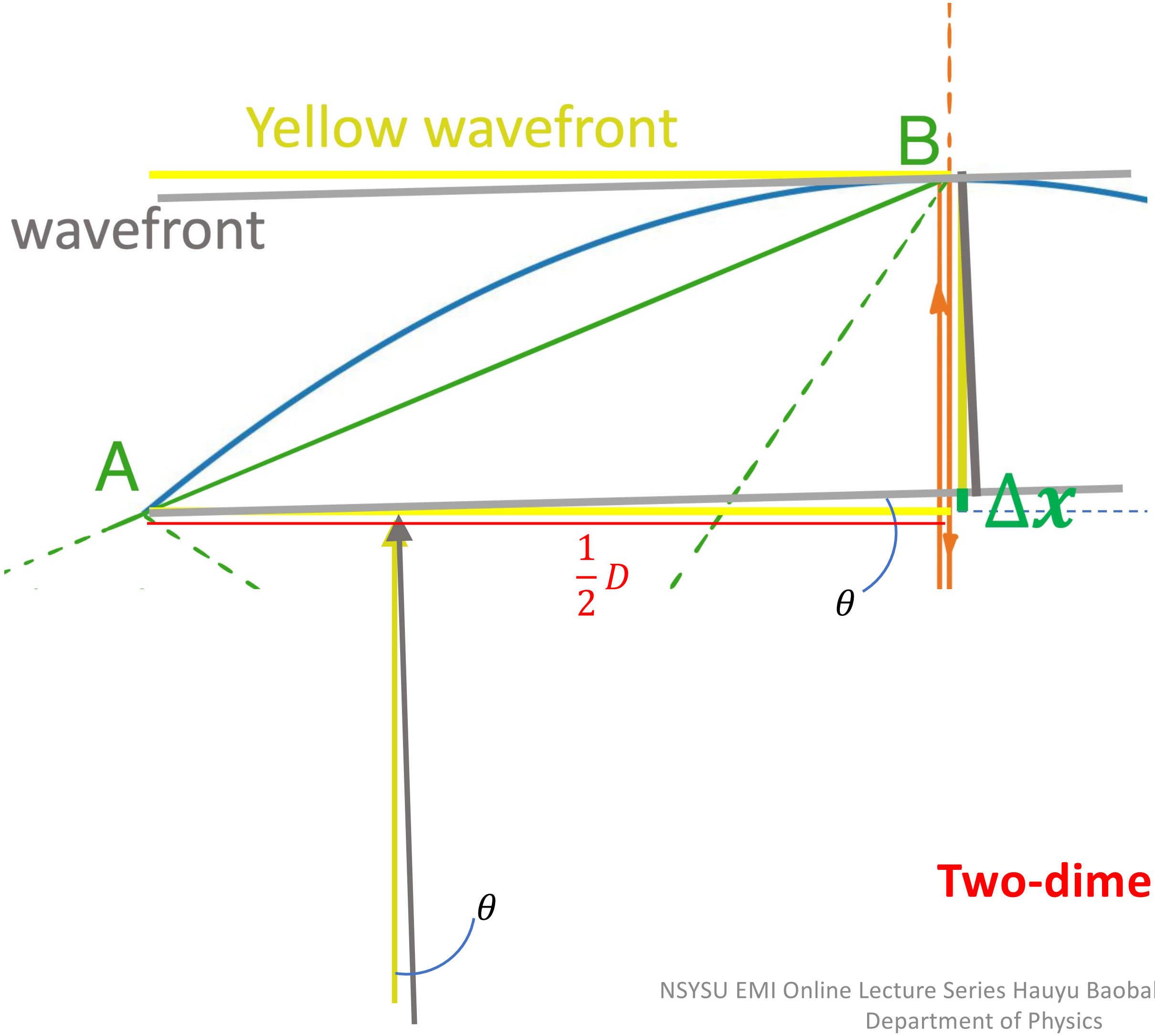
An Introduction to Radio Interferometry

2-4 Observations with single-dish telescopes

You can find relevant material
on my personal webpage



How do we find the first minimum?

The trick we knew when working on single-slit.

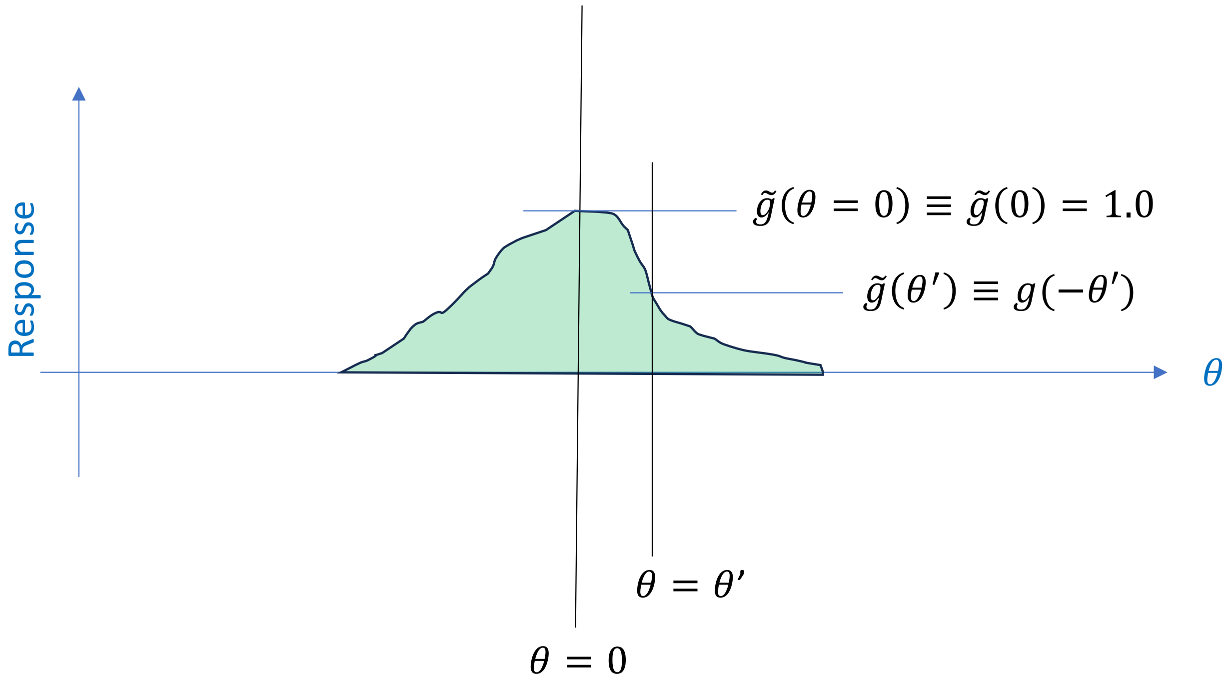
Path length difference: $\Delta x = x_1 - x_2 \sim \frac{1}{2} D\theta$

