**Definition 37.25.** Given a topological space, $(E, \mathcal{O})$, an *arc (or path)* is a continuous map, $\gamma\colon [a,b] \to E$, where $[a,b]$ is a closed interval of the real line, $\mathbb{R}$. The point $\gamma(a)$ is the *initial point* of the arc and the point $\gamma(b)$ is the *terminal point* of the arc. We say that $\gamma$ *is an arc joining* $\gamma(a)$ *and* $\gamma(b)$. See Figure 37.26. An arc is a *closed curve* if $\gamma(a) = \gamma(b)$. The set $\gamma([a,b])$ is the *trace* of the arc $\gamma$.

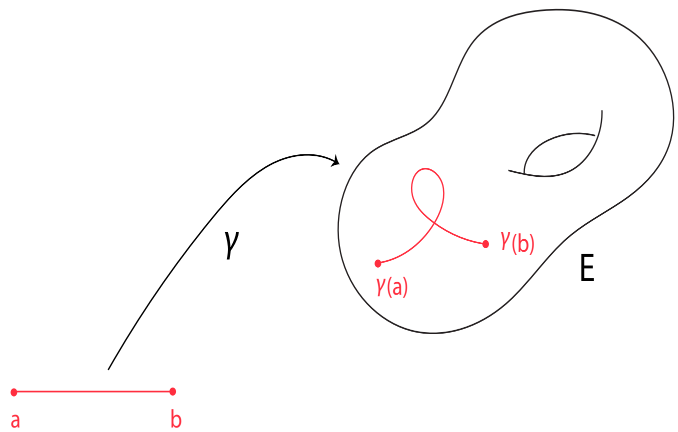


Figure 37.26: Let $E$ be the torus with subspace topology induced from $\mathbb{R}^3$ with red arc $\gamma([a,b])$. The torus is both arcwise connected and locally arcwise connected.

Typically, $a = 0$ and $b = 1$.

One should not confuse an arc, $\gamma\colon [a,b] \to E$, with its trace. For example, $\gamma$ could be constant, and thus, its trace reduced to a single point.

An arc is a *Jordan arc* if $\gamma$ is a homeomorphism onto its trace. An arc, $\gamma\colon [a,b] \to E$, is a *Jordan curve* if $\gamma(a) = \gamma(b)$ and $\gamma$ is injective on $[a,b)$. Since $[a,b]$ is connected, by Proposition 37.18, the trace $\gamma([a,b])$ of an arc is a connected subset of $E$.

Given two arcs $\gamma\colon [0,1] \to E$ and $\delta\colon [0,1] \to E$ such that $\gamma(1) = \delta(0)$, we can form a new arc defined as follows:

**Definition 37.26.** Given two arcs, $\gamma\colon [0,1] \to E$ and $\delta\colon [0,1] \to E$, such that $\gamma(1) = \delta(0)$, we can form their *composition (or product)*, $\gamma\delta$,, defined such that

$$\gamma\delta(t) = \begin{cases} \gamma(2t) & \text{if } 0 \le t \le 1/2; \\ \delta(2t-1) & \text{if } 1/2 \le t \le 1. \end{cases}$$

The *inverse*, $\gamma^{-1}$, *of the arc*, $\gamma$, is the arc defined such that $\gamma^{-1}(t) = \gamma(1-t)$, for all $t \in [0,1]$.

It is trivially verified that Definition 37.26 yields continuous arcs.