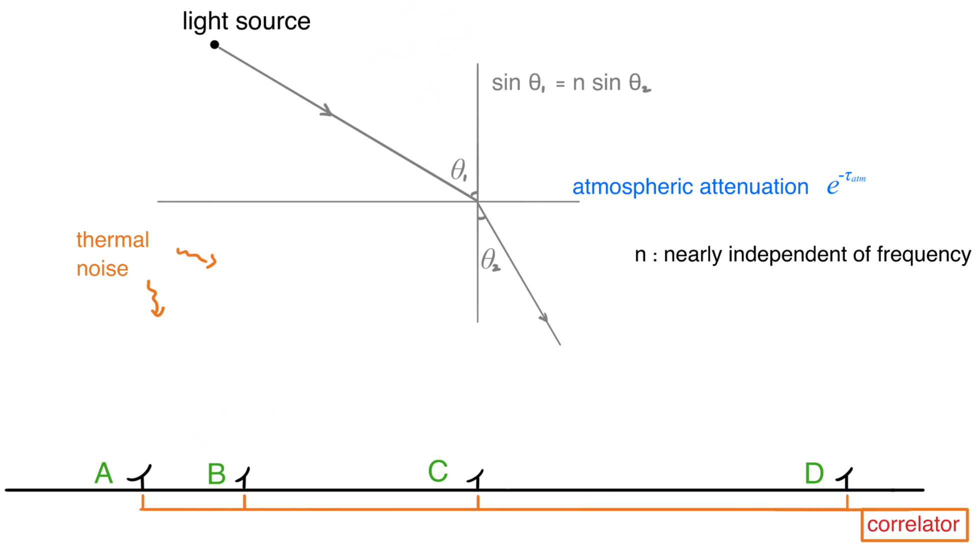
An Introduction to Radio Interferometry

4-3 Inversion of complex visibility

You can find relevant material
on my personal webpage

NSYSU EMI Online Lecture Series Hauyu Baobab Liu (呂浩宇),
Department of Physics

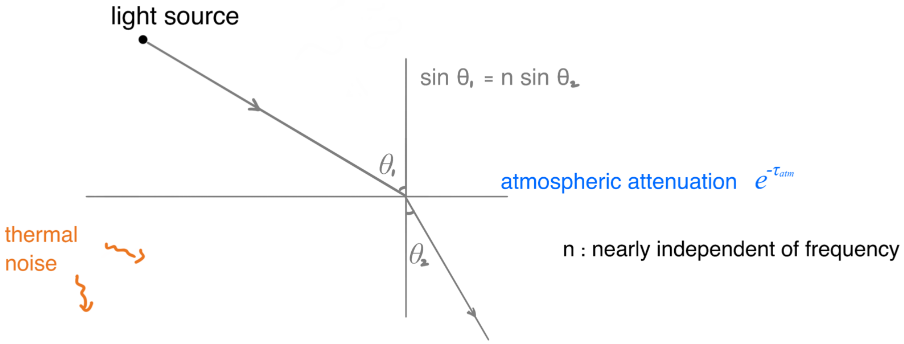
1D linear radio interferometer

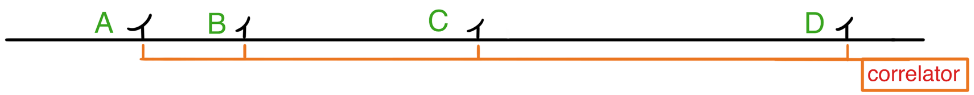


Evaluating Complex visibility for continuous intensity distribution

$$V_{ij} = \int A(\theta)\widetilde{P(\theta)}e^{i2\pi u_{ij}\theta}d\theta$$

1D linear radio interferometer





Evaluating Complex visibility for continuous intensity distribution

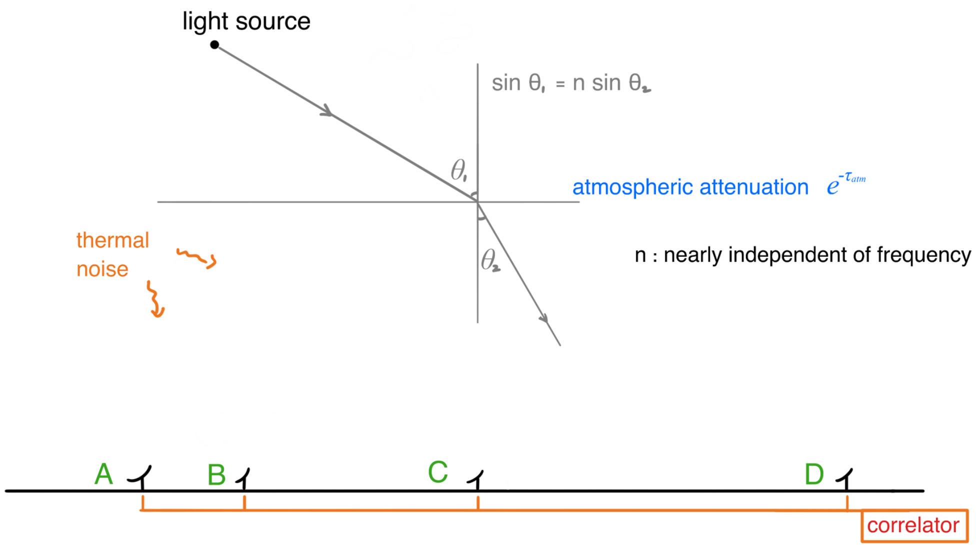
$$V(u) = \underbrace{\int A(\theta)\widetilde{P(\theta)}e^{i2\pi u\theta}d\theta}_{\textbf{Mathematical form of fourier transform}}$$

