

# An Introduction to Radio Interferometry

## 3-1 Interference pattern of a double slit



You can find relevant material  
on my personal webpage

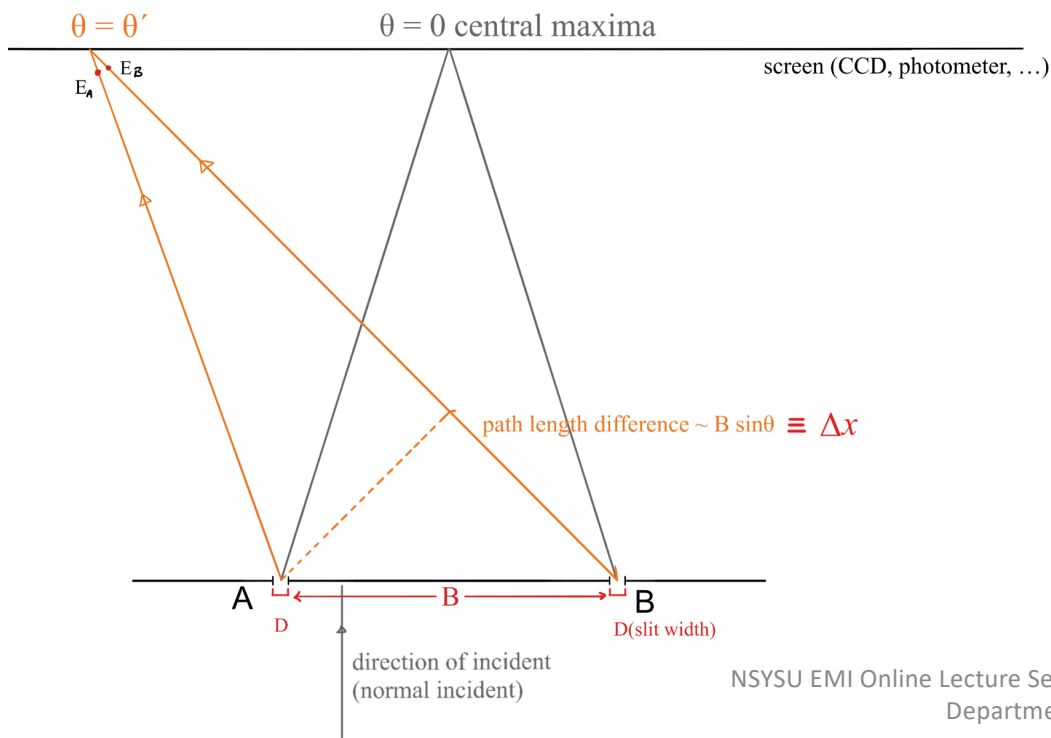
NSYSU EMI Online Lecture Series Haiyu Baobab Liu (吕浩宇),  
Department of Physics

1. EM-wave at long-distance limit: plane wave  $E = E_0 \cos(kx - \omega t + \phi_0)$
2. Energy flux density  $\propto E_0^2$

Lecture Unit 1-2

$$\begin{aligned} \text{Single slit field} &= -\frac{\sin\left(\frac{1}{2}kD \sin \theta'\right)}{\frac{1}{2}k \sin \theta'} \cos\left(kx - \omega t + \frac{1}{2}kD \sin \theta'\right) \\ &\equiv \underbrace{\sqrt{\tilde{P}(\theta)}}_{\text{Amplitude modulation}} \cos\left(\underbrace{kx - \omega t + \phi_s}_{\text{Phase modulation}}\right) \end{aligned}$$

Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area  
(at the angle of emergence  $\theta = \theta'$ )

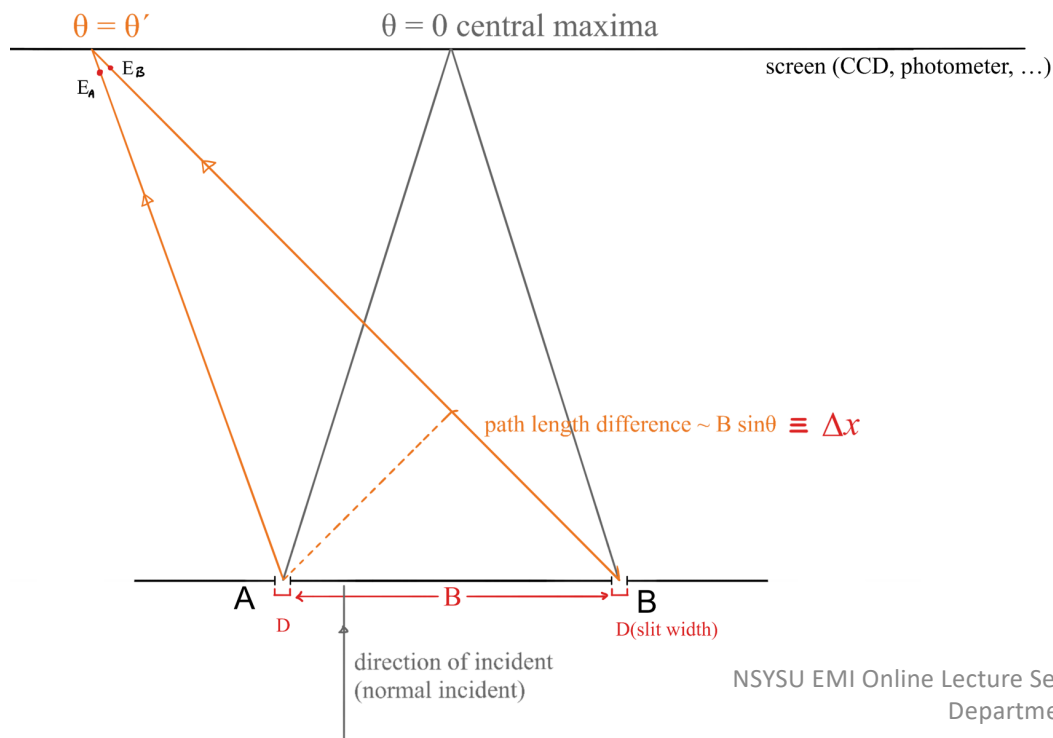
$$\begin{aligned} &\propto [E_A + E_B]^2 \\ &= \tilde{P}(\theta) [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \\ &= \tilde{P}(\theta) \{ [\cos(kx - \omega t + \phi_0)]^2 \\ &\quad + [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \\ &\quad + 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \} \end{aligned}$$

