Principle of Interferometric Data Calibration

lecture 4.
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Observations can be subject to thermal fluctuation and systematic errors. After taking the raw observational data, we need to
1. characterize the thermal fluctuation (introduce later)
2. calibrate out the systematic errors.

→ concept.
1. Identify the origin and form of the systematic errors
2. Based on empirical / theoretical models to reference the calibration factors.
3. There are always some effects that we don't understand or we miss, which make the data look like outliers. We have to flag the data as outliers, and then exclude them from both the calibration procedure and scientific analysis.

For example, we can compare our ruler with the international prototype metre to calibrate the ticks of the ruler.

Source of systematic errors

Atmospheric effects: (or environmental effects, e.g. temperature)	rapid time variation. In most cases, the atmospheric effects have weak dependence on frequency. For example, the index of refraction (n) of the atmosphere is almost independent of frequency in (sub)millimeter and centimeter bands.
Instrumental effects:	The response of modern instruments are relatively stationary (i.e. no strong dependence on time). The instrumental response can change with frequency.

We need to understand these effects, and then plan our observations such that these effect can be calibrated

Potential Issues During the Modern Observations.

Bugs or mechanical problems: In idealized worlds these shouldn't happen. In real life, we need the awareness of that such problems occur all the time. It can lead to source tracking errors (e.g. antennae look at the blanck sky), source labeling errors, frequency tuning errors or frequency tuning out of lock, phase referencing error, data transfer problems (e.g. data not stored), etc. Many of such issues will be reported in the observing logs, as long as the telescope operator notice them early. But some of the problems can still can be missed. So it is always good to inspect the data visually and carefully, and pick out all of those problematic data. Do this right after your observations if possible. In case that you need to contact the observatory to repeat your failed observations.

Note that big telescopes cannot slew fast due to the large inertia. When target sources are at high elevations, there can be source-tracking errors, leading to degraded amplitude. Should flag such data

Deformation of mechanical structures: structures of a telescope can deform due to gravitational force, wind, non-uniform temperature distribution (e.g. due to sunshine, the heat accumulation in the antenna cabin), and so on. There are two effects:

First, the deformed telescope surface will have a different effective surface area (or say, surface efficiency) from the original design. For a large telescope dish with Az-El mount, this can the gravitational deformation can lead to an elevation dependent surface efficiency, which may be corrected with a quasi-empirical model. The calibration factor for this effect is the gain-elevation curve or gain curve for simplicity. In most modern observatories, the gain-elevation curve is stationary and has been tabulated by the observatory. They can be applied (if necessary) with the built-in function of the official data reduction software during the post-processing. This is usually the very first step in the data calibration procedure (if necessary).

◎ In the same stage, when using a digital correlator, we may also apply another correction factor called re-quantizer table to take into account the loss of efficiency during the digitization.

Second, the pointing direction of the telescope may also be offset from the expected direction. How serious is the effect and how frequently we need to calibrate the effect depends on the infrastructure of the observatory and the site condition. The procedure to calibrate this effect is called reference pointing calibration, which makes the telescope(s) scan on a bright point source (e.g. a bright quasar) over a Az-El range and then base on the built-in quasi-empirical model to correct the correction terms. This can be done either in the single-dish mode or interferometric mode observations.

Owing to the Az-El scanning, the visibility amplitudes will show harmonic variations during the reference pointing calibrations. The reference pointing calibration data are mostly taken off-axis and thus cannot be used in any other calibration procedure. If you identify such scans in your data, you need to flag them.

Usually, the Submillimeter Array (SMA) needs a reference pointing calibration right at the beginning of your observing track. We also often make another reference pointing calibration after the target source transit.

Without referencing pointing calibration, the pointing error of the JVLA is about 2~3 arcminutes. When your primary beam FWHM is much larger than the native pointing errors (e.g. when observing at 4, P, L, S, C, X, and Ku bands), it is not particular necessary to carry out reference pointing calibrations unless you need high precision (or if there is any concern with polarization accuracy). At >20 GHz frequencies, JVLA needs a reference pointing calibration everytime you slew the telescopes by more than 20 degrees to compensate the gravitational deformation. Even you stay with more or less the same pointing direction, there needs a pointing calibration every ~40 minutes in night time and 20~30 minute in day time.

