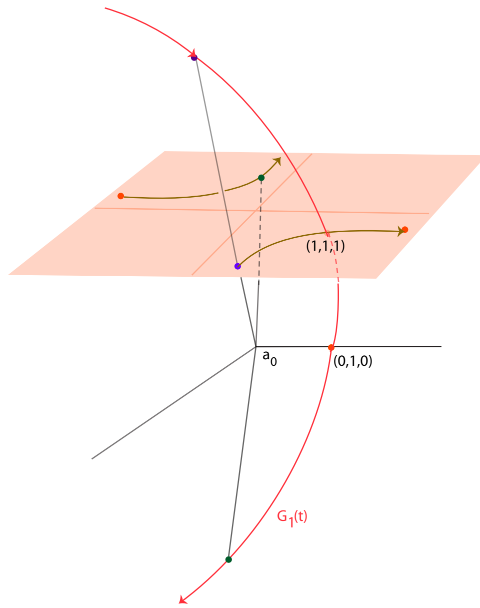

Figure 26.2: A central projection in $\mathbb{A}^3$ through a_0 of the parabola $G_1(t)$ onto the hyperplane $x_3 = 1$.

frame for $\mathcal{E}$. We want to determine the coordinates of the central projection $p(x)$ of a point $x \in \mathcal{E}$ onto the hyperplane H of equation $x_{n+1} = 1$ (the center of projection being a_0). If

$$x = a_0 + x_1 e_1 + \cdots + x_n e_n + x_{n+1} e_{n+1},$$

assuming that $x_{n+1} \neq 0$; a point on the line passing through a_0 and x has coordinates of the form $(\lambda x_1, \ldots, \lambda x_{n+1})$; and $p(x)$, the central projection of x onto the hyperplane H of equation $x_{n+1} = 1$, is the intersection of the line from a_0 to x and this hyperplane H. Thus we must have $\lambda x_{n+1} = 1$, and the coordinates of $p(x)$ are

$$\left(\frac{x_1}{x_{n+1}}, \ldots, \frac{x_n}{x_{n+1}}, 1 \right).$$

Note that $p(x)$ is undefined when $x_{n+1} = 0$. In projective spaces, we can make sense of such points.

The above calculation confirms that $G(t)$ is a central projection of $G_1(t)$. Similarly, if we define the curve F_1 in $\mathbb{A}^3$ by

$$F_1(t) = a_0 + (1 - t^2) e_1 + 2t e_2 + (1 + t^2) e_3,$$

the central projection of the polynomial curve F_1 (again, a parabola in $\mathbb{A}^3$) onto the plane of equation $x_3 = 1$ is the circle F.