- Invariant subspace.
- f-conductor of u into W; conductor of u into W.
- Diagonalizable linear maps.
- Commuting families of linear maps.
- Primary decomposition.
- Generalized eigenvectors.
- Nilpotent linear map.
- Normal form of a nilpotent linear map.
- Jordan decomposition.
- Jordan block.
- Jordan matrix.
- Jordan normal form.
- Systems of first-order linear differential equations.

31.8 Problems

Problem 31.1. Given a linear map $f\colon E \to E$, prove that the set $\mathrm{Ann}(f)$ of polynomials that annihilate f is an ideal.

Problem 31.2. Provide the details of Proposition 31.3.

Problem 31.3. Prove that the f-conductor $S_f(u, W)$ is an ideal in $K[X]$ (Proposition 31.4).

Problem 31.4. Prove that the polynomials $g_1, \ldots, g_k$ used in the proof of Theorem 31.10 are relatively prime.

Problem 31.5. Find the minimal polynomial of the matrix

$$A = \begin{pmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{pmatrix}.$$

Problem 31.6. Find the Jordan decomposition of the matrix

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}.$$