- *Least squares problems*.
- Existence of a least squares solution of smallest norm (Theorem 23.1).
- The *pseudo-inverse* A^+ of a matrix A.
- The least squares solution of smallest norm is given by the pseudo-inverse (Theorem 23.2)
- Projection properties of the pseudo-inverse.
- The pseudo-inverse of a normal matrix.
- The *Penrose characterization* of the pseudo-inverse.
- Data compression and SVD.
- Best approximation of rank $< r$ of a matrix.
- *Principal component analysis*.
- Review of basic statistical concepts: *mean, variance, covariance*, *covariance matrix*.
- Centered data, *centroid*.
- The *principal components* (*PCA*).
- The *Rayleigh–Ritz theorem* (Theorem 23.10).
- The main theorem: *SVD yields PCA* (Theorem 23.11).
- Best affine approximation.
- SVD yields a best affine approximation (Theorem 23.12).
- Face recognition, eigenfaces.

23.7 Problems

Problem 23.1. Consider the overdetermined system in the single variable x:

$$a_1 x = b_1, \ldots, a_m x = b_m,$$

with $a_1^2 + \cdots + a_m^2 \neq 0$. Prove that the least squares solution of smallest norm is given by

$$x^+ = \frac{a_1 b_1 + \cdots + a_m b_m}{a_1^2 + \cdots + a_m^2}.$$