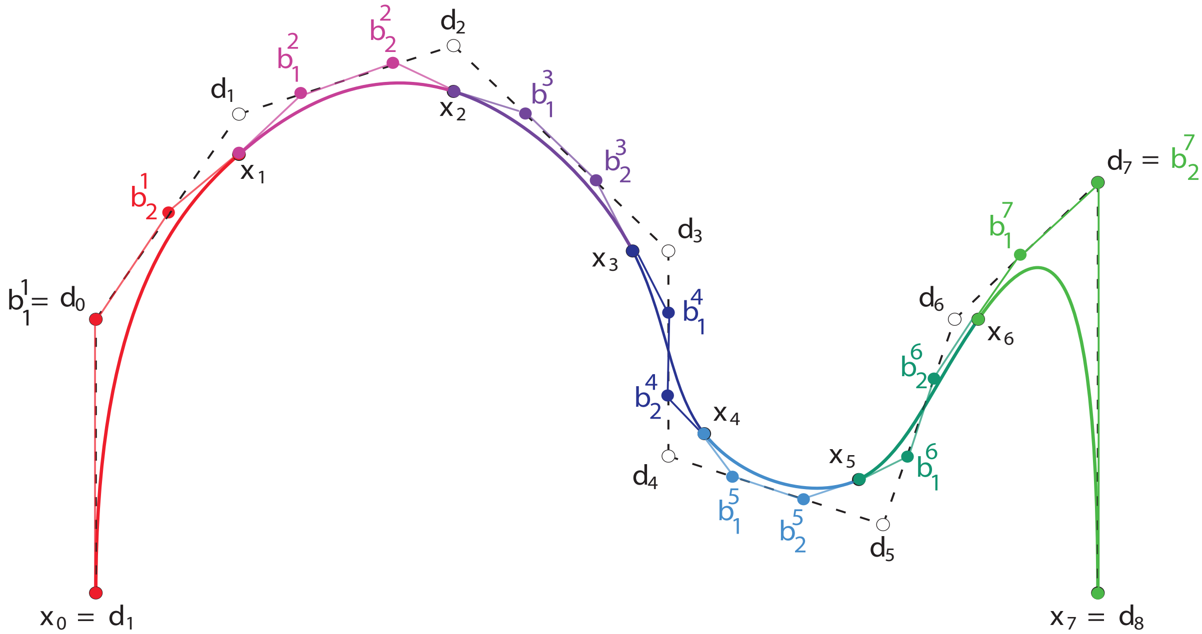


Figure 8.5: A $C^2$ cubic interpolation of $x_0, x_1,\ x_2, x_3,\ x_4, x_5,\ x_6, x_7$ with associated color coded Bézier cubics.

(2) One does not solve (large) linear systems by computing determinants (using Cramer's formulae) since this method requires a number of additions (resp. multiplications) proportional to $(n+1)!$ (resp. $(n+2)!$).

The key idea on which most direct methods (as opposed to iterative methods, that look for an approximation of the solution) are based is that if $A$ is an upper-triangular matrix, which means that $a_{ij} = 0$ for $1 \leq j < i \leq n$ (resp. lower-triangular, which means that $a_{ij} = 0$ for $1 \leq i < j \leq n$), then computing the solution $x$ is trivial. Indeed, say $A$ is an upper-triangular matrix

$$A = \begin{pmatrix} a_{1\,1} & a_{1\,2} & \cdots & a_{1\,n-2} & a_{1\,n-1} & a_{1\,n} \\ 0 & a_{2\,2} & \cdots & a_{2\,n-2} & a_{2\,n-1} & a_{2\,n} \\ 0 & 0 & \ddots & \vdots & \vdots & \vdots \\ & & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & a_{n-1\,n-1} & a_{n-1\,n} \\ 0 & 0 & \cdots & 0 & 0 & a_{n\,n} \end{pmatrix}.$$

Then $\det(A) = a_{1\,1}a_{2\,2}\cdots a_{n\,n} \neq 0$, which implies that $a_{i\,i} \neq 0$ for $i = 1, \ldots, n$, and we can solve the system $Ax = b$ from bottom-up by *back-substitution*. That is, first we compute