that for every optimal solution $(w, b, \eta, \epsilon, \xi)$ of Problem (SVM_{s3}) with $w \neq 0$ and $\eta > 0$, if

$$(p_{sf} + q_{sf})/(p + q) < \nu < 1,$$

then some u_{i_0} or some v_{j_0} is a support vector.

(4) **Basic Quadratic Soft margin ν-SVM Problem** (SVM_{s4}). This is the version of Problem ($\mathrm{SVM}_{s2'}$) in which instead of using the linear function $K_s \begin{pmatrix} \epsilon^\top & \xi^\top \end{pmatrix} \mathbf{1}_{p+q}$ as a regularizing function we use the quadratic function $K(\|\epsilon\|_2^2 + \|\xi\|_2^2)$. The optimization problem is

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2} w^\top w + (p+q)K_s\left(-\nu\eta + \frac{1}{p+q}(\epsilon^\top \epsilon + \xi^\top \xi)\right) \\
&\text{subject to} \\
&\qquad w^\top u_i - b \geq \eta - \epsilon_i, \qquad i = 1, \ldots, p \\
&\qquad -w^\top v_j + b \geq \eta - \xi_j, \qquad j = 1, \ldots, q \\
&\qquad \eta \geq 0,
\end{aligned}$$

where ν and K_s are two given positive constants. As we saw earlier, theoretically, it is convenient to pick $K_s = 1/(p + q)$. When writing a computer program, it is preferable to assume that K_s is arbitrary. In this case ν needs to be replaced by $(p + q)K_s\nu$ in all the formulae obtained with $K_s = 1/(p + q)$.

In this method, it is no longer necessary to require $\epsilon \geq 0$ and $\xi \geq 0$, because an optimal solution satisfies these conditions.

One of the advantages of this methods is that ϵ is determined by λ, ξ is determined by μ, and η and b are determined by λ and μ. We can omit the constraint $\eta \geq 0$, because for an optimal solution it can be shown using duality that $\eta \geq 0$; see Section 54.14. For K_s and ν fixed, if Program (SVM_{s4}) has an optimal solution, then it is unique; see Theorem 54.8.

A drawback of Program (SVM_{s4}) is that for fixed K_s, the quantity $\delta = \eta/\|w\|$ and the hyperplanes $H_{w,b}$, $H_{w,b+\eta}$ and $H_{w,b-\eta}$ are *independent* of ν. This is shown in Theorem 54.8. Thus this method is less flexible than ($\mathrm{SVM}_{s2'}$) and (SVM_{s3}).

It is shown in Section 54.9 that the dual is given by