

- (a)  $f = g$  and  $\tau = 0$  iff  $f$  has some fixed point, i.e., iff  $\text{Fix}(f) \neq \emptyset$ .
- (b) If  $f$  has no fixed points, i.e.,  $\text{Fix}(f) = \emptyset$ , then  $\dim(\text{Ker}(\vec{f} - \text{id})) \geq 1$ .

The remarks made in the Euclidean case also apply to the Hermitian case. In particular, the fact that  $E$  has finite dimension is only used to prove (b).

A version of the Cartan–Dieudonné also holds for affine isometries, but it may not be possible to get rid of Hermitian reflections entirely.

**Theorem 28.14.** *Let  $E$  be an affine Hermitian space of dimension  $n \geq 1$ . Every affine isometry in  $\mathbf{Is}(n, \mathbb{C})$  can be written as the composition of at most  $2n - 1$  affine isometries if it has a fixed point, or else as the composition of at most  $2n + 1$  affine isometries, where all these isometries are affine hyperplane reflections except for possibly one affine Hermitian reflection. When  $n \geq 2$ , the identity is the composition of any reflection with itself.*

*Proof.* The proof is very similar to the proof of Theorem 27.11, except that it uses Theorem 28.5 instead of Theorem 27.1. The details are left as an exercise.  $\square$

When  $n \geq 3$ , as in the Euclidean case, we can characterize the affine isometries in  $\mathbf{SE}(n, \mathbb{C})$  in terms of flips, and we can even bound the number of flips by  $2n - 2$ .

**Theorem 28.15.** *Let  $E$  be a Hermitian affine space of dimension  $n \geq 3$ . Every rigid motion  $f \in \mathbf{SE}(E, \mathbb{C})$  is the composition of an even number of affine flips  $f = f_{2k} \circ \cdots \circ f_1$ , where  $k \leq n - 1$ .*

*Proof.* It is very similar to the proof of theorem 27.12, but it uses Proposition 28.6 instead of Proposition 27.5. The details are left as an exercise.  $\square$

A more detailed study of the rigid motions of Hermitian spaces of dimension 2 and 3 would seem worthwhile, but we are not aware of any reference on this subject.