This is the $(p+q+2) \times 2(p+q)$ matrix A given by

$$A = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\ I_p & 0_{p,q} & I_p & 0_{p,q} \\ 0_{q,p} & I_q & 0_{q,p} & I_q \end{pmatrix}.$$

We leave it as an exercise to prove that A has rank $p+q+2$. The right-hand side is

$$c = \begin{pmatrix} 0 \\ 1 \\ K\mathbf{1}_{p+q} \end{pmatrix}.$$

The symmetric positive semidefinite $(p+q)\times(p+q)$ matrix P defining the quadratic functional is

$$P = 2X^\top X, \quad \text{with} \quad X = \begin{pmatrix} -u_1 & \cdots & -u_p & v_1 & \cdots & v_q \end{pmatrix},$$

and

$$q = 0_{p+q}.$$

Since there are $2(p+q)$ Lagrange multipliers $(\lambda, \mu, \alpha, \beta)$, the $(p+q) \times (p+q)$ matrix $X^\top X$ must be augmented with zero's to make it a $2(p+q) \times 2(p+q)$ matrix P_a given by

$$P_a = \begin{pmatrix} X^\top X & 0_{p+q,p+q} \\ 0_{p+q,p+q} & 0_{p+q,p+q} \end{pmatrix},$$

and similarly q is augmented with zeros as the vector $q_a = 0_{2(p+q)}$.

Since the constraint $w^\top w \leq 1$ causes troubles, we trade it for a different objective function in which $-\delta$ is replaced by $(1/2)\left\|w\right\|_2^2$. This way we are left with purely affine constraints. In the next section we discuss a generalization of Problem (SVM$_{h2}$) obtained by adding a linear regularizing term.

54.3 Soft Margin Support Vector Machines; (SVM$_{s2}$)

In this section we consider the generalization of Problem (SVM$_{h2}$) where we minimize $(1/2)w^\top w$ by adding the "regularizing term" $K\Big(\sum_{i=1}^p \epsilon_i + \sum_{j=1}^q \xi_j,\Big)$ for some $K > 0$. Recall that the margin δ is given by $\delta = 1/\left\|w\right\|$.

Soft margin SVM (SVM$_{s2}$):

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}w^\top w + K\begin{pmatrix}\epsilon^\top & \xi^\top\end{pmatrix}\mathbf{1}_{p+q} \\ &\text{subject to} \\ &\qquad w^\top u_i - b \geq 1 - \epsilon_i, \quad \epsilon_i \geq 0 \qquad i = 1, \ldots, p \\ &\qquad -w^\top v_j + b \geq 1 - \xi_j, \quad \xi_j \geq 0 \qquad j = 1, \ldots, q. \end{aligned}$$