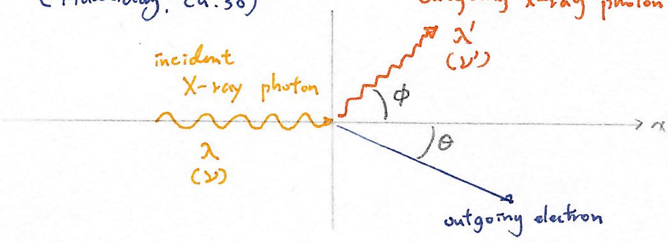
呂浩宇

General Physics (II) Quantum Physics

Compton scattering (Arthur Compton 1922)
(Halliday. ch.38)



He also verified Maxwell's prediction that light is EM wave.

discovered by Heinrich Hertz (1887)

$\boxed{\text{photoelectric effect experimentally implies that the energy of a photon is } h\nu}$

Planck constant

Assumption made by Max Planck in 1870s in the derivation of the formula to describe black body radiation.

energy conservation: (rest energy of the electron has been subtracted from the l.h.s. & r.h.s. of the equation)

$$\underbrace{h\nu}_{\text{energy of incoming photon}} = \overbrace{h\nu'}^{\text{energy of outgoing photon}} + \underbrace{mc^2(\gamma - 1)}_{\text{energy of outgoing electron}}, \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

v: speed of outgoing electron

momentum conservation:

horizontal: $$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + \gamma m v \cos\theta$$

vertical: $$0 = \frac{h\nu'}{c}\sin\phi - \gamma m v \sin\theta$$

There are 3 equations for us to elliminate 3 of the 5 independent variables $(\nu, \nu', v, \phi, \theta)$

Defining $\Delta\lambda \equiv \lambda' - \lambda$, with a little (unimportant) algebra, we got:

$$\underbrace{\Delta\lambda}_{\text{compton shift}} = \frac{h}{mc}(1 - \cos\phi)$$

If we place detectors at various outgoing angle ϕ, this theory predict that we should detect the scattered light at different wavelength.
Note that this theory is only true when the hypothesis of photons is true, which is distinct from the prediction of the classical EM wave propagation.

photons behave like particles. This is a problem!
Can particles behave like wave?

de Broglie (1923) : $\lambda = \dfrac{h}{|\vec{p}|}$ → verified by
(wavelength)

- L.H. Germer & C.J. Davisson (1927)
- G.P. Thomson

利用 X-ray Bragg diffraction 測量 crystal 晶面間距，再用電子來驗證電子的 diffraction pattern

粒子：侷限於小的空間中，具有一定的動量與能量。
其位置與速度隨時間之變化由 equation of motion 決定。

波：瀰漫於空間中之物理量，由波函數 $\psi(\vec{r}, t)$ 描述。
波函數隨時間的演化由 wave equation 決定。

General Physics (II) Quantum Physics

Experiments that show photons behave like particles.

1. Compton scattering (Arthur Compton 1922)

2. Blackbody radiation (the earliest development about QM, since 1859)
   ↳ even before we know the light is EM wave.

   (i) Before 1900, Gustav Kirchhoff

   Argued based on thermal physics, that the intensity of blackbody radiation only depends on the temperature T and frequency ν.

   (ii) mid-1880s, { Ludwig Boltzmann, Josef Stefan

   Argued that the energy density of blackbody emission u [$\mathrm{Joul\ cm^{-3}}$] is proportional to intensity. In addition, its value is

   $$\underbrace{u = \sigma}_{\text{Stefan-Boltzmann constant}} T^4 \quad \text{——— Stefan-Boltzmann law.}$$

   (iii) Wilhelm Wien

   Attempted to derive the blackbody intensity based on statistical physics, assuming that the photons are similar to classical gas particles.

   He obtained the Wien's law which is consistent with observations at short wavelength

   $$I_{\text{Wien}}(\lambda, T) = b\,\lambda^{-5} \exp(-a/\lambda T)$$

   ↳ b, a: constants to be determined experimentally.

   (iv) Early 1900, Max Planck.

