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General Physics (II) Physical Optics (物理光學)

Halliday & Resnick, 11ed., ch. 35

物理光學以光的波動性理解, 幾何光學中光的反射則以粒子性想像

A. Huygens' principle (Christian Huygens, 1678)

(Note there was already a concept that light speed is not infinite
It is c in vacuum and $\frac{c}{n}$ in a medium with an index of refraction n)

This principle tells how to predict the location and shape of the wavefront at any time, if its present location and shape is known. Based on the modern understanding, light is EM wave. the wavefront is a 2D surface in a 3D space, that has a constant phase.

在正式的電磁學推導中, 此 principle 為 Green's function 之直接結果

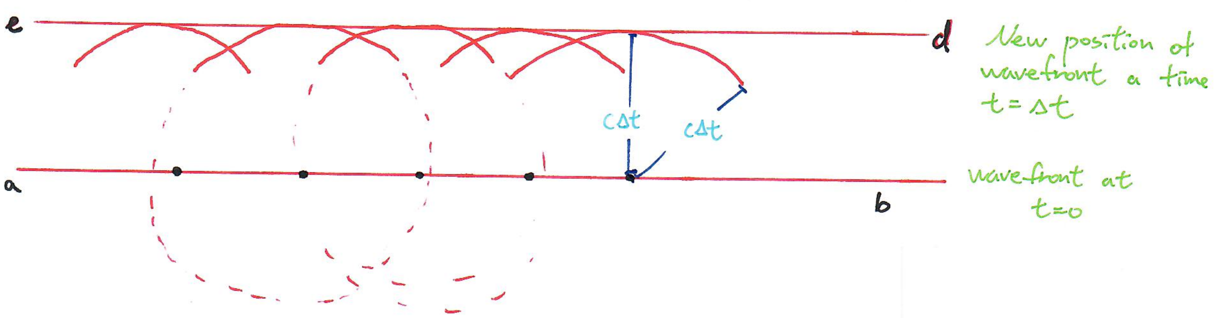
To predict the wavefront after time Δt

(1) assume that any ~~location~~ on ~~the~~ present wavefront is a new source.

(2) draw a sphere around each location, that has radius $\Delta t\left(\frac{c}{n}\right)$

(3) find the surface that is tangent to the spheres you drew. that is the new wavefront.

※真空的 index of refraction $n=1$. 光密介質 $n>1$, 在其中波行進速度因與介質交互作用而變慢. 波長變短但頻率不變



B. The law of refraction

※假設波的頻率及相位不在光波穿入/出不同介質時有不連續性

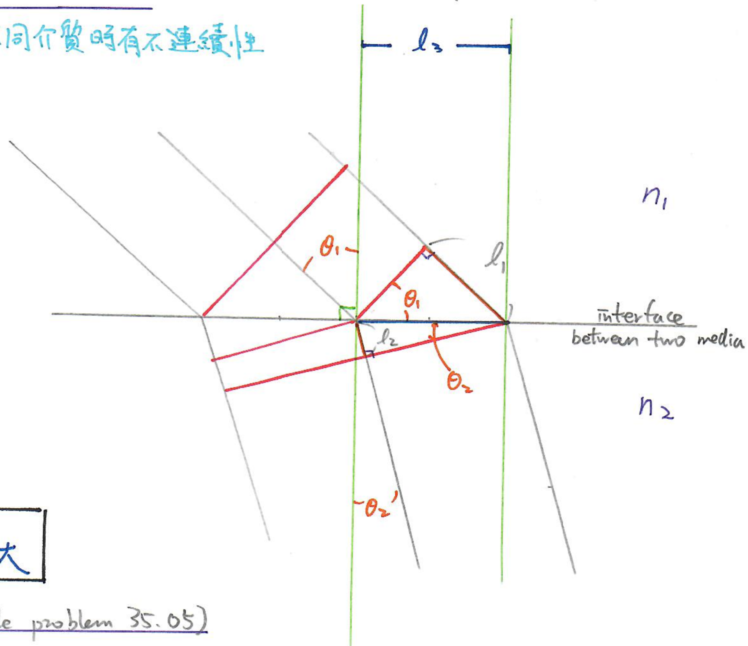
$$l_1 = \left(\frac{c}{n_1}\right)\Delta t$$

