and thus

$$\lambda \in \{z \in \mathbb{C} \mid |z - a_{k\,k}| \leq R'_k(A)\} \subseteq G(A),$$

as claimed.

(1) Strict row diagonal dominance implies that 0 does not belong to any of the Gershgorin discs, so all eigenvalues of A are nonzero, and A is invertible.

(2) If A is strictly row diagonally dominant and $a_{i\,i} > 0$ for $i = 1, \ldots, n$, then each of the Gershgorin discs lies strictly in the right half-plane, so every eigenvalue of A has a strictly positive real part. $\square$

In particular, Theorem 15.9 implies that if a symmetric matrix is strictly row diagonally dominant and has strictly positive diagonal entries, then it is positive definite. Theorem 15.9 is sometimes called the *Gershgorin–Hadamard theorem*.

Since A and $A^\top$ have the same eigenvalues (even for complex matrices) we also have a version of Theorem 15.9 for the discs of radius

$$C'_j(A) = \sum_{\substack{i=1\\ i\neq j}}^{n} |a_{i\,j}|,$$

whose domain $G(A^\top)$ is given by

$$G(A^\top) = \bigcup_{i=1}^{n} \{z \in \mathbb{C} \mid |z - a_{i\,i}| \leq C'_i(A)\}.$$

Figure 15.2 shows $G(A^\top)$ for $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & i & 6 \\ 7 & 8 & 1+i \end{pmatrix}$.

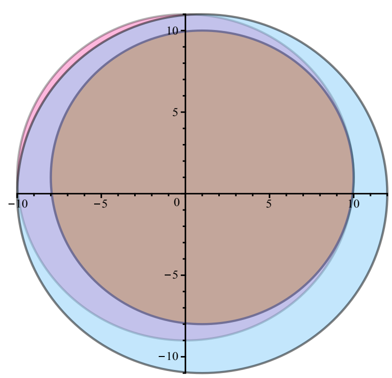

Figure 15.2: Let A be the 3×3 matrix specified at the end of Definition 15.5. For this particular A, we find that $C'_1(A) = 11$, $C'_2(A) = 10$, and $C'_3(A) = 9$. The pale blue disk is $|z - 1| \leq 11$, the pink disk is $|z - i| \leq 10$, the ocher disk is $|z - 1 - i| \leq 9$, and $G(A^\top)$ is the union of these three disks.