

## 14.8 Summary

The main concepts and results of this chapter are listed below:

- *Semilinear maps.*
- *Sesquilinear forms; Hermitian forms.*
- *Quadratic form* associated with a sesquilinear form.
- *Polarization identities.*
- *Positive* and *positive definite* Hermitian forms; *pre-Hilbert spaces*, *Hermitian spaces*.
- *Gram matrix* associated with a Hermitian product.
- The *Cauchy–Schwarz inequality* and the *Minkowski inequality*.
- *Hermitian inner product*, *Hermitian norm*.
- The *parallelogram law*.
- The musical isomorphisms  $\flat: \overline{E} \rightarrow E^*$  and  $\sharp: E^* \rightarrow \overline{E}$ ; Theorem 14.6 ( $E$  is finite-dimensional).
- The *adjoint* of a linear map (with respect to a Hermitian inner product).
- Existence of orthonormal bases in a Hermitian space (Proposition 14.11).
- *Gram–Schmidt orthonormalization procedure*.
- *Linear isometries (unitary transformations)*.
- The *unitary group*, *unitary matrices*.
- The *unitary group*  $\mathbf{U}(n)$ .
- The *special unitary group*  $\mathbf{SU}(n)$ .
- *QR-Decomposition* for arbitrary complex matrices.
- The *Hadamard inequality* for complex matrices.
- The *Hadamard inequality* for Hermitian positive semidefinite matrices.
- Orthogonal projections and involutions; orthogonal reflections.
- Dual norms.
- Nuclear norm (also called trace norm).
- Matrix completion.