In order to eliminate x from the second and third row, we subtract the first row from the second and we subtract twice the first row from the third:

$$\begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ & & & & 2z & = & 0 \\ & & 3y & + & 6z & = & -1. \end{array}$$

Now the trouble is that y does not occur in the second row; so, we can't eliminate y from the third row by adding or subtracting a multiple of the second row to it. The remedy is simple: Permute the second and the third row! We get the system:

$$\begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ & & 3y & + & 6z & = & -1 \\ & & & & 2z & = & 0, \end{array}$$

which is already in triangular form. Another example where some permutations are needed is:

$$\begin{array}{ccccccc} & & & & z & = & 1 \\ -2x & + & 7y & + & 2z & = & 1 \\ 4x & - & 6y & & & = & -1. \end{array}$$

First we permute the first and the second row, obtaining

$$\begin{array}{ccccccc} -2x & + & 7y & + & 2z & = & 1 \\ & & & & z & = & 1 \\ 4x & - & 6y & & & = & -1, \end{array}$$

and then we add twice the first row to the third, obtaining:

$$\begin{array}{ccccccc} -2x & + & 7y & + & 2z & = & 1 \\ & & & & z & = & 1 \\ & & 8y & + & 4z & = & 1. \end{array}$$

Again we permute the second and the third row, getting

$$\begin{array}{ccccccc} -2x & + & 7y & + & 2z & = & 1 \\ & & 8y & + & 4z & = & 1 \\ & & & & z & = & 1, \end{array}$$

an upper-triangular system. Of course, in this example, z is already solved and we could have eliminated it first, but for the general method, we need to proceed in a systematic fashion.

We now describe the method of *Gaussian elimination* applied to a linear system $Ax = b$, where A is assumed to be invertible. We use the variable k to keep track of the stages of elimination. Initially, $k = 1$.