An Introduction to Radio Interferometry

5-4 Passband and complex gain calibration

You can find relevant material
on my personal webpage

NSYSU EMI Online Lecture Series Hauyu Baobab Liu (呂浩宇),
Department of Physics

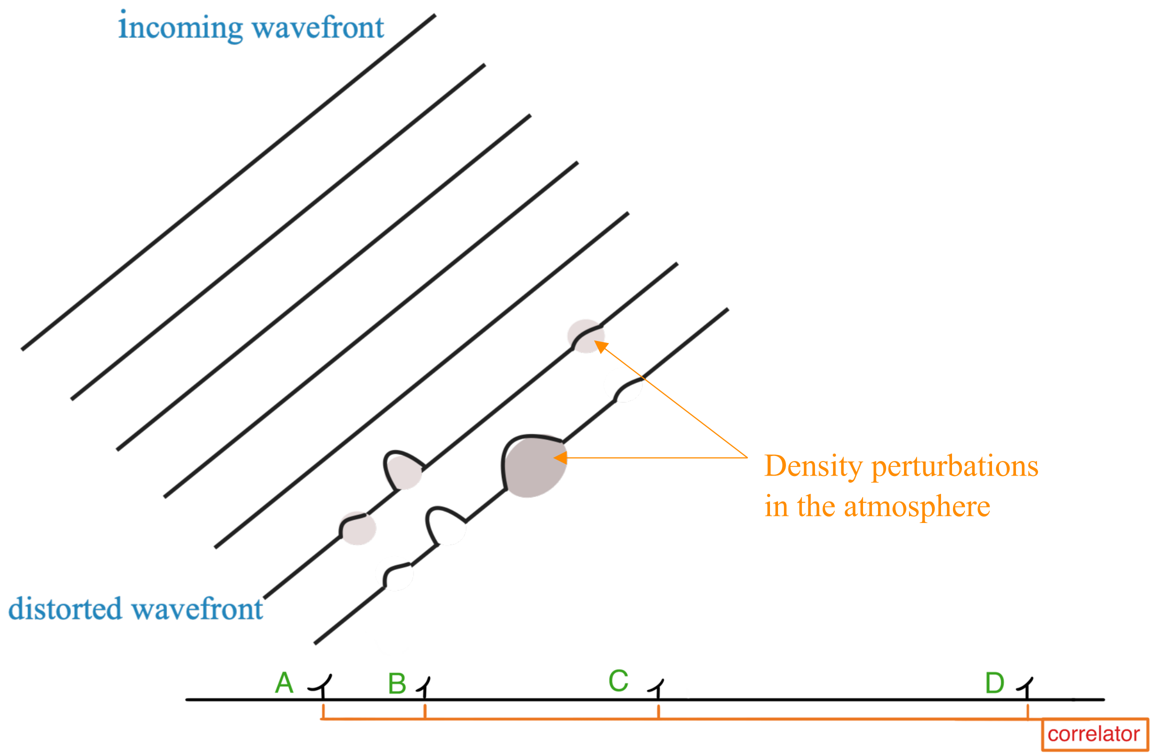
Complex gain calibration

1. Calibrate time dependent visibility phase errors

2. Calibrate time depedent visibility amplitude errors

Complex gain calibration

1. Calibrate time dependent visibility phase errors



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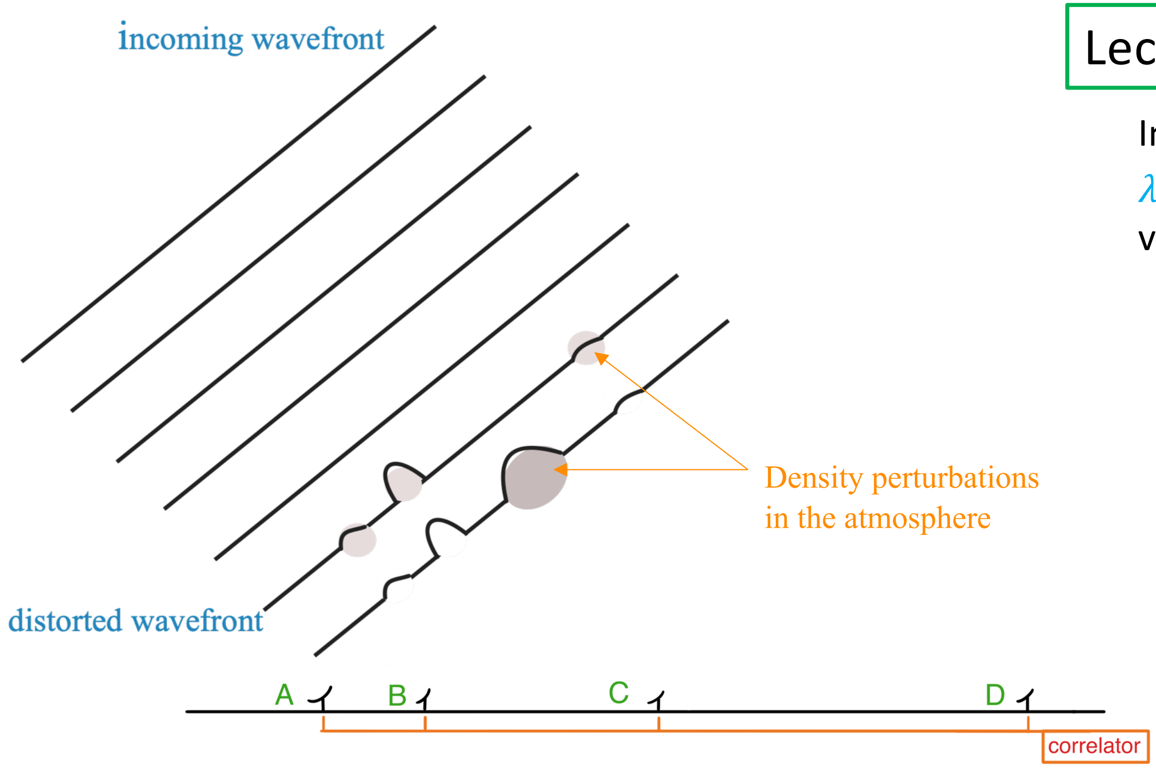