First zero: $\mathrm{k}\Delta x = \frac{1}{2} kD\theta = \pi$

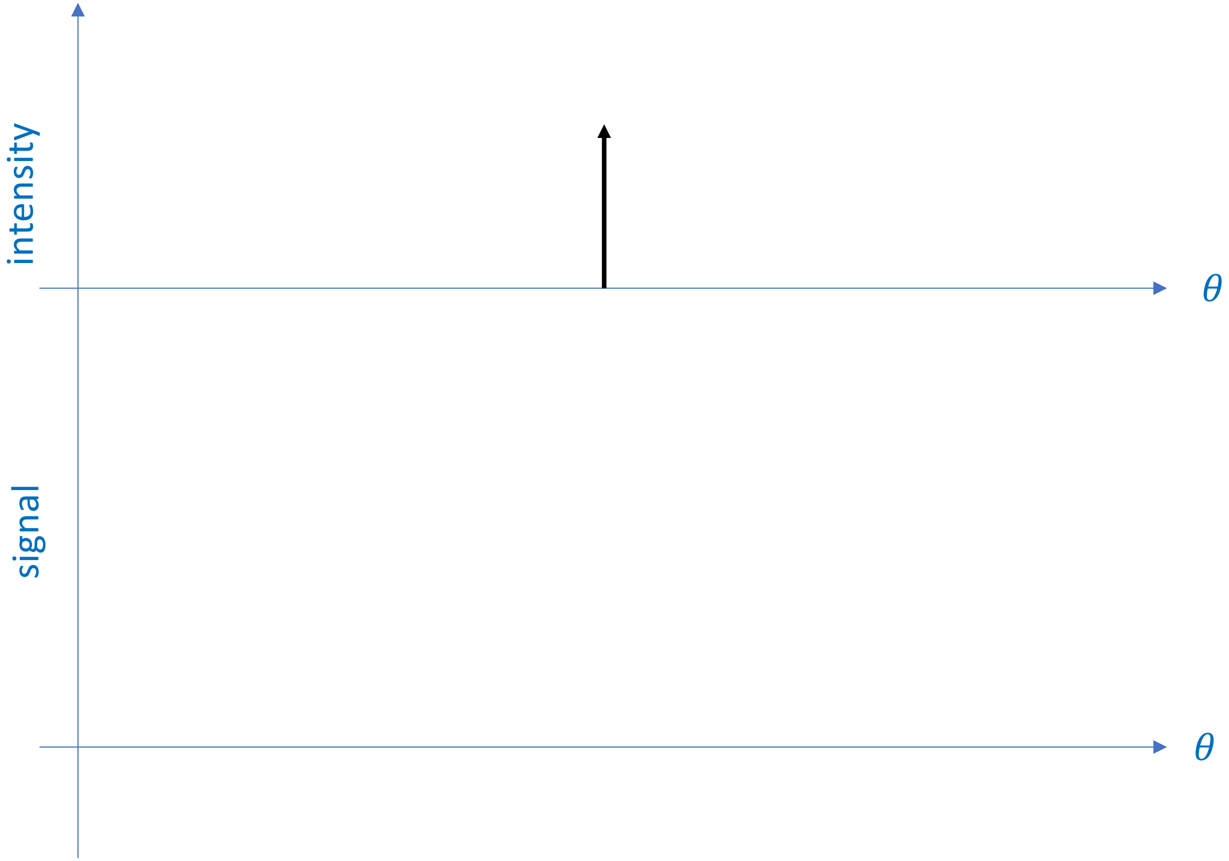
$$\Longrightarrow \frac{1}{2}\frac{2\pi}{\lambda} D\theta = \pi \Longrightarrow \theta = \frac{\lambda}{D}$$

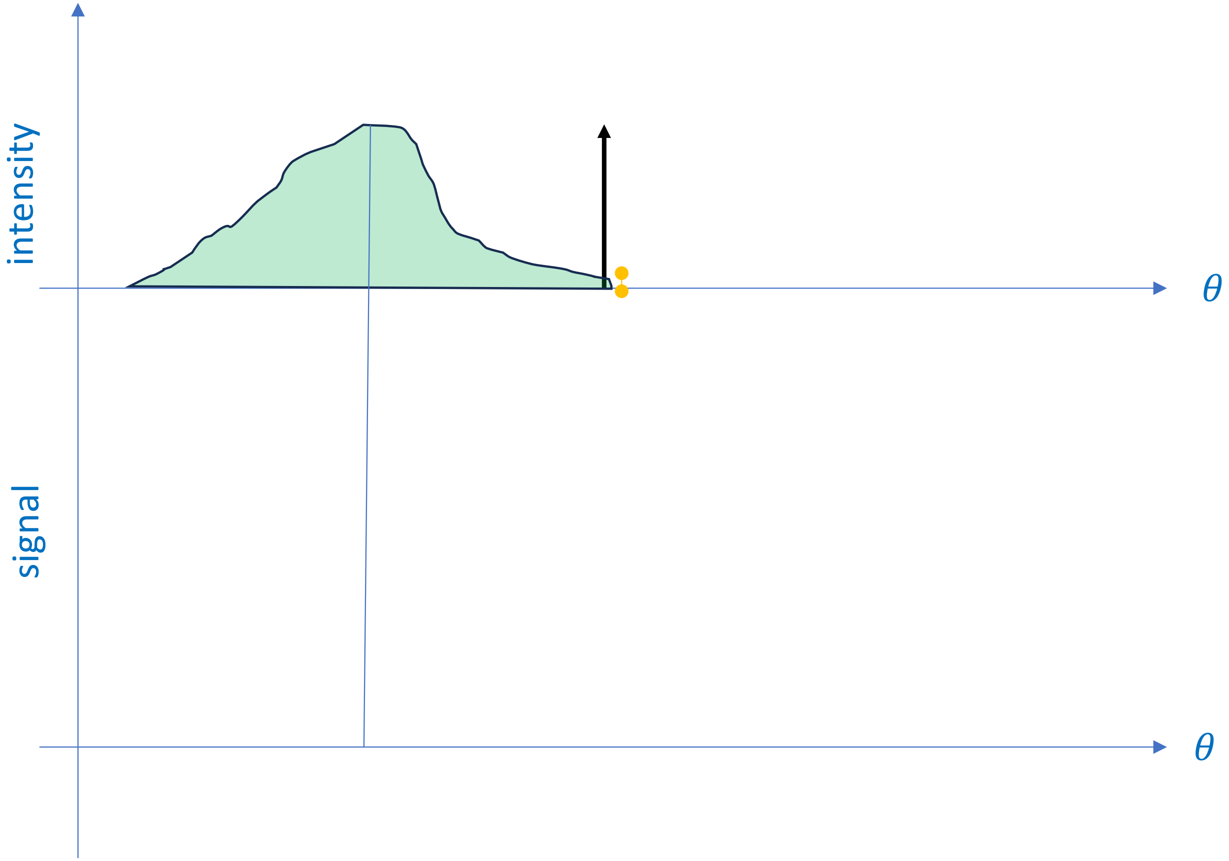
Two-dimensional dish: FWHM $= 1.22\frac{\lambda}{D}$

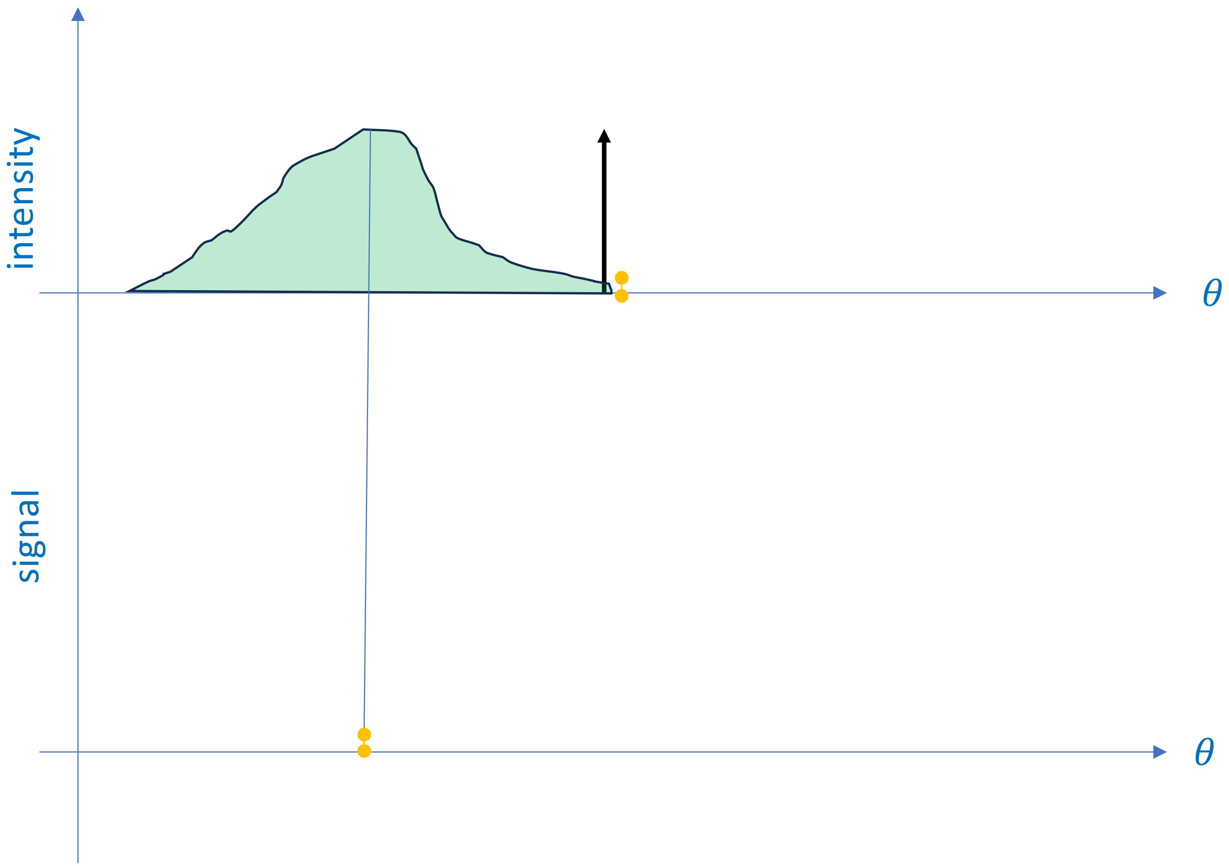
Response of a (one-dimensional) single dish telescope

