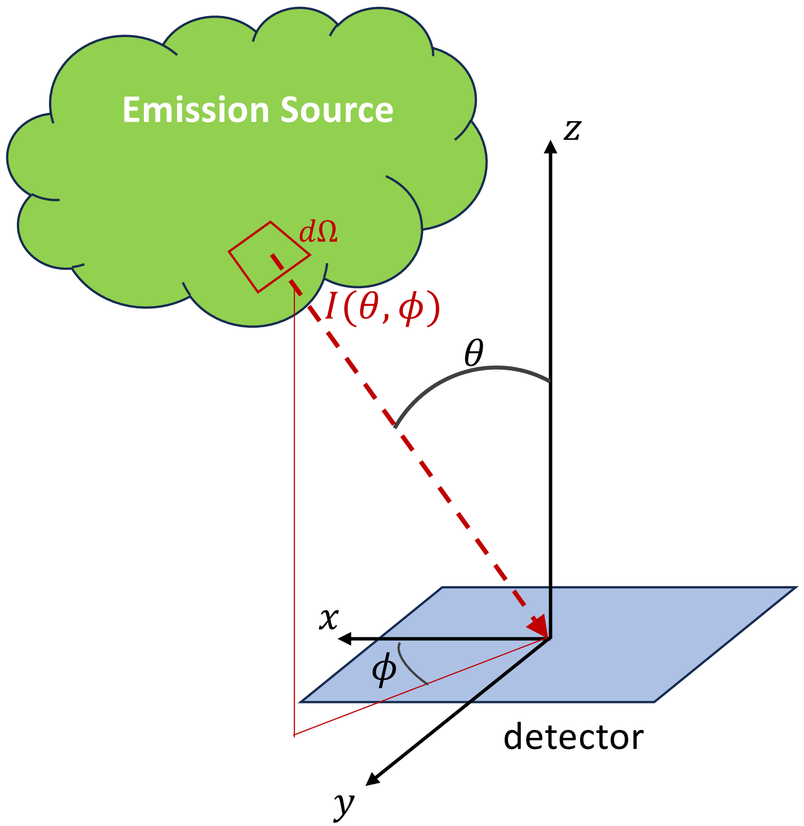
An Introduction to Radio Interferometry

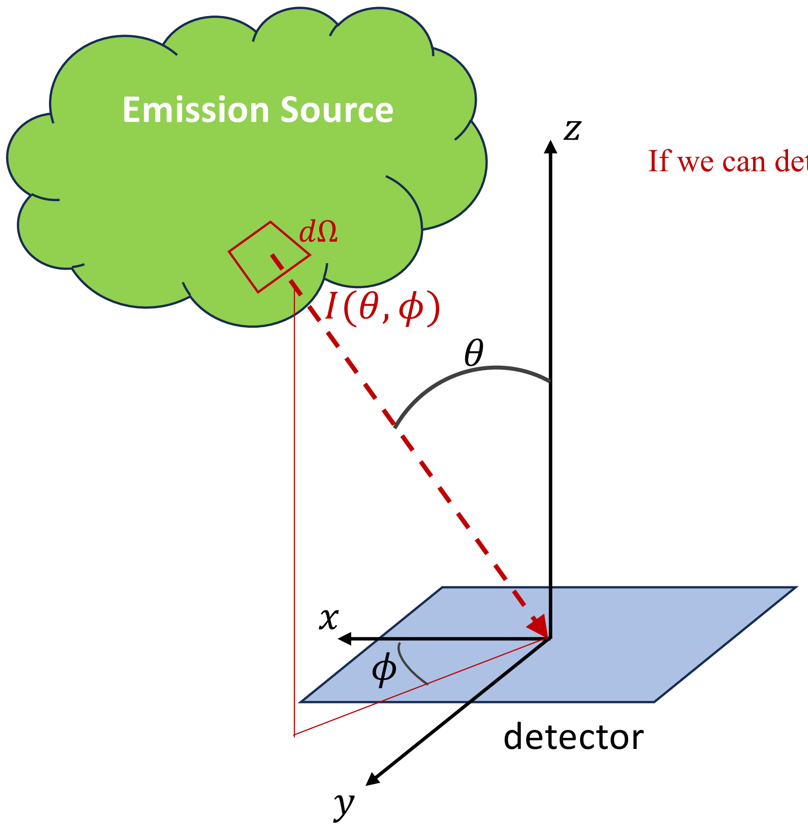
1-5 Antenna response function and the Jy/beam unit

You can find relevant material on my personal webpage



Flux density (incoming energy per unit time, frequency, and area) received by a CCD pixel:

$$F_\nu = \int I_\nu(\theta,\phi)\cos\theta\, d\Omega$$



Flux density (incoming energy per unit time, frequency, and area) received by a CCD pixel:

$$F_\nu = \frac{1}{2} \int I_\nu(\theta, \phi) \cos\theta \, d\Omega$$

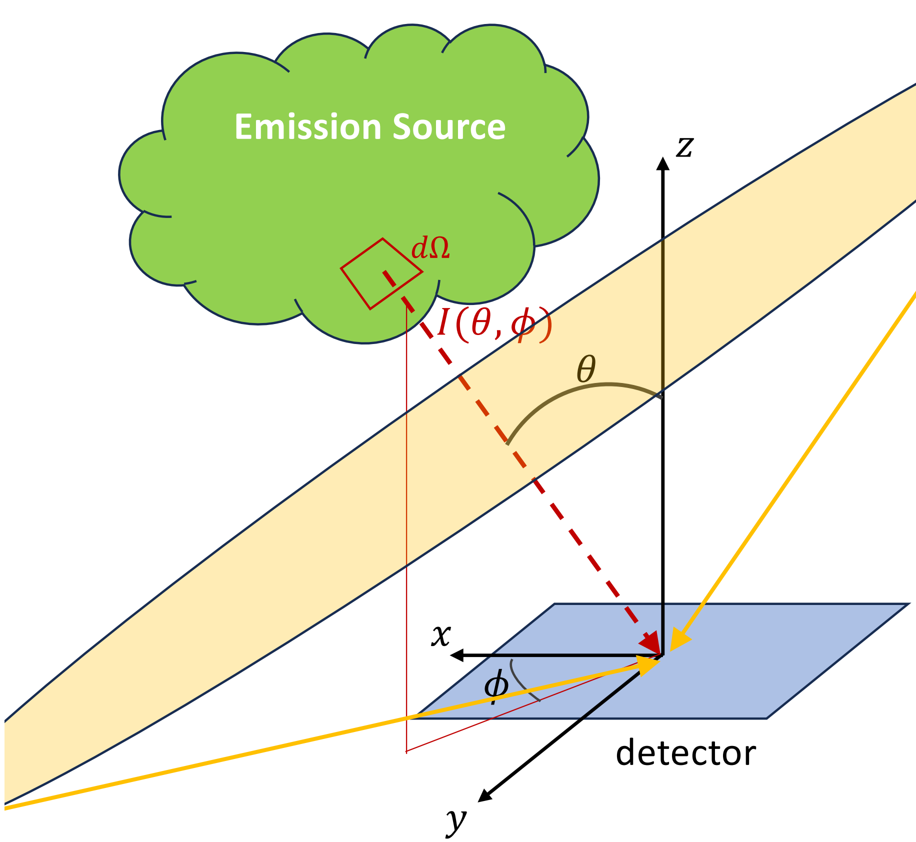
If we can detect one of the two polarizations

Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = \frac{1}{2} A \int I_\nu(\theta, \phi) \cos\theta \, d\Omega$$

surface area of the detector

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.

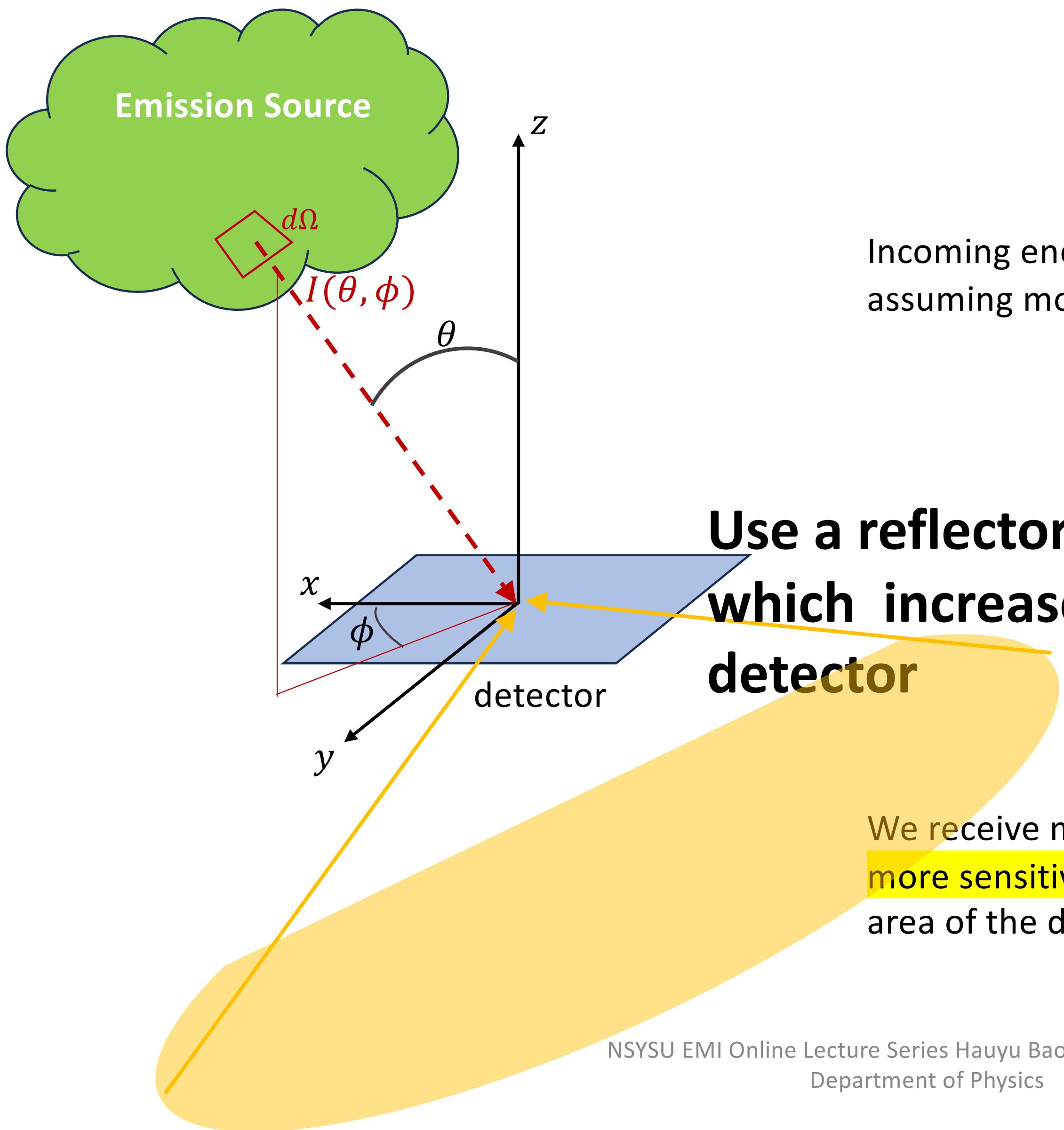


Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = \frac{1}{2} A_e \int I_\nu(\theta,\phi) \cos\theta \, d\Omega$$

Use a lense to collect light from a larger area, which increases the effective surface area of the detector

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.

