Understanding the complexity of linear programing, in particular of the simplex algorithm, is still ongoing. The interested reader is referred to Matousek and Gardner [123] (Chapter 5, Section 5.9) for some pointers.

In the next section we consider important theoretical criteria for determining whether a set of constraints $Ax \leq b$ and $x \geq 0$ has a solution or not.

46.6 Summary

The main concepts and results of this chapter are listed below:

- Degenerate and nondegenerate basic feasible solution.
- Pivoting step.
- Pivot rule.
- Cycling.
- Bland's rule, Dantzig's rule, steepest edge rule, random edge rule, largest increase rule, lexicographic rule.
- Phase I and Phase II of the simplex algorithm.
- eta matrix, eta factorization.
- Revised simplex method.
- Reduced cost.
- Full tableaux.
- The Hirsch conjecture.

46.7 Problems

Problem 46.1. In Section 46.2 prove that if Case (A) arises, then the basic feasible solution u is an optimal solution. Prove that if Case (B1) arises, then the linear program is unbounded. Prove that if Case (B3) arises, then (u^+, K^+) is a basic feasible solution.

Problem 46.2. In Section 46.2 prove that the following equivalences hold:

$$\begin{aligned}
&\text{Case (A)} \iff B = \emptyset, \qquad \text{Case (B)} \iff B \neq \emptyset\\
&\text{Case (B1)} \iff B_1 \neq \emptyset\\
&\text{Case (B2)} \iff B_2 \neq \emptyset\\
&\text{Case (B3)} \iff B_3 \neq \emptyset.
\end{aligned}$$