 | 6 | 14 |
| 8 | 7 | 12 |
| 9 | 8 | 10 |
| 10 | 9 | 16 |
| 11 | 10 | 16 |
| 12 | 11 | 12 |
| 13 | 12 | 18 |
| 14 | 13 | 10 |
| 15 | 14 | 12 |
| 16 | 15 | 14 |
| 17 | 16 | 10 |
| 18 | 17 | 17 |
| 19 | 18 | 8 |
| 20 | 19 | 13 |

np chart

**Fig. 8.1: Partial screenshot of the spreadsheet for given data.**

**Step 2:** We find the total number of defective mobile phones by typing "=Sum(B2:B31)" in Cell B2 and then clicking on ***Enter*** as shown in Fig. 8.2.

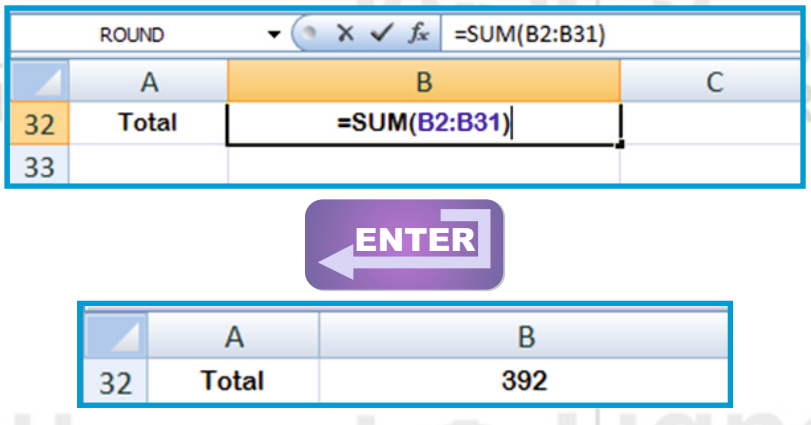



**Fig. 8.2**

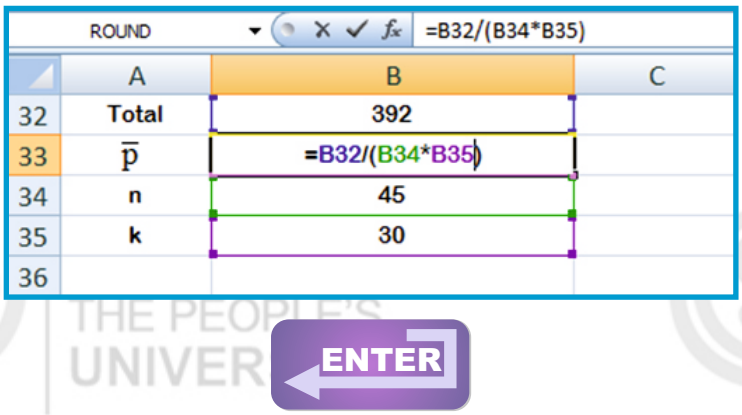
**Step 3:** After calculating the value of $\sum_{i=1}^{30} d_i$, we put the values of n and k in Cells B34 and B35, respectively (Fig. 8.3).

| | A | B |
|---|---|---|
| 32 | Total | 392 |
| 33 | | |
| 34 | n | 45 |
| 35 | k | 30 |

**Fig. 8.3**

The values of $\sum_{i=1}^{30} d_i$, n and k are given in Cells B32, B34 and B35, respectively.

**Step 4:** We now use equation (1) and compute the average fraction defective $\bar{p}$ by typing "=B32/(B34*B35)" in Cell B33 and clicking on ***Enter*** as shown in Fig. 8.4.




| | A | B |
|---|---|---|
| 32 | Total | 392 |
| 33 | $\bar{p}$ | 0.290 |
| 34 | n | 45 |
| 35 | k | 30 |

**Fig. 8.4**

**Step 5:** We determine the values of centre line, upper and lower control limits in Columns C, D and E, respectively. We can calculate these limits from equations (2) to (4) by typing

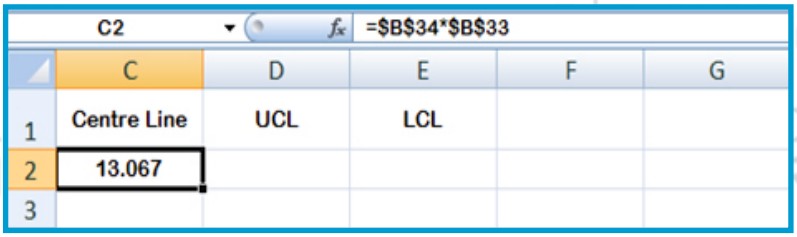
i) "=$B$34*$B$33" in Cell C2 to find centre line as shown in Fig. 8.5a.

ii) "=$B$34*$B$33+3* Sqrt(($B$34*$B$33*(1-$B$33)))" in Cell D2 as shown in Fig. 8.5b.

