Proof. We can prove the above identity assuming that x^* and y^* are of the form e_I^* and e_J^* using Proposition 34.18 and leave the details as an exercise for the reader. $\square$

Thus, $\lrcorner\colon E \times \bigwedge^{q+1} E^* \longrightarrow \bigwedge^q E^*$ is almost an anti-derivation, except that the sign $(-1)^s$ is applied to the wrong factor.

We have a similar identity for the other version of the left hook

$$\lrcorner\colon E^* \times \bigwedge^{q+1} E \longrightarrow \bigwedge^{q} E,$$

namely

$$u^* \lrcorner (x \wedge y) = (-1)^s (u^* \lrcorner x) \wedge y + x \wedge (u^* \lrcorner y)$$

for every $u^* \in E^*$, $x \in \bigwedge^{q+1-s} E$, and $y \in \bigwedge^s E$.

An application of this formula when $q = 3$ and $s = 2$ yields an interesting equation. In this case, $u^* \in E^*$ and $x, y \in \bigwedge^2 E$, so we get

$$u^* \lrcorner (x \wedge y) = (u^* \lrcorner x) \wedge y + x \wedge (u^* \lrcorner y).$$

In particular, for $x = y$, since $x \in \bigwedge^2 E$ and $u^* \lrcorner x \in E$, Proposition 34.12 implies that $(u^* \lrcorner x) \wedge x = x \wedge (u^* \lrcorner x)$, and we obtain

$$u^* \lrcorner (x \wedge x) = 2((u^* \lrcorner x) \wedge x). \tag{$\dagger$}$$

As a consequence, $(u^* \lrcorner x) \wedge x = 0$ iff $u^* \lrcorner (x \wedge x) = 0$. We will use this identity together with Proposition 34.25 to prove that a 2-vector $x \in \bigwedge^2 E$ is decomposable iff $x \wedge x = 0$.

It is also possible to define a *right interior product or right hook* $\llcorner$, using multiplication on the left rather than multiplication on the right. Then we use the maps

$$\llcorner\colon \bigwedge^{p+q} E^* \times \bigwedge^{p} E \longrightarrow \bigwedge^{q} E^*$$

to make the following definition.

Definition 34.10. Let $u \in \bigwedge^p E$ and $z^* \in \bigwedge^{p+q} E^*$. We define $z^* \llcorner u \in \bigwedge^q E^*$ to be the q-vector uniquely defined as

$$\langle z^* \llcorner u, v\rangle = \langle z^*, u \wedge v\rangle, \qquad \text{for all } v \in \textstyle\bigwedge^q E.$$

This time we can prove that

$$z^* \llcorner (u \wedge v) = (z^* \llcorner u) \llcorner v,$$

so the family of operators $\llcorner_{p,q}$ defines a right action

$$\llcorner\colon \bigwedge E^* \times \bigwedge E \longrightarrow \bigwedge E^*$$