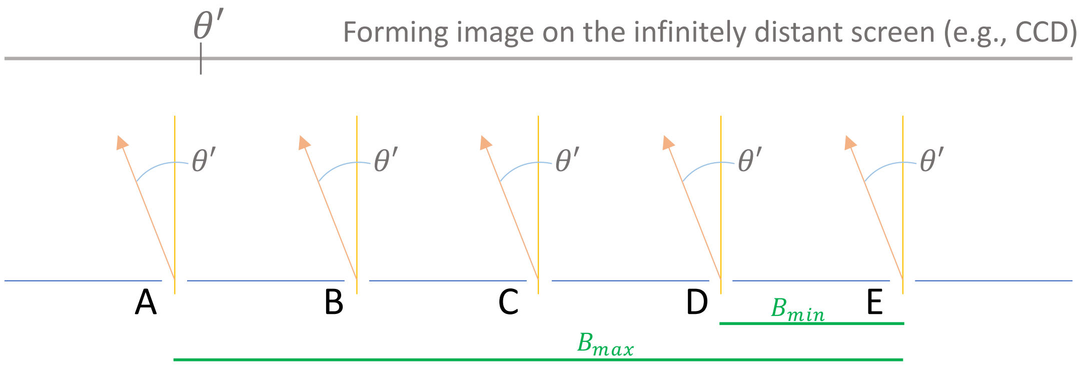
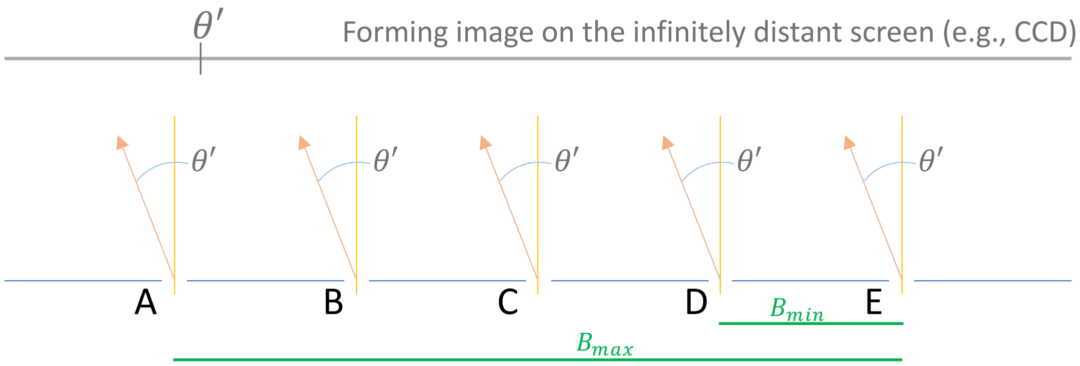
An Introduction to Radio Interferometry

4-2 Complex visibilities of an interferometric array

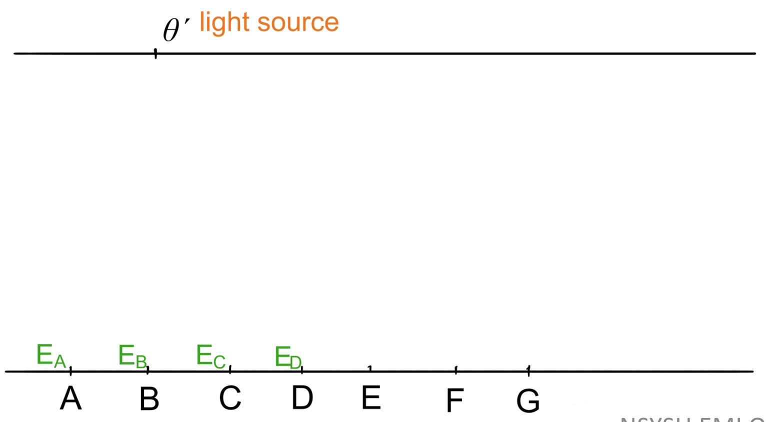
You can find relevant material
on my personal webpage

