

Obviously, $I_{\lambda>0} = I_\lambda \cup K_\lambda$ and $I_{\mu>0} = I_\mu \cup K_\mu$, so $p_f \leq p_m$ and $q_f \leq q_m$. *Intuitively a blue point that fails the margin is on the wrong side of the blue margin and a red point that fails the margin is on the wrong side of the red margin.* The points in $I_{\lambda>0}$ not in K_λ are on the blue margin and the points in $I_{\mu>0}$ not in K_μ are on the red margin. There are $p - p_m$ points u_i classified correctly on the blue side and outside the δ -slab and there are $q - q_m$ points v_j classified correctly on the red side and outside the δ -slab.

It is easy to show that we have the following bounds on K :

$$\max \left\{ \frac{1}{2p_m}, \frac{1}{2q_m} \right\} \leq K \leq \min \left\{ \frac{1}{2p_f}, \frac{1}{2q_f} \right\}.$$

These inequalities restrict the choice of K quite heavily.

It will also be useful to understand how points are classified in terms of ϵ_i (or ξ_j).

- (1) If $\epsilon_i > 0$, then by complementary slackness $\lambda_i = K$, so the i th equation is active and by (2) above,

$$w^\top u_i - b - \delta = -\epsilon_i.$$

Since $\epsilon_i > 0$, the point u_i is within the open half space bounded by the blue margin hyperplane $H_{w,b+\delta}$ and containing the separating hyperplane $H_{w,b}$; if $\epsilon_i \leq \delta$, then u_i is classified correctly, and if $\epsilon_i > \delta$, then u_i is misclassified.

Similarly, if $\xi_j > 0$, then v_j is within the open half space bounded by the red margin hyperplane $H_{w,b-\delta}$ and containing the separating hyperplane $H_{w,b}$; if $\xi_j \leq \delta$, then v_j is classified correctly, and if $\xi_j > \delta$, then v_j is misclassified.

- (2) If $\epsilon_i = 0$, then the point u_i is correctly classified. If $\lambda_i = 0$, then by (3) above, u_i is in the closed half space on the blue side bounded by the blue margin hyperplane $H_{w,b+\delta}$. If $\lambda_i > 0$, then by (1) and (2) above, the point u_i is on the blue margin.

Similarly, if $\xi_j = 0$, then the point v_j is correctly classified. If $\mu_j = 0$, then v_j is in the closed half space on the red side bounded by the red margin hyperplane $H_{w,b-\delta}$, and if $\mu_j > 0$, then the point v_j is on the red margin.

It shown in Section 54.2 how the dual program is solved using ADMM from Section 52.6. If the primal problem is solvable, this yields solutions for λ and μ .

If the optimal value is 0, then $\gamma = 0$ and $X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0$, so in this case it is not possible to determine w . However, if the optimal value is > 0 , then once a solution for λ and μ is obtained, by $(*_w)$, we have

$$\gamma = \frac{1}{2} \left((\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \right)^{1/2}$$

$$w = \frac{1}{2\gamma} \left(\sum_{i=1}^p \lambda_i u_i - \sum_{j=1}^q \mu_j v_j \right),$$