Newton's method can be used to compute the value $P(\alpha)$ at some α of the interpolant $P(X)$ taking the values $\beta_1, \ldots, \beta_{m+1}$ for the (distinct) arguments $\alpha_1, \ldots, \alpha_{m+1}$. We also mention that inductive methods for computing $P(\alpha)$ without first computing the coefficients of the Newton interpolant exist, for example, Aitken's method. For this method, the reader is referred to Farin [58].

It has been observed that Lagrange interpolants oscillate quite badly as their degree increases, and thus, this makes them undesirable as a stable method for interpolation. A standard example due to Runge, is the function

$$f(x) = \frac{1}{1 + x^2},$$

in the interval $[-5, +5]$. Assuming a uniform distribution of points on the curve in the interval $[-5, +5]$, as the degree of the Lagrange interpolant increases, the interpolant shows wilder and wilder oscillations around the points $x = -5$ and $x = +5$. This phenomenon becomes quite noticeable beginning for degree 14, and gets worse and worse. For degree 22, things are quite bad! Equivalently, one may consider the function

$$f(x) = \frac{1}{1 + 25x^2},$$

in the interval $[-1, +1]$.

We now consider a more general interpolation problem which will lead to the Hermite polynomials.

We consider the following interpolation problem:

Given a sequence $(\alpha_1, \ldots, \alpha_{m+1})$ of pairwise distinct scalars in K, integers $n_1, \ldots, n_{m+1}$ where $n_j \geq 0$, and $m+1$ sequences $(\beta_j^0, \ldots, \beta_j^{n_j})$ of scalars in K, letting

$$n = n_1 + \cdots + n_{m+1} + m,$$

find a polynomial P of degree $\leq n$, such that

$$\begin{array}{rcr}
P(\alpha_1) = \beta_1^0, & \ldots & P(\alpha_{m+1}) = \beta_{m+1}^0, \\
\mathrm{D}^1 P(\alpha_1) = \beta_1^1, & \ldots & \mathrm{D}^1 P(\alpha_{m+1}) = \beta_{m+1}^1, \\
& \ldots & \\
\mathrm{D}^i P(\alpha_1) = \beta_1^i, & \ldots & \mathrm{D}^i P(\alpha_{m+1}) = \beta_{m+1}^i, \\
& \ldots & \\
\mathrm{D}^{n_1} P(\alpha_1) = \beta_1^{n_1}, & \ldots & \mathrm{D}^{n_{m+1}} P(\alpha_{m+1}) = \beta_{m+1}^{n_{m+1}}.
\end{array}$$

Note that the above equations constitute $n+1$ constraints, and thus, we can expect that there is a unique polynomial of degree $\leq n$ satisfying the above problem. This is indeed the case and such a polynomial is called a *Hermite polynomial*. We call the above problem the *Hermite interpolation problem*.