Question No 1 State whether the following statements are True or False. Give reason in support of your answer.

(a) The solution of a transportation problem with 5 rows (supplies) and 4 (columns) destinations is feasible if number of possible allocations are 8.

Solution $\boxed{\text{The Statement is TRUE}}$

It is given that

no. of rows $(m) = 5$

no. of columns $(n) = 4$

No. of possible allocations $= m+n-1$

$= 5+4-1$

$= 8$

(b) The moving averages of suitable period in a time-series are free from the influences of seasonal and cyclic variations.

Solution The method of moving average is used to eliminate the short term variations i.e seasonal and cyclic variations.

A moving average is a calculation used to analyse data points by creating a series of averages of different subsets of the full data set.

$\boxed{\text{The Statement is TRUE}}$

(c) If the basic solutions for a system of equations are $(-2, 0, 1)$, $(0, 1, 3)$, $(-2, 3, 0)$ then only $(0, 1, 3)$ is feasible.

<u>Solution</u> :-

$\boxed{\text{The statement is TRUE}}$

A basic solution for which all the basic variables are non-negative is called a <u>basic feasible solution</u>.

So, If the basic solution for a system of equations are $(-2, 0, 1)$ $(0, 1, 3)$ $(-2, 3, 0)$ then only $(0, 1, 3)$ is feasible system.

(d) In the Stepwise selection method of multiple regression model, once a variable enters in the model then it always remains in the model.

Solution $\boxed{\text{The statement is FALSE}}$

In the forward selection method, once a variable enters the model, it stays. whereas in stepwise selection method, a variable is excluded from the model if found insignificant.

(e) An enterprise requires 1000 units per month. The ordering cost is estimated to be 50 per order. The purchase price is 20 per unit and the carrying cost per unit is 10% of it. then the economic lot size to be ordered is 175.

Solution $\boxed{\text{The Statement is FALSE}}$

It is given that

$D =$ Monthly demand $\times 12$

$D = 1000 \times 12$

$D = 12000$ units per year.

$C_o = 50$ per order

$C = 20$ per unit

$C_h = 1 + 10\%$ of C

$= 1 + \left(\frac{10 \times 20}{100}\right)$

$= 1 + 2 = 3$

Economic lot size

$$Q^* = \sqrt{\frac{2 D C_o}{C_h}} = \sqrt{\frac{2 \times 12000 \times 50}{3}}$$

$Q^* = 632.45 \approx 632.$

Question No 2 Use the penalty (Big M) method to solve the following LP problem:

Minimize $z = 5x_1 + 3x_2$

Subject to the constraints

$$2x_1 + 4x_2 \leq 12$$

$$2x_1 + 2x_2 = 10$$

$$5x_1 + 2x_2 \geq 10$$

$$x_1, x \geq 0$$

Solution :→ This problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate:

1. As the constraint→1 is of type '≤' we should add slack variable S_1
2. As the constraint→2 is of type '=' we should add artificial variable A_1
3. As the constraint→3 is of type '≥' we should subtract surplus variable S_2 and artificial variable A_2

By adding surplus variable S_1, S_2 the given LPP can be reformulated as:-

$$\text{Max } z' = -5x_1 - 3x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$$

Subject to,

$$2x_1 + 4x_2 + S_1 = 12$$
$$2x_1 + 2x_2 + A_1 = 10$$
$$5x_1 + 2x_2 - S_2 + A_2 = 10$$
$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

An initial basic solution is given by

$$x_1 = x_2 = 0, \; S_1 = 12, \; A_1 = 10, \; A_2 = 10$$

writing in matrix form $AX = B$

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & S_1 & S_2 & A_1 & A_2 \end{array} \\ \begin{pmatrix} 2 & 4 & 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 5 & 2 & 0 & -1 & 0 & 1 \end{pmatrix} \end{array} \begin{pmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \\ A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 12 \\ 10 \\ 10 \end{pmatrix}$$

			C_j	-5	-3	0	0	$-M$	$-M$	
	C_B	B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	R/R X_B/x_i
R_1 –	0	S_1	12	2	4	1	0	0	0	$12/2 = 6$
R_2 –	$-M$	A_1	10	2	2	0	0	1	0	$10/2 = 5$
R_3	$-M$	A_2	10	5	2	0	-1	0	1	$10/5 = 2$
	$Z_j - C_j$			$5-7m$	$3-4m$	0	M	0	0	

Calculating $\boxed{z_j - c_j = C_B x_j - c_j}$

① $z_1 - c_1 = C_B x_1 - c_1$
$= (0\times2 - M\times2 - M\times5) - (5) = 5 - 7M$

② $z_2 - c_2 = C_B x_2 - c_2$
$= (0\times4 - M\times2 - M\times2 - (-3) = 3 - 4M$

③ $z_3 - c_3 = C_B x_3 - c_3$
$= (0\times1 - M\times0 - M\times0) - 0 = 0$

④ $z_4 - c_4 = C_B S_2 - c_4$
$= (0\times0 - M\times0 - M\times(-1)) - 0 = M$

⑤ $z_5 - c_5 = C_B A_1 - c_5$
$= (0\times0 - M\times1 - M\times0) - (-M) = 0$

⑥ $z_6 - c_6 = C_B A_2 - c_6$
$= (0\times0 - M\times0 - M\times1) - (-M) = 0$

Since some of $z_j - c_j \leq 0$, the current feasible solution is not optimum choose the most negative $z_j - c_j = 5 - 7M$.

