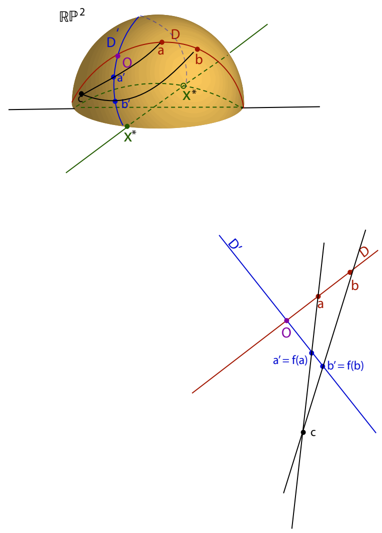


Figure 26.12: An illustration of the perspectivity construction of Proposition 26.6.

$h(d) = d$. Thus by Proposition 26.6, the projectivity $h\colon \langle a, a'\rangle \to \langle c, c'\rangle$ is a perspectivity. Since

$$\begin{aligned} h(a) &= g(f(a)) = g(b) = c, \\ h(a') &= g(f(a')) = g(b') = c', \end{aligned}$$

the intersection $q$ of $\langle a, c\rangle$ and $\langle a', c'\rangle$ is the center of the perspectivity $h$. Also note that the point $m = \langle a, a'\rangle \cap \langle p, r\rangle$ and its image $h(m)$ are both on the line $\langle p, r\rangle$, since $r$ is the center of $f$ and $p$ is the center of $g$. Since $h$ is a perspectivity of center $q$, the line $\langle m, h(m)\rangle = \langle p, r\rangle$ passes through $q$, which proves the proposition. $\square$

Desargues's theorem is illustrated in Figure 26.13. It can also be shown that every projectivity between two distinct lines is the composition of two perspectivities (not in a unique way). An elegant proof of Pappus's theorem can also be given using perspectivities.

## 26.6 Finding a Homography Between Two Projective Frames

In this section we present a method for finding the matrix (up to a scalar) of the unique homography (bijective projective transformation) mapping one projective frame to an other projective frame. This problem arises notably in computer vision in the context of image morphing.

We begin with the simple case of two nondegenerate quadrilatrerals $([p_1], [p_2], [p_3], [p_4])$ and $[(q_1], [q_2], [q_3], [q_4])$ in $\mathbb{RP}^2$, that is, two projective frames, which means that $(p_1, p_2, p_3)$