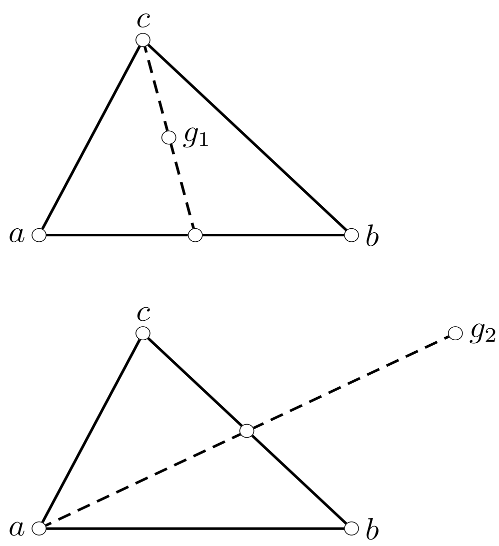

Figure 24.11: Barycenters, $g_1 = \frac{1}{4}a + \frac{1}{4}b + \frac{1}{2}c, \quad g_2 = -a + b + c$

since the sum on the left-hand side is obtained by expanding $(t + (1 - t))^3 = 1$ using the binomial formula. Thus,

$$(1-t)^3\, a + 3t(1-t)^2\, b + 3t^2(1-t)\, c + t^3\, d$$

is a well-defined affine combination. Then, we can define the curve $F\colon \mathbb{A} \to \mathbb{A}^2$ such that

$$F(t) = (1-t)^3\, a + 3t(1-t)^2\, b + 3t^2(1-t)\, c + t^3\, d.$$

Such a curve is called a *Bézier curve*, and (a, b, c, d) are called its *control points*. Note that the curve passes through a and d, but generally not through b and c. It can be sbown that any point $F(t)$ on the curve can be constructed using an algorithm performing affine interpolation steps (the *de Casteljau algorithm*).

24.5 Affine Subspaces

In linear algebra, a (linear) subspace can be characterized as a nonempty subset of a vector space closed under linear combinations. In affine spaces, the notion corresponding to the notion of (linear) subspace is the notion of affine subspace. It is natural to define an affine subspace as a subset of an affine space closed under affine combinations.

Definition 24.3. Given an affine space $\langle E, \overrightarrow{E}, +\rangle$, a subset V of E is an *affine subspace* (*of* $\langle E, \overrightarrow{E}, +\rangle$) if for every family of weighted points $((a_i, \lambda_i))_{i\in I}$ in V such that $\sum_{i\in I} \lambda_i = 1$, the barycenter $\sum_{i\in I} \lambda_i a_i$ belongs to V.