terms of b and η. Let K_λ and K_μ be the sets of indices corresponding to points failing the margin,

$$\begin{aligned}
K_\lambda &= \{i \in \{1, \ldots, p\} \mid \lambda_i = K_s\} \\
K_\mu &= \{j \in \{1, \ldots, q\} \mid \mu_j = K_s\}.
\end{aligned}$$

By definition $p_f = |K_\lambda|, q_f = |K_\mu|$. Then for every $i \in K_\lambda$ we have

$$\epsilon_i = \eta + b - w^\top u_i$$

and for every $j \in K_\mu$ we have

$$\xi_j = \eta - b + w^\top v_j.$$

Using the above formulae we obtain

$$\begin{aligned}
\sum_{i=1}^p \epsilon_i + \sum_{j=1}^q \xi_j &= \sum_{i \in K_\lambda} \epsilon_i + \sum_{j \in K_\mu} \xi_j \\
&= \sum_{i \in K_\lambda} (\eta + b - w^\top u_i) + \sum_{j \in K_\mu} (\eta - b + w^\top v_j) \\
&= (p_f + q_f)\eta + (p_f - q_f)b + w^\top \left(\sum_{j \in K_\mu} v_j - \sum_{i \in K_\lambda} u_i \right)
\end{aligned}$$

Substituting this expression in $(*)$ we obtain

$$\begin{aligned}
(p+q)K_s \nu \eta &= K_s \left(\sum_{i=1}^p \epsilon_i + \sum_{j=1}^q \xi_j \right) + \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(X^\top X + \begin{pmatrix} \mathbf{1}_p \mathbf{1}_p^\top & -\mathbf{1}_p \mathbf{1}_q^\top \\ -\mathbf{1}_q \mathbf{1}_p^\top & \mathbf{1}_q \mathbf{1}_q^\top \end{pmatrix} \right) \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\
&= K_s \left((p_f + q_f)\eta + (p_f - q_f)b + w^\top \left(\sum_{j \in K_\mu} v_j - \sum_{i \in K_\lambda} u_i \right) \right) \\
&\quad + \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(X^\top X + \begin{pmatrix} \mathbf{1}_p \mathbf{1}_p^\top & -\mathbf{1}_p \mathbf{1}_q^\top \\ -\mathbf{1}_q \mathbf{1}_p^\top & \mathbf{1}_q \mathbf{1}_q^\top \end{pmatrix} \right) \begin{pmatrix} \lambda \\ \mu \end{pmatrix},
\end{aligned}$$

which yields

$$\begin{aligned}
((p+q)\nu - p_f - q_f)\eta &= (p_f - q_f)b + w^\top \left(\sum_{j \in K_\mu} v_j - \sum_{i \in K_\lambda} u_i \right) \\
&\qquad + \frac{1}{K_s} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(X^\top X + \begin{pmatrix} \mathbf{1}_p \mathbf{1}_p^\top & -\mathbf{1}_p \mathbf{1}_q^\top \\ -\mathbf{1}_q \mathbf{1}_p^\top & \mathbf{1}_q \mathbf{1}_q^\top \end{pmatrix} \right) \begin{pmatrix} \lambda \\ \mu \end{pmatrix}.
\end{aligned}$$

We show in Proposition 54.5 that $p_f + q_f \leq (p+q)\nu$, so if $\nu > (p_f + q_f)/(p+q)$, we can solve for η in terms of b, w, and λ, μ. But b and w are expressed in terms of λ, μ as

$$\begin{aligned}
w &= -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\
b &= -\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = -\mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu
\end{aligned}$$