3.1. MOTIVATIONS: LINEAR COMBINATIONS, LINEAR INDEPENDENCE, RANK57

Another important application of the SVD is principal component analysis (or PCA), an important tool in data analysis.

Yet another fruitful way of interpreting the resolution of the system Ax = b is to view this problem as an intersection problem. Indeed, each of the equations

$$x_1 + 2x_2 - x_3 = 1$$
$$2x_1 + x_2 + x_3 = 2$$
$$x_1 - 2x_2 - 2x_3 = 3$$

defines a subset of \mathbb{R}^3 which is actually a *plane*. The first equation

$$x_1 + 2x_2 - x_3 = 1$$

defines the plane H_1 passing through the three points (1,0,0), (0,1/2,0), (0,0,-1), on the coordinate axes, the second equation

$$2x_1 + x_2 + x_3 = 2$$

defines the plane H_2 passing through the three points (1,0,0), (0,2,0), (0,0,2), on the coordinate axes, and the third equation

$$x_1 - 2x_2 - 2x_3 = 3$$

defines the plane H_3 passing through the three points (3,0,0), (0,-3/2,0), (0,0,-3/2), on the coordinate axes. See Figure 3.1.

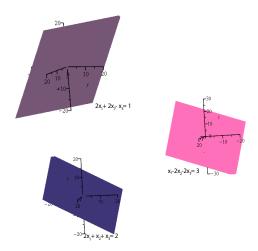


Figure 3.1: The planes defined by the preceding linear equations.