

Proof. The fact that h leaves the set of circular points $\{I, J\}$ fixed means that either $h(I) = I$ and $h(J) = J$ or $h(I) = J$ and $h(J) = I$. If we define I' and J' by

$$I' = (1, -\epsilon i, 0) \quad \text{and} \quad J' = (1, \epsilon i, 0)$$

where $\epsilon = \pm 1$, then the fact that h leaves the set of circular points $\{I, J\}$ fixed is equivalent to

$$h(I) = I' \quad \text{and} \quad h(J) = J'.$$

Assume that h is represented by the invertible matrix

$$A = \begin{pmatrix} a & a' & a'' \\ b & b' & b'' \\ c & c' & c'' \end{pmatrix}.$$

Then $h(I) = I'$ and $h(J) = J'$ means that there is some nonzero scalars $\lambda, \mu \in \mathbb{C}$ such

$$\begin{pmatrix} a & a' & a'' \\ b & b' & b'' \\ c & c' & c'' \end{pmatrix} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ -\epsilon i \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a & a' & a'' \\ b & b' & b'' \\ c & c' & c'' \end{pmatrix} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ \epsilon i \\ 0 \end{pmatrix}.$$

We obtain the following equations:

$$\begin{array}{ll} \lambda = a - ia' & \mu = a + ia' \\ -\lambda\epsilon i = b - ib' & \mu\epsilon i = b + ib' \\ 0 = c + ic' & 0 = c - ic'. \end{array}$$

By adding the two equations on the first row we obtain

$$\lambda + \mu = 2a,$$

by subtracting the first equation from the second on the second row we obtain

$$(\lambda + \mu)\epsilon i = 2ib',$$

so we get

$$b' = \epsilon a.$$

By subtracting the first equation from the second on the first row we obtain

$$\mu - \lambda = 2ia',$$

and by adding the equations on the second row we obtain

$$(\mu - \lambda)\epsilon i = 2b,$$