

Thus, we obtain a simple expression for $\|w\|^2$ in terms of λ and μ .

The vectors u_i and v_j for which the i -th inequality is active and the $(p+j)$ th inequality is active are called *support vectors*. For every vector u_i or v_j that is not a support vector, the corresponding inequality is inactive, so $\lambda_i = 0$ and $\mu_j = 0$. Thus we see that *only the support vectors contribute to a solution*. If we can *guess* which vectors u_i and v_j are support vectors, namely, those for which $\lambda_i \neq 0$ and $\mu_j \neq 0$, then for each support vector u_i we have an equation

$$-\sum_{j=1}^p u_i^\top u_j \lambda_j + \sum_{k=1}^q u_i^\top v_k \mu_k + b + 1 = 0,$$

and for each support vector v_j we have an equation

$$\sum_{i=1}^p v_j^\top u_i \lambda_i - \sum_{k=1}^q v_j^\top v_k \mu_k - b + 1 = 0,$$

with $\lambda_i = 0$ and $\mu_j = 0$ for all non-support vectors, so together with the Equation $(*_2)$ we have a linear system with an equal number of equations and variables, which is solvable if our separation problem has a solution. Thus, in principle we can find λ , μ , and b by solving a linear system.

Remark: We can first solve for λ and μ (by eliminating b), and by $(*_1)$ and since $w \neq 0$, there is at least some nonzero λ_{i_0} and thus some nonzero μ_{j_0} , so the corresponding inequalities are equations

$$\begin{aligned} -\sum_{j=1}^p u_{i_0}^\top u_j \lambda_j + \sum_{k=1}^q u_{i_0}^\top v_k \mu_k + b + 1 &= 0 \\ \sum_{i=1}^p v_{j_0}^\top u_i \lambda_i - \sum_{k=1}^q v_{j_0}^\top v_k \mu_k - b + 1 &= 0, \end{aligned}$$

so b is given in terms of λ and μ by

$$b = \frac{1}{2}(u_{i_0}^\top + v_{j_0}^\top) \left(\sum_{i=1}^p \lambda_i u_i - \sum_{j=1}^p \mu_j v_j \right).$$

Using the dual of the Lagrangian, we can solve for λ and μ , but typically b is not determined, so we use the above method to find b .

The above nondeterministic procedure in which we guess which vectors are support vectors is not practical. We will see later that a practical method for solving for λ and μ consists in maximizing the dual of the Lagrangian.