1. EM-wave at long-distance limit: plane wave  $E = E_0 \cos(kx - \omega t + \phi_0)$
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Lecture Unit 1-2

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Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area  
(at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto [E_A + E_B]^2 \\ &= \tilde{P}(\theta) [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \\ &= \tilde{P}(\theta) \{ [\cos(kx - \omega t + \phi_0)]^2 \\ &\quad + [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \\ &\quad + 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \} \end{aligned}$$

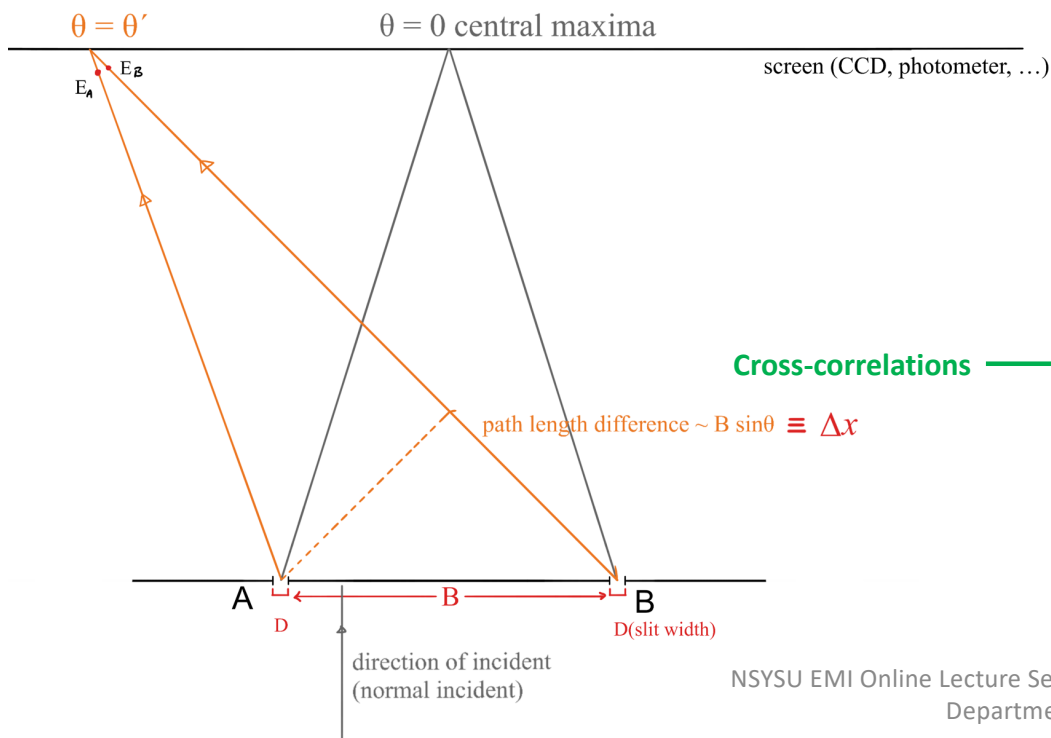
$$\langle \cdot \rangle \equiv \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} \cdot dt$$

1. EM-wave at long-distance limit: plane wave  $E = E_0 \cos(kx - \omega t + \phi_0)$
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Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &\quad \propto \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &\quad = \langle [\cos(kx - \omega t + \phi_0)]^2 \rangle \\ &\quad \quad + \langle [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \quad \left. \vphantom{\langle [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle} \right\} \text{Auto-correlations} \\ &\quad \quad + \langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle \quad \leftarrow \text{Cross-correlations} \end{aligned}$$

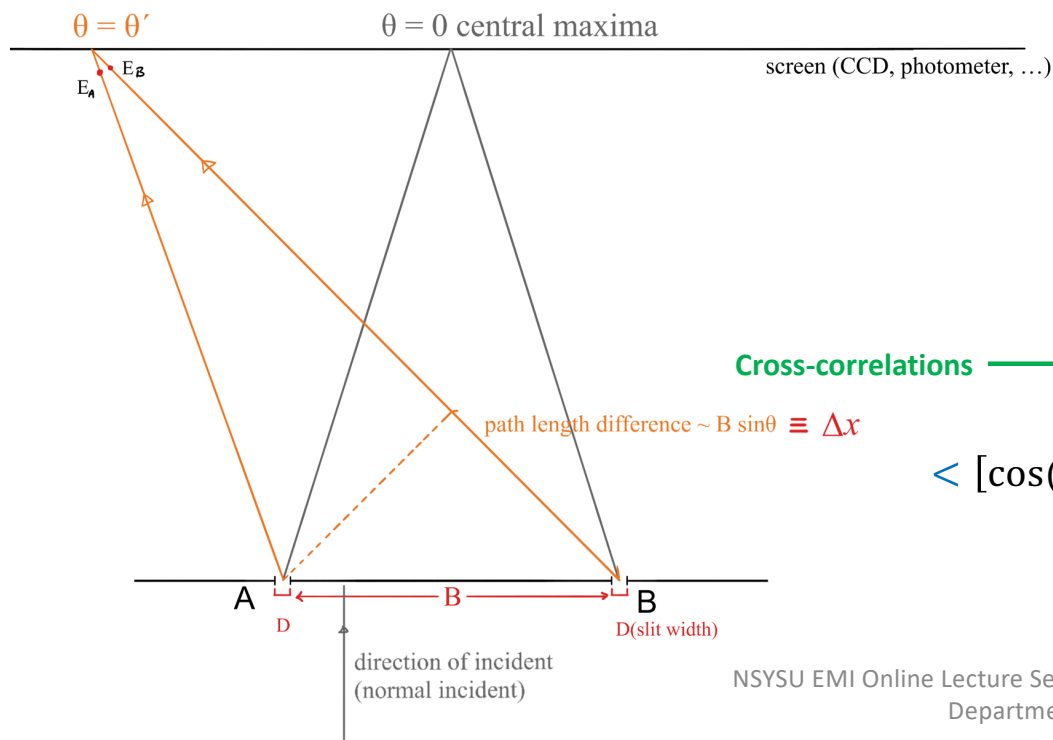
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Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &\propto \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \langle [\cos(kx - \omega t + \phi_0)]^2 \rangle + \langle [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle + \langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle \end{aligned}$$

