An Introduction to Radio Interferometry

3-4 Complex Visibility (2D)

You can find relevant material on my personal webpage

When there is a continuous distribution of incoherent sources

$$< [E_A + E_B]^2 >_{cos}^{cross} \propto \int P(\theta)A(\theta)\cos(2\pi u\theta)\, d\theta$$

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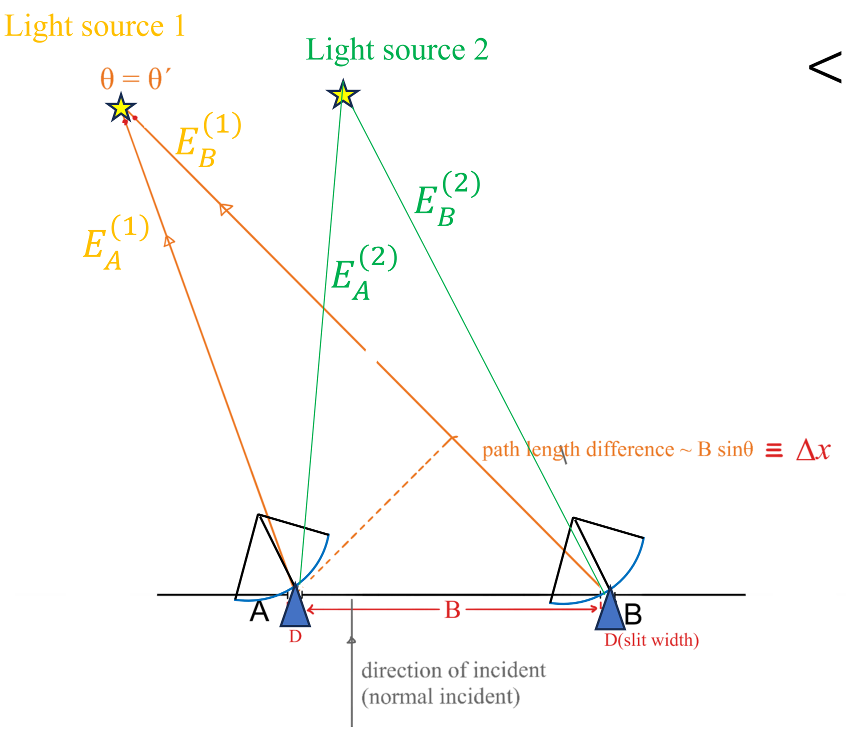

Complex visibility $V_{AB} = < [E_A + E_B]^2 >_{cos}^{cross} + i < [E_A + E_B]^2 >_{sin}^{cross}$

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Lecture Unit 3-3

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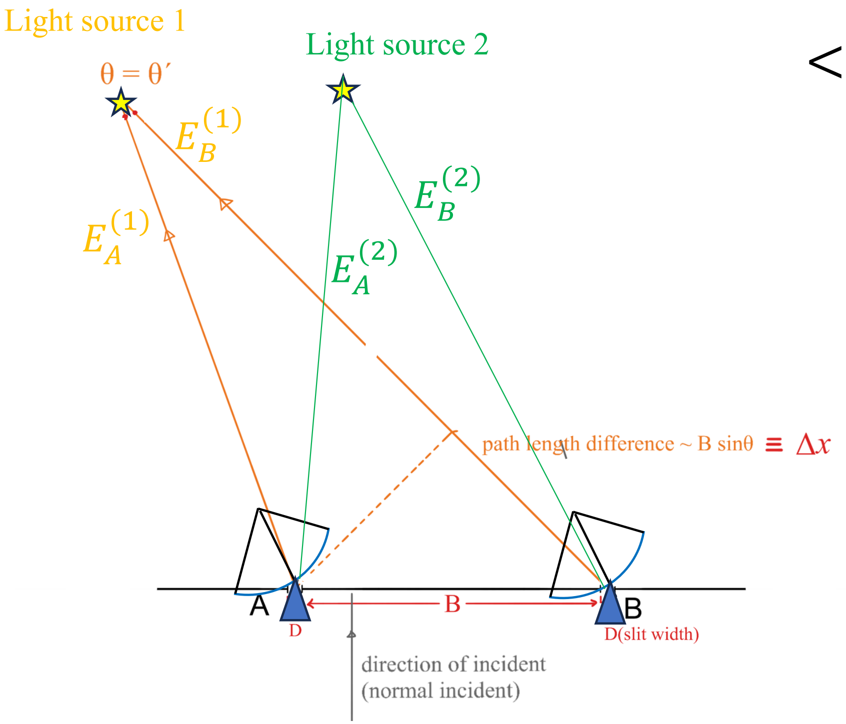

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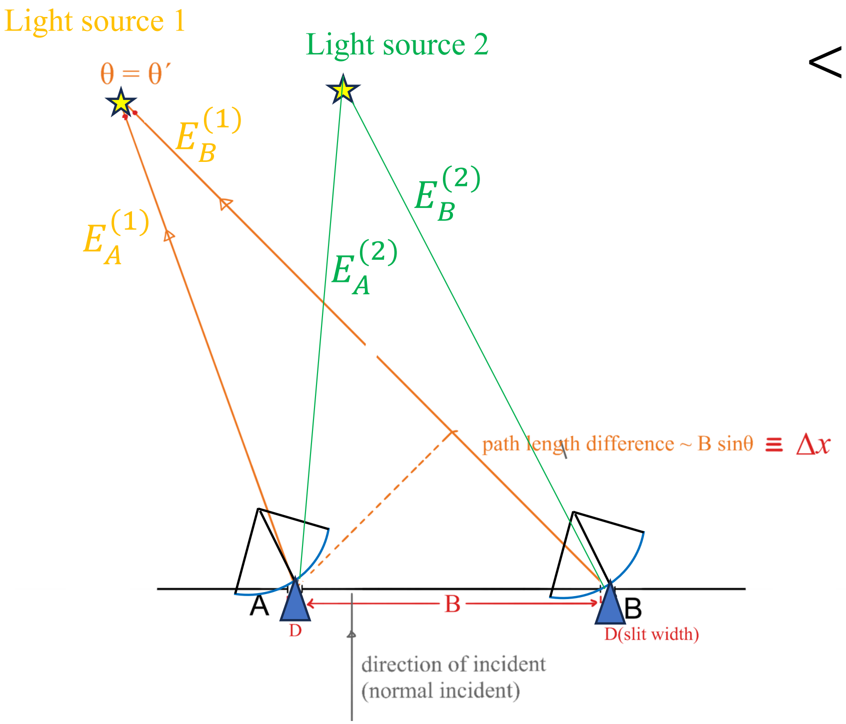

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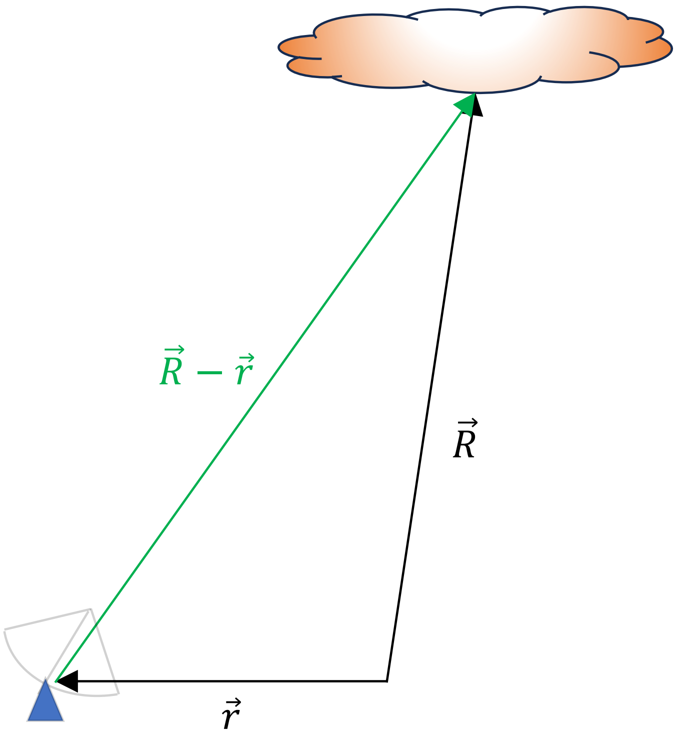

