

Figure 26.15: Case (2)

(with respect to the affine frames $(p_3, (p_1 - p_3, p_2 - p_3))$ and $(q_3, (q_1 - q_3, q_2 - q_3))$). Two possibilities occur exactly as in Cases (2) and (3) depending on the position of p_4 with respect to the line $\langle p_1, p_2 \rangle$ and on the position of q_4 with respect to the line $\langle q_1, q_2 \rangle$. The first possibility is illustrated by the top of Figure 26.17, while the second is illustrated by the bottom of Figure 26.17.

Thus, if both (p_1, p_2, p_3, p_4) and (q_1, q_2, q_3, q_4) satisfy the conditions listed above, there is no point at infinity inside of the convex hull of the quadrangle (p_1, p_2, p_3, p_4) .

It remains to prove that the image of the convex hull of (p_1, p_2, p_3, p_4) is the convex hull of (q_1, q_2, q_3, q_4) .

Proposition 26.15. If both (p_1, p_2, p_3, p_4) and (q_1, q_2, q_3, q_4) satisfy the conditions of Proposition 26.14, then the image of the convex hull of (p_1, p_2, p_3, p_4) under the unique projective map mapping (p_1, p_2, p_3, p_4) to (q_1, q_2, q_3, q_4) is the convex hull of (q_1, q_2, q_3, q_4)

Proof. It suffices to show that the restriction of our projective transformation maps a line segment to the convex hull of the images of the endpoints of this segment. Thus, the problem