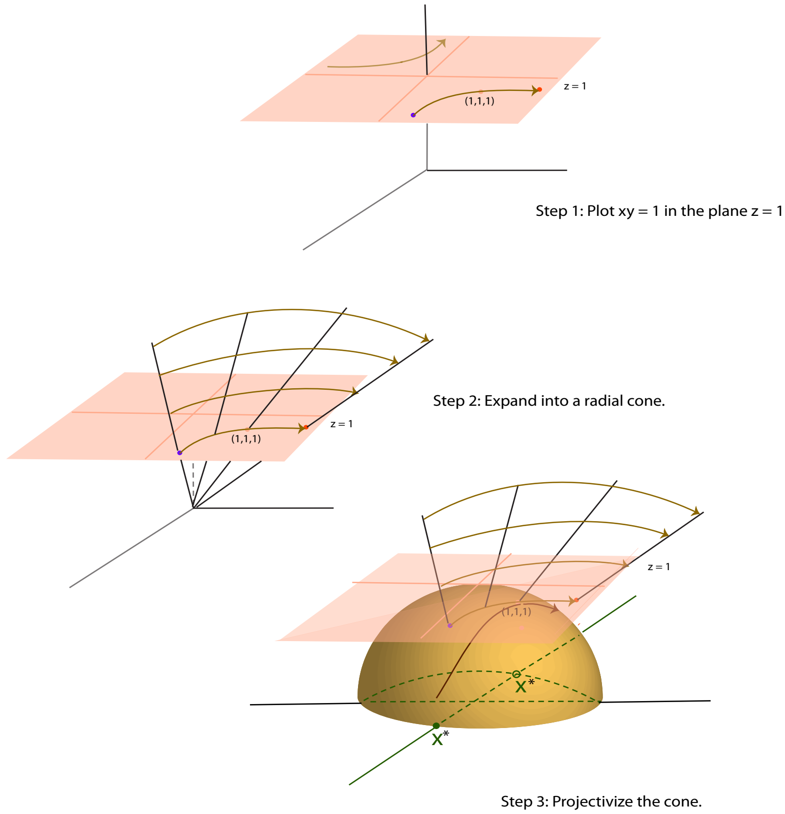

Figure 26.10: A three step process for constructing $V(P)$ where P is the homogenous conic $xy = z$. In Step 2, we convert to homogenous coordinates via the transformation $x \to x/z$, $y \to y/z$.

unique conic (among many sources, see Samuel [142], Section 1.7, Theorem 17, or Coxeter [45], Theorem 6.56, where a geometric construction is given in Section 6.6).

In fact, if we pick a projective frame (a_1, a_2, a_3, a_4) in $\mathbb{CP}^2$ (or $\mathbb{RP}^2$), and if the five points p_1, p_2, p_3, p_4, p_5 have homogeneous coordinates $p_i = (x_i, y_i, z_i)$ for $i = 1, \ldots, 5$ and (x, y, z) are variables, then it is an easy exercise to show that the equation of the unique conic C passing through the points p_1, p_2, p_3, p_4, p_5 is given by

$$\begin{vmatrix} x^2 & xy & y^2 & xz & yz & z^2 \\ x_1^2 & x_1y_1 & y_1^2 & x_1z_1 & y_1z_1 & z_1^2 \\ x_2^2 & x_2y_2 & y_2^2 & x_2z_2 & y_2z_2 & z_2^2 \\ x_3^2 & x_3y_3 & y_3^2 & x_3z_3 & y_3z_3 & z_3^2 \\ x_4^2 & x_4y_4 & y_4^2 & x_4z_4 & y_4z_4 & z_4^2 \\ x_5^2 & x_5y_5 & y_5^2 & x_5z_5 & y_5z_5 & z_5^2 \end{vmatrix} = 0.$$

The polynomial obtained by expanding the above determinant according to the first row is a homogeneous polynomial of degree 2 in the variables x, y, z, and it is not the zero polynomial