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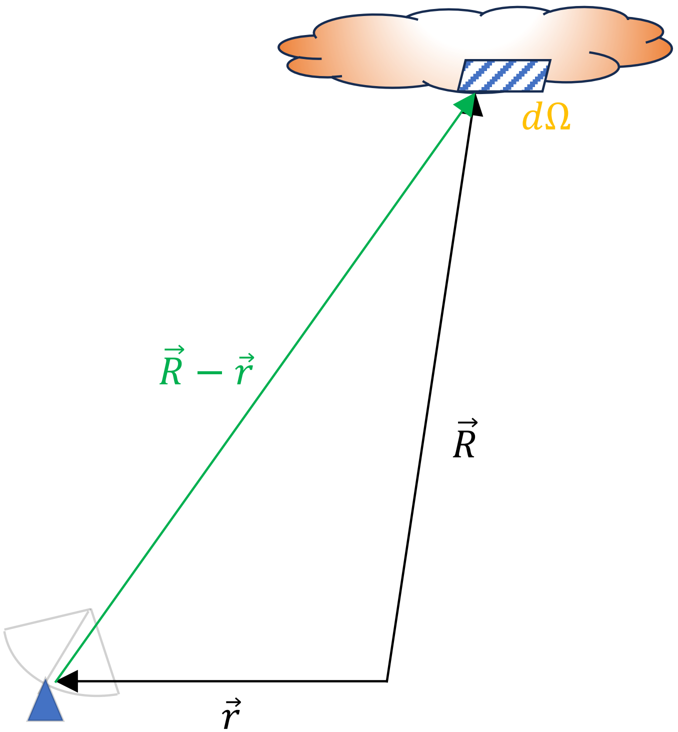
Lecture Unit 3-3



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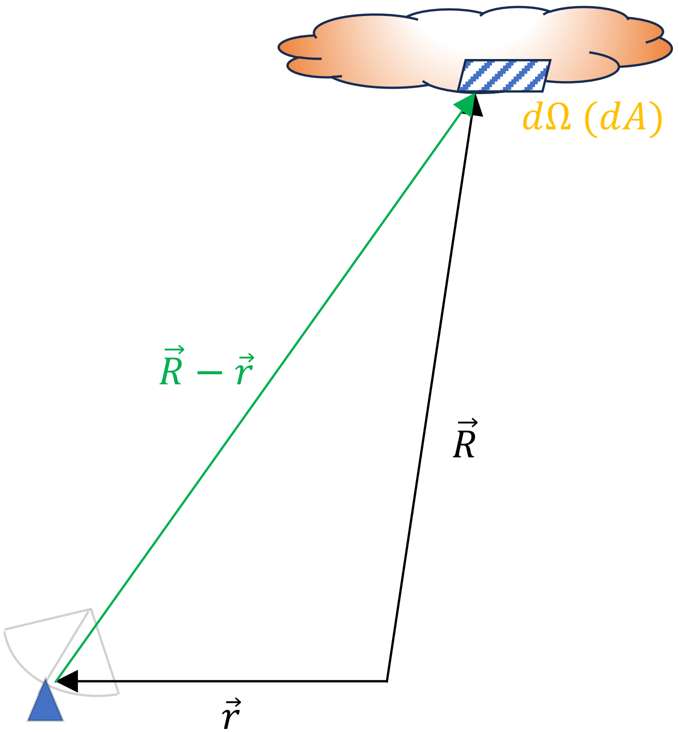

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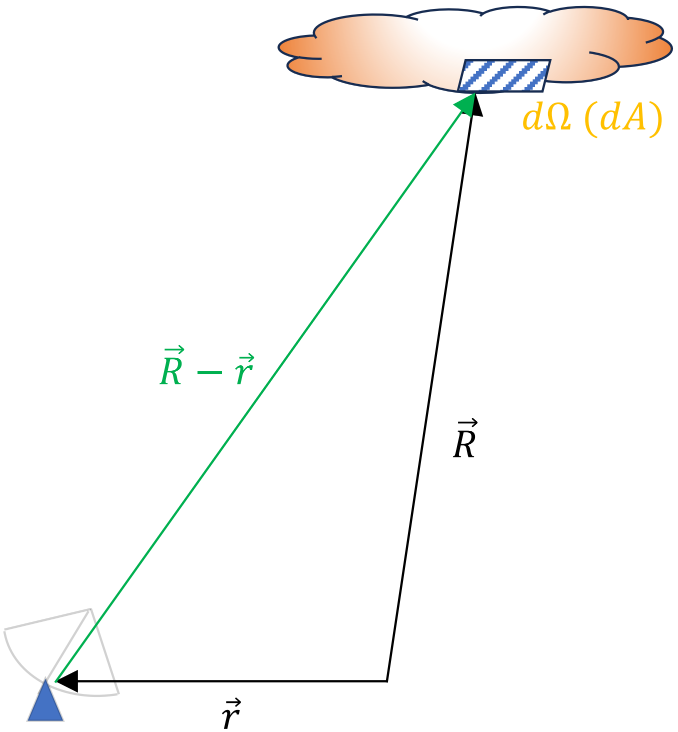

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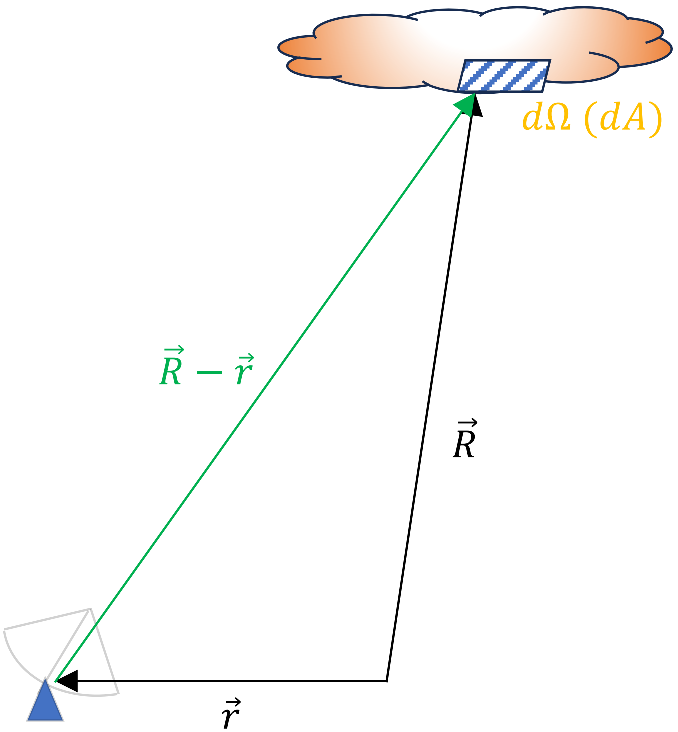

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Proportional constant to describe field strength contributed from a specific location $\vec{R}$

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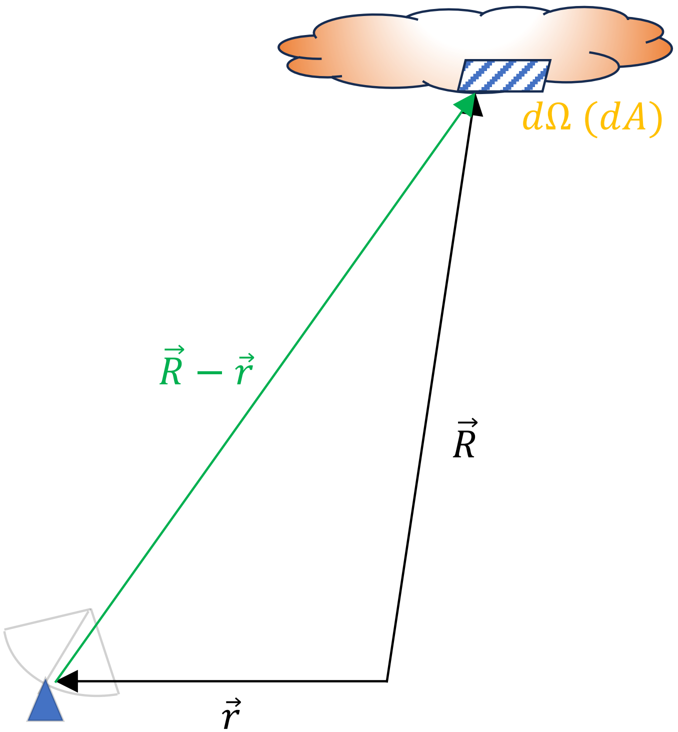