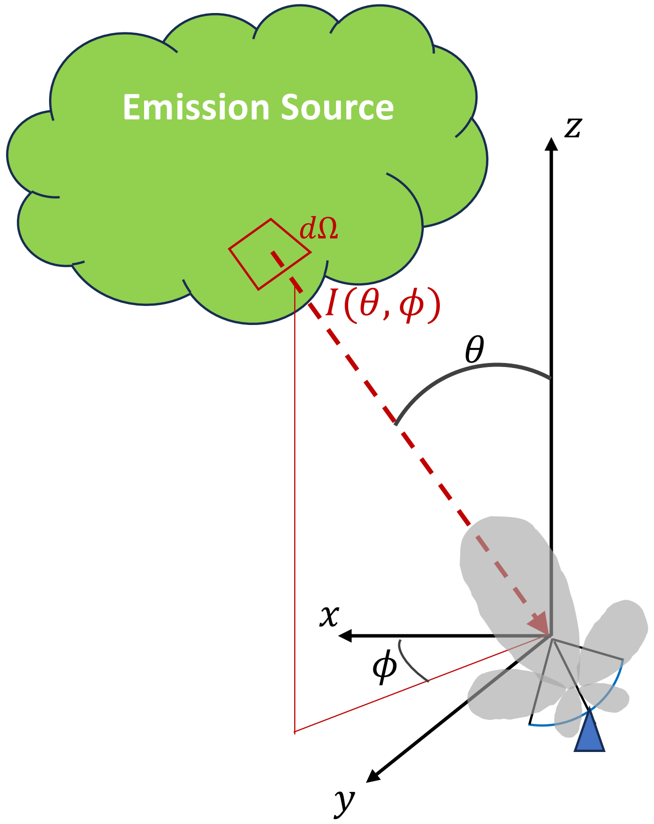


Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = \frac{1}{2} A_e \int I_\nu(\theta, \phi) \cos\theta \, d\Omega$$

Use a reflector to collect light from a larger area, which increases the effective surface area of the detector

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.



Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = \frac{1}{2} A_e \int I_\nu(\theta,\phi)\cos\theta \, \underbrace{P(\theta,\phi)}_{} \, d\Omega$$

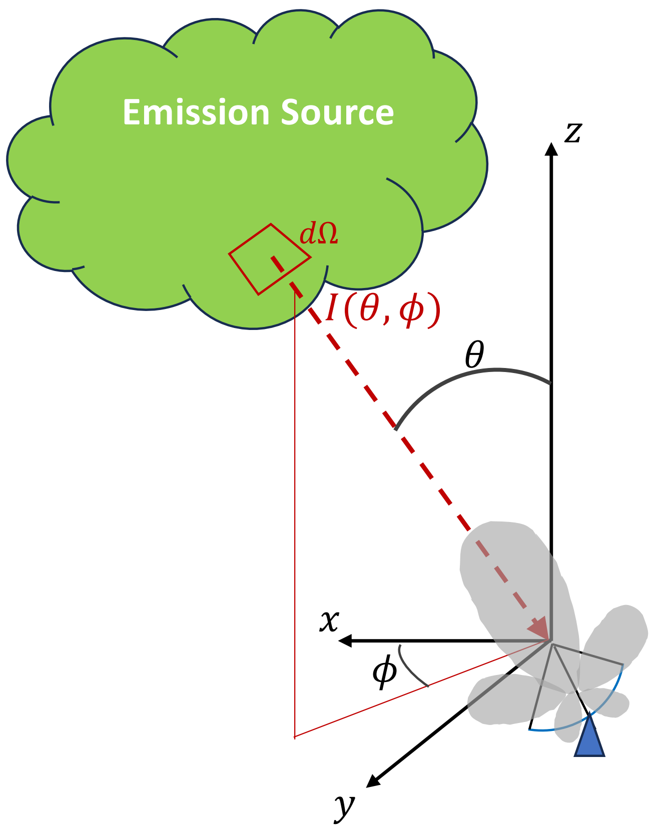
Response function of the observing device.
Normalization: peak = 1.0 (i.e., 100% of incoming intensity can be detected.)

The optics determines the response function.
With a telescope, we:

(1) Receive light from smaller solid angle(s) (usually, provides better angular resolution)

(2) Have a bigger collecting area

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.



Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = A_e \int I_\nu(\theta,\phi) \cos\theta \, \underbrace{P(\theta,\phi)}_{} \, d\Omega \equiv kT_A$$

Boltzmann constant

Antenna temperature

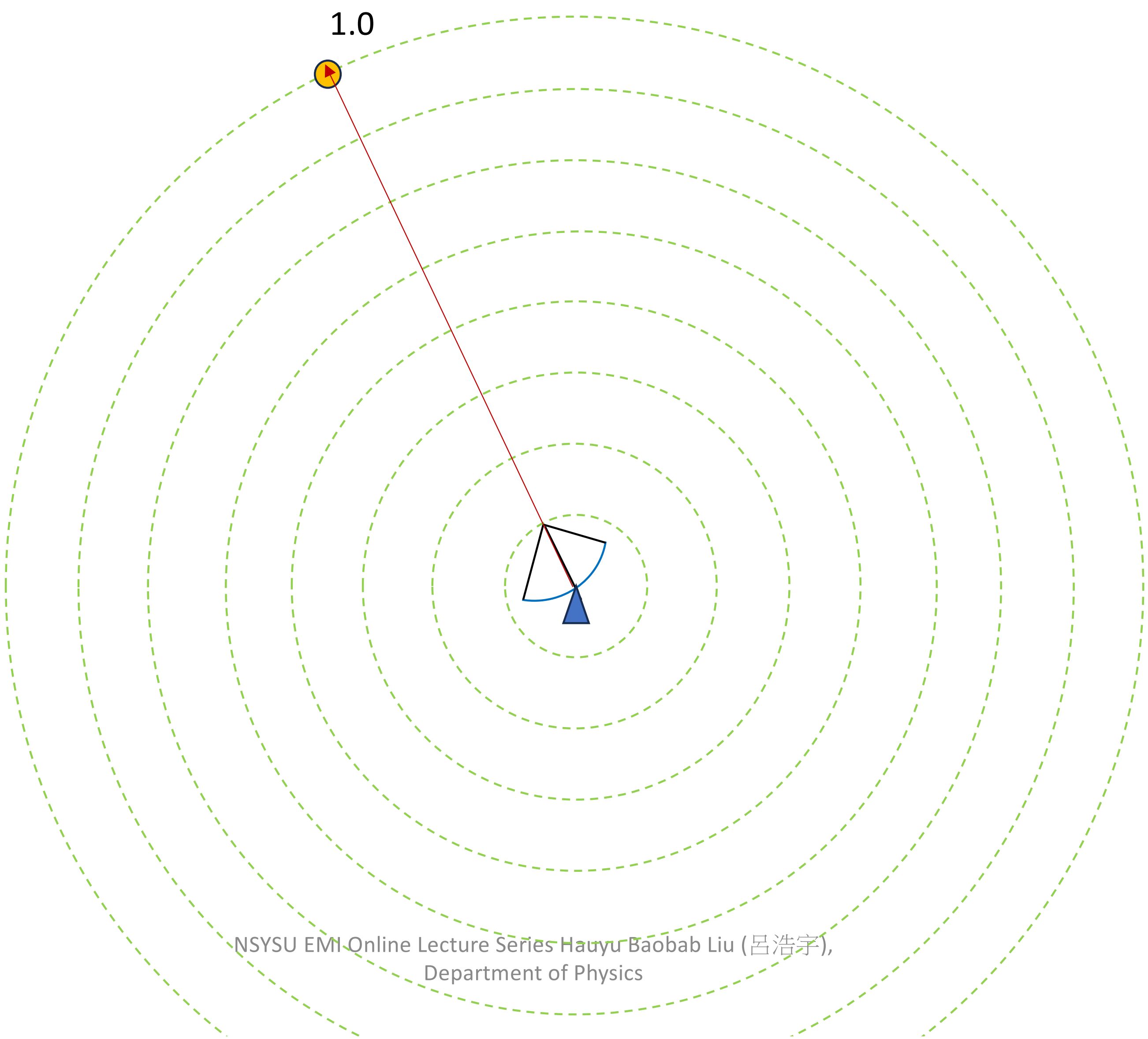
Response function of the observing device.
Normalization: peak = 1.0 (i.e., 100% of incoming intensity can be detected.)

