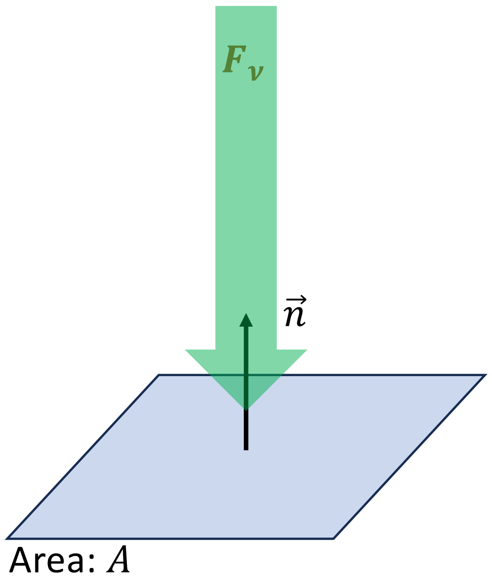
An Introduction to Radio Interferometry

1-4 Intensity and brightness temperature

You can find relevant material
on my personal webpage

Flux density as a vector field

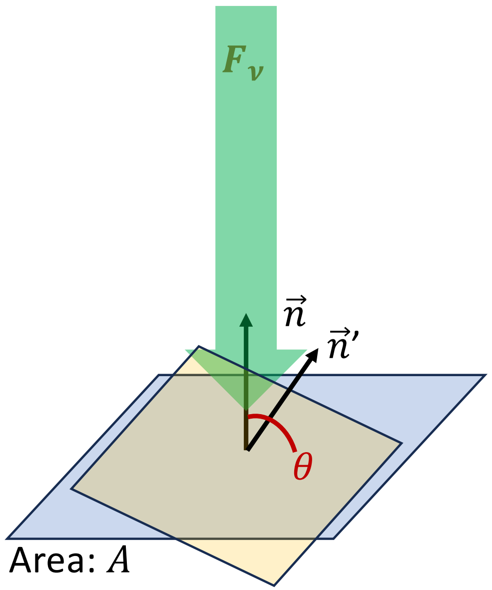


Normal incidence

Energy passing a CCD pixel that has area A in a unit of time

$$F_\nu A$$

Flux density as a vector field



Normal incidence

Energy passing a CCD pixel that has area A in a unit of time

$$F_\nu A$$

Inclined CCD pixel

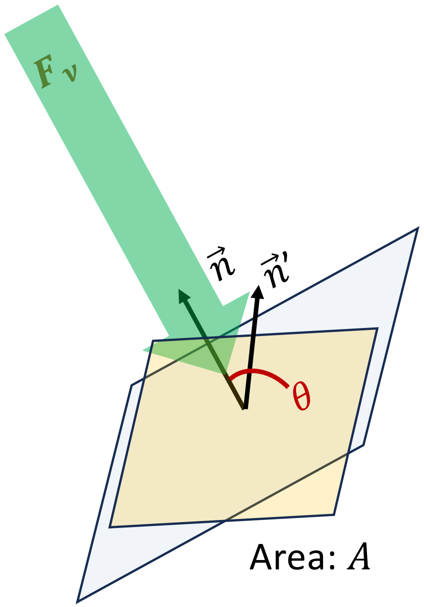
Energy passing a CCD pixel that has area A in a unit of time

$$F_\nu A\left(\vec{n}\cdot\overrightarrow{n'}\right) = F_\nu A\cos\theta$$

$$\vec{n}\cdot\overrightarrow{n'} = \cos\theta$$

Flux density as a vector field

Inclined incident photon streams



Energy passing a CCD pixel that has area A in a unit of time

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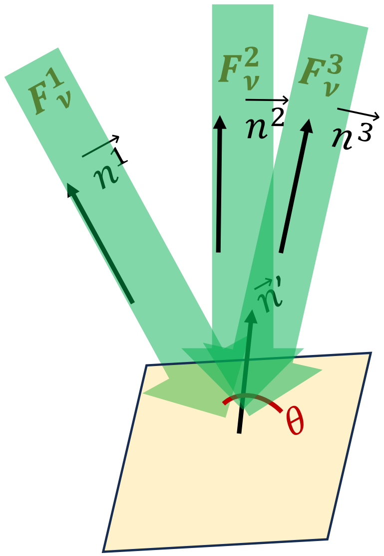
Flux density as a vector field

