Since the form u_1^* is defined by the conditions $u_1^*(u_1) = 1, u_1^*(u_2) = 0, u_1^*(u_3) = 0, u_1^*(u_4) = 0$, it is represented by a row vector $(\lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4)$ such that

$$\begin{pmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4\end{pmatrix}\begin{pmatrix}1 & -3 & 3 & -1\\ 0 & 3 & -6 & 3\\ 0 & 0 & 3 & -3\\ 0 & 0 & 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}.$$

This implies that u_1^* is the first row of the inverse of B_4. Since

$$B_4^{-1} = \begin{pmatrix}1 & 1 & 1 & 1\\ 0 & 1/3 & 2/3 & 1\\ 0 & 0 & 1/3 & 1\\ 0 & 0 & 0 & 1\end{pmatrix},$$

the linear forms $(u_1^*, u_2^*, u_3^*, u_4^*)$ correspond to the rows of B_4^{-1}. In particular, u_1^* is represented by $(1 \ 1 \ 1 \ 1)$.

The above method works for any n. Given any basis $(u_1, \ldots, u_n)$ of $\mathbb{R}^n$, if P is the $n \times n$ matrix whose jth column is u_j, then the dual form u_i^* is given by the ith row of the matrix P^{-1}.

When E is of finite dimension n and $(u_1, \ldots, u_n)$ is a basis of E, by Theorem 11.4 (1), the family $(u_1^*, \ldots, u_n^*)$ is a basis of the dual space E^*. Let us see how the coordinates of a linear form $\varphi^* \in E^*$ over the dual basis $(u_1^*, \ldots, u_n^*)$ vary under a change of basis.

Let $(u_1, \ldots, u_n)$ and $(v_1, \ldots, v_n)$ be two bases of E, and let $P = (a_{i\,j})$ be the change of basis matrix from $(u_1, \ldots, u_n)$ to $(v_1, \ldots, v_n)$, so that

$$v_j = \sum_{i=1}^n a_{i\,j} u_i,$$

and let $P^{-1} = (b_{i\,j})$ be the inverse of P, so that

$$u_i = \sum_{j=1}^n b_{j\,i} v_j.$$

For fixed j, where $1 \leq j \leq n$, we want to find scalars $(c_i)_{i=1}^n$ such that

$$v_j^* = c_1 u_1^* + c_2 u_2^* + \cdots + c_n u_n^*.$$

To find each c_i, we evaluate the above expression at u_i. Since $u_i^*(u_j) = \delta_{i\,j}$ and $v_i^*(v_j) = \delta_{i\,j}$, we get

$$\begin{aligned} v_j^*(u_i) &= (c_1 u_1^* + c_2 u_2^* + \cdots + c_n u_n^*)(u_i) = c_i\\ v_j^*(u_i) &= v_j^*(\sum_{k=1}^n b_{k\,i} v_k) = b_{j\,i}, \end{aligned}$$