It is easy to see that a subspace F of E is indeed a vector space, since the restriction of $+\colon E \times E \to E$ to $F \times F$ is indeed a function $+\colon F \times F \to F$, and the restriction of $\cdot\colon K \times E \to E$ to $K \times F$ is indeed a function $\cdot\colon K \times F \to F$.

Since a subspace F is nonempty, if we pick any vector $u \in F$ and if we let $\lambda = \mu = 0$, then $\lambda u + \mu u = 0u + 0u = 0$, so *every subspace contains the vector* 0.

The following facts also hold. The proof is left as an exercise.

Proposition 3.4.

(1) The intersection of any family (even infinite) of subspaces of a vector space E is a subspace.

(2) Let F be any subspace of a vector space E. For any nonempty finite index set I, if $(u_i)_{i\in I}$ is any family of vectors $u_i \in F$ and $(\lambda_i)_{i\in I}$ is any family of scalars, then $\sum_{i\in I} \lambda_i u_i \in F$.

The subspace $\{0\}$ will be denoted by (0), or even 0 (with a mild abuse of notation).

Example 3.3.

1. In $\mathbb{R}^2$, the set of vectors $u = (x, y)$ such that

$$x + y = 0$$

is the subspace illustrated by Figure 3.9.

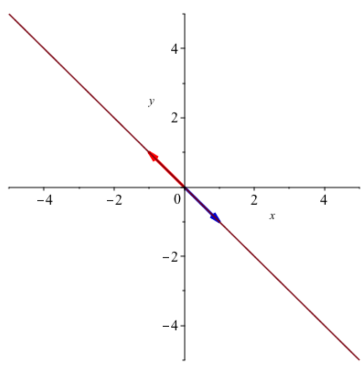

Figure 3.9: The subspace $x + y = 0$ is the line through the origin with slope -1. It consists of all vectors of the form $\lambda(-1, 1)$.

2. In $\mathbb{R}^3$, the set of vectors $u = (x, y, z)$ such that

$$x + y + z = 0$$

is the subspace illustrated by Figure 3.10.