with a $\pm$ in front when the plane is nonoriented. Observe that this formula allows the definition of the angle of two complex lines (possibly a complex number) and the notion of orthogonality of complex lines. In this case, note that the isotropic lines are orthogonal to themselves!

The definition of orthogonality of two lines D_1, D_2 in terms of (D_1, D_2, D_I, D_J) forming a harmonic division can be used to give elegant proofs of various results. Cayley's formula can even be used in computer vision to explain modeling and calibrating cameras! (see Faugeras [59]). As an illustration, consider a triangle (a, b, c), and recall that the line a' passing through a and orthogonal to (b, c) is called the *altitude of* a, and similarly for b and c. It is well known that the altitudes a', b', c' intersect in a common point called the *orthocenter* of the triangle (a, b, c). This can be shown in a number of ways using the circular points. Indeed, letting bc_∞, ab_∞, ac_∞, a'_∞, b'_∞, and c'_∞ denote the points at infinity of the lines $\langle b, c\rangle$, $\langle a, b\rangle$, $\langle a, c\rangle$, a', b', and c', we have

$$[bc_\infty, a'_\infty, I, J] = -1, \quad [ab_\infty, c'_\infty, I, J] = -1, \quad [ac_\infty, b'_\infty, I, J] = -1,$$

and it is easy to show that there is an involution σ of the line at infinity such that

$$\begin{aligned}
\sigma(I) &= J, \\
\sigma(J) &= I, \\
\sigma(bc_\infty) &= a'_\infty, \\
\sigma(ab_\infty) &= c'_\infty, \\
\sigma(ac_\infty) &= b'_\infty.
\end{aligned}$$

Then, it can be shown that the lines a', b', c' are concurrent. For more details and other results, notably on the conics, see Sidler [161], Berger [12], and Samuel [142].

The generalization of what we just did to real Euclidean spaces $(E, \overrightarrow{E})$ of dimension n is simple. Let $(a_0, \ldots, a_{n+1})$ be any projective frame for $\widetilde{E}_{\mathbb{C}}$ such that $(a_0, \ldots, a_{n-1})$ arises from an orthonormal basis $(u_1, \ldots, u_n)$ of $\overrightarrow{E}$ and the hyperplane at infinity H corresponds to $x_{n+1} = 0$ (where $(x_1, \ldots, x_{n+1})$ are the homogeneous coordinates of a point with respect to $(a_0, \ldots, a_{n+1})$). Consider the points belonging to the intersection of the real quadric Σ of equation

$$x_1^2 + \cdots + x_n^2 - x_{n+1}^2 = 0$$

with the hyperplane at infinity $x_{n+1} = 0$. For such points,

$$x_1^2 + \cdots + x_n^2 = 0 \quad \text{and} \quad x_{n+1} = 0.$$

Such points belong to a quadric called the *absolute quadric* of $\widetilde{E}_{\mathbb{C}}$, and denoted by Ω. Any line containing any point on the absolute quadric is called an *isotropic line*. Then, given any two coplanar lines D_1 and D_2 in E, these lines intersect the hyperplane at infinity H in two points $(D_1)_\infty$ and $(D_2)_\infty$, and the line Δ joining $(D_1)_\infty$ and $(D_2)_\infty$ intersects the absolute