

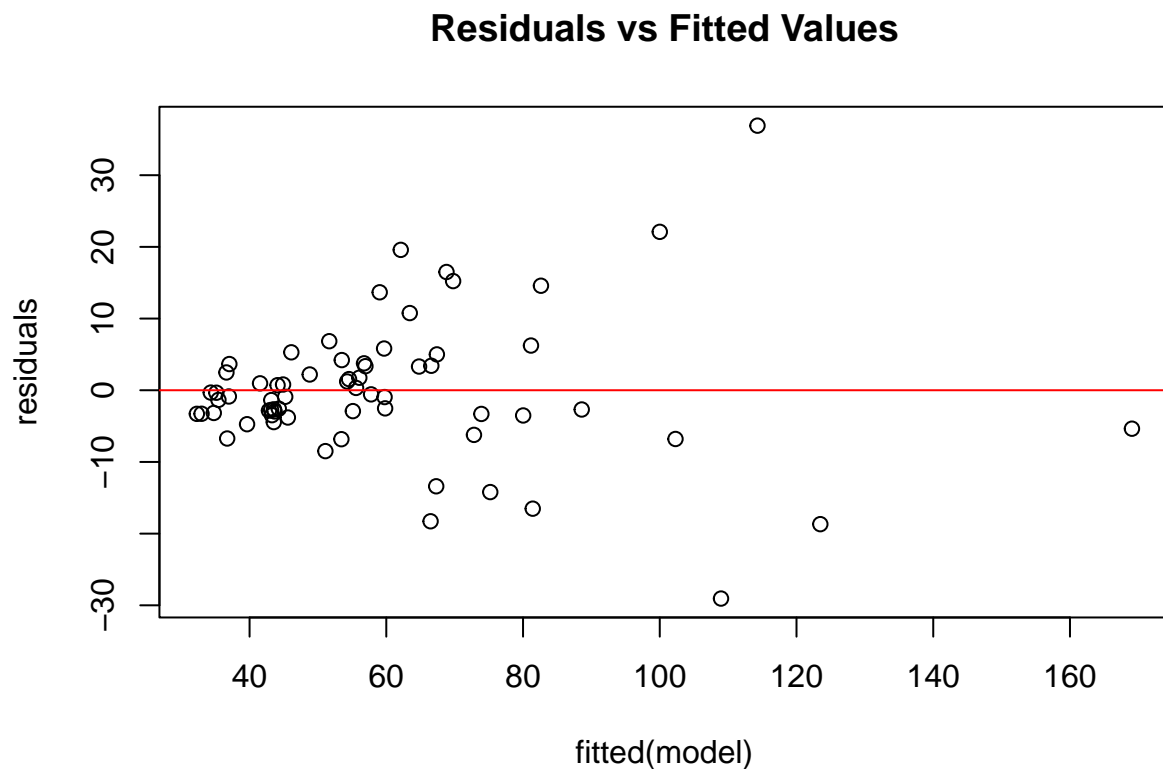
q5

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a)

```
data <- read.csv('employee-salaries.csv')
model <- lm(Y ~ factor(data$Degree) + X3 + X4, data=data)
residuals <- resid(model)
plot(fitted(model), residuals, main="Residuals vs Fitted Values")
abline(h = 0, col = "red")
```



The plot shows unconstant variance in the models residuals increase as  $\hat{Y}$  increases. b)

```
ordered_data <- data[order(fitted(model)), ]
library(ALSM)
```

```
## Loading required package: leaps
```

```
## Loading required package: SuppDists
```

```
## Loading required package: car
```

```
## Loading required package: carData
```