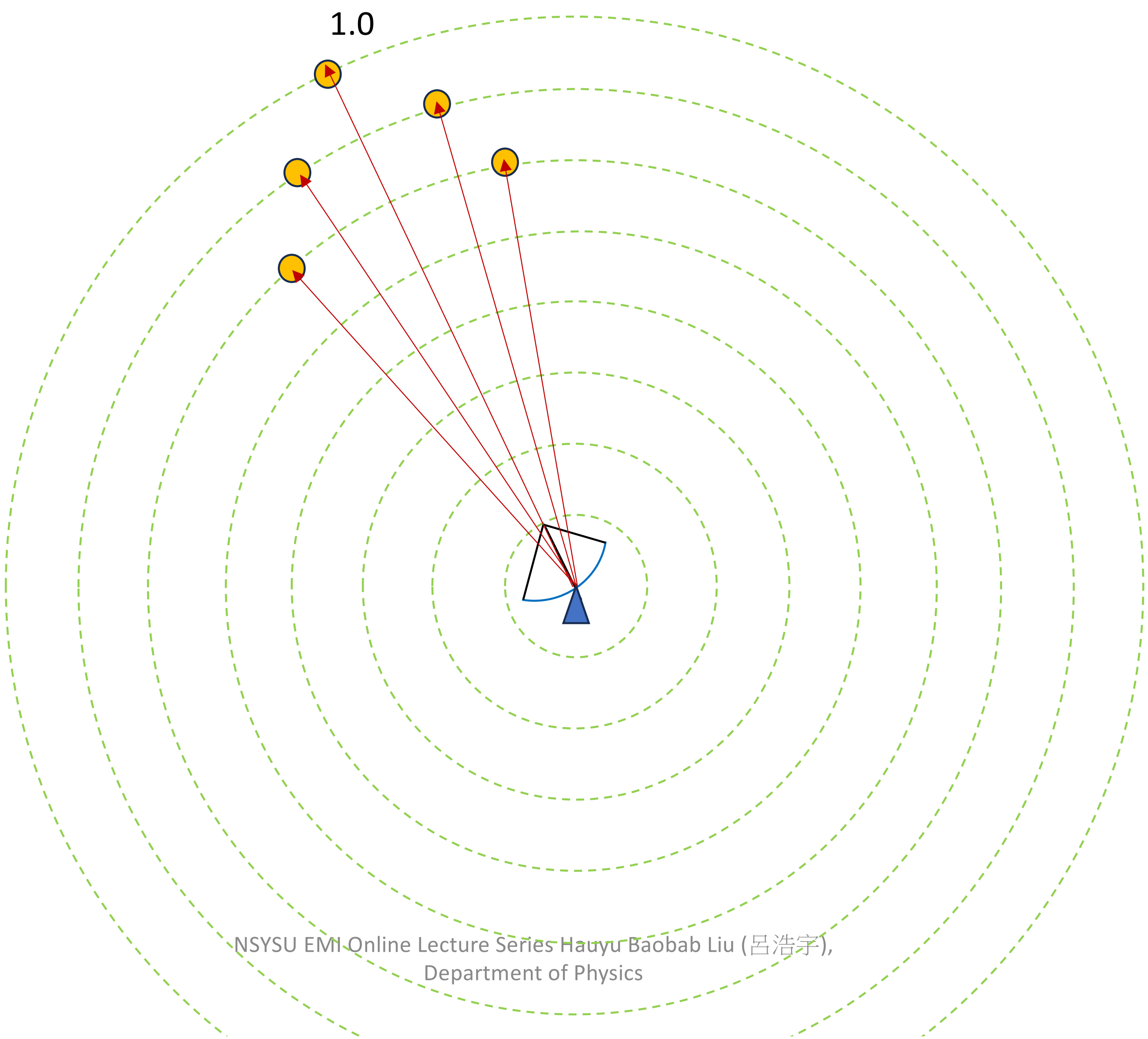
The optics determines the response function. With a telescope, we:

