Problem 8.4. Solve the system

$$\begin{pmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

using the LU-factorization of Example 8.1.

Problem 8.5. Apply **rref** to the matrix

$$A_2 = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 2 & 3 \\ -1 & 0 & 1 & -1 \\ -2 & -1 & 3 & 0 \end{pmatrix}.$$

Problem 8.6. Apply **rref** to the matrix

$$\begin{pmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \\ 16 & 25 & 36 & 49 \end{pmatrix}.$$

Problem 8.7. (1) Prove that the dimension of the subspace of 2×2 matrices A, such that the sum of the entries of every row is the same (say c_1) and the sum of entries of every column is the same (say c_2) is 2.

(2) Prove that the dimension of the subspace of 2×2 matrices A, such that the sum of the entries of every row is the same (say c_1), the sum of entries of every column is the same (say c_2), and $c_1 = c_2$ is also 2. Prove that every such matrix is of the form

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix},$$

and give a basis for this subspace.

(3) Prove that the dimension of the subspace of 3×3 matrices A, such that the sum of the entries of every row is the same (say c_1), the sum of entries of every column is the same (say c_2), and $c_1 = c_2$ is 5. Begin by showing that the above constraints are given by the set of equations

$$\begin{pmatrix} 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 1 & 1 & -1 & 0 & 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{31} \\ a_{32} \\ a_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$