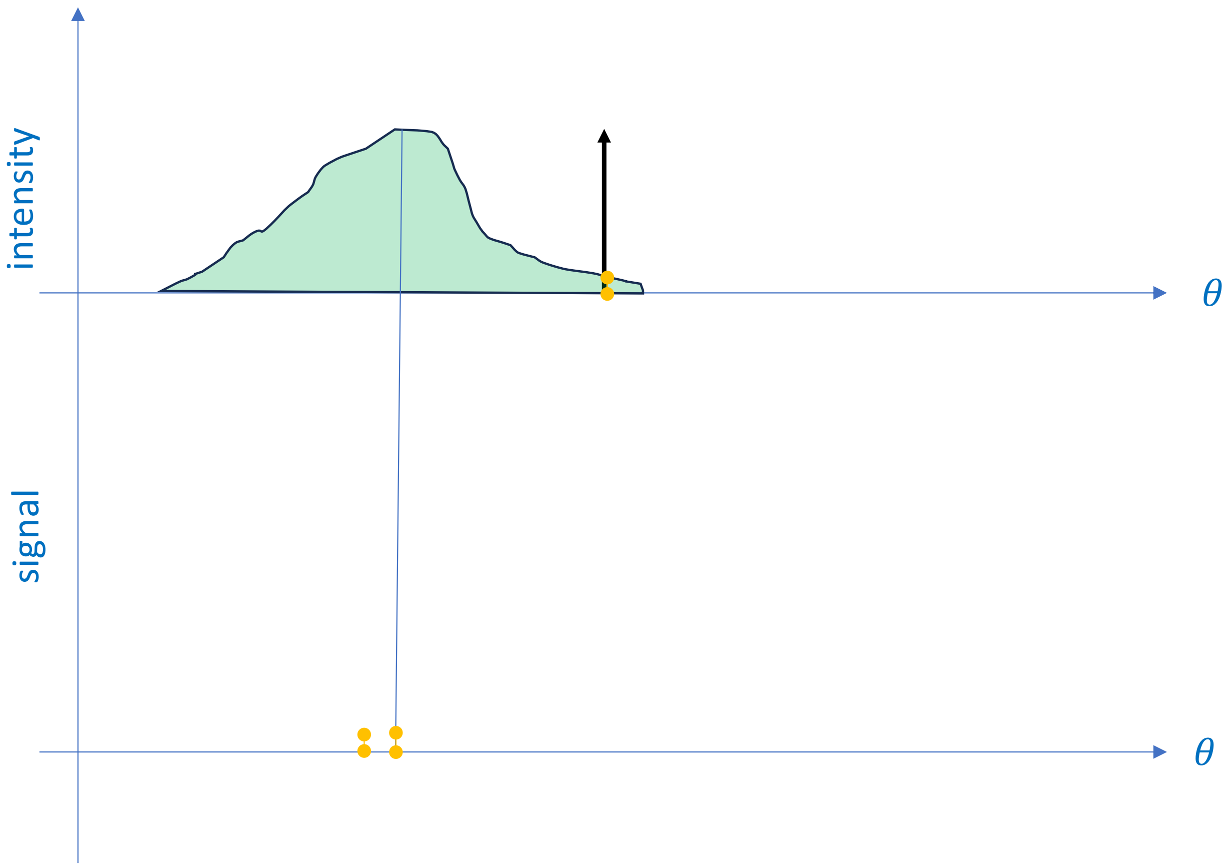


Flux density distribution of a point-source




intensity
signal
θ
θ


intensity
signal
θ
θ


intensity
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θ
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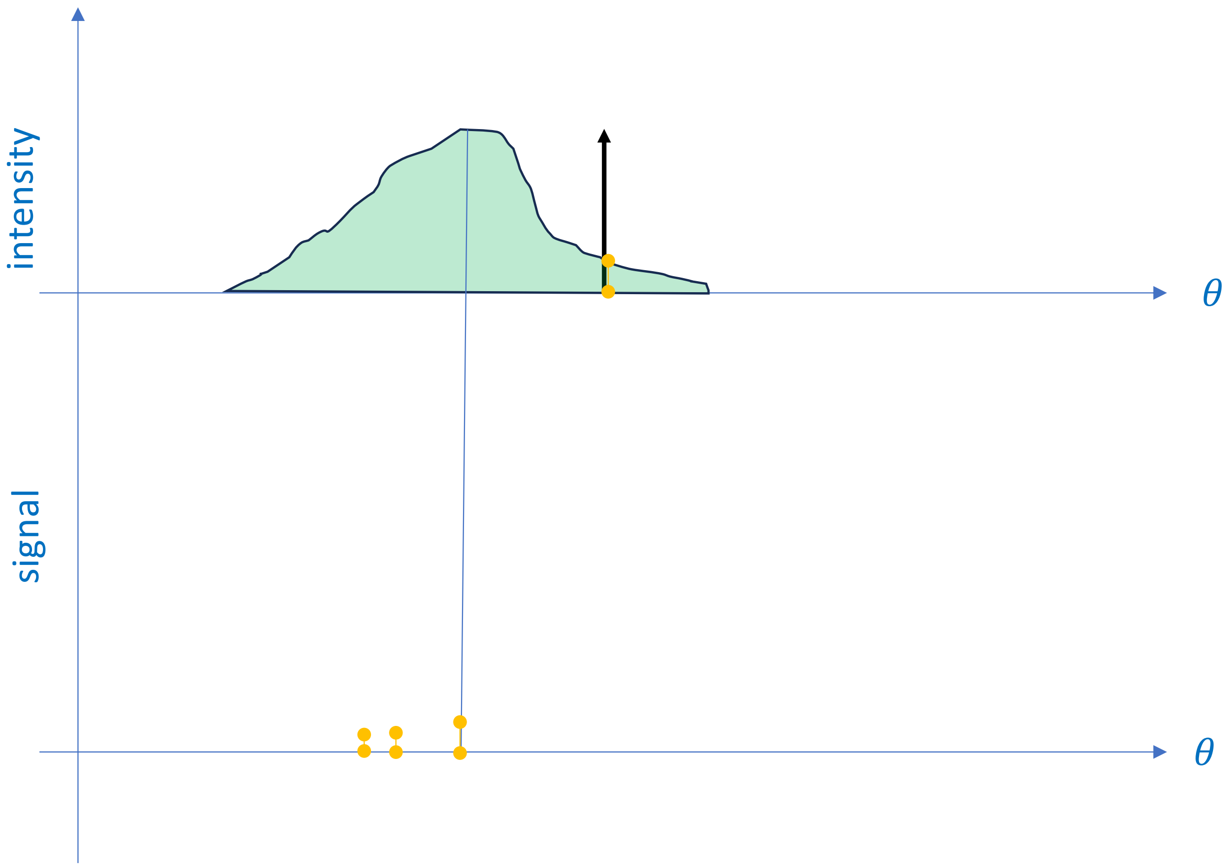
intensity
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θ
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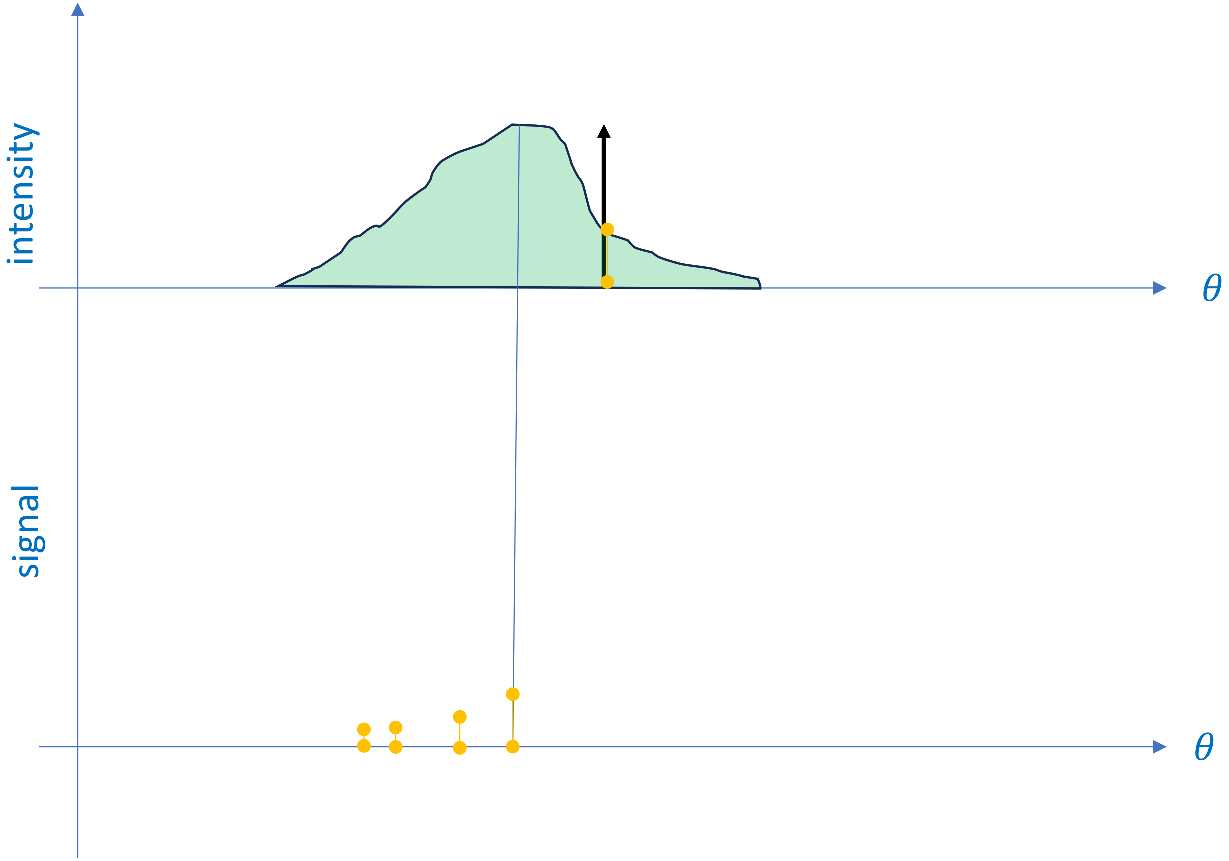
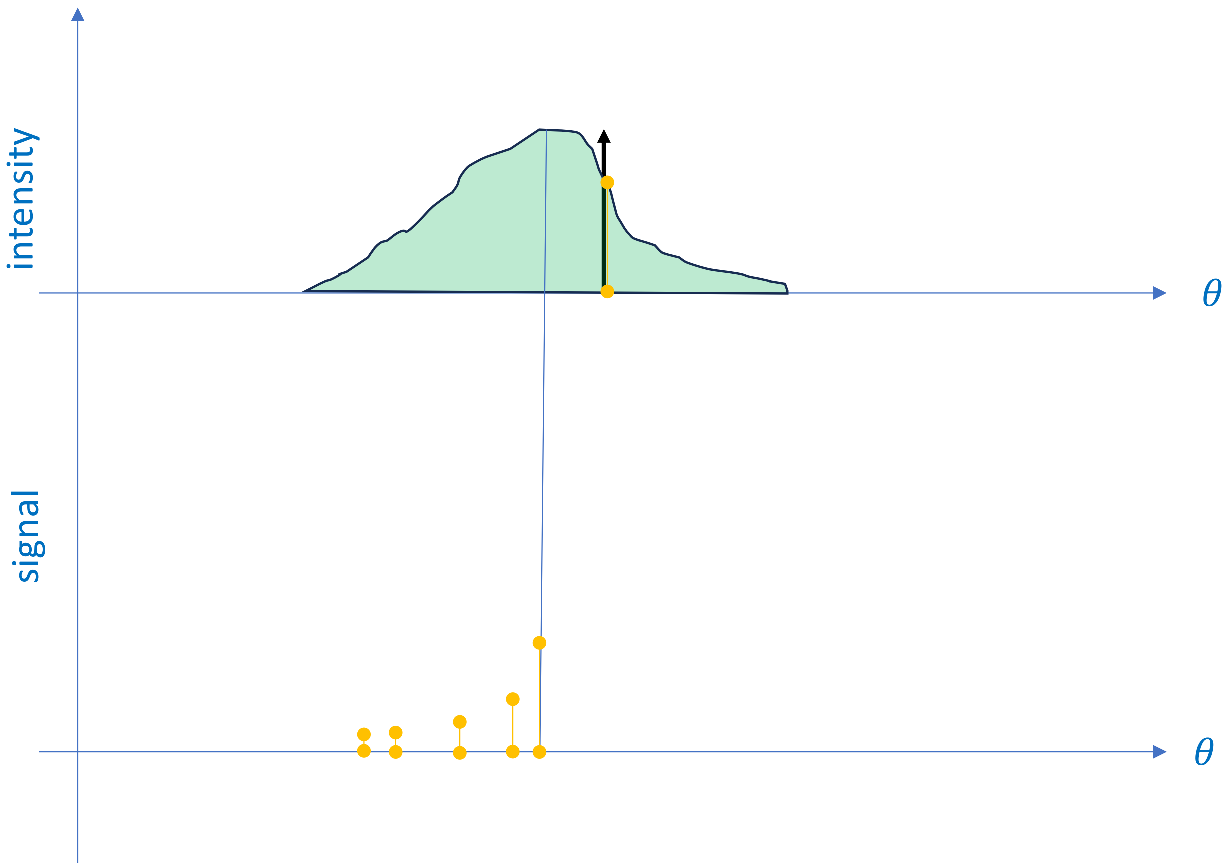

