The interest of Proposition 54.4 lies in the fact that it allows us to compute b and η knowing only a *single* support vector.

In practice we can only find support vectors of type 1 so Proposition 54.4 is useful if we can only find some blue support vector of type 1 *or* some red support vector of type 1.

As earlier, if we define I_λ and I_μ as

$$\begin{aligned} I_\lambda &= \{i \in \{1, \ldots, p\} \mid 0 < \lambda_i < K_s\} \\ I_\mu &= \{j \in \{1, \ldots, q\} \mid 0 < \mu_j < K_s\}, \end{aligned}$$

then we have the following cases to compute η and b.

(1) If $I_\lambda \neq \emptyset$ and $I_\mu \neq \emptyset$, then

$$\begin{aligned} b &= w^\top \left(\Big(\sum_{i \in I_\lambda} u_i\Big)/|I_\lambda| + \Big(\sum_{j \in I_\mu} v_j\Big)/|I_\mu| \right)/2 \\ \eta &= w^\top \left(\Big(\sum_{i \in I_\lambda} u_i\Big)/|I_\lambda| - \Big(\sum_{j \in I_\mu} v_j\Big)/|I_\mu| \right)/2. \end{aligned}$$

(2) If $I_\lambda \neq \emptyset$ and $I_\mu = \emptyset$, then

$$\begin{aligned} b &= -\eta + w^\top \Big(\sum_{i \in I_\lambda} u_i\Big)/|I_\lambda| \\ ((p+q)\nu - 2q_f)\eta &= (p_f - q_f) w^\top \Big(\sum_{i \in I_\lambda} u_i\Big)/|I_\lambda| + w^\top \Big(\sum_{i \in K_\mu} v_j - \sum_{i \in K_\lambda} u_i\Big) \\ &\quad + (p+q) \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix}. \end{aligned}$$

(3) If $I_\lambda = \emptyset$ and $I_\mu \neq \emptyset$, then

$$\begin{aligned} b &= \eta + w^\top \Big(\sum_{j \in I_\mu} v_j\Big)/|I_\mu| \\ ((p+q)\nu - 2p_f)\eta &= (p_f - q_f) w^\top \Big(\sum_{j \in I_\mu} v_j\Big)/|I_\mu| + w^\top \Big(\sum_{i \in K_\mu} v_j - \sum_{i \in K_\lambda} u_i\Big) \\ &\quad + (p+q) \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix}. \end{aligned}$$

The above formulae correspond to $K_s = 1/(p+q)$. In general we have to replace the rightmost $(p+q)$ by $1/K_s$.