We refer to the above model of $\mathbf{P}(E)$ as the *hyperplane model*. In this model some hyperplane H_∞ (through the origin) in $\mathbb{R}^{n+1}$ is singled out, and the points of $\mathbf{P}(E)$ arising from the hyperplane H_∞ are declared to be "points at infinity." The purpose of the affine hyperplane H parallel to H_∞ and distinct from H_∞ is to get images for the other points in $\mathbf{P}(E)$ (i.e., those that arise from lines not contained in H_∞). It should be noted that the choice of which points should be considered as infinite is relative to the choice of H_∞. Viewing certain points of $\mathbf{P}(E)$ as points at infinity is convenient for getting a mental picture of $\mathbf{P}(E)$, but there is nothing intrinsic about that. Points of $\mathbf{P}(E)$ are all equal, and unless some additional structure in introduced in $\mathbf{P}(E)$ (such as a hyperplane), a point in $\mathbf{P}(E)$ doesn't know whether it is infinite! The notion of point at infinity is really an affine notion. This point will be made precise in Section 26.8.

Again, for $\mathbb{RP}^n = \mathbf{P}(\mathbb{R}^{n+1})$, instead of considering the hyperplane H, we can consider the n-sphere S^n of center 0 and radius 1, i.e., the set of points $(x_1, \ldots, x_{n+1})$ such that

$$x_1^2 + \cdots + x_n^2 + x_{n+1}^2 = 1.$$

In this case, every line D through the center of the sphere intersects the sphere S^n in two antipodal points a_+ and a_-. The projective space $\mathbb{RP}^n$ is the quotient space obtained from the sphere S^n by identifying antipodal points a_+ and a_-. It is hard to visualize such an object! We call this model of $\mathbf{P}(E)$ the *spherical model*. See Figure 26.4.

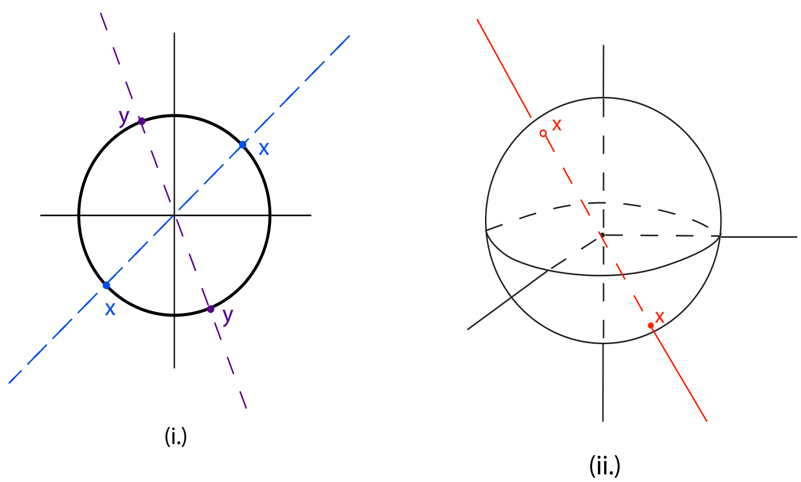

Figure 26.4: The spherical model representations of $\mathbb{RP}^1$ and $\mathbb{RP}^2$.

A more subtle construction consists in considering the (upper) half-sphere instead of the sphere, where the upper half-sphere S^n_+ is set of points on the sphere S^n such that $x_{n+1} \geq 0$. This time, every line through the center intersects the (upper) half-sphere in a single point, except on the boundary of the half-sphere, where it intersects in two antipodal points a_+ and a_-. Thus, the projective space $\mathbb{RP}^n$ is the quotient space obtained from the (upper)