Lecture Unit 1-1

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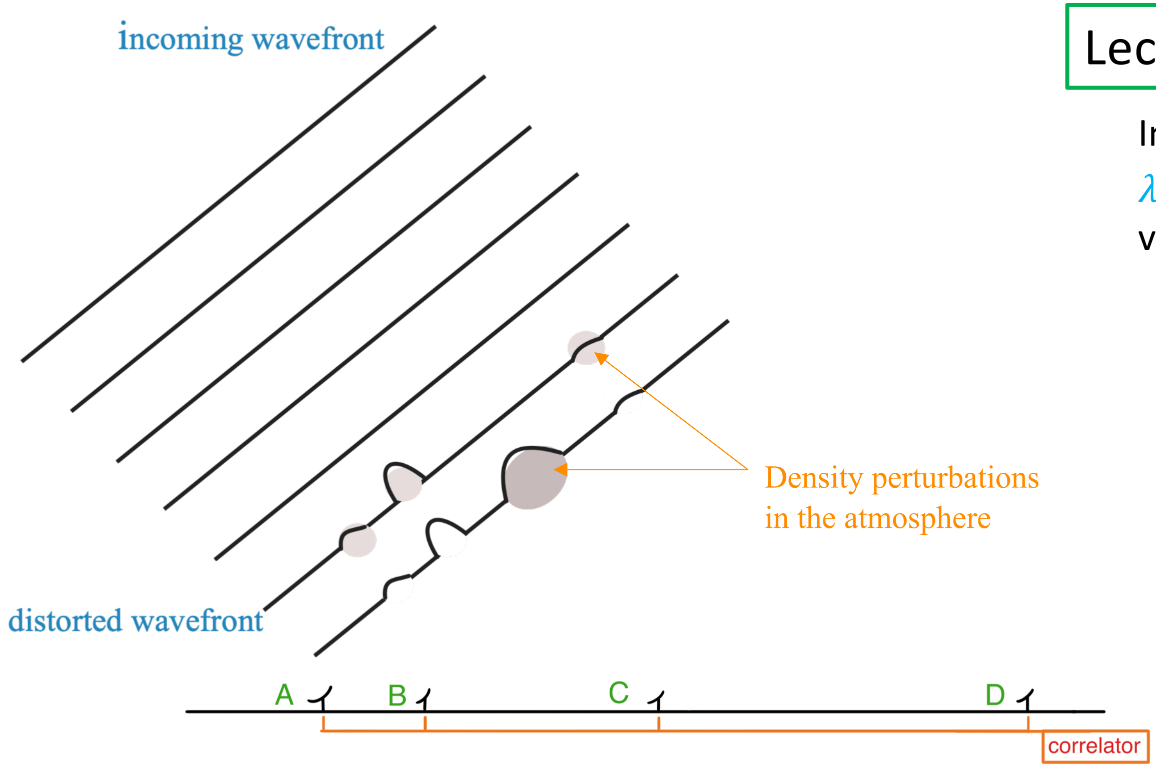

Lecture Unit 1-1

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$$N \equiv (n - n_0) \cdot 10^6$$

dry air component : $N_d \sim 2 \cdot 10^5 \rho$

water vapor component. : $N_{wv} = 1.7 \cdot 10^9 \frac{\rho_{WV}}{T_{atm}}$

$$PWV \equiv \frac{1}{\rho_w} \int \rho_{WV} dz$$

(ρ, ρ_{WV}: mass density in unit of g/cm^3;
ρ_W: mass density of liquid water, z: air thickness)

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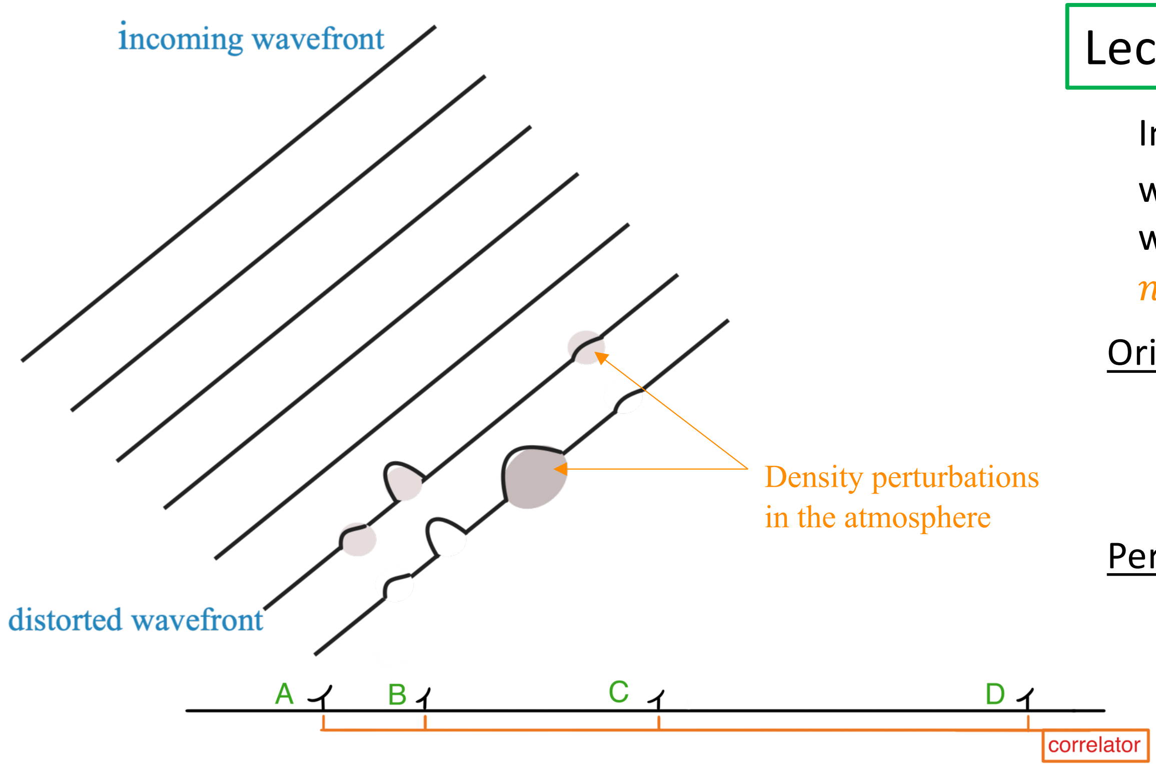

