If we express the coordinates of the vectors p_i and q_i over the canonical basis as

$$p_i = (p_i^1, \dots, p_i^n, p_i^{n+1}), \qquad q_i = (q_i^1, \dots, q_i^n, q_i^{n+1}), \quad i = 1, \dots, n+2,$$

then we have the following result.

Proposition 26.11. *With respect to the canonical basis $\mathcal{E} = (e_1, \dots, e_{n+1})$, the matrix $A_{\mathcal{E}}$ of the unique homography h of $\mathbb{P}(E)$ where E is a K-vector space of dimension $n+1$, mapping the projective frame $([p_1], \dots, [p_{n+1}], [p_{n+2}])$ to the projective frame $[([q_1], \dots, [q_{n+1}], [q_{n+2}])$ is given by*

$$A_{\mathcal{E}} = \begin{pmatrix} q_1^1 & \cdots & q_n^1 & q_{n+1}^1 \\ \vdots & \ddots & \vdots & \vdots \\ q_1^n & \cdots & q_n^n & q_{n+1}^n \\ q_1^{n+1} & \cdots & q_n^{n+1} & q_{n+1}^{n+1} \end{pmatrix} \begin{pmatrix} \frac{\lambda_1}{\alpha_1} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \frac{\lambda_n}{\alpha_n} & 0 \\ 0 & \cdots & 0 & \frac{\lambda_{n+1}}{\alpha_{n+1}} \end{pmatrix} \begin{pmatrix} p_1^1 & \cdots & p_n^1 & p_{n+1}^1 \\ \vdots & \ddots & \vdots & \vdots \\ p_1^n & \cdots & p_n^n & p_{n+1}^n \\ p_1^{n+1} & \cdots & p_n^{n+1} & p_{n+1}^{n+1} \end{pmatrix}^{-1},$$

where $(\alpha_1, \dots, \alpha_{n+1})$ and $(\lambda_1, \dots, \lambda_{n+1})$ are the solutions of the systems

$$\begin{pmatrix} p_1^1 & \cdots & p_n^1 & p_{n+1}^1 \\ \vdots & \ddots & \vdots & \vdots \\ p_1^n & \cdots & p_n^n & p_{n+1}^n \\ p_1^{n+1} & \cdots & p_n^{n+1} & p_{n+1}^{n+1} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \\ \alpha_{n+1} \end{pmatrix} = \begin{pmatrix} p_{n+2}^1 \\ \vdots \\ p_{n+2}^n \\ p_{n+2}^{n+1} \end{pmatrix}$$

and

$$\begin{pmatrix} q_1^1 & \cdots & q_n^1 & q_{n+1}^1 \\ \vdots & \ddots & \vdots & \vdots \\ q_1^n & \cdots & q_n^n & q_{n+1}^n \\ q_1^{n+1} & \cdots & q_n^{n+1} & q_{n+1}^{n+1} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \lambda_{n+1} \end{pmatrix} = \begin{pmatrix} q_{n+2}^1 \\ \vdots \\ q_{n+2}^n \\ q_{n+2}^{n+1} \end{pmatrix}.$$

We now consider the special case where the points $([p_1], [p_2], [p_3], [p_4])$ belong to the affine patch of $\mathbb{RP}^2$ corresponding to the plane H of equation $z = 1$. In this case, we may identify $[p_i]$ with p_i, which has coordinates $(p^i, p_i^y, 1)$ with respect to the canonical basis (the p_is are *not* points at infinity; points at infinity are of of form $(x, y, 0)$). Then, the barycentric coordinates $\alpha_1, \alpha_2, \alpha_3$ of p_4 are solutions of the systems

$$\begin{pmatrix} p_1^x & p_2^x & p_3^x \\ p_1^y & p_2^y & p_3^y \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} p_4^x \\ p_4^y \\ 1 \end{pmatrix}.$$

By Proposition 26.9, we obtain the following result.

Proposition 26.12. *With respect to the canonical basis $\mathcal{E} = (e_1, e_2, e_3)$, the matrix $A_{\mathcal{E}}$ of the unique homography h of $\mathbb{RP}^2$ mapping (p_1, p_2, p_4, p_4), points of the affine plane $z = 1$, to $[([q_1], [q_2], [q_3], [q_4])$ is given by*

$$A_{\mathcal{E}} = \begin{pmatrix} q_1^x & q_2^x & q_3^x \\ q_1^y & q_2^y & q_3^y \\ q_1^z & q_2^z & q_3^z \end{pmatrix} \begin{pmatrix} \frac{\lambda_1}{\alpha_1} & 0 & 0 \\ 0 & \frac{\lambda_2}{\alpha_2} & 0 \\ 0 & 0 & \frac{\lambda_3}{\alpha_3} \end{pmatrix} \begin{pmatrix} p_1^x & p_2^x & p_3^x \\ p_1^y & p_2^y & p_3^y \\ 1 & 1 & 1 \end{pmatrix}^{-1}.$$