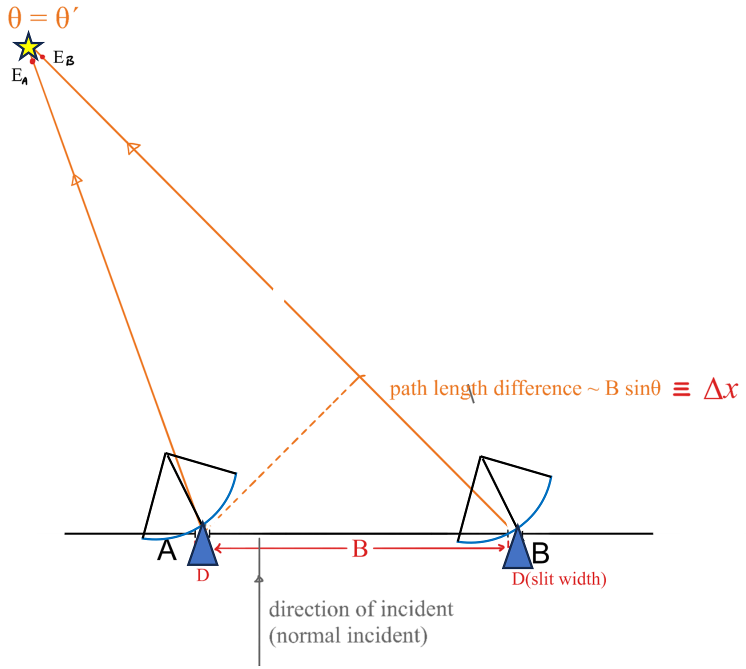
# An Introduction to Radio Interferometry

