and

$$\sum_{i\in I}\lambda_i(b+h(v_i)) = b+\sum_{i\in I}\lambda_i\overrightarrow{b(b+h(v_i))} = b+\sum_{i\in I}\lambda_i h(v_i),$$

we have

$$\begin{aligned}
f\left(\sum_{i\in I}\lambda_i(a+v_i)\right) &= f\left(a+\sum_{i\in I}\lambda_i v_i\right)\\
&= b+h\left(\sum_{i\in I}\lambda_i v_i\right)\\
&= b+\sum_{i\in I}\lambda_i h(v_i)\\
&= \sum_{i\in I}\lambda_i(b+h(v_i))\\
&= \sum_{i\in I}\lambda_i f(a+v_i),
\end{aligned}$$

as claimed. □

Note that the condition $\sum_{i\in I}\lambda_i = 1$ was implicitly used (in a hidden call to Proposition 24.1) in deriving that

$$\sum_{i\in I}\lambda_i(a+v_i) = a+\sum_{i\in I}\lambda_i v_i$$

and

$$\sum_{i\in I}\lambda_i(b+h(v_i)) = b+\sum_{i\in I}\lambda_i h(v_i).$$

As a more concrete example, the map

$$\begin{pmatrix}x_1\\x_2\end{pmatrix}\mapsto\begin{pmatrix}1&2\\0&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}+\begin{pmatrix}3\\1\end{pmatrix}$$

defines an affine map in $\mathbb{A}^2$. It is a "shear" followed by a translation. The effect of this shear on the square (a,b,c,d) is shown in Figure 24.18. The image of the square (a,b,c,d) is the parallelogram (a',b',c',d').

Let us consider one more example. The map

$$\begin{pmatrix}x_1\\x_2\end{pmatrix}\mapsto\begin{pmatrix}1&1\\1&3\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}+\begin{pmatrix}3\\0\end{pmatrix}$$

is an affine map. Since we can write

$$\begin{pmatrix}1&1\\1&3\end{pmatrix}=\sqrt{2}\begin{pmatrix}\sqrt{2}/2&-\sqrt{2}/2\\2/2&\sqrt{2}/2\end{pmatrix}\begin{pmatrix}1&2\\0&1\end{pmatrix},$$