separating hyperplane $H_{w,b}$; if $\epsilon_i \leq \eta$, then u_i is classified correctly, and if $\epsilon_i > \eta$, then u_i is misclassified (u_i lies on the wrong side of the separating hyperplane, the red side).

Similarly, if $\mu_j = K_s$, then

$$w^\top v_j - b + \eta = \xi_j.$$

If $\xi_j = 0$, then the point v_j is on the red margin. If $\xi_j > 0$, then v_j is within the open half space bounded by the red margin hyperplane $H_{w,b-\eta}$ and containing the separating hyperplane $H_{w,b}$; if $\xi_j \leq \eta$, then v_j is classified correctly, and if $\xi_j > \eta$, then v_j is misclassified (v_j lies on the wrong side of the separating hyperplane, the blue side).

(3) If $\lambda_i = 0$, then $\epsilon_i = 0$ and the i-th inequality may or may not be active, so

$$w^\top u_i - b - \eta \geq 0.$$

Thus u_i is in the closed half space on the blue side bounded by the blue margin hyperplane $H_{w,b+\eta}$ (of course, classified correctly).

Similarly, if $\mu_j = 0$, then

$$w^\top v_j - b + \eta \leq 0$$

and v_j is in the closed half space on the red side bounded by the red margin hyperplane $H_{w,b-\eta}$ (of course, classified correctly).

Definition 54.2. The vectors u_i on the blue margin $H_{w,b+\eta}$ and the vectors v_j on the red margin $H_{w,b-\eta}$ are called *support vectors*. Support vectors correspond to vectors u_i for which $w^\top u_i - b - \eta = 0$ (which implies $\epsilon_i = 0$), and vectors v_j for which $w^\top v_j - b + \eta = 0$ (which implies $\xi_j = 0$). Support vectors u_i such that $0 < \lambda_i < K_s$ and support vectors v_j such that $0 < \mu_j < K_s$ are *support vectors of type 1*. Support vectors of type 1 play a special role so we denote the sets of indices associated with them by

$$\begin{aligned} I_\lambda &= \{i \in \{1, \ldots, p\} \mid 0 < \lambda_i < K_s\} \\ I_\mu &= \{j \in \{1, \ldots, q\} \mid 0 < \mu_j < K_s\}. \end{aligned}$$

We denote their cardinalities by $numsvl_1 = |I_\lambda|$ and $numsvm_1 = |I_\mu|$. Support vectors u_i such that $\lambda_i = K_s$ and support vectors v_j such that $\mu_j = K_s$ are *support vectors of type 2*. Those support vectors u_i such that $\lambda_i = 0$ and those support vectors v_j such that $\mu_j = 0$ are called *exceptional support vectors*.

The vectors u_i for which $\lambda_i = K_s$ and the vectors v_j for which $\mu_j = K_s$ are said to *fail the margin*. The sets of indices associated with the vectors failing the margin are denoted by

$$\begin{aligned} K_\lambda &= \{i \in \{1, \ldots, p\} \mid \lambda_i = K_s\} \\ K_\mu &= \{j \in \{1, \ldots, q\} \mid \mu_j = K_s\}. \end{aligned}$$

We denote their cardinalities by $p_f = |K_\lambda|$ and $q_f = |K_\mu|$.