Find a function $u \in V$ such that

$$\begin{aligned}
\frac{d^2}{dt^2}\langle u, v\rangle + a(u, v) = 0, \qquad \text{for all } v \in V \quad \text{and all } t \geq 0 \\
u(x, 0) = u_{i,0}(x), \quad x \in \Omega \quad \text{(intitial condition)}, \\
\frac{\partial u}{\partial t}(x, 0) = u_{i,1}(x), \quad x \in \Omega \quad \text{(intitial condition)},
\end{aligned}$$

where $a \colon V \times V \to \mathbb{R}$ is the bilinear form given by

$$a(u, v) = \int_\Omega \left(\frac{\partial u}{\partial x_1}\frac{\partial v}{\partial x_1} + \frac{\partial u}{\partial x_2}\frac{\partial v}{\partial x_2} \right) dx,$$

and

$$\langle u, v\rangle = \int_\Omega uvdx.$$

As usual, we find approximations of our problem by using finite dimensional subspaces V_a of V. Picking some basis $(w_1, \ldots, w_n)$ of V_a, and triangulating Ω, as before, we obtain the equation

$$\begin{aligned}
A\frac{d^2\mathbf{u}}{dt^2} + K\mathbf{u} = 0, \\
u(x, 0) = u_{a,0}(x), \quad x \in \Gamma, \\
\frac{\partial u}{\partial t}(x, 0) = u_{a,1}(x), \quad x \in \Gamma,
\end{aligned}$$

where $A = (\langle w_i, w_j\rangle)$ and $K = (a(w_i, w_j))$ are two symmetric positive definite matrices.

In principle, the problem is solved, but, it may be difficult to find good spaces V_a, good triangulations of Ω, and good bases of V_a, to be able to compute the matrices A and K, and to ensure that they are sparse.