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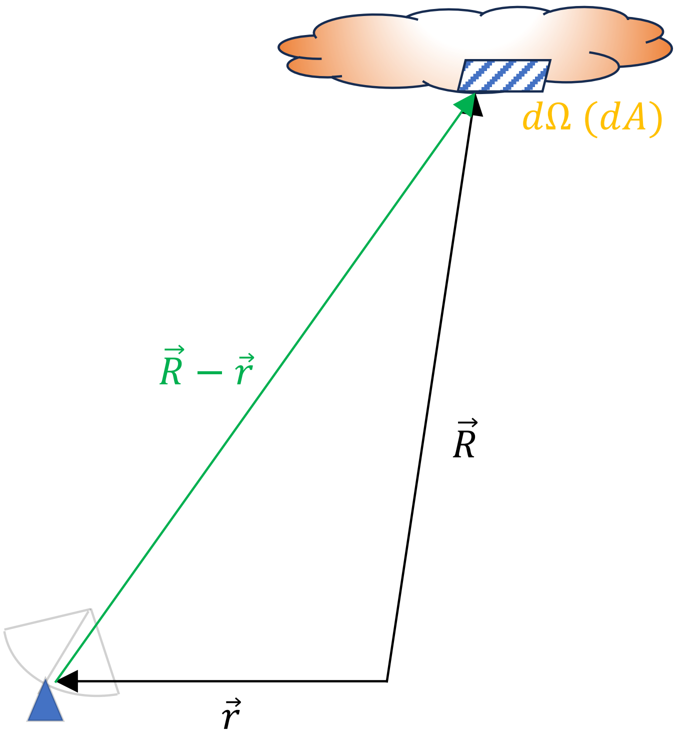

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Solution of EM wave propagation. Physically, only the real part matter. This expression makes the calculation concise.

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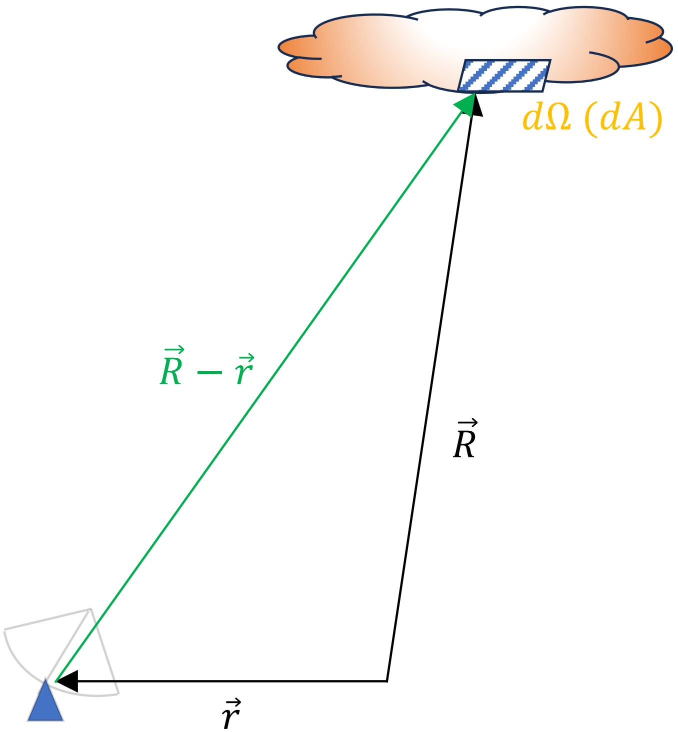

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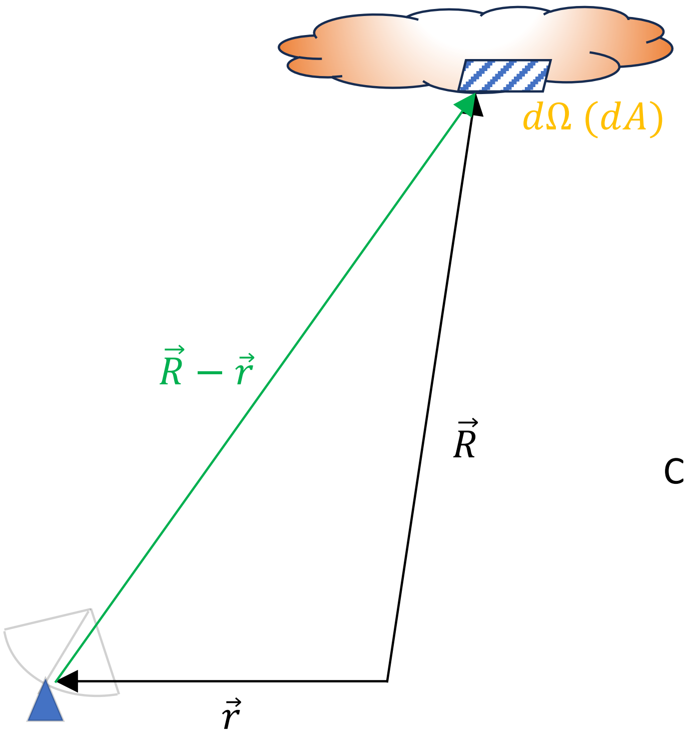

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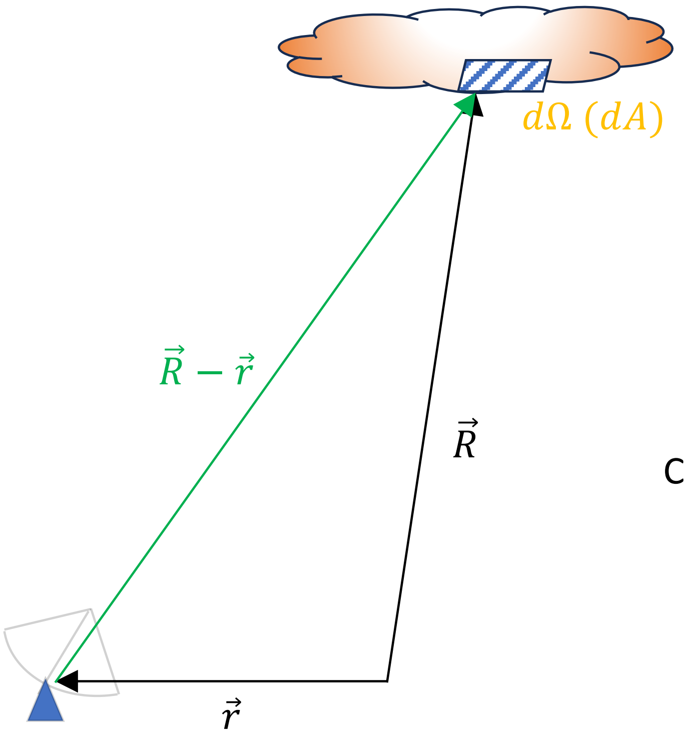

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Ensure auto-correlation to be real and positive

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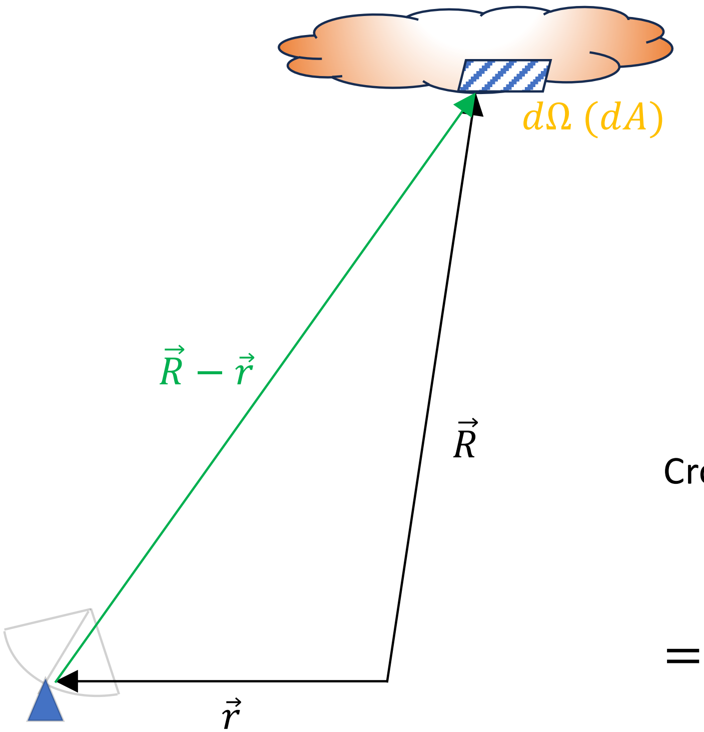