Atmospheric emission and absorption (attenuation)

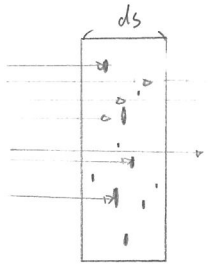
Atmospheric effect is contributed mostly from the troposphere, which is below 7~10 km elevation

Material with non-zero temperature emits black body radiation. So does the atmosphere. If it only emits radiation, its temperature will gradually drop to zero. The Kirchhoff's law implies that the atmosphere also absorp some radiation such that it can reach thermal equilibrium with the ambient radiation field.

The most important absorbers in the (sub)millimeter and radio bands include water vapour (e.g. at 22 GHz and 183 GHz) and O_2 (e.g. at 60 GHz and 118 GHz).

How we can formulate extinction?

Let's imagine a lot of photons are penetrating through a slab of atmosphere that has thickness ds. Only a fraction can penetrate through successfully, which depends on the cross-section of the absorber in this slab



The overall cross-section of the absorbers depends on the thickness of the slab. We can define it with a proportional factor. Then fractionally, the reduced intensity (or number of photons) can be expressed as (note the negative sign is needed)

$$-\frac{\Delta I}{I} = \underbrace{\kappa}_{\text{proportional factor}} \overbrace{\Delta S}^{\text{thickness}}$$

intensity

Defining: $\Delta\tau \equiv \kappa \Delta S$ and then making ΔS infinitelly small, we obtain

τ is called optial depth

$$-\frac{dI}{d\tau} = I \Rightarrow \int \frac{dI}{I} = \int -d\tau + \text{const.}$$

$$\Rightarrow \ln I = -\tau + \text{const.}$$

$$\Rightarrow I = e^{\text{const.}} e^{-\tau}$$

adopting the boundary condition $I = \underbrace{I_0}_{\text{unattenuated intensity}}$ when $\tau = 0$

$$\Rightarrow I = I_0 e^{-\tau_{atm}}$$

$\Rightarrow$ when light propagates through absorbing media, the intensity decay exponentially with optial depth.

Defining the extinction coefficient: $G_{atm} = e^{-\tau_{atm}}$

Form of atmospheric emission (expressed by brightness temperature)

here I assume that the temperature of the atmosphere is a constant, and then give the solution of the Equation of radiative transfer:

$$\underbrace{T_B^{atm}}_{\text{observed brightness temperature of the atmosphere}} = \underbrace{T_{atm}}_{\text{temperature of the atmosphere}} \left(1 - e^{-\overbrace{\tau_{atm}}^{\text{optial depth of the atmosphere}}}\right)$$

how to qualitatively understand this?

- high optial depth limit: ($\tau_{atm} \to \infty$) = $\begin{cases} G_{atm} \to 0 : \text{celestial source cannot be seen} \\ T_B^{atm} \to T_{atm} : \text{black body radiation} \end{cases}$
- low optial depth limit ($\tau_{atm} \to 0$): $T_B^{atm} \sim \left(1 - \left(1 - \tau_{atm} + \frac{1}{2!}\tau_{atm}^2 + \cdots\right)\right)$

$$\sim \underbrace{T_{atm}\tau_{atm}}$$

this can be comprehended by that when the optial depth is low enough such that the absorbers are not obscuring each other, and if the effective emission surface is linearly proportional to the absorption surface, then the brightness temperature is proportional to the temperature of the atmosphere and the optial depth.

The flux calibration procedure will first reference the unattenuated target source brightness temperature with the noise power of a known (to some extend) noise load

(i) $\underbrace{T_{source}}_{\text{unattenuated source brightness temperature}} G_{atm} = T_{source} e^{-\tau_{atm}}$

(ii) $T_B^{atm} + T_{rec}$: the combination of atmospheric brightness temperature and receiver noise.

