Let E be the vector space (over $\mathbb{R}$) consisting of all homogeneous polynomials of degree 2 in x, y, z of the form

$$ax^2 + ay^2 + bxz + cyz + dz^2$$

(plus the null polynomial). The projective space $\mathbf{P}(E)$ consists of all equivalence classes

$$[P]_\sim = \{\lambda P \mid \lambda \neq 0\},$$

where $P(x, y, z)$ is a nonnull homogeneous polynomial in E. We want to give a geometric interpretation of the points of the projective space $\mathbf{P}(E)$. In order to do so, pick some projective frame (a_1, a_2, a_3, a_4) for the projective plane $\mathbb{RP}^2$, and associate to every $[P] \in \mathbf{P}(E)$ the subset of $\mathbb{RP}^2$ known as its its *zero locus (or zero set, or variety)* $V([P])$, and defined such that

$$V([P]) = \{a \in \mathbb{RP}^2 \mid P(x, y, z) = 0\},$$

where (x, y, z) are homogeneous coordinates for a.

As explained earlier, we also use the simpler notation

$$V([P]) = \{(x, y, z) \in \mathbb{RP}^2 \mid P(x, y, z) = 0\}.$$

Actually, in order for $V([P])$ to make sense, we have to check that $V([P])$ does not depend on the representative chosen in the equivalence class $[P] = \{\lambda P \mid \lambda \neq 0\}$. This is because

$$P(x, y, z) = 0 \quad \text{iff} \quad \lambda P(x, y, z) = 0 \quad \text{when } \lambda \neq 0.$$

For simplicity of notation, we also denote $V([P])$ by $V(P)$. We also have to check that if $(\lambda x, \lambda y, \lambda z)$ are other homogeneous coordinates for $a \in \mathbb{RP}^2$, where $\lambda \neq 0$, then

$$P(x, y, z) = 0 \quad \text{iff} \quad P(\lambda x, \lambda y, \lambda z) = 0.$$

However, since $P(x, y, z)$ is homogeneous of degree 2, we have

$$P(\lambda x, \lambda y, \lambda z) = \lambda^2 P(x, y, z),$$

and since $\lambda \neq 0$,

$$P(x, y, z) = 0 \quad \text{iff} \quad \lambda^2 P(x, y, z) = 0.$$

The above argument applies to any homogeneous polynomial $P(x_1, \ldots, x_n)$ in n variables of any degree m, since

$$P(\lambda x_1, \ldots, \lambda x_n) = \lambda^m P(x_1, \ldots, x_n).$$

Thus, we can associate to every $[P] \in \mathbf{P}(E)$ the curve $V(P)$ in $\mathbb{RP}^2$. One might wonder why we are considering only homogeneous polynomials of degree 2, and not arbitrary polynomials of degree 2? The first reason is that the polynomials in x, y, z of degree 2 do **not** form a vector space. For example, if $P = x^2 + x$ and $Q = -x^2 + y$, the polynomial $P + Q = x + y$ is not of degree 2. We could consider the set of polynomials of degree ≤ 2,