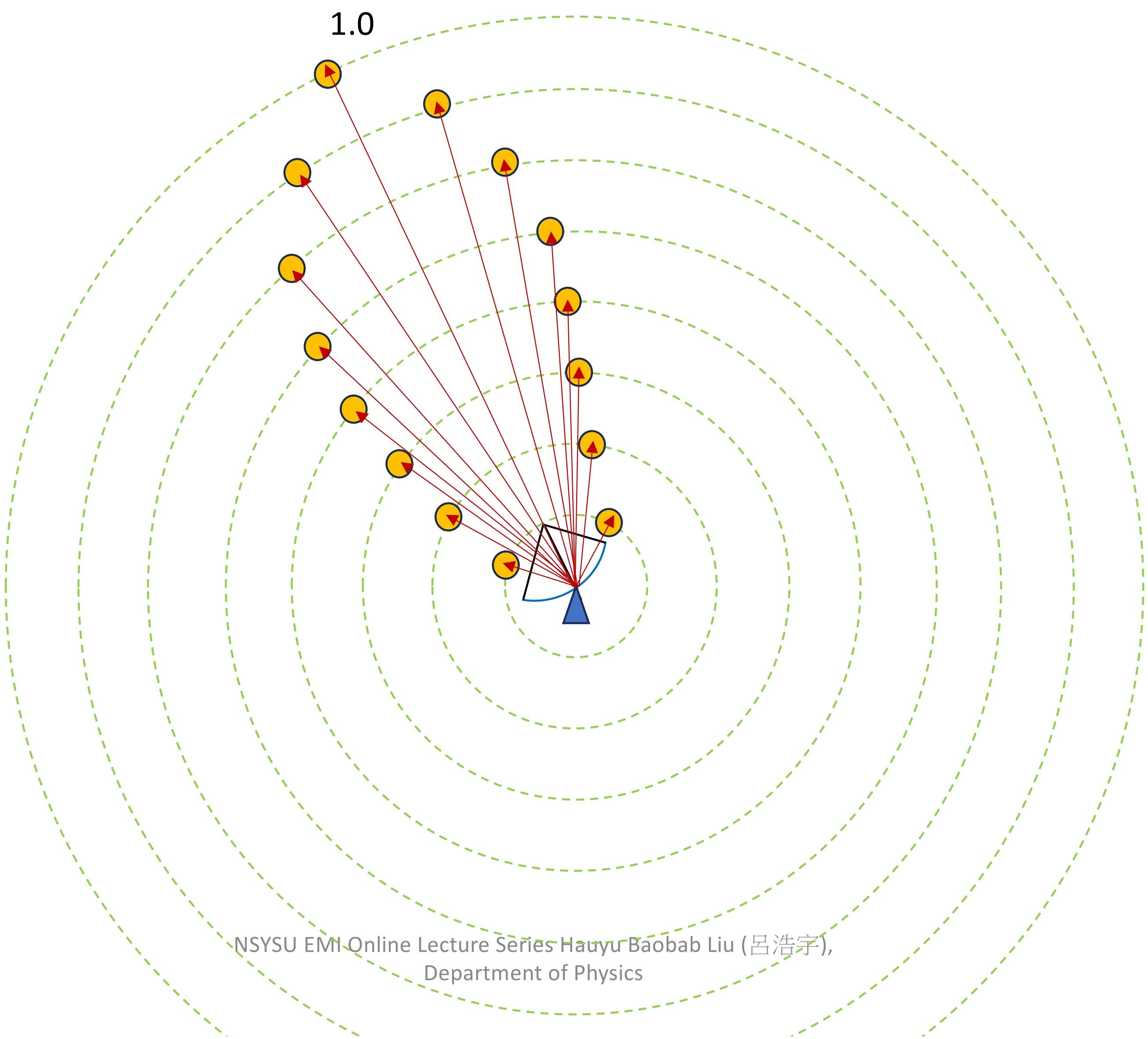
(1) Receive light from smaller solid angle(s) (usually, provides better angular resolution)

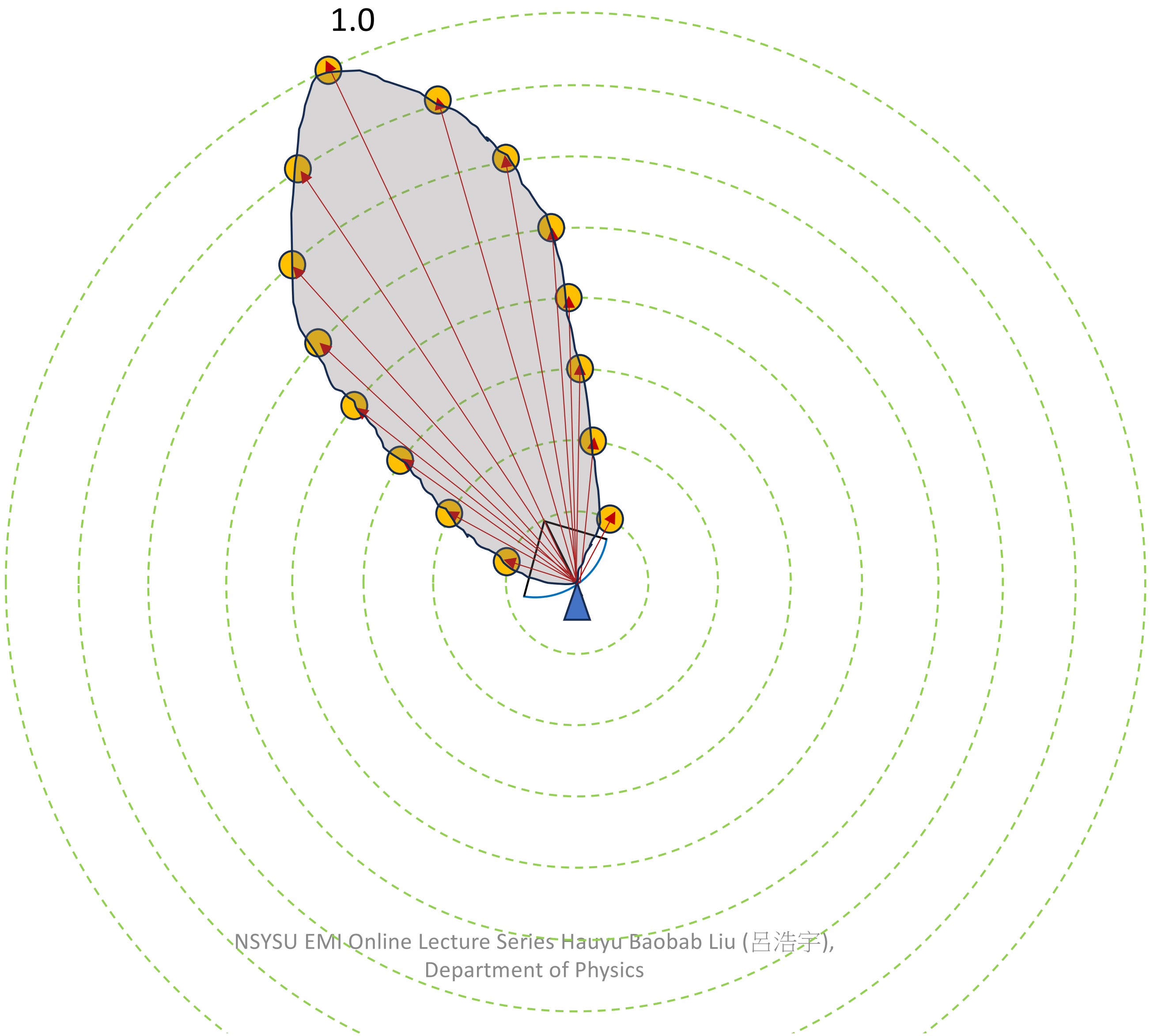
(2) Have a bigger collecting area

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.


