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General Physics (II) Basic Circuits

※串.並聯電阻,電容,及電感在實務應用中非常實用,計算原理推導相對簡單,有心往科学或公程性質職涯發展之同学應閱讀課本修習. 本普通物理課程僅介紹元件之物理原理.

Resistance, Capacitance, and Inductance
電阻 電容 電感

本節僅討論 steady state 之情形. (電位,電流等物理性質不隨時間明顯變化)

Definition of current: for an arbitrary plane, if charge dq pass through that plane in a unit time dt, then the current through that plane is defined as:

$$i = \frac{dq}{dt}$$

SI 制單位: 1 ampere = 1 A = 1 coulomb/s

When drawing a circuit,
- if the charge carriers carry positive charge, we draw arrows in the directions of charge carriers' motion.
- if the charge carriers carry negative charge (e.g. electrons), we draw arrows in the directions of charge carriers' motion.

Definition of current density: $i = \int \underbrace{\vec{J}}_{\text{current density}} \cdot d\vec{A}$

$$\vec{J} = \underbrace{(n}_{\text{number density of charge}}\underbrace{e)}_{1.602\cdot 10^{-19}\,C \text{ (positive) elementary charge}}\underbrace{\vec{v}_d}_{\text{drift velocity (漂移速率) of charge carriers}}$$

A. Resistance, Ohm's law, and Power in Electric circuits
在電路中以 ⩘⩘⩘ 表示　　歐姆定律

If we apply a potential difference V between two points on a conductor and detect the resulting current i, the resistance is defined as:

resistance: $\underbrace{R}_{\text{與外加電壓如何施加於待測導件(e.g. 連接處之接觸面積)}} = \dfrac{V \leftarrow \text{potential difference}}{i \leftarrow \text{current}}$

物理:形成單位電流所需要之電壓

SI 制單位: 1 ohm = 1 Ω = 1 volt per ampere = 1 V/A

Similarly, we can define:

resistivity: $\rho = \dfrac{E}{J}$ (ρ: resistivity)

物理:形成單位電流密度所需要之電場

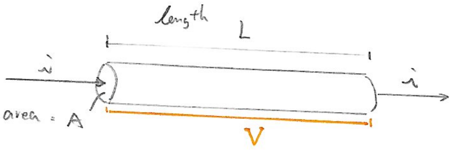
一般向量形式: $\vec{E} = \rho \vec{J}$

SI 制單位: $\dfrac{[E]}{[J]} = \dfrac{V/m}{A/m^2} = \dfrac{V}{A} m = \Omega \cdot m$

and we can correspondingly define conductivity: $\sigma = \dfrac{1}{\rho} \Rightarrow \vec{J} = \sigma \vec{E}$

物理:單位電場所能造成之電流密度

The relation between these quantities.



$E = V/L$, $J = i/A$

$$\rho = \frac{E}{J} = \frac{V/L}{i/A} = \frac{\overbrace{(iR)}^{V}A}{iL} \Rightarrow R = \rho\frac{L}{A}$$

給定 resistivity, 長度愈長電阻愈大, 截面積愈大電阻愈小

對於大部份導体, 溫度愈高, ρ 值愈大

銅的 resistivity 滿足線性經驗公式: $\rho - \rho_0 = \rho_0 \underbrace{\alpha}_{\text{temperature coefficient of resistivity}} (T - T_0)$

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Ohm's law: A conducting device obeys Ohm's law when the resistance of the device is independent of the $\underbrace{\text{magnitude}}_{\text{大小}}$ and $\underbrace{\text{polarity}}_{\text{方向}}$ of the applied potential difference.

注意 $R=\frac{V}{i}$ 僅為電阻之定義. Ohm's law 為 R 不為 V 的函數之非一般情形.
Ohm's law 不為 fundamental 之物理定律

B. Power in Electric Circuits

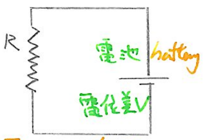
電阻造成電子在高電位與低電位間的能量差以熱耗散的方式逸散. 電容及電感則可將能量儲存於電場或磁場中.

