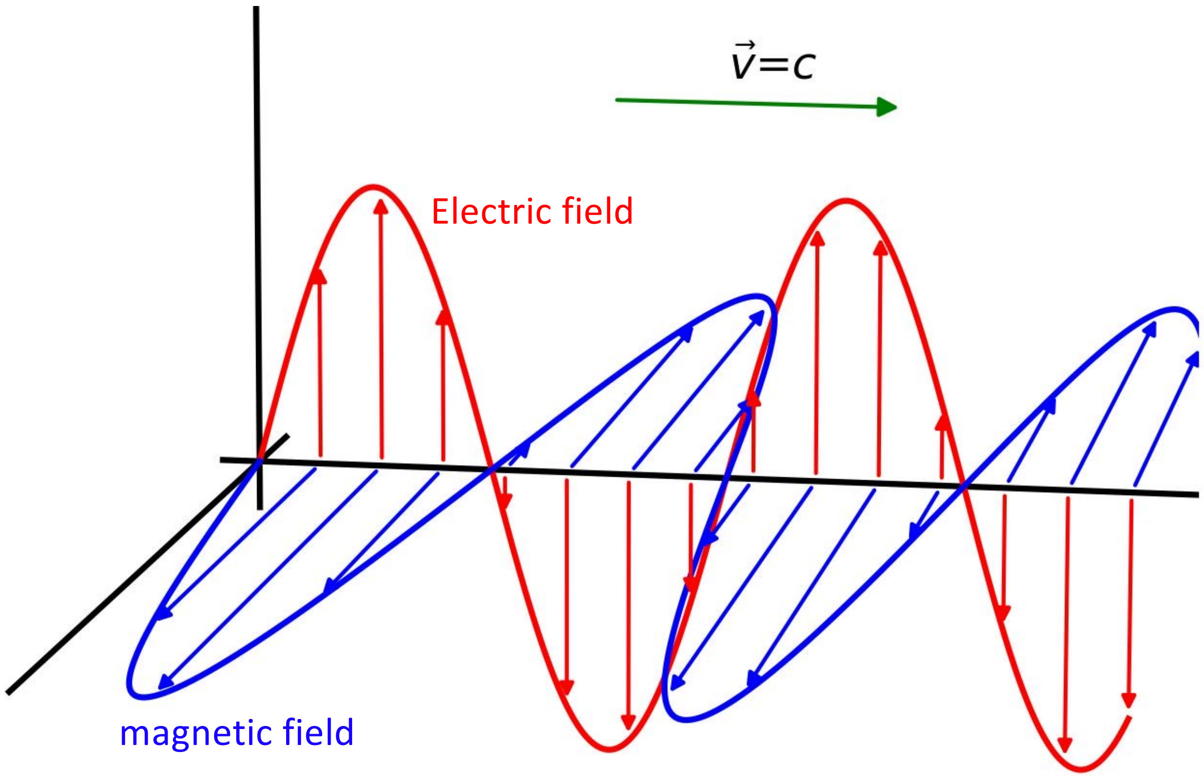
An Introduction to Radio Interferometry

1-3 Energy flux density

You can find relevant material
on my personal webpage

How can we observe? (again)



When the electromagnetic wave passes through a charged particle (e.g., electron or other ions), the charged particle is moved due to the Lorentz force (electromagnetic force).

The capability of moving a charged particle (i.e., doing work) means there is energy stored in the form of electric and magnetic field energy, which is propagated along with the EM waves.

EM waves propagate a certain amount of energy in the direction of $\vec{E} \times \vec{B}$, where $\vec{E}$ and $\vec{B}$ are the electric and magnetic field components of the propagating wave, respectively.

Energy Conservation

Current density

Power in a unit volume $\vec{J} \cdot \vec{E} \sim \left. \frac{dQ}{dA} \right/ _{dt} E \sim \frac{1}{dAds} [(EdQ) \frac{ds}{dt}]$

Work
in a unit of time

Volume

With the help of Maxwell's equations, we can re-expresss work in the following conservation law:

Locally, the change of energy density is balanced by the energy transportation and work.

