(b) If there is some entry a_{1k} in the first row such that a_{11} does not divide a_{1k}, then pick such an entry (say, with the smallest index j such that $\sigma(a_{1j})$ is minimal), and divide a_{1k} by a_{11}; that is, find b_k and b_{1k} such that

$$a_{1k} = a_{11}b_k + b_{1k}, \quad \text{with} \quad \sigma(b_{1k}) < \sigma(a_{11}).$$

Subtract b_k times column 1 from column k and permute column k and column 1, to obtain a matrix of the form

$$M = \begin{pmatrix} b_{1k} & a_{k2} & \cdots & a_{kn} \\ b_{2k} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{mk} & a_{m2} & \cdots & a_{mn} \end{pmatrix}.$$

Go back to Step 2b.

(c) If a_{11} divides every (nonzero) entry a_{1j} for $j \geq 2$, say $a_{1j} = a_{11}b_j$, then subtract b_j times column 1 from column j for $j = 2, \ldots, n$; go to Step 3.

As in Step 2a, whenever we return to the beginning of Step 2b, we have $\sigma(b_{1k}) < \sigma(a_{11})$. Therefore, after a finite number of steps, we must exit Step 2b with a matrix in which all entries in row 1 but the first are zero.

Step 3. This step is reached only if the only nonzero entry in the first row is a_{11}.

(i) If

$$M = \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & Y & \\ 0 & & & \end{pmatrix}$$

go to Step 4.

(ii) If Step 2b ruined column 1 which now contains some nonzero entry below a_{11}, go back to Step 2a.

We perform a sequence of alternating steps between Step 2a and Step 2b. Because the σ-value of the $(1,1)$-entry strictly decreases whenever we reenter Step 2a and Step 2b, such a sequence must terminate with a matrix of the form

$$M = \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & Y & \\ 0 & & & \end{pmatrix}$$

Step 4. If a_{11} divides all entries in Y, stop.