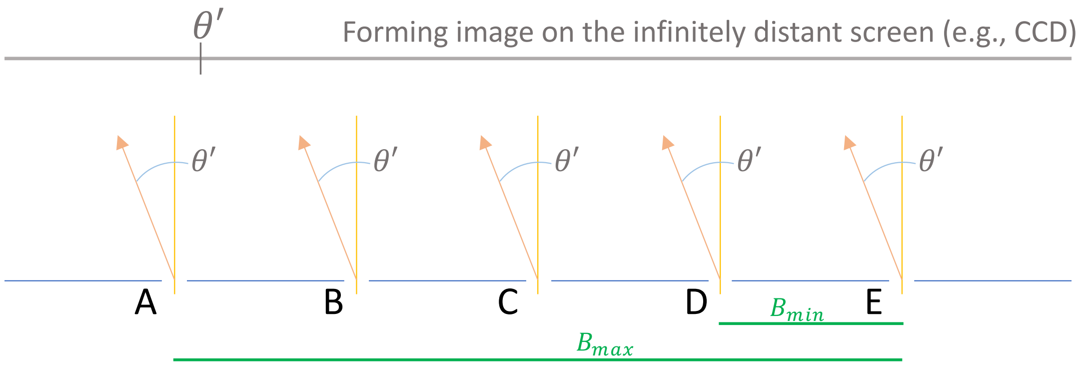


θ'
Forming image on the infinitely distant screen (e.g., CCD)
θ'
θ'
θ'
θ'
θ'
A
B
C
D
B_{min}
E
B_{max}



Interferometric array observation towards a infinitely distant, monochromatic source





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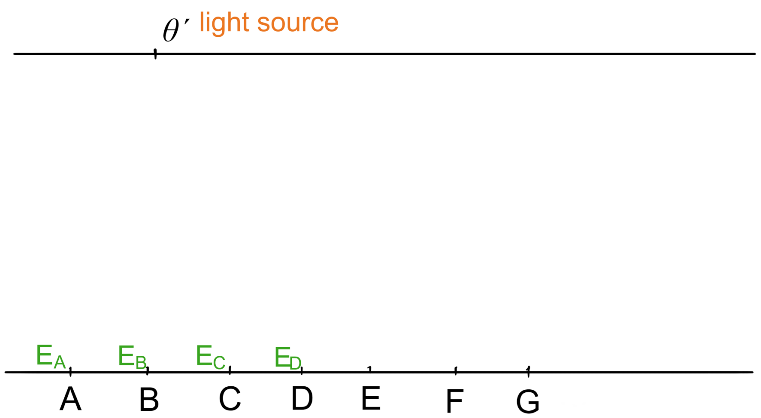
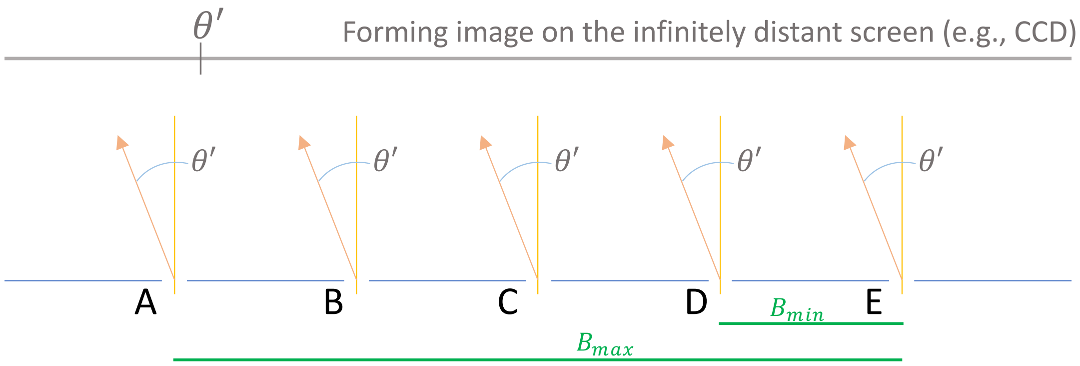


$$E_A = E_0 \cos(kx - \omega t + \phi_0)$$

With correlators, we can evaluate each of the following terms:

$$< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 >$$
$$=< {E_A}^2 >+< {E_B}^2 >+< {E_C}^2 >+< {E_D}^2 >+< {E_E}^2 >$$
$$+< 2E_A E_B >+< 2E_A E_C >+< 2E_A E_D >+< 2E_A E_E >$$
$$+< 2E_B E_C >+< 2E_B E_D >+< 2E_B E_E >$$
$$+< 2E_C E_D >+< 2E_C E_E >$$
$$+< 2E_D E_E >$$



Interferometric array observation towards a infinitely distant, monochromatic source