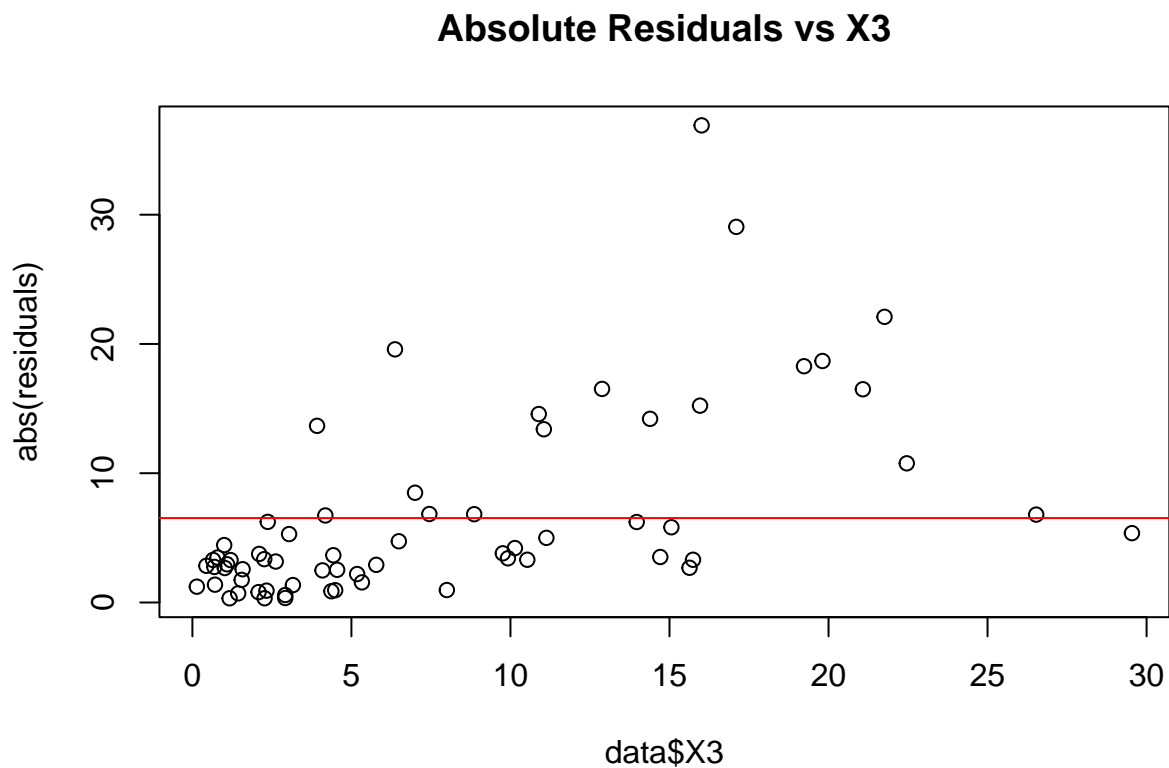
```
threshold <- data[order(fitted(model)), ][33,1]
group <- rep(1, length(data$Y))
group[fitted(model) <= threshold] <- 0
bftest(model, group, alpha=0.01)
```

```
##      t.value      P.Value alpha df
## [1,] 3.930298 0.000213524 0.01 63
```

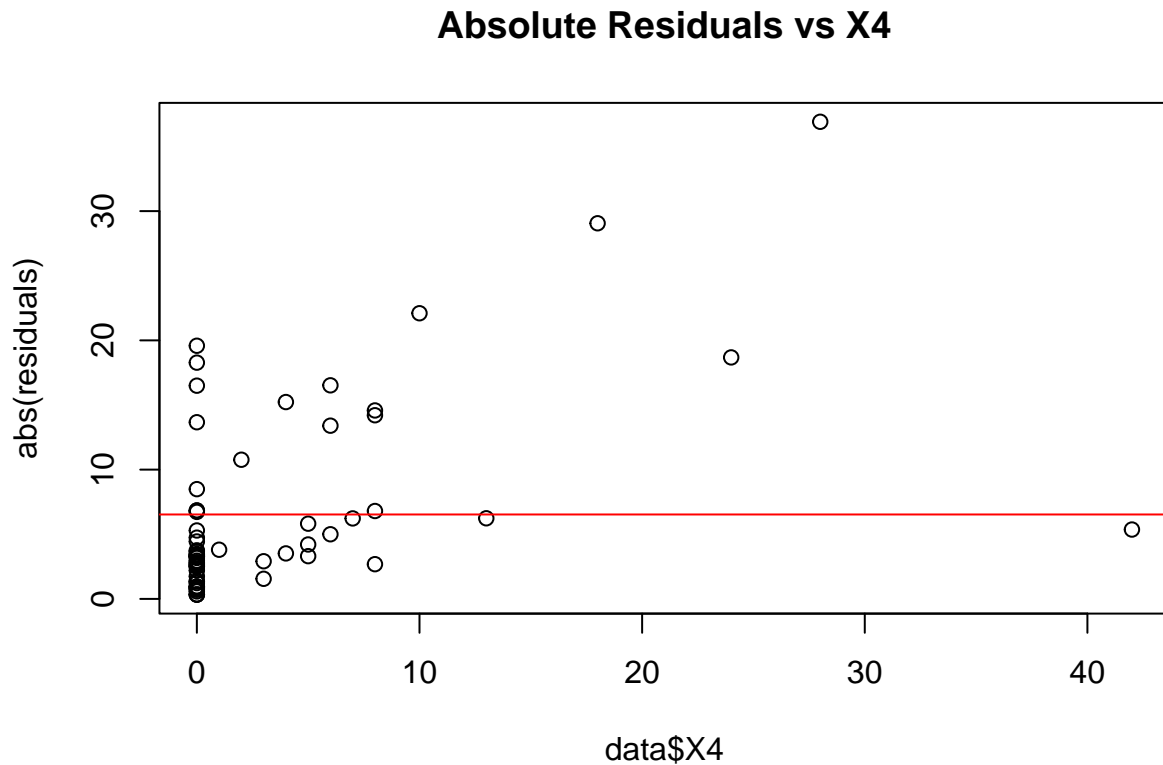
The p-value is way smaller than the significance interval alpha so we reject the null hypothesis of constant variances in favor of the alternative hypothesis of heteroscedasticity.

