with $K_s = 1/(p+q)$.

Tracing through the derivation of the dual program, we obtain

Dual of the Soft margin kernel SVM (SVM$_{s3}$):

$$
\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(\mathbf{K} + \begin{pmatrix}\mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top \\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\
&\text{subject to} \\
&\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = \nu \\
&\qquad 0 \leq \lambda_i \leq K_s, \quad i = 1, \ldots, p \\
&\qquad 0 \leq \mu_j \leq K_s, \quad j = 1, \ldots, q,
\end{aligned}
$$

where $\mathbf{K}$ is the kernel matrix of Section 54.1.

We obtain

$$
\begin{aligned}
w &= \sum_{i=1}^p \lambda_i \varphi(u_i) - \sum_{j=1}^q \mu_j \varphi(v_j) \\
b &= -\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j.
\end{aligned}
$$

The classification function

$$f(x) = \operatorname{sgn}(\langle w, \varphi(x)\rangle - b)$$

is given by

$$f(x) = \operatorname{sgn}\left(\sum_{i=1}^p \lambda_i(\kappa(u_i, x) + 1) - \sum_{j=1}^q \mu_j(\kappa(v_j, x) + 1)\right).$$

54.10 Classification of the Data Points in Terms of ν (SVM$_{s3}$)

The equations (†) and the box inequalities

$$0 \leq \lambda_i \leq K_s, \quad 0 \leq \mu_j \leq K_s$$

also imply the following facts (recall that $\delta = \eta / \|w\|$):

Proposition 54.5. *If Problem* (SVM$_{s3}$) *has an optimal solution with $w \neq 0$ and $\eta > 0$ then the following facts hold:*