Consequently we find that

$$x^{k+3} = z^{k+2} + \frac{1}{\rho}\lambda^{k+2} - \frac{1}{\rho} = x^{k+2} - \frac{1}{\rho}.$$

By induction, we obtain

$$x^{k+3} = x^2 - \frac{k+1}{\rho}, \quad \text{for all } k \geq 0,$$

which shows that x^{k+3} goes to $-\infty$ when k goes to infinity, and since $x^{k+2} = z^{k+2}$, similarly z^{k+3} goes to $-\infty$ when k goes to infinity.

52.5 Stopping Criteria

Going back to Inequality (A2),

$$p^{k+1} - p^* \leq -(\lambda^{k+1})^\top r^{k+1} - \rho(B(z^{k+1} - z^k))^\top(-r^{k+1} + B(z^{k+1} - z^*)), \tag{A2}$$

using the fact that $Ax^* + Bz^* - c = 0$ and $r^{k+1} = Ax^{k+1} + Bz^{k+1} - c$, we have

$$\begin{aligned} -r^{k+1} + B(z^{k+1} - z^*) &= -Ax^{k+1} - Bz^{k+1} + c + B(z^{k+1} - z^*) \\ &= -Ax^{k+1} + c - Bz^* \\ &= -Ax^{k+1} + Ax^* = -A(x^{k+1} - x^*), \end{aligned}$$

so (A2) can be rewritten as

$$p^{k+1} - p^* \leq -(\lambda^{k+1})^\top r^{k+1} + \rho(B(z^{k+1} - z^k))^\top A(x^{k+1} - x^*),$$

or equivalently as

$$p^{k+1} - p^* \leq -(\lambda^{k+1})^\top r^{k+1} + (x^{k+1} - x^*)^\top \rho A^\top B(z^{k+1} - z^k). \tag{s_1}$$

We define the *dual residual* as

$$s^{k+1} = \rho A^\top B(z^{k+1} - z^k),$$

the quantity $r^{k+1} = Ax^{k+1} + Bz^{k+1} - c$ being the *primal residual*. Then (s_1) can be written as

$$p^{k+1} - p^* \leq -(\lambda^{k+1})^\top r^{k+1} + (x^{k+1} - x^*)^\top s^{k+1}. \tag{s}$$

Inequality (s) shows that when the residuals r^k and s^k are small, then p^k is close to p^* from below. Since x^* is unknown, we can't use this inequality, but if we have a guess that $\left\|x^k - x^*\right\| \leq d$, then using Cauchy–Schwarz we obtain

$$p^{k+1} - p^* \leq \left\|\lambda^{k+1}\right\| \left\|r^{k+1}\right\| + d\left\|s^{k+1}\right\|.$$