$$l_2 = \left(\frac{c}{n_2}\right)\Delta t$$

$$\therefore \frac{l_1}{\sin\theta_1} = \frac{l_2}{\sin\theta_2} = l_3$$

Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



$\boxed{\text{※ 速大} \longrightarrow \text{角大、波長大}}$

C. Reflection (參考 Halliday ed 11. Sample problem 35.05)

1. 入射角等於反射角
2. 由 $\left\{\begin{matrix}\text{大}\ n \\ \text{小}\ n\end{matrix}\right.$ 介質入射, 受到 $\left\{\begin{matrix}\text{小}\ n \\ \text{大}\ n\end{matrix}\right.$ 介質反射, 在介面處反射波相位 $\left\{\begin{matrix}\text{不變} \\ \text{與入射波差}\ 180^\circ\ (\text{即}\ \pi)\end{matrix}\right.$

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在實驗中,干涉為測量距離或長度之最精確方法之一,如 Michelson's exp.

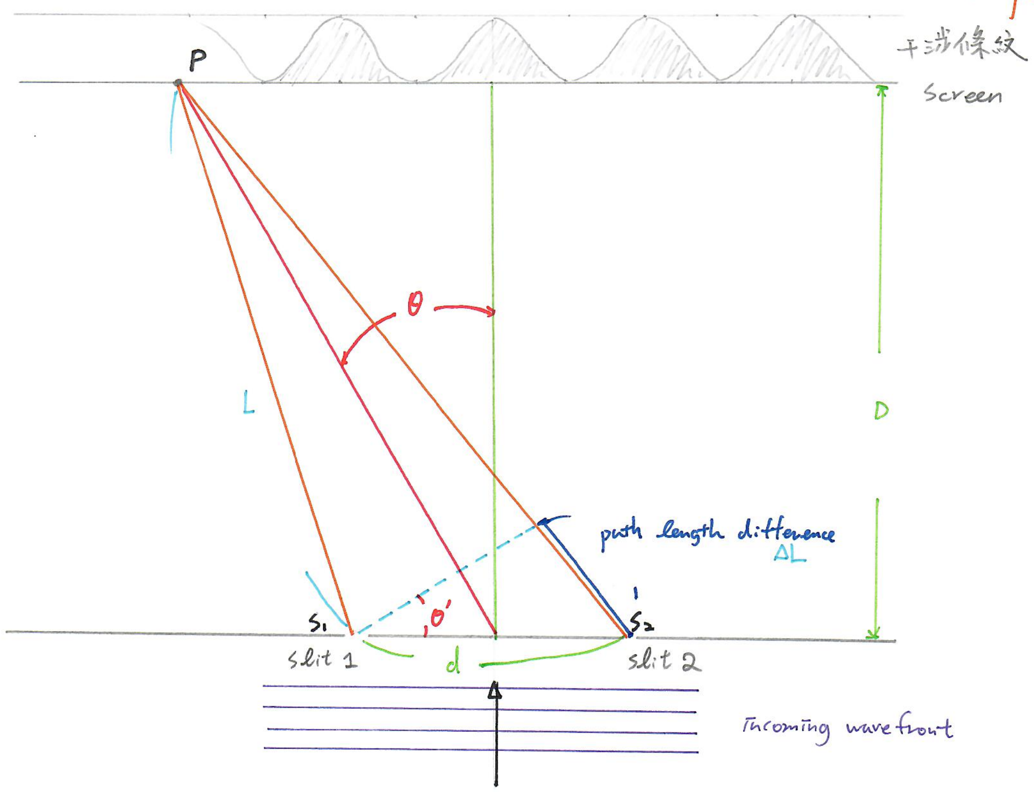
D. Diffraction (繞射) and Interference (干涉)

(推導中利用 Huygens' principle,即古代人對 Green's function 之理解)
應用中,繞射常限制光束之集中程度,干涉影響影像之 dynamic range (造成 artifact)

(一) Young's Interference Experiment (楊氏雙狹縫實驗)

Thomas Young, 1801

術語: interference pattern 干涉條紋 is composed of
- bright bands (or bright fringes, or maxima) 亮紋
- dark bands (or dark fringes, or minima) 暗紋



when $D \gg d$ and θ is small, the path length difference ΔL can be approximated by $\Delta L \sim d\sin\theta$

$\Rightarrow$ locations of {maxima, minima} in terms of angle of emergence θ:
- fully constructive interference 完全相長干涉: $\Delta L = d\sin\theta = m\lambda$, $m = 0, 1, 2, \cdots$
- fully destructive interference 完全相消干涉: $\Delta L = d\sin\theta = (m+\frac{1}{2})\lambda$, $m = 0, 1, 2, \cdots$

angle of emergence θ can be obtained by taking arcsine.

