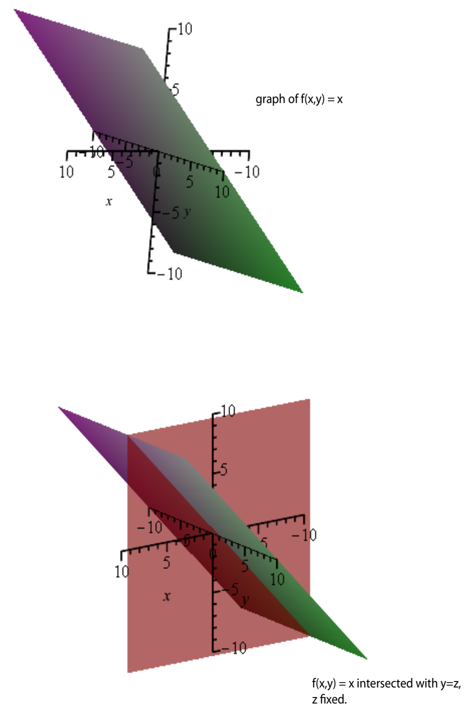

Figure 52.4: A graphical representation of the Example 52.5. This is an illustration of the x minimization step when z is held fixed. Since the intersection of the two planes is an unbounded line, we "see" that minimizing over x yields $-\infty$.

(4) Bertsekas discusses ADMM in [17], Sections 2.2 and 5.4. His formulation of ADMM is slightly different, namely

$$\begin{aligned} &\text{minimize} && f(x) + g(z) \\ &\text{subject to} && Ax = z. \end{aligned}$$

Bertsekas states a convergence result for this version of ADMM under the hypotheses that either $\mathrm{dom}(f)$ is compact or that $A^\top A$ is invertible, and that a saddle point exists; see Proposition 5.4.1. The proof is given in Bertsekas [20], Section 3.4, Proposition 4.2. It appears that the proof makes use of gradients, so it is not clear that it applies in the more general case where f and g are not differentiable.

(5) Versions of ADMM are discussed in Gabay [69] (Sections 4 and 5). They are more general than the version discussed here. Some convergence proofs are given, but because Gabay's framework is more general, it is not clear that they apply to our setting. Also, these proofs rely on earlier result by Lions and Mercier, which makes the comparison difficult.