whose vertices are a_0, a_1, a_2, a_3, including boundary points (faces and edges). The set

$$\{a_0 + \lambda_1 \overrightarrow{a_0 a_1} + \cdots + \lambda_n \overrightarrow{a_0 a_n} \mid \text{where } 0 \leq \lambda_i \leq 1 \ (\lambda_i \in \mathbb{R})\}$$

is called the *parallelotope spanned by* $(a_0, \ldots, a_n)$. When E has dimension 2, a parallelotope is also called a *parallelogram*, and when E has dimension 3, a *parallelepiped*. Figure 24.17 shows the convex hulls and associated parallelotopes for $|I| = 0, 1, 2, 3$.

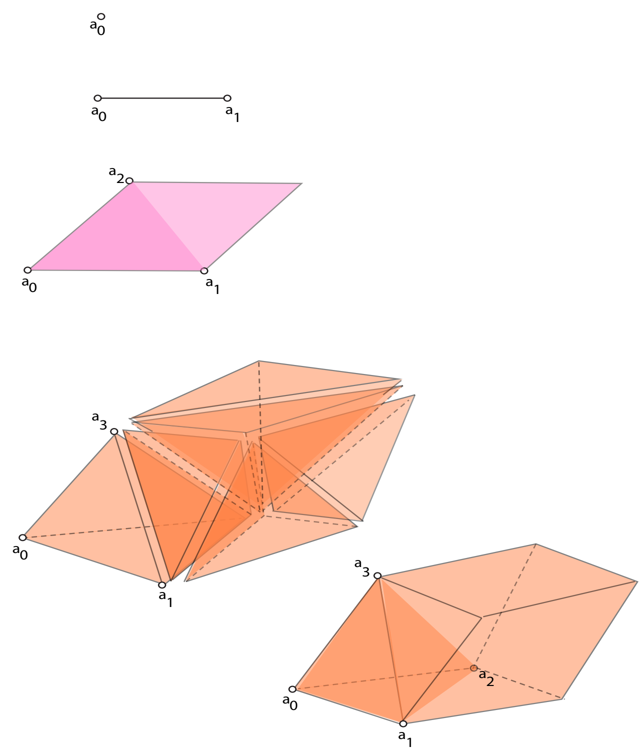

Figure 24.17: Examples of affine frames, convex hulls, and their associated parallelotopes.

More generally, we say that a subset V of E is *convex* if for any two points $a, b \in V$, we have $c \in V$ for every point $c = (1-\lambda)a + \lambda b$, with $0 \leq \lambda \leq 1$ $(\lambda \in \mathbb{R})$.

Points are not vectors! The following example illustrates why treating points as vectors may cause problems. Let a, b, c be three affinely independent points in $\mathbb{A}^3$. Any point x in the plane (a, b, c) can be expressed as

$$x = \lambda_0 a + \lambda_1 b + \lambda_2 c,$$

where $\lambda_0 + \lambda_1 + \lambda_2 = 1$. How can we compute $\lambda_0, \lambda_1, \lambda_2$? Letting $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3)$, $c = (c_1, c_2, c_3)$, and $x = (x_1, x_2, x_3)$ be the coordinates of a, b, c, x in the standard frame of $\mathbb{A}^3$, it is tempting to solve the system of equations