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In a medium with index of refraction $n = n_0 + \delta n$, the wavelength λ is changed to λ_0/n, where λ_0 is the wavelength in the vacuum. Let the index of refraction $n_0 = 1$.

<u>Original phase change over a path length Δx</u>:

$$k\Delta x = \frac{2\pi}{\lambda_0}\Delta x = 2\pi \underbrace{\frac{\Delta x}{\lambda_0}} \equiv \varphi_0$$

We can equivalently say the path length is <u>how many wavelengths</u>

<u>Perturbed phase change over a path length Δx</u>:

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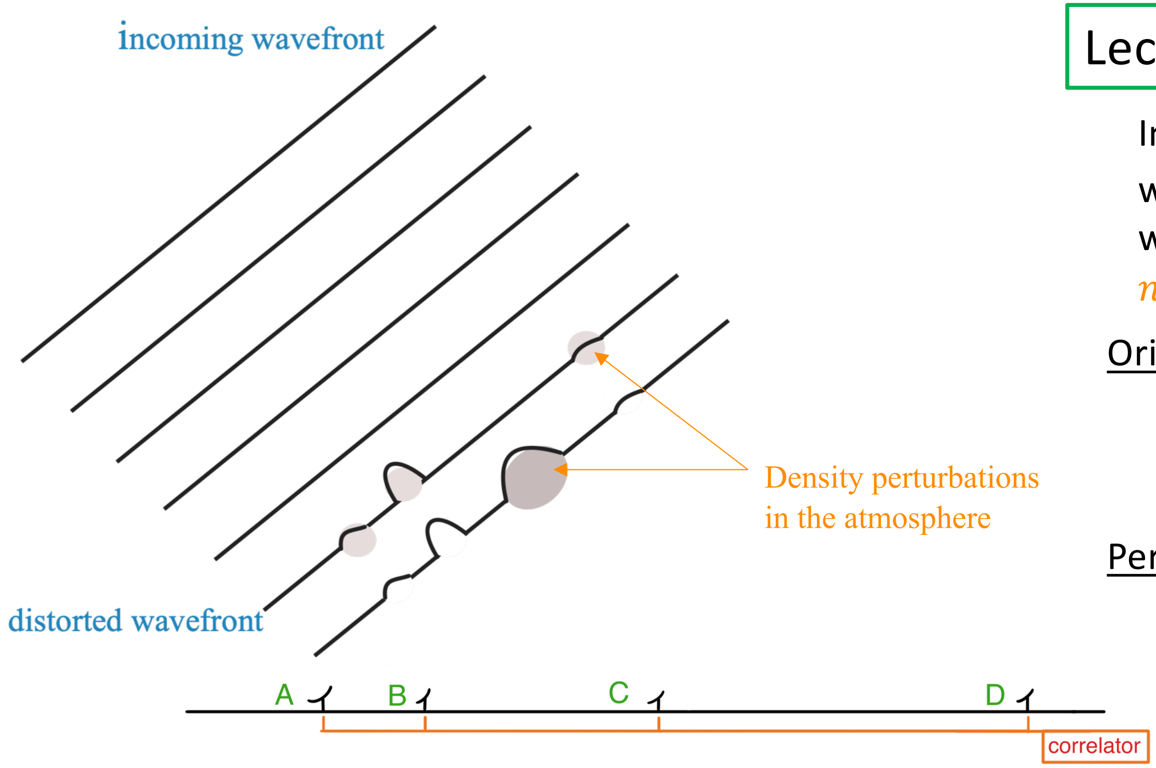

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(iii) An extra delay of $\Delta x \delta n / c$ (e.g., 5 ns)

Complex gain calibration

Considering the complex visibilty measured from two antennae (i.e., 1 baseline), recalling

Lecture Unit 3-1 (slide page 11)

Number of incoming photons in a unit of time and a unit area (at the angle of emergence $\theta = \theta'$)

$$\propto < [E_A + E_B]^2 >$$
$$= < [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 >$$
$$= \left.\begin{array}{l} 1/2 \\ + 1/2 \end{array}\right] \text{Auto-correlations}$$
$$+ < \underbrace{\boxed{2\underbrace{\cos(kx - \omega t + \phi_0)}_{\varphi} \underbrace{\cos(k(x + \Delta x) - \omega t + \phi_0)}_{\psi}}}_{\cos(k\Delta x) + \cancel{\cos(2kx - 2\omega t + 2\phi_0 + k\Delta x)}} >$$

Cross-correlations

1. Calibrate time dependent visibility phase errors

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$$= \frac{1}{2} + \frac{1}{2}$$ **Auto-correlations**

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Cross-correlations

Considering baseline length B

double slit: flux density at the angle of emergence $\theta' \sim {}^{\Delta x}/_{B}$

interferometer: signal of a celestial source at direction $\theta' \sim {}^{\Delta x}/_{B}$

Lecture Unit 3-2

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In interferometric observations, phase errors make us think that the direction of the target source is changed ($\sim {}^{\Delta x \delta n}/_{B}$) instaneously.