## 3-3 Complex Visibility (1D)

You can find relevant material
on my personal webpage

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

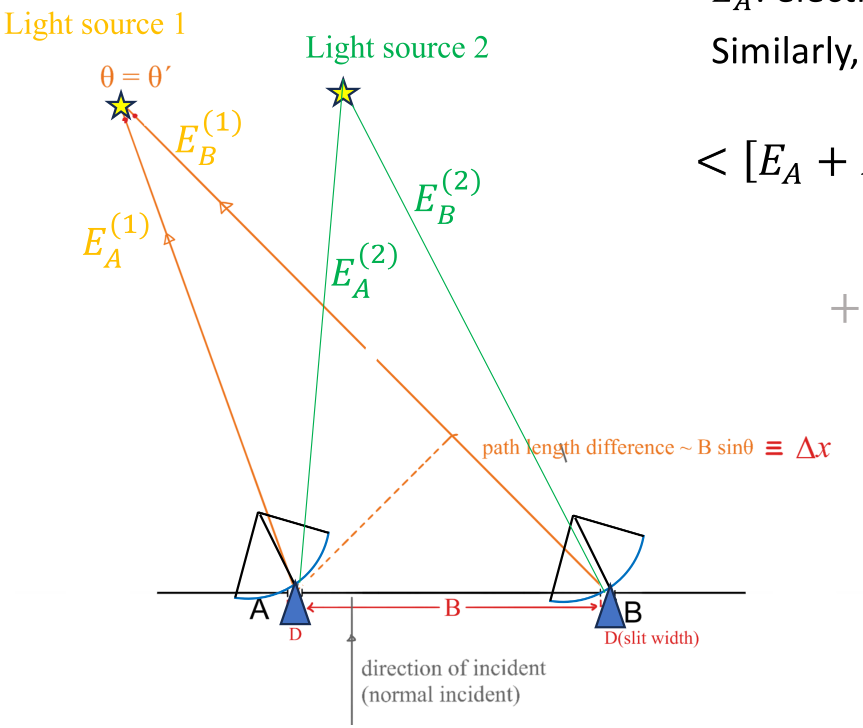



# When there is one target source

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

# When there are two incoherent sources




$E_A$: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

$$< [E_A + E_B]^2 > \; = < {E_A^{(1)}}^2 > + < {E_B^{(1)}}^2 > + < 2E_A^{(1)}E_B^{(1)} >$$

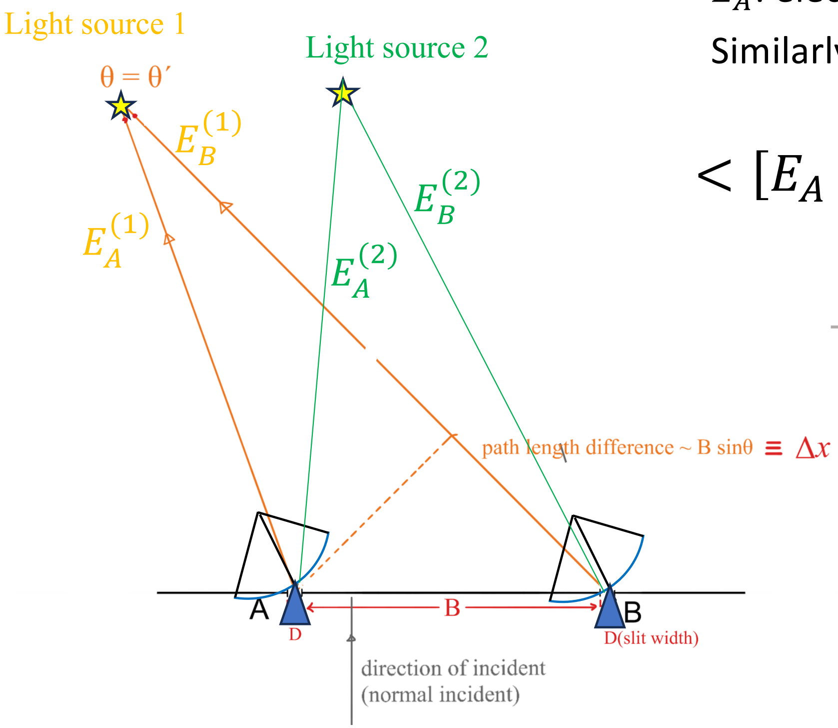
$$+ < {E_A^{(2)}}^2 > + < {E_B^{(2)}}^2 > + < 2E_A^{(2)}E_B^{(2)} >$$

$$+ < 2E_A^{(1)}E_A^{(2)} > + < 2E_A^{(1)}E_B^{(2)} > + < 2E_B^{(1)}E_A^{(2)} > + < 2E_B^{(1)}E_B^{(2)} >$$

<u>Cosine correlation:</u> $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

<u>Sine correlation:</u> $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

# When there are two <u>incoherent</u> sources




$E_A$: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

$$< [E_A + E_B]^2 > \; = < {E_A^{(1)}}^2 > + < {E_B^{(1)}}^2 > + < 2E_A^{(1)} E_B^{(1)} >$$

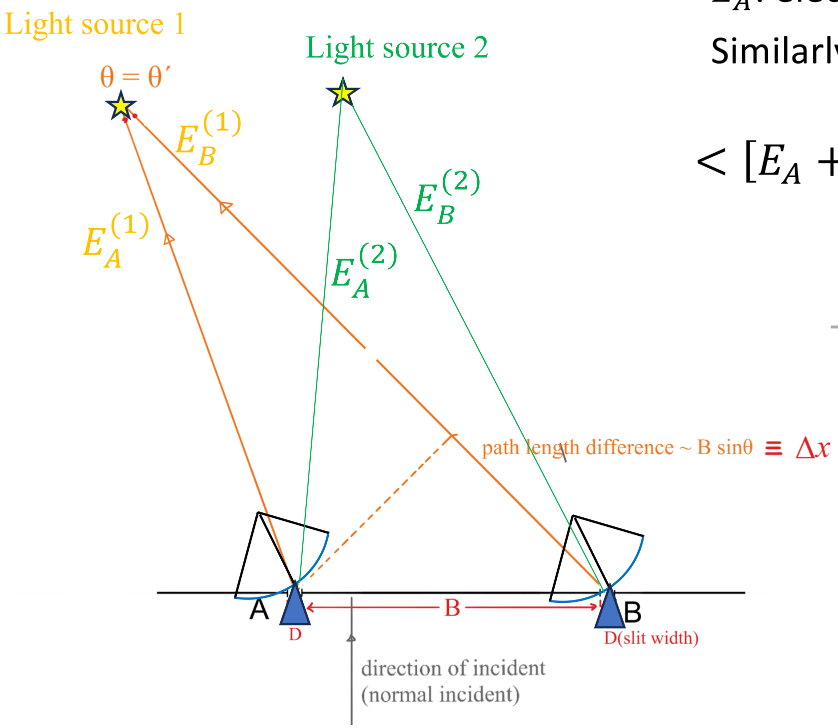
$$+ < {E_A^{(2)}}^2 > + < {E_B^{(2)}}^2 > + < 2E_A^{(2)} E_B^{(2)} >$$