For convenience, we define

$$\underbrace{T_{sys}^{eff}}_{\text{effective system temperature}} = e^{\tau_{atm}} \overbrace{\left[\left(1 - e^{-\tau_{atm}}\right) T_{atm} + T_{rec}\right]}^{T_{sys}}$$

effective system temperature (note that the SMA and ALMA community directly call this term system temperature)

at >200GHz, T_{atm} contributes very significantly to T_{sys}^{eff}

How to measure T_{sys} ? [unknowns are T_{sys} (or T_{sys}^{eff}) and G_{atm}]

measurement 1: observing a black body that has temperature $T = T_{atm}$

$$\underbrace{P_{BB}}_{\text{observing on black body}} = \underbrace{T}_{\text{power received from the black body}} + \underbrace{T_{rec}}_{\text{power emitted from the receiver itself}} = T_{atm} + T_{rec} = T_{atm} \overbrace{- G_{atm} T_{atm} + G_{atm} T_{atm}}^{\text{zero}} + T_{rec}$$

$$= \overbrace{(1 - G_{atm}) T_{atm} + T_{rec}}^{T_{sys}} + G_{atm} T_{atm}$$

$$= T_{sys} + G_{atm} T_{atm} = G_{atm} \underbrace{G_{atm}^{-1} T_{sys}}_{T_{sys}^{eff}} + G_{atm} T_{atm}$$

$$= G_{atm} (T_{sys}^{eff} + T_{atm})$$

measurement 2: observing on the blank sky

$$P_{CMB} = \underbrace{G_{atm} T_{CMB}}_{\text{attenuated cosmic microwave background (by atmosphere)}} + T_{sys} = G_{atm} (T_{CMB} + T_{sys}^{eff}) \Longrightarrow \frac{P_{CMB}}{G_{atm}} = (T_{sys}^{eff} + T_{CMB})$$

$$\Rightarrow P_{BB} - P_{CMB} = G_{atm} (T_{sys}^{eff} + T_{atm} - T_{sys}^{eff} - T_{CMB}) = G_{atm} (T_{atm} - T_{CMB})$$

$$\frac{1}{G_{atm}} = \frac{T_{atm} - T_{CMB}}{P_{BB} - P_{CMB}}$$

$$T_{sys}^{eff} = \frac{P_{CMB}}{G_{atm}} - T_{CMB}$$

$$\Downarrow$$

$$T_{sys}^{eff} = \frac{T_{atm} - T_{CMB}}{P_{BB} - P_{CMB}} P_{CMB} - T_{CMB}$$

$T_{atm} \sim 273K$, $T_{CMB} = 2.73K$,
P_{BB} and P_{CMB} are measured. (in arbitrary unit)

At 225 GHz frequency, when PWV ~ 0.5, the optical depth $\tau_{atm} \sim 0.036$
(i.e., $T_{atm} \sim 9.8K$),
The typical receiver temperature is 50 ~ 100 K ※ look at this value during your observing run.

In the millimeter/submillimeter observations, there is usually a calibration step called Tsys application. We multiply the correlator output by $T_{sys}^{eff} = e^{\tau_{atm}} T_{sys}$, which is compensating the atmospheric attenuation $e^{\tau_{atm}}$ and then scale all of the measurements with Tsys. [When performing absolute flux calibration, we try to use scans that observe the absolute flux calibrator and complex gain calibrator at comparable elevations to minimize the bias due to the $(1 - e^{-\tau_{atm}}) T_{atm}$ term]

※ Errors in T_{sys}^{eff} can lead to very serious absolute flux errors after applying Tsys. Therefore, we need to visually inspect the T_{sys}^{eff} measurements and make sure that nothing look abnormal.
Otherwise, we need to either fix/replace the abnormal data or flag those data.