$\cos(k\Delta x)$

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Lecture Unit 3-2

1. Calibrate time dependent visibility phase errors

Estimates:

Synthesized beam width $\Delta\theta \sim {}^{\lambda}/_{B} \sim 1$ arcsec

phase error $2\pi \frac{\Delta x \delta n}{\lambda_0} \sim \frac{\pi}{6} = 30°$, i.e., $\underbrace{\frac{\Delta x \delta n}{\lambda_0} \sim \frac{1}{12}}$

Path length changed by ~10% of a wavelength

<u>angular offset of the target source:</u>

$\sim {}^{\Delta x \delta n}/_{B} = \frac{\Delta x \delta n}{\lambda_0} \frac{\lambda_0}{B} \sim \frac{1}{12} \Delta\theta = \frac{1}{12}$ arcsec

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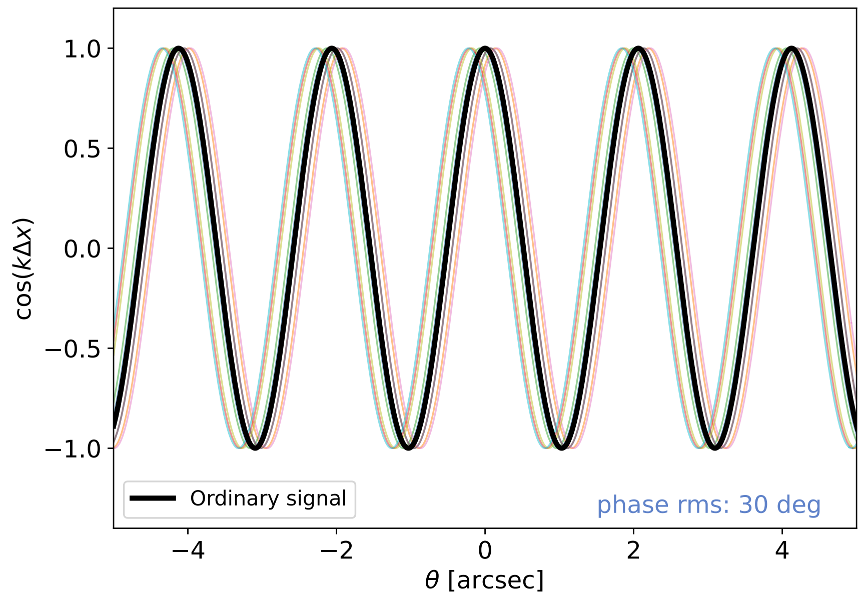
Lecture Unit 3-2

Effect of stochastic phase errors in time-averaged signal
(standard deviation of phase error: 30°)



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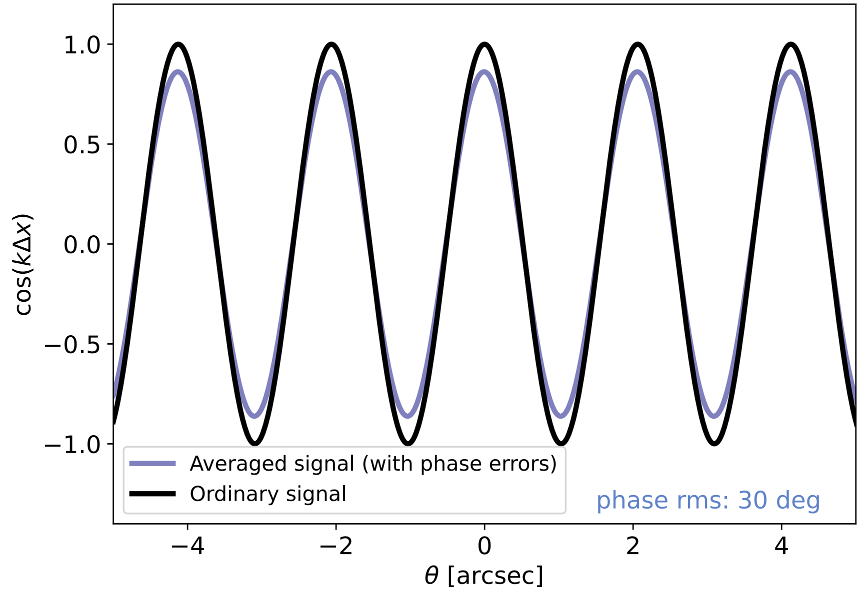
Effect of stochastic phase errors in time-averaged signal
(averaged signal has the same phase, lower amplitude)