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$$\begin{aligned}
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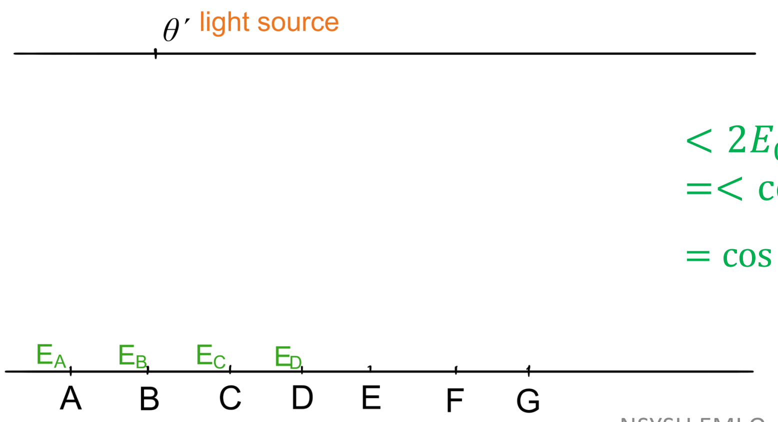
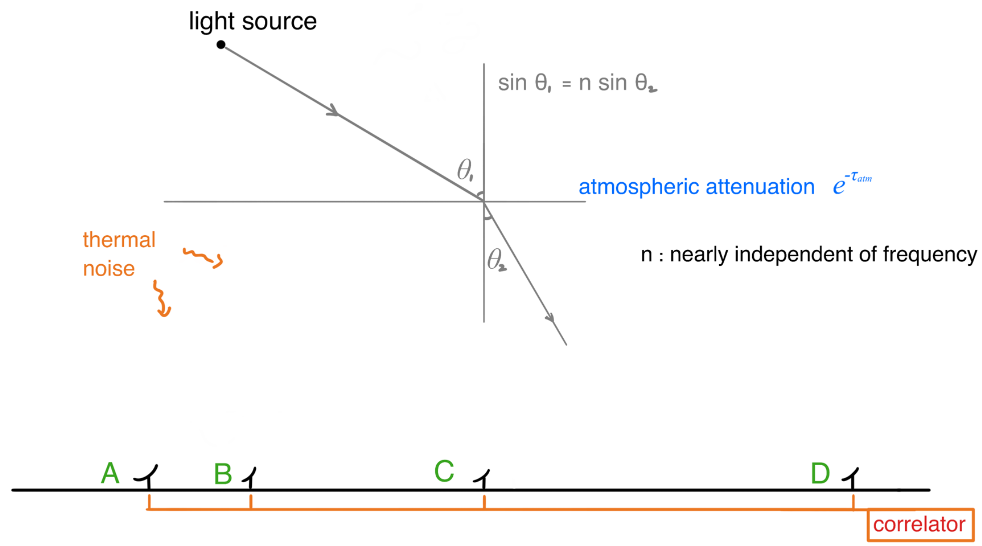
&=< {E_A}^2 >+< {E_B}^2 >+< {E_C}^2 >+< {E_D}^2 >+< {E_E}^2 > \\
&\quad +< 2E_A E_B >+< 2E_A E_C >+< 2E_A E_D >+< 2E_A E_E > \\
&\quad +< 2E_B E_C >+< 2E_B E_D >+< 2E_B E_E > \\
&\quad +< 2E_C E_D >+< 2E_C E_E > \\
&\quad +< 2E_D E_E >
\end{aligned}$$



Form of each cross-term

$$\begin{aligned}
&< 2{E_0}^2 \cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) > \\
&=< \cos(k\Delta x) >+< \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) >, \qquad \Delta x = nB_{min} \sin\theta' \\
&= \cos\left(\frac{2\pi}{\lambda} nB_{min} \sin\theta'\right) \sim \cos\left(\frac{2\pi}{\lambda} nB_{min} \theta'\right)
\end{aligned}$$

1D linear radio interferometer



$E_A = E_0 \cos(kx - \omega t + \phi_0)$

With correlators, we can evaluate each of the following terms:

$$
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&< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 > \\
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&\quad +< 2E_AE_B >+< 2E_AE_C >+< 2E_AE_D >+< 2E_AE_E > \\
&\quad +< 2E_BE_C >+< 2E_BE_D >+< 2E_BE_E > \\
&\quad +< 2E_CE_D >+< 2E_CE_E > \\
&\quad +< 2E_DE_E >
\end{aligned}
$$

