Quadrics, projective, affine, and Euclidean, have been thoroughly investigated. Among many sources, the reader is referred to Berger [11], Samuel [142], Tisseron [175], Fresnel [65], and Vienne [185].

We could also investigate algebraic plane curves of any degree m, by letting E be the vector space of homogeneous polynomials of degree m in x, y, z (plus the null polynomial). The zero locus $V(P)$ of P is defined just as before as

$$V(P) = \{(x\colon y\colon z) \in \mathbb{RP}^2 \mid P(x, y, z) = 0\}.$$

Observe that when $m = 1$, since homogeneous polynomials of degree 1 are linear forms, we are back to the case where $E = (\mathbb{R}^3)^*$, the dual space of $\mathbb{R}^3$, and $\mathbf{P}(E)$ can be identified with the set of lines in $\mathbb{RP}^2$. But when $m \geq 3$, things are even worse regarding the injectivity of the map $[P] \mapsto V(P)$. For instance, both $P = xy^2$ and $Q = x^2y$ define the same union of two lines. It is necessary to consider *irreducible* curves, i.e., curves that are defined by irreducible polynomials, and to work over the field $\mathbb{C}$ of complex numbers (recall that a polynomial P is irreducible if it cannot be written as the product $P = Q_1Q_2$ of two polynomials Q_1, Q_2 of degree ≥ 1). We refer the reader to Fischer's book for a beautiful (and very clear) introduction to algebraic curves [62]. The next step is Fulton [66].

We can also investigate algebraic surfaces in $\mathbb{RP}^3$ (or $\mathbb{CP}^3$), by letting E be the vector space of homogeneous polynomials of degree m in four variables x, y, z, t (plus the null polynomial). We can also consider the zero locus of a set of equations

$$\mathcal{E} = \{P_1 = 0,\ P_2 = 0,\ \ldots,\ P_n = 0\},$$

where $P_1, \ldots, P_n$ are homogeneous polynomials of degree m in x, y, z, t, defined as

$$V(\mathcal{E}) = \{(x\colon y\colon z\colon t) \in \mathbb{RP}^3 \mid P_i(x, y, z, t) = 0,\ 1 \leq i \leq n\}.$$

This way, we can also deal with space curves.

Finally, we can consider homogeneous polynomials $P(x_1, \ldots, x_{N+1})$ in $N + 1$ variables and of degree m (plus the null polynomial), and study the subsets of $\mathbb{RP}^N$ or $\mathbb{CP}^N$ (or more generally of $\mathbb{P}^N_K$, for an arbitrary field K), defined as the zero locus of a set of equations

$$\mathcal{E} = \{P_1 = 0,\ P_2 = 0,\ \ldots,\ P_n = 0\},$$

where $P_1, \ldots, P_n$ are homogeneous polynomials of degree m in the variables $x_1, \ldots, x_{N+1}$. For example, it turns out that the set of lines in $\mathbb{RP}^3$ forms a surface of degree 2 in $\mathbb{RP}^5$ (the Klein quadric). However, all this would really take us too far into algebraic geometry, and we simply refer the interested reader to Hulek [97], Fulton [66], and Harris [87].

We now consider projective maps.