2D interferometric array

$$V_\nu(u,v,w) \sim \overbrace{\iint I_\nu(\ell,m)\frac{e^{-2\pi i(u\ell+vm+w)}}{1}\,d\ell\,dm}^{\text{Fourier transform of intensity distribution}}$$

$$= e^{-2\pi iw}\iint I_\nu(\ell,m)\,e^{-2\pi i(\vec{u}\cdot\vec{\sigma})}d\ell\,dm$$

1D linear radio interferometer



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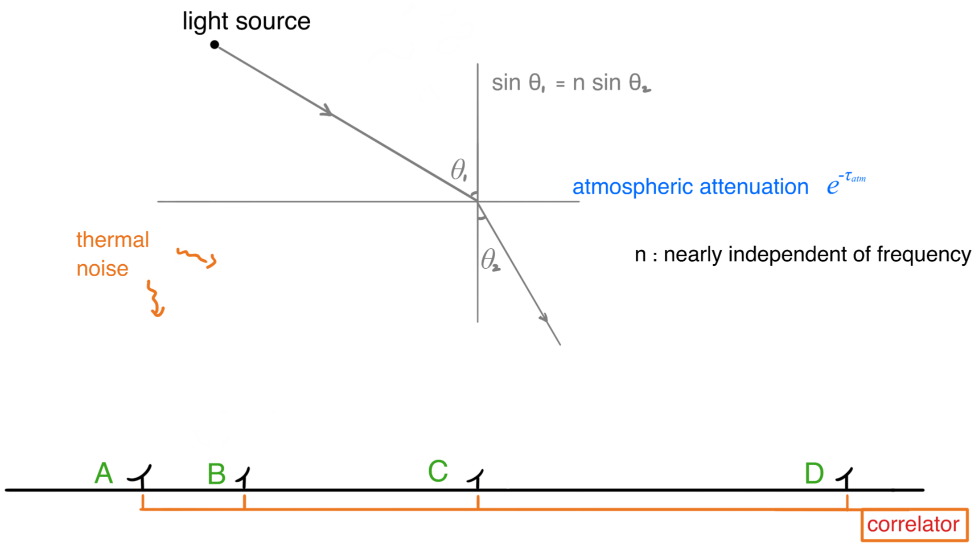
What we directly measure

1D linear radio interferometer



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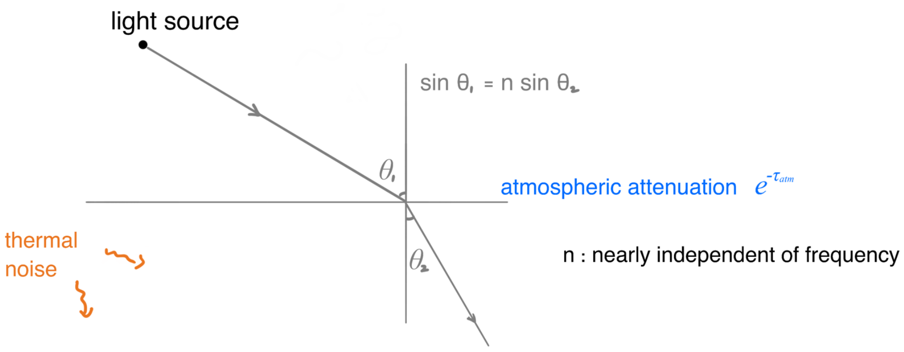
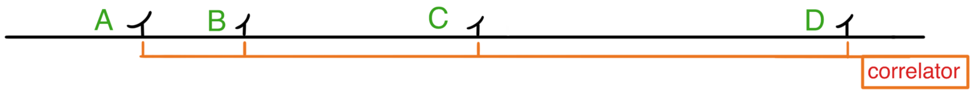
What we are interested in

1D linear radio interferometer





If $V(u)$ (or $V_\nu(u,v,w)$) can be completely sampled (i.e., measured at any u (or (u,v,w)))

