Proof. (1) Consider the (usual) reflection about the hyperplane orthogonal to $w = v - e^{-i\theta}u$. We have

$$s(u) = u - 2\,\frac{(u \cdot (v - e^{-i\theta}u))}{\|v - e^{-i\theta}u\|^2}\,(v - e^{-i\theta}u).$$

We need to compute

$$-2u \cdot (v - e^{-i\theta}u) \quad \text{and} \quad (v - e^{-i\theta}u) \cdot (v - e^{-i\theta}u).$$

Since $u \cdot v = e^{i\theta}|u \cdot v|$, we have

$$e^{-i\theta}u \cdot v = |u \cdot v| \quad \text{and} \quad e^{i\theta}v \cdot u = |u \cdot v|.$$

Using the above and the fact that $\|u\| = \|v\|$, we get

$$\begin{aligned} -2u \cdot (v - e^{-i\theta}u) &= 2e^{i\theta}\,\|u\|^2 - 2u \cdot v, \\ &= 2e^{i\theta}(\|u\|^2 - |u \cdot v|), \end{aligned}$$

and

$$\begin{aligned} (v - e^{-i\theta}u) \cdot (v - e^{-i\theta}u) &= \|v\|^2 + \|u\|^2 - e^{-i\theta}u \cdot v - e^{i\theta}v \cdot u, \\ &= 2(\|u\|^2 - |u \cdot v|), \end{aligned}$$

and thus,

$$-2\,\frac{(u \cdot (v - e^{-i\theta}u))}{\|(v - e^{-i\theta}u)\|^2}\,(v - e^{-i\theta}u) = e^{i\theta}(v - e^{-i\theta}u).$$

But then,

$$s(u) = u + e^{i\theta}(v - e^{-i\theta}u) = u + e^{i\theta}v - u = e^{i\theta}v,$$

and $s(u) = e^{i\theta}v$, as claimed.

(2) This part is easier. Consider the Hermitian reflection

$$\rho_{v,\theta}(u) = u + (e^{i\theta} - 1)\,\frac{(u \cdot v)}{\|v\|^2}\,v.$$

We have

$$\begin{aligned} \rho_{v,\theta}(v) &= v + (e^{i\theta} - 1)\,\frac{(v \cdot v)}{\|v\|^2}\,v, \\ &= v + (e^{i\theta} - 1)v, \\ &= e^{i\theta}v. \end{aligned}$$

Thus, $\rho_{v,\theta}(v) = e^{i\theta}v$. Since $\rho_{v,\theta}$ is linear, changing the argument v to $e^{i\theta}v$, we get

$$\rho_{v,-\theta}(e^{i\theta}v) = v,$$

and thus, $\rho_{v,-\theta} \circ s(u) = v$. $\square$