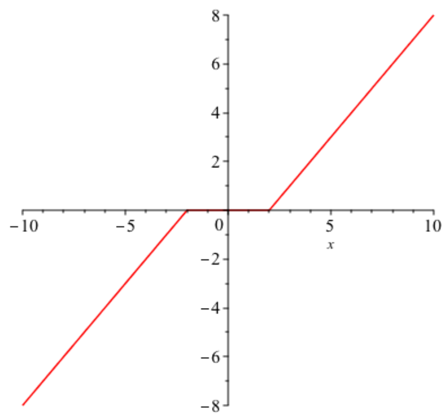

Figure 52.6: The graph of S_c (when $c = 2$).

One can check that

$$S_c(v) = (v - c)_+ - (-v - c)_+,$$

and also

$$S_c(v) = (1 - c/|v|)_+ v, \quad v \neq 0,$$

which shows that S_c is a *shrinkage operator* (it moves a point toward zero).

The operator S_c is extended to vectors in $\mathbb{R}^n$ component wise, that is, if $x = (x_1, \ldots, x_n)$, then

$$S_c(x) = (S_c(x_1), \ldots, S_c(x_n)).$$

We now consider several ℓ^1-norm problems.

(1) *Least absolute deviation.*

This is the problem of minimizing $\|Ax - b\|_1$, rather than $\|Ax - b\|_2$. Least absolute deviation is more robust than least squares fit because it deals better with outliers. The problem can be formulated in ADMM form as follows:

$$\begin{aligned} &\text{minimize} && \|z\|_1 \\ &\text{subject to} && Ax - z = b, \end{aligned}$$

with $f = 0$ and $g = \| \, \|_1$. As usual, we assume that A is an $m \times n$ matrix of rank n, so that $A^\top A$ is invertible. ADMM (in scaled form) can be expressed as

$$\begin{aligned} x^{k+1} &= (A^\top A)^{-1} A^\top (b + z^k - u^k) \\ z^{k+1} &= S_{1/\rho}(Ax^{k+1} - b + u^k) \\ u^{k+1} &= u^k + Ax^{k+1} - z^{k+1} - b. \end{aligned}$$