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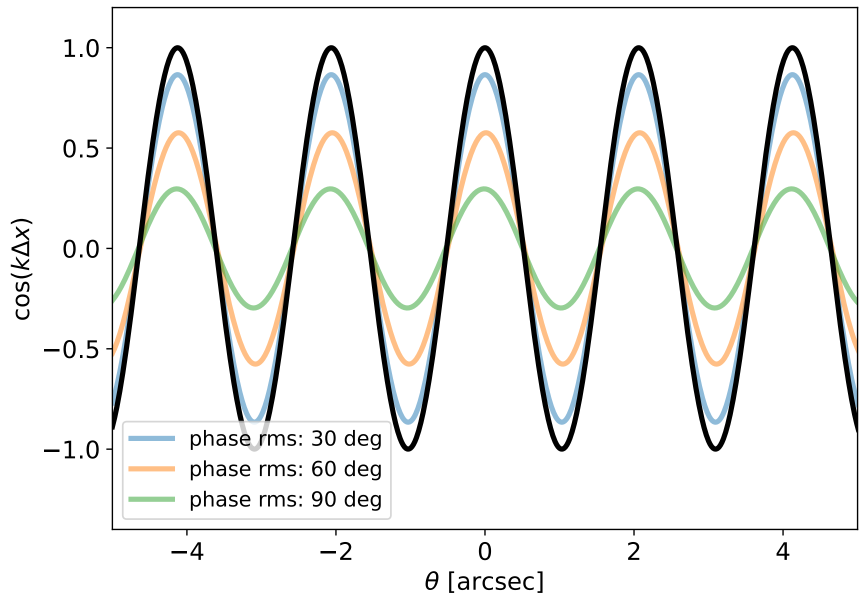

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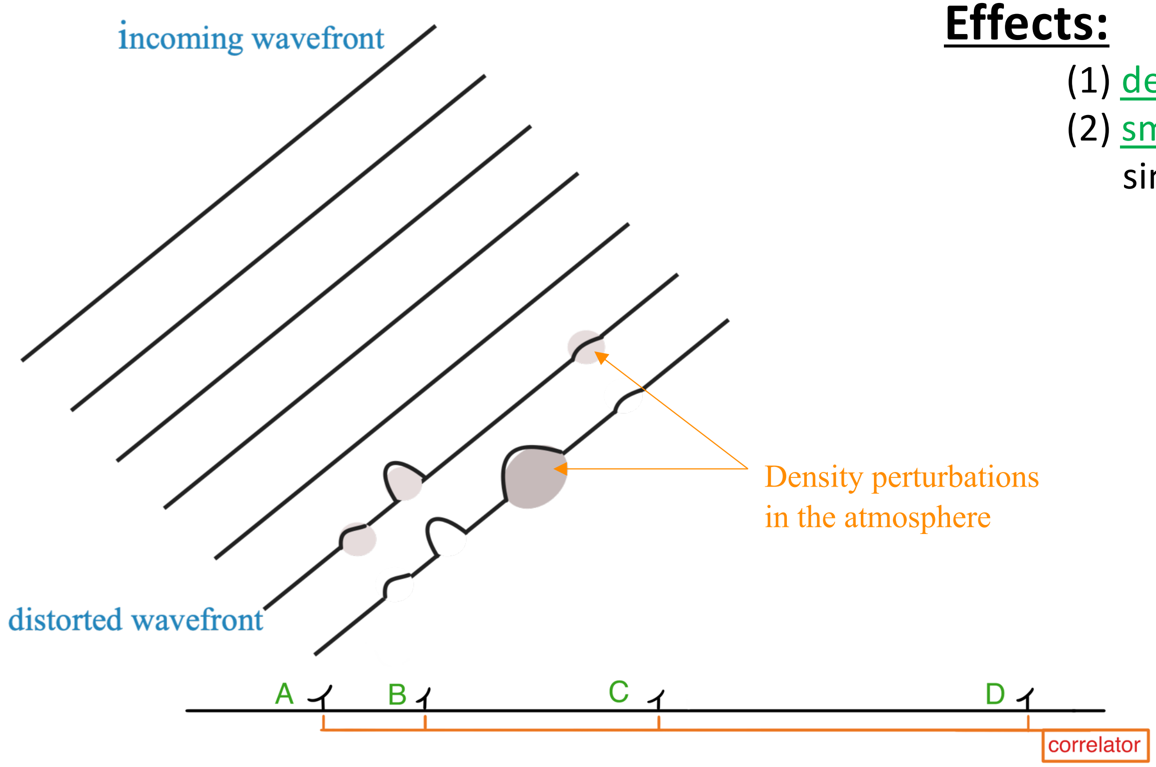
Complex gain calibration

1. Calibrate time dependent visibility phase errors

Effects:

(1) decoherence [loss of visibility amplitude]
(2) smearing [reducing angular resolution,
similar to the effect of seeing in optical observations.]