intensity
θ
signal
θ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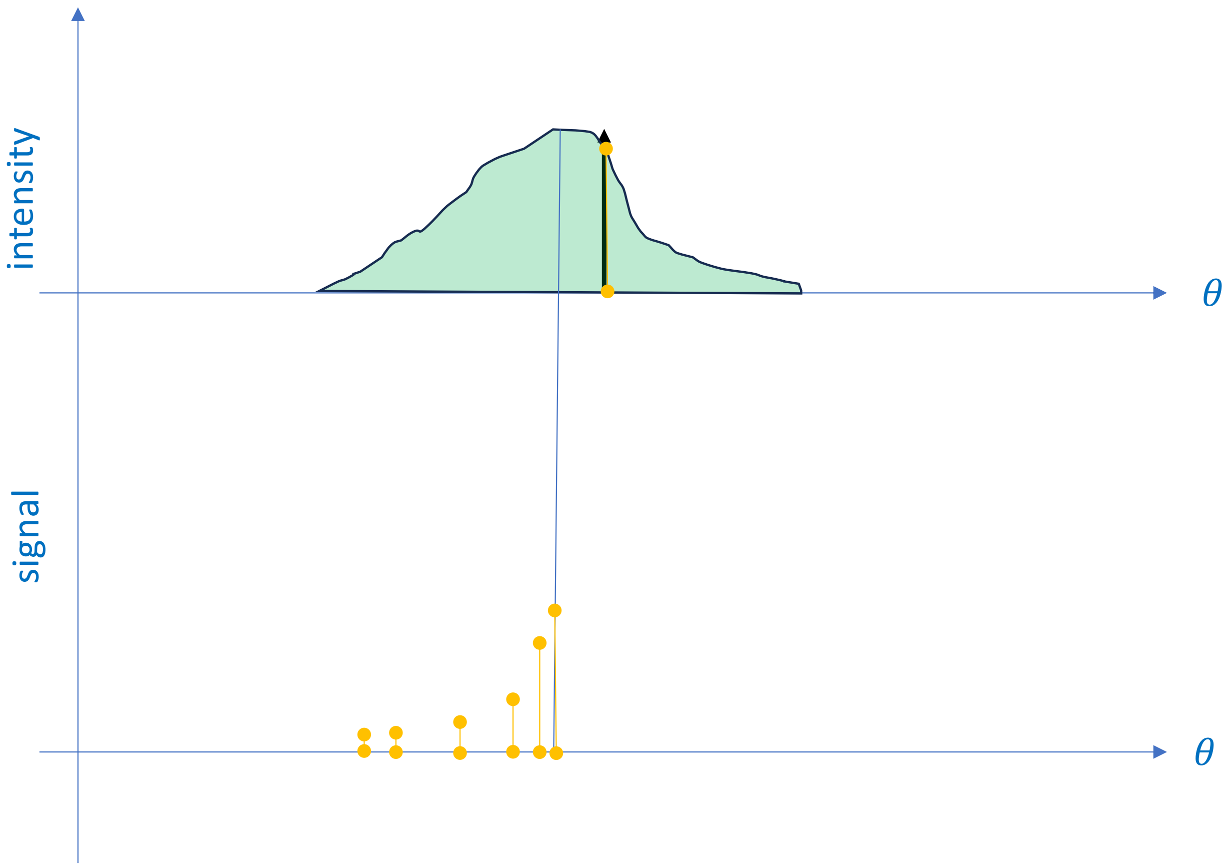
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θ