※ If P_{CMB} is significantly contaminated by the molecular line emission in the Milky Way, the derived T_{sys}^{eff} can by incorrectly interpreted when applying to data.

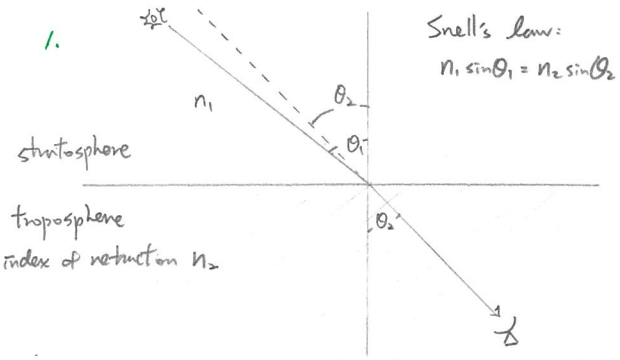
Besides attenuation, other atmospheric effects include

1. pointing errors (due to Snell's law): this can be corrected based on some knowledge/model of the index of refraction n. The residual errors can be corrected when perform the reference pointing calibration
2. wave front distortion that leads to
   - (i) decoherence: loss of visibility amplitude
   - (ii) reduced angular resolution: seeing effect

   → these errors also lead to imperfectness in the imaging procedures. As a result, the r.m.s. noise is also increased.

wave front → a surface of constant phase

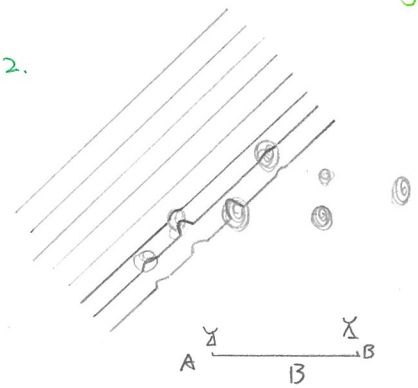
1.



Snell's law:
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

(when the target is at 45° elevation, the order of $\delta\theta$ is ~1 arcmin)

2.



atmosphere of non-uniform density

when arriving the two antennae, the phase offset $\phi_A(t)$ and $\phi_B(t)$ needs to be considered, instead a simple ϕ_0

phase change over a traveling distance D: $\phi = 2\pi \cdot \left(\frac{nD}{\lambda}\right)$ → phase error is inversely proportional to λ

electrical path length: $L_e \equiv nD$

Index of refraction of the atmosphere n.

usually expressed as the difference with respect to vacuum ($n_{vacuum} \equiv 1$) in parts per million

$$N \equiv (n-1) \cdot 10^6$$

- dry air component: $N_d = 2.2 \cdot 10^5 \rho$ (mass density in unit of $g\,cm^{-3}$)
- water vapor component: $N_w = 1.7 \cdot 10^9 \cdot \frac{P_{wv}}{T_{atm}}$ (P_{wv}: water vapor) $\quad P_{wv} = \frac{PWV}{D}$

→ possible to gauge phase error if there is a way to directly measure H_2O column density in the line of sight: Water vapour radiometer

$$\Rightarrow \boxed{L_e^{pwv} = 1.7 \cdot 10^3 \frac{PWV}{T_{atm}}} \quad \text{and} \quad \boxed{\phi = \frac{12.6}{\lambda}\pi \cdot PWV}$$

assuming $T_{atm} = 270K$

trigonometric identity
$\cos\theta\cos\varphi = \frac{1}{2}(\cos(\theta-\varphi) + \cos(\theta+\varphi))$

consider the cross-correlation.

$$\propto \cos(kx - \omega t + \phi_A(t)) \cos(k(x+\Delta x) - \omega t + \phi_B(t))$$

$$= \cos(-k\Delta x + (\phi_A(t) - \phi_B(t))) + \underbrace{\cos(2kx - 2\omega t + \phi_A(t) + \phi_B(t) + k\Delta x)}_{\text{long-time average is zero}}$$

In a short snapshot observations, this term has a similar effect with $-k\Delta x$, It means that when we have $\phi_A(t) - \phi_B(t)$ term, we will feel that the direction of incoming light is changing with time (i.e., seeing effect)

If you insert $\phi_A(t)$ and $\phi_B(t)$ in the derivation of the complex visibility of a point-source (delta function), you will find that the term $\phi_A(t) - \phi_B(t)$ contributes to a visibility-phase offset.