術語: when m = a specific integer n

亮紋 / 暗紋: nth-order {bright, dark} fringe

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Formal evaluation of intensity pattern (in the limit of $D \gg d$)

Here we first claim that the slit width is infinitely narrow.

But this claim is in fact not necessary.

※ the phases at slit S_1 and S_2 should be different by a constant (i.e., not changing over time)

In this case, we call the phase from slit S_1 and S_2 to be completely coherent.

Example of coherent source: laser

At the screen, the electric field of the EM-wave from slit S_1 and S_2 are:

$$\begin{cases} \text{from } S_1: E_1 = E_0 \cos(kL - \omega t + \underbrace{\phi_0}_{\text{phase offset of the wave emerging at the slit}}) \\ S_2: E_2 = E_0 \cos(k(L+\Delta L) - \omega t + \phi_0) \end{cases}$$

On the screen, at the angle of emergence $\theta = \theta'$, $\Delta L \sim d \sin\theta'$

the intensity is proportional to $(E_1 + E_2)^2$

$$\propto \left.\begin{matrix} \cos^2(kL - \omega t + \phi_0) + \\ \cos^2(k(L+\Delta L) - \omega t + \phi_0) + \end{matrix}\right\} \text{auto-correlation term}$$

$$\text{cross-correlation term} \left\{ 2\cos(kL - \omega t + \phi_0)\cos(k(L+\Delta L) - \omega t + \phi_0) \right.$$

When we observe the interference pattern, we place a detector (or an array of detectors, e.g. CCD) on the screen, and take a long exposure. Then what we actually observe is the time averaged quantity $\underbrace{\lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{t=0}^{t=\Delta t} dt}$

long: means the exposure time is much longer than the period of the EM wave

一般以中括號表示: $\langle \quad \rangle$

The contribution of the auto-correlation terms to the time averaged total intensity

$$\langle \cos^2(kL - \omega t + \phi_0) \rangle = \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{t=0}^{t=\Delta t} \cos^2(\underbrace{kL - \omega t + \phi_0}_{x})\, dt$$

$$= \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{t=0}^{t=\Delta t} \frac{1}{2}\left(\underbrace{\cos^2 x - \sin^2 x}_{\cos 2x} + \underbrace{\sin^2 x + \cos^2 x}_{1.0} \right) dt = \frac{1}{\Delta t}\frac{1}{2}\Delta t = \frac{1}{2}$$

$\cos 2x$: t 的餘弦函數在 $\Delta t \to \infty$ 時積分貢獻為 0

Similarly: $\langle \cos^2(k(L+\Delta L) - \omega t + \phi_0) \rangle = \frac{1}{2}$

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The contribution of the cross-correlation to the time averaged total intensity

$$\langle 2\cos(kL-\omega t+\phi_0)\cos(k(L+\Delta L)-\omega t+\phi_0)\rangle$$

$$= 2\lim_{\Delta t\to\infty}\frac{1}{\Delta t}\int_{t=0}^{t=\Delta t}\cos(kL-\omega t+\phi_0)\cos(k(L+\Delta L)-\omega t+\phi_0)\,dt$$