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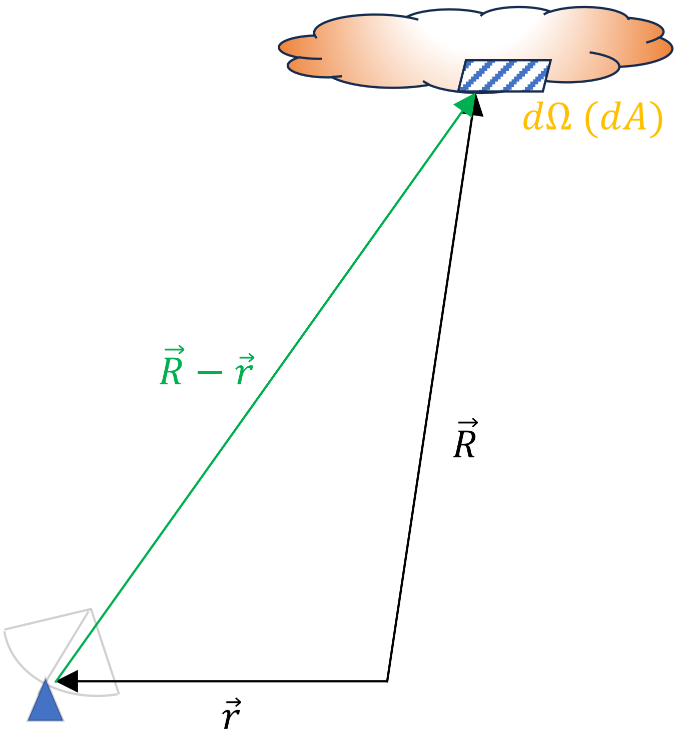

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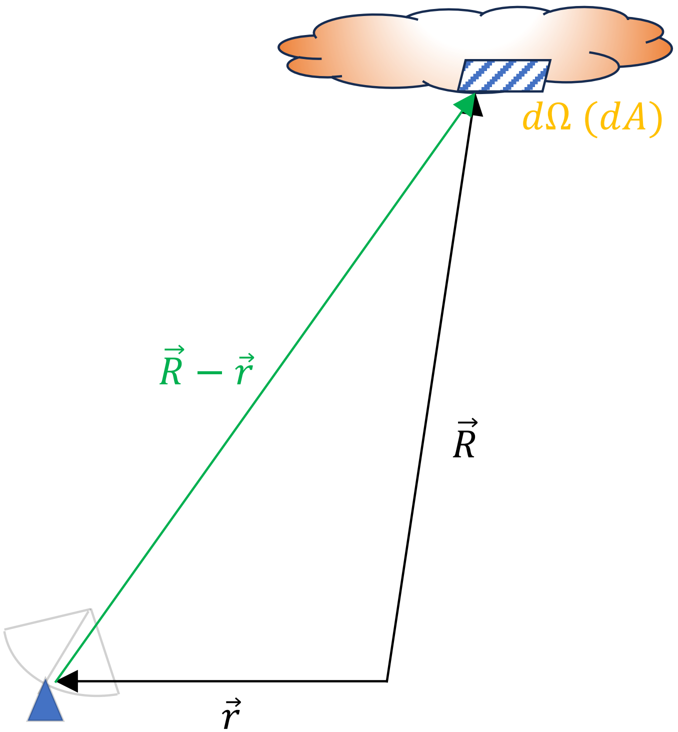

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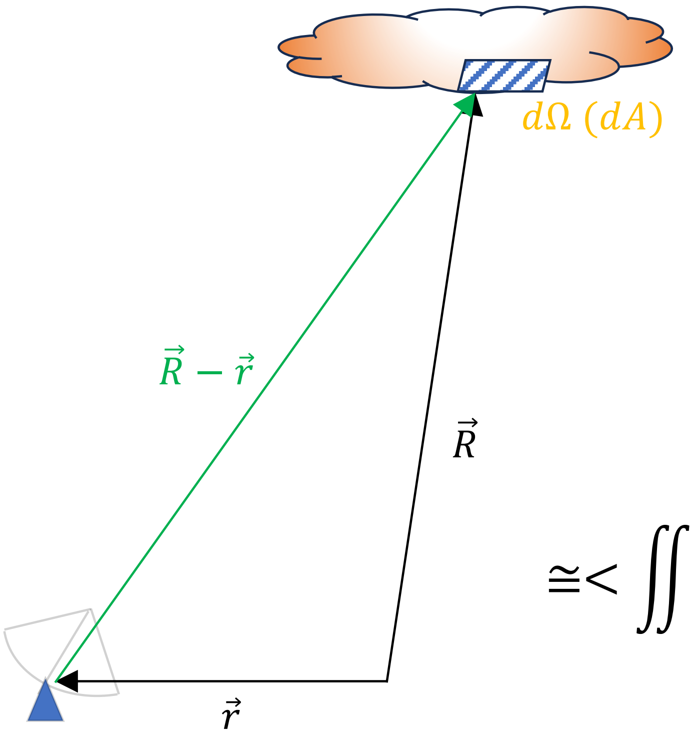

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When $|\vec{R}| \gg |\vec{r}|$, $|\vec{R}-\vec{r}| \cong |\vec{R}| - \frac{\vec{R}\cdot\vec{r}}{|\vec{R}|}$ (first order taylor expansion)

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$$\cong< \iint \varepsilon_\nu(\vec{R_1})\varepsilon^*{}_\nu(\vec{R_2}) \frac{e^{ik|\vec{R_1}|}e^{-ik\vec{s_1}\cdot\vec{r_1}}}{|\vec{R_1}|}\frac{e^{-ik|\vec{R_2}|}e^{ik\vec{s_2}\cdot\vec{r_2}}}{|\vec{R_2}|} |\vec{R_1}|^2|\vec{R_2}|^2 d\Omega_1 d\Omega_2 >$$

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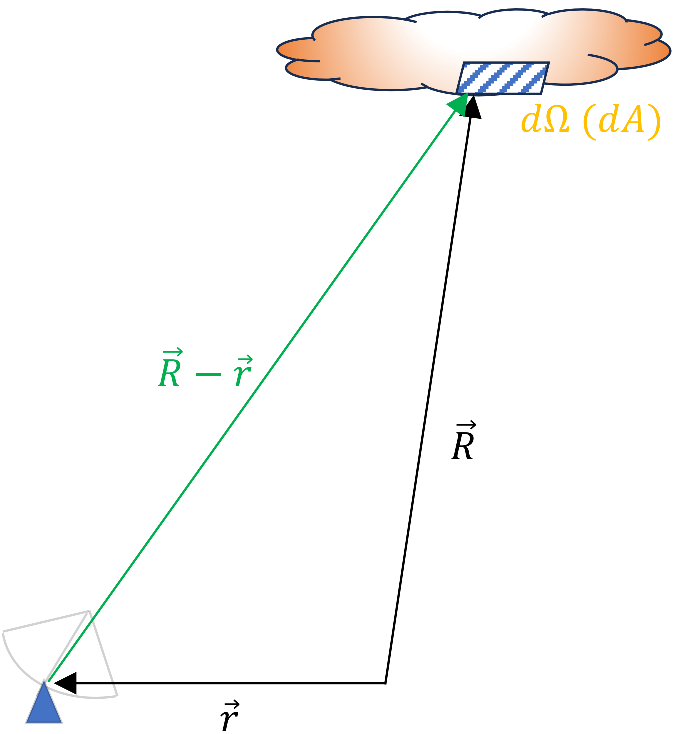

In the 1D case with 2 sources, when we focus on incoherent sources

$$\begin{aligned} <[E_A + E_B]^2> &= <{E_A^{(1)}}^2> + <{E_B^{(1)}}^2> + \boxed{<2E_A^{(1)}E_B^{(1)}>} \\ &+ <{E_A^{(2)}}^2> + <{E_B^{(2)}}^2> + \boxed{<2E_A^{(2)}E_B^{(2)}>} \\ &\cancel{+<2E_A^{(1)}E_A^{(2)}>+<2E_A^{(1)}E_B^{(2)}>+<2E_B^{(1)}E_A^{(2)}>+<2E_B^{(1)}E_B^{(2)}>} \end{aligned}$$

