(1) Consider the matrices of the form

$$R_c^{i,j} = \begin{pmatrix} 1 & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & 0 & 0 & \cdots & 0 & i & & & \\ & & & 0 & 1 & \cdots & 0 & 0 & & & \\ & & & \vdots & \vdots & \ddots & \vdots & \vdots & & & \\ & & & 0 & 0 & \cdots & 1 & 0 & & & \\ & & & i & 0 & \cdots & 0 & 0 & & & \\ & & & & & & & & 1 & & \\ & & & & & & & & & \ddots & \\ & & & & & & & & & & 1 \end{pmatrix}.$$

Prove that $(R_c^{i,j})^* R_c^{i,j} = I$ and $\det(R_c^{i,j}) = +1$. Use the matrices $R^{i,j}, R_c^{i,j} \in \mathbf{SU}(n)$ and the matrices $(R^{i,j} - (R^{i,j})^*)/2$ (from Problem 12.12) to form the real part of a skew-Hermitian matrix and the matrices $(R_c^{i,j} - (R_c^{i,j})^*)/2$ to form the imaginary part of a skew-Hermitian matrix. Deduce that the matrices in $\mathbf{SU}(n)$ span all skew-Hermitian matrices.

(2) Consider matrices of the form

Type 1

$$S_c^{1,2} = \begin{pmatrix} 0 & -i & 0 & 0 & \dots & 0 \\ i & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & -1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}.$$

Type 2

$$S_c^{i,i+1} = \begin{pmatrix} -1 & & & & & & & & \\ & 1 & & & & & & & \\ & & \ddots & & & & & & \\ & & & 1 & & & & & \\ & & & & 0 & -i & & & \\ & & & & i & 0 & & & \\ & & & & & & 1 & & \\ & & & & & & & \ddots & \\ & & & & & & & & 1 \end{pmatrix}.$$