Auto-correlations

Cross-correlations  $\longrightarrow$

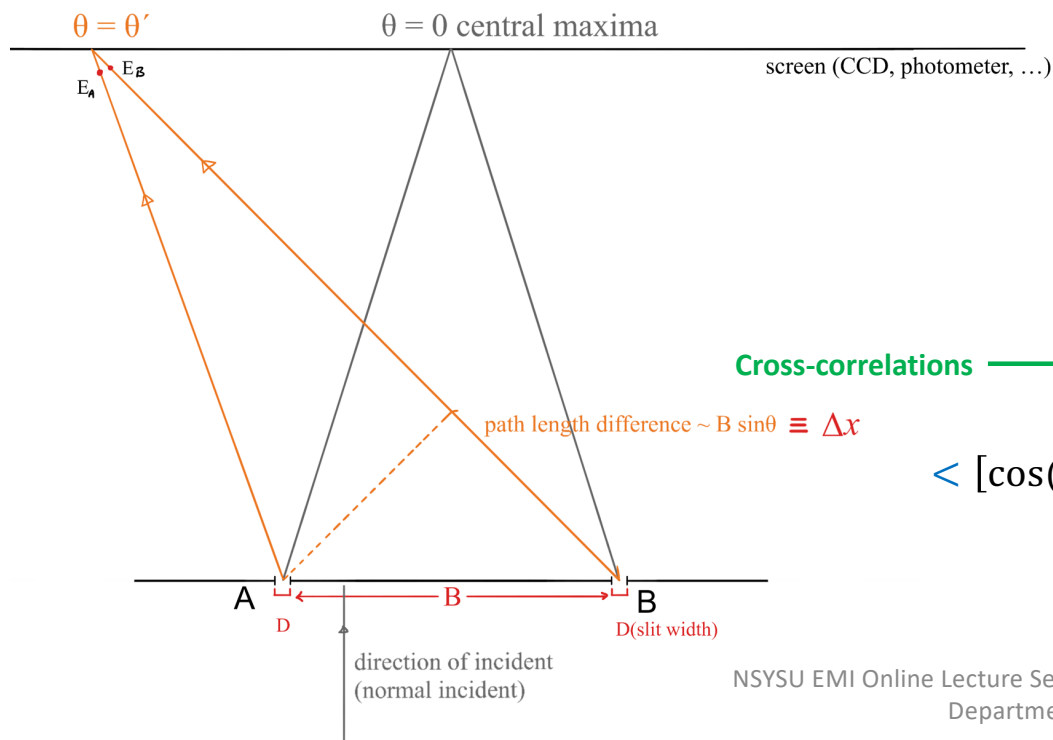
$$\begin{aligned} \langle [\cos(kx - \omega t + \phi_0)]^2 \rangle &\equiv \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} [\cos(\underbrace{kx - \omega t + \phi_0}_{\beta})]^2 dt \\ &= \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} \frac{1}{2} [(\underbrace{\cos^2 \beta + \sin^2 \beta}_1) - (\underbrace{\cos^2 \beta - \sin^2 \beta}_{\cos 2\beta})] dt \end{aligned}$$

1. EM-wave at long-distance limit: plane wave  $E = E_0 \cos(kx - \omega t + \phi_0)$
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Lecture Unit 1-2

$$\begin{aligned} \text{Single slit field} &= -\frac{\sin\left(\frac{1}{2}kD \sin \theta'\right)}{\frac{1}{2}k \sin \theta'} \cos\left(kx - \omega t + \frac{1}{2}kD \sin \theta'\right) \\ &\equiv \underbrace{\sqrt{\tilde{P}(\theta)}}_{\text{Amplitude modulation}} \cos\left(kx - \omega t + \underbrace{\phi_s}_{\text{Phase modulation}}\right) \end{aligned}$$

Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &\propto \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \langle [\cos(kx - \omega t + \phi_0)]^2 \rangle + \langle [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle + \langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle \end{aligned}$$

Auto-correlations

Cross-correlations

$$\begin{aligned} \langle [\cos(kx - \omega t + \phi_0)]^2 \rangle &\equiv \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} [\cos(kx - \omega t + \phi_0)]^2 dt \\ &= \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} \frac{1}{2} [(\cos^2 \beta + \sin^2 \beta) - \cos 2\beta] dt \end{aligned}$$