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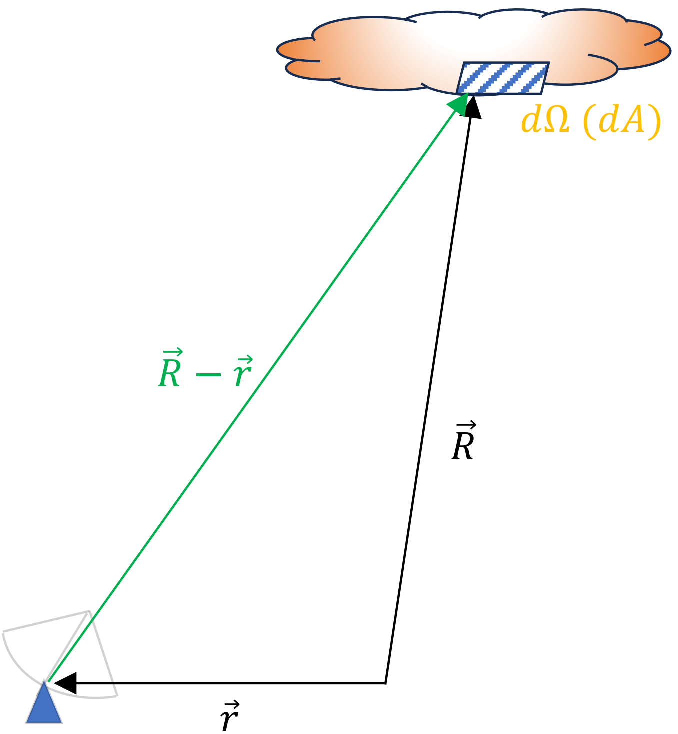

In the 1D case with 2 sources, when we focus on incoherent sources

$$< [E_A + E_B]^2 > \; = < E_A^{(1)^2} > + < E_B^{(1)^2} > + \boxed{< 2E_A^{(1)}E_B^{(1)} >}$$
$$+ < E_A^{(2)^2} > + < E_B^{(2)^2} > + \boxed{< 2E_A^{(2)}E_B^{(2)} >}$$
$$\cancel{+ < 2E_A^{(1)}E_A^{(2)} > + < 2E_A^{(1)}E_B^{(2)} > + < 2E_B^{(1)}E_A^{(2)} > + < 2E_B^{(1)}E_B^{(2)} >}$$

In the 2D case, with continuous intensity distribution, considering incoherent sources is effectively considering

$$< \varepsilon_\nu(\overrightarrow{R_1})\varepsilon^*{}_\nu(\overrightarrow{R_2}) > = 0 \quad \text{for} \quad \overrightarrow{R_1} \neq \overrightarrow{R_2}$$

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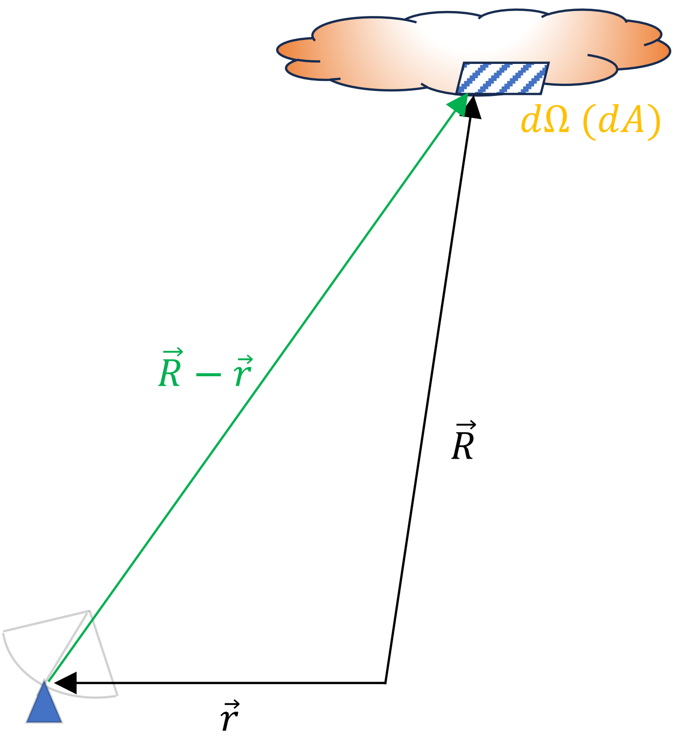

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This can be achieved by implementing Dirac delta function.

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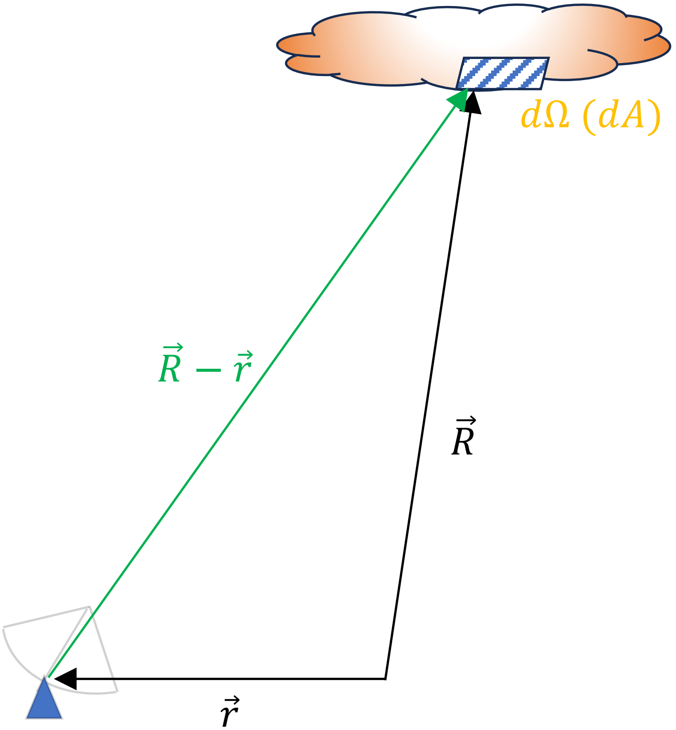

Considering only incoherent sources, eventually, we will pick out the term at $\overrightarrow{R_1} = \overrightarrow{R_2} = \vec{R}$ (one $d\Omega$ will be ellimated by integrating delta function)

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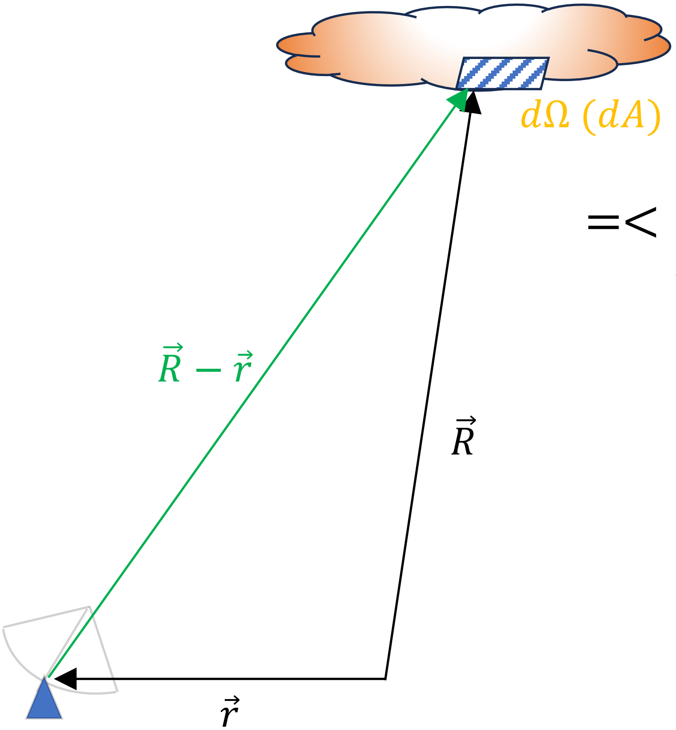

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Defining unit vector $\vec{s} \equiv \frac{\vec{R}}{|\vec{R}|}$, solid angle $d\Omega = \frac{dA}{|\vec{R}|^2}$

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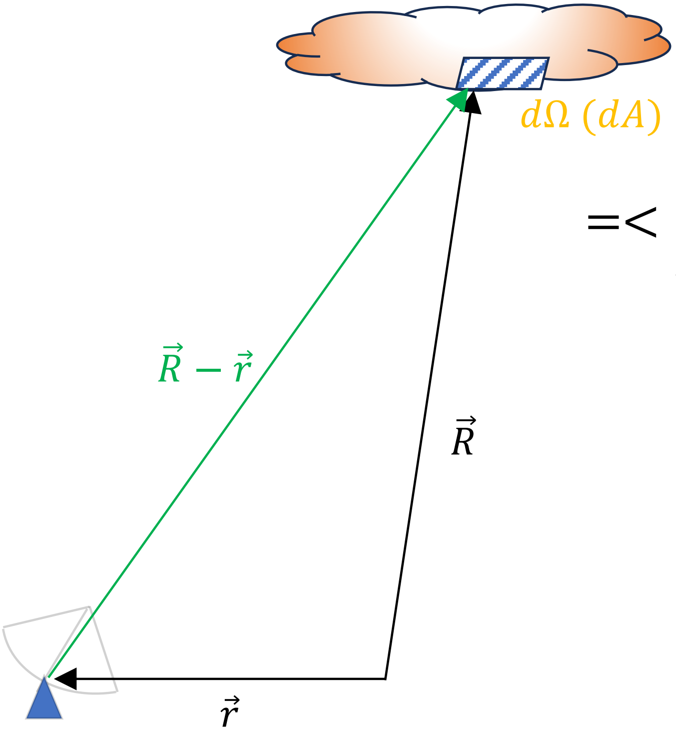