→ phase errors that can be expressed this way is referred to as antenna based phase errors.

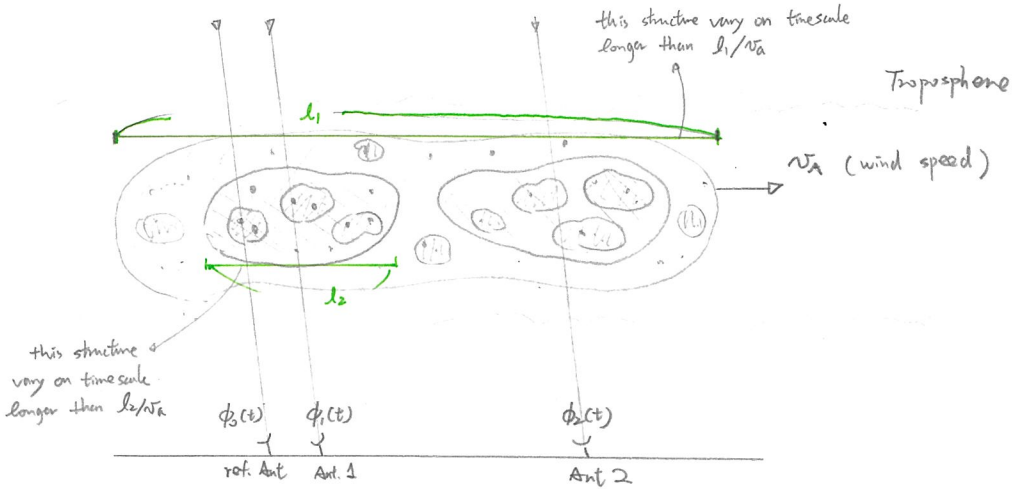
Such phase errors are referred to as tropospheric phase fluctuation. In modern interferometric observations, the time dependent phase errors are dominated by tropospheric phase fluctuations.

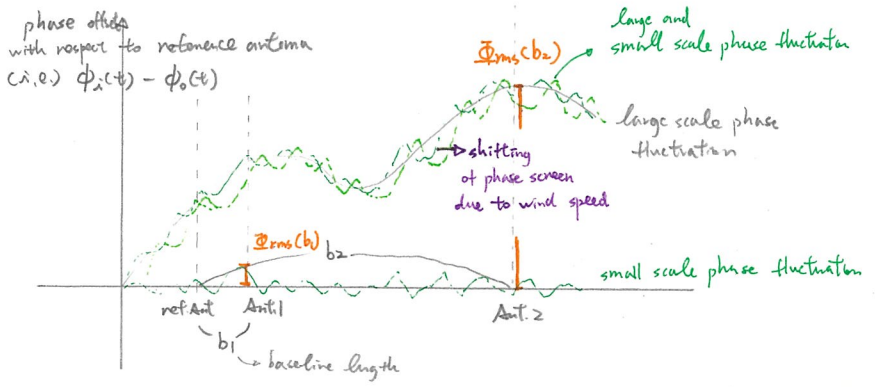
Tropospheric phase model: Taylor hypothesis (frozen screen approximation)

tropospheric phase fluctuation is due to turbulence in the troposphere.

The turbulence intensity is low, the turbulence is approximately stationary and homogeneous.

The turbulence field is unchanged over the atmospheric boundary layer time scale $\frac{b}{v_a}$, where b is the baseline length, and v_a is the wind speed.





order of phase change rate at baseline b : $\Phi_{rms}(b) \cdot \frac{\Delta x}{\Delta t} \sim \Phi_{rms} v_a$

↳ longer baselines see the more rapid phase variations.

