if there is some rotation r such that $r(D_1) = D_2$ and $r(D_3) = D_4$.

It can be verified that the set of (oriented) angles of lines is a group isomorphic to the quotient group $\mathbf{SO}(2)/\{\mathrm{id}, -\mathrm{id}\}$, also denoted by $\mathbf{PSO}(2)$. In order to define the measure of the angle of two lines, the Euclidean plane E must be oriented. The measure of the angle $\widehat{D_1 D_2}$ of two lines is defined up to $k\pi$ ($k \in \mathbb{Z}$). The angle of two lines has a measure that is either θ or $\pi - \theta$, where $\theta \in [0, \pi[$, depending on the orientation of the plane. We now go back to the circular points.

Let (a_0, a_1, a_2, a_3) be any projective frame for $\widetilde{E}_{\mathbb{C}}$ such that (a_0, a_1) arises from an orthonormal basis (u_1, u_2) of $\overrightarrow{E}$ and the line at infinity H corresponds to $z = 0$ (where (x, y, z) are the homogeneous coordinates of a point w.r.t. (a_0, a_1, a_2, a_3)). Consider the points belonging to the intersection of the real conic Σ of equation

$$x^2 + y^2 - z^2 = 0$$

with the line at infinity $z = 0$. For such points, $x^2 + y^2 = 0$ and $z = 0$, and since

$$x^2 + y^2 = (y - ix)(y + ix),$$

we get exactly two points I and J of homogeneous coordinates $(1, -i, 0)$ and $(1, i, 0)$. The points I and J are called the *circular points*, or the *absolute points*, of $\widetilde{E}_{\mathbb{C}}$. They are complex points at infinity. Any line containing either I or J is called an *isotropic line*.

What is remarkable about I and J is that they allow the definition of the angle of two lines in terms of a certain cross-ratio. Indeed, consider two distinct real lines D_1 and D_2 in E, and let D_I and D_J be the isotropic lines joining $D_1 \cap D_2$ to I and J. We will compute the cross-ratio $[D_1, D_2, D_I, D_J]$. For this, we simply have to compute the cross-ratio of the four points obtained by intersecting D_1, D_2, D_I, D_J with any line not passing through $D_1 \cap D_2$. By changing frame if necessary, so that $D_1 \cap D_2 = a_0$, we can assume that the equations of the lines D_1, D_2, D_I, D_J are of the form

$$\begin{aligned} y &= m_1 x, \\ y &= m_2 x, \\ y &= -ix, \\ y &= ix, \end{aligned}$$

leaving the cases $m_1 = \infty$ and $m_2 = \infty$ as a simple exercise. If we choose $z = 0$ as the intersecting line, we need to compute the cross-ratio of the points $(D_1)_\infty = (1, m_1, 0)$, $(D_2)_\infty = (1, m_2, 0)$, $I = (1, -i, 0)$, and $J = (1, i, 0)$, and we get

$$[D_1, D_2, D_I, D_J] = [(D_1)_\infty, (D_2)_\infty, I, J] = \frac{(-i - m_1)}{(i - m_1)} \frac{(i - m_2)}{(-i - m_2)},$$

that is,

$$[D_1, D_2, D_I, D_J] = \frac{m_1 m_2 + 1 + i(m_2 - m_1)}{m_1 m_2 + 1 - i(m_2 - m_1)}.$$