Definition 8.6. An $m \times n$ matrix A is a *reduced row echelon matrix* iff the following conditions hold:

(a) The first nonzero entry in every row is 1. This entry is called a *pivot*.

(b) The first nonzero entry of row $i+1$ is to the right of the first nonzero entry of row i.

(c) The entries above a pivot are zero.

If a matrix satisfies the above conditions, we also say that it is in *reduced row echelon form*, for short *rref*.

Note that Condition (b) implies that the entries below a pivot are also zero. For example, the matrix

$$A = \begin{pmatrix} 1 & 6 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

is a reduced row echelon matrix. In general, a matrix in *rref* has the following shape:

$$\begin{pmatrix} 1 & 0 & 0 & \times & \times & 0 & 0 & \times \\ 0 & 1 & 0 & \times & \times & 0 & 0 & \times \\ 0 & 0 & 1 & \times & \times & 0 & 0 & \times \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \times \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \times \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

if the last row consists of zeros, or

$$\begin{pmatrix} 1 & 0 & 0 & \times & \times & 0 & 0 & \times & 0 & \times \\ 0 & 1 & 0 & \times & \times & 0 & 0 & \times & 0 & \times \\ 0 & 0 & 1 & \times & \times & 0 & 0 & \times & 0 & \times \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \times & 0 & \times \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \times & \times & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \times \end{pmatrix}$$

if the last row contains a pivot.

The following proposition shows that every matrix can be converted to a reduced row echelon form using row operations.

Proposition 8.14. *Given any $m \times n$ matrix A, there is a sequence of row operations $E_1, \ldots, E_k$ such that if $P = E_k \cdots E_1$, then $U = PA$ is a reduced row echelon matrix.*