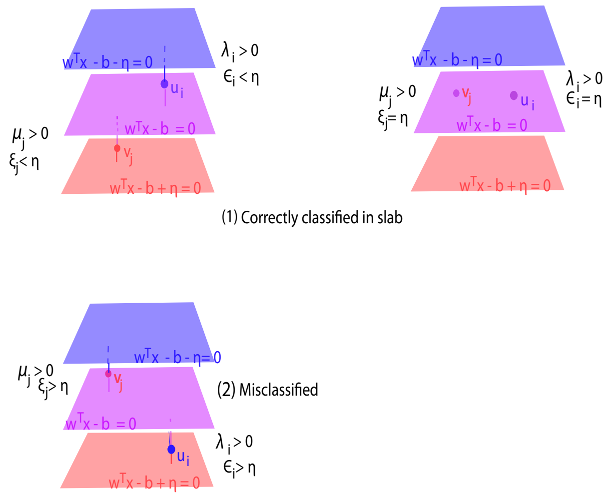

Figure 54.21: The classification of points for SVM$_{s4}$ when the Lagrange multipliers are positive. The left illustration of Figure (1) is when u_i is inside the margin yet still on the correct side of the separating hyperplane $w^\top x - b = 0$. Similarly, v_j is inside the margin on the correct side of the separating hyperplane. The right illustration depicts u_i and v_j on the separating hyperplane. Figure (2) illustrations a misclassification of u_i and v_j.

and since

$$w = -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix}$$

we get

$$\begin{aligned} \nu\eta &= K_s(\epsilon^\top \epsilon + \xi^\top \xi) + \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(X^\top X + \frac{1}{4K_s} I_{p+q} \right) \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\ &= \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(X^\top X + \frac{1}{2K_s} I_{p+q} \right) \begin{pmatrix} \lambda \\ \mu \end{pmatrix}. \end{aligned}$$

The above confirms that at optimality we have $\eta \geq 0$.

Remark: If we do not assume that $K_s = 1/(p+q)$, then the above formula must be replaced by

$$(p+q)K_s \nu\eta = \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(X^\top X + \frac{1}{2K_s} I_{p+q} \right) \begin{pmatrix} \lambda \\ \mu \end{pmatrix}.$$

Since η is determined independently of the existence of support vectors, the margin hyperplane $H_{w,b+\eta}$ may not contain any point u_i and the margin hyperplane $H_{w,b-\eta}$ may not contain any point v_j.

We can solve for b using some active constraint corresponding to any i_0 such that $\lambda_{i_0} > 0$ and any j_0 such that $\mu_{j_0} > 0$ (by a previous remark, the constraints imply that such i_0 and