It will also be useful to understand how points are classified in terms of ϵ_i (or ξ_j).

(1) If $\epsilon_i > 0$, then by complementary slackness $\lambda_i = K_s$, so the ith equation is active and by (2) above,

$$w^\top u_i - b - \eta = -\epsilon_i.$$

Since $\epsilon_i > 0$, the point u_i is strictly within the open half space bounded by the blue margin hyperplane $H_{w,b+\eta}$ and containing the separating hyperplane $H_{w,b}$ (excluding the blue hyperplane $H_{w,b+\eta}$); if $\epsilon_i \leq \eta$, then u_i is classified correctly, and if $\epsilon_i > \eta$, then u_i is misclassified.

Similarly, if $\xi_j > 0$, then v_j is strictly within the open half space bounded by the red margin hyperplane $H_{w,b-\eta}$ and containing the separating hyperplane $H_{w,b}$ (excluding the red hyperplane $H_{w,b-\eta}$); if $\xi_j \leq \eta$, then v_j is classified correctly, and if $\xi_j > \eta$, then v_j is misclassified.

(2) If $\epsilon_i = 0$, then the point u_i is correctly classified. If $\lambda_i = 0$, then by (3) above, u_i is in the closed half space on the blue side bounded by the blue margin hyperplane $H_{w,b+\eta}$. If $\lambda_i > 0$, then by (1) and (2) above, the point u_i is on the blue margin.

Similarly, if $\xi_j = 0$, then the point v_j is correctly classified. If $\mu_j = 0$, then v_j is in the closed half space on the red side bounded by the red margin hyperplane $H_{w,b-\eta}$, and if $\mu_j > 0$, then the point v_j is on the red margin.

Also observe that if $\lambda_i > 0$, then u_i is in the closed half space bounded by the blue hyperplane $H_{w,b+\eta}$ and containing the separating hyperplane $H_{w,b}$ (including the blue hyperplane $H_{w,b+\eta}$).

Similarly, if $\mu_j > 0$, then v_j is in the closed half space bounded by the red hyperplane $H_{w,b+\eta}$ and containing the separating hyperplane $H_{w,b}$ (including the red hyperplane $H_{w,b+\eta}$).

Definition 54.3. Vectors u_i such that $\lambda_i > 0$ and vectors v_j such that $\mu_j > 0$ are said to *have margin at most* δ. The sets of indices associated with these vectors are denoted by

$$\begin{aligned}
I_{\lambda>0} &= \{i \in \{1, \ldots, p\} \mid \lambda_i > 0\} \\
I_{\mu>0} &= \{j \in \{1, \ldots, q\} \mid \mu_j > 0\}.
\end{aligned}$$

We denote their cardinalities by $p_m = |I_{\lambda>0}|$ and $q_m = |I_{\mu>0}|$.

Vectors u_i such that $\epsilon_i > 0$ and vectors v_j such that $\xi_j > 0$ are said to *strictly fail the margin*. The corresponding sets of indices are denoted by

$$\begin{aligned}
E_\lambda &= \{i \in \{1, \ldots, p\} \mid \epsilon_i > 0\} \\
E_\mu &= \{j \in \{1, \ldots, q\} \mid \xi_j > 0\}.
\end{aligned}$$

We write $p_{sf} = |E_\lambda|$ and $q_{sf} = |E_\mu|$.