$\therefore$ x_1 variables enters the basis and the artificial variable A_2 leaves the basis.

		C_j	-5	-3	0	0	$-M$	$-M$	
C_B	B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	
0	S_1	2	0	2	1	0	-1	0	
$-M$	A_1	3	0	-1	-1	-2	-1	0	
-5	x_1	2	1	$2/5$	0	$-7/5$	0	$1/5$	
$Z_j - C_j$			0	$M+1$	M	$2M+7$	$2M$	$M-1$	

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - (R_3 - 3)$

$R_3 \to R_3/5$

① $Z_1 - C_1 = C_B X_1 - C_1$
$= (0\times0 - M\times0 - 5\times1) + 5 = 0$

② $Z_2 - C_2 = C_B X_2 - C_2$
$= (0\times2 - M(-1) - 5\times\frac{2}{5}) + 3 = M+1$

③ $Z_3 - C_3 = C_B X_3 - C_3$
$= (0\times1 - M(-1) - 5\times0) - 0 = M$

④ $Z_4 - C_4 = C_B X_4 - C_4$
$= (0\times0 - M(-2) - 5\times(-\frac{7}{5})) - (-M) = 2M+7$

⑤ $Z_5 - C_5 = C_B X_5 - C_5 = (0\times-1 - M(-1) - 5\times0) + M = 2M$

⑥ $Z_6 - C_6 = C_B X_6 - C_6 = (0\times0 - M\times0 - 5\times\frac{1}{5}) + M = M-1$

Since all $z_j - c_j \geq 0$, the solution is optimum and is given by

$$x_1 = 2, \; x_2 = 0$$

but also we noticed that the artificial variable appears as a basic variable and that too not at the zero level as $A_1 = 3$ and is non-zero.

Therefore the given LPP does not posses any feasible solution.

Question No3 A company has three production facilities S_1, S_2 and S_3 with production capacity of 7, 9 and 18 units (in 100s) per week of a product, respectively. These units are to be shipped to four warehouses D_1, D_2, D_3 and D_4 with requirements of 5, 6, 7 and 14 units (in 100s) per week respectively. The transportation costs (in Rs) per unit between factories to warehouses are given in the table below:

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Obtain optimal solution by the MODI method.

Solution:- Total Supply/capacity $= 7+9+18 = 34$

Total Demand $= 5+8+7+14 = 34$

The problem is balanced transportation problem as the total supply is equal to total demand, we find the initial basic feasible solution as given in the following table:

let the penalty in VAM's method is denoted by P_1, P_2, P_3 and P_4. Applying VAM method for optimality.

	D_1	D_2	D_3	D_4				P_1	P_2	P_3	P_4	P_5
S_1	5 19			2 10	~~7~~	~~2~~	0	9	9	40	40	–
S_2	70	30	7 40	2 60	~~9~~	~~7~~	0	10	20	20	20	20
S_3	40	8 8	70	10 20	~~18~~	~~10~~	0	12	20	50 ←	–	–
	~~5~~ 0	~~8~~ 0	~~7~~ 0	~~14~~ ~~4~~ ~~2~~ 0								
P_1	21	22↑	10	10								
P_2	21↑	–	10	10								
P_3	–	–	10	10								
P_4	–	–	10	50↑								
P_5	–	–	40	60↑								

Conditions for performing optimality test.

1. It contains exactly $m+n-1$ allocations where m and n represents the number

of rows and columns respectively of the transportation table.

2. these allocations are independent.

∴ In our case there are 6 positive independent allocations which are equal to $m+n-1$.

This ensures that the solution is non-degenerate basic feasible solution.

Transportation cost $= 19\times5 + 10\times2 + 40\times7 + 60\times2 + 8\times8 + 20\times10$

$= 95 + 20 + 280 + 120 + 64 + 200$

$=$ Rs. 779.

Hence the optimality test can be performed.

Let us apply the optimality test.

Optimality Test By MODI METHOD

Now, we perform optimality test by applying the MODI method.

Writing the cost matrix for only allocated cells.

Table 1
Cost matrix

19			10	u_1
		40	60	u_2
	8		20	u_3
v_1	v_2	v_3	v_4	

let us denote the row number by u_1, u_2, u_3 and column number by v_1, v_2, v_3 and v_4 such that

$u_1 + v_1 = 19$, $u_1 + v_4 = 12$, $u_2 + v_3 = 40$

$u_2 + v_4 = 60$, $u_3 + v_2 = 8$, $u_3 + v_4 = 20$

* Taking $u_1 = 0$,
from above equation $v_1 = 19$, $v_4 = 12$

$u_2 = 60 - 12 = 48$ $\quad v_3 = 40 - 48 = -8$ $\quad u_3 = 20 - 12 = 8$

$v_2 = 8 - 8 = 0$

Using the above values in the table 1 using $C_{ij} = u_i + v_j$ and puts dots in the already filled cells so that these cells are not considered again.