Inclined, discrete incident photon streams

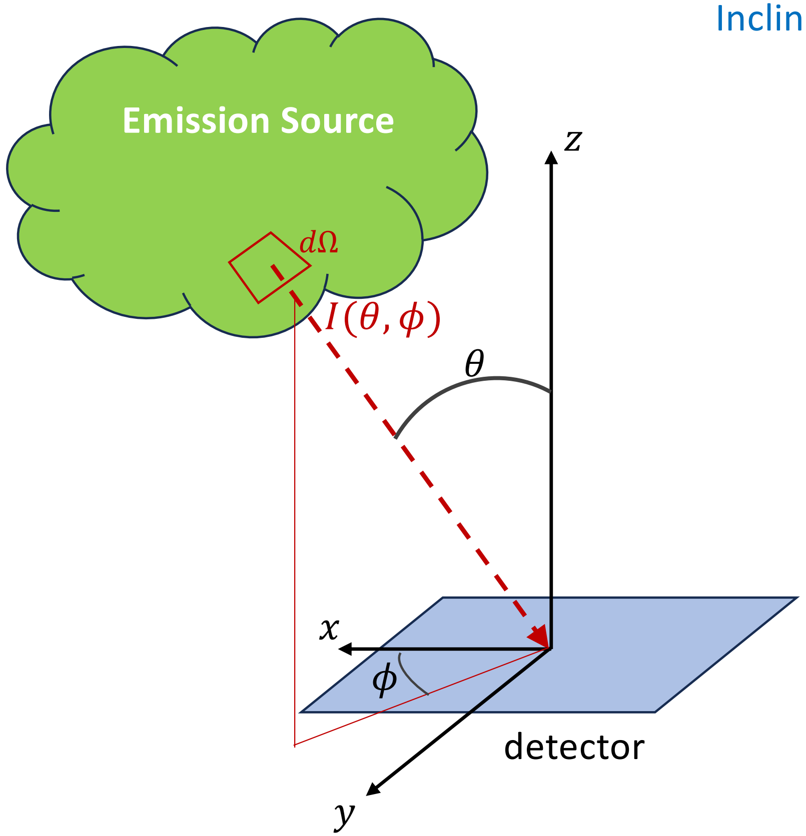


Energy passing a CCD pixel that has area A in a unit of time

$$-\sum_i F_\nu^i A \left(\overrightarrow{n^i} \cdot \overrightarrow{n'}\right) = -\sum_i F_\nu^i\, A \cos\theta^i$$

$$\overrightarrow{n^i} \cdot \overrightarrow{n'} = \cos\theta^i$$

Continuous incident light



Inclined, discrete incident photon streams

Energy passing a CCD pixel that has area A in a unit of time

$$-\sum_i F_\nu^i A\left(\vec{n^i}\cdot\vec{n'}\right) = -\sum_i F_\nu^i\, A\cos\theta^i$$

infinitesimal solid angle: $d\Omega \equiv \sin\theta\, d\theta d\phi$

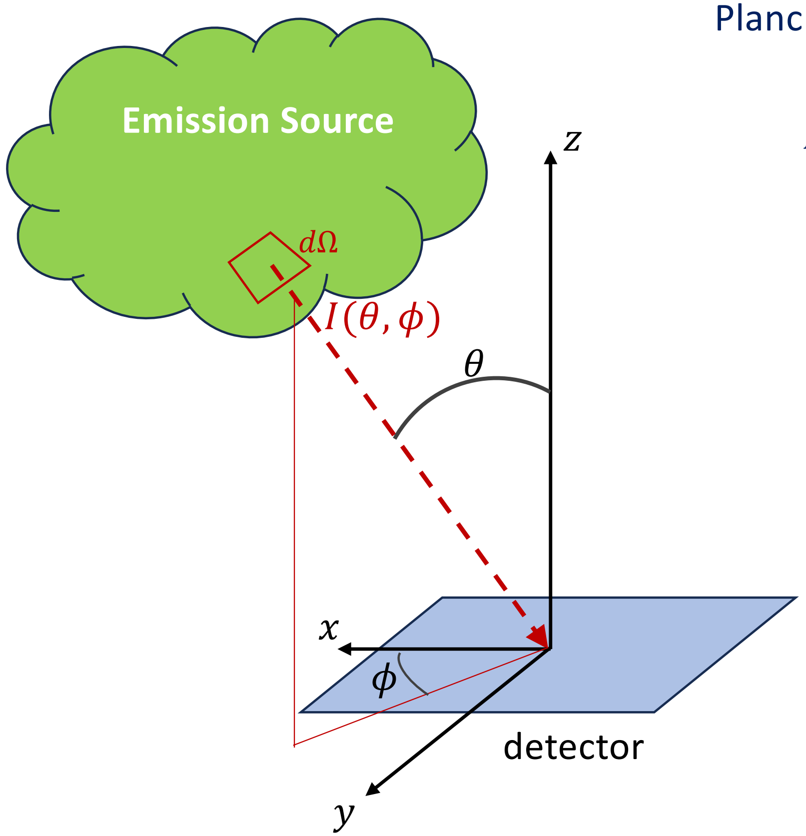
Intensity $I(\theta,\phi)$: the amount of energy through a unit of area A in a unit of time t from a unit solid angle $d\Omega$ around the direction (θ,ϕ) [i.e., flux density per unit solid angle]