1.0


1.0

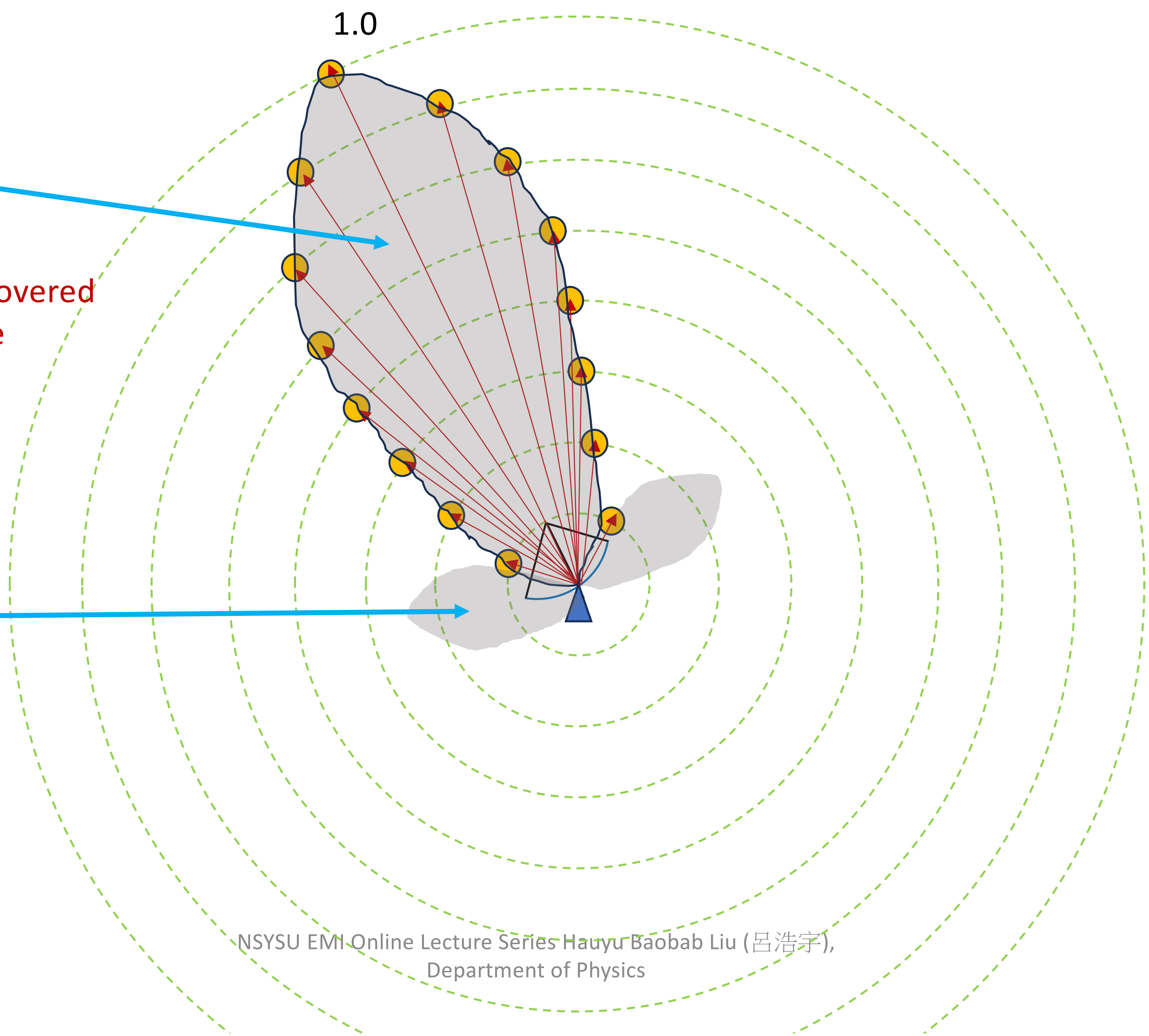

1.0

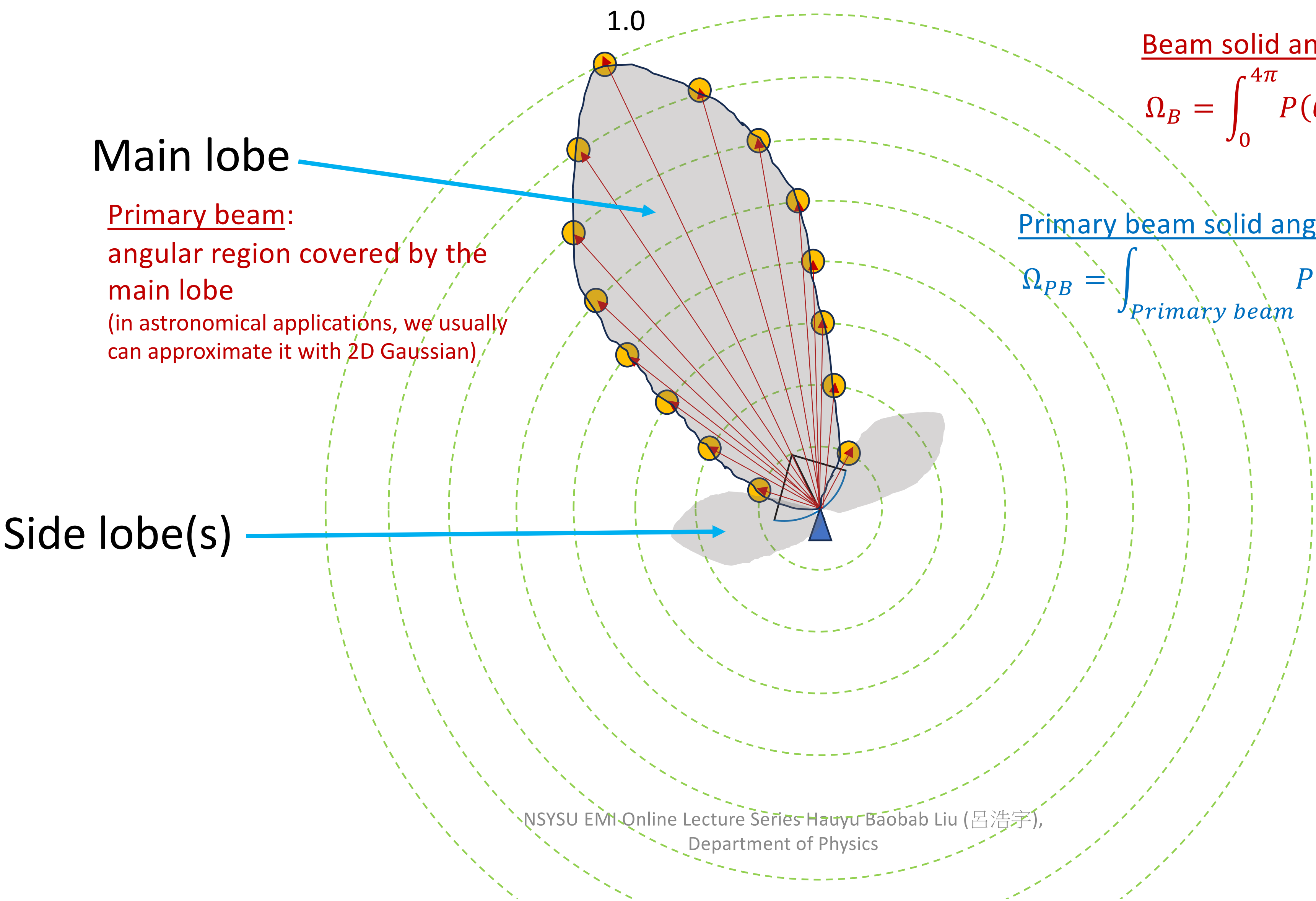

1.0

Main lobe

<u>Primary beam</u>:
angular region covered by the main lobe

Side lobe(s)





Primary beam:
angular region covered by the main lobe
(in astronomical applications, we usually can approximate it with 2D Gaussian)

Beam solid angle:

$$\Omega_B = \int_0^{4\pi} P(\theta, \phi) \sin\theta \, d\theta d\phi$$

Primary beam solid angle:

$$\Omega_{PB} = \int_{Primary\ beam} P(\theta, \phi) \sin\theta \, d\theta d\phi$$

Example: 2D Gaussian beam

with Full width at half maximum (FWHM) θ_a and θ_b in the major and minor axes

(the relation between Gaussian standard deviation σ ang FWHM θ

$\theta = 2\sqrt{2ln2}\sigma \sim 2.35\sigma$)

Beam solid angle:

$$\Omega_B = \int_0^{4\pi} P(\theta,\phi)\sin\theta\, d\theta d\phi$$