$dq = i\,dt$

$$\underbrace{dU}_{\text{在時間經過dt, 因電荷移動而造成之系統能量差}} = \underbrace{dq}_{\text{在時間經過dt, 由高電位處流到低電位處之電荷量}}\, V = i\,dt\,\underbrace{V}_{\text{電位差}}$$

$\Rightarrow$ Power $P=\frac{dU}{dt}=iV$

(功率, 即單位時間之 electric energy transfer)



另: 電池的正負兩極稱為電池的 terminal

正/負極: positive/negative terminal

若存在電阻, 代入電阻定義 $R=\frac{V}{i}$

$$\begin{cases} V=iR \Rightarrow P=i^2R \\ i=\frac{V}{R} \Rightarrow P=V^2/R \end{cases}$$

電阻造成熱耗散之計算方式

C. Capacitor (電容, 用以儲存電能)

在電路圖中以 —||— 表示

When a capacitor is charged, the charges $+q$ and $-q$ become spatially separated.
We refer to the charge of a capacitor as being q.

In most cases, capacitors are made of conductors, for example, parallel plates (平行板). In other words, they are equipotential surfaces. When a potential difference V between two conducting surfaces is maintained when the conductor is charged, with a charge q, we can relate q and V by

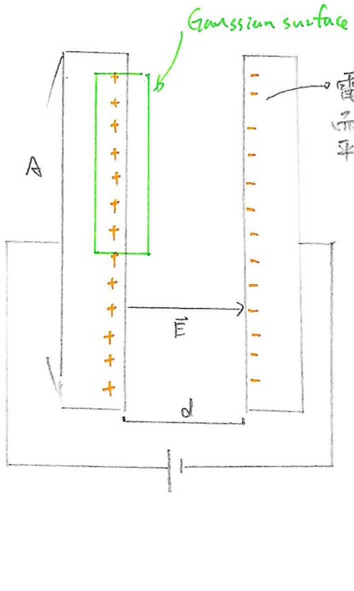
$$q = \underbrace{C}_{\text{capacitance}}V$$

物理意: 每增加一伏特的單位差可以多儲存的電量

SI 制單位: 1 farad = 1 F = 1 coulomb per volt = 1 C/V

Example: parallel-plate capacitor (平行板電容) ⟶ 將平行板近似於無窮大

(見 page 3 之推導)



if the surface area (one side) of the plate is A

according to Gauss's law:

$\epsilon_0 E = \frac{q}{A}$ for any location in between the two conducting plates

potential difference:

$$V = -\int_-^+ \vec{E}\cdot d\vec{s} = Ed \Rightarrow E=\frac{V}{d}$$

$$\epsilon_0\frac{V}{d}=\frac{q}{A} \Rightarrow q=\underbrace{\frac{\epsilon_0 A}{d}}_{\text{capacitance } C}V$$

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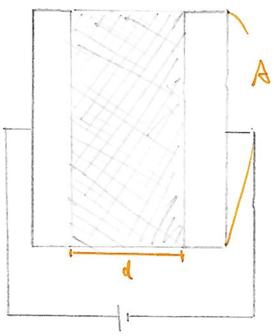
Example: capacitor with a Dielectric

(電介質 → 可被外電場偏極化之物質)

Michael Faraday (1837) = inserting certain materials in between the parallel plates can make the capacitance increased by a numerical factor $\underset{\hookrightarrow \text{dielectric constant}}{\mathcal{K}} \equiv \frac{\epsilon}{\epsilon_0}$ (見 page 5 最底下)

except that when V is greater than the breakdown potential V_{max}, the dielectric material will break down a form a conducting path.



$$C = \mathcal{K} \underbrace{\frac{\epsilon_0 A}{d}}_{\text{沒有電介質情形之 conductance}}$$

Proof. modify the differential form of Gauss law from $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

to $\underbrace{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon}}_{\text{見 page 5 最底下}} = \frac{\epsilon_0}{\epsilon} \frac{\rho}{\epsilon_0} = \frac{1}{\mathcal{K}} \frac{\rho}{\epsilon_0}$

the remaining derivation of charge density distribution and potential difference is the same as the previous example. #

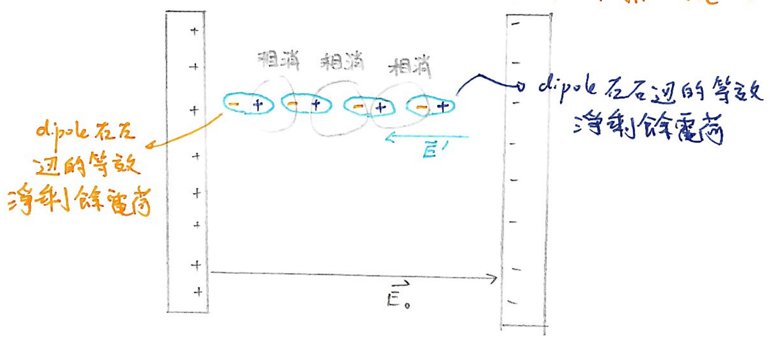
$$\Downarrow$$

$$\vec{\nabla} \cdot \underbrace{(\mathcal{K} \vec{E})}_{} = \frac{\rho}{\epsilon_0}$$