•	0	−8	•	$u_1 = 0$
67	48	•	•	$u_2 = 48$
27	•	0	•	$u_3 = 8$
$v_1 = 19$	$v_2 = 0$	$v_3 = -8$	$v_4 = 12$	

Now, Subtracting these value from the corresponding value of the original cost matrix. we have the net evaluations

i.e $\Delta_{ij} = C_{ij} - (u_i + v_j)$

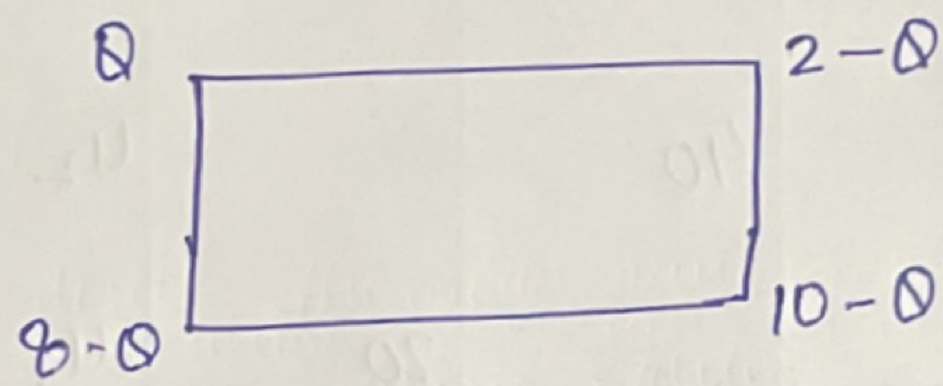
•	30 − 0 = 30	50 + 8 = 58	•
70 − 67 = 3	30 − 48 = −18	•	•
40 − 27 = 13	•	70 − 0 = 70	•

In the above table,

Cell (P_2, D_2) has the most negative opportunity cost.

Therefore, the transportation cost can be reduced by making allocation to this unoccupied cell. This means that if one unit is shifted to this unoccupied cell through the closed loop formed, beginning from this cell.

Closed loop for cell (P_2, D_2)



Maximum no. of units can be allocated to cell (P_2, D_2)

$$Q = \min \begin{cases} \text{the no. of units } (P_2, D_4) = 2 \\ \text{the no of units } (P_3, D_2) = 8 \end{cases} = 2$$

So with the improved allocations:-

19 (6)	30	50	10 (2)
70	30 (2)	40 (7)	60
40	8 (8)	70	20 (10)

Thus, the total cost of transportation for this set.

$= (19\times5) + (2\times10) + (30\times2) + (40\times7) + (8\times8) + (20\times10)$

$= 95 + 20 + 60 + 280 + 64 + 200$

$= 719.$

Let us apply the optimality test to the improved solution.

19			10	u_1
	30	40		u_2
	8		20	u_3
v_1	v_2	v_3	v_4	

Let us denote the row numbers by u_1, u_2, u_3 and column numbers v_1, v_2, v_3 and v_4 such that

$u_1 + v_1 = 19$, $u_1 + v_4 = 10$, $u_2 + v_2 = 30$

$u_2 + v_3 = 40$, $u_3 + v_2 = 8$, $u_3 + v_4 = 20$

Taking $u_1 = 0$, we have $v_1 = 19$, $v_4 = 10$

Also, $u_3 = 20 - 10$, $v_2 = 8 - 10$
$= 10$ $\quad = -2$

$u_2 = 30 + 2 = 32$, $v_3 = 40 - 32 = -8$

Using these value, we fill all the unoccupied cells. $\boxed{C_{ij} = u_i + v_j}$

	$v_1 = 19$	$v_2 = -2$	$v_3 = -8$	$v_4 = 10$
$u_1 = 0$	•	-2	-8	•
$u_2 = 32$	51	•	•	42
$u_3 = 10$	29	•	2	•

Now, subtracting these value from the corresponding values of the original matrix.

•	32	58	•
19	•	•	
11	•	68	•

Since none of net evaluation is negative, this solution is optimal. Thus the total minimum transportation cost is

$\boxed{= \text{Rs. } 719}$

Question No 4 Four professor are capable of teaching any one of four different courses. Class preparation time in hours for different topics varies from professor to professor and is given in the table below:

Professor	A	B	C	D
Linear Programming	2	15	13	4
Queuing theory	10	4	14	15
Transportation Problem	9	14	16	13
Regression Analysis	7	8	11	9

Each professor is assigned only one course. Determine an assignment schedule so as to minimise the total course preparation time for all course.

Solution :→

$\boxed{\text{STEP 1}}$:- Subtract the minimum element of each row from every element of that row.

row	A	B	C	D
1	0	13	11	2
2	6	0	10	11
3	0	5	7	4
4	0	1	4	2

Step 2 :→ let us now subtract the minimum element of each column from every element of that column in the resulting matrix.