Energy density

$$\frac{\partial u}{\partial t} + \nabla \cdot \vec{S} = -\vec{J} \cdot \vec{E}$$

Poynting vector $\vec{S} = \vec{E} \times \vec{H}$, $\overrightarrow{H} = \frac{\vec{B}}{\mu}$, where μ is the permeability

Energy in Electric Field

Electric potential at location $\overrightarrow{x_i}$: $$\Phi(\overrightarrow{x_i}) = \frac{1}{4\pi\epsilon_0}\sum_j \frac{q_j}{|\vec{x}_j - \vec{x}_i|}$$

Electric field: $$\vec{E} = -\nabla\Phi$$

Energy in Electric Field (Discrete charges)

Electric potential at location $\overrightarrow{x_i}$: $\Phi(\overrightarrow{x_i}) = \dfrac{1}{4\pi\epsilon_0}\sum_j \dfrac{q_j}{|\vec{x}_j - \vec{x}_i|}$

Electric field: $\vec{E} = -\nabla\Phi$

Electric potential energy of charge $\overrightarrow{q_i}$: $W_i = q_i\Phi(\overrightarrow{x_i}) = \dfrac{q_i}{4\pi\epsilon_0}\sum_{j\neq i}\dfrac{q_j}{|\overrightarrow{x_i} - \overrightarrow{x_j}|}$

Total electric potential energy : $W = \dfrac{1}{2}\sum_i\sum_j \dfrac{1}{4\pi\epsilon_0}\dfrac{q_i q_j}{|\overrightarrow{x_i} - \overrightarrow{x_j}|}$

Energy in Electric Field (Continuous charges)

Electric field: $\vec{E} = -\nabla\Phi$

Total electric potential energy : $W = \frac{1}{2}\int \rho(\vec{x})\Phi(\vec{x})d^3x = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\int\int \rho(\vec{x})\frac{\rho(\vec{x'})}{|\vec{x} - \vec{x'}|}d^3x d^3x'$

Energy in Electric Field (Continuous charges)

Electric field: $\vec{E} = -\nabla\Phi$

Total electric potential energy : $W = \frac{1}{2}\int \rho(\vec{x})\Phi(\vec{x})d^3x = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\int\int \rho(\vec{x})\frac{\rho(\vec{x'})}{|\vec{x} - \vec{x'}|}d^3xd^3x'$

Gauss law: $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Energy in Electric Field (Continuous charges)

Electric field: $\vec{E} = -\nabla\Phi$

Total electric potential energy : $W = \frac{1}{2}\int \rho(\vec{x})\Phi(\vec{x})d^3x = \frac{\epsilon_0}{2}\int (\nabla\cdot\vec{E})\Phi(\vec{x})d^3x$

Gauss law: $\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

Energy in Electric Field (Continuous charges)

Electric field: $\vec{E} = -\nabla\Phi$

Total electric potential energy : $W = \frac{1}{2}\int \rho(\vec{x})\Phi(\vec{x})d^3x = \frac{\epsilon_0}{2}\int (\nabla\cdot\vec{E})\Phi(\vec{x})d^3x$

Integration by part

$$= \frac{\epsilon_0}{2}\int \vec{E}\cdot\big(-\nabla\Phi(\vec{x})\big)d^3x = \frac{\epsilon_0}{2}\int |E|^2 d^3x$$

Energy in Electric Field (Continuous charges)

Electric field: $\vec{E} = -\nabla\Phi$

Total electric potential energy : $W_E = \frac{1}{2}\int \rho(\vec{x})\Phi(\vec{x})d^3x = \frac{\epsilon_0}{2}\int (\nabla\cdot\vec{E})\Phi(\vec{x})d^3x$

Integration by part

$$= \frac{\epsilon_0}{2}\int \vec{E}\cdot\big(-\nabla\Phi(\vec{x})\big)d^3x = \frac{\epsilon_0}{2}\int |E|^2 d^3x$$

From the symmetry of Maxwell's equations we expect magnetic field energy to have a similar form $W_B = \frac{1}{2\mu_0}\int |B|^2 d^3x$

Some Incomplete Physical Arguments that Help

Energy in Electric Field (Continuous charges)