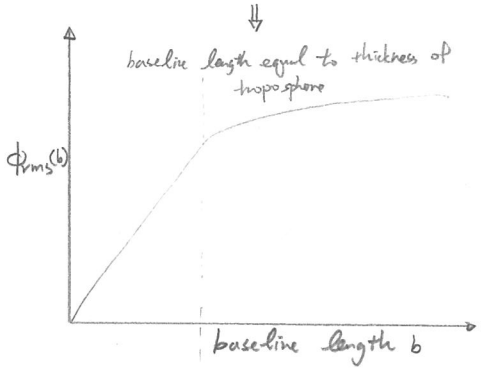
Kolmogorov turbulence theory

$$\Phi_{rms}(b) = \frac{K}{\lambda_{mm}} b^{\alpha} \quad [\text{deg}]$$

K: a constant depending on site condition. $K=300$ for JVLA

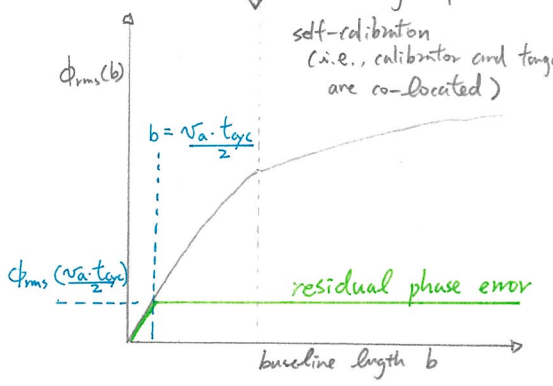
α:
- $\alpha = \frac{5}{6}$: baselines shorter than the thickness of turbulent layer
- $\alpha = \frac{1}{3}$: baselines longer than the thickness of turbulence layer

⇓



⇓

after gain phase self-calibration (i.e., calibrator and target are co-located)



effect of gain phase calibration : calibrating out the phase fluctuations on larger than $v_a \cdot t_{cyc}$ scales. i.e., for baselines longer than $b_{off} = \frac{1}{2} v_a t_{cyc} + d$ the rms phase error will saturate to

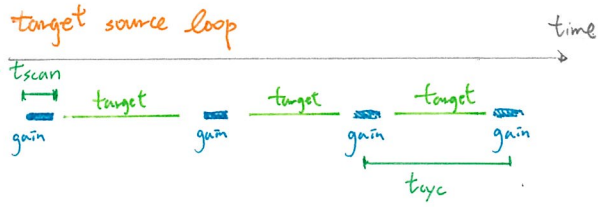
$\Phi_{rms}(\frac{1}{2} v_a t_{cyc} + d)$

d: physical distance between the calibrator and target source in the troposphere

Complex gain phase calibration

Observing a source (usually a point source) that has known structure and location, which has very small angular separation from the target source. (to allow tracing the same atmospheric phase fluctuation, and to minimize slewing time)

We call such calibration source gain calibrator, which are high redshift quasars in most cases. The observations on the gain calibrator is repeated every cycle time t_{cyc} to allow interpolating the calibration solutions onto the target source scans.



Effect of residual phase error

(assuming gaussian random phase fluctuation)

- decoherence : attenuating visibility amplitude (after time averaging) by $e^{-\phi_{rms}^2/2}$, where ϕ_{rms} is in unit of radian
- seeing

Gain calibrators need to have stationary (over a night) flux density such that we can also use it to calibrate time dependent amplitude errors.

→ recommended scan length : given the relevant observing bandwidth, for every baseline (i.e., the visibility taken with every two antennae) for every polarization, the scan length should allow the S/N of that scan to be 5~10

(do not use unnecessarily long scan length. Otherwise, there is no time to integrate [illegible])

Frequency dependent amplitude and phase response: passband calibration

This is usually related to instrumental effect. For example, if the signal path in an antenna has some length errors (e.g. due to thermal expansion/contraction), it can induce a delay error. In the visibility, this is manifested as the additive phase error that depends linearly on frequency. In most cases, the passband phase/amplitude are stationary over a night's observations. We can calibrate out this effect by observing a bright point-like source. (It is ok to have time-variable flux density. It is usuable as long as it is bright) that is at a known coordinate.