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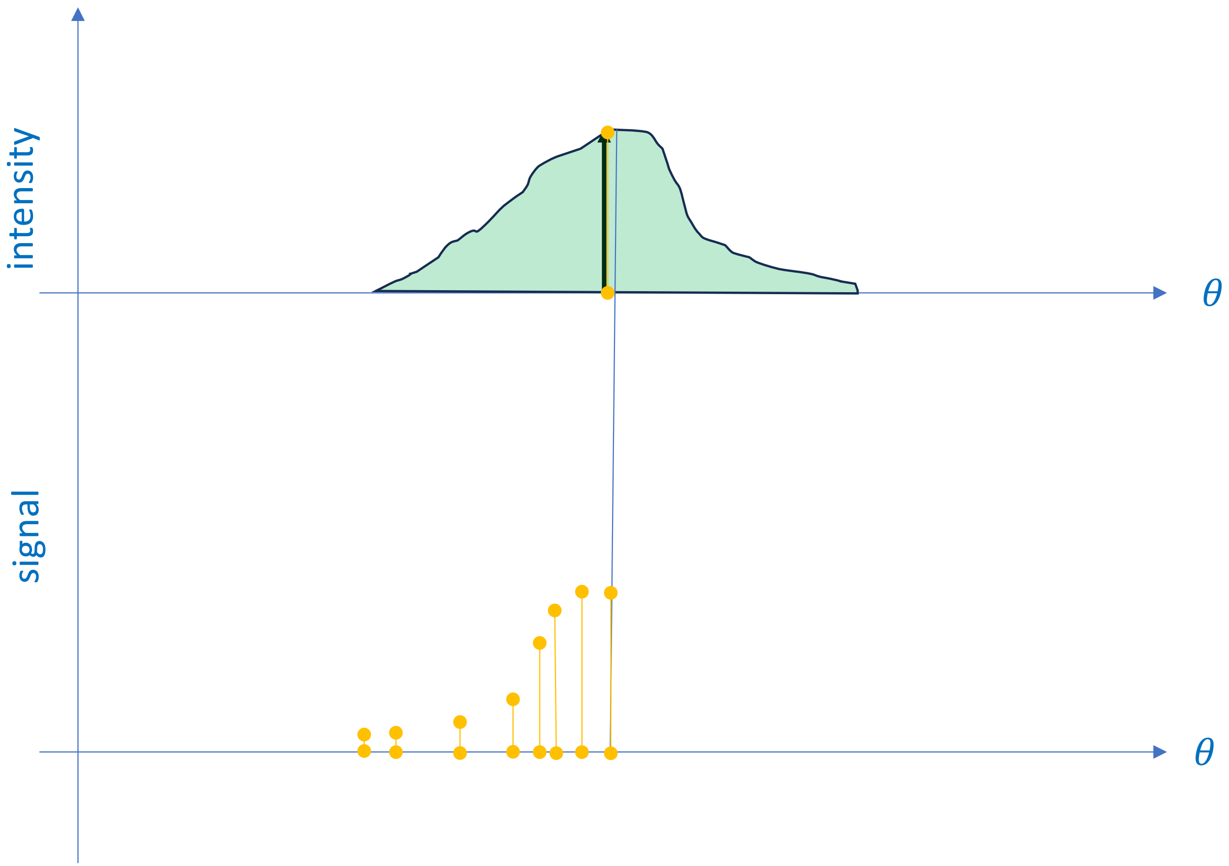
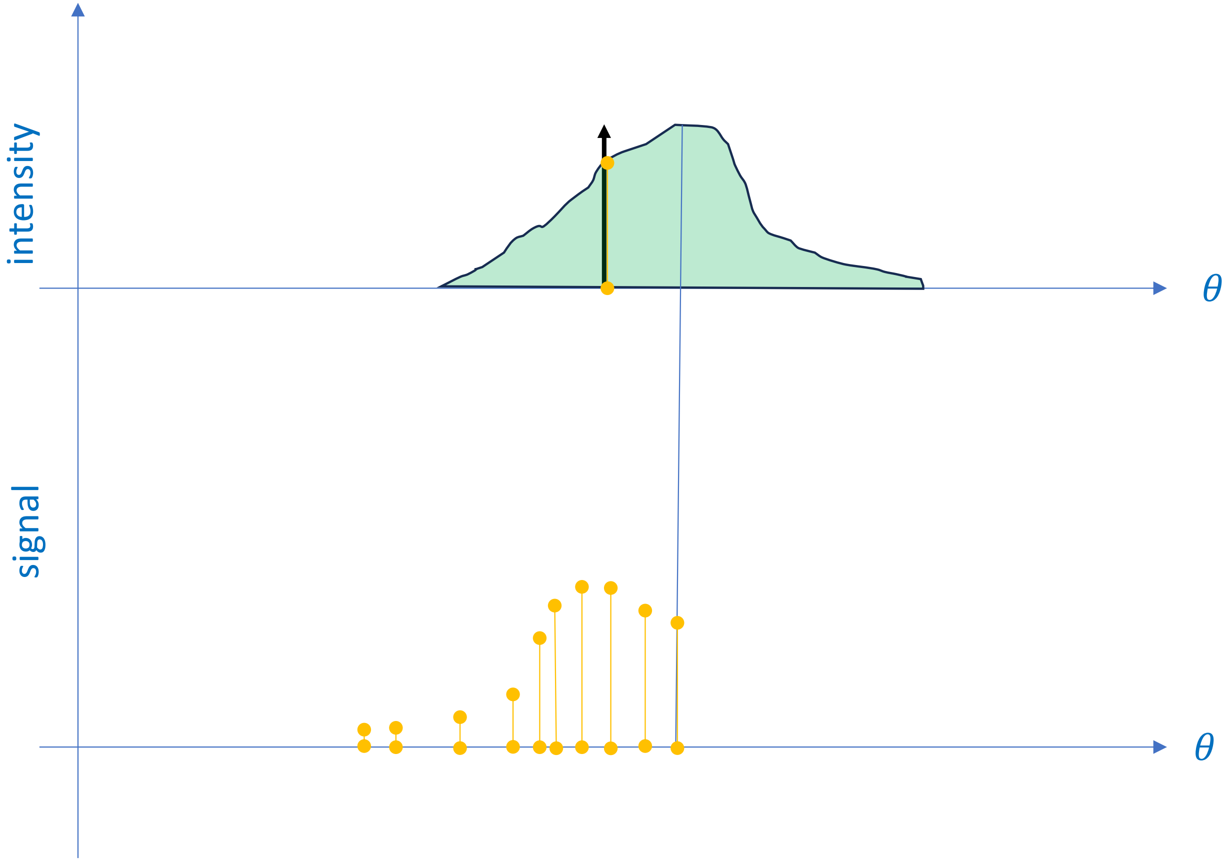