Evaluating Complex visibility for continuous intensity distribution

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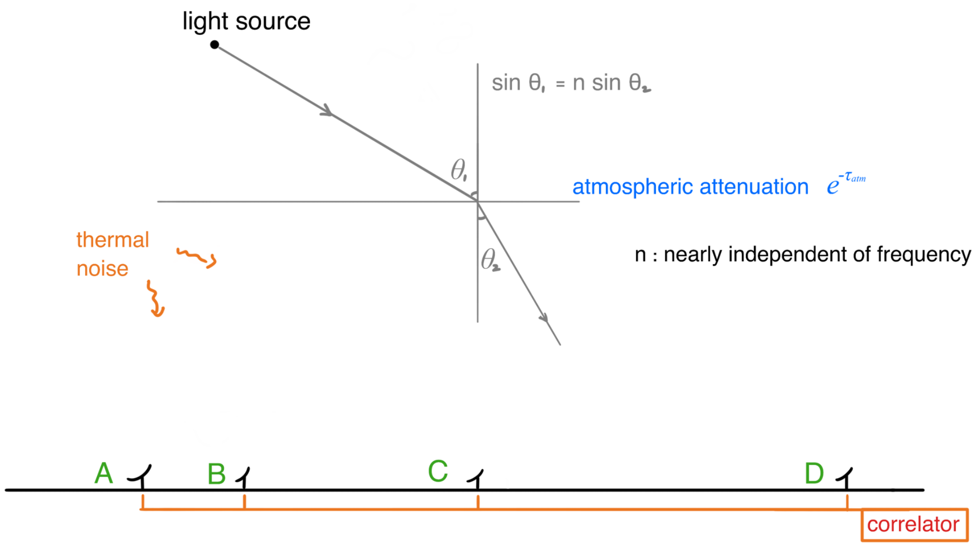

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Mathematical form of fourier transform

Inverse fourier transform

$$A(\theta')\widetilde{P(\theta')} = \int V(u) e^{-i2\pi u\theta'} du$$

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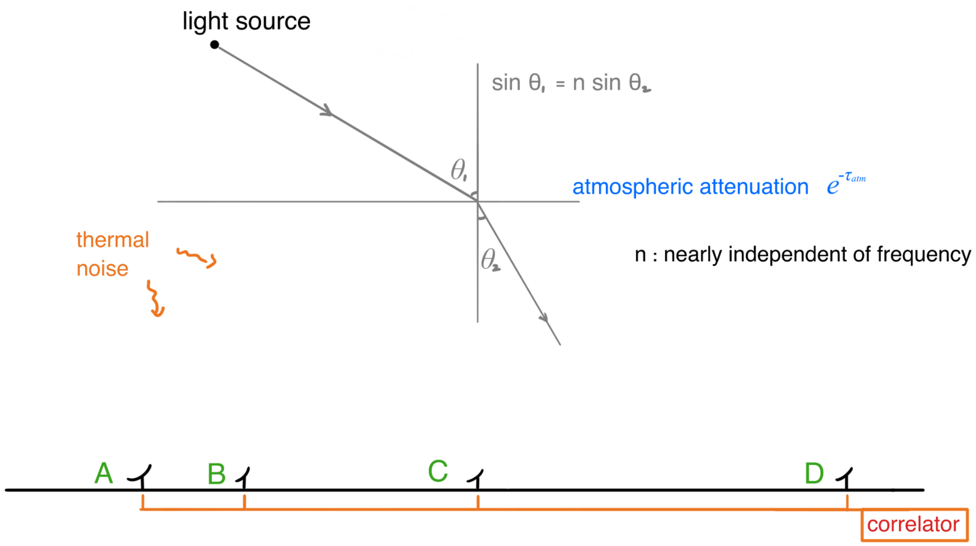

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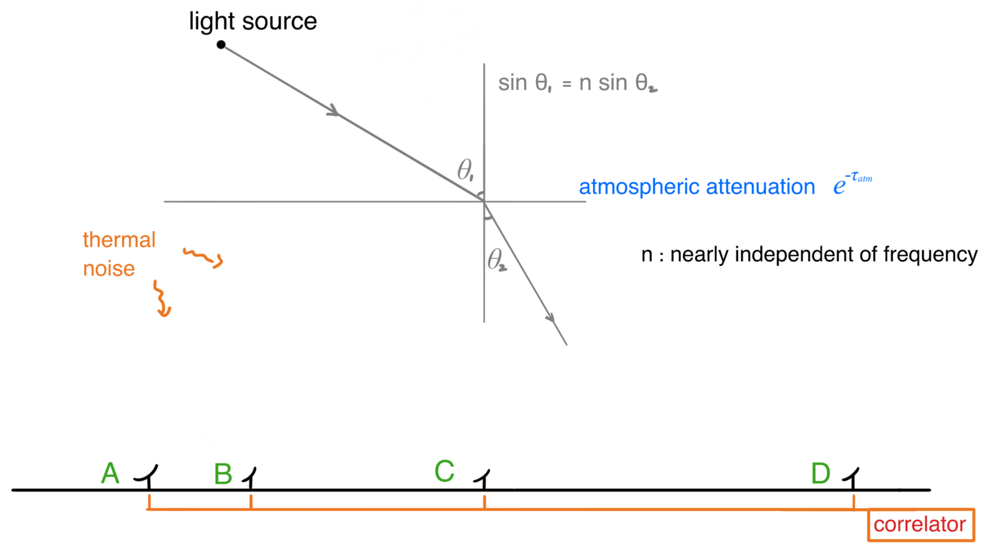
Requiring measurements at arbitrary antenna-separations, even at infinitely small antenna-separations.