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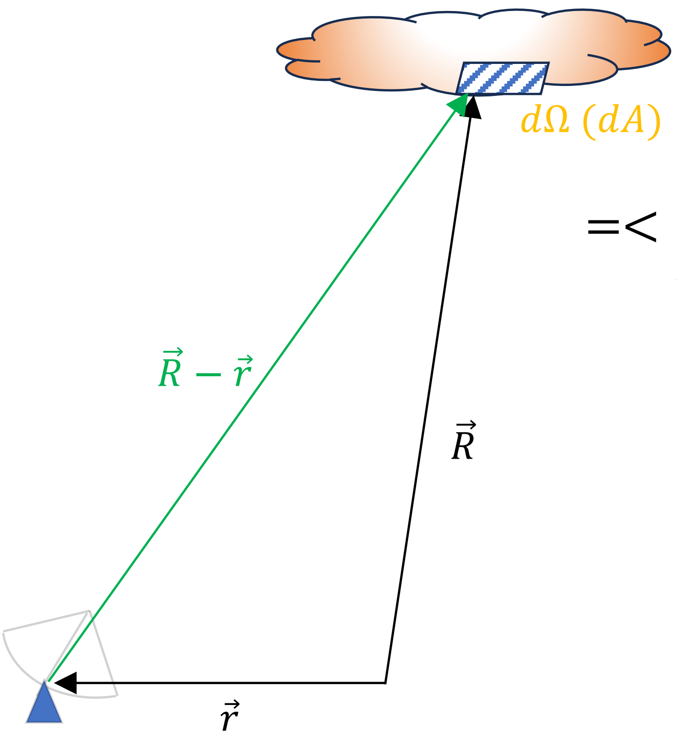

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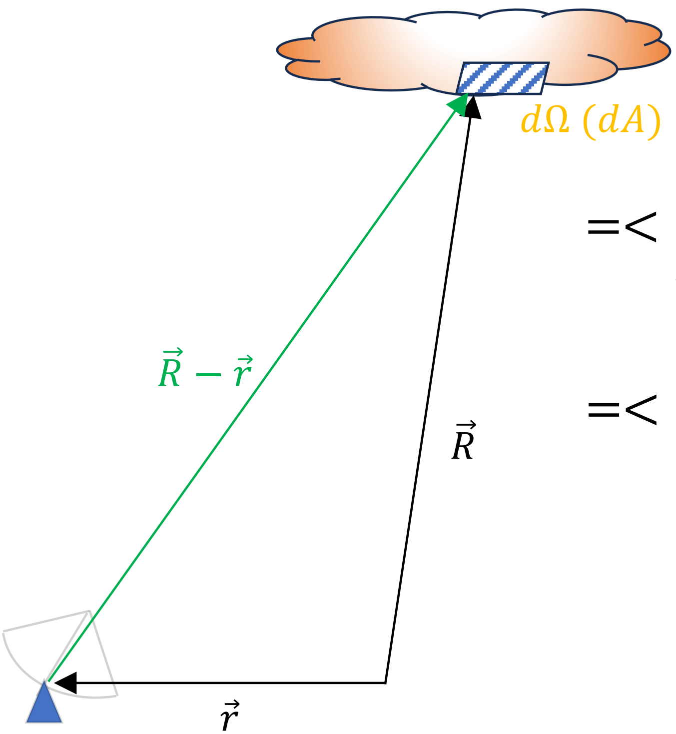

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$$= < \iint \varepsilon_\nu(\vec{R})\varepsilon^*{}_\nu(\vec{R}) |\vec{R}|^2 e^{-ik\vec{s}\cdot(\overrightarrow{r_1}-\overrightarrow{r_2})} d\Omega >$$

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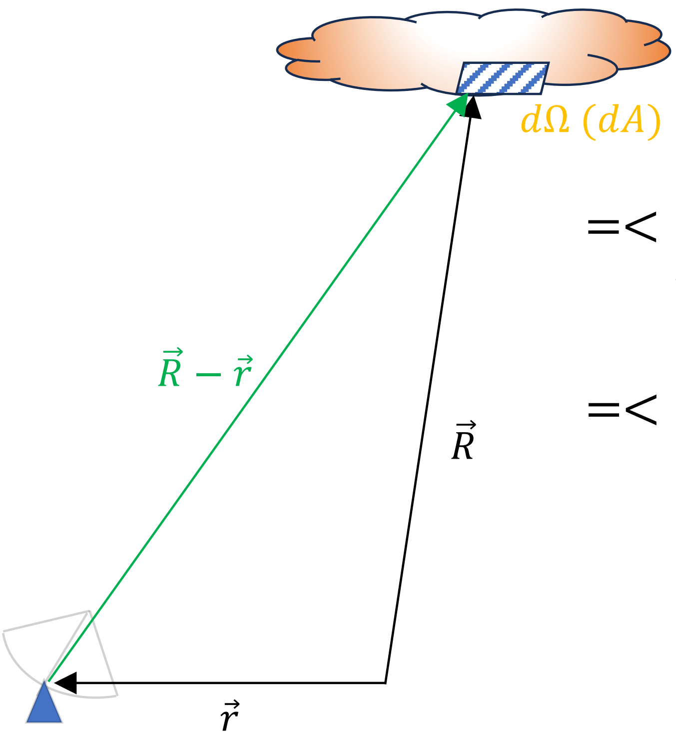

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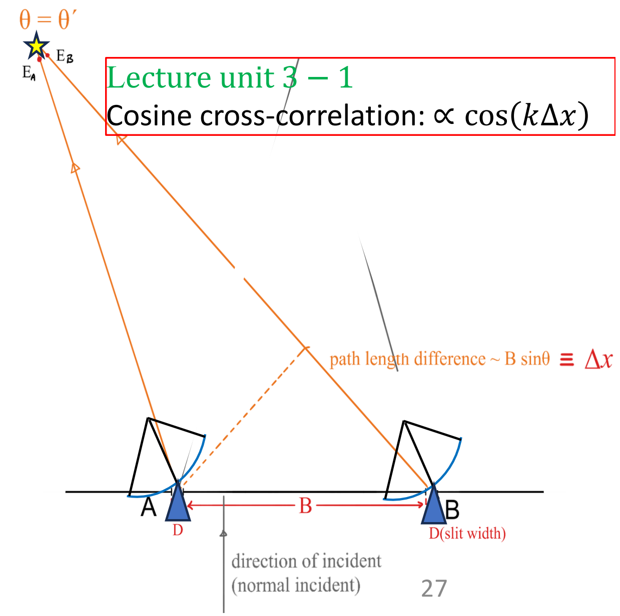

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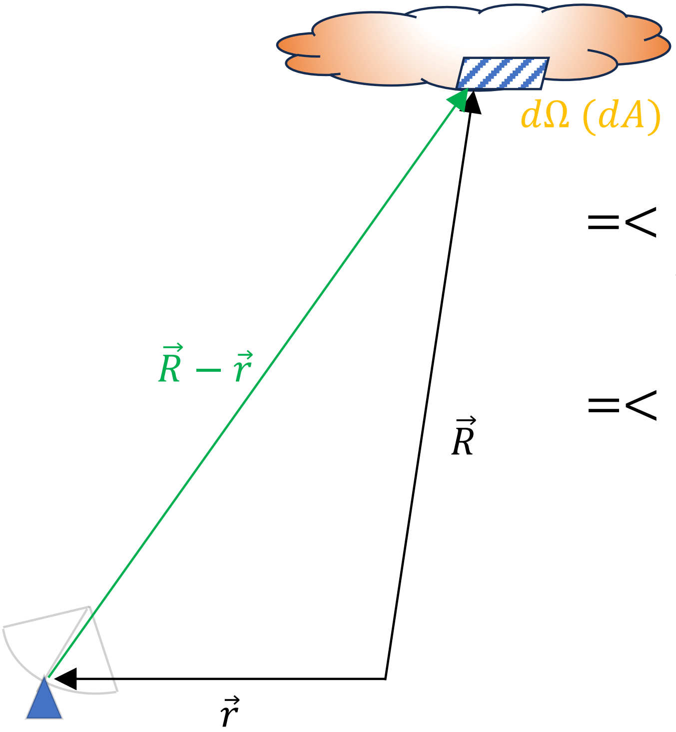