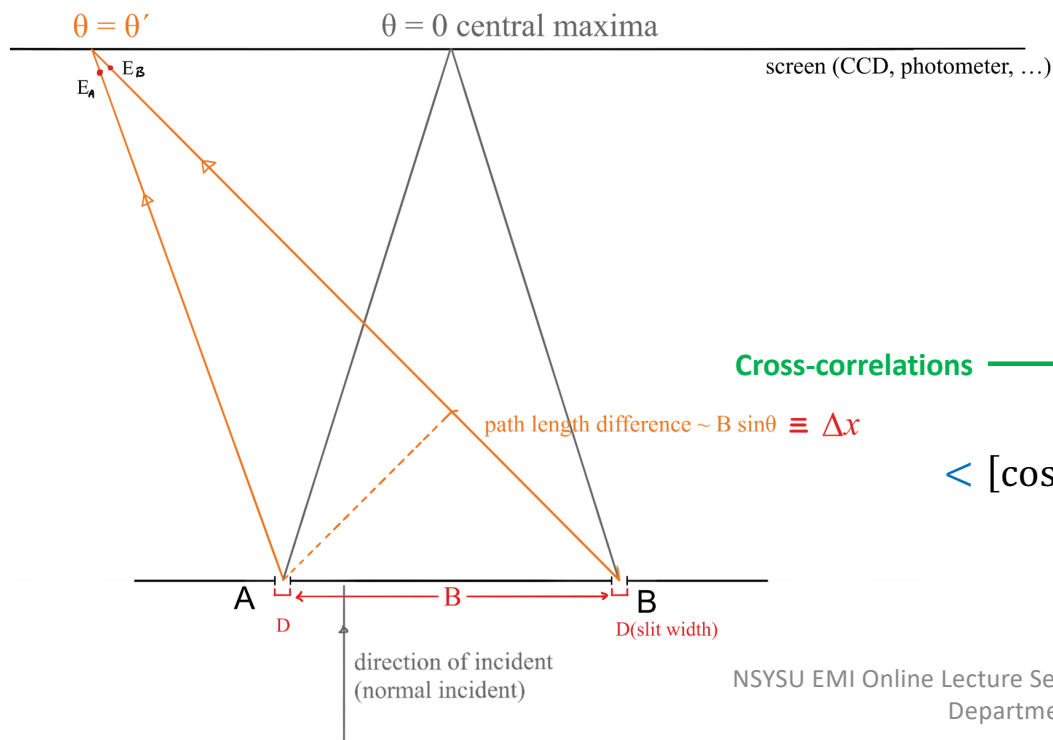
1

1. EM-wave at long-distance limit: plane wave  $E = E_0 \cos(kx - \omega t + \phi_0)$
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Lecture Unit 1-2

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Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &\propto \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \langle [\cos(kx - \omega t + \phi_0)]^2 \rangle \\ &\quad + \langle [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \quad \left. \vphantom{\langle [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle} \right\} \text{Auto-correlations} \\ &\quad + \langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle \quad \leftarrow \text{Cross-correlations} \end{aligned}$$

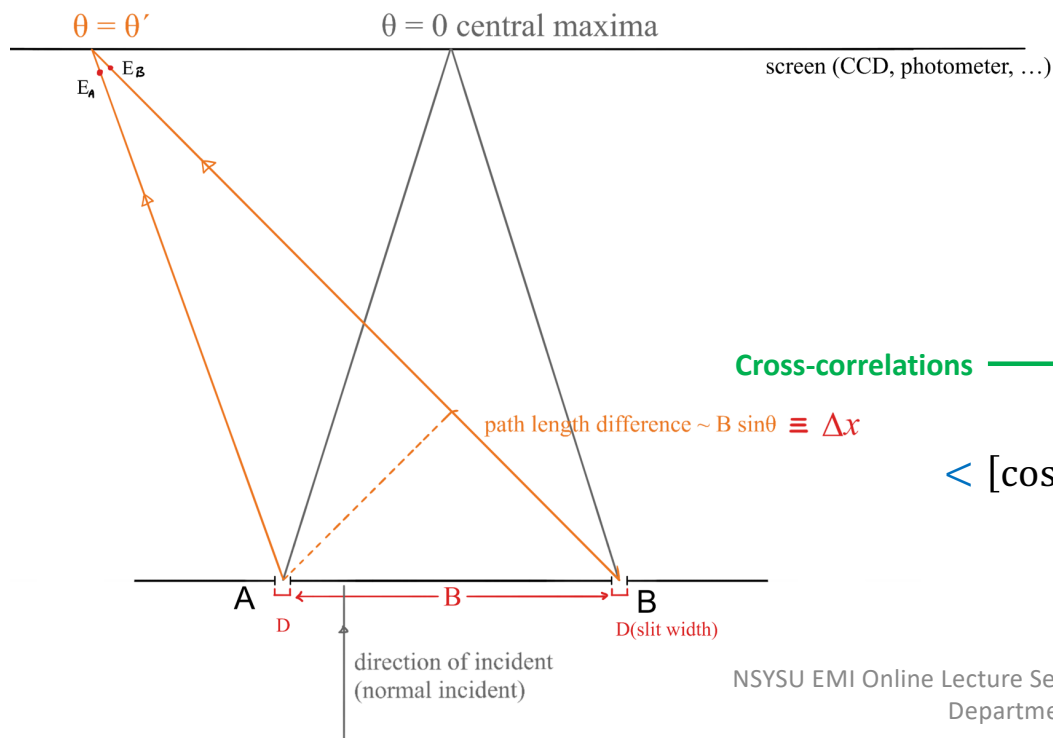
$$\begin{aligned} \langle [\cos(kx - \omega t + \phi_0)]^2 \rangle &\equiv \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} [\underbrace{\cos(kx - \omega t + \phi_0)}_{\beta}]^2 dt \\ &= \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} \frac{1}{2} [(\underbrace{\cos^2 \beta + \sin^2 \beta}_{1})] dt = \frac{1}{\Delta t} \frac{1}{2} \left( \frac{1}{2} \Delta t - \frac{-1}{2} \Delta t \right) \\ &= \frac{1}{2} \end{aligned}$$

1. EM-wave at long-distance limit: plane wave  $E = E_0 \cos(kx - \omega t + \phi_0)$
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Lecture Unit 1-2

$$\begin{aligned} \text{Single slit field} &= -\frac{\sin\left(\frac{1}{2}kD \sin \theta'\right)}{\frac{1}{2}k \sin \theta'} \cos\left(kx - \omega t + \frac{1}{2}kD \sin \theta'\right) \\ &\equiv \underbrace{\sqrt{\tilde{P}(\theta)}}_{\text{Amplitude modulation}} \cos\left(kx - \omega t + \underbrace{\phi_s}_{\text{Phase modulation}}\right) \end{aligned}$$

Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &\propto \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \left. \begin{aligned} &\cdot \frac{1}{2} \\ &+ \frac{1}{2} \end{aligned} \right\} \text{Auto-correlations} \end{aligned}$$

$$\text{Cross-correlations} \longrightarrow + \langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle$$

$$\langle [\cos(kx - \omega t + \phi_0)]^2 \rangle \equiv \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} [\cos(kx - \omega t + \phi_0)]^2 dt$$

$$\begin{aligned} &= \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} \frac{1}{2} [(\cos^2 \beta + \sin^2 \beta)] dt = \frac{1}{\Delta t} \frac{1}{2} \left( \frac{1}{2} \Delta t - \frac{-1}{2} \Delta t \right) \\ &= \frac{1}{2} \end{aligned}$$

