Then by $(*_1)$ we obtain

$$w = \frac{2(u-v)}{(u-v)^\top (u-v)}.$$

We verify easily that

$$2(u_1 - v_1)x_1 + \cdots + 2(u_n - v_n)x_n = (u_1^2 + \cdots + u_n^2) - (v_1^2 + \cdots + v_n^2)$$

is the equation of the bisector hyperplane between u and v; see Figure 50.16.

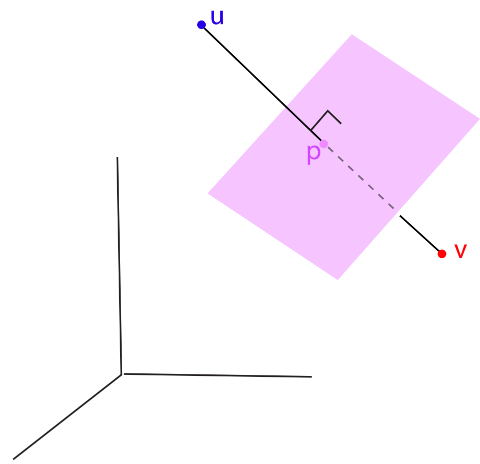

Figure 50.16: In $\mathbb{R}^3$, the solution to Hard Margin SVM_{h2} for the points u and v is the purple perpendicular planar bisector of $u - v$.

In the next section we will derive the dual of the optimization problem discussed in this section. We will also consider a more flexible solution involvlng a *soft margin*.

50.7 Lagrangian Duality and Saddle Points

In this section we investigate methods to solve the *Minimization Problem (P):*

$$\begin{aligned} &\text{minimize} \quad J(v) \\ &\text{subject to} \quad \varphi_i(v) \le 0, \quad i = 1, \ldots, m. \end{aligned}$$

It turns out that under certain conditions the original Problem (P), called *primal problem*, can be solved in two stages with the help another Problem (D), called the *dual problem*. The Dual Problem (D) is a *maximization problem* involving a function G, called the *Lagrangian*