

## 4.5 Summary

The main concepts and results of this chapter are listed below:

- The representation of linear maps by *matrices*.
- The *matrix representation mapping*  $M: \text{Hom}(E, F) \rightarrow M_{n,p}$  and the representation isomorphism (Proposition 4.2).
- *Change of basis matrix* and Proposition 4.5.

## 4.6 Problems

**Problem 4.1.** Prove that the column vectors of the matrix  $A_1$  given by

$$A_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 7 \\ 1 & 3 & 1 \end{pmatrix}$$

are linearly independent.

Prove that the coordinates of the column vectors of the matrix  $B_1$  over the basis consisting of the column vectors of  $A_1$  given by

$$B_1 = \begin{pmatrix} 3 & 5 & 1 \\ 1 & 2 & 1 \\ 4 & 3 & -6 \end{pmatrix}$$

are the columns of the matrix  $P_1$  given by

$$P_1 = \begin{pmatrix} -27 & -61 & -41 \\ 9 & 18 & 9 \\ 4 & 10 & 8 \end{pmatrix}.$$

Give a nontrivial linear dependence of the columns of  $P_1$ . Check that  $B_1 = A_1 P_1$ . Is the matrix  $B_1$  invertible?

**Problem 4.2.** Prove that the column vectors of the matrix  $A_2$  given by

$$A_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 3 \end{pmatrix}$$

are linearly independent.