# Solution of the Legible Math Problem

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## System of Equations

$$\begin{aligned}
z\,e\,r\,o &= 0,\\
o\,n\,e &= 1,\\
t\,w\,o &= 2,\\
t\,h\,r\,e\,e &= 3,\\
f\,o\,u\,r &= 4,\\
f\,i\,v\,e &= 5,\\
s\,i\,x &= 6,\\
s\,e\,v\,e\,n &= 7,\\
e\,i\,g\,h\,t &= 8,\\
n\,i\,n\,e &= 9,\\
t\,e\,n &= 10,\\
e\,l\,e\,v\,e\,n &= 11,\\
n\,e\,g\,a\,t\,i\,v\,e &= -1.
\end{aligned}$$

**1. From the first equation:**

$$z\,e\,r\,o = 0$$

$$\text{Since } e, r, o \neq 0, \text{ we conclude } z = 0.$$

**2. Introduce free parameters:**

$$o,\ e,\ r,\ v$$

be four arbitrary nonzero real numbers.

**3. From the equation**

$$o\,n\,e = 1 \quad\Longrightarrow\quad n = \frac{1}{oe}.$$

**4. From the equation**

$$t\,e\,n = 10 \quad\Longrightarrow\quad t = 10\,o.$$

**5. From the equation**

$$(10o)\,w\,o = 2 \quad\Longrightarrow\quad w = \frac{1}{5\,o^2}.$$

**6. From the equation**

$$(10o)\,h\,r\,e^2 = 3 \quad\Longrightarrow\quad h = \frac{3}{10\,o\,r\,e^2}.$$

**7. From the equation**

$$\left(\frac{1}{oe}\right) i \left(\frac{1}{oe}\right) e = 9 \quad\Longrightarrow\quad i = 9\,o^2 e.$$

**8. From the equation**

$$f\,(9o^2e)\,v\,e = 5 \quad\Longrightarrow\quad f = \frac{5}{9\,o^2\,e^2\,v}.$$

**9. From the equation**

$$f\,o\,u\,r = 4 \quad\Longrightarrow\quad u = \frac{4}{f\,o\,r} = \frac{36\,o\,e^2\,v}{5\,r}.$$

**10. From the equation**

$$s\,e^2\,v\,n = 7 \quad\Longrightarrow\quad s = \frac{7\,o}{e\,v}.$$

**11. From the equation**

$$\left(\frac{7o}{e\,v}\right)(9o^2e)\,x = 6 \quad\Longrightarrow\quad x = \frac{2\,v}{21\,o^3}.$$

**12. From the equation**

$$e\,(9o^2e)\,g\left(\frac{3}{10\,o\,r\,e^2}\right)(10o)=8 \quad\Longrightarrow\quad g=\frac{8\,r}{27\,o^2}.$$

**13. From the equation**

$$e^3\,l\,v\,n=11 \quad\Longrightarrow\quad l=\frac{11\,o}{e^2\,v}.$$

**14. From the equation**

$$\left(\frac{1}{oe}\right)e^2\left(\frac{8r}{27\,o^2}\right)a\,(10o)\,(9o^2e)\,v=-1 \quad\Longrightarrow\quad a=-\frac{3}{80\,e^2\,r\,v}.$$

# General Solution Summary

$$\begin{array}{llll}
z=0, & o,e,r,v \text{ free parameters,} & & \\
n=\dfrac{1}{oe}, & t=10\,o, & w=\dfrac{1}{5\,o^2}, & h=\dfrac{3}{10\,o\,r\,e^2}, \\
i=9\,o^2e, & f=\dfrac{5}{9\,o^2\,e^2\,v}, & u=\dfrac{36\,o\,e^2\,v}{5\,r}, & s=\dfrac{7\,o}{e\,v}, \\
x=\dfrac{2\,v}{21\,o^3}, & g=\dfrac{8\,r}{27\,o^2}, & l=\dfrac{11\,o}{e^2\,v}, & a=-\dfrac{3}{80\,e^2\,r\,v}.
\end{array}$$

# Concrete Example

We choose

$$o=1,\quad e=1,\quad r=1,\quad v=1.$$

Then

$$\begin{array}{llll}
z=0, & & & \\
o=1, & e=1, & r=1, & v=1, \\
n=1, & t=10, & w=\dfrac{1}{5}, & h=\dfrac{3}{10}, \\
i=9, & f=\dfrac{5}{9}, & u=\dfrac{36}{5}, & s=7, \\
x=\dfrac{2}{21}, & g=\dfrac{8}{27}, & l=11, & a=-\dfrac{3}{80}.
\end{array}$$

## Why not Twelve?

**Theorem.** *The system of word-product equations admits solutions in real numbers, but it is* impossible *to extend it by*

$$t\,w\,e\,l\,v\,e = 12.$$

*Proof.* From the equations

$$o\,n\,e = 1 \quad \text{and} \quad z\,e\,r\,o = 0,$$

since $o, n, e \neq 0$ (else $o\,n\,e$ could not be 1), we must have

$$z = 0, \quad o = n = e = 1.$$

Next, from

$$t\,e\,n = 10 \quad \Longrightarrow \quad t \cdot 1 \cdot 1 = 10 \quad \Longrightarrow \quad t = 10,$$

and from

$$t\,w\,o = 2 \quad \Longrightarrow \quad 10\,w \cdot 1 = 2 \quad \Longrightarrow \quad w = \frac{1}{5}.$$

Likewise, from

$$e\,l\,e\,v\,e\,n = 11 \quad \Longrightarrow \quad 1 \cdot l \cdot 1 \cdot v \cdot 1 \cdot 1 = 11 \quad \Longrightarrow \quad l\,v = 11.$$

Hence the would-be twelve product is

$$t\,w\,e\,l\,v\,e = \big(t\big)\big(w\big)\big(e\big)\big(l\,v\big)\big(e\big) = 10 \times \frac{1}{5} \times 1 \times 11 \times 1 = 2 \times 11 = 22.$$

This shows that in any solution of the original system one necessarily has

$$t\,w\,e\,l\,v\,e = 22 \neq 12.$$

Therefore the equation $t\,w\,e\,l\,v\,e = 12$ cannot be satisfied simultaneously with the first eleven equations. $\square$