with $\lambda \geq 0, \nu^+ \geq 0, \nu^- \geq 0$, and if we introduce $\nu_j = \nu_j^+ - \nu_j^-$ we obtain the following KKT conditions for programs with explicit equality constraints:

$$J'_u + \sum_{i=1}^m \lambda_i (\varphi'_i)_u + \sum_{j=1}^p \nu_j (\psi'_j)_u = 0,$$

and

$$\sum_{i=1}^m \lambda_i \varphi_i(u) = 0$$

with $\lambda \geq 0$ and $\nu \in \mathbb{R}^p$ arbitrary.

Let us now assume that the functions φ_i and ψ_j are *convex*. As we explained just after Definition 50.6, nonaffine equality constraints are never qualified. Thus, in order to generalize Theorem 50.6 to explicit equality constraints, we assume that the *equality constraints* ψ_j are *affine*.

Theorem 50.18. *Let $\varphi_i \colon \Omega \to \mathbb{R}$ be m convex inequality constraints and $\psi_j \colon \Omega \to \mathbb{R}$ be p affine equality constraints defined on some open convex subset Ω of a finite-dimensional Euclidean vector space V (more generally, a real Hilbert space V), let $J \colon \Omega \to \mathbb{R}$ be some function, let U be given by*

$$U = \{x \in \Omega \mid \varphi_i(x) \leq 0,\ \ \psi_j(x) = 0,\ \ 1 \leq i \leq m,\ 1 \leq j \leq p\},$$

and let $u \in U$ be any point such that the functions φ_i and J are differentiable at u, and the functions ψ_j are affine.

(1) If J has a local minimum at u with respect to U, and if the constraints are qualified, then there exist some vectors $\lambda \in \mathbb{R}^m_+$ and $\nu \in \mathbb{R}^p$, such that the KKT condition hold:

$$J'_u + \sum_{i=1}^m \lambda_i(u) (\varphi'_i)_u + \sum_{j=1}^p \nu_j (\psi'_j)_u = 0,$$

and

$$\sum_{i=1}^m \lambda_i(u) \varphi_i(u) = 0, \quad \lambda_i \geq 0, \quad i = 1, \ldots, m.$$

Equivalently, in terms of gradients, the above conditions are expressed as

$$\nabla J_u + \sum_{i=1}^m \lambda_i \nabla(\varphi_i)_u + \sum_{j=1}^p \nu_j \nabla(\psi_j)_u = 0$$

and

$$\sum_{i=1}^m \lambda_i(u) \varphi_i(u) = 0, \quad \lambda_i \geq 0, \quad i = 1, \ldots, m.$$