(I) Three real eigenvalues α, β, γ. The matrix Γ has the form

$$\Gamma = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix},$$

with $\alpha, \beta, \gamma \in \mathbb{R}$ nonzero and all distinct. As illustrated in Figure 26.24, the homography h has three fixed points P, Q, R, forming a triangle. The sides (lines) of this triangle are invariant under h. The restriction of h to each of these sides is hyperbolic.

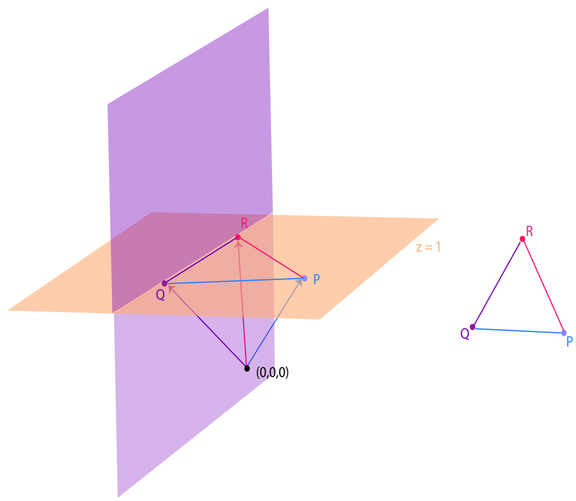

Figure 26.24: Case (I): The left figure is the hyperplane representation of $\mathbb{RP}^2$ and a homography with fixed points P, Q, R. The purple (linear) hyperplane maps to itself in a manner which is not the identity.

(II) One real eigenvalue α and two complex conjugate eigenvalues. Then Γ has the form

$$\Gamma = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & -\gamma \\ 0 & \gamma & \beta \end{pmatrix},$$

with $\alpha, \gamma \in \mathbb{R}$ nonzero. The homography h, which is illustrated in Figure 26.25, has one fixed point P, and a line Δ invariant under h and not containing P. The restriction of h to Δ is elliptic.