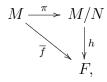
we have

$$\overline{f}((u_1, \dots, u_i + v_i, \dots, u_n) - (u_1, \dots, u_i, \dots, u_n) - (u_1, \dots, v_i, \dots, u_n))
= f(u_1, \dots, u_i + v_i, \dots, u_n) - f(u_1, \dots, u_i, \dots, u_n) - f(u_1, \dots, v_i, \dots, u_n)
= f(u_1, \dots, u_i, \dots, u_n) + f(u_1, \dots, v_i, \dots, u_n) - f(u_1, \dots, u_i, \dots, u_n)
- f(u_1, \dots, v_i, \dots, u_n)
= 0.$$

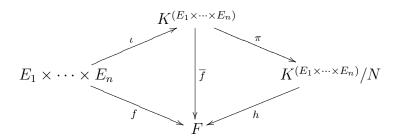
But then, $\overline{f}: M \to F$ factors through M/N, which means that there is a unique linear map $h: M/N \to F$ such that $\overline{f} = h \circ \pi$ making the following diagram commute



by defining $h([z]) = \overline{f}(z)$ for every $z \in M$, where [z] denotes the equivalence class in M/N of $z \in M$. Indeed, the fact that \overline{f} vanishes on N insures that h is well defined on M/N, and it is clearly linear by definition. Since $f = \overline{f} \circ \iota$, from the equation $\overline{f} = h \circ \pi$, by composing on the right with ι , we obtain

$$f = \overline{f} \circ \iota = h \circ \pi \circ \iota = h \circ \varphi,$$

as in the following commutative diagram.



We now prove the uniqueness of h. For any linear map $f_{\otimes} : E_1 \otimes \cdots \otimes E_n \to F$ such that $f = f_{\otimes} \circ \varphi$, since the vectors $u_1 \otimes \cdots \otimes u_n$ generate $E_1 \otimes \cdots \otimes E_n$ and since $\varphi(u_1, \ldots, u_n) = u_1 \otimes \cdots \otimes u_n$, the map f_{\otimes} is uniquely defined by

$$f_{\otimes}(u_1\otimes\cdots\otimes u_n)=f(u_1,\ldots,u_n)$$

Since $f = h \circ \varphi$, the map h is unique, and we let $f_{\otimes} = h$.

The map φ from $E_1 \times \cdots \times E_n$ to $E_1 \otimes \cdots \otimes E_n$ is often denoted by ι_{\otimes} , so that

$$\iota_{\otimes}(u_1,\ldots,u_n)=u_1\otimes\cdots\otimes u_n.$$