Problem 18.4. Prove that if A is upper Hessenberg, then the matrices A_k obtained by applying the QR-algorithm are also upper Hessenberg.

Problem 18.5. Prove the *implicit Q theorem*. This theorem says that if A is reduced to upper Hessenberg form as $A = UHU^*$ and if H is unreduced ($h_{i+1i} \neq 0$ for $i = 1, \ldots, n-1$), then the columns of index $2, \ldots, n$ of U are determined by the first column of U up to sign;

Problem 18.6. Read Section 7.5 of Golub and Van Loan [80] and implement their version of the QR-algorithm with shifts.

Problem 18.7. If an Arnoldi iteration has a breakdown at stage n, that is, $h_{n+1} = 0$, then we found the first unreduced block of the Hessenberg matrix H. Prove that the eigenvalues of H_n are eigenvalues of A.

Problem 18.8. Prove Theorem 18.6.

Problem 18.9. Implement GRMES and test it on some linear systems.

Problem 18.10. State and prove versions of Proposition 18.5 and Theorem 18.6 for the Lanczos iteration.

Problem 18.11. Prove the results about the power iteration method stated in Section 18.7.

Problem 18.12. Prove the results about the inverse power iteration method stated in Section 18.7.

Problem 18.13. Implement and test the power iteration method and the inverse power iteration method.

Problem 18.14. Read Lecture 27 in Trefethen and Bau [176] and implement and test the Rayleigh quotient iteration method.