and $R = PQ$ such that

$$c_k = \sum_{i+j=k} a_i b_j.$$

We define the polynomial e_k such that $e_k(k) = 1$ and $e_k(i) = 0$ for $i \neq k$. We also denote e_0 by 1 when $k = 0$. Given a polynomial P, the $a_k = P(k) \in A$ are called the *coefficients of* P. If P is not the null polynomial, there is a greatest $n \geq 0$ such that $a_n \neq 0$ (and thus, $a_k = 0$ for all $k > n$) called the *degree of* P and denoted by $\deg(P)$. Then, P is written uniquely as

$$P = a_0 e_0 + a_1 e_1 + \cdots + a_n e_n.$$

When P is the null polynomial, we let $\deg(P) = -\infty$.

There is an injection of A into $\mathcal{P}_A(1)$ given by the map $a \mapsto a1$ (recall that 1 denotes e_0). There is also an injection of $\mathbb{N}$ into $\mathcal{P}_A(1)$ given by the map $k \mapsto e_k$. Observe that $e_k = e_1^k$ (with $e_1^0 = e_0 = 1$). In order to alleviate the notation, we often denote e_1 by X, and we call X a *variable (or indeterminate)*. Then, $e_k = e_1^k$ is denoted by X^k. Adopting this notation, given a nonnull polynomial P of degree n, if $P(k) = a_k$, P is denoted by

$$P = a_0 + a_1 X + \cdots + a_n X^n,$$

or by

$$P = a_n X^n + a_{n-1} X^{n-1} + \cdots + a_0,$$

if this is more convenient (the order of the terms does not matter anyway). Sometimes, it will also be convenient to write a polynomial as

$$P = a_0 X^n + a_1 X^{n-1} + \cdots + a_n.$$

The set $\mathcal{P}_A(1)$ is also denoted by $A[X]$ and a polynomial P may be denoted by $P(X)$. In denoting polynomials, we will use both upper-case and lower-case letters, usually, P, Q, R, S, p, q, r, s, but also f, g, h, etc., if needed (as long as no ambiguities arise).

Given a nonnull polynomial P of degree n, the nonnull coefficient a_n is called the *leading coefficient of* P. The coefficient a_0 is called the *constant term* of P. A polynomial of the form $a_k X^k$ is called a *monomial*. We say that $a_k X^k$ *occurs* in P if $a_k \neq 0$. A nonzero polynomial P of degree n is called a *monic polynomial (or unitary polynomial, or monic)* if $a_n = 1$, where a_n is its leading coefficient, and such a polynomial can be written as

$$P = X^n + a_{n-1} X^{n-1} + \cdots + a_0 \qquad \text{or} \qquad P = X^n + a_1 X^{n-1} + \cdots + a_n.$$

The choice of the variable X to denote e_1 is standard practice, but there is nothing special about X. We could have chosen Y, Z, or any other symbol, as long as no ambiguities arise.