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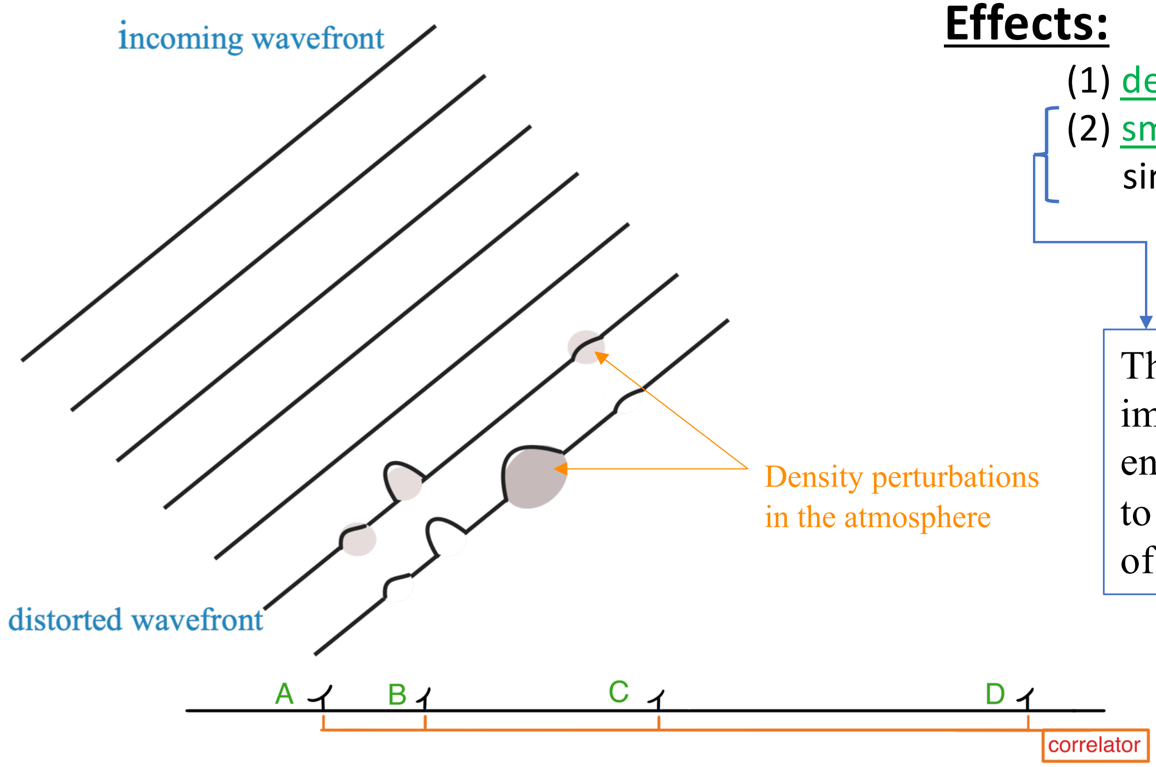

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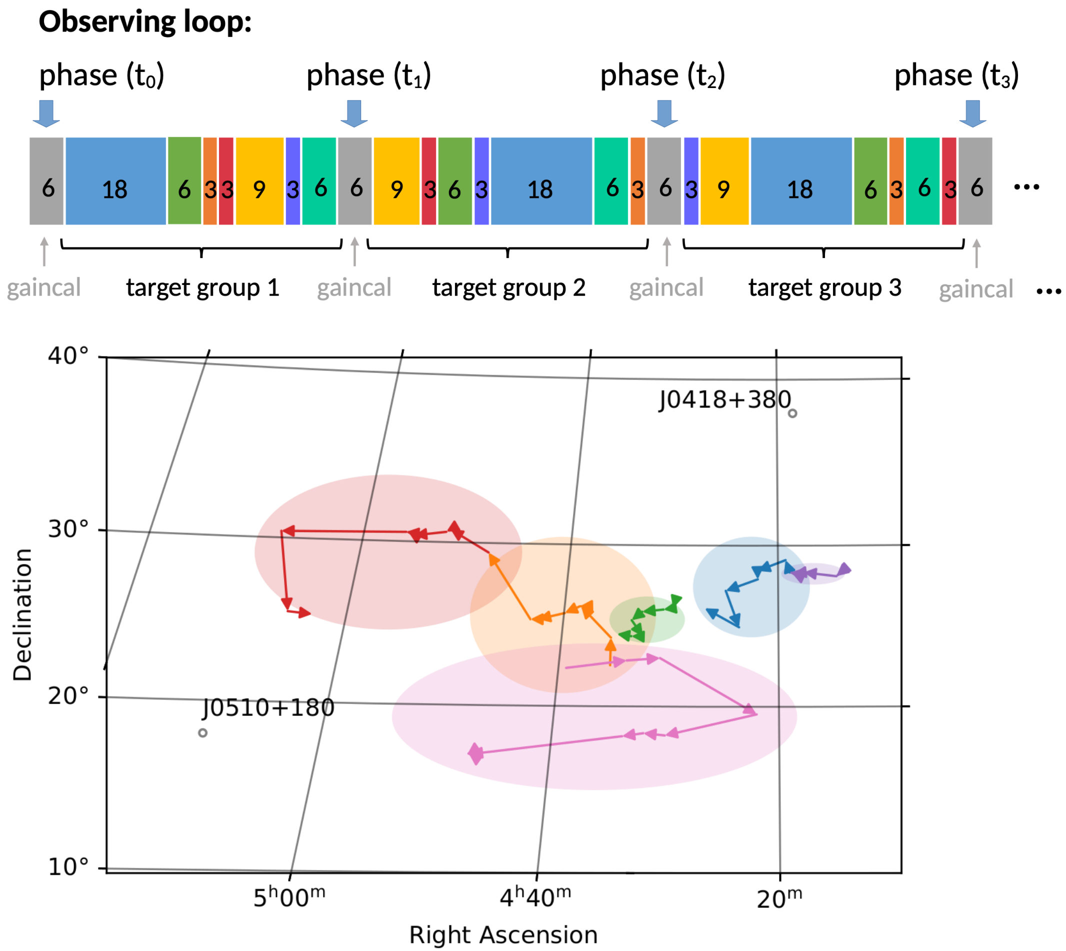
This further induces artifacts when you are performing imaging (i.e., deconvolving the dirty images), in turns enhance the rms noise level. The images that are subject to this issue seriously are dynamic range limited instead of thermal noise limited.