	A	B	C	D
1	0	13	7	0
2	6	0	6	9
3	0	5	3	2
4	0	1	0	0

Step 3 :→ Starting from first row onward, we draw a rectangle around the 0 in each row having a single zero and cross all other zeros.

	A	B	C	D
1	~~0~~	13	7	0
2	6	$\boxed{0}$	6	9
3	$\boxed{0}$	5	3	2
4	~~0~~	1	0	0

Step 4 :→ In the first and fourth row, there is no single zero. Hence we move column wise.

	A	B	C	D
1	~~0~~	13	7	$\boxed{0}$
2	6	$\boxed{0}$	6	9
3	$\boxed{0}$	5	3	2
4	~~0~~	1	$\boxed{0}$	~~0~~

The assignment jobs are made on the basis of the cells corresponding to the zeros around which rectangles have been drawn. Therefore the optimum solution for the problem is

$$\boxed{1 \rightarrow D,\ 2 \rightarrow B,\ 3 \rightarrow A,\ 4 \rightarrow C}$$

The minimum time taken in developing the total course preparation is

$$= 90 + 90 + 110 + 80$$

$$= 370 \text{ hours.}$$

Question No5 In a railway marshalling yard, goods trains arrive at a rate 36 trains per day. Assuming that the inter-arrival and service time distributions both follow exponential distribution with an average of 30 minutes, calculate the following

(i) Traffic intensity

(ii) The mean queue length

(iii) Probability that the queue size exceeds.

Solution We are given that

Arrival Rate $(\lambda) = 36$ trains/day

Service Rate $(\mu) = 1$ per 30 minutes.

$= \frac{60}{30}$ per hour $= \frac{60}{30} \times 24$ per day

$= 48$ trains/day

(i) Traffic intensity

$$\boxed{\text{Traffic intensity } (\rho) = \frac{\lambda}{\mu}}$$

$$\rho = \frac{36}{48} = 0.75$$

(ii) Mean length of the system

$$\boxed{L_s = \frac{\lambda}{\mu - \lambda}}$$

$$L_s = \frac{36}{48-36} = \frac{36}{12} = 3 \text{ trains.}$$

(iii) Probability that the queue size exceeds t.

In the question it is not given so I am assuming an integer 't'