$$+ < 2E_A^{(1)} E_A^{(2)} > + < 2E_A^{(1)} E_B^{(2)} > + < 2E_B^{(1)} E_A^{(2)} > + < 2E_B^{(1)} E_B^{(2)} >$$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

# When there are two incoherent sources




$E_A$: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

$$< [E_A + E_B]^2 > \; = \; < {E_A^{(1)}}^2 > + < {E_B^{(1)}}^2 > + < 2E_A^{(1)}E_B^{(1)} >$$

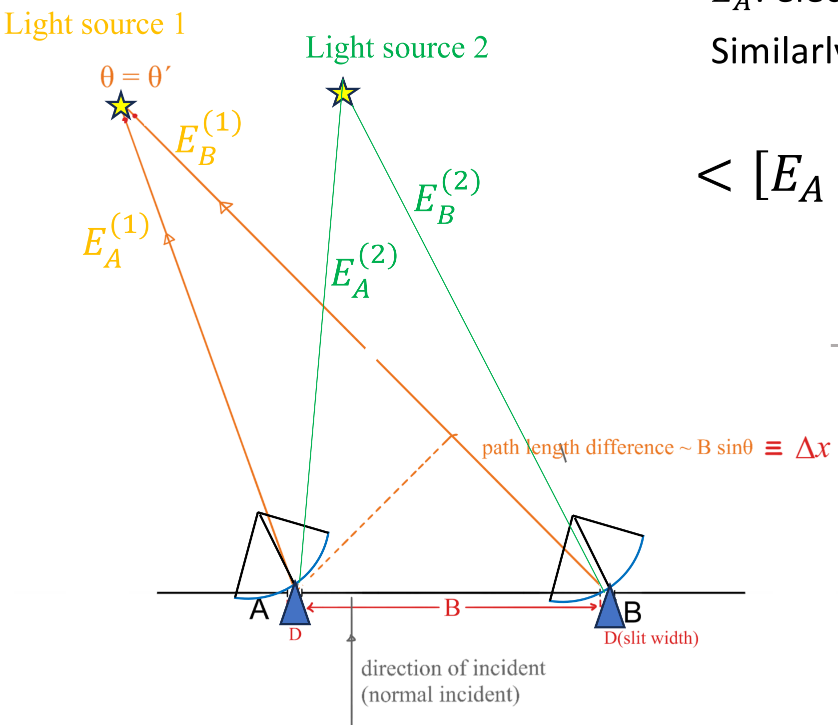
$$+ < \boldsymbol{{E_A^{(2)}}^2} > + < \boldsymbol{{E_B^{(2)}}^2} > + < \boldsymbol{2E_A^{(2)}E_B^{(2)}} >$$

$$+ < 2E_A^{(1)}E_A^{(2)} > + < 2E_A^{(1)}E_B^{(2)} > + < 2E_B^{(1)}E_A^{(2)} > + < 2E_B^{(1)}E_B^{(2)} >$$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

# When there are two incoherent sources




$E_A$: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

$$< [E_A + E_B]^2 > \ = \ < \boldsymbol{E_A^{(1)}}^2 > + < \boldsymbol{E_B^{(1)}}^2 > + < \boldsymbol{2E_A^{(1)}E_B^{(1)}} >$$