Complex gain calibration



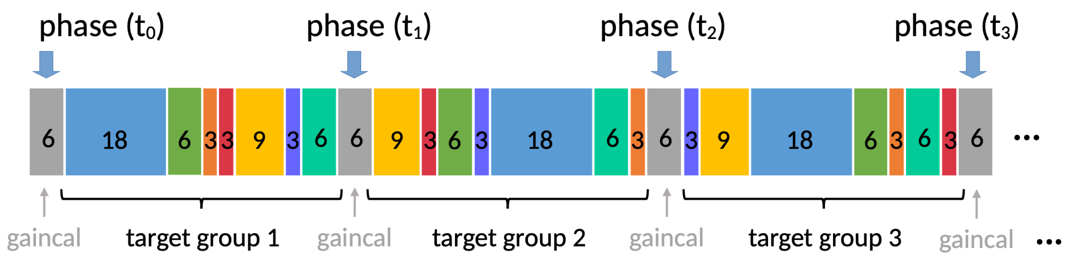
(Chung, Chia-Ying, Master's thesis, 2023, NTU)

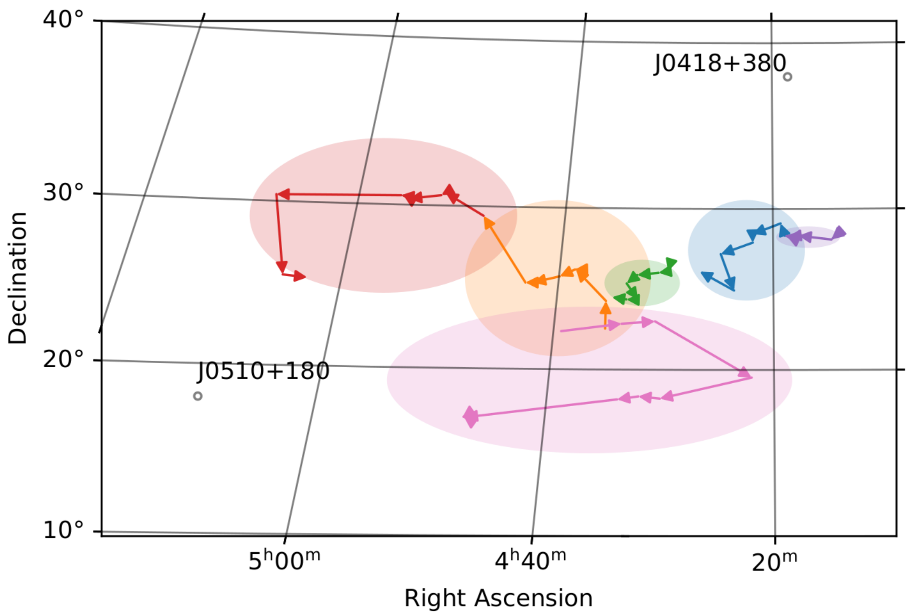
<u>Calibration strategy</u>:

observe a bright, point-like celestial source at known coordinates every few or few tens minutes [calibration duty cycle-time depends on the site condition and baseline length (related to the coherent length of the tropospheric turbulence)]. We refer to this point-like celestial source as complex gain calibrator.

Complex gain calibration

Observing loop:





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Check this link for a basic introduction to the tropospheric turbulence

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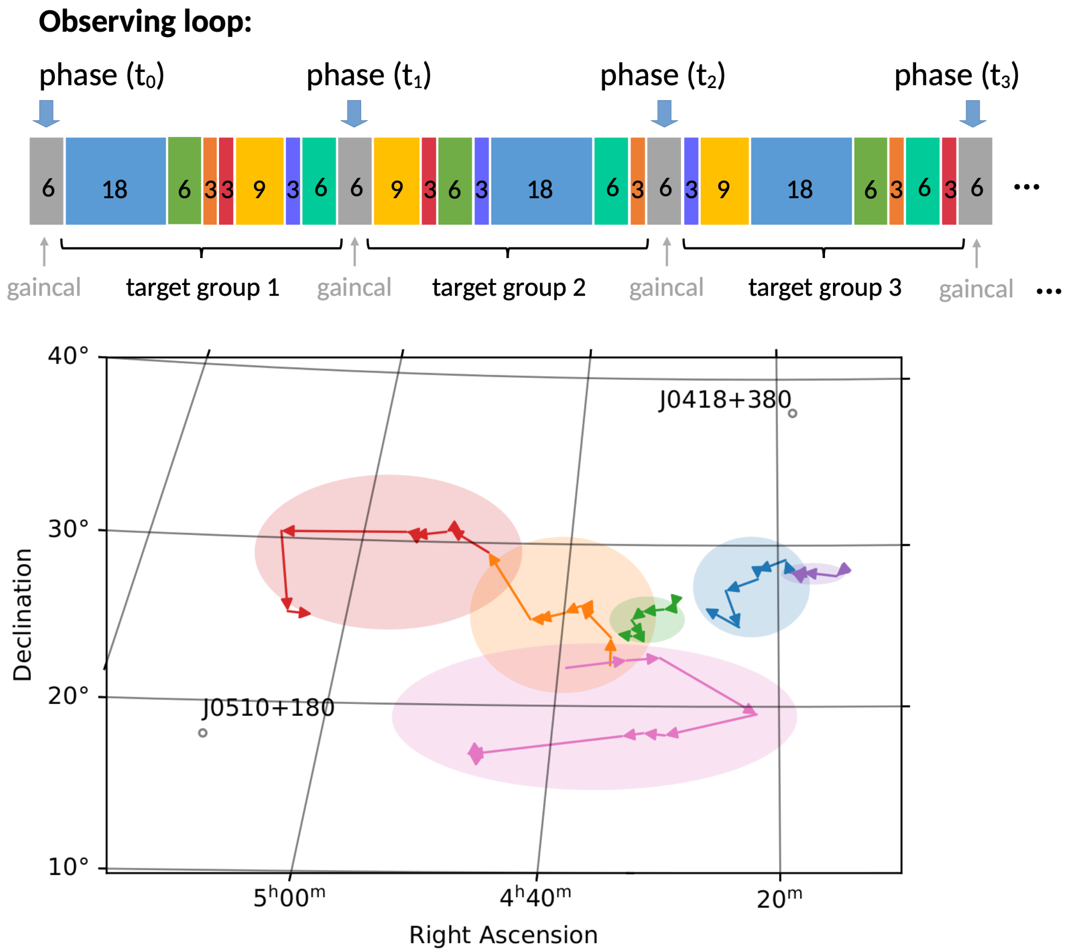

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The complex gain calibrator is usually a quasar, which is a distant, active supermassive black hole. The flux density of a quasar varies at the characteristic timescale of the period at the innermost stable circular orbit (t_{ISCO}), which is proportional to the black hole mass.