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$$= < \iint \underbrace{\varepsilon_\nu(\vec{R})\varepsilon^*_\nu(\vec{R})\, |\vec{R}|^2}_{Intensity\ \ I_\nu(\vec{s})} e^{-ik\vec{s}\cdot(\overrightarrow{r_1}-\overrightarrow{r_2})} d\Omega >$$

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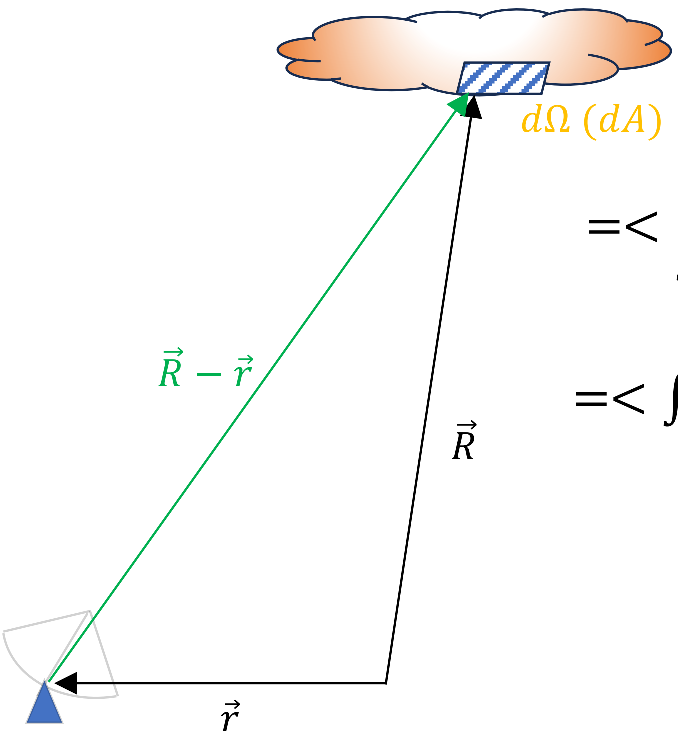

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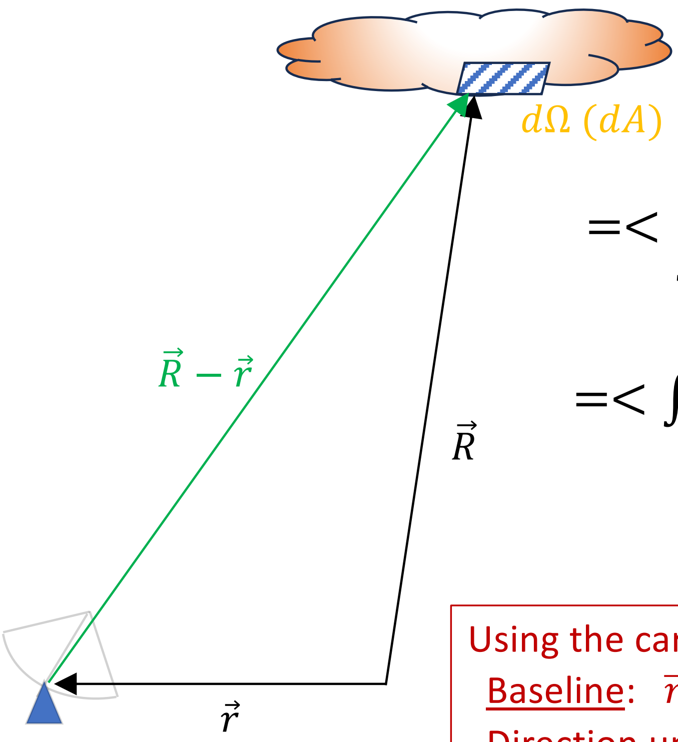
Defining unit vector $\vec{s} \equiv \frac{\vec{R}}{|\vec{R}|}$, solid angle $d\Omega = \frac{dA}{|\vec{R}|^2}$

$$\cong < \iint \varepsilon_\nu(\vec{R_1})\varepsilon^*{}_\nu(\vec{R_2}) \frac{e^{ik|\vec{R_1}|}e^{-ik\vec{s_1}\cdot\vec{r_1}}}{|\vec{R_1}|}\frac{e^{-ik|\vec{R_2}|}e^{ik\vec{s_2}\cdot\vec{r_2}}}{|\vec{R_2}|}\,|\vec{R_1}|^2|\vec{R_2}|^2 d\Omega_1 d\Omega_2 >$$



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$$= < \iint \underbrace{\varepsilon_\nu(\vec{R})\varepsilon^*{}_\nu(\vec{R})\,|\vec{R}|^2}_{Intensity\ \ I_\nu(\vec{s})} e^{-ik\vec{s}\cdot(\vec{r_1}-\vec{r_2})} d\Omega > = < \iint I_\nu(\vec{s})\, e^{-i2\pi\vec{s}\cdot\frac{(\vec{r_1}-\vec{r_2})}{\lambda}} d\Omega >$$

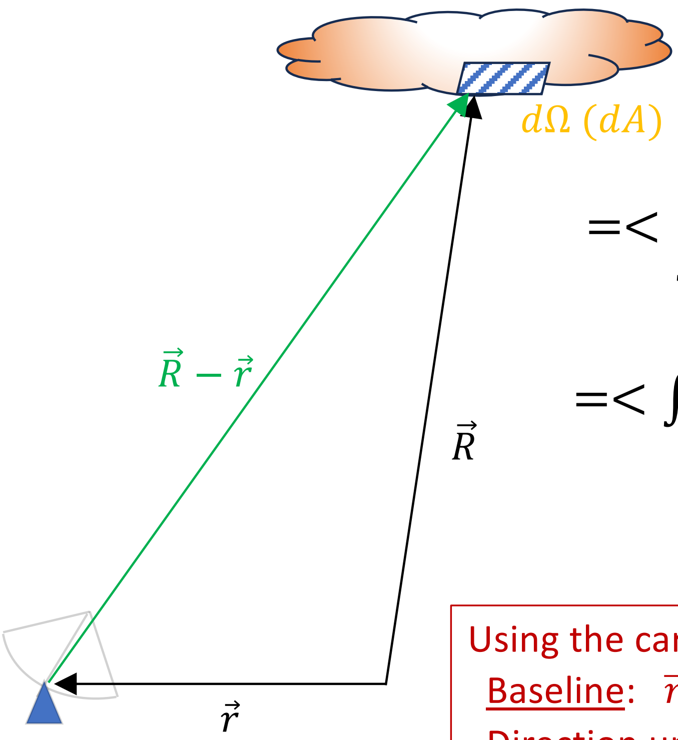
Using the cartesian coordinate system:

<u>Baseline</u>: $\vec{r_1} - \vec{r_2} \equiv (\lambda u, \lambda v, \lambda w)$

<u>Direction unit vector</u>: $\vec{s} = (\ell, \mathrm{m}, \sqrt{1 - \ell^2 - m^2})$

Defining unit vector $\vec{s} \equiv \frac{\vec{R}}{|\vec{R}|}$, solid angle $d\Omega = \frac{dA}{|\vec{R}|^2}$

$$\cong < \iint \varepsilon_\nu(\vec{R_1})\varepsilon^*_\nu(\vec{R_2}) \frac{e^{ik|\vec{R_1}|}e^{-ik\vec{s_1}\cdot\vec{r_1}}}{|\vec{R_1}|} \frac{e^{-ik|\vec{R_2}|}e^{ik\vec{s_2}\cdot\vec{r_2}}}{|\vec{R_2}|} |\vec{R_1}|^2 |\vec{R_2}|^2 d\Omega_1 d\Omega_2 >$$



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$$= < \iint \underbrace{\varepsilon_\nu(\vec{R})\varepsilon^*_\nu(\vec{R}) \, |\vec{R}|^2}_{Intensity \;\; I_\nu(\vec{s})} e^{-ik\vec{s}\cdot(\vec{r_1}-\vec{r_2})} d\Omega > = < \iint I_\nu(\vec{s}) \, e^{-i2\pi\vec{s}\cdot\frac{(\vec{r_1}-\vec{r_2})}{\lambda}} d\Omega >$$

$$\Rightarrow V_\nu(u,v,w) = \iint I_\nu(\ell,m) \frac{e^{-2\pi i\left(u\ell+vm+w\sqrt{1-\ell^2-m^2}\right)}}{\sqrt{1-\ell^2-m^2}} d\ell \, dm$$

