Rather than using W^{-1} to convert a vector u to a vector c of coefficients over the Haar basis, and the matrix W to reconstruct the vector u from its Haar coefficients c, we can use faster algorithms that use averaging and differencing.

If c is a vector of Haar coefficients of dimension 2^n, we compute the sequence of vectors $u^0, u^1, \ldots, u^n$ as follows:

$$\begin{aligned}
u^0 &= c \\
u^{j+1} &= u^j \\
u^{j+1}(2i-1) &= u^j(i) + u^j(2^j + i) \\
u^{j+1}(2i) &= u^j(i) - u^j(2^j + i),
\end{aligned}$$

for $j = 0, \ldots, n-1$ and $i = 1, \ldots, 2^j$. The reconstructed vector (signal) is $u = u^n$.

If u is a vector of dimension 2^n, we compute the sequence of vectors $c^n, c^{n-1}, \ldots, c^0$ as follows:

$$\begin{aligned}
c^n &= u \\
c^j &= c^{j+1} \\
c^j(i) &= (c^{j+1}(2i-1) + c^{j+1}(2i))/2 \\
c^j(2^j + i) &= (c^{j+1}(2i-1) - c^{j+1}(2i))/2,
\end{aligned}$$

for $j = n-1, \ldots, 0$ and $i = 1, \ldots, 2^j$. The vector over the Haar basis is $c = c^0$.

We leave it as an exercise to implement the above programs in `Matlab` using two variables u and c, and by building iteratively 2^j. Here is an example of the conversion of a vector to its Haar coefficients for $n = 3$.

Given the sequence $u = (31, 29, 23, 17, -6, -8, -2, -4)$, we get the sequence

$$\begin{aligned}
c^3 &= (31, 29, 23, 17, -6, -8, -2, -4) \\
c^2 &= \left(\frac{31+29}{2}, \frac{23+17}{2}, \frac{-6-8}{2}, \frac{-2-4}{2}, \frac{31-29}{2}, \frac{23-17}{2}, \frac{-6-(-8)}{2}, \frac{-2-(-4)}{2}\right) \\
&= (30, 20, -7, -3, 1, 3, 1, 1) \\
c^1 &= \left(\frac{30+20}{2}, \frac{-7-3}{2}, \frac{30-20}{2}, \frac{-7-(-3)}{2}, 1, 3, 1, 1\right) \\
&= (25, -5, 5, -2, 1, 3, 1, 1) \\
c^0 &= \left(\frac{25-5}{2}, \frac{25-(-5)}{2}, 5, -2, 1, 3, 1, 1\right) = (10, 15, 5, -2, 1, 3, 1, 1)
\end{aligned}$$

so $c = (10, 15, 5, -2, 1, 3, 1, 1)$. Conversely, given $c = (10, 15, 5, -2, 1, 3, 1, 1)$, we get the