We can simplify this problem by maximizing over the variable $\mu \in \mathbb{R}$. For fixed λ, the objective function is maximized when the derivative is zero, that is,

$$-1 + e^{-\mu-1} \sum_{i=1}^{n} e^{-(A^i)^\top \lambda} = 0,$$

which yields

$$\mu = \log\left(\sum_{i=1}^{n} e^{-(A^i)^\top \lambda}\right) - 1.$$

By plugging the above value back into the objective function of the dual, we obtain the following program:

$$\begin{aligned} &\text{maximize} \quad -b^\top \lambda - \log\left(\sum_{i=1}^{n} e^{-(A^i)^\top \lambda}\right) \\ &\text{subject to} \quad \lambda \geq 0. \end{aligned}$$

The entropy minimization problem is another problem for which Theorem 50.18 applies, and thus can be solved using the dual program. Indeed, the Lagrangian of the primal program is given by

$$L(x, \lambda, \mu) = \sum_{i-1}^{n} x_i \log x_i + \lambda^\top (Ax - b) + \mu(\mathbf{1}^\top x - 1).$$

Using the second derivative criterion for convexity, we see that $L(x, \lambda, \mu)$ is strictly convex for $x \in \mathbb{R}^n_+$ and is bounded below, so it has a unique minimum which is obtain by setting the gradient ∇L_x to zero. We have

$$\nabla L_x = \begin{pmatrix} \log x_1 + 1 + (A^1)^\top \lambda + \mu \\ \vdots \\ \log x_n + 1 + (A^n)^\top \lambda + \mu. \end{pmatrix}$$

so by setting ∇L_x to 0 we obtain

$$x_i = e^{-((A^n)^\top \lambda + \mu + 1)}, \quad i = 1, \ldots, n. \tag{$*$}$$

By Theorem 50.18, since the objective function is convex and the constraints are affine, if the primal has a solution then so does the dual, and λ and μ constitute an optimal solution of the dual, then $x = (x_1, \ldots, x_n)$ given by the equations in $(*)$ is an optimal solution of the primal.

Other examples are given in Boyd and Vandenberghe; see [29], Section 5.1.6.

The derivation of the dual function of Problem (SVM$_{h1}$) from Section 50.5 involves a similar type of reasoning.