Indeed, we have $\epsilon_i\alpha_i = 0$ for $i = 1, \ldots, p$ and $\xi_j\beta_j = 0$ for $j = 1, \ldots, q$. Also, if $\lambda_i > 0$, then corresponding constraint is active, and similarly if $\mu_j > 0$. Since $\lambda_i + \alpha_i = K$, it follows that $\epsilon_i\alpha_i = 0$ iff $\epsilon_i(K - \lambda_i) = 0$, and since $\mu_j + \beta_j = K$, we have $\xi_j\beta_j = 0$ iff $\xi_j(K - \mu_j) = 0$. Thus if $\epsilon_i > 0$, then $\lambda_i = K$, and if $\xi_j > 0$, then $\mu_j = K$. Consequently, if $\lambda_i < K$, then $\epsilon_i = 0$ and $u_i$ is correctly classified, and similarly if $\mu_j < K$, then $\xi_j = 0$ and $v_j$ is correctly classified.

We have a classification of the points $u_i$ and $v_j$ in terms of $\lambda$ and $\mu$ obtained from the classification given in Section 54.1 by replacing $\delta$ with 1. Since it is so similar, it is omitted. Let us simply recall that the vectors $u_i$ on the blue margin and the vectors $v_j$ on the red margin are called *support vectors*; these are the vectors $u_i$ for which $w^\top u_i - b - 1 = 0$ (which implies $\epsilon_i = 0$), and the vectors $v_j$ for which $w^\top v_j - b + 1 = 0$ (which implies $\xi_j = 0$). Those support vectors $u_i$ such that $\lambda_i = 0$ and those support vectors such that $\mu_j = 0$ are called *exceptional support vectors*.

We also have the following classification of the points $u_i$ and $v_j$ terms of $\epsilon_i$ (or $\xi_j$) obtained by replacing $\delta$ with 1.

(1) If $\epsilon_i > 0$, then by complementary slackness $\lambda_i = K$, so the $i$th equation is active and by (2) above,
$$w^\top u_i - b - 1 = -\epsilon_i.$$
Since $\epsilon_i > 0$, the point $u_i$ is within the open half space bounded by the blue margin hyperplane $H_{w,b+1}$ and containing the separating hyperplane $H_{w,b}$; if $\epsilon_i \leq 1$, then $u_i$ is classified correctly, and if $\epsilon_i > 1$, then $u_i$ is misclassified.

Similarly, if $\xi_j > 0$, then $v_j$ is within the open half space bounded by the red margin hyperplane $H_{w,b-1}$ and containing the separating hyperplane $H_{w,b}$; if $\xi_j \leq 1$, then $v_j$ is classified correctly, and if $\xi_j > 1$, then $v_j$ is misclassified.

(2) If $\epsilon_i = 0$, then the point $u_i$ is correctly classified. If $\lambda_i = 0$, then by (3) above, $u_i$ is in the closed half space on the blue side bounded by the blue margin hyperplane $H_{w,b+\eta}$. If $\lambda_i > 0$, then by (1) and (2) above, the point $u_i$ is on the blue margin.

Similarly, if $\xi_j = 0$, then the point $v_j$ is correctly classified. If $\mu_j = 0$, then $v_j$ is in the closed half space on the red side bounded by the red margin hyperplane $H_{w,b-\eta}$, and if $\mu_j > 0$, then the point $v_j$ is on the red margin. See Figure 54.5 (3).

Vectors $u_i$ for which $\lambda_i = K$ and vectors $v_j$ such that $\xi_j = K$ are said to *fail the margin*.

It is shown in Section 54.4 how the dual program is solved using ADMM from Section 52.6. If the primal problem is solvable, this yields solutions for $\lambda$ and $\mu$.

**Remark:** The hard margin Problem (SVM$_{h2}$) corresponds to the special case of Problem (SVM$_{s2}$) in which $\epsilon = 0$, $\xi = 0$, and $K = +\infty$. Indeed, in Problem (SVM$_{h2}$) the terms