or

$$\int_0^1 (u'v' + cuv)dx = \int_0^1 fv dx, \quad \text{for all } v \in V. \tag{$**$}$$

Thus, it is natural to introduce the bilinear form $a\colon V \times V \to \mathbb{R}$ given by

$$a(u,v) = \int_0^1 (u'v' + cuv)dx, \quad \text{for all } u, v \in V,$$

and the linear form $\widetilde{f}\colon V \to \mathbb{R}$ given by

$$\widetilde{f}(v) = \int_0^1 f(x)v(x)dx, \quad \text{for all } v \in V.$$

Then, $(**)$ becomes

$$a(u,v) = \widetilde{f}(v), \quad \text{for all } v \in V.$$

We also introduce the *energy function* J given by

$$J(v) = \frac{1}{2}a(v,v) - \widetilde{f}(v) \quad v \in V.$$

Then, we have the following theorem.

Theorem 19.1. *Let u be any solution of the boundary problem (BP).*

(1) Then we have

$$a(u,v) = \widetilde{f}(v), \quad \text{for all } v \in V, \tag{WF}$$

where

$$a(u,v) = \int_0^1 (u'v' + cuv)dx, \quad \textit{for all } u, v \in V,$$

and

$$\widetilde{f}(v) = \int_0^1 f(x)v(x)dx, \quad \textit{for all } v \in V.$$

(2) If $c(x) \geq 0$ for all $x \in [0,1]$, then a function $u \in V$ is a solution of (WF) iff u minimizes $J(v)$, that is,

$$J(u) = \inf_{v \in V} J(v),$$

with

$$J(v) = \frac{1}{2}a(v,v) - \widetilde{f}(v) \quad v \in V.$$

Furthermore, u is unique.