Sgr A* : $M{\sim}4\cdot 10^{6}\ M_{\odot}$, $t_{ISCO}{\sim}12$ minutes

Typical quasar. : $M = 10^{8}{\sim}10^{10}\ M_{\odot}$, $t_{ISCO}{\sim}0.2{\sim}20$ days

Complex gain self-calibration

Do it only when you know what you are doing

Rely on the redundant measurements of an interferometric array to iteratively solve the target source structures and the phase/amplitude errors.

There is no gaurantee that the iterations will converge correctly.

During the self-calibration, we will likely lose the absolute astrometry.

We can perform phase-only self-calibration, which is relatively robust. The amplitude self-calibration is under-constrained. By performing amplitude self-calibration, you may artificially create/remove some intensity structures. Be really cautionary and conservative when performing amplitude self-calibration.

Complex gain self-calibration

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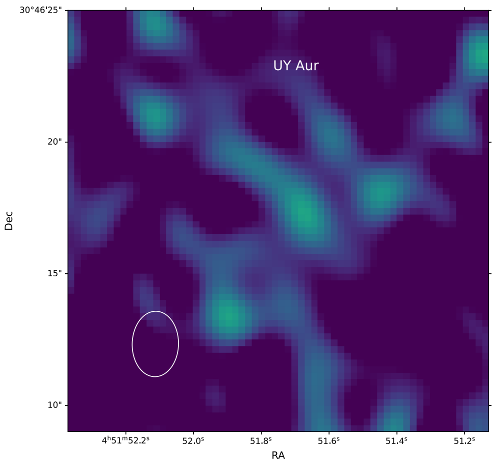
To take care of the Inevitable imperfection of the complex gain calibration

If the gain-phase self-calibration is converging correctly, and if there is no massive flagging over the iterations of self-calibration (e.g., due to a lot of baselines did not detect a strong signal and thus are not self-calibratable), the peak intensity should increase with iteration. At the same time, the noise level should decrease with iteration. If there is massive flagging, just be careful about what you are doing.

without phase selcal



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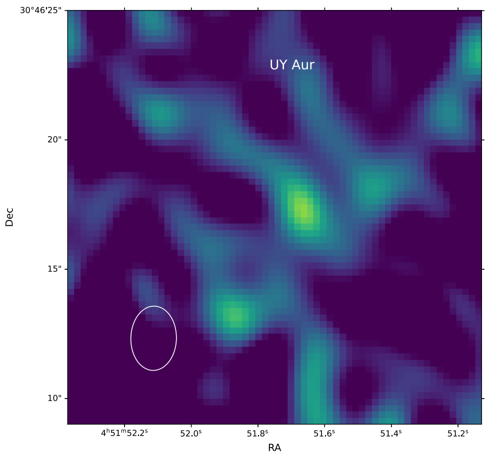

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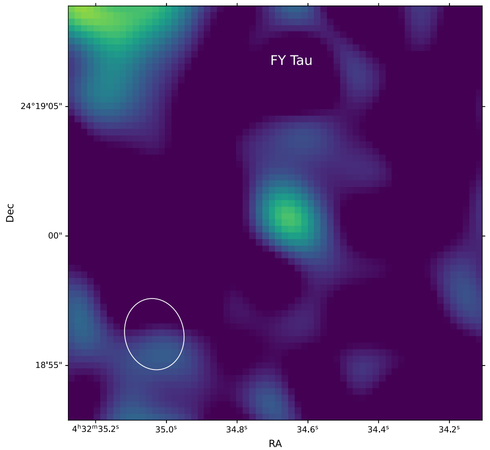

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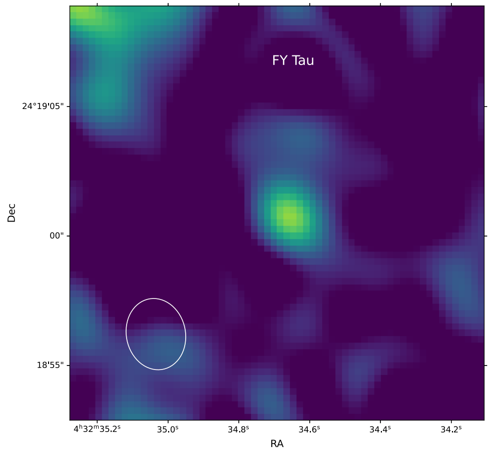

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Passband calibration

1. Calibrate frequency dependent visibility phase errors

2. Calibrate frequency depedent visibility amplitude errors

1. We need to observe a bright source at the beginning and/or end of our observing run as the passband calibrator; we need to observe a bright and stationary quasar that is close to our target source as the complex gain calibrator.
2. Phase errors can lead to smearing of the images, imaging artifacts, and degradation of the visibility amplitudes.
3. Gain-phase self-calibration may remove the residual phase errors. But we need to be very careful when performing self-calibration.