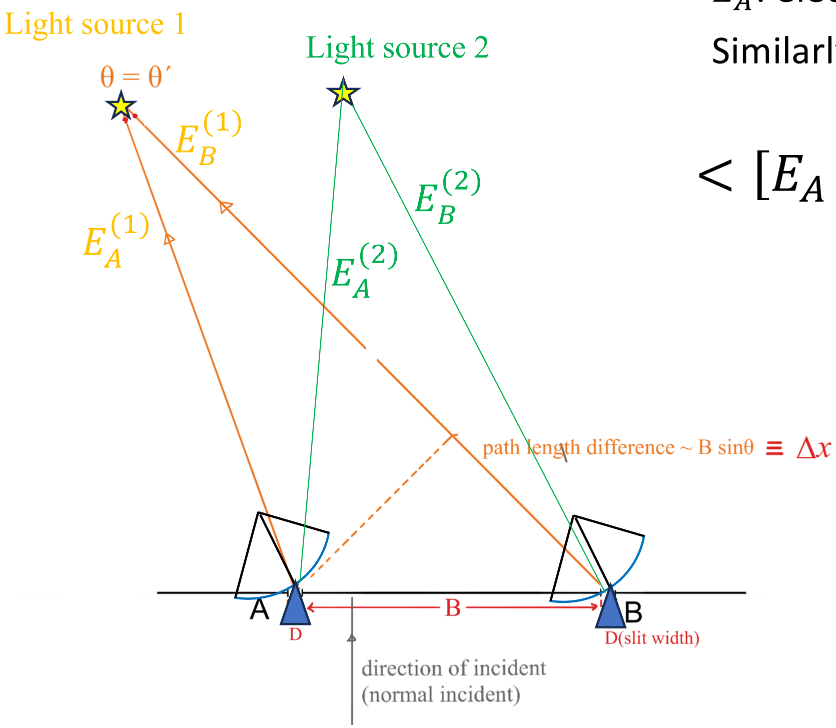
$$+ < \boldsymbol{E_A^{(2)}}^2 > + < \boldsymbol{E_B^{(2)}}^2 > + < \boldsymbol{2E_A^{(2)}E_B^{(2)}} >$$

$$\underbrace{\cancel{+ < 2E_A^{(1)}E_A^{(2)} > + < 2E_A^{(1)}E_B^{(2)} > + < 2E_B^{(1)}E_A^{(2)} > + < 2E_B^{(1)}E_B^{(2)} >}}_{\text{Definition of incoherent sources}}$$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

# When there are two incoherent sources




$E_A$: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,
Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

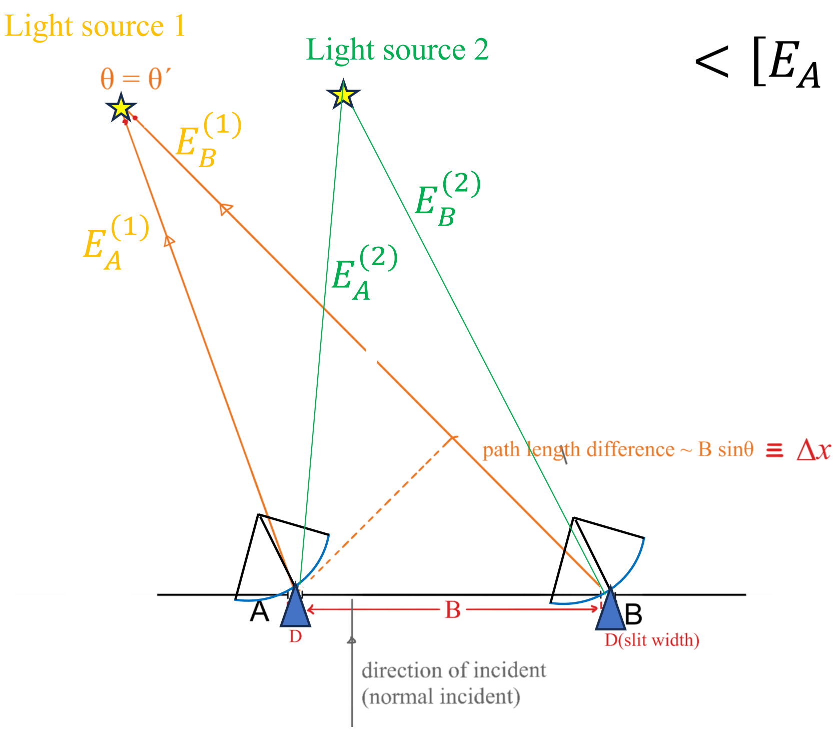
$$< [E_A + E_B]^2 > \; = \; < \boldsymbol{E_A^{(1)}}^2 > + < \boldsymbol{E_B^{(1)}}^2 > + < 2\boldsymbol{E_A^{(1)}}\boldsymbol{E_B^{(1)}} > + < \boldsymbol{E_A^{(2)}}^2 > + < \boldsymbol{E_B^{(2)}}^2 > + < 2\boldsymbol{E_A^{(2)}}\boldsymbol{E_B^{(2)}} >$$

