6. It is not hard to show that every 2×2 rotation matrix $R \in \mathbf{SO}(2)$ can be written as
$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad \text{with} \quad 0 \leq \theta < 2\pi.$$
Then $\mathbf{SO}(2)$ can be considered as a subgroup of $\mathbf{SO}(3)$ by viewing the matrix
$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$
as the matrix
$$Q = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$
7. The set of 2×2 upper-triangular matrices of the form
$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \quad a, b, c \in \mathbb{R},\ a, c \neq 0$$
is a subgroup of the group $\mathbf{GL}(2, \mathbb{R})$.
8. The set V consisting of the four matrices
$$\begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix}$$
is a subgroup of the group $\mathbf{GL}(2, \mathbb{R})$ called the *Klein four-group*.

Definition 2.5. If H is a subgroup of G and $g \in G$ is any element, the sets of the form gH are called *left cosets of H in G* and the sets of the form Hg are called *right cosets of H in G*. The left cosets (resp. right cosets) of H induce an equivalence relation $\sim$ defined as follows: For all $g_1, g_2 \in G$,
$$g_1 \sim g_2 \quad \text{iff} \quad g_1 H = g_2 H$$
(resp. $g_1 \sim g_2$ iff $Hg_1 = Hg_2$). Obviously, $\sim$ is an equivalence relation.

Now, we claim the following fact:

Proposition 2.7. *Given a group G and any subgroup H of G, we have $g_1 H = g_2 H$ iff $g_2^{-1} g_1 H = H$ iff $g_2^{-1} g_1 \in H$, for all $g_1, g_2 \in G$.*

Proof. If we apply the bijection $L_{g_2^{-1}}$ to both $g_1 H$ and $g_2 H$ we get $L_{g_2^{-1}}(g_1 H) = g_2^{-1} g_1 H$ and $L_{g_2^{-1}}(g_2 H) = H$, so $g_1 H = g_2 H$ iff $g_2^{-1} g_1 H = H$. If $g_2^{-1} g_1 H = H$, since $1 \in H$, we get $g_2^{-1} g_1 \in H$. Conversely, if $g_2^{-1} g_1 \in H$, since H is a group, the left translation $L_{g_2^{-1} g_1}$ is a bijection of H, so $g_2^{-1} g_1 H = H$. Thus, $g_2^{-1} g_1 H = H$ iff $g_2^{-1} g_1 \in H$. $\square$