The unique linear map $\overrightarrow{f} : \overrightarrow{E} \to \overrightarrow{E'}$ given by Proposition 24.8 is called the *linear map associated with the affine map f*.

Note that the condition

$$f(a + v) = f(a) + \overrightarrow{f}(v),$$

for every $a \in E$ and every $v \in \overrightarrow{E}$, can be stated equivalently as

$$f(x) = f(a) + \overrightarrow{f}(\overrightarrow{ax}), \quad \text{or} \quad \overrightarrow{f(a)f(x)} = \overrightarrow{f}(\overrightarrow{ax}),$$

for all $a, x \in E$. Proposition 24.8 shows that for any affine map $f\colon E \to E'$, there are points $a \in E$, $b \in E'$, and a unique linear map $\overrightarrow{f} : \overrightarrow{E} \to \overrightarrow{E'}$, such that

$$f(a + v) = b + \overrightarrow{f}(v),$$

for all $v \in \overrightarrow{E}$ (just let $b = f(a)$, for any $a \in E$). Affine maps for which $\overrightarrow{f}$ is the identity map are called *translations*. Indeed, if $\overrightarrow{f} = \mathrm{id}$,

$$\begin{aligned} f(x) &= f(a) + \overrightarrow{f}(\overrightarrow{ax}) = f(a) + \overrightarrow{ax} = x + \overrightarrow{xa} + \overrightarrow{af(a)} + \overrightarrow{ax} \\ &= x + \overrightarrow{xa} + \overrightarrow{af(a)} - \overrightarrow{xa} = x + \overrightarrow{af(a)}, \end{aligned}$$

and so

$$\overrightarrow{xf(x)} = \overrightarrow{af(a)},$$

which shows that f is the translation induced by the vector $\overrightarrow{af(a)}$ (which does not depend on a).

Since an affine map preserves barycenters, and since an affine subspace V is closed under barycentric combinations, the image $f(V)$ of V is an affine subspace in E'. So, for example, the image of a line is a point or a line, and the image of a plane is either a point, a line, or a plane.

It is easily verified that the composition of two affine maps is an affine map. Also, given affine maps $f\colon E \to E'$ and $g\colon E' \to E''$, we have

$$g(f(a + v)) = g\Big(f(a) + \overrightarrow{f}(v)\Big) = g(f(a)) + \overrightarrow{g}\Big(\overrightarrow{f}(v)\Big),$$

which shows that $\overrightarrow{g \circ f} = \overrightarrow{g} \circ \overrightarrow{f}$. It is easy to show that an affine map $f\colon E \to E'$ is injective iff $\overrightarrow{f} : \overrightarrow{E} \to \overrightarrow{E'}$ is injective, and that $f\colon E \to E'$ is surjective iff $\overrightarrow{f} : \overrightarrow{E} \to \overrightarrow{E'}$ is surjective. An affine map $f\colon E \to E'$ is constant iff $\overrightarrow{f} : \overrightarrow{E} \to \overrightarrow{E'}$ is the null (constant) linear map equal to 0 for all $v \in \overrightarrow{E}$.

If E is an affine space of dimension m and $(a_0, a_1, \ldots, a_m)$ is an affine frame for E, then for any other affine space F and for any sequence $(b_0, b_1, \ldots, b_m)$ of $m + 1$ points in F, there