Correlation of multiple incoherent sources can be evaluated by summing over the contribution of individual sources.

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

# When there are more incoherent sources




$$< [E_A + E_B]^2 >_{cos}^{cross} \propto \sum_{i=0}^{N-1} \widetilde{P(\theta_i')}\ [\underbrace{A_i}\cos(2\pi u\theta_i')]$$

A coefficient to describe how bright is source $i$

Dimension: flux density

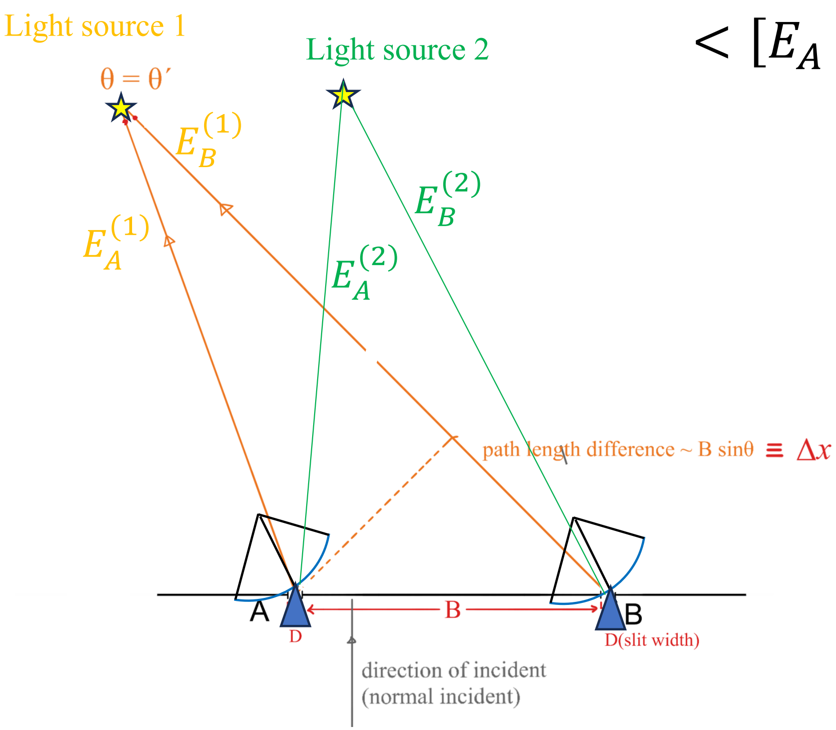
Looking at cross correlations at this moment

(i.e., neglecting auto-correlation terms)

