2. Find if there exist some *optimal value* ω_0 of $\omega \in I$, so that

$$\rho(\mathcal{L}_{\omega_0}) = \inf_{\omega \in I} \rho(\mathcal{L}_\omega).$$

We will give partial answers to the above questions in the next section.

It is also possible to extend the methods of this section by using *block decompositions* of the form $A = D - E - F$, where D, E, and F consist of blocks, and D is an invertible block-diagonal matrix. See Figure 10.1.

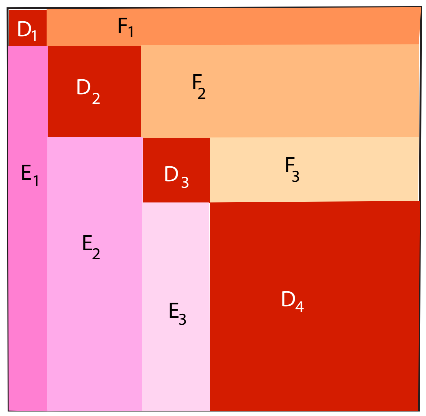

Figure 10.1: A schematic representation of a block decomposition $A = D - E - F$, where $D = \cup_{i=1}^4 D_i$, $E = \cup_{i=1}^3 E_i$, and $F = \cup_{i=1}^3 F_i$.

10.4 Convergence of the Methods of Gauss–Seidel and Relaxation

We begin with a general criterion for the convergence of an iterative method associated with a (complex) Hermitian positive definite matrix, $A = M - N$. Next we apply this result to the relaxation method.

Proposition 10.5. *Let A be any Hermitian positive definite matrix, written as*

$$A = M - N,$$

with M invertible. Then $M^* + N$ is Hermitian, and if it is positive definite, then

$$\rho(M^{-1}N) < 1,$$

so that the iterative method converges.