

- Cauchy family, summable family.
- Bessel inequality.
- The Hilbert space  $\ell^2(K)$ .
- Parseval identity.
- Zorn's lemma.
- Riesz–Fischer theorem.
- Legendre polynomials.

## A.4 Problems

**Problem A.1.** Prove that the subspace consisting of sequences  $(z_k)_{k \in K}$  such that  $z_k = 0$  except perhaps for finitely many  $k$  is a dense subspace of  $\ell^2(K)$ .

**Problem A.2.** If  $V$  is any nonempty subset of  $E$ , prove that  $V^\perp$  is closed (even if  $V$  is not) and that  $V^{\perp\perp}$  is the closure of  $V$  (see the remarks following Proposition A.5).