吕浩宇

(Calibration) Error and Error Recognition

After observations, we received the raw (unprocessed) complex visibility that requires calibrations. In the data processing, the goal is to remove the systematic errors, $\epsilon(u,v)$, which can be additive or multiplicative

additive errors

$$\underbrace{\tilde{V}(u,v)}_{\text{observed complex visibility}} = \overbrace{V(u,v)}^{\text{actual complex visibility}} + \underbrace{\epsilon(u,v)}_{\text{error}}$$

The additive errors may be contributed by (e.g. $\overbrace{\text{Radio Frequency) Interference (RFI)}}^{\text{human communication signal}}$, antennae cross-talk, additive correlator offset, receiver noise. The effect is superimposing F.T. $(\epsilon(u,v))$ on top of the image F.T. $(\epsilon(u,v))$.

multiplicative errors

$$\tilde{V}(u,v) = V(u,v) \cdot \epsilon(u,v) \Rightarrow \text{the image } \overbrace{F.T.(\tilde{V}(u,v)) = \underbrace{F.T.(V(u,v))}_{\text{true image}} * \underbrace{F.T.(\epsilon(u,v))}_{\text{error image}}}^{\text{Convolution theorem}}$$

The multiplicative errors may be contributed by atmospheric and/or ionospheric amplitude/phase errors, etc. When the calibration is imperfect, which is always the case, the form of the residual calibration errors are still in multiplicative form.

→ errors that are related to certain antennae (e.g. the gain of a certain antenna) is **antenna-based**. errors that are only present in certain baselines are **baseline-based**. The baseline-based errors may be due to malfunction of correlator, which is small in most cases in the modern observatories.

complex baseline-based gain : $g_{ij}(t)$

t → specific time t

ij → indicating antennae i and j

When observing a point-like calibrator that has flux density S, and when the additive errors (e.g. thermal noise, which can be averaged out) and the multiplicative errors (e.g. may be calibrated out) are negligible, the complex baseline-based gain can be derived by

$$g_{ij}(t) = \frac{\tilde{V}_{ij}(t)}{S}$$

antenna-based complex gain

It can be factored into

$$\begin{cases} \text{amplitude term: } A_{ij}(t) = \overbrace{a_i(t)}^{\text{gain amplitude of antenna } i}\ \overbrace{a_j(t)}^{\text{gain-amplitude of antenna } j} \\ \text{phase term: } \Phi_{ij}(t) = \underbrace{\phi_i(t) - \phi_j(t)}_{} \end{cases}$$

↳ gain-phase of antenna i, and j, e.g. due to the (atmospheric) path-length difference observed at these two antennae

note that phase errors only appear in the form of phase difference. Therefore, we have the degree-of-freedom to choose a phase reference. Usually, we set the phase of a reference antenna to be zero

For a point-like calibrator at the phase referencing center (i.e., phase $\Phi_{ij} = 0$) that has amplitude S, the effects of these antenna-based errors is to make the detected complex visibility

$$\tilde{V}(u,v) = S\, a_i(t)\, a_j(t)\, e^{i(\phi_i(t) - \phi_j(t))}$$

Recognizing errors.

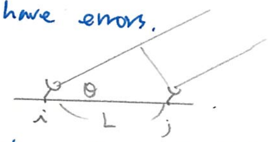
Errors are only recognizable when we observe sources with known location and structures, for example, the high-z quasars which are point-like sources in most cases. We can diagnose errors either in the visibility (i.e. Fourier transformed) domain, or in the image domain.

The errors that have large effects on one or a few visibility points. are easily seen in the visibility domain. The errors that have effects on a large number of visibilities (e.g. all the visibilities that are associated with one antenna) may be more easily seen in the image domain.

Recognizing $G(u,v)$ in the visibility domain

Calibration sources with simple structures should show consistent visibility (i.e. no rapid changes) in the time domain, frequency domain, and when comparing polarizations.

When we see phase drift in the time domain for a few baselines, the location of one antenna may have errors.



The path length difference error

$(L+\Delta L)\sin\theta - L\sin\theta$

$= \Delta L \sin\theta$ is varying with θ, thus is varying with time.

when all baselines show time drift, the information of the calibration source location may be wrong.