   Assumed the walls in a blackbody chamber can only emit and absorb EM wave of frequency ν by energy in unit of $\underline{h\nu}$

   such energy units were named "quanta" by Planck

   Based on this assumption, Planck obtained the correct form of blackbody intensity by applying methods of statistical physics.

   $$I(\lambda, T) = \frac{b}{\lambda^5}\,\frac{1}{\exp(a/\lambda T) - 1}$$

   詮釋量子效應為 h 為有限大之結果. 若 $h \to 0$, 即光子能量單位可為無窮小, 則 blackbody radiation formula 趨近於 Rayleigh-Jeans 推導出的古典型式, 在短波長處積分發散

   (v) Einstein (1905)

   At a certain frequency, the energy of EM wave can only be integer times $h\nu$. This predicts the photoelectric effect (光電效應) that can be tested by further experiments.

3. Photoelectric effect (Heinrich Hertz, then Robert Millikan in 1904-1913)
   電子僅能吸收單一光子

   classical expectation {
   1. Accumulating the incoming radiative energy with time can eventually excite photoelectron.
   2. There can be a time lag between illumination and the ejection of photoelectron.

   actual case: {
   1. Capability of exciting photoelectron only relates to frequency, not intensity. Kinetic energy of photoelectron depends on frequency and the surface work-function.
   2. Ejection of photoelectron is instantaneous.

呂喆宇

General Physics (II) Quantum Physics.

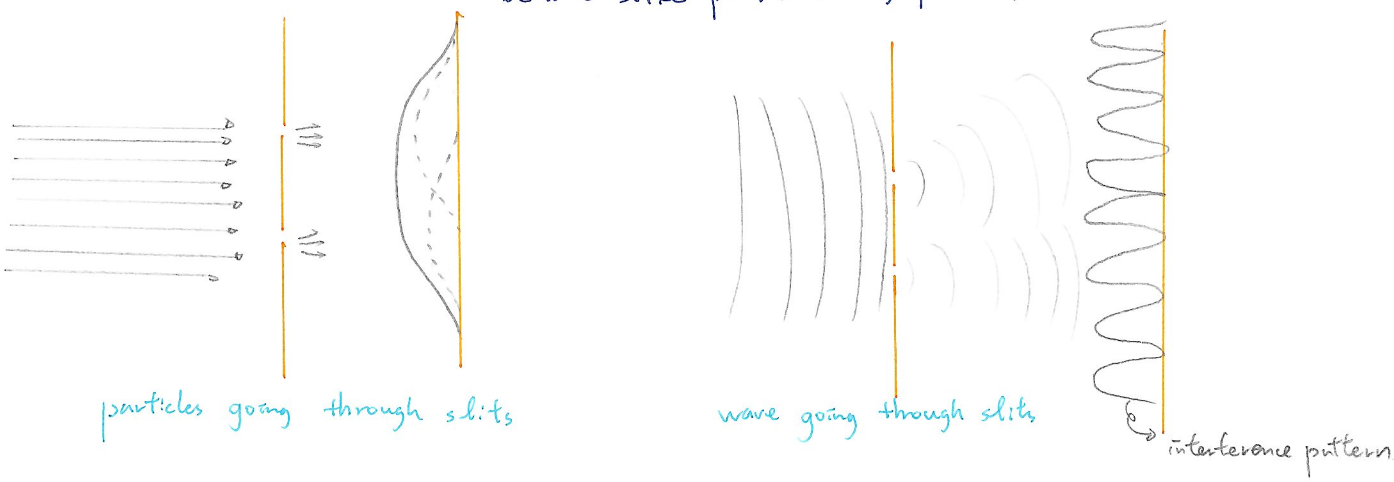
(此處講一下光的双狹縫實驗, Halliday ch. 35, 36)

Problem: if photon behave like particles, then how we understand the inteference of light in the double slit experiments?

(i) the interference pattern emerge even when there is only one photon at a time

(ii) when we try to change the aparatus such that we can know which slit the photon is going through, the interference pattern disappear.

The way we measure somehow "decide" whether light behave like particles (e.g. photons) or waves.



particles going through slits

wave going through slits

Question: can particle behaves like wave?

Louis de Broglie: matter has wave-like property. The wavelength is

$$\lambda = \frac{h}{|\vec{p}|}$$ de Broglie wavelength.

(i) C.J. Davisson & L.H. Germer, G.P. Thomson — observing the interference pattern of electrons scattered by planes in crystals.

(ii) slit experiments (see textbook) Tonomura et al. (1989)

Max Born's hypothesis: each photon is associated a wave ψ, called probability amplitude (机率幅)

電場正比於電子出現之機率幅. 机率正比於机率幅平方

In the EM wave, the $|E|^2$ determines the probability for a photon to appear. Can electrons be described by another sort of probability wave?

