after dividing row 3 by $-1/2$, subtracting row 3 from row 1, and subtracting $(3/2) \times$ row 3 from row 2.

It is clear that columns $1, 2$ and 4 are linearly independent, that column 3 is a linear combination of columns 1 and 2, and that column 5 is a linear combination of columns $1, 2, 4$.

In general, the sequence of steps leading to a reduced echelon matrix is not unique. For example, we could have chosen 1 instead of 2 as the second pivot in matrix A_2. Nevertheless, *the reduced row echelon matrix obtained from any given matrix is unique*; that is, it does not depend on the the sequence of steps that are followed during the reduction process. This fact is not so easy to prove rigorously, but we will do it later.

If we want to solve a linear system of equations of the form $Ax = b$, we apply elementary row operations to both the matrix A and the right-hand side b. To do this conveniently, we form the *augmented matrix* (A, b), which is the $m \times (n+1)$ matrix obtained by adding b as an extra column to the matrix A. For example if

$$A = \begin{pmatrix} 1 & 0 & 2 & 1 \\ 1 & 1 & 5 & 2 \\ 1 & 2 & 8 & 4 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 5 \\ 7 \\ 12 \end{pmatrix},$$

then the augmented matrix is

$$(A, b) = \begin{pmatrix} 1 & 0 & 2 & 1 & 5 \\ 1 & 1 & 5 & 2 & 7 \\ 1 & 2 & 8 & 4 & 12 \end{pmatrix}.$$

Now for any matrix M, since

$$M(A, b) = (MA, Mb),$$

performing elementary row operations on (A, b) is equivalent to simultaneously performing operations on both A and b. For example, consider the system

$$\begin{array}{ccccccccc} x_1 & & & + & 2x_3 & + & x_4 & = & 5 \\ x_1 & + & x_2 & + & 5x_3 & + & 2x_4 & = & 7 \\ x_1 & + & 2x_2 & + & 8x_3 & + & 4x_4 & = & 12. \end{array}$$

Its augmented matrix is the matrix

$$(A, b) = \begin{pmatrix} 1 & 0 & 2 & 1 & 5 \\ 1 & 1 & 5 & 2 & 7 \\ 1 & 2 & 8 & 4 & 12 \end{pmatrix}$$

considered above, so the reduction steps applied to this matrix yield the system

$$\begin{array}{ccccccc} x_1 & & + & 2x_3 & & = & 2 \\ & x_2 & + & 3x_3 & & = & -1 \\ & & & & x_4 & = & 3. \end{array}$$