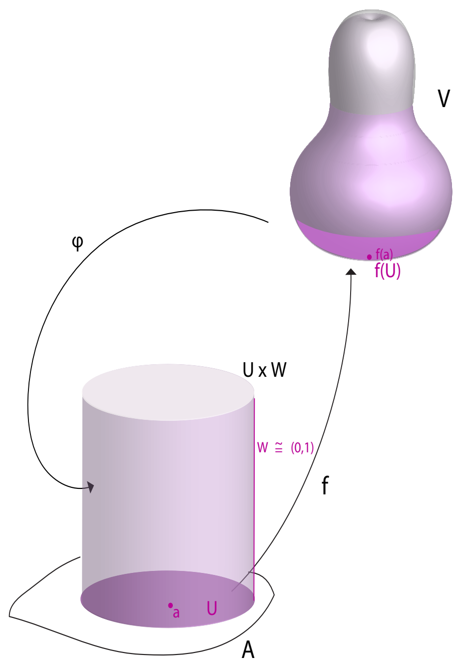

Figure 39.8: Let $n = 2$ and $m = 3$. The immersion maps the purple circular base of the cylinder $U \times W$ to circular cup on the surface of the solid purple gourd.

39.5 Tangent Spaces and Differentials

In this section, we discuss briefly a geometric interpretation of the notion of derivative. We consider sets of points defined by a differentiable function. This is a special case of the notion of a (differential) manifold.

Given two normed affine spaces E and F, let A be an open subset of E, and let $f\colon A \to F$ be a function.

Definition 39.10. Given $f\colon A \to F$ as above, its *graph* $\Gamma(f)$ is the set of all points

$$\Gamma(f) = \{(x, y) \in E \times F \mid x \in A,\, y = f(x)\}.$$

If $\mathrm{D}f$ is defined on A, we say that $\Gamma(f)$ is a *differential submanifold of* $E \times F$ *of equation* $y = f(x)$.

It should be noted that this is a very particular kind of differential manifold.

Example 39.9. If $E = \mathbb{R}$ and $F = \mathbb{R}^2$, letting $f = (g, h)$, where $g\colon \mathbb{R} \to \mathbb{R}$ and $h\colon \mathbb{R} \to \mathbb{R}$, $\Gamma(f)$ is a curve in $\mathbb{R}^3$, of equations $y = g(x)$, $z = h(x)$. When $E = \mathbb{R}^2$ and $F = \mathbb{R}$, $\Gamma(f)$ is a surface in $\mathbb{R}^3$, of equation $z = f(x, y)$.