gives the single relation

$$4e_1 = 0,$$

but the second generator e_2 cannot be eliminated. This matrix presents the module $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}$. On the other hand, the 1×2 matrix

$$R_2 = \begin{pmatrix} 4 & 0 \end{pmatrix}$$

gives two relations

$$4e_1 = 0,$$
$$0 = 0,$$

so the second generator can be eliminated and R_2 presents the module $\mathbb{Z}/4\mathbb{Z}$.

The rules of Proposition 35.8 make it possible to simplify a presentation matrix quite a lot in some cases. For example, consider the relation matrix

$$R = \begin{pmatrix} 3 & 8 & 7 & 9 \\ 2 & 4 & 6 & 6 \\ 1 & 2 & 2 & 1 \end{pmatrix}.$$

By subtracting 2 times row 3 from row 2 and subtracting 3 times row 3 from row 1, we get

$$\begin{pmatrix} 0 & 2 & 1 & 6 \\ 0 & 0 & 2 & 4 \\ 1 & 2 & 2 & 1 \end{pmatrix}.$$

After deleting column 1 and row 3, we get

$$\begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 4 \end{pmatrix}.$$

By subtracting 2 times row 1 from row 2, we get

$$\begin{pmatrix} 2 & 1 & 6 \\ -4 & 0 & -8 \end{pmatrix}.$$

After deleting column 2 and row 1, we get

$$\begin{pmatrix} -4 & -8 \end{pmatrix}.$$

By subtracting 2 times column 1 from column 2, we get

$$\begin{pmatrix} -4 & 0 \end{pmatrix}.$$

Finally, we can drop the second column and we get

$$(4),$$