In reality, an interferometric array can only take discretized samples at certain antenna-separations. In addition, the shortest possible (projected) antenna-separation is limited by the sizes of the reflector.

1D linear radio interferometer



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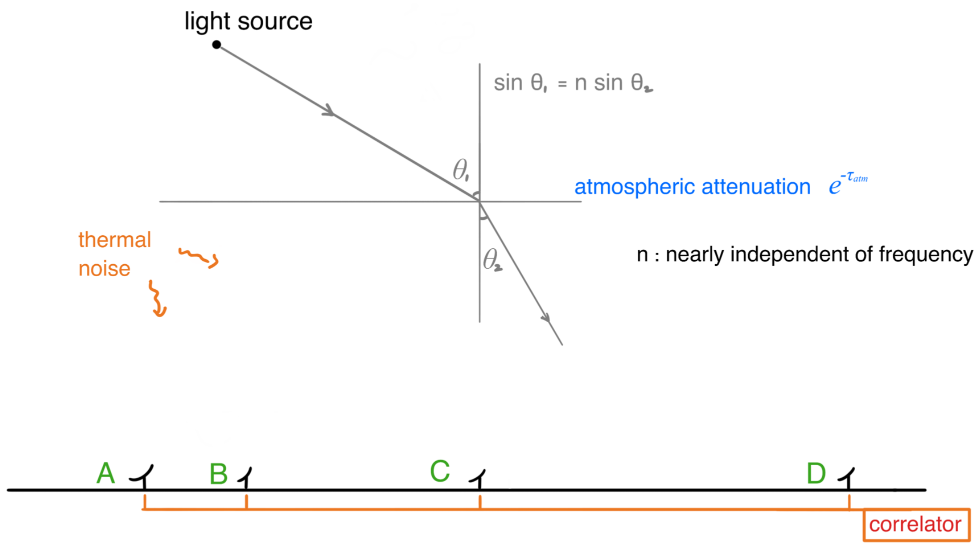
Sampling function (at $u_k, k = 1,2,3, \ldots, M$)

$$S(u) \equiv \sum_{k=1}^{M} \delta(u - u_k)$$

1D linear radio interferometer



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Naiive inverse fourier transform

$$\left[A(\theta')\widetilde{P(\theta')}\right]^D \equiv \int S(u)V(u)e^{-i2\pi u\theta'}du$$

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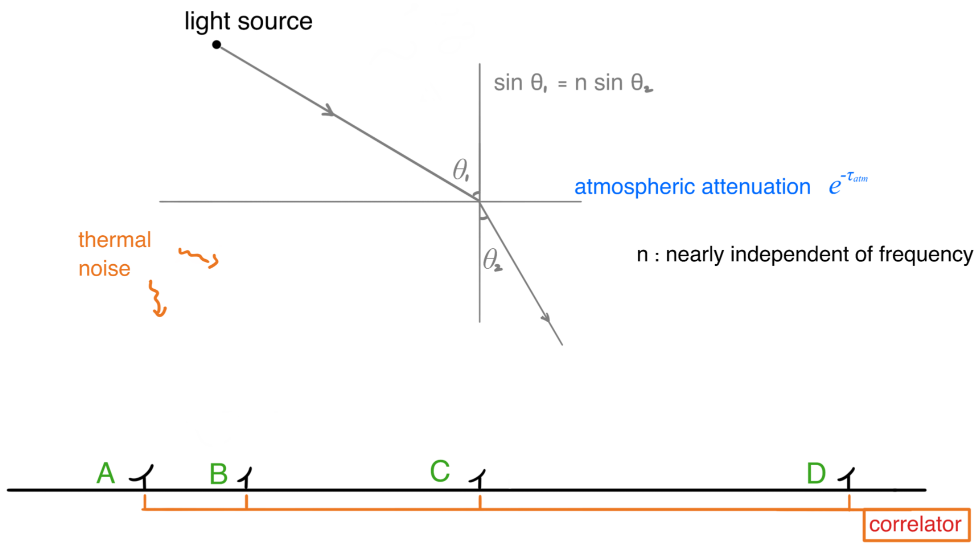

