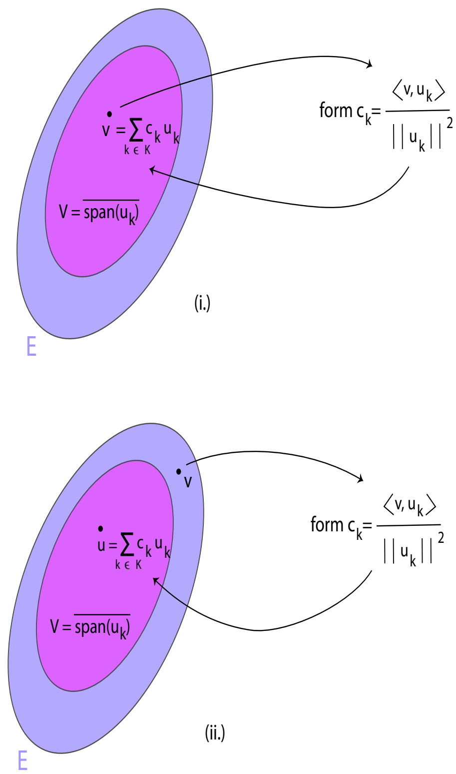

Figure A.1: A schematic illustration of Proposition A.2. Figure (i.) illustrates Condition (2b), while Figure (ii.) illustrates Condition (3). Note E is the purple oval and V is the magenta oval. In both cases, take a vector of E, form the Fourier coefficients c_k, then form the Fourier series $\sum_{k\in K} c_k u_k$. Condition (2b) ensures v equals its Fourier series since $v \in V$. However, if $v \notin V$, the Fourier series does not equal v. Eventually, we will discover that $V = E$, which implies that that Fourier series converges to its vector v.

for any finite subset I of K. We claim that $v - u_I$ is orthogonal to u_i for every $i \in I$. Indeed,

$$\begin{aligned}
\langle v - u_I, u_i \rangle &= \left\langle v - \sum_{j\in I} c_j u_j, u_i \right\rangle \\
&= \langle v, u_i \rangle - \sum_{j\in I} c_j \langle u_j, u_i \rangle \\
&= \langle v, u_i \rangle - c_i \|u_i\|^2 \\
&= \langle v, u_i \rangle - \langle v, u_i \rangle = 0,
\end{aligned}$$