General expression for the cross (cosine) correlation of antennae i and j:

$$
\begin{aligned}
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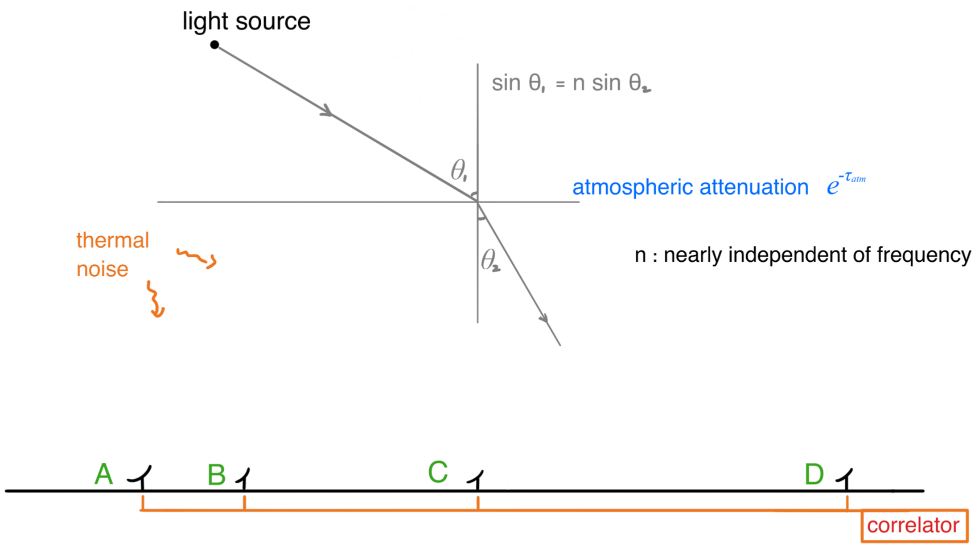

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&\quad +< 2E_B E_C >+< 2E_B E_D >+< 2E_B E_E > \\
&\quad +< 2E_C E_D >+< 2E_C E_E > \\
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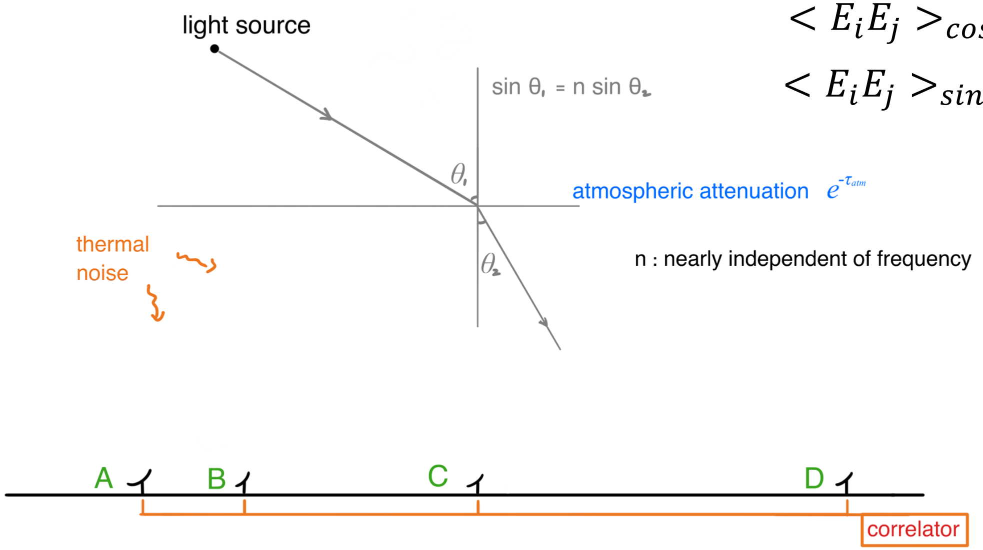
&< 2{E_0}^2 \cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) > \\
&=< \cos(k\Delta x) >+< \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) >, \\
&= \cos\left(\frac{2\pi}{\lambda} B_{ij} \sin\theta'\right) \sim \cos\left(2\pi \frac{B_{ij}}{\lambda} \theta'\right) \equiv \cos(2\pi u_{ij} \theta')
\end{aligned}
$$

$$\Delta x = \underbrace{B_{ij}}_{\text{Separation of antennae i and j}} \sin\theta'$$

