

```
function u = relax3(A,b,u,omega)
    n = size(A,1);
    for i = 1:n
        u(i,1) = u(i,1) + omega*(-A(i,:)*u + b(i))/A(i,i);
    end
end
```

Observe that function `relax3` is obtained from the function `GaussSeidel3` by simply inserting ω in front of the expression $(-A(i,:) * u + b(i))/A(i,i)$. In order to run m iteration steps, run the following function:

```
function u = relax(A,b,u0,omega,m)
    u = u0;
    for j = 1:m
        u = relax3(A,b,u,omega);
    end
end
```

Example 10.3. Consider the same linear system as in Examples 10.1 and 10.2, whose solution is

$$x_1 = 11, x_2 = -3, x_3 = 7, x_4 = -4.$$

After 10 relaxation iterations with $\omega = 1.1$, we find the approximate solution

$$x_1 = 11.0026, x_2 = -2.9968, x_3 = 7.0024, x_4 = -3.9989.$$

After 10 iterations with $\omega = 1.2$, we find the approximate solution

$$x_1 = 11.0014, x_2 = -2.9985, x_3 = 7.0010, x_4 = -3.9996.$$

After 10 iterations with $\omega = 1.3$, we find the approximate solution

$$x_1 = 10.9996, x_2 = -3.0001, x_3 = 6.9999, x_4 = -4.0000.$$

After 10 iterations with $\omega = 1.27$, we find the approximate solution

$$x_1 = 11.0000, x_2 = -3.0000, x_3 = 7.0000, x_4 = -4.0000,$$

correct up to at least four decimals. We observe that for this example the method of relaxation with $\omega = 1.27$ converges faster than the method of Gauss–Seidel. This observation will be confirmed by Proposition 10.10.

What remains to be done is to find conditions that ensure the convergence of the relaxation method (and the Gauss–Seidel method), that is:

1. Find conditions on ω , namely some interval $I \subseteq \mathbb{R}$ so that $\omega \in I$ implies $\rho(\mathcal{L}_\omega) < 1$; we will prove that $\omega \in (0, 2)$ is a necessary condition.