Problem 34.11. Show that

$$(u^* \wedge v^*) \lrcorner z = u^* \lrcorner (v^* \lrcorner z),$$

whenever $u^* \in \bigwedge^k E^*$, $v^* \in \bigwedge^{p-k} E^*$, and $z \in \bigwedge^{p+q} E$.

Problem 34.12. Prove Statement (3) of Proposition 34.18.

Problem 34.13. Prove Proposition 34.19.

Also prove the identity

$$u^* \lrcorner (x \wedge y) = (-1)^s (u^* \lrcorner x) \wedge y + x \wedge (u^* \lrcorner y),$$

where $u^* \in E^*$, $x \in \bigwedge^{q+1-s} E$, and $y \in \bigwedge^s E$.

Problem 34.14. Use the Grassmann-Plücker's equations prove that if $\dim(E) = n$, then every tensor in $\bigwedge^{n-1}(E)$ is decomposable.

Problem 34.15. Recall that the map

$$\mu_F \colon \left(\bigwedge^n (E^*)\right) \otimes F \longrightarrow \mathrm{Alt}^n(E; F)$$

is defined on generators by

$$\mu_F((v_1^* \wedge \cdots \wedge v_n^*) \otimes f)(u_1, \ldots, u_n) = (\det(v_j^*(u_i))f,$$

with $v_1^*, \ldots, v_n^* \in E^*$, $u_1, \ldots, u_n \in E$, and $f \in F$.

Given any three vector spaces, F, G, H, and any bilinear map $\Phi \colon F \times G \to H$, for all $\omega \in (\bigwedge^n (E^*)) \otimes F$ and all $\eta \in (\bigwedge^n (E^*)) \otimes G$ prove that

$$\mu_H(\omega \wedge_\Phi \eta) = \mu_F(\omega) \wedge_\Phi \mu_G(\eta).$$