Convolution theorem

Lecture Unit 2-4

$$F.T.(f * g) = F.T.(f) \cdot F.T.(g)$$

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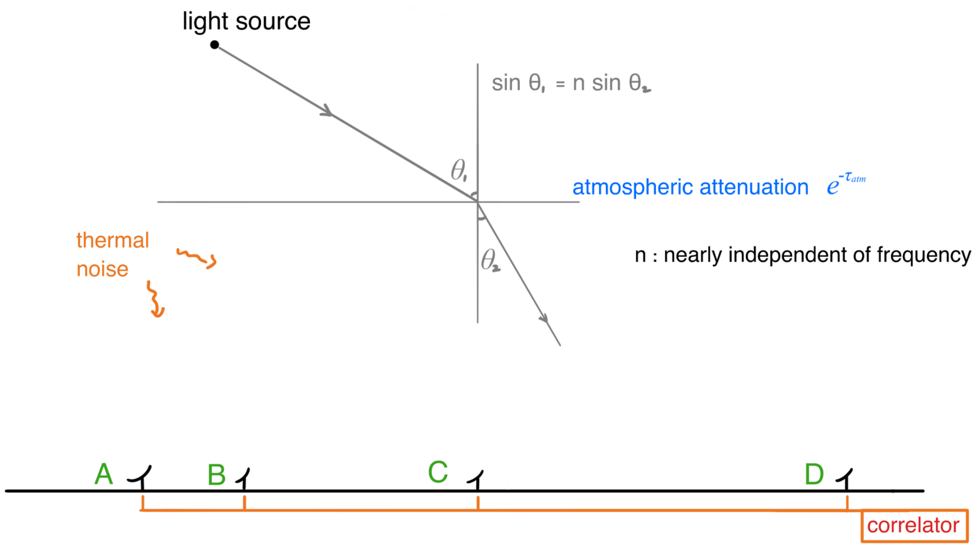
Dirty image

1D linear radio interferometer



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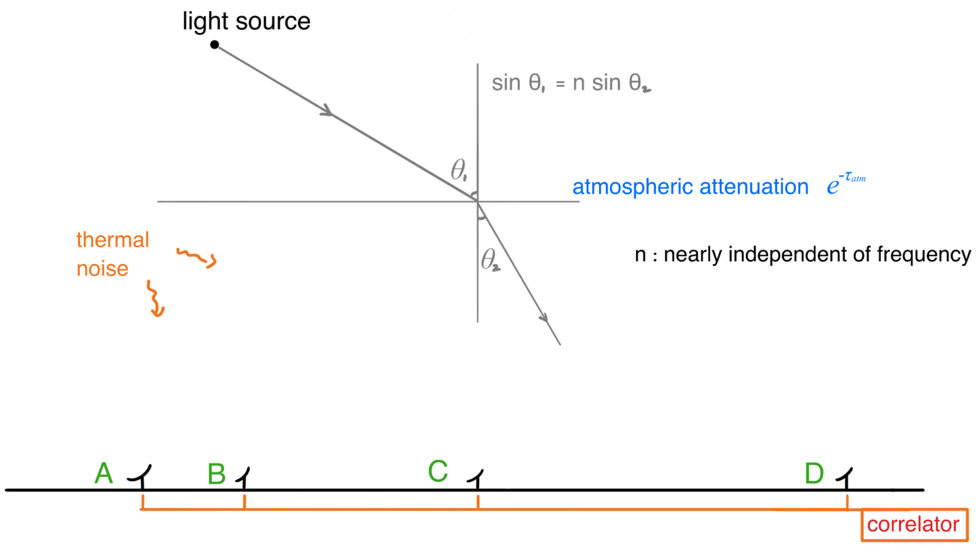

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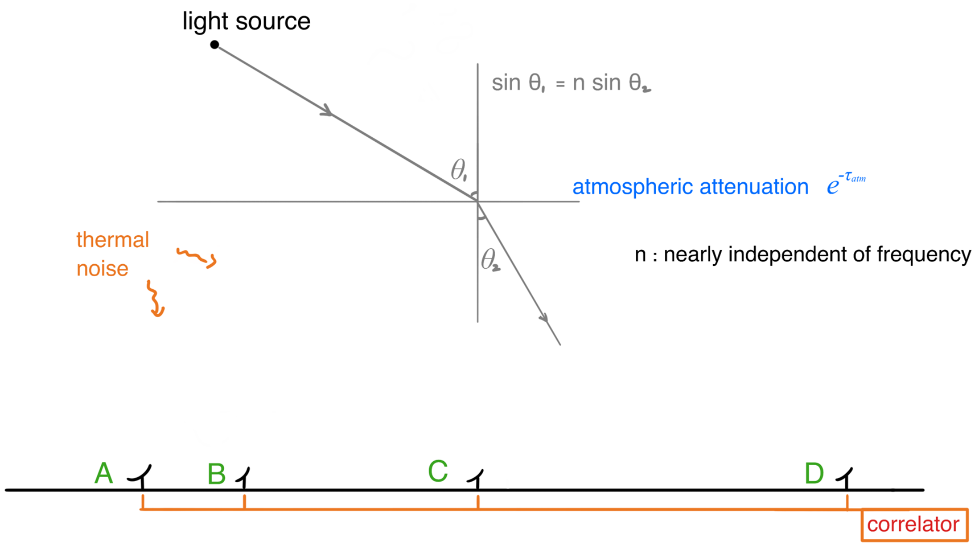

