An Introduction to Noise

In auto-correlation (e.g. in single-dish observations)

electric field propagated from target source

if the noise and target sources are not correlated

$$\text{power} \propto \langle (E_i + n_i)^2 \rangle = \langle E_i^2 \rangle + \langle n_i^2 \rangle + 2\overbrace{\langle E_i n_i \rangle}^{=0} \propto (T_{A_i} + T_{sys_i})\Delta\nu$$

T_{sys_i}: system temperature

T_{A_i}: antenna temperature

n_i: contribution from noise sources, e.g.

1. receiver
2. ground
3. (i) celestial source
   (ii) atmospheric emission

Noise makes a net bias to the power.

Cross-correlation

※ a strategy to characterize a Gaussian random process:
(1) knowing the standard deviation of the process (2) knowing how many independent sampling we get.

$$\frac{t}{\Delta t} = \frac{t}{\frac{1}{\Delta\nu}} = t\,\Delta\nu$$

$\Delta\nu$: bandwidth

Δt: the smallest time unit we can resolve

$$\text{power} \propto \langle (E_i + n_i)(E_j + n_j) \rangle$$

$$= \langle E_i E_j \rangle + \underbrace{\langle E_i n_j \rangle}_{=0} + \underbrace{\langle n_i E_j \rangle}_{=0} + \underbrace{\langle n_i n_j \rangle}_{=0}$$

After a long-term time-averaging, noise makes no net bias to auto-correlation.

$\Longrightarrow$ In each measurement, $\langle E_i E_j \rangle$ is a random process that the mean value is identical to that when there is no noise. When there is noise, the uncertainty of $\langle E_i E_j \rangle$ can be expressed by the standard deviation σ of $\langle E_i E_j \rangle$:

$$\sigma^2 \propto \left\langle \left([(E_i + n_i)(E_j + n_j)] - \langle E_i E_j \rangle \right)^2 \right\rangle$$

$$= \langle [(E_i + n_i)(E_j + n_j)]^2 \rangle$$

$$-2 \langle \underbrace{[(E_i + n_i)(E_j + n_j)] \langle E_i E_j \rangle}_{= -2\langle E_i E_j \rangle^2} \rangle$$

$$+ \langle \langle E_i E_j \rangle^2 \rangle$$

$= -2\langle E_i E_j \rangle^2$, if $\langle n_i \rangle = 0$, $\langle n_j \rangle = 0$, $\langle n_i n_j \rangle = 0$ — white noise fulfill this assumption.

$$= \langle [(E_i + n_i)(E_j + n_j)]^2 \rangle - \langle E_i E_j \rangle^2$$

high noise limit, $n \gg E$

$$\sim \langle n_i^2 n_j^2 \rangle = \langle n_i^2 \rangle \langle n_j^2 \rangle \propto T_{sys}^2$$

identity for random variables:

$$\langle z_1 z_2 z_3 z_4 \rangle = \langle z_1 z_2 \rangle \langle z_3 z_4 \rangle + \langle z_1 z_3 \rangle \langle z_2 z_4 \rangle + \langle z_1 z_4 \rangle \langle z_2 z_3 \rangle$$

rms noise of a visibility:

$$\Delta S_{ij}\,[\mathrm{Jy}] = \frac{1}{\eta_s} \frac{T_{sys}/K}{\sqrt{2\,\Delta\tau\,\Delta\nu}}$$

K: a coefficient to convert from T_{sys} to flux density

$\Delta\nu$: integration bandwidth

$\Delta\tau$: integration time

$\eta_s < 1$, for considering any effect which artificially degrade the correlation.

In imaging

characterizing how difficult it is to distinguish a real signal from noise.

$$\Delta I_m = \frac{1}{\eta_s} \frac{T_{sys}/K}{\sqrt{2\,\Delta\nu\,\Delta\tau}\,\sqrt{\frac{1}{2}N(N-1)}}$$

N: number of antennae. $C_2^N = \frac{1}{2}N(N-1)$: number of baselines.