Using the cartesian coordinate system:

Baseline: $\vec{r_1} - \vec{r_2} \equiv (\lambda u, \lambda v, \lambda w)$

Direction unit vector: $\vec{s} = (\ell, \mathrm{m}, \sqrt{1-\ell^2-m^2})$

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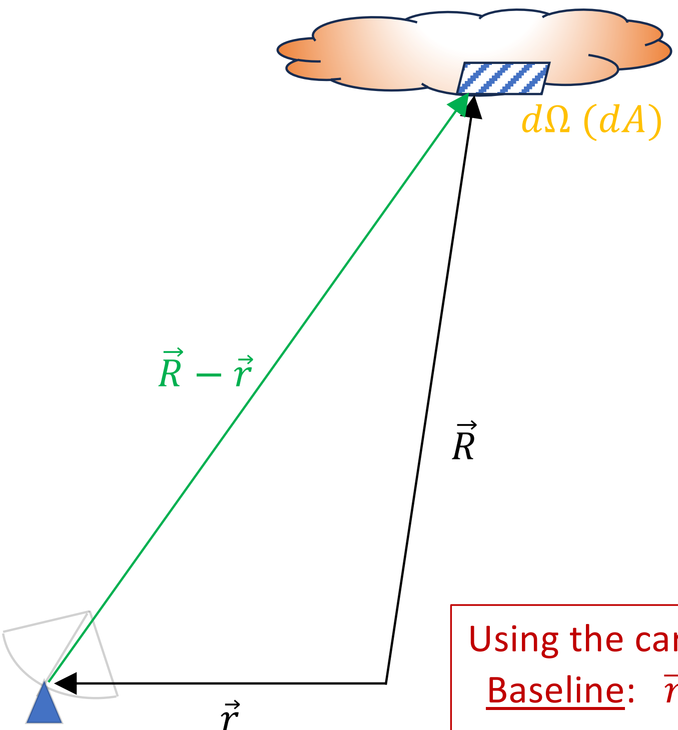


Using the cartesian coordinate system:

(small incident angle limit)

Baseline: $\vec{r_1}-\vec{r_2} \equiv (\lambda u, \lambda v, \lambda w)$

Direction unit vector: $\vec{s} = \left(\ell, \mathrm{m}, \sqrt{1-\ell^2-m^2}\right) \equiv \vec{s_0} + \vec{\sigma}$

Unit vector towards a conveniently defined source center (phase referencing center)

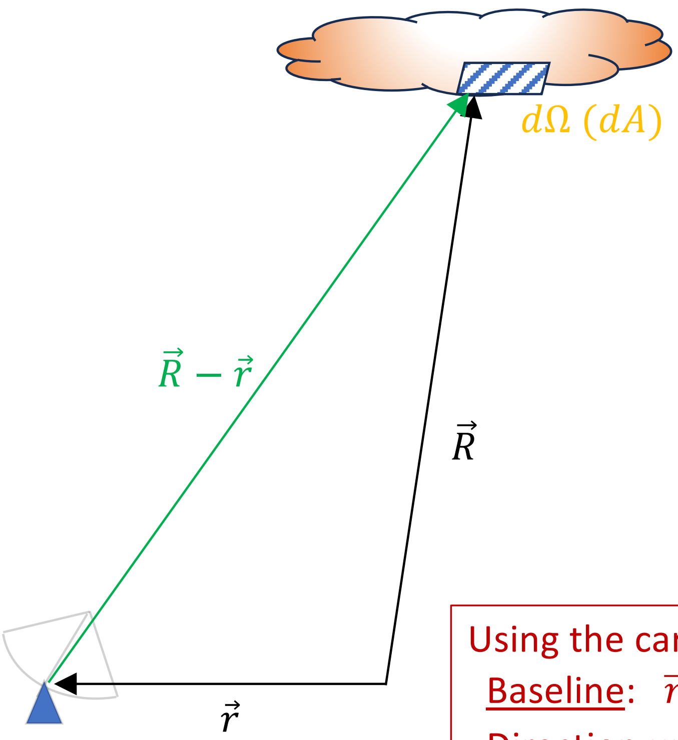
angulr offset

$$V_\nu(u,v,w) = \iint I_\nu(\ell,m) \frac{e^{-2\pi i\left(u\ell+vm+w\sqrt{1-\ell^2-m^2}\right)}}{\sqrt{1-\ell^2-m^2}}\, d\ell\, dm$$

If the angular scale of the target source is not large, we can choose the cartesian coordinate system that ℓ and m are both $\ll 1$

$$V_\nu(u,v,w) \sim \iint I_\nu(\ell,m) \frac{e^{-2\pi i(u\ell+vm+w)}}{1}\, d\ell\, dm$$

$$= e^{-2\pi i w} \iint I_\nu(\ell,m)\, e^{-2\pi i(u\ell+vm)} d\ell\, dm$$



Using the cartesian coordinate system: (small incident angle limit)

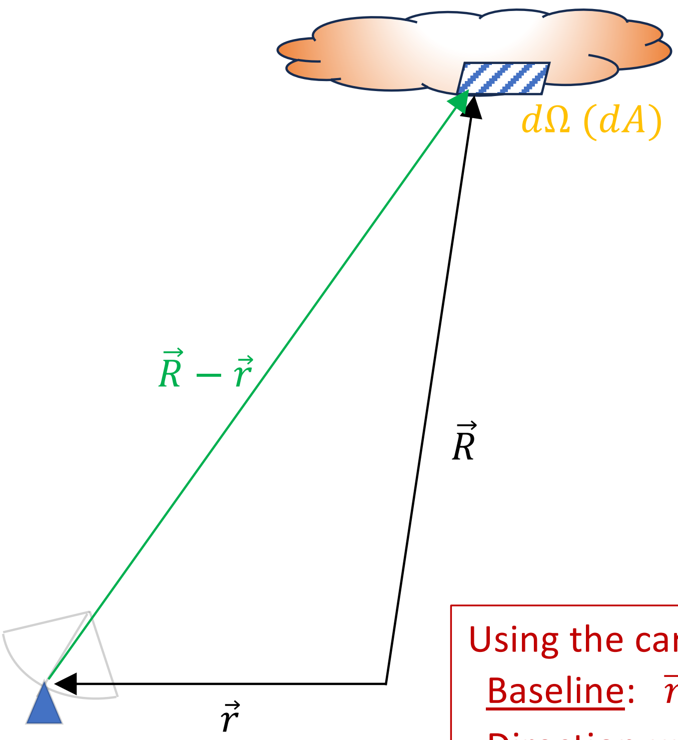
Baseline: $\vec{r_1} - \vec{r_2} \equiv (\lambda u, \lambda v, \lambda w)$

Direction unit vector: $\vec{s} = \left(\ell, \mathrm{m}, \sqrt{1-\ell^2-m^2}\right) \equiv \vec{s_0} + \vec{\sigma}$

Unit vector towards a conveniently defined source center (phase referencing center)

angulr offset

$$V_\nu(u,v,w) = \iint I_\nu(\ell,m)\frac{e^{-2\pi i\left(u\ell+vm+w\sqrt{1-\ell^2-m^2}\right)}}{\sqrt{1-\ell^2-m^2}}\,d\ell\,dm$$



If the angular scale of the target source is not large, we can choose the cartesian coordinate system that ℓ and m are both $\ll 1$

$$V_\nu(u,v,w) \sim \iint I_\nu(\ell,m)\frac{e^{-2\pi i(u\ell+vm+w)}}{1}\,d\ell\,dm$$

$$= e^{-2\pi i w}\underbrace{\iint I_\nu(\ell,m)\,e^{-2\pi i(u\ell+vm)}\,d\ell\,dm}_{\text{Fourier transform of intensity distribution}}$$

Using the cartesian coordinate system: (small incident angle limit)

<u>Baseline</u>: $\vec{r_1}-\vec{r_2} \equiv (\lambda u, \lambda v, \lambda w)$

<u>Direction unit vector</u>: $\vec{s} = \left(\ell, \mathrm{m}, \sqrt{1-\ell^2-m^2}\right) \equiv \vec{s_0} + \vec{\sigma}$

Unit vector towards a conveniently defined source center (phase referencing center)

angulr offset