The fact that the map $\varphi\colon E^n \to \bigwedge^n(E)$ is alternating and multilinear can also be expressed as follows:

$$\begin{aligned} u_1 \wedge \cdots \wedge (u_i + v_i) \wedge \cdots \wedge u_n &= (u_1 \wedge \cdots \wedge u_i \wedge \cdots \wedge u_n) \\ &\quad + (u_1 \wedge \cdots \wedge v_i \wedge \cdots \wedge u_n), \\ u_1 \wedge \cdots \wedge (\lambda u_i) \wedge \cdots \wedge u_n &= \lambda(u_1 \wedge \cdots \wedge u_i \wedge \cdots \wedge u_n), \\ u_{\sigma(1)} \wedge \cdots \wedge u_{\sigma(n)} &= \mathrm{sgn}(\sigma)\, u_1 \wedge \cdots \wedge u_n, \end{aligned}$$

for all $\sigma \in \mathfrak{S}_n$.

The map φ from E^n to $\bigwedge^n(E)$ is often denoted $\iota_\wedge$, so that

$$\iota_\wedge(u_1, \ldots, u_n) = u_1 \wedge \cdots \wedge u_n.$$

Theorem 34.4 implies the following result.

Proposition 34.5. *There is a canonical isomorphism*

$$\mathrm{Hom}(\bigwedge^n(E), F) \cong \mathrm{Alt}^n(E; F)$$

between the vector space of linear maps $\mathrm{Hom}(\bigwedge^n(E), F)$ *and the vector space of alternating multilinear maps* $\mathrm{Alt}^n(E; F)$*, given by the linear map* $- \circ \varphi$ *defined by* $\mapsto h \circ \varphi$*, with* $h \in \mathrm{Hom}(\bigwedge^n(E), F)$*. In particular, when* $F = K$*, we get a canonical isomorphism*

$$\left(\bigwedge^n(E)\right)^* \cong \mathrm{Alt}^n(E; K).$$

Definition 34.3. Tensors $\alpha \in \bigwedge^n(E)$ are called *alternating n-tensors* or *alternating tensors of degree n* and we write $\deg(\alpha) = n$. Tensors of the form $u_1 \wedge \cdots \wedge u_n$, where $u_i \in E$, are called *simple (or decomposable) alternating n-tensors*. Those alternating n-tensors that are not simple are often called *compound alternating n-tensors*. Simple tensors $u_1 \wedge \cdots \wedge u_n \in \bigwedge^n(E)$ are also called *n-vectors* and tensors in $\bigwedge^n(E^*)$ are often called *(alternating) n-forms*.

Given a linear map $f\colon E \to E'$, since the map $\iota'_\wedge \circ (f \times f)$ is bilinear and alternating, there is a unique linear map $f \wedge f\colon \bigwedge^2(E) \to \bigwedge^2(E')$ making the following diagram commute:

$$\begin{tikzcd} E^2 \arrow[r, "\iota_\wedge"] \arrow[d, "f\times f"'] & \bigwedge^2(E) \arrow[d, "f\wedge f"] \\ (E')^2 \arrow[r, "\iota'_\wedge"'] & \bigwedge^2(E'). \end{tikzcd}$$

The map $f \wedge f\colon \bigwedge^2(E) \to \bigwedge^2(E')$ is determined by

$$(f \wedge f)(u \wedge v) = f(u) \wedge f(v).$$