

Problem 45.7. The n -dimensional *hypercube* is the subset C_n of \mathbb{R}^n given by

$$C_n = \{(x_1, \dots, x_n) \in \mathbb{R}^n \mid |x_i| \leq 1, 1 \leq i \leq n\}.$$

(1) Prove that C_n is convex and that it is the convex hull of the 2^n vectors $(\pm 1, \dots, \pm 1)$, $i = 1, \dots, n$.

(2) Prove that C_n is the intersection of $2n$ half spaces and determine the hyperplanes defining these half-spaces.

Remark: The volume of C_n is 2^n .