SI Unit: Joul s^{-1} m^{-2} Hz^{-1} Sr^{-1}

Flux density normal to our CCD pixel:

$$F_\nu = \int I_\nu(\theta,\phi)\cos\theta\, d\Omega$$

Emission of extended black body



Planck function: intensity of a black body at temperature T and frequency ν

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

h: Planck's constant ($6.62607015 \times 10^{-34}$ m^2 kg s^{-1})

k: Boltzmann constant (1.380649×10^{-23} m^2 kg s^{-2} K^{-1})

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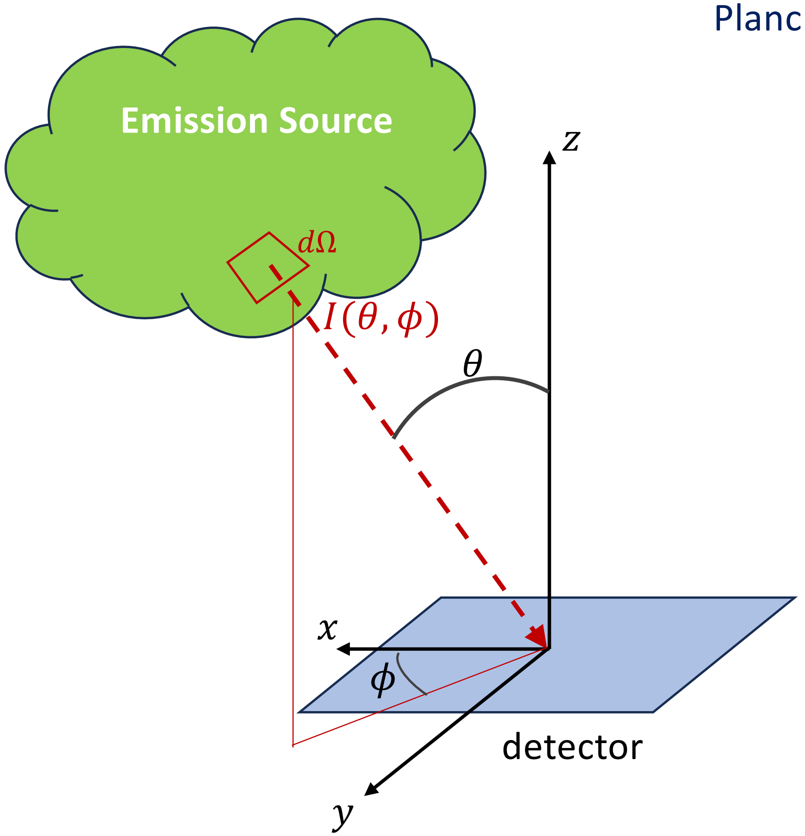

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$$(h\nu \ll kT) \sim \frac{2\nu^2}{c^2} kT$$

(Rayleigh-Jeans limit, i.e., high temperature or long wavelength limit)