c)

```
plot(data$X3, abs(residuals), main="Absolute Residuals vs X3")
abline(h = mean(abs(residuals)), col = "red")
```



```
plot(data$X4, abs(residuals), main="Absolute Residuals vs X4")
abline(h = mean(abs(residuals)), col = "red")
```

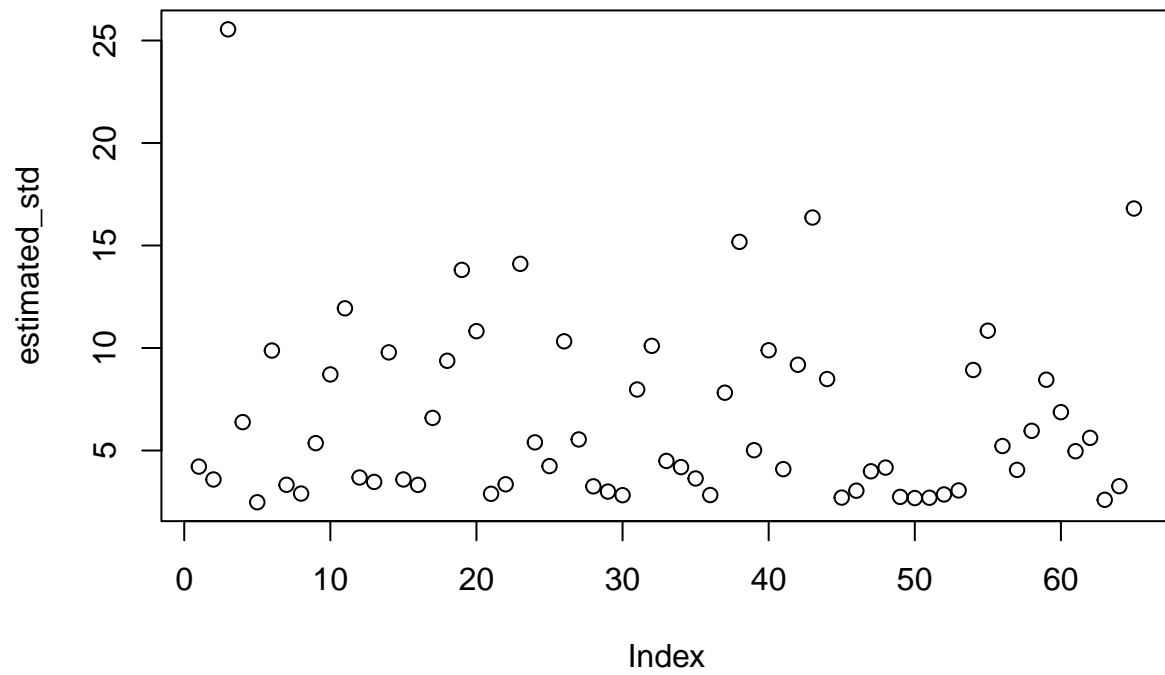


Both plots show increasing variance, ie standard deviation of error terms, as predictors increase since dots deviates from the mean of absolute residuals.

d)

