

$$\begin{aligned}\lambda + \alpha &= K_s \mathbf{1}_p \\ \mu + \beta &= K_s \mathbf{1}_q \\ \mathbf{1}_p^\top \lambda &= \mathbf{1}_q^\top \mu,\end{aligned}$$

and

$$\mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu = K_m + \gamma. \quad (*_\gamma)$$

The second and third equations are equivalent to the box constraints

$$0 \leq \lambda_i, \mu_j \leq K_s, \quad i = 1, \dots, p, \quad j = 1, \dots, q,$$

and since  $\gamma \geq 0$  equation  $(*_\gamma)$  is equivalent to

$$\mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu \geq K_m.$$

Plugging back  $w$  from  $(*_w)$  into the Lagrangian, after simplifications we get

$$\begin{aligned}G(\lambda, \mu, \alpha, \beta) &= \frac{1}{2} (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} - (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\ &= -\frac{1}{2} (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix},\end{aligned}$$

so the dual function is independent of  $\alpha, \beta$  and is given by

$$G(\lambda, \mu) = -\frac{1}{2} (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix}.$$

The dual program is given by

$$\text{maximize} \quad -\frac{1}{2} (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix}$$

subject to

$$\sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0$$

$$\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq K_m$$

$$0 \leq \lambda_i \leq K_s, \quad i = 1, \dots, p$$

$$0 \leq \mu_j \leq K_s, \quad j = 1, \dots, q.$$

Finally, the dual program is equivalent to the following minimization program: