Dual of Soft margin kernel SVM (SVM_{s2}):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\mathbf{K}\begin{pmatrix}\lambda\\ \mu\end{pmatrix} - \begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\mathbf{1}_{p+q}\\
&\text{subject to}\\
&\qquad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0\\
&\qquad 0 \leq \lambda_i \leq K, \quad i = 1, \ldots, p\\
&\qquad 0 \leq \mu_j \leq K, \quad j = 1, \ldots, q,
\end{aligned}$$

where $\mathbf{K}$ is the $\ell \times \ell$ kernel symmetric matrix (with $\ell = p + q$) given at the end of Section 54.1. We also find that

$$w = \sum_{i=1}^p \lambda_i \varphi(u_i) - \sum_{j=1}^q \mu_j \varphi(v_j),$$

so

$$b = \frac{1}{2}\bigg(\sum_{i=1}^p \lambda_i(\kappa(u_i, u_{i_0}) + \kappa(u_i, v_{j_0})) - \sum_{j=1}^q \mu_j(\kappa(v_j, u_{i_0}) + \kappa(v_j, v_{j_0}))\bigg),$$

and the classification function

$$f(x) = \mathrm{sgn}(\langle w, \varphi(x)\rangle - b)$$

is given by

$$\begin{aligned}
f(x) = \mathrm{sgn}\bigg(&\sum_{i=1}^p \lambda_i(2\kappa(u_i, x) - \kappa(u_i, u_{i_0}) - \kappa(u_i, v_{j_0}))\\
&- \sum_{j=1}^q \mu_j(2\kappa(v_j, x) - \kappa(v_j, u_{i_0}) - \kappa(v_j, v_{j_0}))\bigg).
\end{aligned}$$

54.4 Solving SVM (SVM_{s2}) Using ADMM

In order to solve (SVM_{s2}) using ADMM we need to write the matrix corresponding to the constraints in equational form,

$$\begin{aligned}
\sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j &= 0\\
\lambda_i + \alpha_i &= K, \quad i = 1, \ldots, p\\
\mu_j + \beta_j &= K, \quad j = 1, \ldots, q.
\end{aligned}$$