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

When there is a continuous distribution of incoherent sources



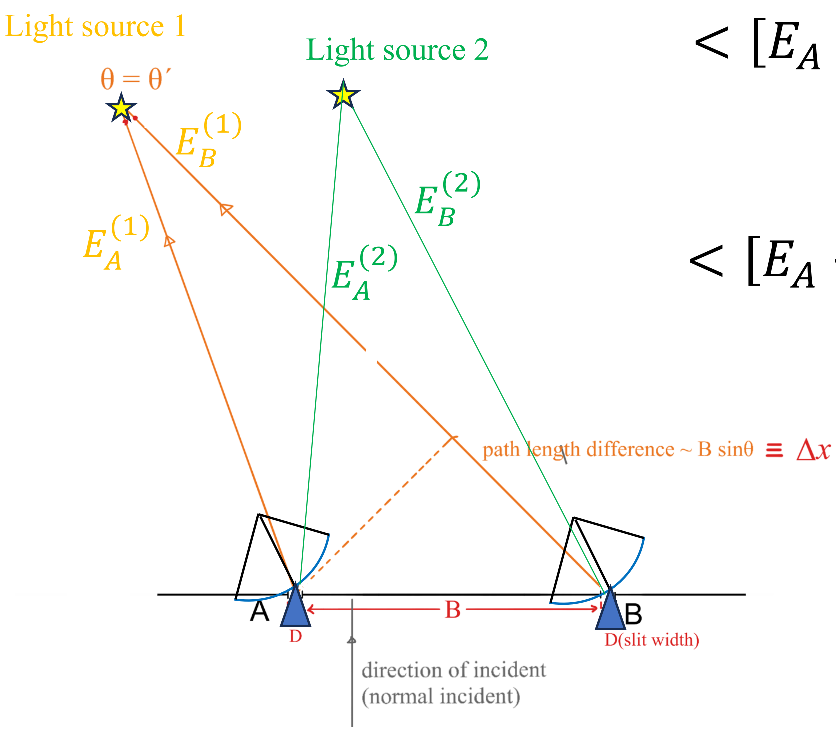

$$< [E_A + E_B]^2 >_{cos}^{cross} \propto \int \tilde{P}(\theta)\underbrace{A(\theta)}\cos(2\pi u\theta)\, d\theta$$

A coefficient to describe the intensity of the target source at incident angle $\theta$
Dimension: intensity

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

When there is a continuous distribution of incoherent sources




$$< [E_A + E_B]^2 >_{cos}^{cross} \propto \int \tilde{P}(\theta)\underbrace{A(\theta)}\cos(2\pi u\theta)\, d\theta$$

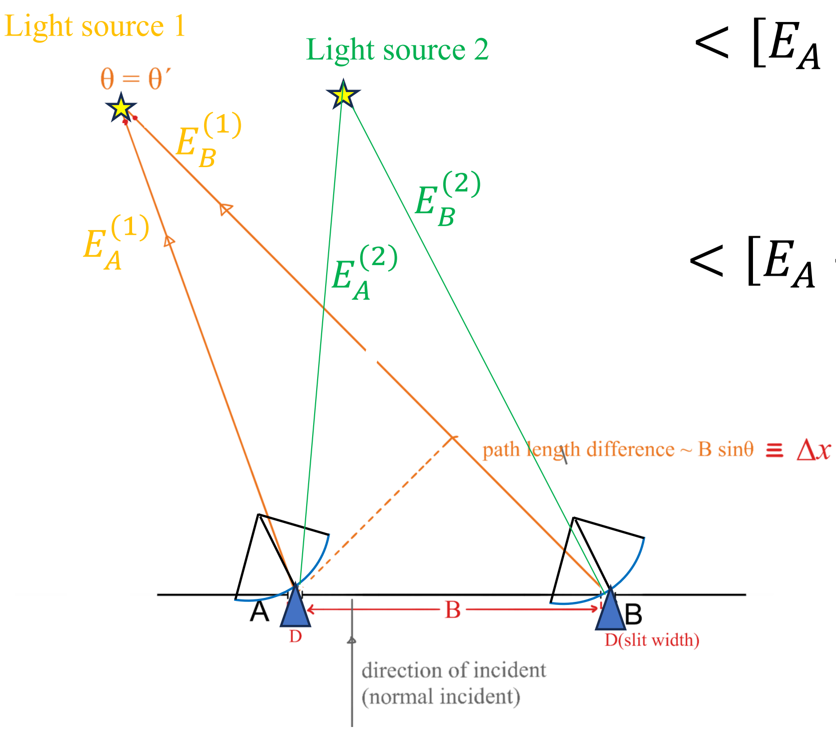
A coefficient to describe the intensity of the target source at incident angle $\theta$

$$< [E_A + E_B]^2 >_{sin}^{cross} \propto \int \tilde{P}(\theta)A(\theta)\sin(2\pi u\theta)\, d\theta$$

Cosine correlation: $<[E_A+E_B]^2>_{cos} \sim \tilde{P}(\theta)[1+\cos(2\pi u\theta)]$

Sine correlation: $<[E_A+E_B]^2>_{sin} \sim \tilde{P}(\theta)[1+\sin(2\pi u\theta)]$

