Proposition 34.20. *The following identities hold:*

$$\begin{aligned} z^* \llcorner u &= (-1)^{pq}\, u \lrcorner z^* \quad \text{for all } u \in \textstyle\bigwedge^p E \text{ and all } z^* \in \textstyle\bigwedge^{p+q} E^* \\ z \llcorner u^* &= (-1)^{pq}\, u^* \lrcorner z \quad \text{for all } u^* \in \textstyle\bigwedge^p E^* \text{ and all } z \in \textstyle\bigwedge^{p+q} E. \end{aligned}$$

Therefore the left and right hooks are not independent, and in fact each one determines the other. As a consequence, we can restrict our attention to only one of the hooks, for example the left hook, but there are a few situations where it is nice to use both, for example in Proposition 34.23.

A version of Proposition 34.18 holds for right hooks, but beware that the indices in $\rho_{L-H,H}$ are permuted. This permutation has to do with the fact that the left hook and the right hook are related *via* a sign factor.

Proposition 34.21. *For any basis $(e_1, \ldots, e_n)$ of E the following properties hold:*

(1) For the right hook

$$\llcorner : \bigwedge^{p+q} E \times \bigwedge^{p} E^* \longrightarrow \bigwedge^{q} E$$

we have

$$\begin{aligned} e_L \llcorner e_H^* &= 0 \quad \text{if } H \not\subseteq L \\ e_L \llcorner e_H^* &= \rho_{H,L-H} e_{L-H} \quad \text{if } H \subseteq L. \end{aligned}$$

(2) For the right hook

$$\llcorner : \bigwedge^{p+q} E^* \times \bigwedge^{p} E \longrightarrow \bigwedge^{q} E^*$$

we have

$$\begin{aligned} e_L^* \llcorner e_H &= 0 \quad \text{if } H \not\subseteq L \\ e_L^* \llcorner e_H &= \rho_{H,L-H} e_{L-H}^* \quad \text{if } H \subseteq L. \end{aligned}$$

Remark: Our definition of left hooks as left actions $\lrcorner : \bigwedge^p E \times \bigwedge^{p+q} E^* \longrightarrow \bigwedge^q E^*$ and $\lrcorner : \bigwedge^p E^* \times \bigwedge^{p+q} E \longrightarrow \bigwedge^q E$ and right hooks as right actions $\llcorner : \bigwedge^{p+q} E^* \times \bigwedge^p E \longrightarrow \bigwedge^q E^*$ and $\llcorner : \bigwedge^{p+q} E \times \bigwedge^p E^* \longrightarrow \bigwedge^q E$ is identical to the definition found in Fulton and Harris [68] (Appendix B). However, the reader should be aware that this is not a universally accepted notation. In fact, the left hook $u^* \lrcorner z$ defined in Bourbaki [25] is our right hook $z \llcorner u^*$, up to the sign $(-1)^{p(p-1)/2}$. This has to do with the fact that Bourbaki uses a different pairing which also involves an extra sign, namely

$$\langle v^*, u^* \lrcorner z\rangle = (-1)^{p(p-1)/2} \langle u^* \wedge v^*, z\rangle.$$