(1) Consider combinations $\sum_{i\in I} \lambda_i u_i$ for which

$$\sum_{i\in I} \lambda_i = 1.$$

These are called *affine combinations*. One should realize that every linear combination $\sum_{i\in I} \lambda_i u_i$ can be viewed as an affine combination. For example, if k is an index not in I, if we let $J = I \cup \{k\}$, $u_k = 0$, and $\lambda_k = 1 - \sum_{i\in I} \lambda_i$, then $\sum_{j\in J} \lambda_j u_j$ is an affine combination and

$$\sum_{i\in I} \lambda_i u_i = \sum_{j\in J} \lambda_j u_j.$$

However, we get new spaces. For example, in $\mathbb{R}^3$, the set of all affine combinations of the three vectors $e_1 = (1,0,0), e_2 = (0,1,0)$, and $e_3 = (0,0,1)$, is the plane passing through these three points. Since it does not contain $0 = (0,0,0)$, it is not a linear subspace.

(2) Consider combinations $\sum_{i\in I} \lambda_i u_i$ for which

$$\lambda_i \geq 0, \quad \text{for all } i \in I.$$

These are called *positive* (or *conic*) *combinations*. It turns out that positive combinations of families of vectors are *cones*. They show up naturally in convex optimization.

(3) Consider combinations $\sum_{i\in I} \lambda_i u_i$ for which we require (1) *and* (2), that is

$$\sum_{i\in I} \lambda_i = 1, \quad \text{and} \quad \lambda_i \geq 0 \quad \text{for all } i \in I.$$

These are called *convex combinations*. Given any finite family of vectors, the set of all convex combinations of these vectors is a *convex polyhedron*. Convex polyhedra play a very important role in convex optimization.

Remark: The notion of linear combination can also be defined for infinite index sets I. To ensure that a sum $\sum_{i\in I} \lambda_i u_i$ makes sense, we restrict our attention to families of finite support.

Definition 3.5. Given any field K, a family of scalars $(\lambda_i)_{i\in I}$ has *finite support* if $\lambda_i = 0$ for all $i \in I - J$, for some finite subset J of I.

If $(\lambda_i)_{i\in I}$ is a family of scalars of finite support, for any vector space E over K, for any (possibly infinite) family $(u_i)_{i\in I}$ of vectors $u_i \in E$, we define the linear combination $\sum_{i\in I} \lambda_i u_i$ as the finite linear combination $\sum_{j\in J} \lambda_j u_j$, where J is any finite subset of I such that $\lambda_i = 0$ for all $i \in I - J$. In general, results stated for finite families also hold for families of finite support.