

in the blue open half-space bounded by the margin hyperplane $H_{w,b+\eta}$ (not containing the separating hyperplane $H_{w,b}$). See Figure 54.20.

Similarly, if $\mu_j = 0$, then $\xi_j = 0$ and the inequality $-w^\top v_j + b - \eta \geq 0$ holds. If equality holds then v_j is a support vector on the red margin (the hyperplane $H_{w,b-\eta}$). Otherwise v_j is in the red open half-space bounded by the margin hyperplane $H_{w,b-\eta}$ (not containing the separating hyperplane $H_{w,b}$). See Figure 54.20.

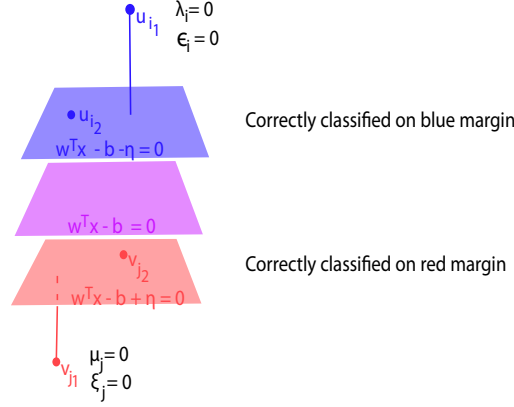


Figure 54.20: When $\lambda_i = 0$, u_i is correctly classified on or outside the blue margin. When $\mu_j = 0$, v_j is correctly classified on or outside outside the red margin.

- (2) If $\lambda_i > 0$, then $\epsilon_i = \lambda_i / (2K_s) > 0$. The corresponding constraint is active, so we have

$$w^\top u_i - b = \eta - \epsilon_i.$$

If $\epsilon_i \leq \eta$, then the points u_i is inside the slab bounded by the blue margin hyperplane $H_{w,b+\eta}$ and the separating hyperplane $H_{w,b}$. If $\epsilon_i > \eta$, then the point u_i belongs to the open half-space bounded by the separating hyperplane and containing the red margin hyperplane (the red side); it is misclassified. See Figure 54.21.

Similarly, if $\mu_j > 0$, then $\xi_j = \mu_j / (2K_s) > 0$. The corresponding constraint is active, so we have

$$-w^\top v_j + b = \eta - \xi_j.$$

If $\xi_j \leq \eta$, then the points v_j is inside the slab bounded by the red margin hyperplane $H_{w,b-\eta}$ and the separating hyperplane $H_{w,b}$. If $\xi_j > \eta$, then the point v_j belongs to the open half-space bounded by the separating hyperplane and containing the blue margin hyperplane (the blue side); it is misclassified. See Figure 54.21.

We can use the fact that the duality gap is 0 to find η . We have

$$\frac{1}{2}w^\top w - \nu\eta + K_s(\epsilon^\top \epsilon + \xi^\top \xi) = -\frac{1}{2} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(X^\top X + \frac{1}{2K_s} I_{p+q} \right) \begin{pmatrix} \lambda \\ \mu \end{pmatrix},$$