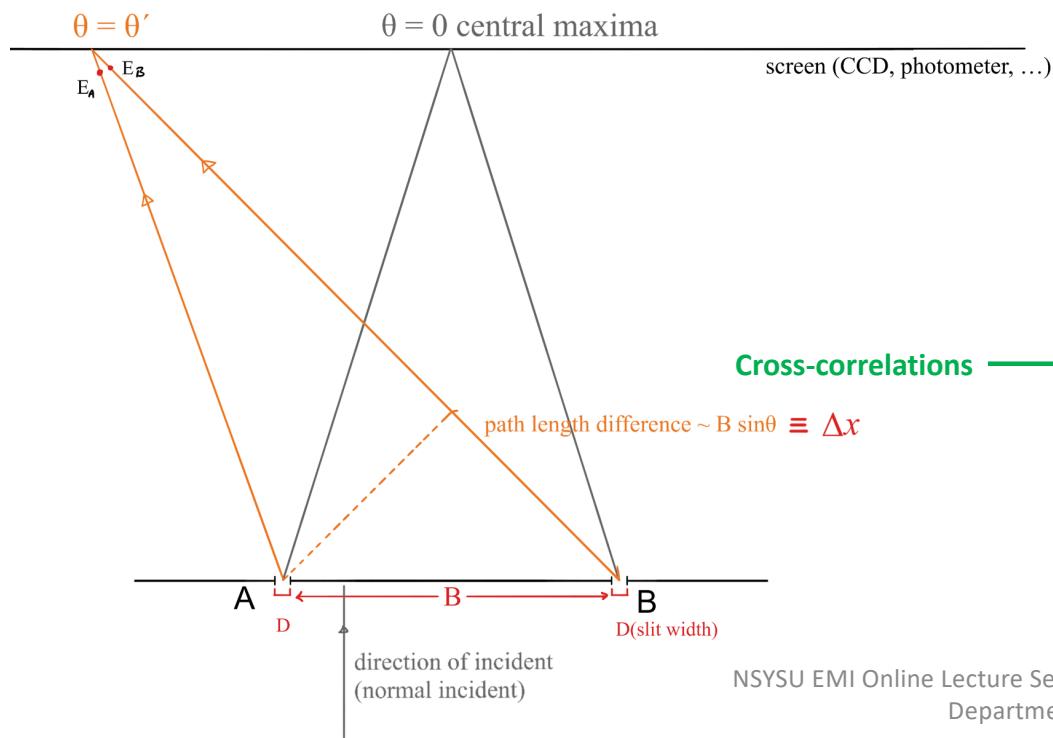


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Lecture Unit 1-2

$$\begin{aligned} \text{Single slit field} &= -\frac{\sin\left(\frac{1}{2}kD \sin \theta'\right)}{\frac{1}{2}k \sin \theta'} \cos\left(kx - \omega t + \frac{1}{2}kD \sin \theta'\right) \\ &\equiv \underbrace{\sqrt{\tilde{P}(\theta)}}_{\text{Amplitude modulation}} \cos\left(kx - \omega t + \underbrace{\frac{1}{2}kD \sin \theta'}_{\text{Phase modulation}}\right) \end{aligned}$$

Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &= \alpha \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \left. \begin{aligned} &\frac{1}{2} \\ &+ \frac{1}{2} \end{aligned} \right\} \text{Auto-correlations} \\ &\quad + \langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle \end{aligned}$$

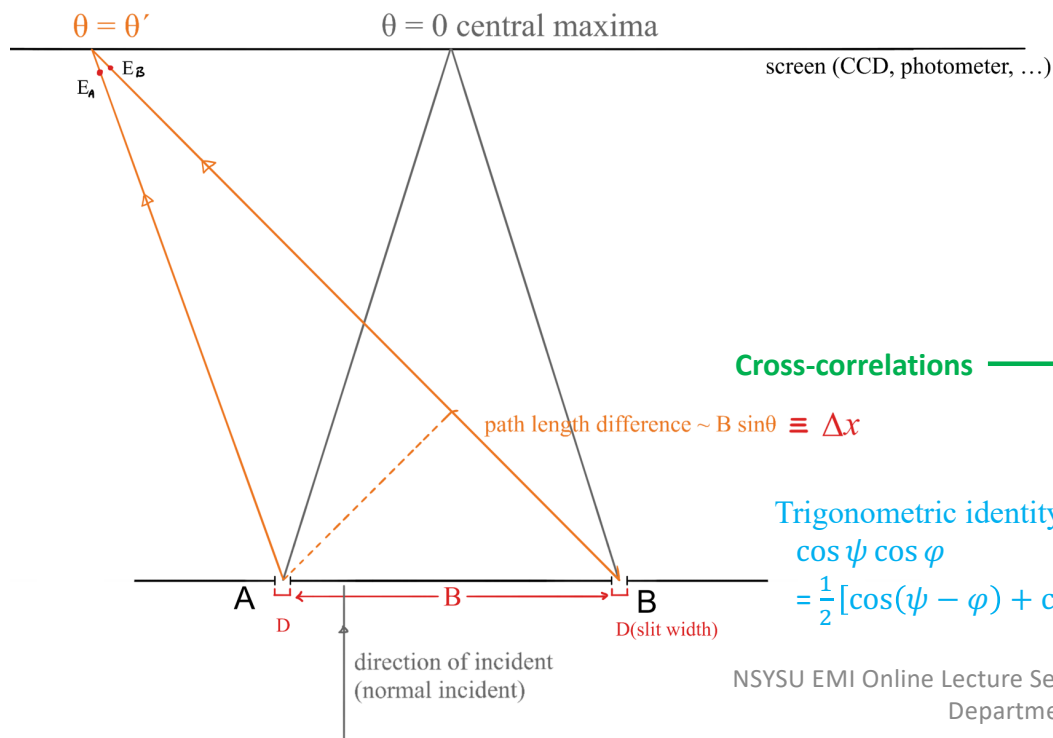
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Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &\propto \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \left. \begin{aligned} &\frac{1}{2} \\ &+ \frac{1}{2} \end{aligned} \right\} \text{Auto-correlations} \\ &\quad + \underbrace{\langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle}_{\psi \quad \phi} \end{aligned}$$