General Physics (II) Quantum Physics

physical Hilbert space : The space of functions that can be either normalized to unity, or to the Dirac delta function.

Postulate : (Max Born, Erwin Schrödinger 1926)

a function in the physical Hilbert space, s.t. there is a concept of probability conservation

Each electron is associated with a $\underbrace{\text{wave function } \psi(\vec{r},t)}_{\text{机率幅}}$ that obeys some $\underbrace{\text{linear}}$ equation. (i.e., Schrödinger equation)

可對比於光波問題中之電場，但 $\psi(\vec{r},t)$ 無相對應之古典場

linearity is a key to allow interference

ψ can be a complex function, with a complex conjugate ψ^*.

The probability of finding a electron at time t in a infinitesimal space $d^3\vec{r}$ is $\underbrace{\overbrace{|\psi(\vec{r},t)|^2}^{\text{机率密度}}\, d^3\vec{r}}_{\text{机率}}$

The electron must be somewhere in the space. So

$$\int_{\text{all space}} |\psi(\vec{r},t)|^2\, d^3\vec{r} = 1$$

Particle may be described as a ψ wave pulse that has a finite extension in space.

making partial t derivative on both side

$$\Rightarrow \int_{\text{all space}} \left(\frac{\partial \psi^*(\vec{r},t)}{\partial t}\psi(\vec{r},t) + \psi^*(\vec{r},t)\frac{\partial \psi(\vec{r},t)}{\partial t} \right) d^3\vec{r} = 0$$

a way to satisfy this equation at all time : requiring $\frac{\partial\psi}{\partial t}$ to be related to ψ in some way.

should still be linear.

related by the "Governing equation", which is the Schrödinger Eq. we are contructing.

建構 Ansatz : ψ 必須以類似電磁波的 sin, cos wave 型式干涉，且的時間微分要為 ψ 的 linear operation

最自然的猜法

先猜 ψ 的型式為 $\cos(kx-\omega t)$, $\sin(kx-\omega t)$ 的線性組合

$$\psi(x,t) = A[\cos(kx-\omega t) + i\sin(kx-\omega t)] = Ae^{i(kx-\omega t)}$$

角頻率，為頻率的2π倍

$$\frac{\partial\psi}{\partial t} = -i\omega\psi(x,t)$$

(what are k and ω ?)

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free electron (自由電子)
(non-relativistic)

wave number

plane wave : $k = \frac{2\pi}{\lambda}$ $\Rightarrow$ $k = 2\pi\left(\frac{p}{h}\right) = \frac{p}{\hbar}$, $\hbar \equiv h/2\pi$

de Broglie wave length $\lambda = \frac{h}{p}$

analogous to light (EM) wave, we expect the energy of the electron to be related to frequency ν by $E = h\nu = \frac{h}{2\pi}(2\pi\nu) = \hbar\omega$ $\Rightarrow$ $\omega = \frac{E}{\hbar}$

$\Longrightarrow$ plane-wave form of the electron wave function

$$\psi(x,t) = \underbrace{A}_{\text{normalization constant}} e^{\frac{i(px - Et)}{\hbar}}$$

For a non-relativistic electron, we expect $E = \frac{1}{2}\frac{p^2}{m}$

$$\Rightarrow \frac{\partial \psi}{\partial t} = -i\omega\psi = -i\frac{E}{\hbar}\psi = -i\frac{p^2}{\hbar\, 2m}\psi$$

Observing that $\frac{\partial^2 \psi}{\partial x^2} = -k^2\psi = -\frac{p^2}{\hbar^2}\psi = -\frac{i}{\hbar}2m\left(-i\frac{p^2}{\hbar\, 2m}\right)\psi = -i\frac{2m}{\hbar}\frac{\partial \psi}{\partial t}$

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi$$

time-indedepdent Schrödinger eqn. for free-electron.

electron in potential

modifying from $E = \frac{1}{2}\frac{p^2}{m}$ to $E = \frac{p^2}{2m} + V$

$$\frac{\partial \psi}{\partial t} = i\omega\psi = -i\frac{E}{\hbar}\psi = -i\frac{1}{\hbar}\left(\frac{p^2}{2m} + V\right)\psi$$

following the derivation as the free-electron case

$$\Rightarrow i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi$$