which yields

$$b = \frac{r_{32} - r_{23}}{4a}, \quad c = \frac{r_{13} - r_{31}}{4a}, \quad d = \frac{r_{21} - r_{12}}{4a}.$$

Case 2. $\mathrm{tr}(R) = -1$, or equivalently $\theta = \pi$. In this case $a = 0$. By equating $R + R^\top$ and $R_q + R_q^\top$, we get

$$\begin{aligned} 4bc &= r_{21} + r_{12} \\ 4bd &= r_{13} + r_{31} \\ 4cd &= r_{32} + r_{23}. \end{aligned}$$

By equating the diagonal terms of R and R_q, we also get

$$\begin{aligned} b^2 &= \frac{1 + r_{11}}{2} \\ c^2 &= \frac{1 + r_{22}}{2} \\ d^2 &= \frac{1 + r_{33}}{2}. \end{aligned}$$

Since $q \neq 0$ and $a = 0$, at least one of b, c, d is nonzero.

If $b \neq 0$, let

$$b = \frac{\sqrt{1 + r_{11}}}{\sqrt{2}},$$

and determine c, d using

$$\begin{aligned} 4bc &= r_{21} + r_{12} \\ 4bd &= r_{13} + r_{31}. \end{aligned}$$

If $c \neq 0$, let

$$c = \frac{\sqrt{1 + r_{22}}}{\sqrt{2}},$$

and determine b, d using

$$\begin{aligned} 4bc &= r_{21} + r_{12} \\ 4cd &= r_{32} + r_{23}. \end{aligned}$$

If $d \neq 0$, let

$$d = \frac{\sqrt{1 + r_{33}}}{\sqrt{2}},$$

and determine b, c using

$$\begin{aligned} 4bd &= r_{13} + r_{31} \\ 4cd &= r_{32} + r_{23}. \end{aligned}$$