

On the other hand, with respect to the dual basis  $(e_1^*, \dots, e_n^*)$  of  $E^*$ , the linear form  $f$  is represented by the column vector

$$\begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{pmatrix}.$$

**Remark:** In many texts using tensors, vectors are often indexed with lower indices. If so, it is more convenient to write the coordinates of a vector  $x$  over the basis  $(u_1, \dots, u_n)$  as  $(x^i)$ , using an upper index, so that

$$x = \sum_{i=1}^n x^i u_i,$$

and in a change of basis, we have

$$v_j = \sum_{i=1}^n a_j^i u_i$$

and

$$x^i = \sum_{j=1}^n a_j^i x'^j.$$

Dually, linear forms are indexed with upper indices. Then it is more convenient to write the coordinates of a covector  $\varphi^*$  over the dual basis  $(u^{*1}, \dots, u^{*n})$  as  $(\varphi_i)$ , using a lower index, so that

$$\varphi^* = \sum_{i=1}^n \varphi_i u^{*i}$$

and in a change of basis, we have

$$u^{*i} = \sum_{j=1}^n a_j^i v^{*j}$$

and

$$\varphi'_j = \sum_{i=1}^n a_j^i \varphi_i.$$

With these conventions, the index of summation appears once in upper position and once in lower position, and the summation sign can be safely omitted, a trick due to *Einstein*. For example, we can write

$$\varphi'_j = a_j^i \varphi_i$$

as an abbreviation for

$$\varphi'_j = \sum_{i=1}^n a_j^i \varphi_i.$$