we get the simplified problem

$$\begin{aligned} \text{minimize} \quad & z^\top C z + 2z^\top b \\ \text{subject to} \quad & z^\top z = s^2,\ z \in \mathbb{R}^m. \end{aligned}$$

Unfortunately, if $b \neq 0$, Proposition 23.10 is no longer applicable. It is still possible to find the minimum of the function $z^\top C z + 2z^\top b$ using Lagrange multipliers, but such a solution is too involved to be presented here. Interested readers will find a thorough discussion in Gander, Golub, and von Matt [75].

## 42.4 Summary

The main concepts and results of this chapter are listed below:

- Quadratic optimization problems; *quadratic functions*.
- Symmetric *positive definite* and *positive semidefinite* matrices.
- The *positive semidefinite cone ordering*.
- Existence of a global minimum when $A$ is symmetric positive definite.
- Constrained quadratic optimization problems.
- *Lagrange multipliers*; *Lagrangian*.
- *Primal* and *dual* problems.
- Quadratic optimization problems: the case of a symmetric invertible matrix $A$.
- Quadratic optimization problems: the general case of a symmetric matrix $A$.
- Adding linear constraints of the form $C^\top x = 0$.
- Adding affine constraints of the form $C^\top x = t$, with $t \neq 0$.
- Maximizing a quadratic function over the unit sphere.
- Maximizing a quadratic function over an ellipsoid.
- Maximizing a Hermitian quadratic form.
- Adding linear constraints of the form $C^\top x = 0$.
- Adding affine constraints of the form $N^\top x = t$, with $t \neq 0$.