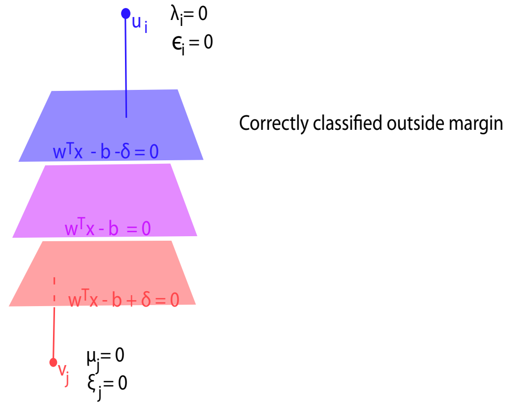

Figure 54.6: When $\lambda_i = 0$, u_i is correctly classified outside the blue margin. When $\mu_j = 0$, v_j is correctly classified outside the red margin.

Definition 54.1. The vectors u_i on the blue margin $H_{w,b+\delta}$ and the vectors v_j on the red margin $H_{w,b-\delta}$ are called *support vectors*. Support vectors correspond to vectors u_i for which $w^\top u_i - b - \delta = 0$ (which implies $\epsilon_i = 0$), and vectors v_j for which $w^\top v_j - b + \delta = 0$ (which implies $\xi_j = 0$). Support vectors u_i such that $0 < \lambda_i < K$ and support vectors v_j such that $0 < \mu_j < K$ are *support vectors of type 1*. Support vectors of type 1 play a special role so we denote the sets of indices associated with them by

$$\begin{aligned} I_\lambda &= \{i \in \{1, \ldots, p\} \mid 0 < \lambda_i < K\} \\ I_\mu &= \{j \in \{1, \ldots, q\} \mid 0 < \mu_j < K\}. \end{aligned}$$

We denote their cardinalities by $numsvl_1 = |I_\lambda|$ and $numsvm_1 = |I_\mu|$. Support vectors u_i such that $\lambda_i = K$ and support vectors v_j such that $\mu_j = K$ are *support vectors of type 2*. Those support vectors u_i such that $\lambda_i = 0$ and those support vectors v_j such that $\mu_j = 0$ are called *exceptional support vectors*.

The vectors u_i for which $\lambda_i = K$ and the vectors v_j for which $\mu_j = K$ are said to *fail the margin*. The sets of indices associated with the vectors failing the margin are denoted by

$$\begin{aligned} K_\lambda &= \{i \in \{1, \ldots, p\} \mid \lambda_i = K\} \\ K_\mu &= \{j \in \{1, \ldots, q\} \mid \mu_j = K\}. \end{aligned}$$

We denote their cardinalities by $p_f = |K_\lambda|$ and $q_f = |K_\mu|$.

Vectors u_i such that $\lambda_i > 0$ and vectors v_j such that $\mu_j > 0$ are said to *have margin at most* δ. The sets of indices associated with these vectors are denoted by

$$\begin{aligned} I_{\lambda>0} &= \{i \in \{1, \ldots, p\} \mid \lambda_i > 0\} \\ I_{\mu>0} &= \{j \in \{1, \ldots, q\} \mid \mu_j > 0\}. \end{aligned}$$

We denote their cardinalities by $p_m = |I_{\lambda>0}|$ and $q_m = |I_{\mu>0}|$.