Then we have

$$\begin{aligned}\begin{pmatrix}-I_n & I_n & X^\top & -X^\top\end{pmatrix}^\top \begin{pmatrix}-I_n & I_n & X^\top & -X^\top\end{pmatrix} &= \begin{pmatrix}-I_n \\ I_n \\ X \\ -X\end{pmatrix}\begin{pmatrix}-I_n & I_n & X^\top & -X^\top\end{pmatrix} \\ &= \begin{pmatrix} I_n & -I_n & -X^\top & X^\top \\ -I_n & I_n & X^\top & -X^\top \\ -X & X & XX^\top & -XX^\top \\ X & -X & -XX^\top & XX^\top \end{pmatrix}.\end{aligned}$$

If we define the symmetric positive semidefinite $2(n+m)\times 2(n+m)$ matrix Q as

$$Q = \begin{pmatrix} I_n & -I_n & -X^\top & X^\top \\ -I_n & I_n & X^\top & -X^\top \\ -X & X & XX^\top & -XX^\top \\ X & -X & -XX^\top & XX^\top \end{pmatrix},$$

then

$$\frac{1}{2}w^\top w = \frac{1}{2}\begin{pmatrix}\beta_+^\top & \beta_-^\top & \mu_+^\top & \mu_-^\top\end{pmatrix} Q \begin{pmatrix}\beta_+ \\ \beta_- \\ \mu_+ \\ \mu_-\end{pmatrix}.$$

As a consequence, using $(*_w)$ and the fact that $\xi = K\mu$, we find that the dual function is given by

$$\begin{aligned} G(\mu, \beta_+, \beta_-) &= \frac{1}{2}\xi^\top\xi - \xi^\top\lambda + \lambda^\top y + w^\top(\alpha_+ - \alpha_- - X^\top\lambda) + \frac{1}{2}Kw^\top w \\ &= \frac{1}{2}\xi^\top\xi - K\xi^\top\mu + K\mu^\top y + Kw^\top(\beta_+ - \beta_- - X^\top\mu) + \frac{1}{2}Kw^\top w \\ &= \frac{1}{2}K^2\mu^\top\mu - K^2\mu^\top\mu + Ky^\top\mu - Kw^\top w + \frac{1}{2}Kw^\top w \\ &= -\frac{1}{2}K^2\mu^\top\mu - \frac{1}{2}Kw^\top w + Ky^\top\mu. \end{aligned}$$

But

$$\mu = \begin{pmatrix} I_m & -I_m\end{pmatrix}\begin{pmatrix}\mu_+ \\ \mu_-\end{pmatrix},$$

so

$$\frac{1}{2}\mu^\top\mu = \frac{1}{2}\begin{pmatrix}\mu_+^\top & \mu_-^\top\end{pmatrix}\begin{pmatrix} I_m & -I_m \\ -I_m & I_m\end{pmatrix}\begin{pmatrix}\mu_+ \\ \mu_-\end{pmatrix},$$