i.e $P(n>t) = \rho^{t+1} = (0.75)^{t+1}$

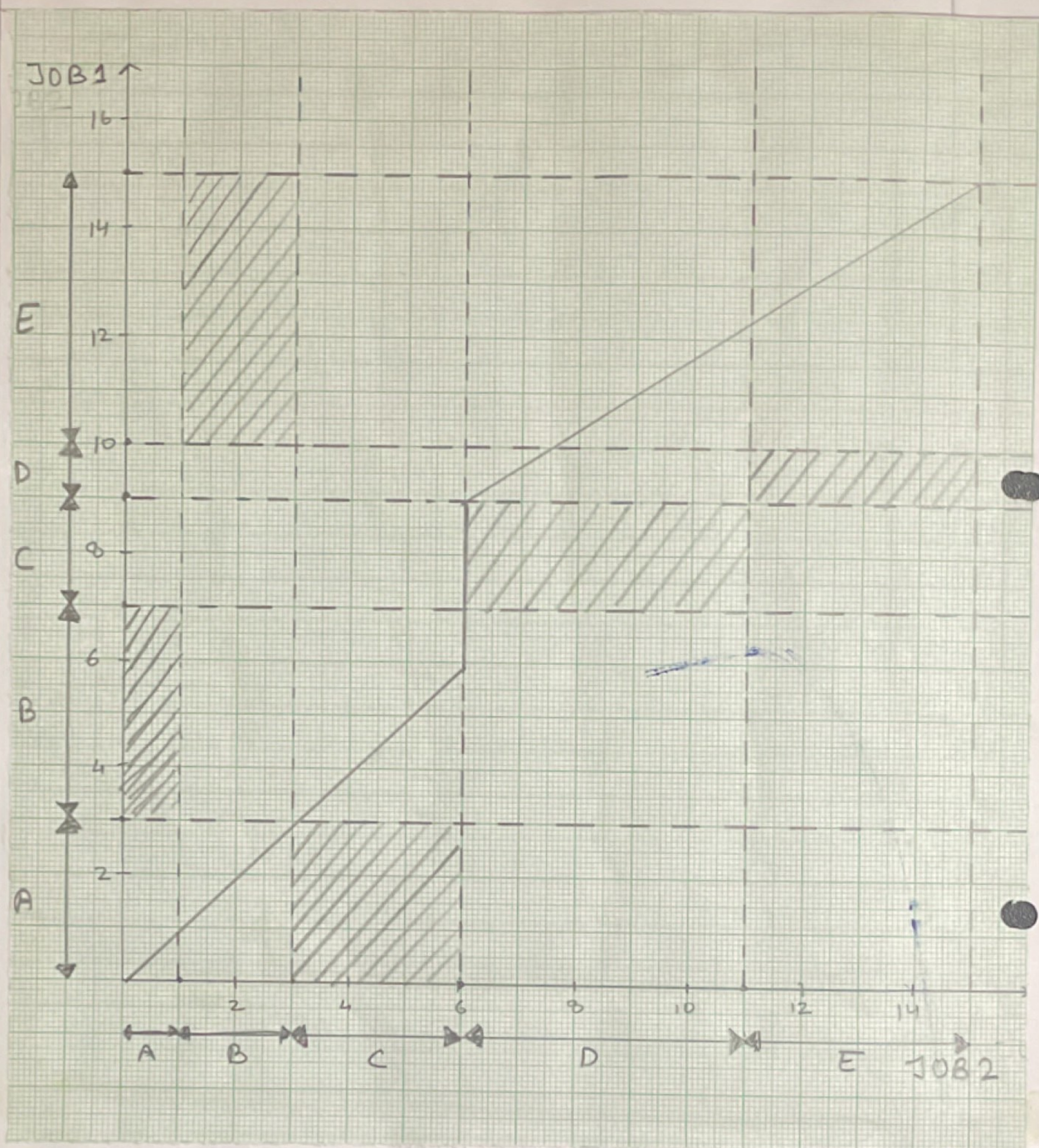
Also, lets $t = 11$

$$\boxed{\rho^{11+1} = (0.75)^{11+1}}$$

Question No 6 Using the graphical method to minimise the time required to process Job 1 and Job 2 on five machines A, B, C, D and E. find the minimum elapsed times an idle times to complete both jobs.

Job 1	Sequence	A	B	C	D	E
	Time (in hrs)	1	2	3	5	4
Job 2	Sequence	C	A	D	E	B
	Time (in hrs)	3	4	2	1	5

Solution:→ Let us draw a set of axes at right angle to each other where the x-axis represents the time of job 1 on different machine while job 2 remains idle and y axis represents the processing time of job 2 while job 1 remains idle. We draw vertical / horizontal lines through processing times. and shade the common area for each machine.



Elapsed time obtained by adding the idle time for Job 1 to its processing time is (15+3)=18 hours.

Elapsed time obtained by adding the idle time for job2 to its processing time is (15+0) =15 hours

Question No 7 A firm wants to know whether there is any linear relationship between the sales (X) and its yearly revenue (Y). The records for 10 years were examined and the following results were obtained.

$\sum X = 265$, $\sum Y = 27.73$, $SS_X = 285.6$,

$SS_Y = 6.978$ and $S_{XY} = 57.456$.

(a) fit a regression line taking Y as the dependent variable and X as the independent variable.

(b) Test whether the sales have any effect on revenue at 5% level of significance.

(c) Comment on the goodness of fit of the regression line.

Solution :→ From the given data, we get the following results

$\sum X = 268$, $\sum Y = 27.73$, $SS_X = 485.6$

$SS_Y = 6.978$, $SS_{XY} = 57.456$ and $n = 10$.

(i) Put the value in regression equation

$$\boxed{\hat{b} = \frac{SS_{XY}}{SS_X}}$$

$$\hat{b} = \frac{57.456}{285.6} = 0.201$$

$$\boxed{\hat{\sigma}^2 = \frac{SS_y - \frac{(SS_{xy})^2}{SS_x}}{n-2}}$$

$$\hat{\sigma}^2 = \frac{6.978 - \frac{(57.456)^2}{485.6}}{10-2}$$

$$= \frac{0.17}{8} = 0.021$$

$$\boxed{\hat{\sigma}^2 = 0.021}$$

$$\boxed{\hat{a} = \bar{Y} - \hat{b}\bar{X}}$$

$$\hat{a} = 2.773 - 0.201 \times 26.8$$

$$\boxed{\hat{a} = -2.6138}$$

Thus, the fitted line is

$$\boxed{\hat{Y} = -2.613 + 0.201X}$$

(iii) We have to test

$H_0 : b = 0$ against $H_1 : b \neq 0$

Equation given:→

$$\boxed{t = \frac{\hat{b} - 0}{\sqrt{\hat{\sigma}^2 / SS_x}}}$$

$$= \frac{0.201 - 0}{\sqrt{0.021/485.6}}$$

$$= \frac{0.201}{\sqrt{0.0000432}} = \frac{0.201}{0.0065}$$

$$t = 30.92$$

where the tabulated value of $t_{0.025}$ on 8 d.f is 2.306.

Since $|t| > 2.306$, the null hypothesis is rejected. This means that the sales force X has a significant effect on sales revenue Y.

(iii) Correlation Coefficient r

$$\boxed{r = \frac{SS_{xy}}{\sqrt{SS_x \times SS_y}}}$$

$$r = \frac{57.456}{\sqrt{205.6 \times 6.970}}$$

$$r = \frac{57.456}{58.210}$$

$$r = 0.987$$

Hence the coefficient of determination $R^2 = r^2 = 0.974$. Thus 97% of the variability is accounted for by x. Hence the regression model gives a very good fit.

