Definition 45.1. *A Linear Program* (P) is the following kind of optimization problem:

$$\begin{aligned}
&\text{maximize} \quad cx \\
&\text{subject to} \\
&\quad a_1 x \leq b_1 \\
&\quad \ldots \\
&\quad a_m x \leq b_m \\
&\quad x \geq 0,
\end{aligned}$$

where $x \in \mathbb{R}^n$, $c, a_1, \ldots, a_m \in (\mathbb{R}^n)^*$, $b_1, \ldots, b_m \in \mathbb{R}$.

The linear form c defines the *objective function* $x \mapsto cx$ of the Linear Program (P) (from $\mathbb{R}^n$ to $\mathbb{R}$), and the inequalities $a_i x \leq b_i$ and $x_j \geq 0$ are called the *constraints* of the Linear Program (P).

If we define the $m \times n$ matrix

$$A = \begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix}$$

whose rows are the row vectors $a_1, \ldots, a_m$ and b as the column vector

$$b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix},$$

the m inequality constraints $a_i x \leq b_i$ can be written in matrix form as

$$Ax \leq b.$$

Thus the Linear Program (P) can also be stated as the Linear Program (P):

$$\begin{aligned}
&\text{maximize} \quad cx \\
&\text{subject to} \quad Ax \leq b \ \text{and} \ x \geq 0.
\end{aligned}$$

We should note that in many applications, the natural primal optimization problem is actually the *minimization* of some objective function $cx = c_1x_1 + \cdots + c_nx_n$, rather its maximization. For example, many of the optimization problems considered in Papadimitriou and Steiglitz [134] are minimization problems.

Of course, minimizing cx is equivalent to maximizing $-cx$, so our presentation covers minimization too.

Here is an explicit example of a linear program of Type (P):