

- (1) There is a finite sequence of occurrences of Case (B2) and/or Case (B3) ending with an occurrence of Case (A). Then the last vertex produced by the algorithm is an optimal solution. This is what occurred in Examples 46.1 and 46.2.
- (2) There is a finite sequence of occurrences of Case (B2) and/or Case (B3) ending with an occurrence of Case (B1). We conclude that the problem is unbounded, and thus has no solution. This is what occurred in Example 46.3.
- (3) There is a finite sequence of occurrences of Case (B2) and/or Case (B3), followed by an infinite sequence of Case (B3). If this occurs, the algorithm visits the same basis twice. This a phenomenon known as *cycling*. In this eventually the algorithm fails to come to a conclusion.

There are examples for which cycling occur, although this is rare in practice. Such an example is given in Chvatal [40]; see Chapter 3, pages 31-32, for an example with seven variables and three equations that cycles after six iterations under a certain pivot rule.

The third possibility can be avoided by the choice of a suitable pivot rule. Two of these rules are *Bland's rule* and the *lexicographic rule*; see Chvatal [40] (Chapter 3, pages 34-38).

Bland's rule says: choose the smallest of the eligible incoming indices $j^+ \notin K$, and similarly choose the smallest of the eligible outgoing indices $k^- \in K$.

It can be proven that cycling cannot occur if Bland's rule is chosen as the pivot rule. The proof is very technical; see Chvatal [40] (Chapter 3, pages 37-38), Matousek and Gardner [123] (Chapter 5, Theorem 5.8.1), and Papadimitriou and Steiglitz [134] (Section 2.7). Therefore, assuming that some initial basic feasible solution is provided, and using a suitable pivot rule (such as Bland's rule), the simplex algorithm always terminates and either yields an optimal solution or reports that the linear program is unbounded. Unfortunately, Bland's rule is one of the slowest pivot rules.

The choice of a pivot rule affects greatly the number of pivoting steps that the simplex algorithm goes through. It is not our intention here to explain the various pivot rules. We simply mention the following rules, referring the reader to Matousek and Gardner [123] (Chapter 5, Section 5.7) or to the texts cited in Section 44.1.

1. Largest coefficient, or Dantzig's rule.
2. Largest increase.
3. Steepest edge.
4. Bland's Rule.
5. Random edge.