Consequently, if we set $\nabla L_{w,\epsilon,b,\xi,\xi'} = 0$, we obtain the equations

$$\begin{aligned} w &= \sum_{i=1}^m (\mu_i - \lambda_i) x_i = X^\top (\mu - \lambda), \qquad (*_w) \\ C\nu - \gamma - \sum_{i=1}^m (\lambda_i + \mu_i) &= 0 \\ \sum_{i=1}^m (\lambda_i - \mu_i) &= 0 \\ \frac{C}{m} - \lambda - \alpha = 0, &\quad \frac{C}{m} - \mu - \beta = 0. \end{aligned}$$

Substituting the above equations in the second expression for the Lagrangian, we find that the dual function G is independent of the variables γ, α, β and is given by

$$G(\lambda, \mu) = -\frac{1}{2} \sum_{i,j=1}^m (\lambda_i - \mu_i)(\lambda_j - \mu_j) x_i^\top x_j - \sum_{i=1}^m (\lambda_i - \mu_i) y_i$$

if

$$\begin{aligned} \sum_{i=1}^m \lambda_i - \sum_{i=1}^m \mu_i &= 0 \\ \sum_{i=1}^m \lambda_i + \sum_{i=1}^m \mu_i + \gamma &= C\nu \\ \lambda + \alpha = \frac{C}{m}, &\quad \mu + \beta = \frac{C}{m}, \end{aligned}$$

and $-\infty$ otherwise.

The dual program is obtained by maximizing $G(\alpha, \mu)$ or equivalently by minimizing $-G(\alpha, \mu)$, over $\alpha, \mu \in \mathbb{R}^m_+$. Taking into account the fact that $\alpha, \beta \geq 0$ and $\gamma \geq 0$, we obtain the following dual program:

Dual Program for ν-SV Regression:

$$\text{minimize} \quad \frac{1}{2} \sum_{i,j=1}^m (\lambda_i - \mu_i)(\lambda_j - \mu_j) x_i^\top x_j + \sum_{i=1}^m (\lambda_i - \mu_i) y_i$$

subject to

$$\begin{aligned} &\sum_{i=1}^m \lambda_i - \sum_{i=1}^m \mu_i = 0 \\ &\sum_{i=1}^m \lambda_i + \sum_{i=1}^m \mu_i \leq C\nu \\ &0 \leq \lambda_i \leq \frac{C}{m}, \quad 0 \leq \mu_i \leq \frac{C}{m}, \quad i = 1, \ldots, m, \end{aligned}$$