half-sphere $S^n_+$ by identifying antipodal points $a_+$ and $a_-$ on the boundary of the half-sphere. We call this model of $\mathbf{P}(E)$ the *half-spherical model*; see Figure 26.5.

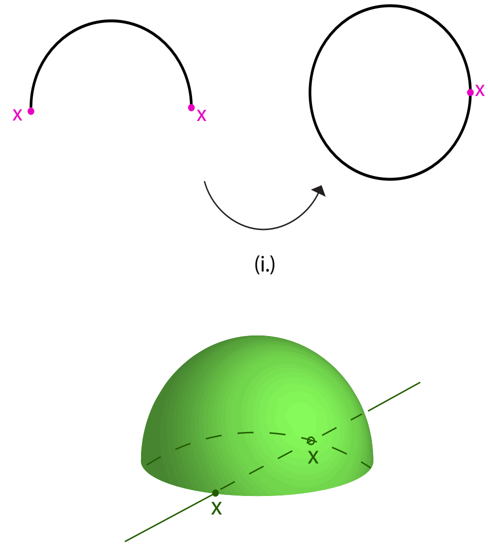


Figure 26.5: The half-spherical model representations of $\mathbb{RP}^1$ and $\mathbb{RP}^2$.

When $n = 2$, we get a circle. When $n = 3$, the upper half-sphere is homeomorphic to a closed disk (say, by orthogonal projection onto the $xy$-plane), and $\mathbb{RP}^2$ is in bijection with a closed disk in which antipodal points on its boundary (a unit circle) have been identified. This is hard to visualize! In this model of the real projective space, projective lines are great semicircles on the upper half-sphere, with antipodal points on the boundary identified. Boundary points correspond to points at infinity. By orthogonal projection, these great semicircles correspond to semiellipses, with antipodal points on the boundary identified. Traveling along such a projective "line," when we reach a boundary point, we "wrap around"! In general, the upper half-sphere $S^n_+$ is homeomorphic to the closed unit ball in $\mathbb{R}^n$, whose boundary is the $(n-1)$-sphere $S^{n-1}$. For example, the projective space $\mathbb{RP}^3$ is in bijection with the closed unit ball in $\mathbb{R}^3$, with antipodal points on its boundary (the sphere $S^2$) identified!

**Remarks:**

(1) A projective space $\mathbf{P}(E)$ has been defined as a *set* without any topological structure. When the field $K$ is either the field $\mathbb{R}$ of reals or the field $\mathbb{C}$ of complex numbers, the vector space $E$ is a topological space. Thus, the projection map $p\colon (E - \{0\}) \to \mathbf{P}(E)$ induces a topology on the projective space $\mathbf{P}(E)$, namely the quotient topology. This means that a subset $V$ of $\mathbf{P}(E)$ is open iff $p^{-1}(V)$ is an open set in $E$. Then, for example, it turns out that the real projective space $\mathbb{RP}^n$ is homeomorphic to the space