into the system

$$\begin{pmatrix} A \\ -c \end{pmatrix} x \leq \begin{pmatrix} b \\ -(\mu + \epsilon) \end{pmatrix}.$$

By Proposition 47.4 (Farkas II), there is some linear form $(\lambda, z) \in (\mathbb{R}^{m+1})^*$ such that $\lambda \geq 0$, $z \geq 0$,

$$\begin{pmatrix} \lambda & z \end{pmatrix} \begin{pmatrix} A \\ -c \end{pmatrix} \geq 0_m^\top,$$

and

$$\begin{pmatrix} \lambda & z \end{pmatrix} \begin{pmatrix} b \\ -(\mu + \epsilon) \end{pmatrix} < 0,$$

which means that

$$\lambda A - zc \geq 0_m^\top, \quad \lambda b - z(\mu + \epsilon) < 0,$$

that is,

$$\begin{aligned} \lambda A &\geq zc \\ \lambda b &< z(\mu + \epsilon) \\ \lambda \geq 0, &\ \ z \geq 0. \end{aligned}$$

On the other hand, since $x^* \geq 0$ is an optimal solution of the system $Ax \leq b$, by Farkas II again (by taking the negation of the equivalence), since $\lambda A \geq 0$ (for the same λ as before), we must have

$$\lambda b \geq 0. \tag{$*_1$}$$

We claim that $z > 0$. Otherwise, since $z \geq 0$, we must have $z = 0$, but then

$$\lambda b < z(\mu + \epsilon)$$

implies

$$\lambda b < 0, \tag{$*_2$}$$

and since $\lambda b \geq 0$ by $(*_1)$, we have a contradiction. Consequently, we can divide by $z > 0$ without changing the direction of inequalities, and we obtain

$$\begin{aligned} \frac{\lambda}{z} A &\geq c \\ \frac{\lambda}{z} b &< \mu + \epsilon \\ \frac{\lambda}{z} &\geq 0, \end{aligned}$$

which shows that $v = \lambda/z$ is a feasible solution of the Dual Problem (D). However, weak duality (Proposition 47.6) implies that $cx^* = \mu \leq yb$ for any feasible solution $y \geq 0$ of the Dual Program (D), so (D) is bounded below and by Proposition 45.1 applied to the version of (D) written as a maximization problem, we conclude that (D) has some optimal solution.