as illustrated by the following commutative diagram.

$$\begin{tikzcd}
E_1 \times \cdots \times E_n \arrow[r, "\varphi_2"] \arrow[dr, "\varphi_1"'] & T_2 \arrow[d, "(\varphi_1)_\otimes"] \\
& T_1
\end{tikzcd}$$

Putting these diagrams together, we obtain the commutative diagrams

$$\begin{tikzcd}
& T_1 \arrow[d, "(\varphi_2)_\otimes"] \\
E_1 \times \cdots \times E_n \arrow[ur, "\varphi_1"] \arrow[r, "\varphi_2"] \arrow[dr, "\varphi_1"'] & T_2 \arrow[d, "(\varphi_1)_\otimes"] \\
& T_1
\end{tikzcd}$$

and

$$\begin{tikzcd}
& T_2 \arrow[d, "(\varphi_1)_\otimes"] \\
E_1 \times \cdots \times E_n \arrow[ur, "\varphi_2"] \arrow[r, "\varphi_1"] \arrow[dr, "\varphi_2"'] & T_1 \arrow[d, "(\varphi_2)_\otimes"] \\
& T_2,
\end{tikzcd}$$

which means that

$$\varphi_1 = (\varphi_1)_\otimes \circ (\varphi_2)_\otimes \circ \varphi_1 \quad \text{and} \quad \varphi_2 = (\varphi_2)_\otimes \circ (\varphi_1)_\otimes \circ \varphi_2.$$

On the other hand, focusing on (T_1, φ_1), we have a multilinear map $\varphi_1 \colon E_1 \times \cdots \times E_n \to T_1$, but the unique linear map $h \colon T_1 \to T_1$ with

$$\varphi_1 = h \circ \varphi_1$$

is $h = \mathrm{id}$, as illustrated by the following commutative diagram

$$\begin{tikzcd}
E_1 \times \cdots \times E_n \arrow[r, "\varphi_1"] \arrow[dr, "\varphi_1"'] & T_1 \arrow[d, "\mathrm{id}"] \\
& T_1,
\end{tikzcd}$$

and since $(\varphi_1)_\otimes \circ (\varphi_2)_\otimes$ is linear as a composition of linear maps, we must have

$$(\varphi_1)_\otimes \circ (\varphi_2)_\otimes = \mathrm{id}.$$