- Newton's method.
- Newton step.
- Newton decrement.
- Damped Newton phase.
- Quadratically convergent phase.
- Self-concordant functions.
- Conjugate gradient method.
- Projected gradient methods.
- Penalty methods.

49.14 Problems

Problem 49.1. Consider the function $J\colon \mathbb{R}^n \to \mathbb{R}$ given by

$$J(v) = \frac{1}{2}\langle Av, v\rangle - \langle b, v\rangle + g(v),$$

where A is a real $n \times n$ symmetric positive definite matrix, $b \in \mathbb{R}^n$, and $g\colon \mathbb{R}^n \to \mathbb{R}$ is a continuous (not necessarily differentiable) convex function such that $g(v) \geq 0$ for all $v \in \mathbb{R}^n$, and let U be a nonempty, bounded, closed, convex subset of $\mathbb{R}^n$.

(1) Prove that there is a unique element $u \in U$ such that

$$J(u) = \inf_{v \in U} J(v).$$

Hint. Prove that J is strictly convex on $\mathbb{R}^n$.

(2) Check that

$$J(v) - J(u) = \langle Au - b, v - u\rangle + g(v) - g(u) + \frac{1}{2}\langle A(v-u), v-u\rangle.$$

Prove that an element $u \in U$ minimizes J in U iff

$$\langle Au - b, v - u\rangle + g(v) - g(u) \geq 0 \quad \text{for all } v \in U.$$

Problem 49.2. Consider n piecewise C^1 functions $\varphi_i\colon [0,1] \to \mathbb{R}$ and assume that these functions are linearly independent and that

$$\sum_{i=1}^{n} \varphi_i(x) = 1 \quad \text{for all } x \in [0,1].$$