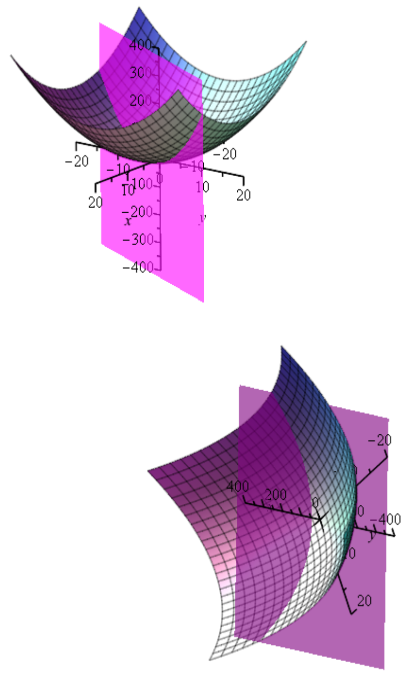

Figure 42.1: Two views of the constrained optimization problem $Q(x_1, x_2) = \frac{1}{2}\big(x_1^2 + x_2^2\big)$ subject to the constraint $2x_1 - x_2 = 5$. The minimum $(x_1, x_2) = (2, -1)$ is the the vertex of the parabolic curve formed the intersection of the magenta planar constraint with the bowl shaped surface.

$\lambda = (\lambda_1, \ldots, \lambda_n)$ called *Lagrange multipliers*, one for each constraint. We form the *Lagrangian*

$$L(x, \lambda) = Q(x) + \lambda^\top (B^\top x - f) = \frac{1}{2} x^\top A^{-1} x - (b - B\lambda)^\top x - \lambda^\top f.$$

We know from Theorem 40.2 that a necessary condition for our constrained optimization problem to have a solution is that $\nabla L(x, \lambda) = 0$. Since

$$\begin{aligned} \frac{\partial L}{\partial x}(x, \lambda) &= A^{-1}x - (b - B\lambda) \\ \frac{\partial L}{\partial \lambda}(x, \lambda) &= B^\top x - f, \end{aligned}$$

we obtain the system of linear equations

$$\begin{aligned} A^{-1}x + B\lambda &= b, \\ B^\top x &= f, \end{aligned}$$