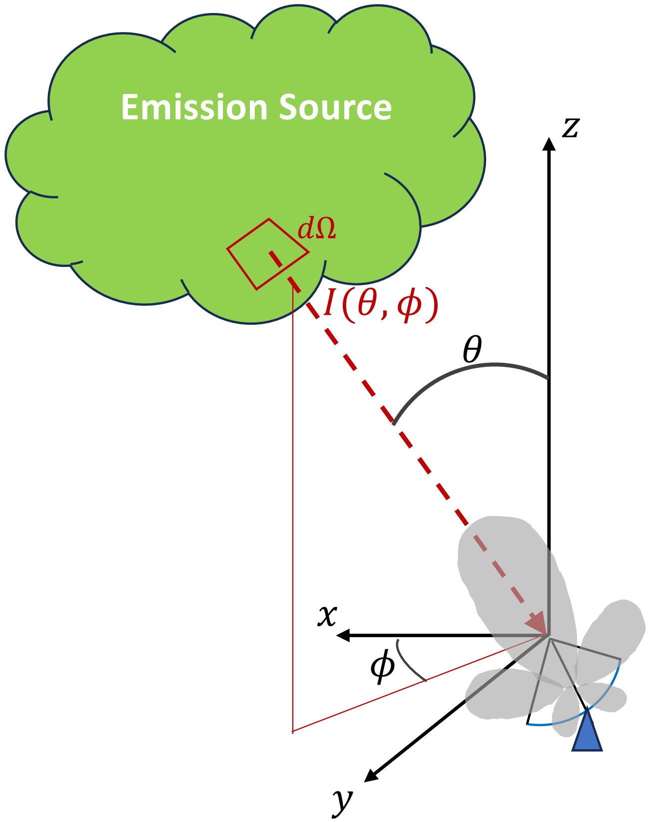
Primary beam solid angle:

$$\Omega_{PB} = \int_{Primary\ beam} P(\theta,\phi)\sin\theta\, d\theta d\phi$$

$$P = e^{\left(\frac{-\xi^2}{2\sigma_a^2}+\frac{-\eta^2}{2\sigma_b^2}\right)} = e^{\left[4ln2\left(\frac{-\xi^2}{(2\sqrt{ln2}\sigma_a)^2}+\frac{-\eta^2}{(2\sqrt{ln2}\sigma_b)^2}\right)\right]} = e^{\left[4ln2\left(\frac{-\xi^2}{(\theta_a)^2}+\frac{-\eta^2}{(\theta_b)^2}\right)\right]}$$

$$\Omega_B = \int e^{\left[4ln2\left(\frac{-\xi^2}{(\theta_a)^2}+\frac{-\eta^2}{(\theta_b)^2}\right)\right]} d\xi d\eta = \frac{\pi\theta_a\theta_b}{4ln2}$$

$$\int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}$$



Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

Boltzmann constant

$$P_\nu = A_e \int I_\nu(\theta,\phi)\cos\theta\, \underbrace{P(\theta,\phi)}\, d\Omega \equiv k T_A$$

