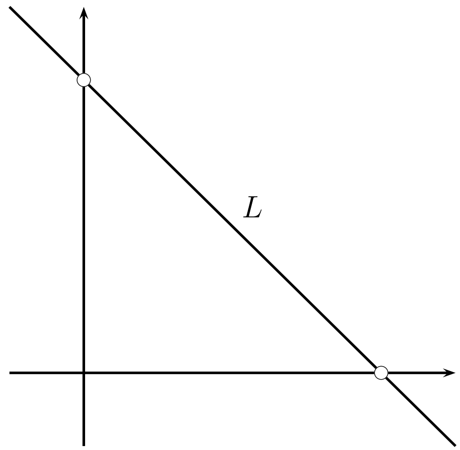

Figure 24.5: An affine space: the line of equation $x + y - 1 = 0$.

24.2 Examples of Affine Spaces

Let us now give an example of an affine space that is not given as a vector space (at least, not in an obvious fashion). Consider the subset L of $\mathbb{A}^2$ consisting of all points (x, y) satisfying the equation

$$x + y - 1 = 0.$$

The set L is the line of slope -1 passing through the points $(1, 0)$ and $(0, 1)$ shown in Figure 24.5.

The line L can be made into an official affine space by defining the action $+\colon L \times \mathbb{R} \to L$ of $\mathbb{R}$ on L defined such that for every point $(x, 1 - x)$ on L and any $u \in \mathbb{R}$,

$$(x, 1 - x) + u = (x + u, 1 - x - u).$$

It is immediately verified that this action makes L into an affine space. For example, for any two points $a = (a_1, 1 - a_1)$ and $b = (b_1, 1 - b_1)$ on L, the unique (vector) $u \in \mathbb{R}$ such that $b = a + u$ is $u = b_1 - a_1$. Note that the vector space $\mathbb{R}$ is isomorphic to the line of equation $x + y = 0$ passing through the origin.

Similarly, consider the subset H of $\mathbb{A}^3$ consisting of all points (x, y, z) satisfying the equation

$$x + y + z - 1 = 0.$$

The set H is the plane passing through the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. The plane H can be made into an official affine space by defining the action $+\colon H \times \mathbb{R}^2 \to H$ of $\mathbb{R}^2$ on