Problem 43.3. Consider the function g of Example 39.11 defined by

$$g(a, b, c) = \log(ac - b^2),$$

where $ac - b^2 > 0$. We found that the Hessian matrix of g is given by

$$Hg(a, b, c) = \frac{1}{(ac - b^2)^2} \begin{pmatrix} -c^2 & 2bc & -b^2 \\ 2bc & -2(b^2 + ac) & 2ab \\ -b^2 & 2ab & -a^2 \end{pmatrix}.$$

Use the Schur complement (of a^2) to prove that the matrix $-Hg(a, b, c)$ is symmetric positive definite if $ac - b^2 > 0$ and $a, c > 0$.