1D linear radio interferometer



Evaluating Complex visibility of a point source at θ'
(i.e., Dirac delta function) taken from antennae i and j

$$< E_i E_j >_{cos} \cong \tilde{P}(\theta') \cos\left(\frac{2\pi}{\lambda} B_{ij}\theta'\right) = \int \delta(\theta - \theta')\, \tilde{P}(\theta) \cos(2\pi u_{ij}\theta)\, d\theta$$

$$< E_i E_j >_{sin} \cong \tilde{P}(\theta') \sin\left(\frac{2\pi}{\lambda} B_{ij}\theta'\right) = \int \delta(\theta - \theta')\, \underbrace{\tilde{P}(\theta)}_{\text{Response function of each single-dish reflector}} \sin(2\pi u_{ij}\theta)\, d\theta$$

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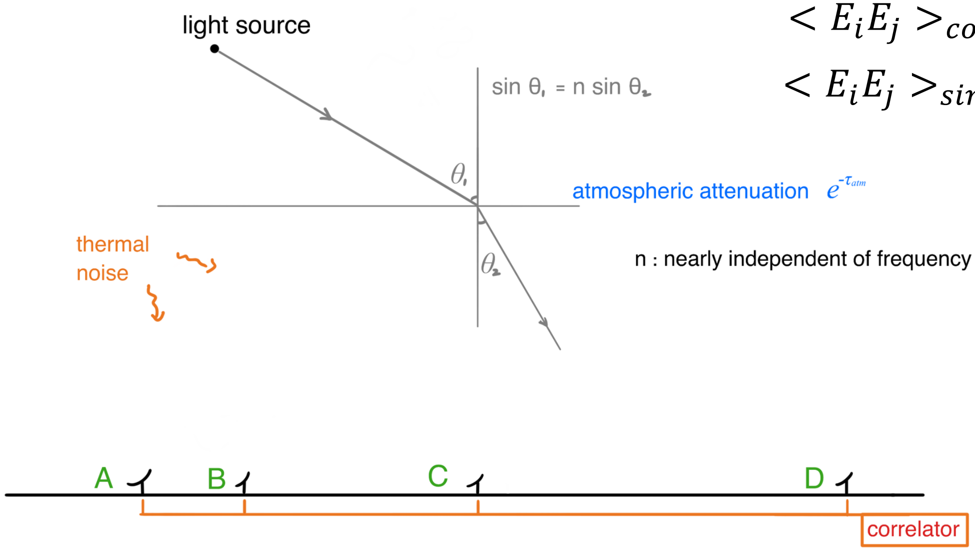

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$$V_{ij} = \int \delta(\theta-\theta')\widetilde{P(\theta)}e^{i2\pi u_{ij}\theta} d\theta$$

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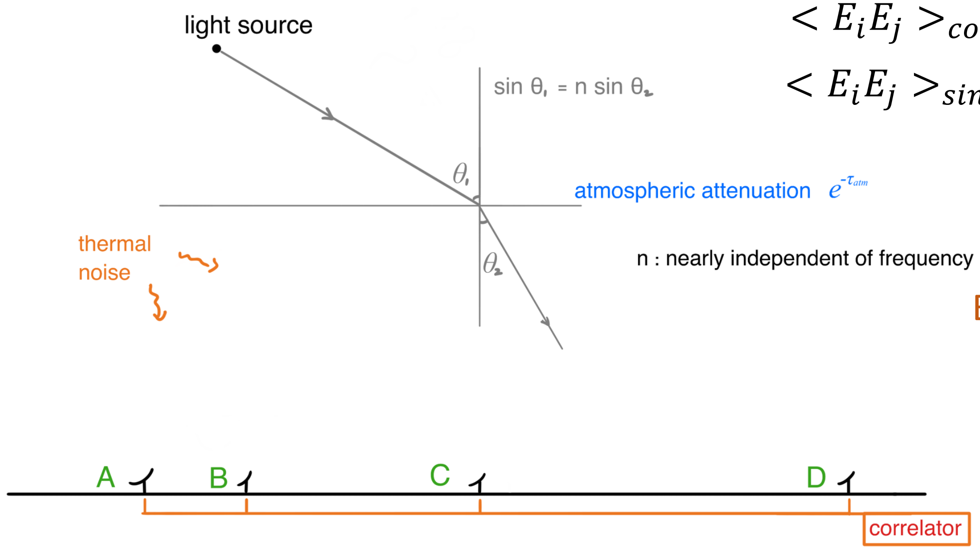

