which, in view of (A3), yields u. The composition of the second with the first mapping is

$$b \mapsto \overrightarrow{ab} \mapsto a + \overrightarrow{ab},$$

which, in view of (A3), yields b. Thus, these compositions are the identity from $\overrightarrow{E}$ to $\overrightarrow{E}$ and the identity from E to E, and the mappings are both bijections.

When we identify E with $\overrightarrow{E}$ via the mapping $b \mapsto \overrightarrow{ab}$, we say that we consider E as the vector space obtained *by taking a as the origin in E*, and we denote it by E_a. Because E_a is a vector space, to be consistent with our notational conventions we should use the notation $\overrightarrow{E_a}$ (using an arrow), instead of E_a. However, for simplicity, we stick to the notation E_a.

Thus, an affine space $\langle E, \overrightarrow{E}, + \rangle$ is a way of defining a vector space structure on a set of points E, without making a commitment to a **fixed** origin in E. Nevertheless, as soon as we commit to an origin a in E, we can view E as the vector space E_a. However, we urge the reader to think of E as a physical set of points and of $\overrightarrow{E}$ as a set of forces acting on E, rather than reducing E to some isomorphic copy of $\mathbb{R}^n$. After all, points are points, and not vectors! For notational simplicity, we will often denote an affine space $\langle E, \overrightarrow{E}, + \rangle$ by $(E, \overrightarrow{E})$, or even by E. The vector space $\overrightarrow{E}$ is called the *vector space associated with E*.

One should be careful about the overloading of the addition symbol $+$. Addition is well-defined on vectors, as in $u + v$; the translate $a + u$ of a point $a \in E$ by a vector $u \in \overrightarrow{E}$ is also well-defined, but addition of points $a + b$ **does not make sense**. In this respect, the notation $b - a$ for the unique vector u such that $b = a + u$ is somewhat confusing, since it suggests that points can be subtracted (but not added!).

Any vector space $\overrightarrow{E}$ has an affine space structure specified by choosing $E = \overrightarrow{E}$, and letting $+$ be addition in the vector space $\overrightarrow{E}$. We will refer to the affine structure $\langle \overrightarrow{E}, \overrightarrow{E}, + \rangle$ on a vector space $\overrightarrow{E}$ as the *canonical (or natural) affine structure on $\overrightarrow{E}$*. In particular, the vector space $\mathbb{R}^n$ can be viewed as the affine space $\langle \mathbb{R}^n, \mathbb{R}^n, + \rangle$, denoted by $\mathbb{A}^n$. In general, if K is any field, the affine space $\langle K^n, K^n, + \rangle$ is denoted by $\mathbb{A}^n_K$. In order to distinguish between the double role played by members of $\mathbb{R}^n$, points and vectors, we will denote points by row vectors, and vectors by column vectors. Thus, the action of the vector space $\mathbb{R}^n$ over the set $\mathbb{R}^n$ simply viewed as a set of points is given by

$$(a_1, \ldots, a_n) + \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} = (a_1 + u_1, \ldots, a_n + u_n).$$

We will also use the convention that if $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$, then the column vector associated with x is denoted by $\mathbf{x}$ (in boldface notation). Abusing the notation slightly, if $a \in \mathbb{R}^n$ is a point, we also write $a \in \mathbb{A}^n$. The affine space $\mathbb{A}^n$ is called the *real affine space of dimension n*. In most cases, we will consider $n = 1, 2, 3$.