In particular, Theorem 3.23 shows a finite-dimensional vector space and its dual E^* have the same dimension.

We explained just after Definition 3.26 that if the space E is finite-dimensional and has a finite basis $(u_1, \ldots, u_n)$, then a linear form $f^* \colon E \to K$ is represented by the *row vector* of coefficients

$$\begin{pmatrix} f^*(u_1) & \cdots & f^*(u_n) \end{pmatrix}. \tag{1}$$

The proof of Theorem 3.23 shows that over the dual basis $(u_1^*, \ldots, u_n^*)$ of E^*, the linear form f^* is represented by the same coefficients, but as the *column vector*

$$\begin{pmatrix} f^*(u_1) \\ \vdots \\ f^*(u_n) \end{pmatrix}, \tag{2}$$

which is the transpose of the row vector in (1).

3.10 Summary

The main concepts and results of this chapter are listed below:

- The notion of a *vector space*.
- *Families* of vectors.
- *Linear combinations* of vectors; *linear dependence* and *linear independence* of a family of vectors.
- Linear *subspaces*.
- *Spanning* (or *generating*) family; *generators*, *finitely generated subspace*; *basis of a subspace*.
- *Every linearly independent family can be extended to a basis* (Theorem 3.7).
- A family B of vectors is a basis iff it is a maximal linearly independent family iff it is a minimal generating family (Proposition 3.8).
- The replacement lemma (Proposition 3.10).
- Any two bases in a finitely generated vector space E have the *same number of elements*; this is the *dimension* of E (Theorem 3.11).
- *Hyperplanes*.
- Every vector has a *unique representation* over a basis (in terms of its coordinates).