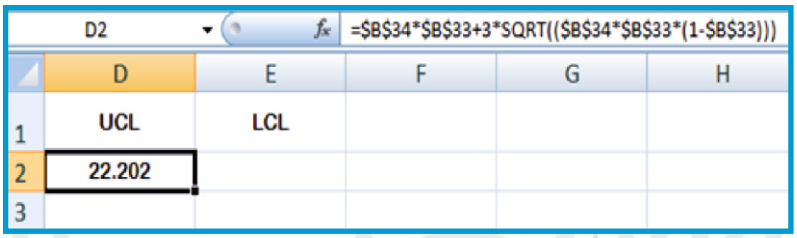
iii) "=$B$34*$B$33-3* Sqrt(($B$34*$B$33*(1-$B$33)))" in Cell E2 as shown in Fig. 8.5c.

The formula with dollar sign ($) is used for an absolute reference.

(a)




(b)




(c)

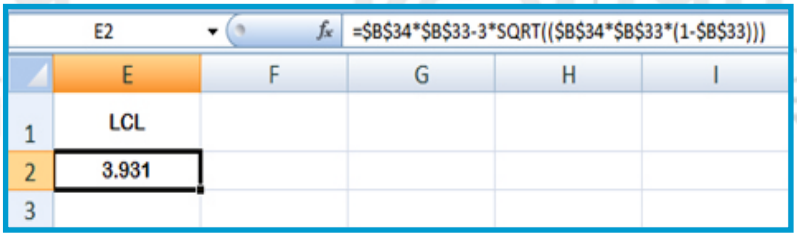



Fig. 8.5

**Step 6:** We first select Cells C2:E2 and drag them down up to Row 31 to get the control limits corresponding to each sample as shown in Fig. 8.6.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Sample No. | Number of Defective Phones | Centre Line | UCL | LCL |
| 2 | 1 | 13 | 13.067 | 22.202 | 3.931 |
| 3 | 2 | 12 | 13.067 | 22.202 | 3.931 |
| 4 | 3 | 13 | 13.067 | 22.202 | 3.931 |
| 5 | 4 | 18 | 13.067 | 22.202 | 3.931 |
| 6 | 5 | 7 | 13.067 | 22.202 | 3.931 |
| 7 | 6 | 14 | 13.067 | 22.202 | 3.931 |
| 8 | 7 | 12 | 13.067 | 22.202 | 3.931 |
| 9 | 8 | 10 | 13.067 | 22.202 | 3.931 |
| 10 | 9 | 16 | 13.067 | 22.202 | 3.931 |
| 11 | 10 | 16 | 13.067 | 22.202 | 3.931 |
| 12 | 11 | 12 | 13.067 | 22.202 | 3.931 |
| 13 | 12 | 18 | 13.067 | 22.202 | 3.931 |
| 14 | 13 | 10 | 13.067 | 22.202 | 3.931 |
| 15 | 14 | 12 | 13.067 | 22.202 | 3.931 |
| 16 | 15 | 14 | 13.067 | 22.202 | 3.931 |
| 17 | 16 | 10 | 13.067 | 22.202 | 3.931 |
| 18 | 17 | 17 | 13.067 | 22.202 | 3.931 |
| 19 | 18 | 8 | 13.067 | 22.202 | 3.931 |
| 20 | 19 | 13 | 13.067 | 22.202 | 3.931 |

np chart

Fig. 8.6

**Step 7:** To plot the np-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1. It means that we

1. select Cells B1:E31,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting np-chart is shown in Fig. 8.7.

