Example 45.1.

$$\begin{aligned}
\text{maximize} \quad & x_1 + x_2 \\
\text{subject to} \quad & \\
& x_2 - x_1 \leq 1 \\
& x_1 + 6x_2 \leq 15 \\
& 4x_1 - x_2 \leq 10 \\
& x_1 \geq 0,\ x_2 \geq 0,
\end{aligned}$$

and in matrix form

$$\begin{aligned}
\text{maximize} \quad & \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\
\text{subject to} \quad & \\
& \begin{pmatrix} -1 & 1 \\ 1 & 6 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 1 \\ 15 \\ 10 \end{pmatrix} \\
& x_1 \geq 0,\ x_2 \geq 0.
\end{aligned}$$

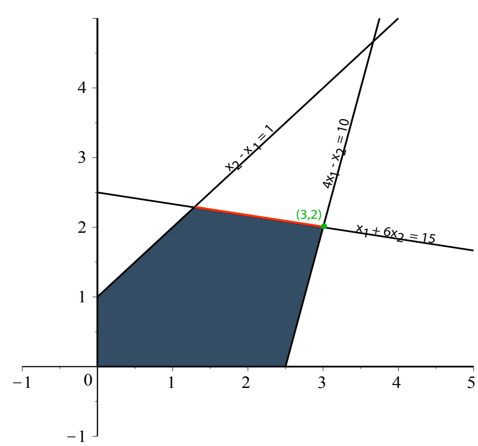

Figure 45.1: The $\mathcal{H}$-polyhedron associated with Example 45.1. The green point $(3,2)$ is the unique optimal solution.

It turns out that $x_1 = 3, x_2 = 2$ yields the maximum of the objective function $x_1 + x_2$, which is 5. This is illustrated in Figure 45.1. Observe that the set of points that satisfy the above constraints is a convex region cut out by half planes determined by the lines of