Electric field: $\vec{E} = -\nabla\Phi$

Total electric potential energy : $W_E = \frac{1}{2}\int \rho(\vec{x})\Phi(\vec{x})d^3x = \frac{\epsilon_0}{2}\int (\nabla\cdot\vec{E})\Phi(\vec{x})d^3x$

Integration by part

$$= \frac{\epsilon_0}{2}\int \vec{E}\cdot\left(-\nabla\Phi(\vec{x})\right)d^3x = \frac{\epsilon_0}{2}\int |E|^2 d^3x$$

From the symmetry of Maxwell's equations we expect magnetic field energy to have a similar form $W_B = \frac{1}{2\mu_0}\int |B|^2 d^3x$

In EM wave

$$\frac{E_m}{B_m} = \frac{1}{\mu_0\epsilon_0 c}$$

Flux density

Energy is proportional to the square of electric or magnetic field strength.

The **energy flux density**, which is the amount of energy passing a unit area in a unit of frequency and time,

Is also proportional to the **square of electric and/or magnetic field strength**.

Electric field: $\vec{E} = -\nabla\Phi$

Total electric potential energy : $W_E = \frac{1}{2}\int \rho(\vec{x})\Phi(\vec{x})d^3x = \frac{\epsilon_0}{2}\int (\nabla\cdot\vec{E})\Phi(\vec{x})d^3x$

$$= \frac{\epsilon_0}{2}\int \vec{E}\cdot\left(-\nabla\Phi(\vec{x})\right)d^3x = \frac{\epsilon_0}{2}\int |E|^2 d^3x$$

From the symmetry of Maxwell's equations we expect magnetic field energy to have a similar form $W_B = \frac{1}{2\mu_0}\int |B|^2 d^3x$

In EM wave

$$\frac{E_m}{B_m} = \frac{1}{\mu_0\epsilon_0 c}$$

Flux density

Energy is proportional to the square of electric or magnetic field strength.
The **energy flux density**, which is the amount of energy passing a unit area in a unit of frequency and time,
Is also proportional to the **square of electric and/or magnetic field strength**.

Flux density F_{ν} :

SI unit: Joul s^{-1} m^{-2} Hz^{-1} = W m^{-2} Hz^{-1}
Often used in astronomical studies: 1 Jy = 10^{-26} W m^{-2} Hz^{-1}

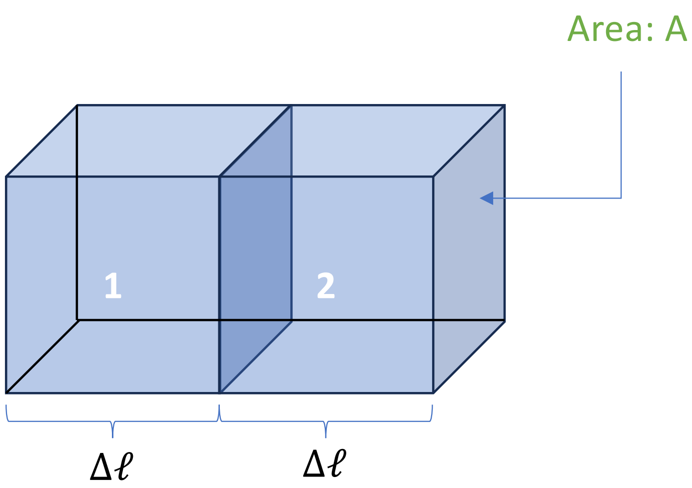
Jansky (the name of the first radio astronomer, Karl Jansky)

Flux density

(Illustrated, transportation of stational electric field)

$$W_E = \frac{\epsilon_0}{2}\int |E|^2 d^3x$$

$$\Delta t = \frac{\Delta \ell}{c}$$



Initial total energy in box 1 $\sim \frac{\epsilon_0}{2} E^2 A\Delta\ell \equiv W$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Flux density: $\frac{W}{A\Delta t} = \frac{cW}{A\Delta\ell} = \frac{1}{2}\epsilon_0 c E^2 = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}|E|^2$