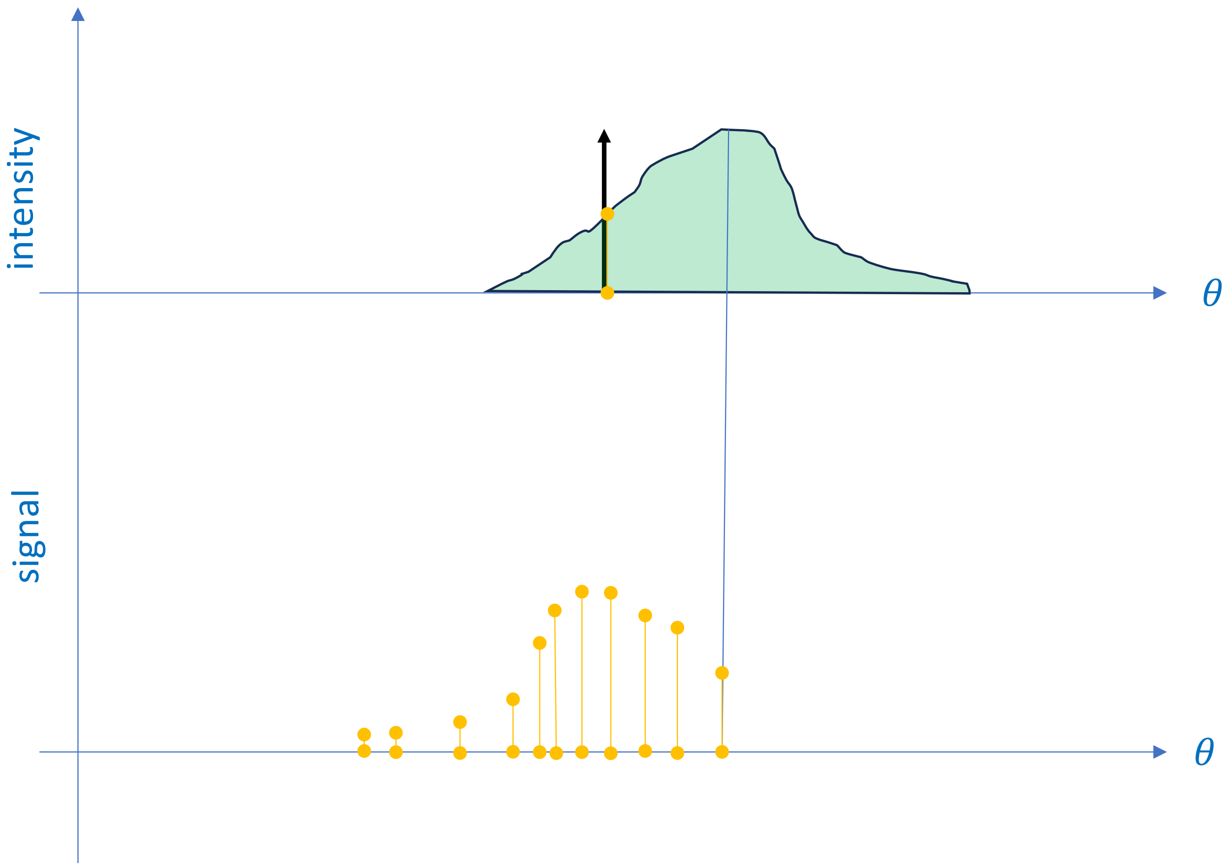
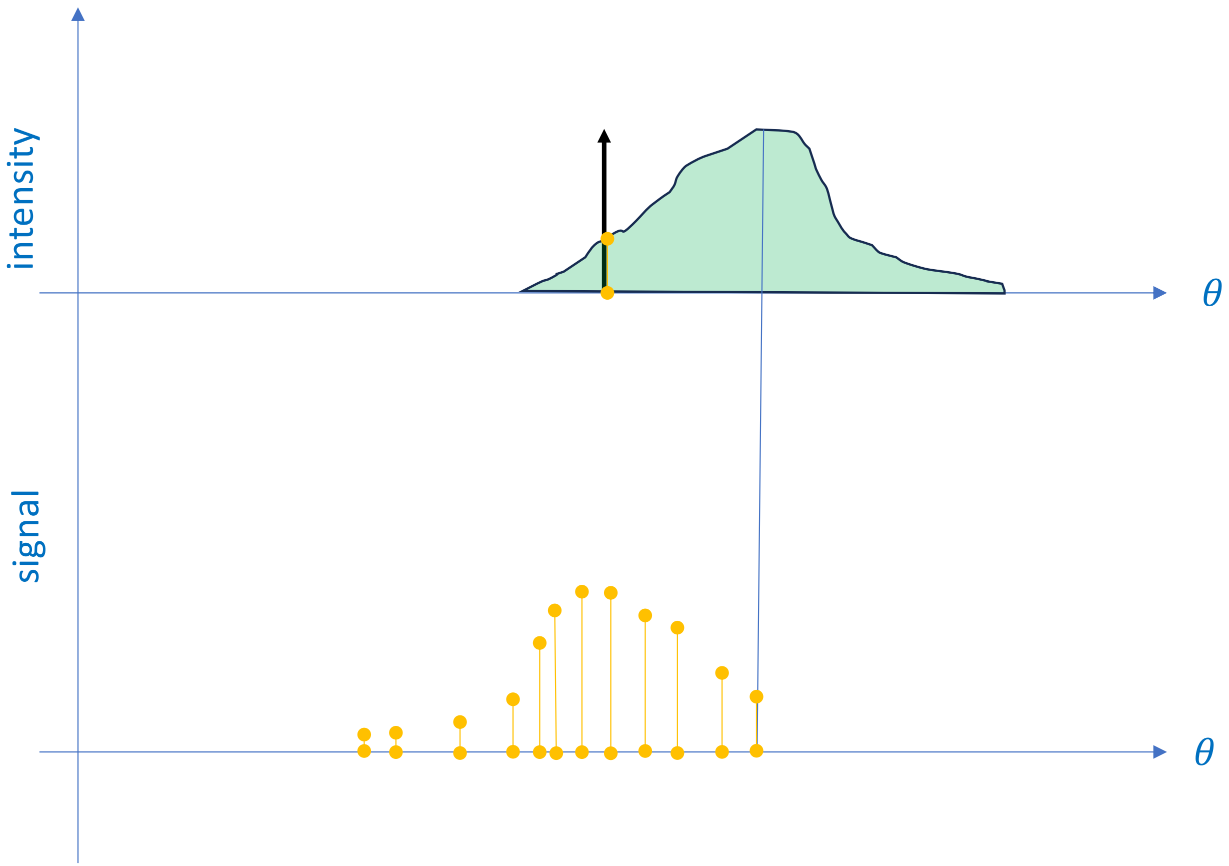
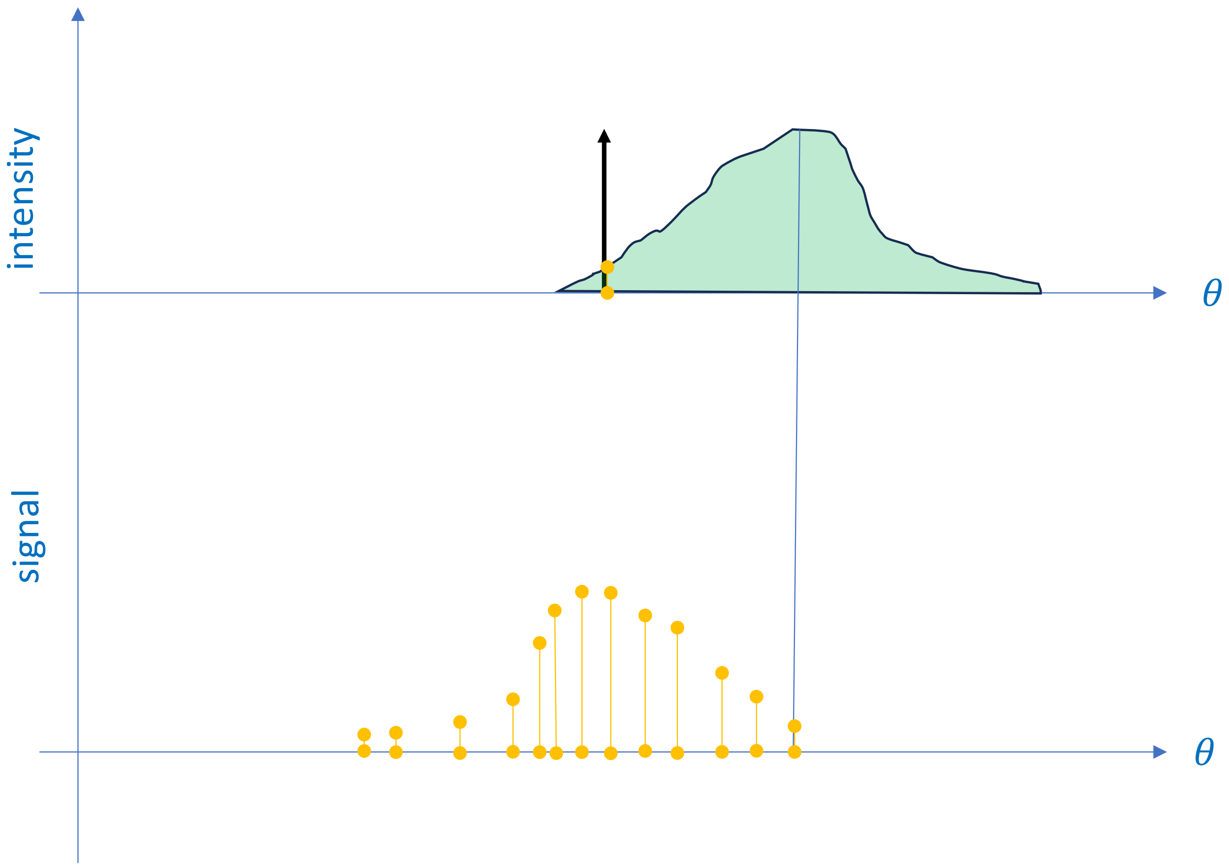
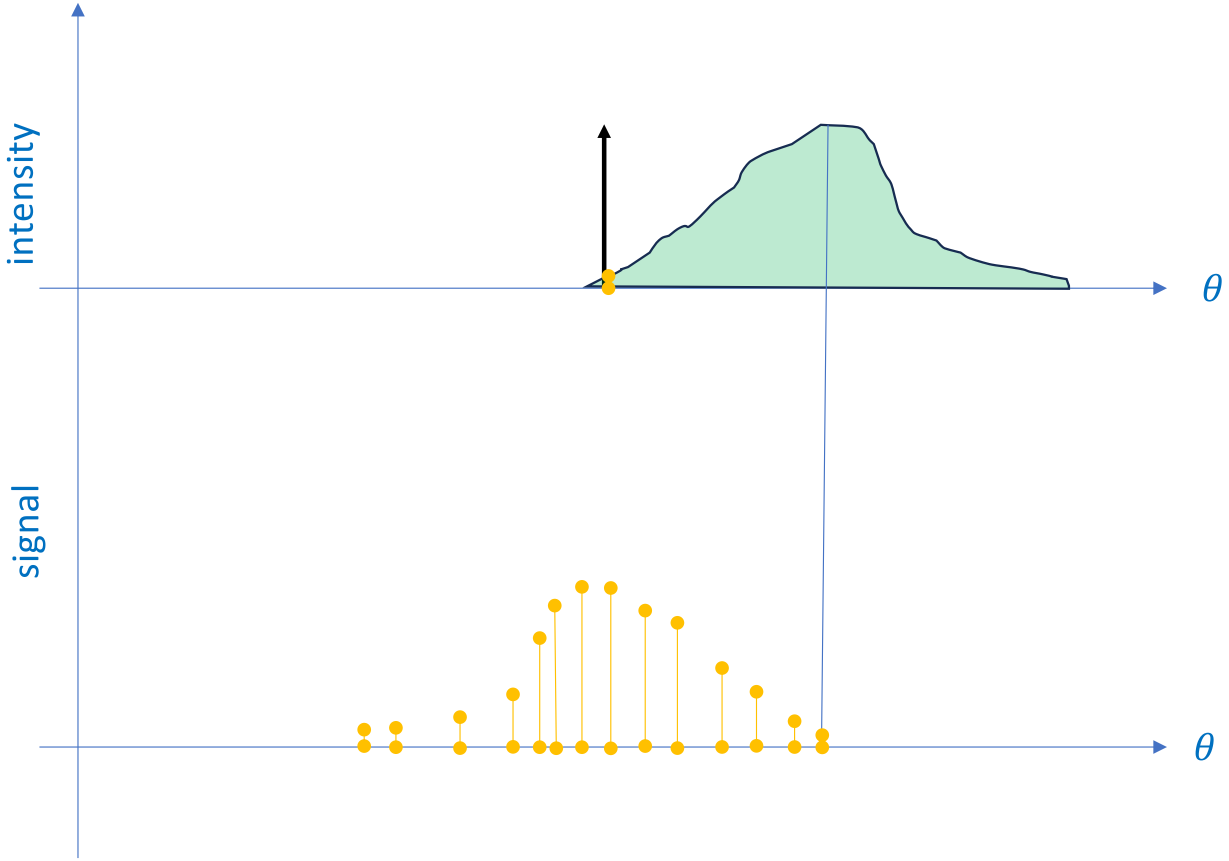