When there is a continuous distribution of incoherent sources




$$<[E_A+E_B]^2>_{cos}^{cross} \propto \int \tilde{P}(\theta)\underbrace{A(\theta)}\cos(2\pi u\theta)\, d\theta$$

A coefficient to describe the intensity of the target source at incident angle $\theta$

$$<[E_A+E_B]^2>_{sin}^{cross} \propto \int \tilde{P}(\theta)A(\theta)\sin(2\pi u\theta)\, d\theta$$

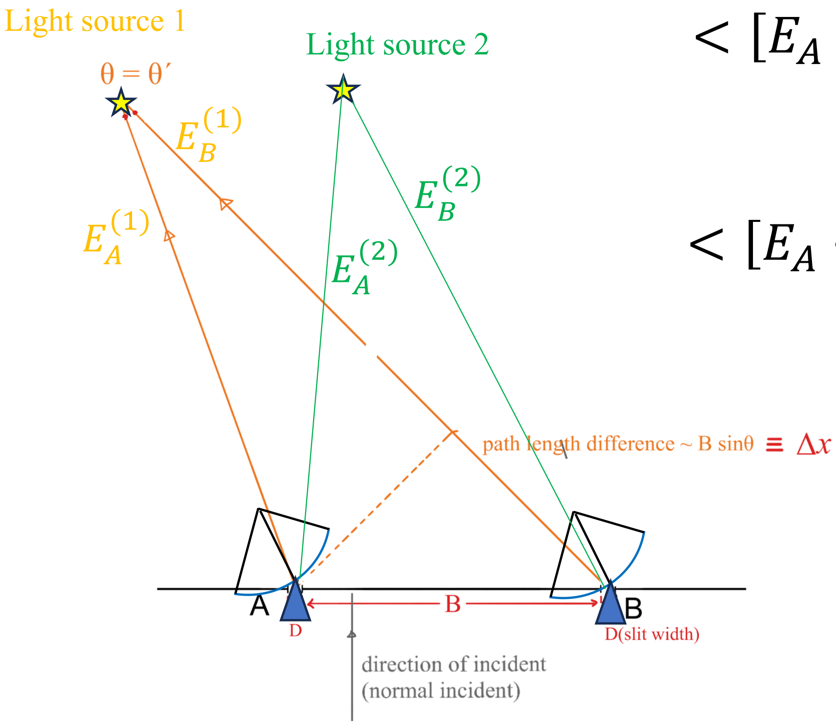
Complex visibility $V_{AB} = <[E_A+E_B]^2>_{cos}^{cross} + i <[E_A+E_B]^2>_{sin}^{cross}$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u \theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u \theta)]$

Complex visibility $V_{AB} = < [E_A + E_B]^2 >_{cos}^{cross} + i < [E_A + E_B]^2 >_{sin}^{cross}$

Intensity distribution of a point-like source: $A(\theta) = \delta(\theta - \theta')$




$$< [E_A + E_B]^2 >_{cos}^{cross} = g \int \tilde{P}(\theta) A(\theta) \cos(2\pi u \theta)\, d\theta$$

$$= g\tilde{P}(\theta')\cos(2\pi u \theta')$$

$$< [E_A + E_B]^2 >_{sin}^{cross} = g\tilde{P}(\theta')\sin(2\pi u \theta')$$

Gain factor

Complex visibility $V_{AB} = g\tilde{P}(\theta')\cos(2\pi u \theta') + ig\tilde{P}(\theta')\sin(2\pi u \theta')$

$$= \underbrace{g\tilde{P}(\theta')}_{\text{Visibility amplitude}} e^{i\underbrace{2\pi u \theta'}_{\text{Visibility phase}}}$$

1. Complex visibility: $V_{AB} = < [E_A + E_B]^2 >_{cos}{}^{cross} + i < [E_A + E_B]^2 >_{sin}{}^{cross}$

2. For a point-like source: $V_{AB} = g\tilde{P}(\theta')\cos(2\pi u\theta') + ig\tilde{P}(\theta')\sin(2\pi u\theta')$

$$= \underbrace{g\tilde{P}(\theta')}_{\text{Visibility amplitude}} e^{i\overbrace{2\pi u\theta'}^{\text{Visibility phase}}}$$