Remark: The terminology *walk* or *chain* is often used instead of *path*, the word path being reserved to the case where the nodes v_i are all distinct, except that $v_0 = v_k$ when the path is closed.

The binary relation on $V \times V$ defined so that u and v are related iff there is a path from u to v is an equivalence relation whose equivalence classes are called the *connected components* of G.

The notion of incidence matrix for an undirected graph is not as useful as in the case of directed graphs

Definition 20.8. Given a graph $G = (V, E)$, with $V = \{v_1, \ldots, v_m\}$, if $E = \{e_1, \ldots, e_n\}$, then the *incidence matrix* $B(G)$ of G is the $m \times n$ matrix whose entries $b_{i\,j}$ are given by

$$b_{i\,j} = \begin{cases} +1 & \text{if } e_j = \{v_i, v_k\} \text{ for some } k \\ 0 & \text{otherwise.} \end{cases}$$

Unlike the case of directed graphs, the entries in the incidence matrix of a graph (undirected) are nonnegative. We usually write B instead of $B(G)$.

Definition 20.9. If $G = (V, E)$ is a directed or an undirected graph, given a node $u \in V$, any node $v \in V$ such that there is an edge (u, v) in the directed case or $\{u, v\}$ in the undirected case is called *adjacent to* u, and we often use the notation

$$u \sim v.$$

Observe that the binary relation $\sim$ is symmetric when G is an undirected graph, but in general it is not symmetric when G is a directed graph.

The notion of adjacency matrix is basically the same for directed or undirected graphs.

Definition 20.10. Given a directed or undirected graph $G = (V, E)$, with $V = \{v_1, \ldots, v_m\}$, the *adjacency matrix* $A(G)$ of G is the symmetric $m \times m$ matrix $(a_{i\,j})$ such that

(1) If G is directed, then

$$a_{i\,j} = \begin{cases} 1 & \text{if there is some edge } (v_i, v_j) \in E \text{ or some edge } (v_j, v_i) \in E \\ 0 & \text{otherwise.} \end{cases}$$

(2) Else if G is undirected, then

$$a_{i\,j} = \begin{cases} 1 & \text{if there is some edge } \{v_i, v_j\} \in E \\ 0 & \text{otherwise.} \end{cases}$$