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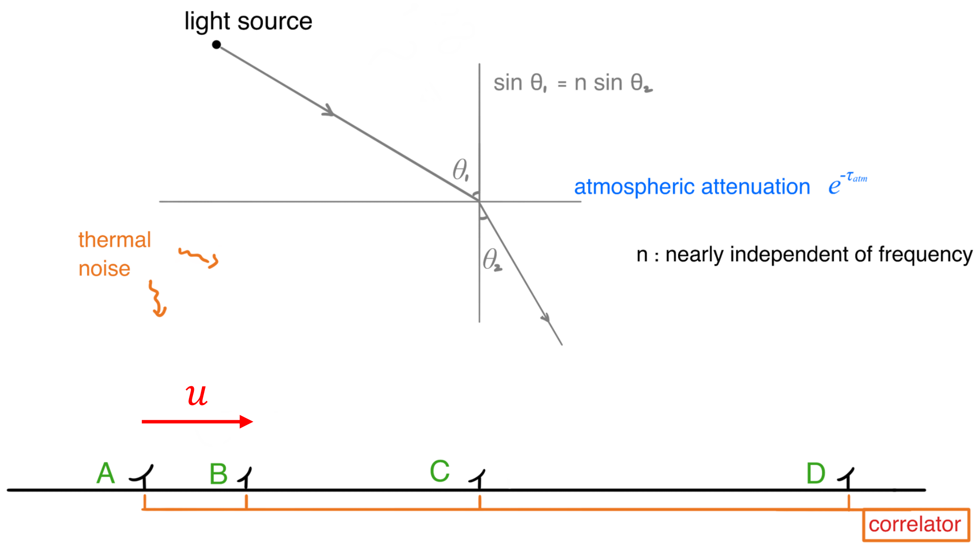

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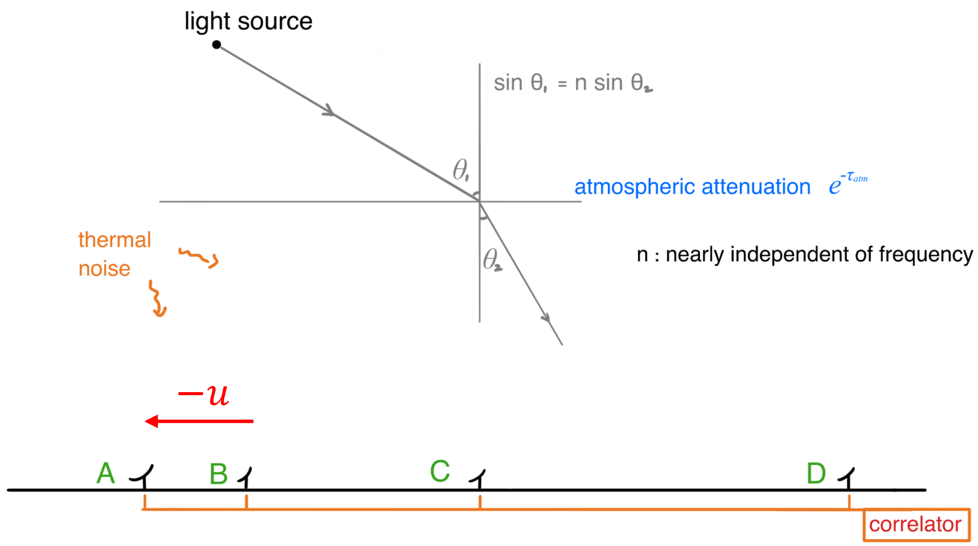

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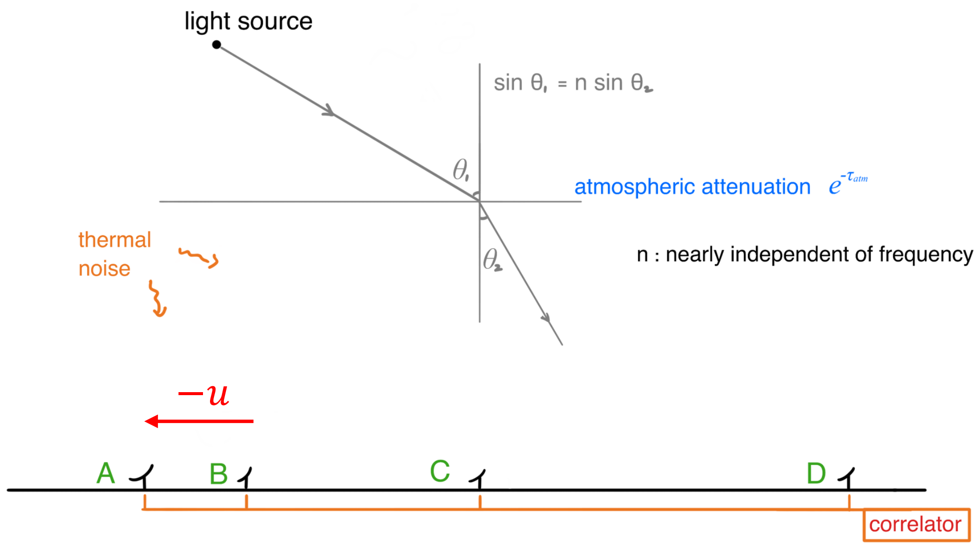