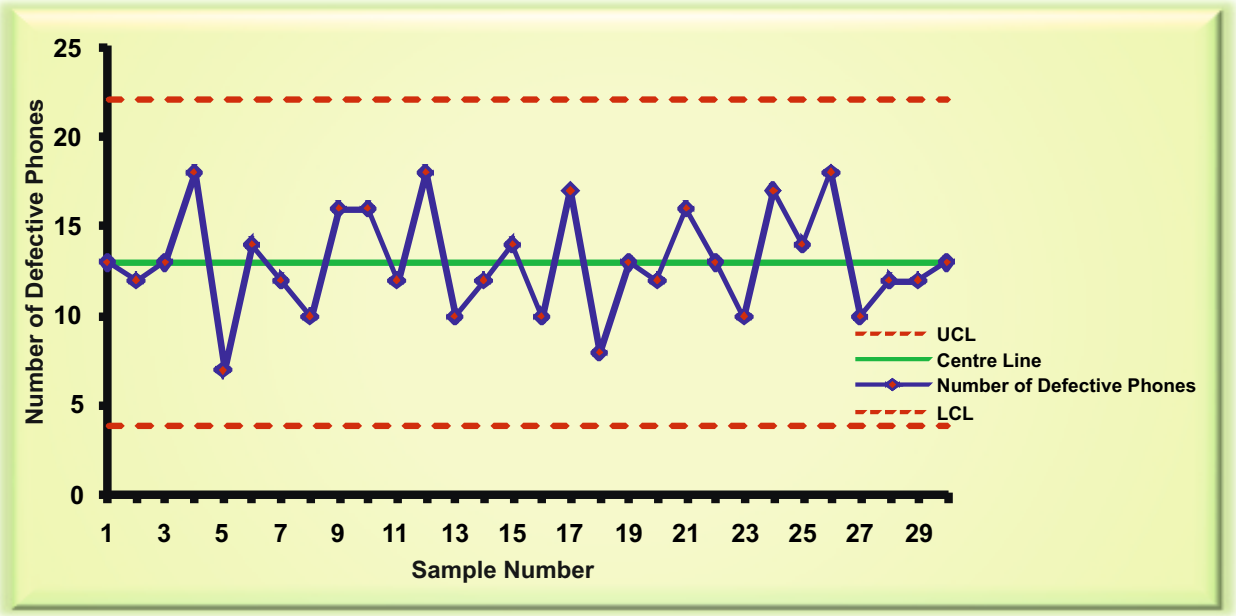



**Fig. 8.7**

## Interpretation

The control chart for defective mobile phones (shown in Fig. 8.7) reveals that the process is under statistical control since no point lies outside the control limits of the np-chart. We may conclude that the process is under statistical control with respect to number of defective mobile phones.

In the next section, we are listing the formulae and procedure of computing the revised control limits for the np-chart if you come across this situation.

## 8.5 REVISED np-CHART

If one or more points lie outside the control limits (UCL or LCL or both), the control chart indicates that the process is not under statistical control. Some assignable causes are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Once this is done, we eliminate the out-of-control samples and calculate the revised centre line, upper and lower control limits for the np-chart using the remaining samples as follows:

**Step 1:** To get the revised limits for np-chart, we first calculate the new $\bar{p}$ as follows:

$$\bar{p}_{new} = \frac{\sum_{i=1}^{k} d_i - \sum_{j=1}^{d} d_j}{n(k-d)} \quad ...(5)$$

where d – number of discarded samples, and

$\sum_{j=1}^{d} d_j$ – sum of number of defectives in the discarded samples.

**Step 2:** After finding $\bar{p}_{new}$, we reconstruct the centre line, upper and lower control limits by replacing $\bar{p}$ by $\bar{p}_{new}$ as follows:

- ✓ Centre line (CL) $= n\bar{p}_{new}$ ...(6)
- ✓ Upper control limit (UCL)

$$= n\bar{p}_{new} + 3\sqrt{n\bar{p}_{new}(1-\bar{p}_{new})} \quad ...(7)$$

- ✓ Lower control limit (LCL)

$$= n\bar{p}_{new} - 3\sqrt{n\bar{p}_{new}(1-\bar{p}_{new})} \quad ..(8)$$

**Step 3:** As discussed in Lab Sessions 2 to 5, we plot the np-chart for the revised control limits until all points lie within UCL and LCL.

**Step 4:** Interpretation of the p-chart

You should do the following exercises to construct the np-chart.

## Activity

Construct the control charts for the number of defectives with the help of MS Excel 2007 and interpret the results for

**A1)** Examples 5 and 6 given in Unit 3 of MSTE-001.

**A2)** Exercises E9 and E10 given in Unit 3 of MSTE-001.

Match the results with the manual calculation done in Unit 3 of MSTE-001.

## Continuous Assessment 8

In the production of tyres, 650 tyres were inspected every day before sending them to finished goods stores. The number of defective tyres found everyday is summarised in Table 2.

Table 2: Number of defective tyres

| Sample No. | Number of Defective Tyres | Sample No. | Number of Defective Tyres |
|---|---|---|---|
| 1 | 70 | 14 | 78 |
| 2 | 74 | 15 | 64 |
| 3 | 58 | 16 | 90 |
| 4 | 61 | 17 | 87 |
| 5 | 65 | 18 | 91 |
| 6 | 108 | 19 | 78 |
| 7 | 82 | 20 | 60 |
| 8 | 56 | 21 | 56 |
| 9 | 80 | 22 | 55 |
| 10 | 90 | 23 | 57 |
| 11 | 71 | 24 | 78 |
| 12 | 75 | 25 | 68 |
| 13 | 77 | | |

Draw the np-chart and comment about the state of the process. Also plot the revised control chart, if necessary.

## Home Work: Do It Yourself

1) Follow the steps explained in Secs. 8.4 and 8.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 8" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.