Since $q_1 \neq q_2$ and $q_1 \neq -q_2$, we have $0 < \Omega < \pi$, so we get

$$q_1^{-1}q_2 = \left[\cos\Omega,\ \sin\Omega\, \frac{(-\sin\theta\cos\varphi\, u + \cos\theta\sin\varphi\, v - \sin\theta\sin\varphi(u\times v)}{\sin\Omega}\right],$$

where the term multiplying $\sin\Omega$ is a unit vector because q_1 and q_2 are unit quaternions, so $q_1^{-1}q_2$ is also a unit quaternion. By $(*_{\log})$, we have

$$(q_1^{-1}q_2)^\lambda = \left[\cos\lambda\Omega,\ \sin\lambda\Omega\, \frac{(-\sin\theta\cos\varphi\, u + \cos\theta\sin\varphi\, v - \sin\theta\sin\varphi(u\times v)}{\sin\Omega}\right].$$

Next we need to compute $q_1(q_1^{-1}q_2)^\lambda$. The scalar part of this product is

$$\begin{aligned} s = \cos\theta\cos\lambda\Omega + \frac{\sin\lambda\Omega}{\sin\Omega}\sin^2\theta\cos\varphi(u\cdot u) - \frac{\sin\lambda\Omega}{\sin\Omega}\sin\theta\sin\varphi\cos\theta(u\cdot v) \\ + \frac{\sin\lambda\Omega}{\sin\Omega}\sin^2\theta\sin\varphi(u\cdot(u\times v)). \end{aligned}$$

Since $u\cdot(u\times v) = 0$, the last term is zero, and since $u\cdot u = 1$ and

$$\sin\theta\sin\varphi(u\cdot v) = \cos\Omega - \cos\theta\cos\varphi,$$

we get

$$\begin{aligned} s &= \cos\theta\cos\lambda\Omega + \frac{\sin\lambda\Omega}{\sin\Omega}\sin^2\theta\cos\varphi - \frac{\sin\lambda\Omega}{\sin\Omega}\cos\theta(\cos\Omega - \cos\theta\cos\varphi) \\ &= \cos\theta\cos\lambda\Omega + \frac{\sin\lambda\Omega}{\sin\Omega}(\sin^2\theta + \cos^2\theta)\cos\varphi - \frac{\sin\lambda\Omega}{\sin\Omega}\cos\theta\cos\Omega \\ &= \frac{(\cos\lambda\Omega\sin\Omega - \sin\lambda\Omega\cos\Omega)\cos\theta}{\sin\Omega} + \frac{\sin\lambda\Omega}{\sin\Omega}\cos\varphi \\ &= \frac{\sin(1-\lambda)\Omega}{\sin\Omega}\cos\theta + \frac{\sin\lambda\Omega}{\sin\Omega}\cos\varphi. \end{aligned}$$

The vector part of the product $q_1(q_1^{-1}q_2)^\lambda$ is given by

$$\begin{aligned} \nu = -\frac{\sin\lambda\Omega}{\sin\Omega}\cos\theta\sin\theta\cos\varphi\, u + \frac{\sin\lambda\Omega}{\sin\Omega}\cos^2\theta\sin\varphi\, v - \frac{\sin\lambda\Omega}{\sin\Omega}\cos\theta\sin\theta\sin\varphi(u\times v) \\ + \cos\lambda\Omega\sin\theta\, u - \frac{\sin\lambda\Omega}{\sin\Omega}\sin^2\theta\cos\varphi(u\times u) + \frac{\sin\lambda\Omega}{\sin\Omega}\cos\theta\sin\theta\sin\varphi(u\times v) \\ - \frac{\sin\lambda\Omega}{\sin\Omega}\sin^2\theta\sin\varphi(u\times(u\times v)). \end{aligned}$$

We have $u\times u = 0$, the two terms involving $u\times v$ cancel out,

$$u\times(u\times v) = (u\cdot v)u - (u\cdot u)v,$$