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Dirty beam

$A(\theta')\widetilde{P(\theta')}$

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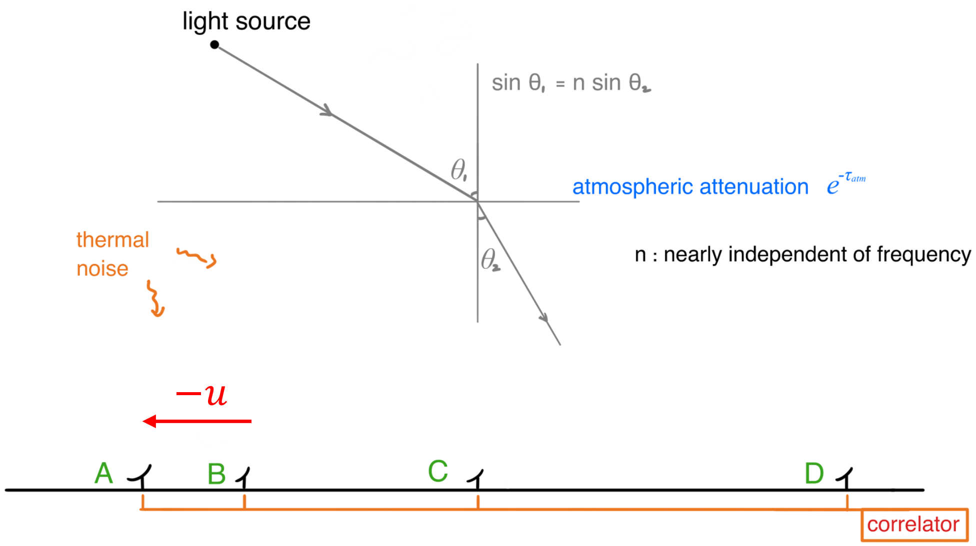

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Resemblance to the diffraction pattern of the diffraction grating

Lecture Unit 4-1

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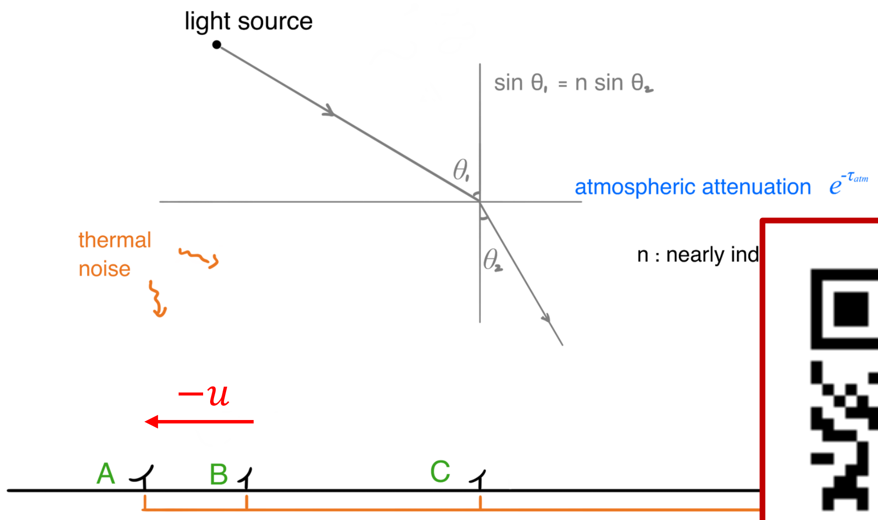

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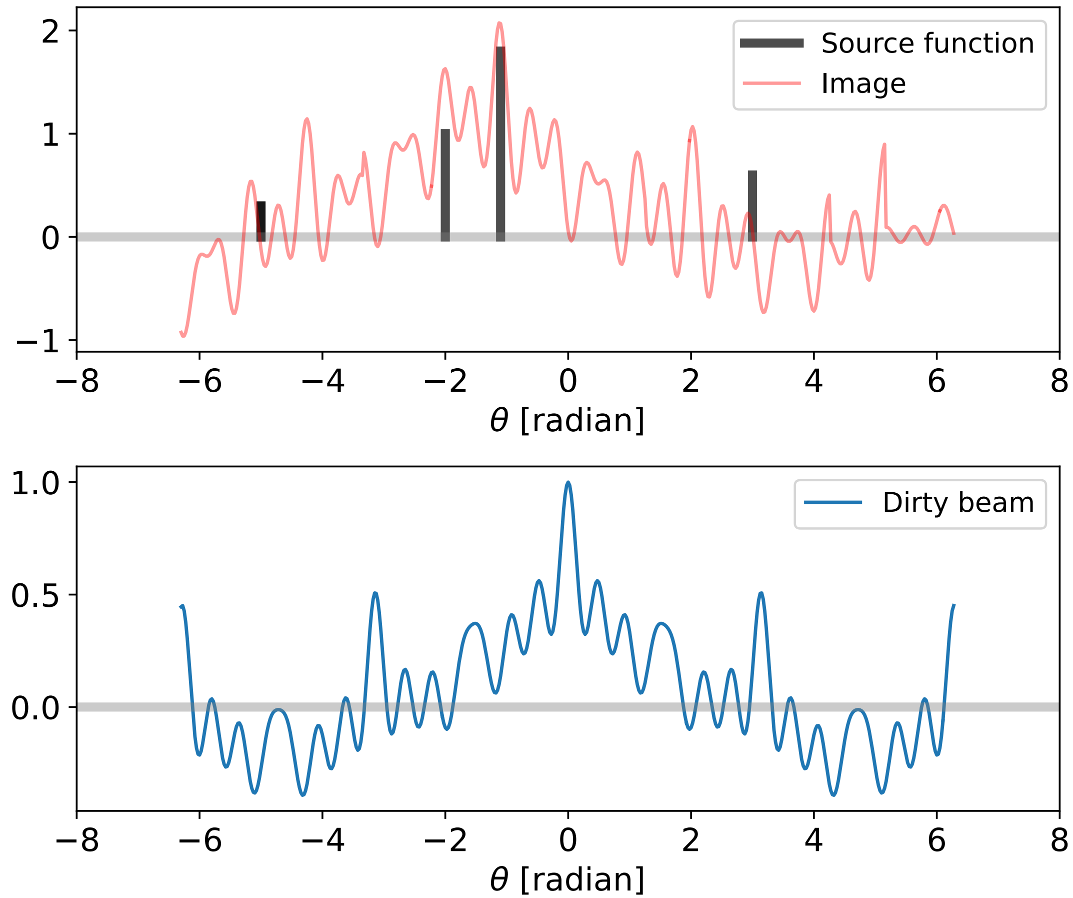