Question No 8 A researcher is interested in developing a linear model for the electricity consumption of a household having an AC (1.5 ton) so that she can predict the electricity consumption. For this purpose, she selects 25 houses and records the electricity consumption (in KWh), size of house (in square feet) and AC hours for one month during summers. The results obtained are

$\hat{B}_0 = 22.38$, $SS(B_0) = 12526.08$, $SS(B_0, B_1, B_2) = 18079.0$

$\hat{B}_1 = 1.6161$, $SS(B_0, B_1) = 17908.47$

$\hat{B}_2 = 0.0144$, $SS(B_0, B_2) = 17125.23$

$\hat{\sigma}^2 = 10.53$, $SE(\hat{B}_1) = 0.17$, $SE(\hat{B}_2) = 0.0035$

Build a regression model by selecting appropiate regressor in the model using the stepwise selection method.

Solution:-

Using the results given above, we calculate the extra sum of squares due to x_1 after accounting for x_0:

$$\boxed{SS(B_1 | B_0) = SS(B_0, B_1) - SS(B_0)}$$

$= 17908.47 - 12526.08$

$= 5382.39$

- The extra sum of squares due to X_2 after accounting for X_0 is

$$\boxed{SS(B_2 | B_0) = SS(B_0, B_2) - SS(B_0)}$$

$= 17125.23 - 12526.08$

$= 4599.15$

- The extra sum of squares due to X_2 after accounting X_0 and X_1

$$\boxed{SS(B_2 | B_0, B_1) = SS(B_0, B_1, B_2) - SS(B_0, B_1)}$$

$= 18079.00 - 17908.47$

$= 170.53.$

- The extra sum of squares due to X_1 and X_2 after accounting for X_0 is

$$\boxed{SS(B_1, B_2 | B_0) = SS(B_0, B_1, B_2) - SS(B_0)}$$

$= 18079.00 - 17125.23$

$= 953.77.$

Stepwise Selection Method.

Here we start with the model with only the intercept i.e

$$Y = B_0 + e.$$

and try to see if regressor variables can be added. For this, we fit two models with one variable.

$$Y = B_0 + B_1 X_1 + e,$$
$$Y = B_0 + B_2 X_2 + e$$

According to the calculations and the model fitted by including the extra variables X_1 over X_0 separately, one at a time.

$$SS(B_1|B_0) = 5382.39$$
$$SS(B_2|B_0) = 4599.15$$

Since the extra sum of squares due to X_1 over X_0 i.e, $SS(B_1|B_0)$ is larger than the extra sum of squares due to X_2 over X_0 i.e $SS(B_2|B_0)$ we now test the hypothesis

$$H_0: B_1 = 0 \text{ against } H_1: B_1 \neq 0$$

and get the t-statistic value as follows:

$$\boxed{t = \frac{\hat{B}_1}{SE(\hat{B}_1)}} = \frac{1.6161}{0.17} = 9.15$$

This value is significant in comparision with the tabulated value $t_{(0.5, 22)} = 2.074$. Thus, the first variable to enter in the model is X_1. Hence at this stage the selected model is

$$Y = B_0 + B_1 X_1 + e.$$

We now check whether the variable X_2 can be included in the model. The extra residual sum of squares due to X_2, after including X_1 is

$$SS(B_2 | B_0, B_1) = 170.53$$

Then we calculate the value of F-statistic

$$F = \frac{170.53}{10.53} = 16.19$$

which is significant. We also test the hypothesis $H_0: B_2 = 0$ against $H_1: B_2 \neq 0$

and get the t-statistic value

$$t = \frac{\hat{B_2}}{SE(\hat{B_2})} = \frac{0.0144}{0.0035} = 4.11$$

which is significant in comparision with the tabulated value $t_{(0.5,22)} = 2.074$. Hence it is worthwhile to include X_2 after X_1 has been included in the model.

At this stage we have to check whether X_1 can be excluded from the model.

For this we calculate the sum of squares due to X_1 over X_0 and X_2, i.e

$SS(B_1 | B_0, B_2)$ as follows:-

$$SS(B_1 | B_0, B_2) = 953.77$$

We now have to test whether this is significant or not. In case it is not significant, we can exclude X_1 from the model. We test its significance against

$\hat{\sigma}^2 = 10.53$,

$$F = 953.77/10.53 = 90.58$$

which is significant. This suggests that we cannot delete any term from the model.

Hence the selected model is

$$\boxed{y = B_0 + B_1 X_1 + B_2 X_2 + e}$$

Question No 9 The following table represents the sales (in thousand) of mobile sets of a shop for 16 quarters over four years.

Year	Quarter			
	Q_1	Q_2	Q_3	Q_4
2011	554	590	616	653
2012	472	501	521	552
2013	501	531	553	595
204	403	448	460	480

(a) Compute the seasonal indicies for four quarters by simple average method.

(b) Obtain deseasonlised values.

Solution

Year / Quarters	2009 (a)	2010 (b)	2011 (c)	2012 (d)	Total(T) = a+b+c+d	Quataly Avg(Q) = T/4	Seasonal index = Q/526.9
I	554	472	301	403	1930	482.5	91.51
II	590	501	531	448	2070	517.5	98.21
III	616	521	553	460	2150	537.5	102.01
IV	653	552	595	480	2280	570.0	108.18
Total				8430	21075		400
Average				2107.5	526.9		100

We are given quarterly prices of a commodity.

