- Saddle point.
- KKT conditions.
- Qualified constraints.
- Duality gap.

51.8 Problems

Problem 51.1. Prove Proposition 51.1.

Problem 51.2. Prove Proposition 51.2.

Problem 51.3. Prove Proposition 51.3.

Problem 51.4. Prove that the convex function defined in Example 51.4 has the property that the limit along any line segment from $(0, 0)$ to a point in the open right half-plane is 0.

Problem 51.5. Check that the normal cone to C at a is a convex cone.

Problem 51.6. Prove that $\partial f(x)$ is closed and convex.

Problem 51.7. For Example 51.6, with $f(x) = \|x\|_\infty$, prove that $\partial f(0)$ is the polyhedron

$$\partial f(0) = \mathrm{conv}\{\pm e_1, \ldots, \pm e_n\}.$$

Problem 51.8. For Example 51.7, with

$$f(x) = \begin{cases} -(1 - |x|^2)^{1/2} & \text{if } |x| \leq 1 \\ +\infty & \text{otherwise.} \end{cases}$$

prove that f is subdifferentiable (in fact differentiable) at x when $|x| < 1$, but $\partial f(x) = \emptyset$ when $|x| \geq 1$, even though $x \in \mathrm{dom}(f)$ for $|x| = 1$

Problem 51.9. Prove Proposition 51.15.

Problem 51.10. Prove that as a convex function of u, the effective domain of the function $u \mapsto f'(x; u)$ is the convex cone generated by $\mathrm{dom}(f) - x$.

Problem 51.11. Prove Proposition 51.28.

Problem 51.12. Prove Proposition 51.33.

Problem 51.13. Prove that Proposition 51.38(2) also holds in the following cases:

(1) C is a $\mathcal{H}$-polyhedron and $\mathbf{relint}(\mathrm{dom}(h)) \cap C \neq \emptyset$

(2) h is polyhedral and $\mathrm{dom}(h) \cap \mathbf{relint}(C) \neq \emptyset$.

(3) Both h and C are polyhedral, and $\mathrm{dom}(h) \cap C \neq \emptyset$.