

# An Introduction to Radio Interferometry

4-4 Generalized sampling function



You can find relevant material  
on my personal webpage

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Sampling function (at  $u_k, k = 1, 2, 3, \dots, M$ )

## Lecture Unit 4-4

$$S(u) \equiv \sum_{k=1}^M \delta(u - u_k)$$

**Naïve inverse fourier transform**

$$[A(\theta') \widetilde{P(\theta')}]^D \equiv \int S(u) V(u) e^{-i2\pi u \theta'} du$$

Generalized sampling function (at  $u_k, k = 1, 2, 3, \dots, M$ )

$$R(u)T(u)D(u)S(u), \quad S(u) \equiv \sum_{k=1}^M \delta(u - u_k)$$

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A quantity to tell effectively (i.e., considering the atmospheric transmission), how much we are affected by thermal noise (emitted by the target source, atmosphere, and our optics).

Reliability weight  $R(u)$  : the value is lower when the **system temperature ( $T_{\text{sys}}$ )** of that specific visibility measurement is higher. There is a commonly adopted functional form of  $R(u_k)$ . We almost always adopt it.

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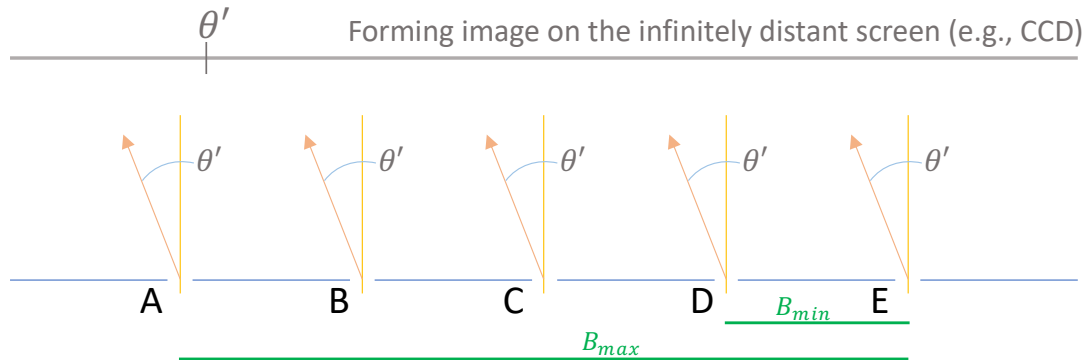
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## Evaluation of diffraction pattern

(proportional to the number of incoming photons in a unit of time and a unit of area)

$$E_A = E_0 \cos(kx - \omega t + \phi_0)$$

Auto-correlation

$$\begin{aligned} \langle (E_A + E_B + E_C + E_D + E_E + \dots)^2 \rangle &= \langle E_A^2 \rangle + \langle E_B^2 \rangle + \langle E_C^2 \rangle + \langle E_D^2 \rangle + \langle E_E^2 \rangle \\ &\quad + \langle 2E_A E_B \rangle + \langle 2E_A E_C \rangle + \langle 2E_A E_D \rangle + \langle 2E_A E_E \rangle \\ &\quad + \langle 2E_B E_C \rangle + \langle 2E_B E_D \rangle + \langle 2E_B E_E \rangle \\ &\quad + \langle 2E_C E_D \rangle + \langle 2E_C E_E \rangle \\ &\quad + \langle 2E_D E_E \rangle \end{aligned}$$

Cross-correlation

4 slit-pairs at shortest separation

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**Uniform weighting** : higher noise, better angular resolution

**Natural weighting** : lowest achievable noise

1. We can apply weightings to individuals of complex visibilities.
2. This is implemented by modifying the sampling function to

$$R(u)T(u)D(u)S(u) \equiv \sum_{k=1}^M \delta(u - u_k) R(u_k) T(u_k) D(u_k)$$

$R(u_k)$  : Reliability weight     $T(u_k)$  : Taper     $D(u_k)$  : Density weighting

3. **Natural weighting**:  $T(u) = D(u) = 1$ . In most cases, it gives the best sensitivity and worst angular resolution. **Uniform weighting** considers  $T(u) = 1$  and  $D(u)$  is inversely proportional to the density of visibility. It normally gives the best angular resolution and worst sensitivity.