1. Calculate the quater wise total of the production amount in 4 years.
2. Calculate averages of all 4 quarters.

Quarter I :- 482.5

Quarter II :- 517.5

Quarter III :- 537.5

Quarter IV :- 570.

3. After calculating the quarterly averages, we calculate the average of all quarterly averages.

$$\bar{\bar{y}} = \frac{1}{4}(482.5 + 517.5 + 537.5 + 570)$$

$$\boxed{\bar{\bar{y}} = 526.9}$$

4. We calculate the seasonal indices

Seasonal index Quarter I $= \frac{482.5}{526.9} \times 100 = 91.57$

Quarter II $= \frac{517.5}{526.9} \times 100 = 98.21$

Quarter III $= \frac{537.5}{526.9} \times 100 = 102.01$

Quarter IV $= \frac{570}{526.9} \times 100 = 108.18$

(b) obtaining Deseonalised value.

$$\boxed{\text{Deseaonalised } z = \frac{Y_t}{S_t} = T_t \times C_t \times I_t}$$

Year X_t		Y_t	S_t	Deseasonalised values $Z_t = \frac{Y_t}{S_t} \times 100$
2013	Q1	554	91.51	$(554/91.5) \times 100 = 605.3$
	Q2	590	98.21	$(590/98.21) \times 100 = 600.7$
	Q3	616	102.01	$(616/102.01) \times 100 = 603.8$
	Q4	653	108.18	$(653/108.18) \times 100 = 603.6$
2012	Q1	472	91.51	$(472/91.51) \times 100 = 515.7$
	Q2	501	98.21	$(501/98.21) \times 100 = 510.13$
	Q3	521	102.01	$(521/102.01) \times 100 = 510.73$
	Q4	552	108.18	$(552/108.18) \times 100 = 510.2$
2013	Q1	501	91.51	$(501/91.51) \times 100 = 547.4$
	Q2	531	98.21	$(531/98.21) \times 100 = 540.6$
	Q3	553	102.01	$(553/102.01) \times 100 = 520.5$
	Q4	595	108.18	$(595/108.18) \times 100 = 550.0$
2014	Q1	403	91.51	$(403/91.51) \times 100 = 440.3$
	Q2	448	98.21	$(448/98.21) \times 100 = 456.16$
	Q3	460	102.01	$(460/102.01) \times 100 = 450.9$
	Q4	480.	108.18	$(480/108.18) \times 100 = 443.7$

For calculating the deseasonalised values, we first calculate the seasonal indices for the given data.

We have already calculated the seasonal indices in the first section of this question.

Deseasonalised values are also calculated by dividing the ~~same~~ time series value by the corresponding indicies S_t.

Question No 10 Seven successive observations on a stationary time series are as follows:

12, 14, 13, 10, 15, 12, 15

(a) Calculate auto-covariances C_0, C_1, C_2, C_3 and C_4.

(b) Calculate auto-correlation coefficients r_1, r_2, r_3 and r_4.

(c) Plot the correlogram.

Solution:—

$$\bar{Y} = \frac{12+14+13+10+15+12+15}{7} = 13$$

SNo.	Y	Y^2	$\bar{Y}=13$ $(Y_t-\bar{Y})$	$(Y_t-\bar{Y})\times(Y_{t+1}-\bar{Y})$	$(Y_t-\bar{Y})\times(Y_{t+2}-\bar{Y})$	$(Y_t-\bar{Y})\times(\bar{Y}_{t+3}-\bar{Y})$	$(Y_t-\bar{Y})\times(\bar{Y}_{t+4}-\bar{Y})$
1	12	144	12−13 = −1				
2	14	196	14−13 = 1	−1×(14×13) = −182			
3	13	169	13−13 = 0	1×(13×13) = 169	−1×(13×13) = −169		
4	10	100	10−13 = −3	0×(10×13) = 0	1×(10×13) = 130	−1×(10×13) = −130	
5	15	225	15−13 = 2	−3×(15×13) = −585	0×(15×13) = 0	1×(15×13) = 195	−1×(15×13) = −195
6	12	144	12−13 = −1	2×(12×13) = 312	−3×(12×13) = −468	0×(12×13) = 0	1×(12×13) = 156
7	15	225	15−13 = −2	−1×(15×13) = −195	2×(15×13) = 390	−3×(15×13) = −585	0×(15×13) = 0
Total	91	1203		−481	−117	−520	−39

(a) Calculating autocovariance.

