$$yA^{2} - c_{2} = (-1/3 \ 0 \ 0) \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} + 3 = \frac{5}{3}, \qquad z^{*}A^{2} = -(-1 \ 3 \ -1) \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = 14,$$

$$yA^4 - c_4 = (-1/3 \ 0 \ 0) \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + 1 = \frac{2}{3}, \qquad z^*A^4 = -(-1 \ 3 \ -1) \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 5,$$

SO

$$\theta^+ = \min\left\{ \ \frac{5}{42}, \frac{2}{15} \right\} = \frac{5}{42},$$

and we conclude that the new feasible solution for (D) is

$$y^{+} = (-1/3 \ 0 \ 0) + \frac{5}{42}(-1 \ 3 \ -1) = (-19/42 \ 5/14 \ -5/42).$$

When we substitute  $y^+$  into (D), we discover that the first two constraints are equalities, and that the new J is  $J = \{1, 2\}$ . The new Reduced Primal (RP2) is

Maximize 
$$-(\xi_1 + \xi_2 + \xi_3)$$

subject to 
$$\begin{pmatrix} 3 & 4 & 1 & 0 & 0 \\ 3 & -2 & 0 & 1 & 0 \\ 6 & 4 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \text{ and } x_1, x_2, \xi_1, \xi_2, \xi_3 \ge 0.$$

Once again, we solve (RP2) via the simplex algorithm, where  $\hat{c} = (0, 0, -1, -1, -1)$ ,  $(x_1, x_2, \xi_1, \xi_2, \xi_3) = (1/3, 0, 1, 0, 2)$  and K = (3, 1, 5). The initial tableau is obtained from the final tableau of the previous (RP1) by adding a column corresponding the the variable  $x_2$ , namely

$$\widehat{A}_K^{-1} A^2 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1/3 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ -2/3 \\ 8 \end{pmatrix},$$

with

$$\overline{c}_2 = c_2 - z^* A^2 = 0 - \begin{pmatrix} -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = 14,$$

and we get

	$x_1$	$x_2$	$\xi_1$	$\xi_2$	$\xi_3$	
3	0	14	0	-4	0	
$\xi_1 = 1$	0	(6)	1	-1	0	
$x_1 = 1/3$	1	-2/3	0	1/3	0	
$\xi_3 = 2$	0	8	0	-2	1	