Program ν-SV Regression Version 2

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}w^\top w + \frac{1}{2}b^2 + C\left(\nu\epsilon + \frac{1}{m}\sum_{i=1}^m(\xi_i + \xi_i')\right)\\
&\text{subject to}\\
&\qquad w^\top x_i + b - y_i \leq \epsilon + \xi_i, \quad \xi_i \geq 0 \qquad i = 1, \ldots, m\\
&\qquad - w^\top x_i - b + y_i \leq \epsilon + \xi_i', \quad \xi_i' \geq 0 \qquad i = 1, \ldots, m,
\end{aligned}$$

minimizing over the variables w, b, ϵ, ξ, and ξ'. The constraint $\epsilon \geq 0$ is omitted since the problem has no solution if $\epsilon < 0$.

We leave it as an exercise to show that the new Lagrangian is

$$\begin{aligned}
L(w, b, \lambda, \mu, \xi, \xi', \epsilon, \alpha, \beta) &= \frac{1}{2}w^\top w + w^\top\left(\sum_{i=1}^m(\lambda_i - \mu_i)x_i\right)\\
&+ \epsilon\left(C\nu - \sum_{i=1}^m(\lambda_i + \mu_i)\right) + \sum_{i=1}^m \xi_i\left(\frac{C}{m} - \lambda_i - \alpha_i\right)\\
&+ \sum_{i=1}^m \xi_i'\left(\frac{C}{m} - \mu_i - \beta_i\right) + \frac{1}{2}b^2 + b\left(\sum_{i=1}^m(\lambda_i - \mu_i)\right) - \sum_{i=1}^m(\lambda_i - \mu_i)y_i.
\end{aligned}$$

If we set the Laplacian $\nabla L_{w,\epsilon,b,\xi,\xi'}$ to zero we obtain the equations

$$\begin{aligned}
w &= \sum_{i=1}^m(\mu_i - \lambda_i)x_i = X^\top(\mu - \lambda) \qquad (*_w)\\
C\nu - \sum_{i=1}^m(\lambda_i + \mu_i) &= 0\\
b + \sum_{i=1}^m(\lambda_i - \mu_i) &= 0\\
\frac{C}{m} - \lambda - \alpha = 0, &\quad \frac{C}{m} - \mu - \beta = 0.
\end{aligned}$$

We obtain the new equation

$$b = -\sum_{i=1}^m(\lambda_i - \mu_i) = -(\mathbf{1}_m^\top\lambda - \mathbf{1}_m^\top\mu) \qquad (*_b)$$

determining b, which replaces the equation

$$\sum_{i=1}^m \lambda_i - \sum_{i=1}^m \mu_i = 0.$$