Problem 52.4. Prove that the soft thresholding operator S_c from Section 52.8 satisfies the equations

$$S_c(v) = (v - c)_+ - (-v - c)_+,$$

and

$$S_c(v) = (1 - c/|v|)_+ v, \quad v \neq 0.$$

Problem 52.5. Rederive the formula

$$S_c(v) = \begin{cases} v - c & \text{if } v > c \\ 0 & \text{if } |v| \leq c \\ v + c & \text{if } v < -c \end{cases}$$

using subgradients.

Problem 52.6. In basis pursuit (see Section 52.8 (2)) prove that

$$x^{k+1} = (I - A^\top (AA^\top)^{-1} A)(z^k - u^k) + A^\top (AA^\top)^{-1} b.$$

Problem 52.7. Implement (in `Matlab`) ADMM applied to lasso regularization as described in Section 52.6 (4). The stopping criterion should be based on feasibility tolerances ϵ^{pri} and ϵ^{dual}, say 10^{-4}, and on a maximum number of iteration steps, say 10000. There is a build in `Matlab` function `wthresh` implementing soft thresholding. You may use the `Matlab` command `randn` to create a random data set X and a random response vector y (see the help menu in `Matlab` under `lasso`). Try various values of ρ and τ. You will observe that the choice of ρ greatly affects the rate of convergence of the procedure.