置入電介質後，電場需變為原本的 $\frac{1}{\mathcal{K}}$ 以抵消 dielectric const. 之效果

微觀理解電場因何會變為原本的 $\frac{1}{\mathcal{K}}$

induced dipole 的電場抵消了平行板上電荷所貢獻的電場



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D. Inductor (電感,可儲存磁能)

在電路圖中以 ℓℓℓℓ 符號表示

此處僅介紹電路学中的 inductance, 而不使用電磁學中原始的定義.
學電磁学時需注意.原始定義與此處定義之關係才不至於混淆

$$\underbrace{\mathcal{E}_L}_{\text{感應電動勢}} = -L \overbrace{\frac{di}{dt}}^{\text{單位時間之電流改變量}}$$

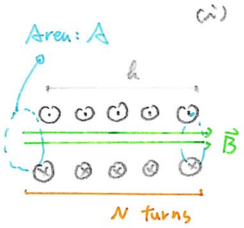
L → inductance: (負)單位電流改變率造成的感應電動勢.

公制單位: $1\ \text{henry} = 1\ H = 1\ T\cdot m^2/A$

比較: (page 12) $V = \frac{1}{C} q$: 電容可理解為單位電荷所能維持之電位差.

Example: inductance of a solenoid

↳ 線圈



(i) Faraday's law (page 10): $\mathcal{E} = -\frac{d\Phi_B}{dt}$

for a solenoid with N turns. N is large (page 9)

$$B = \mu_0 i n = \mu_0 i \frac{N}{h}$$

↳ 單位長度之線圈繞數: $n = \frac{N}{h}$

the magnetic flux through each on of the N current loop:

$$BA = \frac{\mu_0 i}{h} NA$$

⇒ the emf produced by each of the N current loop:

$$\Delta\mathcal{E} = -\frac{d}{dt}(BA) = -\frac{\mu_0 N}{h} A \frac{di}{dt}$$

the overall emf:

$$\mathcal{E} = \sum \Delta\mathcal{E} = -N\frac{\mu_0 N}{h} A \frac{di}{dt} = -h\mu_0 n^2 A \frac{di}{dt}$$

⇒ the inductance: $\underline{L = \mu_0 n^2 h A}$

solenoid電感正比於單位長度繞數之平方, 線圈長度, 及線圈截面積

↓

可重新整理為

$$L = \mu_0 i n \left(\frac{nh}{i} A\right)$$

$$= B\cdot\frac{N}{i} A = \frac{N\Delta\Phi_B}{i}$$

$\Delta\Phi_B$: 每一圈 current loop 造成之磁通量

表示法 $L = \frac{N\Delta\Phi_B}{i}$ 比較接近電磁学中用的 fundamental 定義

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※ 雖然大部分情形，電流因負電荷(即電子)的移動而形成，但習慣上，電路學中之電流箭頭方向仍表示正電流之流向

E. Circuits (電路)

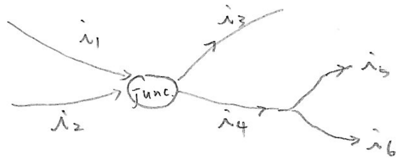
1. **Junction rule** (會合處，交叉點)：進入 junction 之電流總合等於流出 junction 之電流總合
(源自電荷守恆)

2. **Loop rule**：在電路中，若繞行任一迴圈，並將繞行中所遇到之電位變化加總，則加總起來的電位變化為 0
(源自能量守恆)

2.1：**resistance rule**：經過電阻時，順著電流方向電位變化為 $-iR$；沿相反方向則電位變化為 iR

2.2：**emf rule**：electromotive force rule
經過 emf device (如電池) 時，沿 emf 方向電位增加 $\mathcal{E}$，反之則電位減少 $\mathcal{E}$.

