**Dual of the Soft margin kernel SVM** (SVM$_{s4}$):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(\mathbf{K} + \frac{1}{2K_s} I_{p+q}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\
&\text{subject to} \\
&\qquad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0 \\
&\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq \nu \\
&\qquad \lambda_i \geq 0, \quad i = 1, \ldots, p \\
&\qquad \mu_j \geq 0, \quad j = 1, \ldots, q,
\end{aligned}$$

where $\mathbf{K}$ is the kernel matrix of Section 54.1. Then $w$, $b$, and $f(x)$ are obtained exactly as in Section 54.5.

## 54.14 Solving SVM (SVM$_{s4}$) Using ADMM

In order to solve (SVM$_{s4}$) using ADMM we need to write the matrix corresponding to the constraints in equational form,

$$\begin{aligned}
\sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j &= 0 \\
\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j - \gamma &= K_m,
\end{aligned}$$

with $K_m = (p+q)K_s\nu$. This is the $2 \times (p+q+1)$ matrix $A$ given by

$$A = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0 \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top & -1 \end{pmatrix}.$$

We leave it as an exercise to prove that $A$ has rank 2. The right-hand side is

$$c = \begin{pmatrix} 0 \\ K_m \end{pmatrix}.$$

The symmetric positive semidefinite $(p+q)\times(p+q)$ matrix $P$ defining the quadratic functional is

$$P = X^\top X + \frac{1}{2K_s} I_{p+q}, \quad \text{with} \quad X = \begin{pmatrix} -u_1 & \cdots & -u_p & v_1 & \cdots & v_q \end{pmatrix},$$

and

$$q = 0_{p+q}.$$