To allow a successful complex gain calibration, we usually require the gain calibration scans to have $<25\%$ point-to-point amplitude variation, and $<30^\circ$ point-to-point phase variation. otherwise, the interpolation of the calibration solution will be ambiguous. In addition, the residual calibration errors will be large even after calibration

When we see phase wrapping in the frequency domain, the $\underset{\Delta}{\text{path length}}$ (or equivalently, the delay) may have errors. In this case, the phase error depends linearly on $\frac{\Delta}{\lambda} \propto \Delta\nu \propto \nu$

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effects of $\epsilon(u,v)$ in the image domain

When $\epsilon(u,v)$ is real (i.e. there is only amplitude error, but no phase error), the image F.T.($\tilde{V}(u,v)$) is essential the true image convolved with F.T.$\Big(\underbrace{A(u,v)\, e^{i\cdot 0}}_{}\Big)$ = F.T.$(A(u,v))$

↳ image of a point-source located at the phase referencing center

Therefore, when there is amplitude errors, all sources will be surrounded by symmetric side-lobe pattern.

when $\epsilon(u,v)$ is complex (i.e. there is phase error), F.T.($\tilde{V}(u,v)$) is the true image convolved with the image $\underbrace{\text{F.T.}(A\, e^{i\theta})}$

a pattern that is offset from the phase referencing center

Therefore, all sources are surrounded pattern.

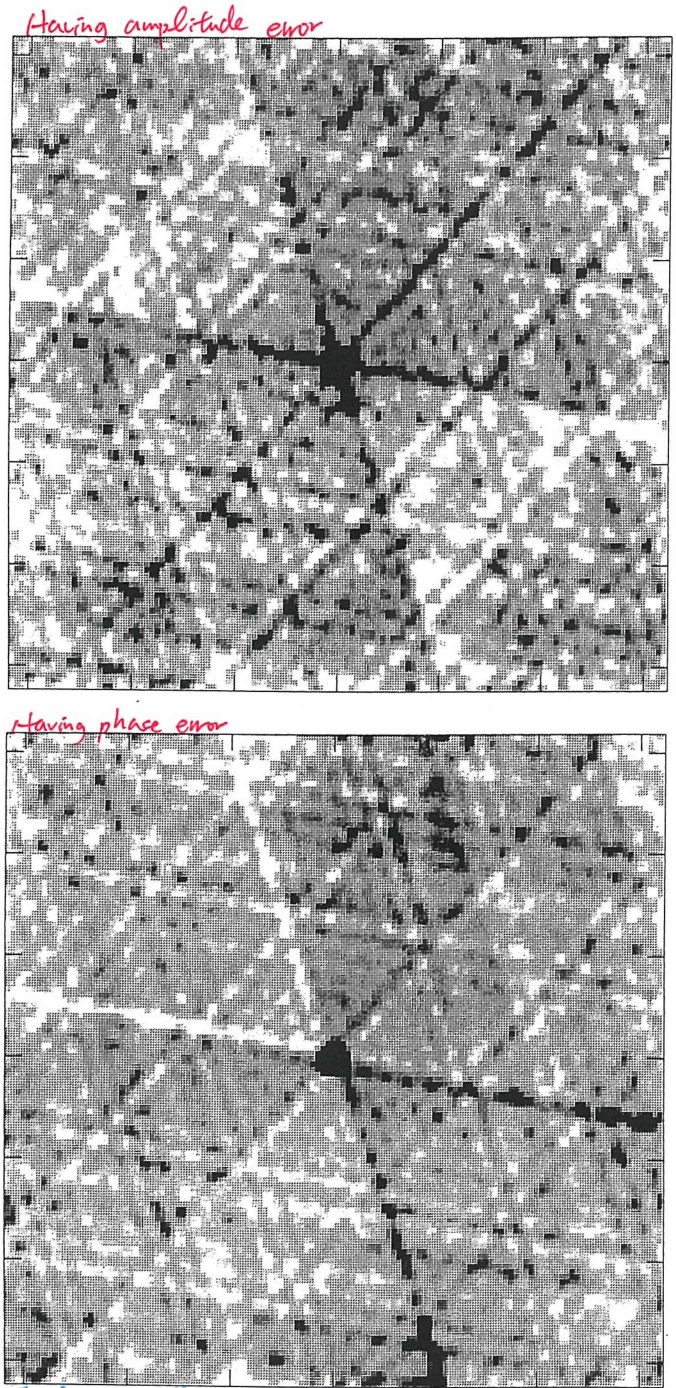
After some long-time integration, the effect of time-dependent phase and amplitude errors is analogous to the effect of seeing in optical observations.

The sources are surrounded by some error patterns. When the target source has spatially extended structures, these patterns can confuse each other.

Visually recognizing such pattern is a way to diagnose and assess the residual calibration errors.

(a)

(b)



Taylor, Carilli, and Perley, Synthesis Imaging in radio astronomy II

Figure 15–3. Images from a "snapshot" observation of a point source **(a)** with a 10% amplitude error on one antenna and **(b)** with a 10% phase error on one antenna.

Figure 15–6. Asymmetric pattern which could be caused by atmospheric phase errors.