Example 1. Junction rule

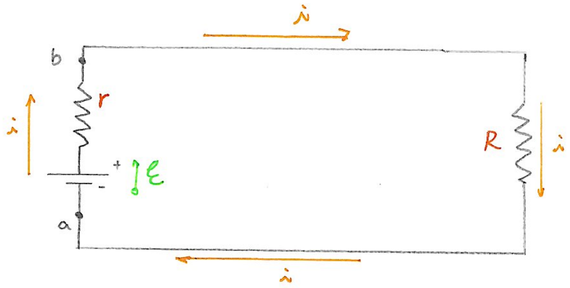


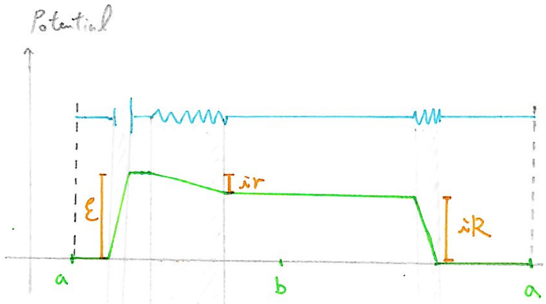
$$i_1 + i_2 = i_3 + i_4 = i_3 + i_5 + i_6$$

Example 2. Loop rule (僅有電池與串聯電阻之情形)

(Halliday textbook, chapter 27)

物理：以局部電位之高低來判定電流的方向。
電位之物理概念：emf device (如電池) 將電荷加速，電荷獲得動能，
電荷流經電阻時因碰撞而損失動能，剩於的動能決定電荷是否應/能跨過之後的電位差
(類比被加速而得到動能的球是否能跨過重力位能差)





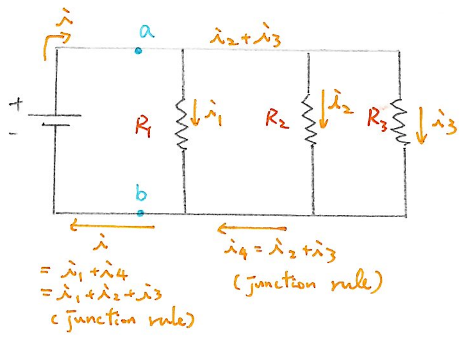
$$\text{loop rule}: \quad +\mathcal{E} - ir - iR = 0$$

$$\Rightarrow i = \frac{\mathcal{E}}{R+r}$$

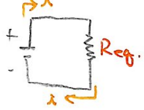
※ 串聯多個電阻形成之等效電阻，為各個電阻值之總合

Example 3. Junction + loop rule (電池及並聯電阻情形)

resistance in parallel



$\equiv$



$$\text{loop rule}: \begin{cases} \mathcal{E} - i_1 R_1 = 0 \\ \mathcal{E} - i_2 R_2 = 0 \\ \mathcal{E} - i_3 R_3 = 0 \end{cases}$$

$$\text{junction rule}: i = i_1 + i_2 + i_3 = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$\Rightarrow$ 可視 a, b 右側為一等效電阻 $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

※ 並聯電阻之等效電阻之倒數，為各別電阻之倒數和.

※ 碰到複雜電路時，先以此類手法簡化電路圖

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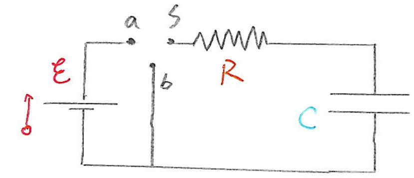
(RLC例式不細講 藉電感講磁場之能量)

Example: RC circuits (有電阻及電容之情形)



(i) S接a不接b, 電容充電
(charging the capacitor with a battery)

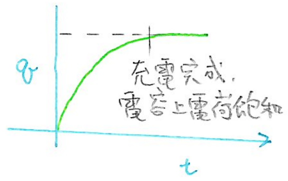
$$\text{loop rule: } \varepsilon - iR - \underbrace{\frac{q}{C}}_{q=CV} = 0$$

電容上之電荷量

$$\Rightarrow \varepsilon - \frac{dq}{dt}R - \frac{1}{C}q = 0$$

: q對時間的一次常微分方程, 若初始條件為電容電荷為0, 則解為

$$q = C\varepsilon(1 - e^{-t/RC})$$



(ii) S接b不接a, 電容放電

$$-iR - \frac{1}{C}q = 0$$

$$\Rightarrow q = q_0 e^{-t/RC}$$

: 電容上電荷呈exponential decay, 其decay之特徵時間為電阻與電容值的乘積.

Example: RL circuits (有電阻及電感之情形)