Because $\lambda_i \mu_i = 0$, the sets K_λ and K_μ are disjoint. Observe that from Definition 56.2 we have $p_f = |K_\lambda|$ and $q_f = |K_\mu|$. Then by $(*_\xi)$ and $(*_{\xi'})$, we have

$$\begin{aligned}\sum_{i=1}^{m}(\xi_i + \xi_i') &= \sum_{i \in K_\lambda} \xi_i + \sum_{j \in K_\mu} \xi_j' \\ &= \sum_{i \in K_\lambda} (w^\top x_i + b - y_i - \epsilon) + \sum_{j \in K_\mu} (-w^\top x_j - b + y_j - \epsilon) \\ &= w^\top \Bigg(\sum_{i \in K_\lambda} x_i - \sum_{j \in K_\mu} x_j\Bigg) - \sum_{i \in K_\lambda} y_i + \sum_{j \in K_\mu} y_j + (p_f - q_f)b - (p_f + q_f)\epsilon.\end{aligned}$$

The optimal value of the dual is given by

$$-\frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(P + \begin{pmatrix}\mathbf{1}_m\mathbf{1}_m^\top & -\mathbf{1}_m\mathbf{1}_m^\top \\ -\mathbf{1}_m\mathbf{1}_m^\top & \mathbf{1}_m\mathbf{1}_m^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} - q^\top\begin{pmatrix}\lambda \\ \mu\end{pmatrix},$$

with

$$q = \begin{pmatrix} y \\ -y \end{pmatrix}.$$

Expressing that the duality gap is zero we have

$$\begin{aligned}&\frac{1}{2}w^\top w + \frac{1}{2}b^2 + C\nu\epsilon + \frac{C}{m}\sum_{i=1}^{m}(\xi_i + \xi_i') \\ &\qquad = -\frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(P + \begin{pmatrix}\mathbf{1}_m\mathbf{1}_m^\top & -\mathbf{1}_m\mathbf{1}_m^\top \\ -\mathbf{1}_m\mathbf{1}_m^\top & \mathbf{1}_m\mathbf{1}_m^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} - q^\top\begin{pmatrix}\lambda \\ \mu\end{pmatrix},\end{aligned}$$

that is,

$$\begin{aligned}&\frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(P + \begin{pmatrix}\mathbf{1}_m\mathbf{1}_m^\top & -\mathbf{1}_m\mathbf{1}_m^\top \\ -\mathbf{1}_m\mathbf{1}_m^\top & \mathbf{1}_m\mathbf{1}_m^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} + C\nu\epsilon \\ &\quad + \frac{C}{m}\Bigg(w^\top\Bigg(\sum_{i \in K_\lambda} x_i - \sum_{j \in K_\mu} x_j\Bigg) - \sum_{i \in K_\lambda} y_i + \sum_{j \in K_\mu} y_j + (p_f - q_f)b - (p_f + q_f)\epsilon\Bigg) \\ &\qquad = -\frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(P + \begin{pmatrix}\mathbf{1}_m\mathbf{1}_m^\top & -\mathbf{1}_m\mathbf{1}_m^\top \\ -\mathbf{1}_m\mathbf{1}_m^\top & \mathbf{1}_m\mathbf{1}_m^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} - q^\top\begin{pmatrix}\lambda \\ \mu\end{pmatrix}.\end{aligned}$$

Solving for ϵ we get

$$\begin{aligned}C\left(\nu - \frac{p_f + q_f}{m}\right)\epsilon &= -\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(P + \begin{pmatrix}\mathbf{1}_m\mathbf{1}_m^\top & -\mathbf{1}_m\mathbf{1}_m^\top \\ -\mathbf{1}_m\mathbf{1}_m^\top & \mathbf{1}_m\mathbf{1}_m^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} - \begin{pmatrix}y^\top & -y^\top\end{pmatrix}\begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\ &\quad - \frac{C}{m}\Bigg(w^\top\Bigg(\sum_{i \in K_\lambda} x_i - \sum_{j \in K_\mu} x_j\Bigg) - \sum_{i \in K_\lambda} y_i + \sum_{j \in K_\mu} y_j + (p_f - q_f)b\Bigg),\end{aligned}$$