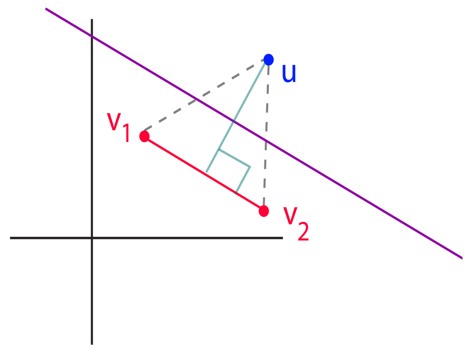

Figure 50.15: The purple line, which is the bisector of the altitude of the isosceles triangle, separates the two red points from the blue point in a manner which satisfies Hard Margin SVM_{h2}.

If $p = q = 1$, we can find a solution explicitly. Then $(*_2)$ yields

$$\lambda = \mu,$$

and if we guess that the constraints are active, the corresponding equality constraints are

$$\begin{aligned}-u^\top u\lambda + u^\top v\mu + b + 1 &= 0\\ u^\top v\lambda - v^\top v\mu - b + 1 &= 0,\end{aligned}$$

so we get

$$\begin{aligned}(-u^\top u + u^\top v)\lambda + b + 1 &= 0\\ (u^\top v - v^\top v)\lambda - b + 1 &= 0,\end{aligned}$$

Adding up the two equations we find

$$(2u^\top v - u^\top u - v^\top v)\lambda + 2 = 0,$$

that is

$$\lambda = \frac{2}{(u-v)^\top (u-v)}.$$

By subtracting the first equation from the second, we find

$$(u^\top u - v^\top v)\lambda - 2b = 0,$$

which yields

$$b = \lambda\frac{(u^\top u - v^\top v)}{2} = \frac{u^\top u - v^\top v}{(u-v)^\top (u-v)}.$$