(2) If A is invertible and the diagonal entries of R are positive, it can be shown that Q and R are unique.

(3) If we allow negative diagonal entries in R, the matrix H_n may be omitted ($H_n = I$).

(4) The method allows the computation of the determinant of A. We have

$$\det(A) = (-1)^m r_{1,1} \cdots r_{n,n},$$

where m is the number of Householder matrices (not the identity) among the H_i.

(5) The "condition number" of the matrix A is preserved (see Strang [170], Golub and Van Loan [80], Trefethen and Bau [176], Kincaid and Cheney [102], or Ciarlet [41]). This is very good for numerical stability.

(6) The method also applies to a rectangular $m \times n$ matrix. If $m \geq n$, then R is an $n \times n$ upper triangular matrix and Q is an $m \times n$ matrix such that $Q^\top Q = I_n$.

The following `Matlab` functions implement the QR-factorization method of a real square (possibly singular) matrix A using Householder reflections

The main function `houseqr` computes the upper triangular matrix R obtained by applying Householder reflections to A. It makes use of the function `house`, which computes a unit vector u such that given a vector $x \in \mathbb{R}^p$, the Householder transformation $P = I - 2uu^\top$ sets to zero all entries in x but the first entry x_1. It only applies if $\|x(2:p)\|_1 = |x_2| + \cdots + |x_p| > 0$. Since computations are done in floating point, we use a tolerance factor *tol*, and if $\|x(2:p)\|_1 \leq tol$, then we return $u = 0$, which indicates that the corresponding Householder transformation is the identity. To make sure that $\|Px\|$ is as large as possible, we pick $uu = x + \mathrm{sign}(x_1)\, \|x\|_2\, e_1$, where $\mathrm{sign}(z) = 1$ if $z \geq 0$ and $\mathrm{sign}(z) = -1$ if $z < 0$. Note that as a result, diagonal entries in R may be negative. We will take care of this issue later.

```
function s = signe(x)
%  if x >= 0, then signe(x) = 1
%  else if x < 0 then signe(x) = -1
%

if x < 0
   s = -1;
else
    s = 1;
end
end
```