Flux density in plane EM Wave

$$\omega = 2\pi\nu$$

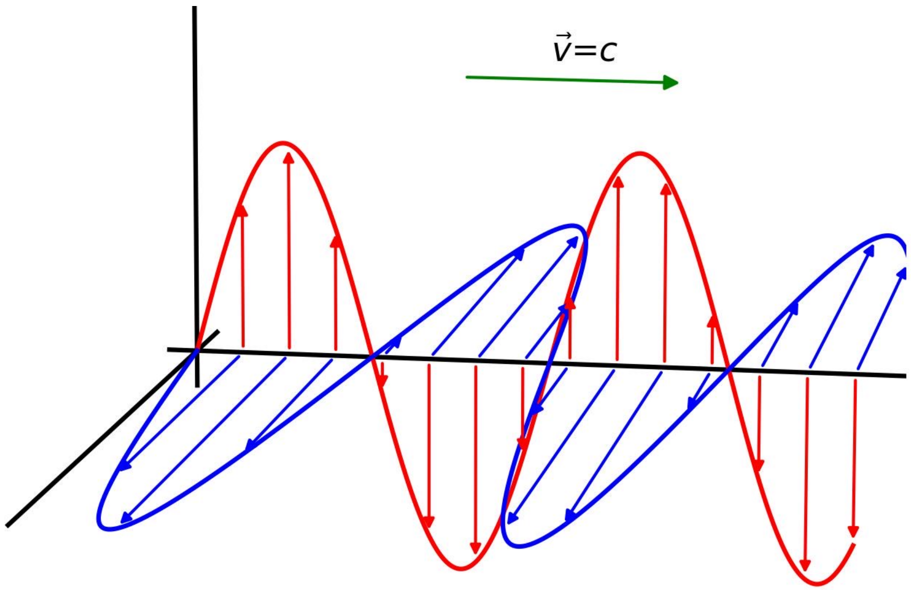
Monochromatic plane wave

$$\begin{cases} E = E_m \sin(kx - \omega t) \\ B = B_m \sin(kx - \omega t) \end{cases}$$

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$k = \frac{2\pi}{\lambda}$$

$$\frac{E_m}{B_m} = \frac{1}{\mu_0 \epsilon_0 c}$$



(time-averaged) Flux density (i.e., time-averaged amplitude of Poynting vector):

$$F = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}\,|E_m|^2$$

Photons

<u>Max Born's hypothesis</u>

1. A photon has energy $\boldsymbol{h\nu}$, and a photon is indivisible.
2. A single photon may be described by the one-photon electric field $\vec{e}(\vec{r},t)$. The probability of this photon to be found at location $\vec{r}$ and time t is proportional to $[\vec{e}(\vec{r},t)]^2 d^3\vec{r}$. $\vec{e}(\vec{r},t)$ satisifies a linear equation of motion.
3. When many photons are involved, the individual one-photon fields somehow combine to create the classical electric field $\vec{E}$.

(time-averaged) number of incoming photons in a unit of time and area **(monochromatic wave)**:

$$F/h\nu = \frac{1}{2h\nu}\sqrt{\frac{\epsilon_0}{\mu_0}}\,|E_m|^2$$

Photons

<u>Max Born's hypothesis</u>

1. A photon has energy $h\nu$, and a photon is indivisible.
2. A single photon may be described by the one-photon electric field $\vec{e}(\vec{r},t)$. The probability of this photon to be found at location $\vec{r}$ and time t is proportional to $[\vec{e}(\vec{r},t)]^2 d^3\vec{r}$. $\vec{e}(\vec{r},t)$ satisifies a linear equation of motion.
3. When many photons are involved, the individual one-photon fields somehow combine to create the classical electric field $\vec{E}$.

(time-averaged) number of incoming photons in a unit of time and area **(monochromatic wave)**:

$$\underbrace{F/h\nu}_{\text{optical detector (e.g., CCD)}} = \frac{1}{2h\nu}\sqrt{\frac{\epsilon_0}{\mu_0}}\,\underbrace{|E_m|^2}_{\text{radio detector}}$$

1. The (plane) EM waves have a sinusoidal form: $E = E_m \sin(kx - \omega t)$

   $B = B_m \sin(kx - \omega t)$

   EM wave can transports energy.

   $\frac{E_m}{B_m} = \frac{1}{\mu_0 \epsilon_0 c}$

2. The time-averaged flux density (energy in a unit time, unit area, and unit frequency) is $F = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}|E_m|^2$