Usually, there is only one or two scans on the passband calibrator(s). The overall integration time needs to allow achieving a $S/N \sim 5\sim10$ in each polarization with every two antennae, and with the frequency width that is relevant to the science purpose. For example, if the observations requires 1 MHz frequency resolution, then we need to achieve a $S/N \sim 5-10$ in every 10 MHz frequency interval.

Absolute flux calibrator

We need to observe an absolute flux standard, namely the absolute flux calibrator to re-scale the visibility amplitudes to the calibrated absolute scales (e.g. in Jy unit). At cm bands (e.g. with the JVLA) this is done by observing a quasar that is known to have stationary flux density (e.g. over >10 yrs timescale). The primary absolute flux calibrators for the JVLA are 3C286, 3C147, and 3C48 quasars. At millimeter/submillimeter bands, the absolute flux calibrators are usually planets or planet moons, of which the flux density of the thermal continuum emission can be modeled based on its distance to the Sun and the earth, and the absorption/scattering opacity at their surface.

i.e. the heating source

We should be careful that some planets have extended angular scale (e.g. Jupiter) and can be largely filtered out in the observations with long baseline. When planning observations, we should avoid using such calibrators since they do not provide good S/N ratios.

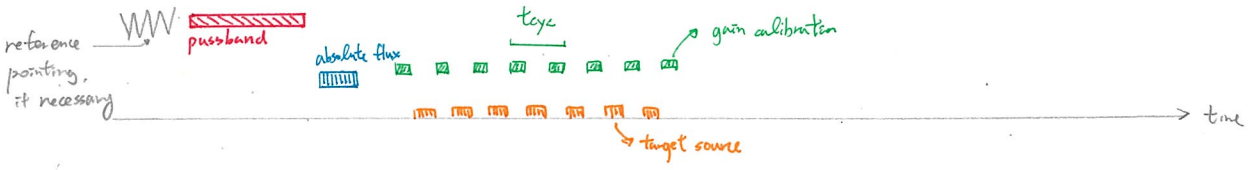
Polarization calibration

When performing polarization observations, we need to calibrate polarization leakage and polarization position angle. This part is complicated and cannot be covered in this lecture. If you're interested in details, please check Sault, R.J. et al. 1996, A&AS, 117, 149

Summary 1: When organizing an observing script, we need to consider

1. scans on the referencing calibrator: depending on the pointing accuracy of the telescopes and the field-of-view. For example, for the SMA, a scan at the begining of the observations and a scan in the midnight is sufficient. For JVLA, more frequent reference pointing calibration is necessary if observe at >30 GHz frequency.
2. absolute flux calibration scans: on compact (marginally resolved or unresolved) source(s) which have known and stationary flux densities. For the SMA, we almost always use planet(s) or planet moons.
3. passband calibration scans: on bright enough compact sources that can locate anywhere on the sky.
4. complex gain calibrator: bright compact sources that locate close to the target sources. The flux density of the complex gain calibrator needs to be stationary over a night's observations. Most of the quasars are OK. Their host SMBH are $\gtrsim 10^8 M_\odot$. Their radio/(sub)millimeter emission are dominated by the synchrotron emission from the relativistic accretion flow close to the event horizon. The characteristic timescale for flux density variation is the orbital timescale at their innermost stable circular orbit.

The typical observing scans can look like



Summary 2: When calibrating the observation data, usually we follow the steps

1. Inspect data and flag bad data (and pointing calibration scans)
2. Apply T_{sys}^{eff}. (and gain curve, requantizer table, etc, if relevant)
3. perform passband calibration
4. (i) perform complex gain calibration
   (ii) reference absolute flux

↓

1. imaging
2. self-calibration (if applicable)