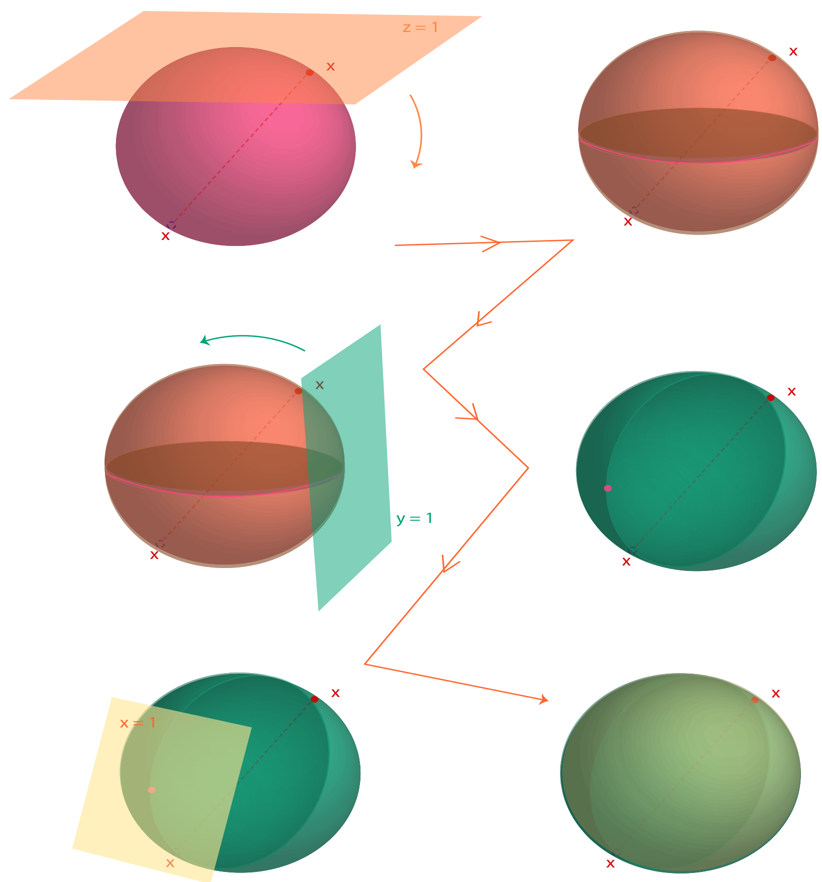

Figure 26.19: The covering of $\mathbb{RP}^2$ by the affine planes $z = 1$, $x = 1$, and $y = 1$. The plane $z = 1$ covers everything but the circle $x^2 + y^2 = 1$ in the xy-plane. The plane $y = 1$ covers that circle modulo the point $(1, 0, 0)$, which is then covered by the plane $x = 1$.

26.8 Projective Completion of an Affine Space

We begin by spelling out the universal property characterizing the projective completion of an affine space $(E, \overrightarrow{E})$. Then, we prove that $\langle \widetilde{E}, \mathbf{P}(\overrightarrow{E}), i\rangle$ where $\widetilde{E} = \mathbf{P}(\widehat{E})$ is the projective space obtained associated with the vector space $\widehat{E}$ obtained from E by the hat construction from Chapter 25 is indeed a projective completion of $(E, \overrightarrow{E})$.

Definition 26.7. Given any affine space E with associated vector space $\overrightarrow{E}$, a *projective completion of the affine space E with hyperplane at infinity* $\mathbf{P}(\mathcal{H})$ is a triple $\langle \mathbf{P}(\mathcal{E}), \mathbf{P}(\mathcal{H}), i\rangle$, where $\mathcal{E}$ is a vector space, $\mathcal{H}$ is a hyperplane in $\mathcal{E}$, $i\colon E \to \mathbf{P}(\mathcal{E})$ is an injective map such that $i(E) = \mathcal{E}_{\mathcal{H}}$ and i is affine (where $\mathcal{E}_{\mathcal{H}} = \mathbf{P}(\mathcal{E}) - \mathbf{P}(\mathcal{H})$ is an affine patch), and for every projective space $\mathbf{P}(F)$ (where F is some vector space), every hyperplane H in F, and every map $f\colon E \to \mathbf{P}(F)$ such that $f(E) \subseteq F_H$ and f is affine (where $F_H = \mathbf{P}(F) - \mathbf{P}(H)$ is an