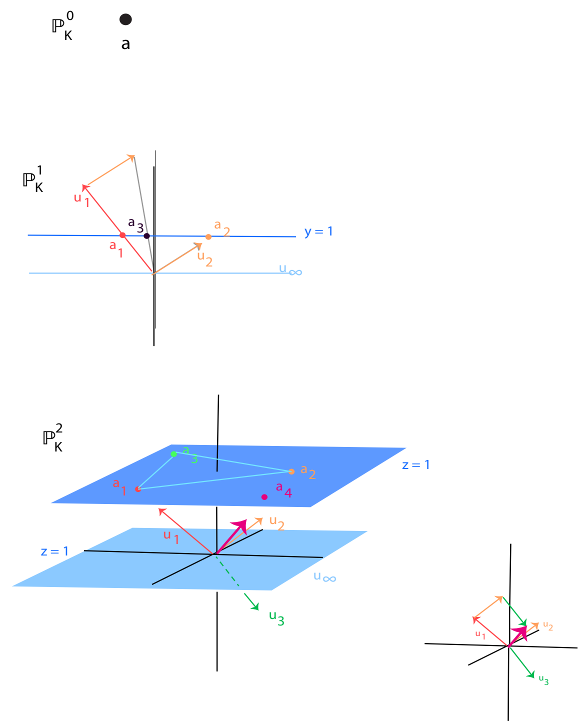

Figure 26.7: The projective frames for projective spaces of dimension 1, 2, and 3.

Definition 26.4. Given a nontrivial vector space E of dimension $n+1$, for any projective frame $(a_i)_{1\leq i\leq n+2}$ of $\mathbf{P}(E)$ and for any point $a \in \mathbf{P}(E)$, the *set of homogeneous coordinates of a with respect to* $(a_i)_{1\leq i\leq n+2}$ is the set of $(n+1)$-tuples

$$\{(\lambda x_1, \ldots, \lambda x_{n+1}) \in K^{n+1} \mid x_i \neq 0 \ \text{ for some } i, \ \lambda \neq 0, \ a = p(x_1 u_1 + \cdots + x_{n+1} u_{n+1})\},$$

where $(u_1, \ldots, u_{n+1})$ is any basis of E associated with $(a_i)_{1\leq i\leq n+2}$.

Given a projective frame $(a_i)_{1\leq i\leq n+2}$ for $\mathbf{P}(E)$, if $(x_1, \ldots, x_{n+1})$ are homogeneous coordinates of a point $a \in \mathbf{P}(E)$, we write $a = (x_1, \ldots, x_{n+1})$, and with a slight abuse of language, we may even talk about a point $(x_1, \ldots, x_{n+1})$ in $\mathbf{P}(E)$ and write $(x_1, \ldots, x_{n+1}) \in \mathbf{P}(E)$.

The special case of the projective line $\mathbb{P}^1_K$ is worth examining. The projective line $\mathbb{P}^1_K$ consists of all equivalence classes $[x, y]$ of pairs $(x, y) \in K^2$ such that $(x, y) \neq (0, 0)$, under the equivalence relation $\sim$ defined such that

$$(x_1, y_1) \sim (x_2, y_2) \quad \text{iff} \quad x_2 = \lambda x_1 \quad \text{and} \quad y_2 = \lambda y_1,$$

for some $\lambda \in K - \{0\}$. When $y \neq 0$, the equivalence class of (x, y) contains the representative $(xy^{-1}, 1)$, and when $y = 0$, the equivalence class of $(x, 0)$ contains the representative $(1, 0)$.