Cross-correlations

Trigonometric identity:

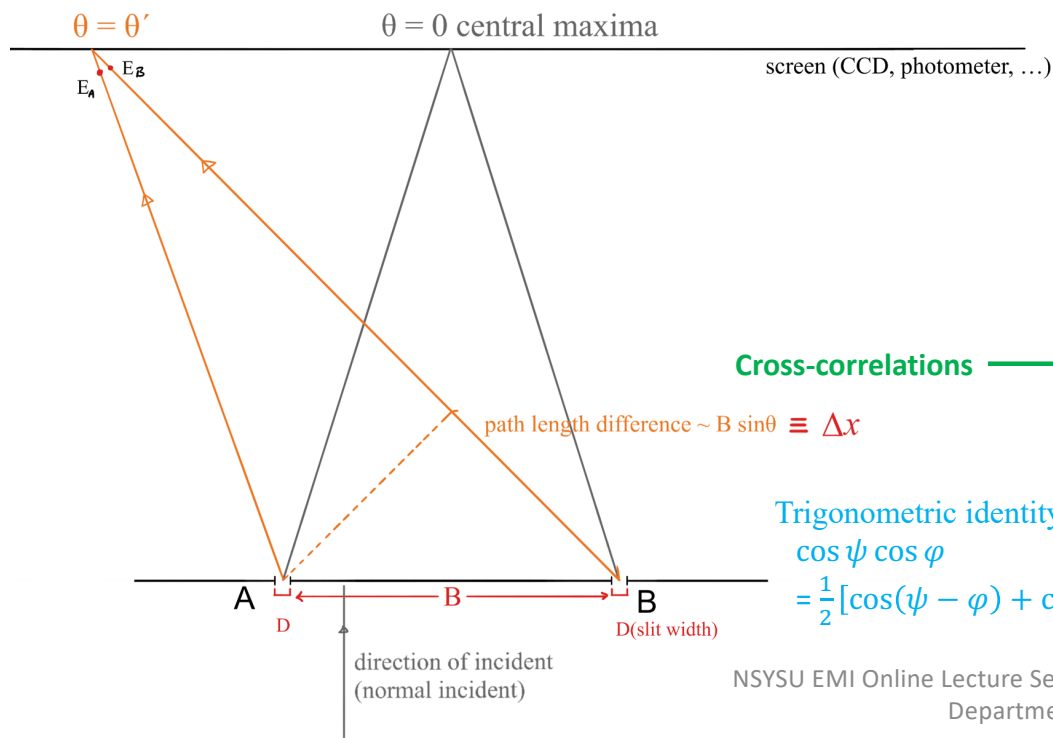
$$\begin{aligned} &\cos \psi \cos \phi \\ &= \frac{1}{2} [\cos(\psi - \phi) + \cos(\psi + \phi)] \end{aligned}$$

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Lecture Unit 1-2

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Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &= \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \left. \begin{aligned} &\frac{1}{2} \\ &+ \frac{1}{2} \end{aligned} \right\} \text{Auto-correlations} \\ &\quad + \underbrace{\langle 2 \cos(\underbrace{kx - \omega t + \phi_0}_{\phi}) \cos(\underbrace{k(x + \Delta x) - \omega t + \phi_0}_{\psi}) \rangle}_{\text{Cross-correlations}} \\ &= \cos(k\Delta x) + \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) \end{aligned}$$

Trigonometric identity:

$$\begin{aligned} &\cos \psi \cos \phi \\ &= \frac{1}{2} [\cos(\psi - \phi) + \cos(\psi + \phi)] \end{aligned}$$

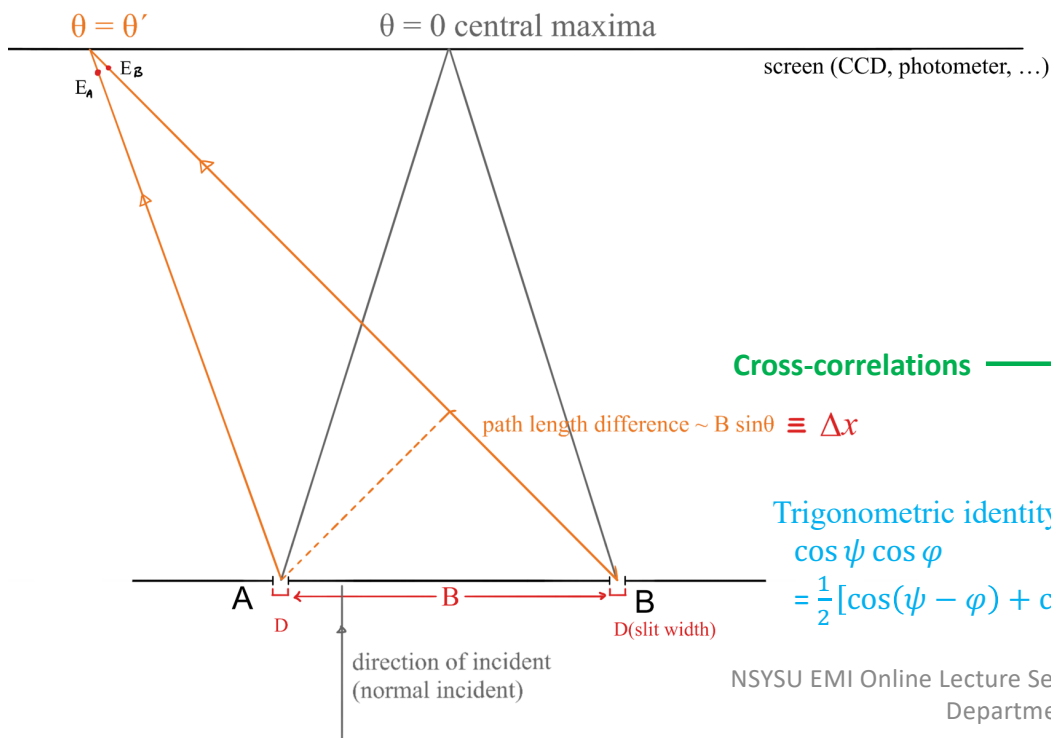
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Lecture Unit 1-2

$$\text{Single slit field} = -\frac{\sin\left(\frac{1}{2}kD \sin \theta'\right)}{\frac{1}{2}k \sin \theta'} \cos\left(kx - \omega t + \frac{1}{2}kD \sin \theta'\right)$$

$$\equiv \underbrace{\sqrt{\tilde{P}(\theta)}}_{\text{Amplitude modulation}} \cos\left(\underbrace{kx - \omega t + \phi_s}_{\text{Phase modulation}}\right)$$

Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\propto \langle [E_A + E_B]^2 \rangle$$

$$= \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle$$

$$= \left. \begin{matrix} 1/2 \\ + 1/2 \end{matrix} \right\} \text{Auto-correlations}$$

$$\text{Cross-correlations} \longrightarrow + \langle \underbrace{2\cos(kx - \omega t + \phi_0)}_{\phi} \underbrace{\cos(k(x + \Delta x) - \omega t + \phi_0)}_{\psi} \rangle$$

Trigonometric identity:

$$\cos \psi \cos \phi = \frac{1}{2} [\cos(\psi - \phi) + \cos(\psi + \phi)]$$

$$\cos(k\Delta x) + \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x)$$

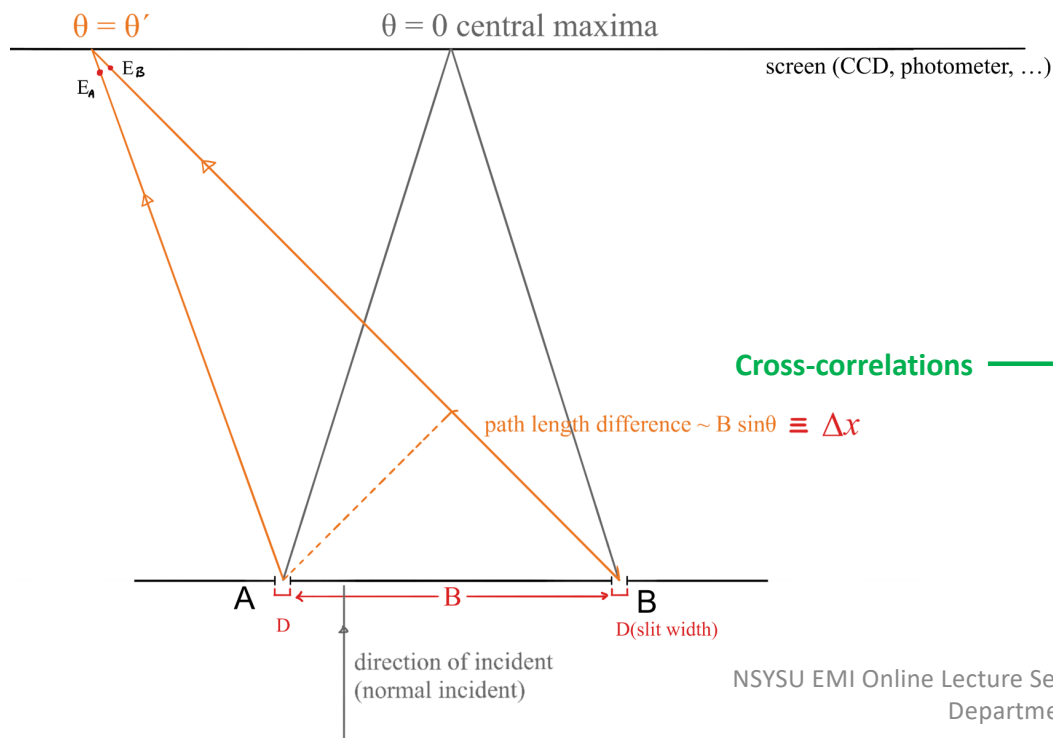
Minima:  $\cos(k\Delta x) = -1$   
 $\Rightarrow k\Delta x = \frac{2\pi}{\lambda} B \sin \theta = m\pi, m = \pm 1, 3, 5, \dots$

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Lecture Unit 2-2



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$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &= \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \left. \begin{aligned} &\frac{1}{2} \\ &+ \frac{1}{2} \end{aligned} \right\} \text{Auto-correlations} \\ &\quad + \underbrace{\langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle}_{\text{Cross-correlations}} \end{aligned}$$

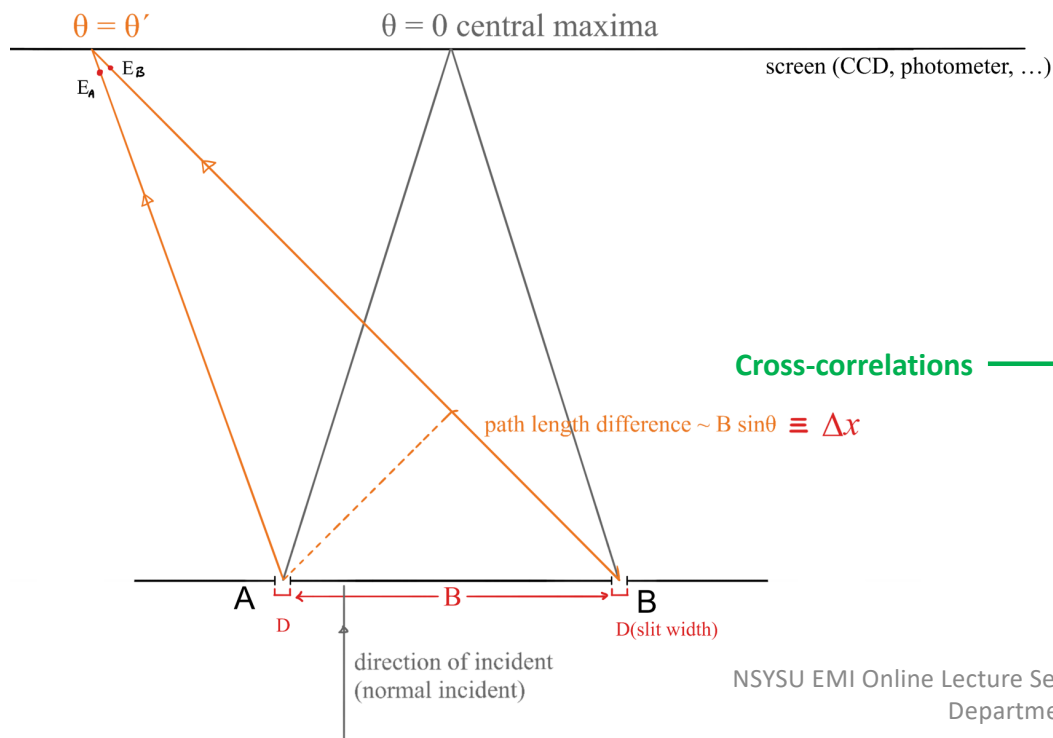
$$\begin{aligned} \langle [E_A + E_B]^2 \rangle &= \tilde{P}(\theta) \left[ 1 + \cos\left(\frac{2\pi}{\lambda} B \sin \theta\right) \right] \\ &\sim \tilde{P}(\theta) \left[ 1 + \cos\left(2\pi \frac{B}{\lambda} \theta\right) \right] \end{aligned}$$