For $K=0$ $$\boxed{C_0 = \frac{\sum_{t=1}^{N-K} (y_i - \bar{y})^2}{N} = \frac{\sum y_i^2 - N\bar{y}^2}{N}}$$

$$C_0 = \frac{1203 - 7 \times 13^2}{7} = \frac{1203 - 1183}{7}$$

$$\boxed{C_0 = 2.857}$$

For $K=1$ $$\boxed{C_1 = \frac{\sum_{t=1}^{6} (Y_t - \bar{Y})(Y_{t+1} - \bar{Y})}{6}}$$

$$C_1 = \frac{-481}{6} = -68.71$$

For $K=2$ $$\boxed{C_2 = \frac{\sum_{t=1}^{5} (Y_t - \bar{Y})(Y_{t+2} - \bar{Y})}{5}} = \frac{-117}{5} =$$

for $K=3$ $$\boxed{C_3 = \frac{\sum_{t=1}^{4} (Y_t - \bar{Y})(Y_{t+3} - \bar{Y})}{4}} = \frac{-520}{4} =$$

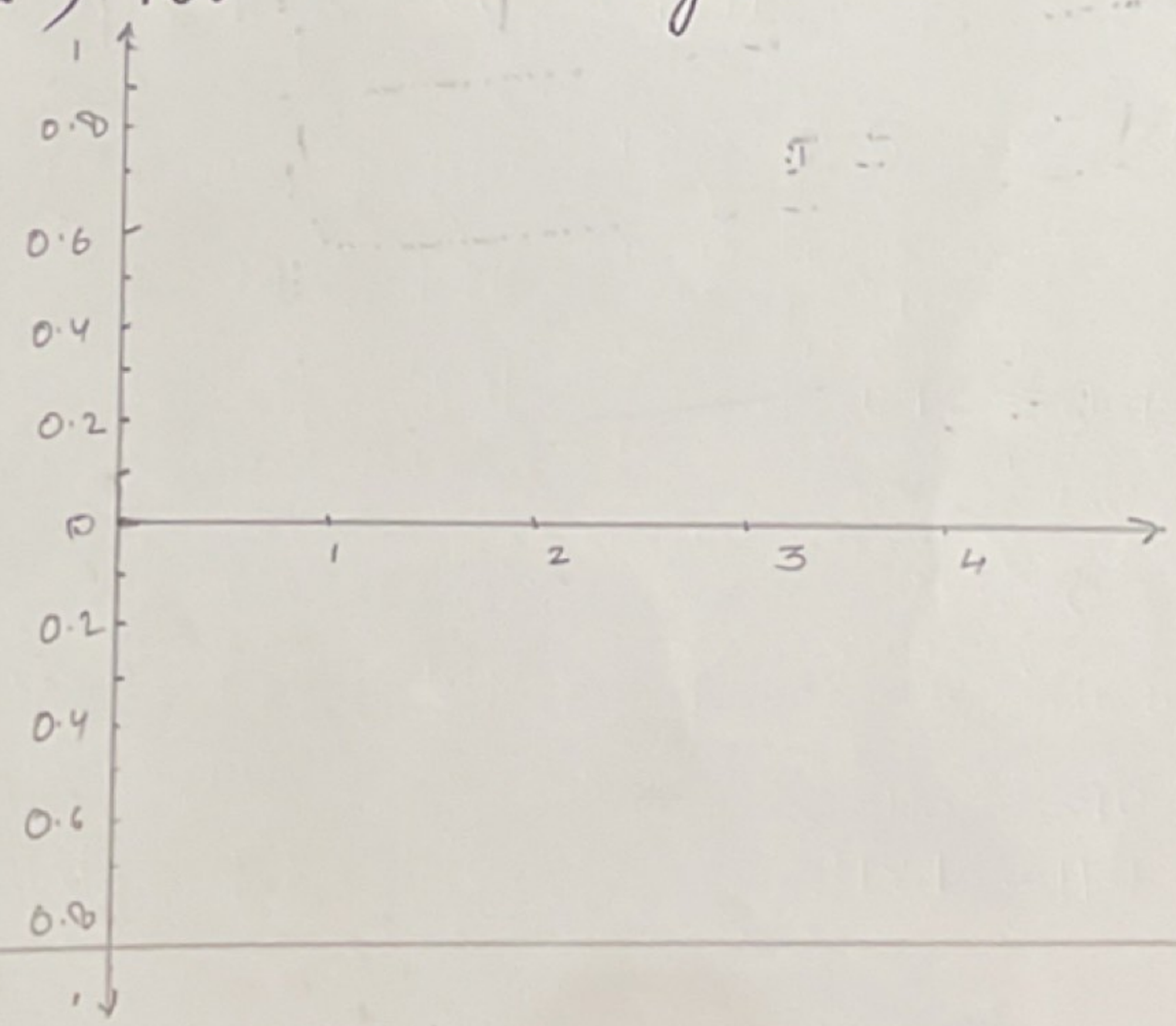
for $K=4$ $$\boxed{C_4 = \frac{\sum_{t=1}^{3} (Y_t - \bar{Y})(Y_{t+4} - \bar{Y})}{3}} = \frac{-39}{3} =$$

(b) Calculating auto-correlation.

* $$\boxed{r_1 = \frac{C_1}{C_0}} = \frac{-68.71}{\text{[illegible]}}$$

$$r_2 = \frac{C_2}{C_0}$$

$$r_3 = \frac{C_3}{C_0}$$

$$r_4 = \frac{C_4}{C_0}$$

(iii) Plot the correlogram.