- Weak duality.
- Strong duality.
- Handling equality constraints in the Lagrangian.
- Dual of the Hard Margin SVM (SVM_{h2}).
- Conjugate functions and Legendre dual functions.
- Dual of the Hard Margin SVM (SVM_{h1}).
- Uzawa's Method.

50.15 Problems

Problem 50.1. Prove (3) and (4) of Proposition 50.11.

Problem 50.2. Assume that in Theorem 50.17, $V = \mathbb{R}^n$, J is elliptic and the constraints φ_i are of the form

$$\sum_{j=1}^{n} c_{ij} v_j \leq d_i,$$

that is, affine. Prove that the Problem (P_μ) has a unique solution which is continuous in μ.

Problem 50.3. (1) Prove that the set of saddle points of a function $L\colon \Omega \times M \to \mathbb{R}$ is of the form $V_0 \times M_0$, for some $V_0 \subseteq \Omega$ and some $M_0 \subseteq M$.

(2) Assume that Ω and M are convex subsets of some normed vector spaces, assume that for any fixed $v \in \Omega$ the map

$$\mu \mapsto L(v, \mu) \quad \text{is concave,}$$

and for any fixed $\mu \in M$ the map

$$v \mapsto L(v, \mu) \quad \text{is convex.}$$

Prove that $V_0 \times M_0$ is convex.

(3) Prove that if for every fixed $\mu \in M$ the map

$$v \mapsto L(v, \mu) \quad \text{is strictly convex,}$$

then V_0 has a most one element.

Problem 50.4. Prove that the conjugate of the function f given by $f(X) = \log\det(X^{-1})$, where X is an $n \times n$ symmetric positive definite matrix, is

$$f^*(Y) = \log\det((-Y)^{-1}) - n,$$

where Y is an $n \times n$ symmetric negative definite matrix.