The new dual program is solved using ADMM. The $(2m+1)\times 4m$ matrix A_3 corresponding to the equational constraints

$$\sum_{i=1}^m \lambda_i + \sum_{i=1}^m \mu_i = C\nu$$
$$\lambda + \alpha = \frac{C}{m}, \quad \mu + \beta = \frac{C}{m},$$

is given by

$$A_3 = \begin{pmatrix} \mathbf{1}_m^\top & \mathbf{1}_m^\top & 0_m^\top & 0_m^\top \\ I_m & 0_{m,m} & I_m & 0_{m,m} \\ 0_{m,m} & I_m & 0_{m,m} & I_m \end{pmatrix}.$$

We leave it as an exercise to show that A_3 has rank $2m+1$. We define the vector c_3 (of dimension $2m+1$) as

$$c_3 = \begin{pmatrix} C\nu \\ \frac{C}{m}\mathbf{1}_{2m} \end{pmatrix}.$$

Since there are $4m$ Lagrange multipliers $(\lambda, \mu, \alpha, \beta)$, we need to pad the $2m \times 2m$ matrix $P_3 = P + \begin{pmatrix} \mathbf{1}_m\mathbf{1}_m^\top & -\mathbf{1}_m\mathbf{1}_m^\top \\ -\mathbf{1}_m\mathbf{1}_m^\top & \mathbf{1}_m\mathbf{1}_m^\top \end{pmatrix}$ with zeros to make it into a $4m \times 4m$ matrix

$$P_{3a} = \begin{pmatrix} P_3 & 0_{2m,2m} \\ 0_{2m,2m} & 0_{2m,2m} \end{pmatrix}.$$

Similarly, we pad q with zeros to make it a vector q_{3a} of dimension $4m$,

$$q_{3a} = \begin{pmatrix} q \\ 0_{2m} \end{pmatrix}.$$

It remains to compute ϵ. Ther are two methods to do this.

The first method assumes the **Standard Margin Hypothesis**, which is that there is some i_0 such that $0 < \lambda_{i_0} < C/m$ *or* there is some j_0 such that $0 < \mu_{j_0} < C/m$; in other words, there is some support vector of type 1. By the complementary slackness conditions, $\xi_{i_0} = 0$ or $\xi'_{j_0} = 0$, so we have either $w^\top x_{i_0} + b - y_{i_0} = \epsilon$ or $-w^\top x_{j_0} - b + y_{j_0} = \epsilon$, which determines ϵ.

Due to numerical instability, when writing a computer program it is preferable to compute the lists of indices I_λ and I_μ given by

$$I_\lambda = \{i \in \{1, \ldots, m\} \mid 0 < \lambda_i < C/m\}$$
$$I_\mu = \{j \in \{1, \ldots, m\} \mid 0 < \mu_j < C/m\}.$$