Antenna temperature

Response function of the observing device.
Normalization: peak = 1.0 (i.e., 100% of incoming intensity can be detected.)

$$\frac{P_\nu}{A_e} = \int I_\nu(\theta,\phi)\cos\theta\, P(\theta,\phi)\, d\Omega \quad \underset{\theta \to 0 \implies \cos\theta \to 1}{\sim} \int I_\nu(\theta,\phi)\, P(\theta,\phi)\, d\Omega$$

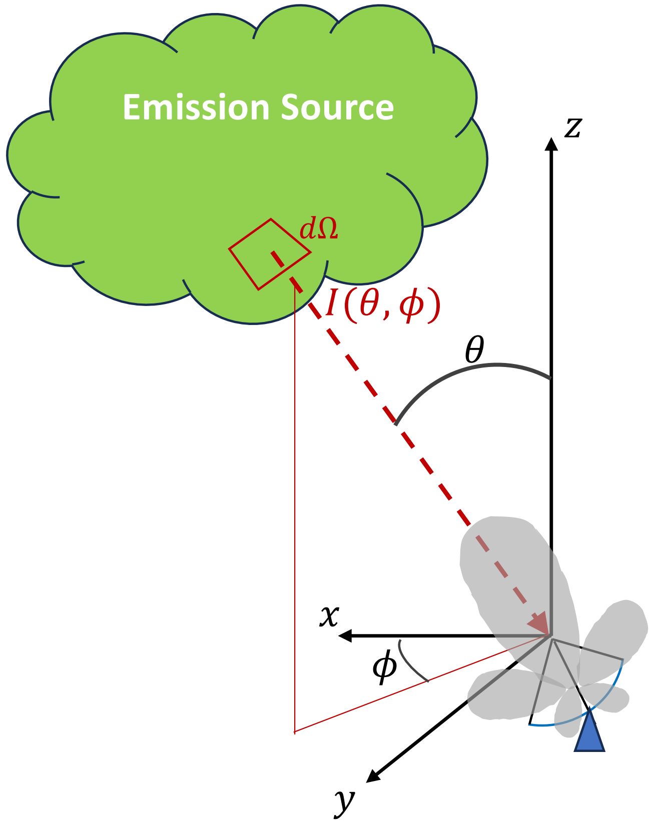
If $I_\nu(\theta,\phi)$ is approximately constant over the beam

Solid angle in Sr unit

$$\underbrace{\frac{P_\nu}{A_e}} = \sim I_\nu \int P(\theta,\phi)\, d\Omega = \underbrace{I_\nu}\overbrace{\Omega_B} = 1\, Jy$$

Flux density

Intensity in Jy Sr[-1] unit



Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = A_e \int I_\nu(\theta,\phi)\cos\theta\, \underbrace{P(\theta,\phi)}\, d\Omega \equiv kT_A$$

Boltzmann constant

Antenna temperature

Response function of the observing device.
Normalization: peak = 1.0 (i.e., 100% of incoming intensity can be detected.)

$$\frac{P_\nu}{A_e} = \int I_\nu(\theta,\phi)\cos\theta\, P(\theta,\phi)\, d\Omega \underset{\theta \to 0 \Longrightarrow \cos\theta \to 1}{\sim} \int I_\nu(\theta,\phi)\, P(\theta,\phi)\, d\Omega$$

If $I_\nu(\theta,\phi)$ is approximately constant over the beam

$$\underbrace{\frac{P_\nu}{A_e}} = \sim I_\nu \int P(\theta,\phi)\, d\Omega = \underbrace{I_\nu}\overbrace{\Omega_B} = \underbrace{I_\nu^{Jy/beam} \times \overbrace{1}} = 1\, Jy$$

Solid angle in Sr unit

Solid angle in beam unit

Flux density

Intensity in Jy Sr^{-1} unit

changing unit

Intensity in Jy beam^{-1} unit

Converting $I_\nu^{Jy/beam}$ to T_B (assuming 2D Gaussian beam)

$$\underbrace{T_B = \frac{\lambda^2}{2k} I_\nu}_{\text{Rayleigh-Jean limit}} = \frac{\lambda^2}{2k}\left(\frac{F_\nu}{\Omega_B}\right) = \frac{\lambda^2}{2k}\left(\frac{F_\nu}{1\ beam}\right)\left(\frac{1\ beam}{\Omega_B}\right)$$

$$= \frac{\lambda^2}{2k} I_\nu^{Jy/beam} \frac{1\ beam}{\Omega_B}$$

(see page 14 for the form of Ω_B)

$$= \frac{\lambda^2}{2 \cdot 1.38 \cdot 10^{-23}} I_\nu^{Jy/beam} \frac{4 ln 2}{\pi \theta_a \theta_b}$$

Rayleigh-Jean limit
(see lecture Unit 1-4)

$$\Longrightarrow \boxed{T_B = 3.2 \cdot 10^{22} \frac{\lambda^2}{\theta_b \theta_b} I_\nu^{Jy/beam}}$$

The key information when we look up data archive

1. Target source name/coordinates
2. Ranges of observing frequency
3. Frequency resolution, angular resolution ($\theta_a \times \theta_b ; P.A.$)
4. The rms noise level in terms of T_B or $I_\nu^{Jy/beam}$
5. Polarization (X, Y, R, L)
6. Maximum recoverable angular scale (only relevant in interferometric observations)

1. An observational device can be characterized with a response function $P(\theta, \phi)$ that the maximum is normalized to 1.0. The form of $P(\theta, \phi)$ is related to the geometry of the optics.

2. Solid angle of Gaussian primary beam:

$$\Omega_{PB} = \int_{Primary\ beam} P(\theta, \phi) \sin\theta \, d\theta d\phi = \frac{\pi \theta_a \theta_b}{4 ln2}$$

   where θ_a, θ_b are the FWHM of the Gaussian in the major and minor axes.

3. The conversion between brightness temperature and the intensity in Jansky per beam unit: $T_B = 3.2 \cdot 10^{22} \frac{\lambda^2}{\theta_b \theta_b} I_\nu^{Jy/beam}$