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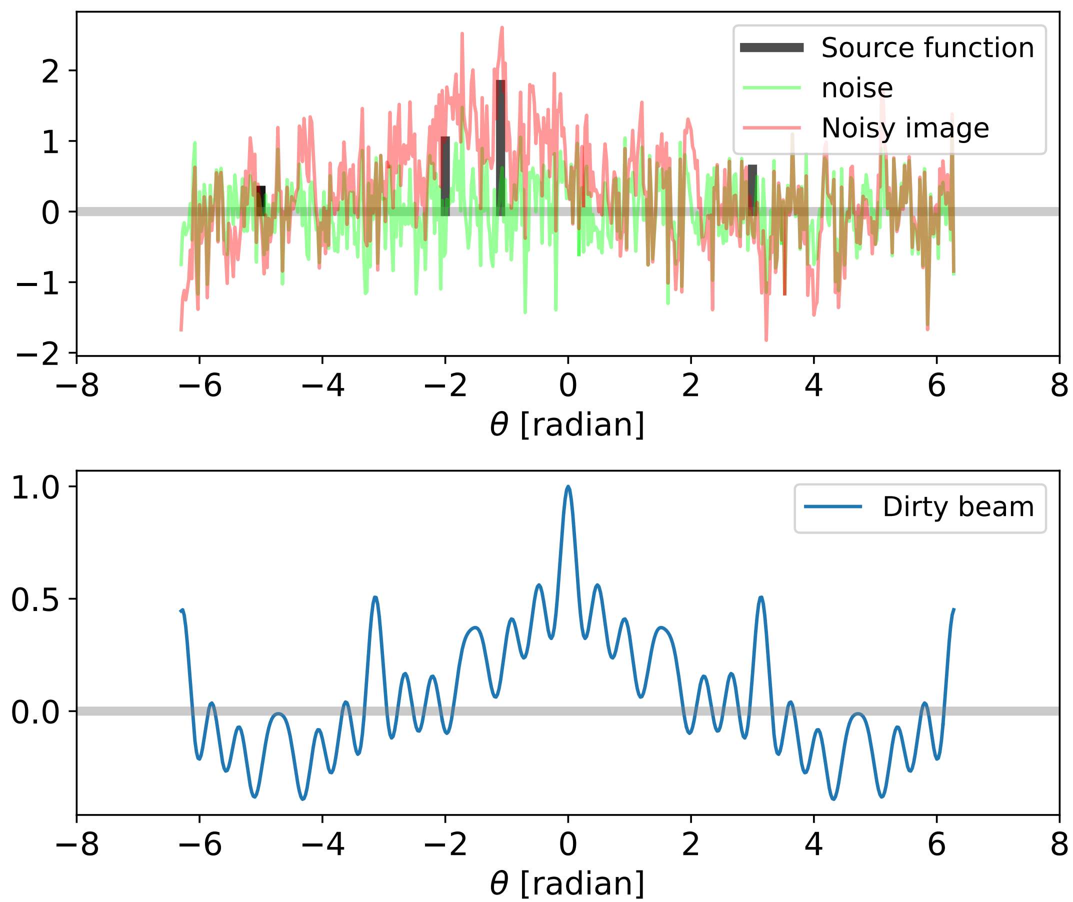

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$\boxed{\text{Need to deconvolve } \left[A(\theta')\widetilde{P(\theta')}\right]^D}$

1. An interferometric array makes discretized samplings on the Fourier transformed (and primary beam attenuated) intensity distribution.
2. The interferometric sampling can be described by a collection of Dirac delta function $$S(u) \equiv \sum_{k=1}^{M} \delta(u - u_k)$$
3. The dirty beam of the interferometric observations is the Fourier formation of $S(u)$.
4. A dirty interferometric image is a convolution of the dirty beam with the actual (primary beam attenuated) intensity distribution.
5. To invert from the dirty image to the actual (primary beam attenuated) intensity distribution, we need to do deconvolution.