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Lecture Unit 1-2

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$\phi$   $\psi$

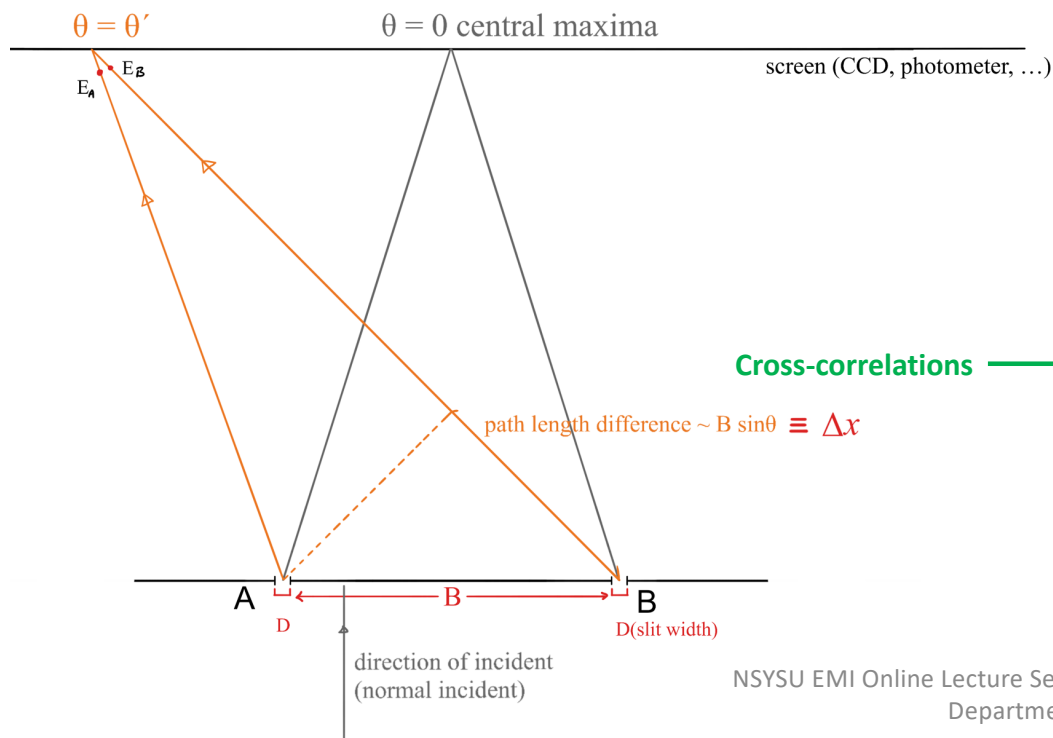
$$\begin{aligned} \langle [E_A + E_B]^2 \rangle &= \tilde{P}(\theta) \left[ 1 + \cos\left(\frac{2\pi}{\lambda} B \sin \theta\right) \right] \\ &\sim \tilde{P}(\theta) \left[ \underbrace{1}_{\text{Auto-correlations}} + \underbrace{\cos\left(2\pi \frac{B}{\lambda} \theta\right)}_{\text{Cross-correlations}} \right] \end{aligned}$$

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2. Energy flux density  $\propto E_0^2$

Lecture Unit 1-2

$$\begin{aligned} \text{Single slit field} &= -\frac{\sin\left(\frac{1}{2}kD \sin \theta'\right)}{\frac{1}{2}k \sin \theta'} \cos\left(kx - \omega t + \frac{1}{2}kD \sin \theta'\right) \\ &\equiv \underbrace{\sqrt{\tilde{P}(\theta)}}_{\text{Amplitude modulation}} \cos\left(\underbrace{kx - \omega t + \phi_s}_{\text{Phase modulation}}\right) \end{aligned}$$

Lecture Unit 2-2



Number of incoming photons in a unit of time and a unit area (at the angle of emergence  $\theta = \theta'$ )

$$\begin{aligned} &\propto \langle [E_A + E_B]^2 \rangle \\ &= \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\ &= \left. \begin{aligned} &\frac{1}{2} \\ &+ \frac{1}{2} \end{aligned} \right\} \text{Auto-correlations} \\ &\quad + \underbrace{\langle 2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) \rangle}_{\text{Cross-correlations}} \end{aligned}$$

$\phi$   $\psi$

$$\begin{aligned} \langle [E_A + E_B]^2 \rangle &= \tilde{P}(\theta) \left[ 1 + \cos\left(\frac{2\pi}{\lambda} B \sin \theta\right) \right] \\ &\sim \tilde{P}(\theta) \left[ 1 + \cos\left(2\pi \frac{B}{\lambda} \theta\right) \right] \sim \tilde{P}(\theta) [1 + \cos(2\pi u \theta)] \end{aligned}$$

## 1. Interference pattern of a double-slit:

$\tilde{P}(\theta) \left[ 1 + \cos \left( 2\pi \frac{B}{\lambda} \theta \right) \right] \sim \tilde{P}(\theta) [1 + \cos(2\pi u \theta)]$ , where  $B$  is the separation of the two slits, and  $\tilde{P}(\theta)$  is the diffraction pattern of a single-slit.