intensity
θ
signal
θ

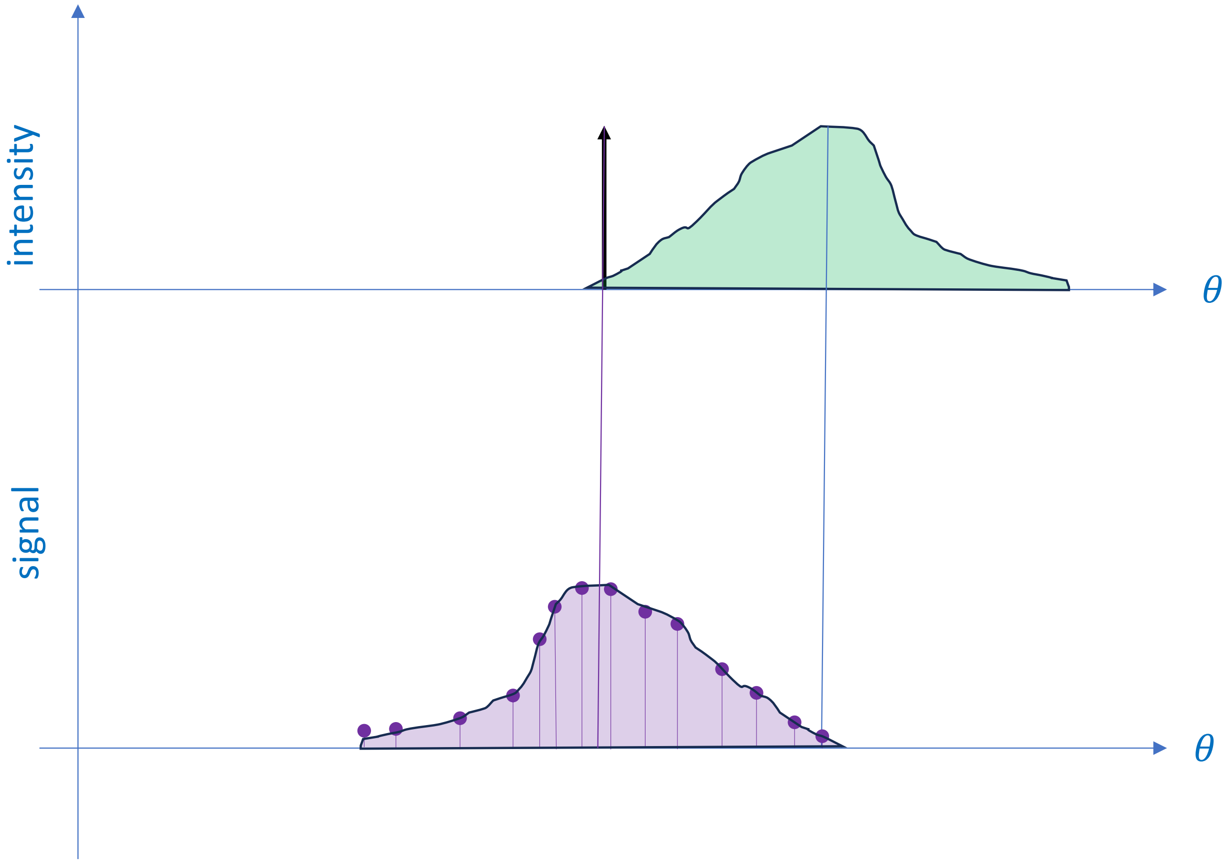

intensity
θ
signal
θ

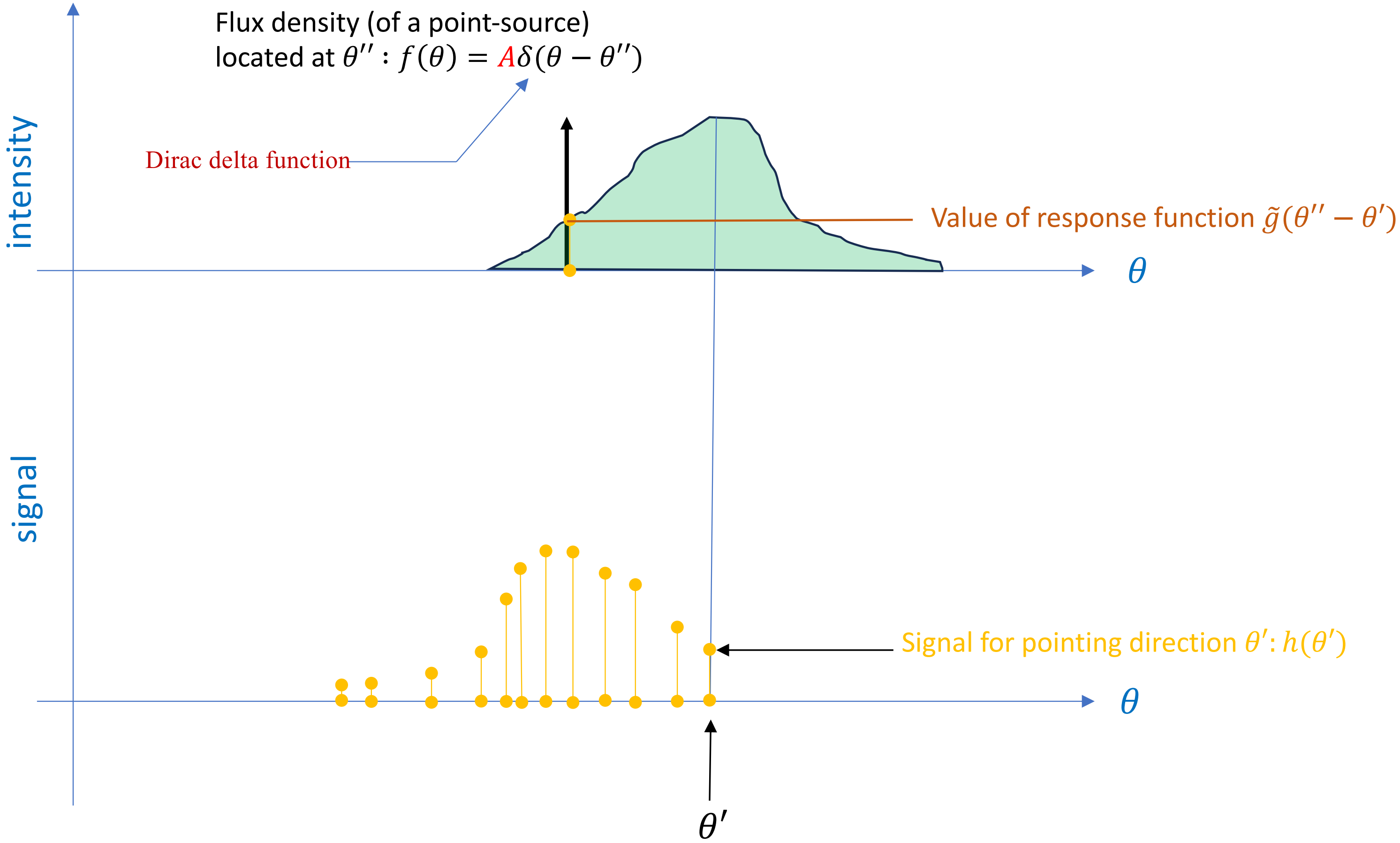

intensity
signal
θ
θ

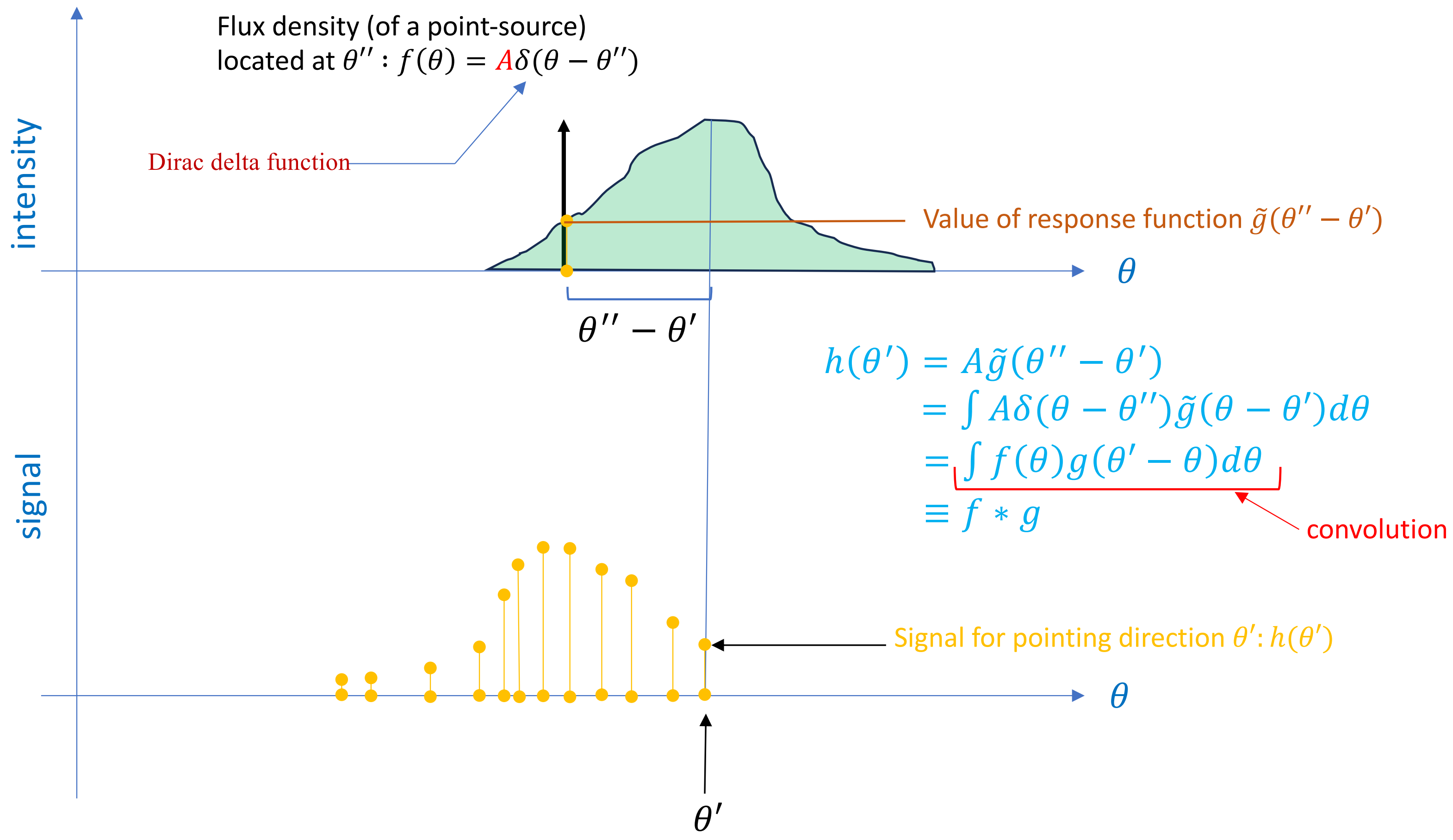

intensity
θ
signal
θ


intensity
signal
θ
θ


intensity
θ
signal
θ


intensity
signal
θ
θ


Flux density (of a point-source)
located at θ'' : $f(\theta) = A\delta(\theta - \theta'')$
Dirac delta function
intensity
Value of response function $\tilde{g}(\theta'' - \theta')$
θ
signal
Signal for pointing direction θ': $h(\theta')$
θ
θ'


intensity
signal
Flux density (of a point-source)
located at θ'' : $f(\theta) = A\delta(\theta - \theta'')$
Dirac delta function
Value of response function $\tilde{g}(\theta'' - \theta')$
θ
$\theta'' - \theta'$
$h(\theta') = A\tilde{g}(\theta'' - \theta')$
$= \int A\delta(\theta - \theta'')\tilde{g}(\theta - \theta')d\theta$
$= \underbrace{\int f(\theta)g(\theta' - \theta)d\theta}_{\text{convolution}}$
$\equiv f * g$
Signal for pointing direction θ': $h(\theta')$
θ
θ'

When there are multiple point sources

$$h(\theta') = \sum A_i\, g(\theta_i'' - \theta')$$
$$= \int \sum A_i\, \delta(\theta - \theta_i'')\tilde{g}(\theta - \theta')d\theta$$
$$= \int f(\theta)g(\theta' - \theta)d\theta$$
$$\equiv f * g$$

$$f(\theta) \equiv \sum A_i \delta(\theta - \theta_i'')$$

