

Dual of Soft margin SVM (SVM_{s2'}):

$$\begin{aligned}
 & \text{minimize} \quad \frac{1}{2} (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\
 & \text{subject to} \\
 & \quad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0 \\
 & \quad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq K_m \\
 & \quad 0 \leq \lambda_i \leq K_s, \quad i = 1, \dots, p \\
 & \quad 0 \leq \mu_j \leq K_s, \quad j = 1, \dots, q.
 \end{aligned}$$

If $(w, \eta, \epsilon, \xi, b)$ is an optimal solution of Problem (SVM_{s2'}) with $w \neq 0$ and $\eta \neq 0$, then the complementary slackness conditions yield a classification of the points u_i and v_j in terms of the values of λ and μ . Indeed, we have $\epsilon_i \alpha_i = 0$ for $i = 1, \dots, p$ and $\xi_j \beta_j = 0$ for $j = 1, \dots, q$. Also, if $\lambda_i > 0$, then the corresponding constraint is active, and similarly if $\mu_j > 0$. Since $\lambda_i + \alpha_i = K_s$, it follows that $\epsilon_i \alpha_i = 0$ iff $\epsilon_i (K_s - \lambda_i) = 0$, and since $\mu_j + \beta_j = K_s$, we have $\xi_j \beta_j = 0$ iff $\xi_j (K_s - \mu_j) = 0$. Thus if $\epsilon_i > 0$, then $\lambda_i = K_s$, and if $\xi_j > 0$, then $\mu_j = K_s$. Consequently, if $\lambda_i < K_s$, then $\epsilon_i = 0$ and u_i is correctly classified, and similarly if $\mu_j < K_s$, then $\xi_j = 0$ and v_j is correctly classified.

We have the following classification which is basically the classification given in Section 54.1 obtained by replacing δ with η (recall that $\eta > 0$ and $\delta = \eta / \|w\|$).

- (1) If $0 < \lambda_i < K_s$, then $\epsilon_i = 0$ and the i -th inequality is active, so

$$w^\top u_i - b - \eta = 0.$$

This means that u_i is on the blue margin (the hyperplane $H_{w, b+\eta}$ of equation $w^\top x = b + \eta$) and is classified correctly.

Similarly, if $0 < \mu_j < K_s$, then $\xi_j = 0$ and

$$w^\top v_j - b + \eta = 0,$$

so v_j is on the red margin (the hyperplane $H_{w, b-\eta}$ of equation $w^\top x = b - \eta$) and is classified correctly.

- (2) If $\lambda_i = K_s$, then the i -th inequality is active, so

$$w^\top u_i - b - \eta = -\epsilon_i.$$

If $\epsilon_i = 0$, then the point u_i is on the blue margin. If $\epsilon_i > 0$, then u_i is within the open half space bounded by the blue margin hyperplane $H_{w, b+\eta}$ and containing the