Since $\max\{2p_f/m, 2q_f/m\} \leq \nu$ implies that $(p_f + q_f)/m \leq \nu$ and $p_{sf} \leq p_f$, $q_{sf} \leq q_f$, we have

$$\nu - \frac{p_{sf} + q_{sf}}{m} \geq 0, \tag{$*_2$}$$

and so $\omega(\theta) \leq \omega(0)$. If inequality $(*_2)$ is strict, then this contradicts the optimality of the original solution. Therefore, $\nu = (p_{sf} + q_{sf})/m$, $\omega(\theta) = \omega(0)$ and $(w, b, \epsilon - \theta, \xi + \theta, \xi' + \theta)$ is an optimal solution such that either

$$w^\top x_i + b - y_i = \epsilon - \theta \quad \text{for some } i \notin (E_\lambda \cup E_\mu)$$

or

$$-w^\top x_i - b + y_i = \epsilon - \theta \quad \text{for some } i \notin (E_\lambda \cup E_\mu).$$

We are now reduced to *Case 1a* or or *Case 2a*.

Case 2. We have

$$\begin{aligned} w^\top x_i + b - y_i &\leq \epsilon & & i \notin (E_\lambda \cup E_\mu) \\ -w^\top x_i - b + y_i &< \epsilon & & i \notin (E_\lambda \cup E_\mu). \end{aligned}$$

Again there are two subcases.

Case 2a. Assume that there is some $i \notin (E_\lambda \cup E_\mu)$ such that $w^\top x_i + b - y_i = \epsilon$. Our strategy is to decrease ϵ and decrease b by a small amount θ in such a way that some inequality $-w^\top x_j - b + y_j < \epsilon$ becomes an equation for some $j \notin (E_\lambda \cup E_\mu)$. Geometrically, this amounts to lowering the separating hyperplane $H_{w,b}$ and decreasing the width of the slab, keeping the blue margin hyperplane unchanged. See Figure 56.10.

The inequalities imply that

$$-\epsilon < w^\top x_i + b - y_i \leq \epsilon.$$

Let us pick θ such that

$$\theta = (1/2)\min\{\epsilon - (-w^\top x_i - b + y_i) \mid i \notin (E_\lambda \cup E_\mu)\}.$$

Our hypotheses imply that $\theta > 0$, and we have $\theta \leq \epsilon$, because $(1/2)(\epsilon - (-w^\top x_i - b + y_i)) \leq \epsilon$ is equivalent to $\epsilon - (-w^\top x_i - b + y_i) \leq 2\epsilon$ which is equivalent to $w^\top x_i + b - y_i \leq \epsilon$ which holds for all $i \notin (E_\lambda \cup E_\mu)$ by hypothesis.

We can write

$$\begin{aligned} w^\top x_i + b - \theta - y_i &= \epsilon - \theta + \xi_i & \xi_i &> 0 & & i \in E_\lambda \\ -w^\top x_j - (b - \theta) + y_j &= \epsilon - \theta + \xi'_j + 2\theta & \xi'_j &> 0 & & j \in E_\mu \\ w^\top x_i + b - \theta - y_i &\leq \epsilon - \theta & & & & i \notin (E_\lambda \cup E_\mu) \\ -w^\top x_i - (b - \theta) + y_i &\leq \epsilon - \theta & & & & i \notin (E_\lambda \cup E_\mu). \end{aligned}$$