General expression (source intensity can be continuous)

$$h(\theta') = \int f(\theta) g(\theta' - \theta) d\theta$$

$$\equiv f * g$$

Convolution theorem

$$h(\theta') = \int f(\theta)g(\theta' - \theta)d\theta$$

$$\equiv f * g$$

$$F.T.(f * g) = F.T.(f) \cdot F.T.(g)$$

$$F.T.(f \cdot g) = F.T.(f) * F.T.(g)$$

$$h(\theta') = \int f(\theta) g(\theta' - \theta) d\theta$$

$$\equiv f * g$$

Fourier transform

$$F.T.(f) \equiv \tilde{f}(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x}\, dx$$

Convolution theorem

$$F.T.(f * g) = F.T.(f) \cdot F.T.(g) \quad \leftrightarrow \quad F.T.(f \cdot g) = F.T.(f) * F.T.(g)$$

$$F.T.(f * g) = \iint f(\theta) g(\theta' - \theta) d\theta\, e^{-2\pi i u \theta'} d\theta'$$

$$= \iint f(\theta) g(\theta' - \theta) d\theta\, e^{-2\pi i u (\theta' - \theta + \theta)} d\theta'$$

$$= \iint f(\theta)\, e^{-2\pi i u \theta} d\theta g(\theta' - \theta) e^{-2\pi i u (\theta' - \theta)} d\theta'$$

$$= \iint \underbrace{f(\theta)\, e^{-2\pi i u \theta} d\theta}_{F.T.(f)} \underbrace{g(\theta' - \theta) e^{-2\pi i u (\theta' - \theta)} d(\theta' \text{-} \theta)}_{F.T.(g)}$$

1. What we observe is a convolution of the source intensity distribution with the response function of the telescope.
2. Convolution theorem:

$$F.T.(f * g) = F.T.(f) \cdot F.T.(g)$$

$$F.T.(f \cdot g) = F.T.(f) * F.T.(g)$$