Remark: For numerical stability, it is preferable to use $w_{k+1} = r_{k+1,k+1}\, e_{k+1} + e^{-i\theta_{k+1}} u''_{k+1}$ instead of $w_{k+1} = r_{k+1,k+1}\, e_{k+1} - e^{-i\theta_{k+1}} u''_{k+1}$. The effect of that choice is that the diagonal entries in R will be of the form $-e^{i\theta_j} r_{j,j} = e^{i(\theta_j+\pi)} r_{j,j}$. Of course, we can make these entries nonegative by applying

$$h_{n+1} = \rho_{e_n,\, \pi-\theta_n} \circ \cdots \circ \rho_{e_1,\pi-\theta_1}$$

after h_n.

As in the Euclidean case, Proposition 28.7 immediately implies the QR-decomposition for arbitrary complex $n \times n$-matrices, where Q is now unitary (see Kincaid and Cheney [102], Golub and Van Loan [80], Trefethen and Bau [176], or Ciarlet [41]).

Proposition 28.8. *For every complex $n \times n$-matrix A, there is a sequence $H_1, \ldots, H_{n-1}$ of matrices, where each H_i is either a Householder matrix or the identity, and an upper triangular matrix R, such that*

$$R = H_{n-1} \cdots H_2 H_1 A.$$

As a corollary, there is a pair of matrices Q, R, where Q is unitary and R is upper triangular, such that $A = QR$ (a QR-decomposition of A). Furthermore, R can be chosen so that its diagonal entries are nonnegative. This can be achieved by a diagonal matrix D with entries such that $|d_{ii}| = 1$ for $i = 1, \ldots, n$, and we have $A = \widetilde{Q}\widetilde{R}$ with

$$\widetilde{Q} = H_1 \cdots H_{n-1} D, \quad \widetilde{R} = D^* R,$$

where $\widetilde{R}$ is upper triangular and has nonnegative diagonal entries

Proof. It is essentially identical to the proof of Proposition 13.4, and we leave the details as an exercise. For the last statement, observe that $h_n \circ \cdots \circ h_1$ is also an isometry. ∎

As in the Euclidean case, the QR-decomposition has applications to least squares problems. It is also possible to convert any complex matrix to bidiagonal form.

28.2 Affine Isometries (Rigid Motions)

In this section, we study very briefly the affine isometries of a Hermitian space. Most results holding for Euclidean affine spaces generalize without any problems to Hermitian spaces.

The characterization of the set of fixed points of an affine map is unchanged. Similarly, every affine isometry f (of a Hermitian space) can be written uniquely as

$$f = t \circ g, \quad \text{with} \quad t \circ g = g \circ t,$$

where g is an isometry having a fixed point, and t is a translation by a vector τ such that $\overrightarrow{f}(\tau) = \tau$, and with some additional nice properties (see Proposition 28.13). A generalization