Using trigonometric identity:

$$\cos\theta\cos\varphi = \frac{1}{2}\left(\cos(\theta-\varphi)+\cos(\theta+\varphi)\right)$$

$$= \lim_{\Delta t\to\infty}\frac{1}{\Delta t}\int_{t=0}^{t=\Delta t}\Big[\underbrace{\cos(k\Delta L)}_{\text{非 t 的函數, 積分簡單}} + \underbrace{\cos(2kL-2\omega t+2\phi_0+k\Delta L)}_{\text{在 }\Delta t\to\infty\text{ 情形積分貢獻為 }0}\Big]\,dt$$

$$= \lim_{\Delta t\to\infty}\frac{1}{\Delta t}\cos(k\Delta L)\,\Delta t$$ recalling page 2, $\Delta L = d\sin\theta'$

$$= \cos(kd\sin\theta') = \cos\left(\frac{d}{\lambda}2\pi\sin\theta'\right)$$

In the limit of $\theta'\to 0$ (小角度出射), $\sin\theta'\to\theta' \Rightarrow \cos\left(\frac{d}{\lambda}2\pi\sin\theta'\right) \sim \cos\left(\frac{2\pi d\theta'}{\lambda}\right)$

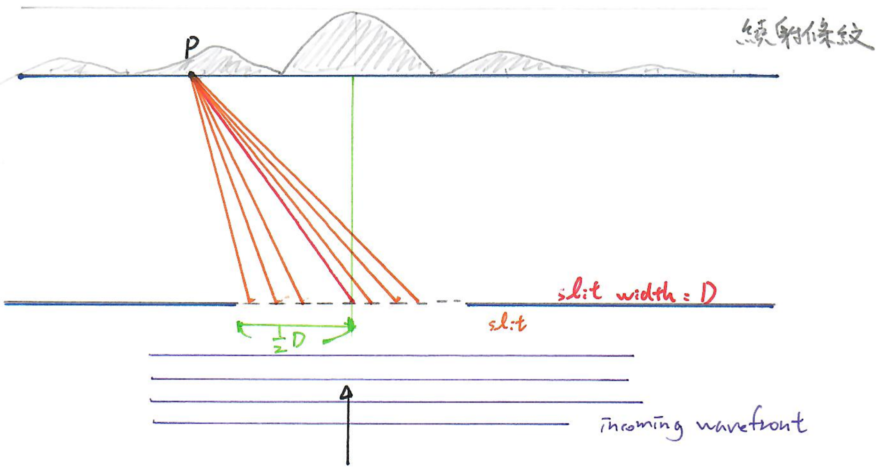
注意此處 d 為狹縫間距 而非微分算符

we see $\begin{cases}\text{maxima} \\ \text{minima}\end{cases}$ when $\cos\left(\frac{2\pi d\theta'}{\lambda}\right) = \begin{cases}1 \\ -1\end{cases}$

在極 $\begin{cases}\text{大} \\ \text{小}\end{cases}$ 處 $\frac{2\pi}{\lambda}d\theta' = m\pi$, $m = \begin{cases}0, 2, 4, \cdots \\ 1, 3, 5, \cdots\end{cases}$

※ 同樣的推導, 在干涉儀實驗設置中非常重要. 由於干涉儀為測量距離的儀器中精度最高的, 故建議熟習此部分.
例子可在我個人網頁中的 Lectures ⟶ Radio Interferometry 講義中找到
Halliday Ed 11 中 §35-5 中的 Michelson's Interferometer 也是很好的例子

(iii) Diffraction (單狹縫繞射) ⟶ 在任何光學元件, 此原理決定解析度, 重要



繞射條紋之計算:
單狹縫上每一小段在屏幕上貢獻之電場的總合取平方

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若直接用積分不會積，就先用差分列式再取極限（雖然取極限要小心，但大部分情況合法）

At emergence angle $\theta = \theta'$, the photon number density at an instance is proportional to

設其有 N 個小段，以 $n=0,1,\cdots,N$ 標注

足標 n 表示第 n 個小段

$$\left[\sum_{n=0}^{N-1} \cos\left(k(L + \Delta L_n) - \omega t \right) \underbrace{\left(\tfrac{1}{N} D \right)}_{\text{每一小段之貢獻正比於該小段之寬度，在此處為常數}} \right]^2$$

$$= \Big[\underbrace{\sum_{n=0}^{N-1} \cos\left(kL + k\frac{nD}{N}\sin\theta' - \omega t \right) \left(\tfrac{1}{N} D \right)}_{} \Big]^2$$

