so by the results of Section 50.4, x^+ is a component of the solution of the KKT-system

$$\begin{pmatrix} P+\rho I & C^\top \\ C & 0 \end{pmatrix} \begin{pmatrix} x^+ \\ y \end{pmatrix} = \begin{pmatrix} -q+\rho v \\ b \end{pmatrix}.$$

The matrix $P+\rho I$ is symmetric positive definite, so the KKT-matrix is invertible.

We can now describe how ADMM is used to solve two common problems of convex optimization.

(1) *Minimization of a proper closed convex function f over a closed convex set C in $\mathbb{R}^n$.* This is the following problem

$$\begin{aligned} &\text{minimize} && f(x) \\ &\text{subject to} && x \in C, \end{aligned}$$

which can be rewritten in ADMM form as

$$\begin{aligned} &\text{minimize} && f(x) + I_C(z) \\ &\text{subject to} && x - z = 0. \end{aligned}$$

Using the scaled dual variable $u = \lambda/\rho$, the augmented Lagrangian is

$$L_\rho(x,z,u) = f(x) + I_C(z) + \frac{\rho}{2}\left\|x - z + u\right\|_2^2 - \frac{\rho}{2}\left\|u\right\|^2.$$

In view of Example 52.8, the scaled form of ADMM for this problem is

$$\begin{aligned} x^{k+1} &= \operatorname*{arg\,min}_x \Big(f(x) + (\rho/2)\left\|x - z^k + u^k\right\|_2^2 \Big) \\ z^{k+1} &= \Pi_C(x^{k+1} + u^k) \\ u^{k+1} &= u^k + x^{k+1} - z^{k+1}. \end{aligned}$$

The x-update involves evaluating a proximal operator. Note that the function f need not be differentiable. Of course, these minimizations depend on having efficient computational procedures for the proximal operator and the projection operator.

(2) *Quadratic Programming, Version 1.* Here the problem is

$$\begin{aligned} &\text{minimize} && \frac{1}{2}x^\top P x + q^\top x + r \\ &\text{subject to} && Ax = b,\ x \geq 0, \end{aligned}$$

where P is an $n \times n$ symmetric positive semidefinite matrix, $q \in \mathbb{R}^n$, $r \in \mathbb{R}$, and A is an $m \times n$ matrix of rank m.