



Figure 50.19: In  $\mathbb{R}^2$  the convex hull of the  $u_i$ s, namely the blue hexagon, is separated from the convex hull of the  $v_j$ s, i.e. the red square, by the purple hyperplane (line) which is the perpendicular bisector to the blue line segment between  $u_i$  and  $v_1$ , where this blue line segment is the shortest distance between the two convex polygons.

denberghe; see [29], Section 5.7:

- (1) Introducing new variables and associated equality constraints.
- (2) Replacing the objective function with an increasing function of the the original function.
- (3) Making explicit constraints implicit, that is, incorporating them into the domain of the objective function.

We only give illustrations of (1) and (2) and refer the reader to Boyd and Vandenberghe [29] (Section 5.7) for more examples of these techniques.

Consider the unconstrained program:

$$\text{minimize } f(Ax + b),$$

where  $A$  is an  $m \times n$  matrix and  $b \in \mathbb{R}^m$ . While the conditions for a zero duality gap are satisfied, the Lagrangian is

$$L(x) = f(Ax + b),$$

so the dual function  $G$  is the constant function whose value is

$$G = \inf_{x \in \mathbb{R}^n} f(Ax + b),$$

which is not useful at all.