在中括號內部，取 $N \longrightarrow \infty$ 極限做計算後再取平方。

在此極限中，$\frac{nD}{N}$ 成為下上界為 $[0, D]$ 之積分變數，$\frac{1}{N}D$ 為 delta 該積分變數。

故在 $N \to \infty$ 之極限，中括號內為簡單的 cos 積分，正比於

$$= \frac{\Big[\overbrace{\sin(kL + kD\sin\theta' - \omega t)}^{\text{上界之貢獻}} - \overbrace{\sin(kL - \omega t)}^{\text{下界之貢獻}} \Big]}{k\sin\theta'}$$

Using the trigonometric identity

$$\sin\theta - \sin\varphi = 2\sin\left(\frac{\theta-\varphi}{2}\right)\cos\left(\frac{\theta+\varphi}{2}\right)$$

Amplitude modulation

$$= -\underbrace{\frac{\sin\left(\frac{1}{2}kD\sin\theta'\right) \cos\left(kL - \omega t + \frac{1}{2}kD\sin\theta'\right)}{\frac{1}{2}k\sin\theta'}}$$

單狹縫造成之 geometric phase modulation，在設計儀器時有時效果重要，需以較正測量消除

此項取平方後再取 $\langle\ \rangle$，得到長時間曝光後測得之平均 photon number density at angle of emergent θ'

此處可看出，amplitude modulation 項為一與時間 t 無關之正比係數，故取 $\langle\ \rangle$ 之計算與雙狹縫例子中的 auto-correlation 項取 $\langle\ \rangle$ 無異。

$\Rightarrow$ diffraction pattern of single slit (time averaged photon number density)

$$\propto \frac{\left(\sin\frac{1}{2}kD\sin\theta'\right)^2}{\underbrace{(k\sin\theta')^2}_{\text{振幅隨 }\sin\theta'\text{ 減小}}}$$

隨 $\sin\theta'$ 變化之正弦型式

極小位置在 ($\theta' \to 0$ 近似)

$$\frac{1}{2}kD\theta' = m\pi, \quad m = 1, 2, 3, \cdots$$

$$\Rightarrow \frac{1}{2}\frac{2\pi}{\lambda}D\theta' = m\pi \Rightarrow \theta' = \frac{\lambda}{D}m$$

$m=1$ 之極小位置決定孔徑成像之解析力。

稱此 θ' 為半寬

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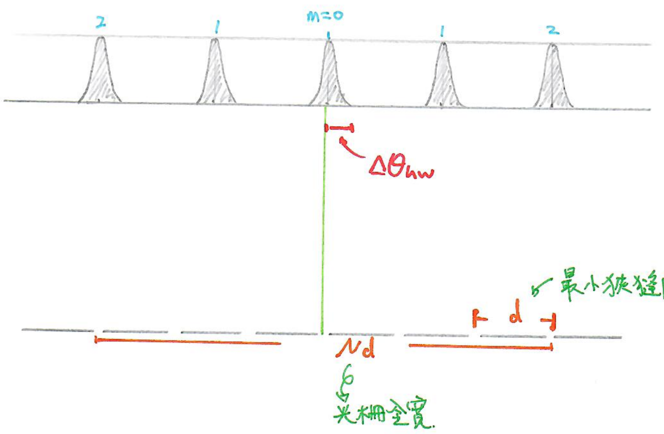
(iii) Diffraction Grating 繞射光柵

多狹縫干涉/繞射條紋　一般應用中,每毫米可有數千個狹縫



最小狹縫間距 (grating spacing)

單色光之繞射條紋,若非單色光,
則不同顏色的光在 $m>0$ 的極大
angle of emergence 錯開.
光柵可應用於分光

(a) 亮紋間距: 由 $d\sin\theta = m\lambda$ 計算.

即計算間距時,只需拿其中兩個最靠近的狹縫計算干涉條紋

若光柵間距已知,可利用亮紋間距計算波長.

(可先用已知波長之入射光,如某種 laser 來較正 d,再以較正結果計算未知波長之入射波之波長)

(b) 亮紋半寬 (half-width: $\Delta\theta_{hw}$)

$m=0$ 處 $(Nd)\sin(\Delta\theta_{hw}) = \lambda \Rightarrow \Delta\theta_{hw} = \dfrac{\lambda}{Nd}$

即計算單一干涉條紋的半寬時,約略視整個光柵為一單狹縫

$m>0$ 處: $\Delta\theta_{hw} = \dfrac{\lambda}{Nd\cos\theta}$