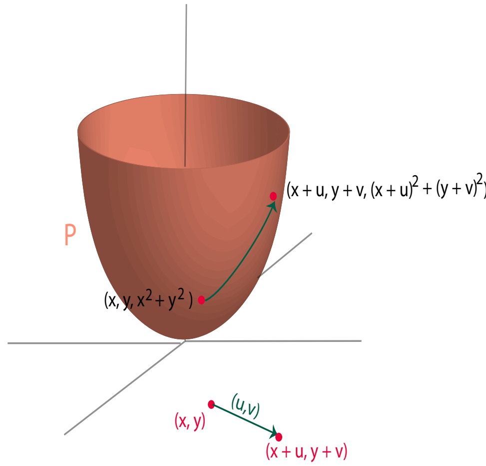

Figure 24.7: The paraboloid of revolution P viewed as a two-dimensional affine space.

by (A2), and thus, by (A3),

$$\overrightarrow{ab} + \overrightarrow{bc} = \overrightarrow{ac},$$

which is known as *Chasles's identity*, and illustrated in Figure 24.8.

Since $a = a + \overrightarrow{aa}$ and by (A1) $a = a + 0$, by (A3) we get

$$\overrightarrow{aa} = 0.$$

Thus, letting $a = c$ in Chasles's identity, we get

$$\overrightarrow{ba} = -\overrightarrow{ab}.$$

Given any four points $a, b, c, d \in E$, since by Chasles's identity

$$\overrightarrow{ab} + \overrightarrow{bc} = \overrightarrow{ad} + \overrightarrow{dc} = \overrightarrow{ac},$$

we have the *parallelogram law*

$$\overrightarrow{ab} = \overrightarrow{dc} \quad \text{iff} \quad \overrightarrow{bc} = \overrightarrow{ad}.$$

24.4 Affine Combinations, Barycenters

A fundamental concept in linear algebra is that of a linear combination. The corresponding concept in affine geometry is that of an *affine combination*, also called a *barycenter*. However, there is a problem with the naive approach involving a coordinate system, as we saw in Section 24.1. Since this problem is the reason for introducing affine combinations, at the