then our linear combination $x_1 u + x_2 v + x_3 w$ can be written in matrix form as

$$x_1 u + x_2 v + x_3 w = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},$$

so our linear system is expressed by

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

or more concisely as

$$Ax = b.$$

Now what if the vectors u, v, w are *linearly dependent*? For example, if we consider the vectors

$$u = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \qquad v = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \qquad w = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix},$$

we see that

$$u - v = w,$$

a nontrivial *linear dependence*. It can be verified that u and v are still linearly independent. Now for our problem

$$x_1 u + x_2 v + x_3 w = b$$

it must be the case that b can be expressed as linear combination of u and v. However, it turns out that u, v, b are linearly independent (one way to see this is to compute the determinant $\det(u, v, b) = -6$), so b cannot be expressed as a linear combination of u and v and thus, our system has *no* solution.

If we change the vector b to

$$b = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix},$$

then

$$b = u + v,$$

and so the system

$$x_1 u + x_2 v + x_3 w = b$$

has the solution

$$x_1 = 1, \quad x_2 = 1, \quad x_3 = 0.$$