Instead of performing a minimization step jointly over x and z, as the method of multipliers would in the step

$$(x^{k+1}, z^{k+1}) = \arg\min_{x,z} L_\rho(x, z, \lambda^k),$$

ADMM first performs an x-minimization step, and then a z-minimization step. Thus x and z are updated in an alternating or sequential fashion, which accounts for the term *alternating direction*.

The algorithm state in ADMM is (z^k, λ^k), in the sense that (z^{k+1}, λ^{k+1}) is a function of (z^k, λ^k). The variable x^{k+1} is an auxiliary variable which is used to compute z^{k+1} from (z^k, λ^k). The roles of x and z are not quite symmetric, since the update of x is done before the update of λ. By switching x and z, f and g and A and B, we obtain a variant of ADMM in which the order of the x-update step and the z-update step are reversed.

Example 52.4. Let us reconsider the problem of Example 52.2 to solve it using ADMM. We formulate the problem as

$$\begin{aligned} &\text{minimize} && 2x + z^2 \\ &\text{subject to} && 2x - z = 0, \end{aligned}$$

with $f(x) = 2x$ and $g(z) = z^2$. The augmented Lagrangian is given by

$$L_\rho(x, z, \lambda) = 2x + z^2 + 2\lambda x - \lambda z + 2\rho x^2 - 2\rho x z + \frac{\rho}{2} z^2.$$

The ADMM steps are as follows. The x-update is

$$x^{k+1} = \arg\min_x \left(2\rho x^2 - 2\rho x z^k + 2\lambda^k x + 2x\right),$$

and since this is a quadratic function in x, its minimum is achieved when the derivative of the above function (with respect to x) is zero, namely

$$x^{k+1} = \frac{1}{2} z^k - \frac{1}{2\rho}\lambda^k - \frac{1}{2\rho}. \tag{1}$$

The z-update is

$$z^{k+1} = \arg\min_z \left(z^2 + \frac{\rho}{2} z^2 - 2\rho x^{k+1} z - \lambda^k z\right),$$

and as for the x-step, the minimum is achieved when the derivative of the above function (with respect to z) is zero, namely

$$z^{k+1} = \frac{2\rho x^{k+1}}{\rho + 2} + \frac{\lambda^k}{\rho + 2}. \tag{2}$$

The λ-update is

$$\lambda^{k+1} = \lambda^k + \rho(2x^{k+1} - z^{k+1}). \tag{3}$$