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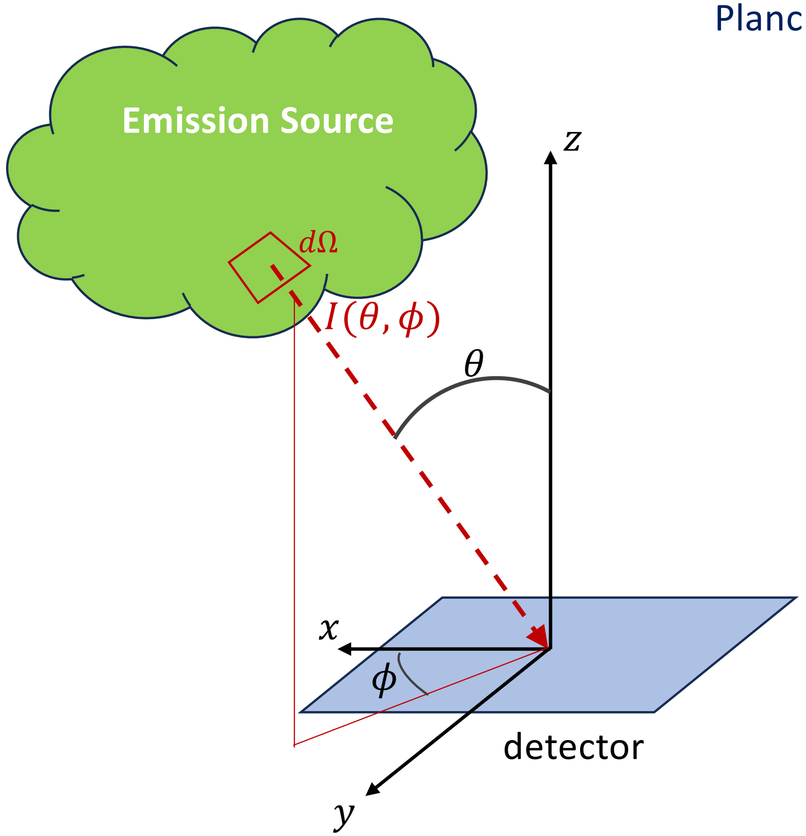

Planck function: intensity of a black body at temperature T and frequency ν

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \sim \frac{2h\nu^3}{c^2}\frac{1}{1+\frac{h\nu}{kT}-1} \sim \frac{2\nu^2}{c^2}kT$$

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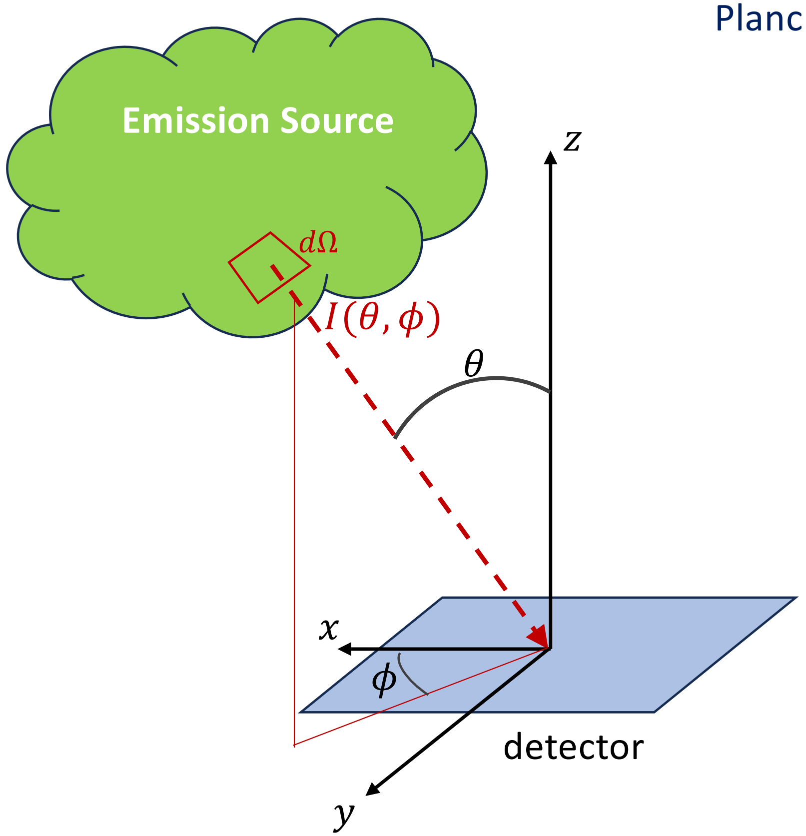