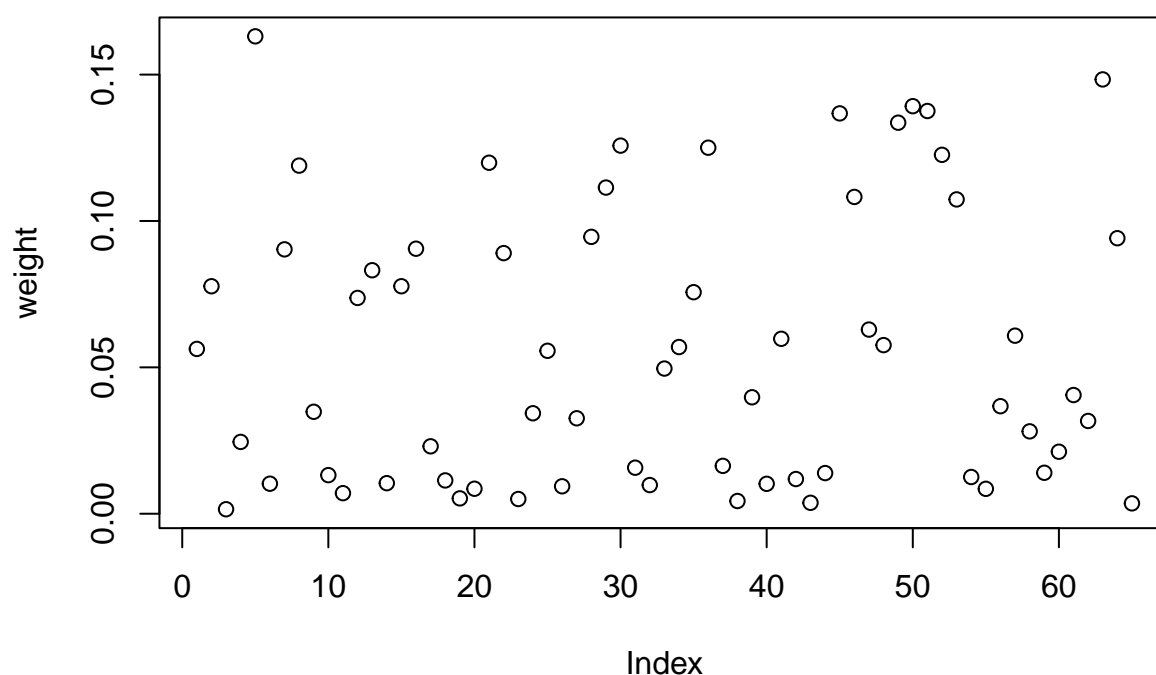
```
sd_model <- lm(abs(residuals) ~ data$X3 + data$X4)
estimated_std <- sd_model$fitted.values
weight <- 1 / (estimated_std)^2
plot(estimated_std, main = 'Estimated standard deviation')
```

## Estimated standard deviation



```
plot(weight, main = 'Weight for all observations')
```

## Weight for all observations



e)

```
wls_model <- lm(Y ~ factor(data$Degree) + X3 + X4, data=data, weights=weight)
wls_model$coefficients
```

```
##      (Intercept) factor(data$Degree)2 factor(data$Degree)3
##      29.425494      10.899639      26.684879
##           X3           X4
##      1.425330      1.723949
```

```
model$coefficients
```

```
##      (Intercept) factor(data$Degree)2 factor(data$Degree)3
##      31.471430      10.811952      22.630727
##           X3           X4
##      1.258124      1.852313
```

All coefficients are discrepant from the ols model except that X1 and intercept don't have a big difference.

f)

```
standard_error_ols <- summary(model)$coefficients[, "Std. Error"]
standard_error_wls <- summary(wls_model)$coefficients[, "Std. Error"]
data.frame(standard_error_ols, standard_error_wls)
```

	standard_error_ols	standard_error_wls
## (Intercept)	2.8690780	1.3616587
## factor(data\$Degree)2	3.2183072	1.4917607
## factor(data\$Degree)3	3.4846124	1.6686145
## X3	0.2272961	0.2002390
## X4	0.2276418	0.3206416

Standard errors of ols is larger than that of wls except in X4, indicating that wls has more accurate models than ols.

g)

```
sd_model_re <- lm(abs(residuals(wls_model)) ~ data$X3 + data$X4)
estimated_std_re <- sd_model_re$fitted.values
weight_re <- 1 / (estimated_std_re)^2
wls_model_re <- lm(Y ~ factor(data$Degree) + X3 + X4, data=data, weights=weight_re)
wls_model_re$coefficients
```

	(Intercept)	factor(data\$Degree)2	factor(data\$Degree)3
##	29.083233	11.007530	26.814222
##	X3	X4	
##	1.490446	1.692183	

In this iterated process, intercept and X2 show relatively large changes; all the rest are roughly the same, indicating that the coefficients are on the way to convergence. In this case, we should perform another iteration to converge even closer to the finest result.