Since by hypothesis $b \geq 0$ and the objective function is bounded above by 0, this linear program has an optimal solution (x_J^*, ξ^*).

If $\xi^* = 0$, then the vector $u^* \in \mathbb{R}^n$ given by $u_J^* = x_J^*$ and $u_N^* = 0_{n-p}$ is an optimal solution of (P).

Otherwise, $\xi^* > 0$ and we have failed to solve $(*_1)$. However we may try to use ξ^* to improve y. For this consider the Dual (DRP) of (RP):

$$\begin{aligned} \text{minimize} \quad & zb \\ \text{subject to} \quad & zA_J \geq 0 \\ & z \geq -\mathbf{1}_m^\top. \end{aligned}$$

Observe that the Program (DRP) has the same objective function as the original Dual Program (D). We know by Theorem 47.12 that the optimal solution (x_J^*, ξ^*) of (RP) yields an optimal solution z^* of (DRP) such that

$$z^* b = -(\xi_1^* + \cdots + \xi_m^*) < 0.$$

In fact, if K^* is the basis associated with (x_J^*, ξ^*) and if we write

$$\widehat{A} = \begin{pmatrix} A_J & I_m \end{pmatrix}$$

and $\widehat{c} = [0_p^\top \;\; -\mathbf{1}^\top]$, then by Theorem 47.12 we have

$$z^* = \widehat{c}_{K^*} \widehat{A}_{K^*}^{-1} = -\mathbf{1}_m^\top - (\overline{c}_{K^*})_{(p+1,\ldots,p+m)},$$

where $(\overline{c}_{K^*})_{(p+1,\ldots,p+m)}$ denotes the row vector of reduced costs in the final tableau corresponding to the last m columns.

If we write

$$y(\theta) = y + \theta z^*,$$

then the new value of the objective function of (D) is

$$y(\theta) b = yb + \theta z^* b, \tag{$*_2$}$$

and since $z^* b < 0$, we have a chance of improving the objective function of (D), that is, decreasing its value for $\theta > 0$ small enough if $y(\theta)$ is feasible for (D). This will be the case iff $y(\theta) A \geq c$ iff

$$yA + \theta z^* A \geq c. \tag{$*_3$}$$

Now since y is a feasible solution of (D) we have $yA \geq c$, so if $z^* A \geq 0$, then $(*_3)$ is satisfied and $y(\theta)$ is a solution of (D) for all $\theta > 0$, which means that (D) is unbounded. But this implies that (P) is not feasible.