for $\mathcal{E}$, a polynomial curve of degree m is a map $F\colon \mathbb{A} \to \mathcal{E}$ such that

$$F(t) = a_0 + F_1(t)e_1 + \cdots + F_n(t)e_n,$$

for all $t \in \mathbb{A}$, where $F_1(t), \ldots, F_n(t)$ are polynomials of degree at most m.

Although many curves can be defined, it is somewhat embarassing that a circle cannot be defined in such a way. In fact, many interesting curves cannot be defined this way, for example, ellipses and hyperbolas. A rather simple way to extend the class of curves defined parametrically is to allow rational functions instead of polynomials. A *parametric rational curve* of degree m is a function $F\colon \mathbb{A} \to \mathcal{E}$ such that

$$F(t) = a_0 + \frac{F_1(t)}{F_{n+1}(t)}e_1 + \cdots + \frac{F_n(t)}{F_{n+1}(t)}e_n,$$

for all $t \in \mathbb{A}$, where $F_1(t), \ldots, F_n(t), F_{n+1}(t)$ are polynomials of degree at most m. For example, a circle in $\mathbb{A}^2$ can be defined by the rational map

$$F(t) = a_0 + \frac{1-t^2}{1+t^2}e_1 + \frac{2t}{1+t^2}e_2.$$

In terms of coordinates, the above curve is given by

$$\begin{aligned} x &= \frac{1-t^2}{1+t^2} \\ y &= \frac{2t}{1+t^2}, \end{aligned}$$

and it is easily checked that $x^2 + y^2 = 1$. Note that the point $(-1, 0)$ is not achieved for any finite value of t, but it is for $t = \infty$.

In the above example, the denominator $F_3(t) = 1 + t^2$ never takes the value 0 when t ranges over $\mathbb{A}$, but consider the following curve in $\mathbb{A}^2$:

$$G(t) = a_0 + \frac{t^2}{t}e_1 + \frac{1}{t}e_2.$$

Observe that $G(0)$ is undefined. In terms of coordinates, the above curve is given by

$$\begin{aligned} x &= \frac{t^2}{t} = t \\ y &= \frac{1}{t}, \end{aligned}$$

so we have $y = 1/x$. The curve defined above is a hyperbola, and for t close to 0, the point on the curve goes toward infinity in one of the two asymptotic directions.