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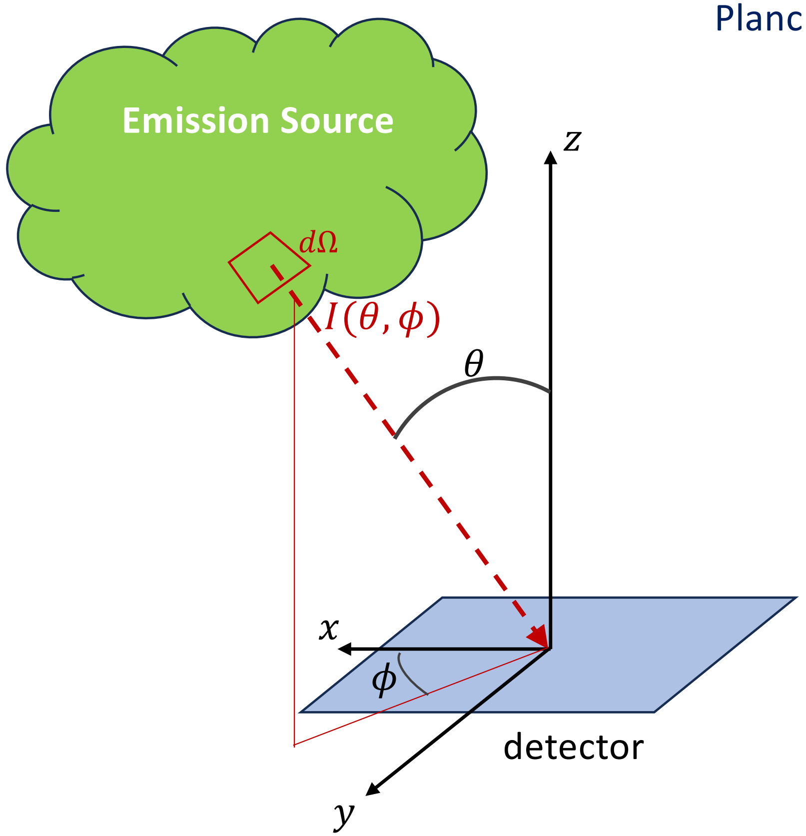
$$B_\nu^{RJ}(T) \sim \frac{2h\nu^3}{c^2} \frac{1}{1+\frac{h\nu}{kT}+\cdots-1} \sim \frac{2\nu^2}{c^2} kT = I_\nu(\theta, \phi)$$

$(h\nu \ll kT) \sim \frac{2\nu^2}{c^2} kT$ (Rayleigh-Jeans limit, i.e., high temperature or long wavelength limit)

We denote the temperature of the black body as T_B in this case and call it the corresponding brightness temperature of the observed in intensity $I_\nu(\theta, \phi)$.

Emission of extended black body



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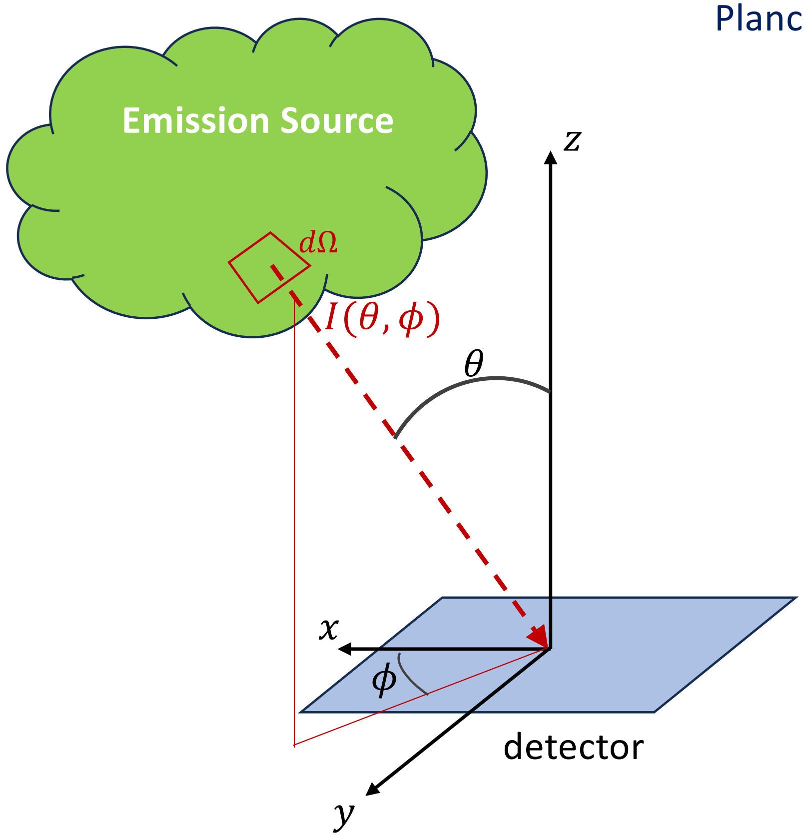
Example of the effect of dust scattering

Emission of extended black body



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$$\boxed{T_B = \frac{c^2}{2k\nu^2} I_\nu = \frac{\lambda^2}{2k} I_\nu}$$

Important!

1. We defined intensity $I(\theta, \phi)$ to describe flux density in a unit solid angle.

2. The flux density $F_\nu = \int I_\nu(\theta, \phi) \cos\theta \, d\Omega$

3. We define brightness temperature $T_B = \frac{c^2}{2k\nu^2} I_\nu = \frac{\lambda^2}{2k} I_\nu$