**Problem 3.18.** Given two vectors spaces $E$ and $F$, let $(u_i)_{i\in I}$ be any basis of $E$ and let $(v_i)_{i\in I}$ be any family of vectors in $F$. Prove that the unique linear map $f\colon E \to F$ such that $f(u_i) = v_i$ for all $i \in I$ is surjective iff $(v_i)_{i\in I}$ spans $F$.

**Problem 3.19.** Let $f\colon E \to F$ be a linear map with $\dim(E) = n$ and $\dim(F) = m$. Prove that $f$ has rank 1 iff $f$ is represented by an $m \times n$ matrix of the form

$$A = uv^\top$$

with $u$ a nonzero column vector of dimension $m$ and $v$ a nonzero column vector of dimension $n$.

**Problem 3.20.** Find a nontrivial linear dependence among the linear forms

$$\varphi_1(x,y,z) = 2x - y + 3z, \quad \varphi_2(x,y,z) = 3x - 5y + z, \quad \varphi_3(x,y,z) = 4x - 7y + z.$$

**Problem 3.21.** Prove that the linear forms

$$\varphi_1(x,y,z) = x + 2y + z, \quad \varphi_2(x,y,z) = 2x + 3y + 3z, \quad \varphi_3(x,y,z) = 3x + 7y + z$$

are linearly independent. Express the linear form $\varphi(x,y,z) = x+y+z$ as a linear combination of $\varphi_1, \varphi_2, \varphi_3$.