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$$V_{ij} = \int \delta(\theta-\theta')\widetilde{P(\theta)}e^{i2\pi u_{ij}\theta}d\theta$$

Evaluating Complex visibility for continuous intensity distribution

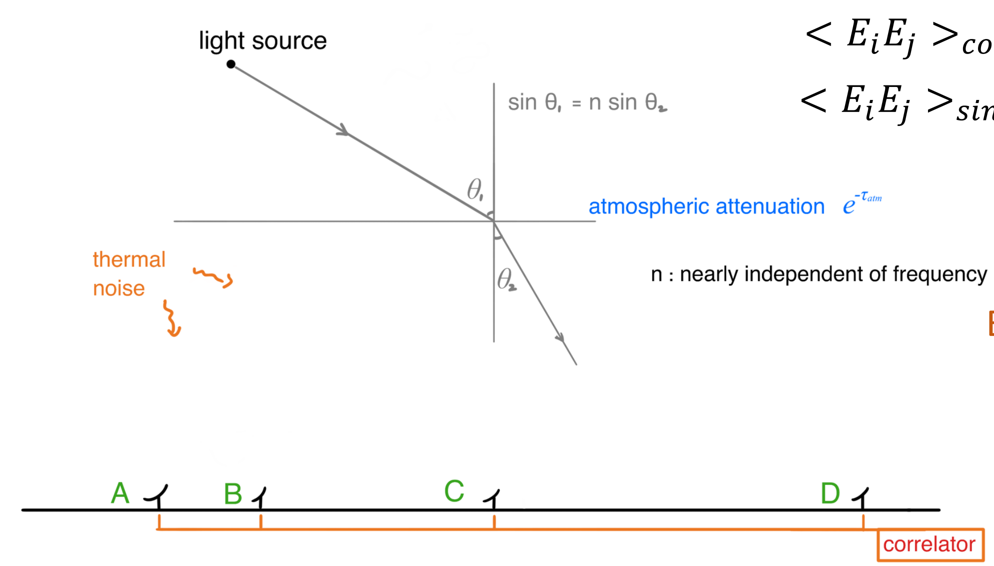
$$V_{ij} = \int A(\theta)\widetilde{P(\theta)}e^{i2\pi u_{ij}\theta}d\theta$$

General expression for the cross (cosine) correlation of antennae i and j:

$$< 2{E_0}^2 \cos(kx-\omega t+\phi_0)\cos(k(x+\Delta x)-\omega t+\phi_0) >$$
$$=< \cos(k\Delta x) >+< \cos(2kx-2\omega t+2\phi_0+k\Delta x) >,$$
$$= \cos\left(\frac{2\pi}{\lambda}B_{ij}\sin\theta'\right) \sim \cos\left(\frac{2\pi}{\lambda}B_{ij}\theta'\right)$$

$$\Delta x = \underbrace{B_{ij}}_{\text{Separation of antennae i and j}} \sin\theta'$$

1D linear radio interferometer



Evaluating Complex visibility of a point source at θ' (i.e., Dirac delta function) taken from antennae i and j

$$< E_i E_j >_{cos} \cong \tilde{P}(\theta')\cos\left(\frac{2\pi}{\lambda}B_{ij}\theta'\right) = \int \delta(\theta-\theta')\,\tilde{P}(\theta)\cos(2\pi u_{ij}\theta)\,d\theta$$

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Evaluating Complex visibility for continuous intensity distribution

$$V_{ij} = \underbrace{\int A(\theta)\widetilde{P(\theta)}e^{i2\pi u_{ij}\theta}\,d\theta}_{\text{Mathematical form of fourier transform}}$$

Mathematical form of fourier transform

General expression for the cross (cosine) correlation of antennae i and j:

$$< 2{E_0}^2\cos(kx-\omega t+\phi_0)\cos(k(x+\Delta x)-\omega t+\phi_0) >$$
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$$\Delta x = \underbrace{B_{ij}}\sin\theta'$$

Separation of antennae i and j

1. The cross-correlations produced from any pair of antennae in an interferometric array are not different from those of a 1D, 2-element interferometer.
2. The integrated measurements of an interferometric array are a collection of the complex visibilities measured from individual baselines (i.e., pair of antennae).
3. The response function of an interferometric array is very similar to the interference pattern of a diffraction grating.