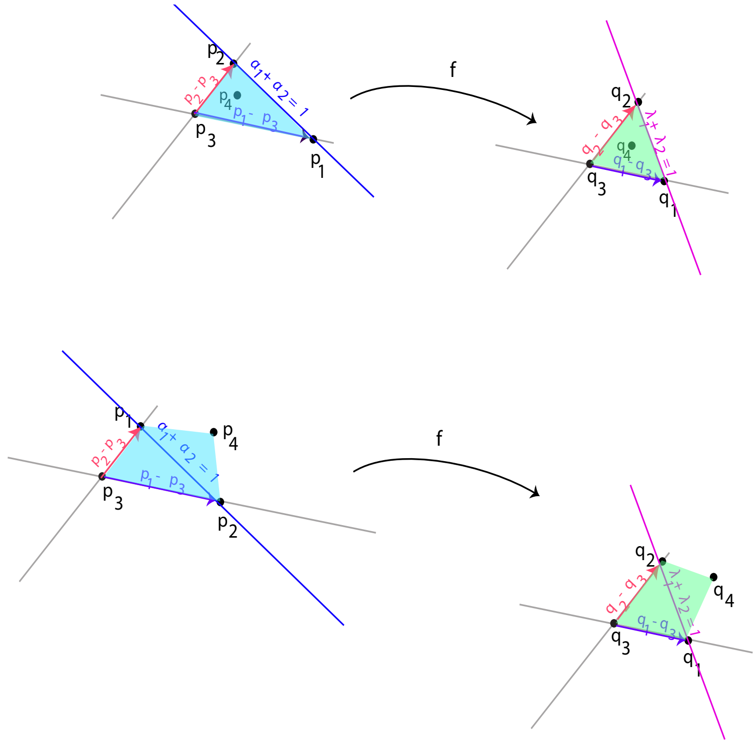

Figure 26.17: Case (4)

In order for our map to be defined for $0 \leq x \leq 1$, $cx + d$ must have a constant sign for $0 \leq x \leq 1$, which means that d and $c + d$ have the same sign. Then,

$$\frac{(ad - bc)x}{d(cx + d)}$$

and

$$\frac{(ad - bc)(x - 1)}{(c + d)(cx + d)}$$

have opposite signs when $0 < x < 1$, which means that the image of $[0, 1]$ is the interval $[b/d, (a + b)/(c + d)]$ (or $[(a + b)/(c + d), b/d]$). $\square$

We now consider the projective completion of an affine space. First, we introduce the notion of affine patch.

26.7 Affine Patches

Given an affine space E with associated vector space $\overrightarrow{E}$, we can form the vector space $\widehat{E}$, the homogenized version of E, and then, the projective space $\mathbf{P}(\widehat{E})$ induced by $\widehat{E}$. This