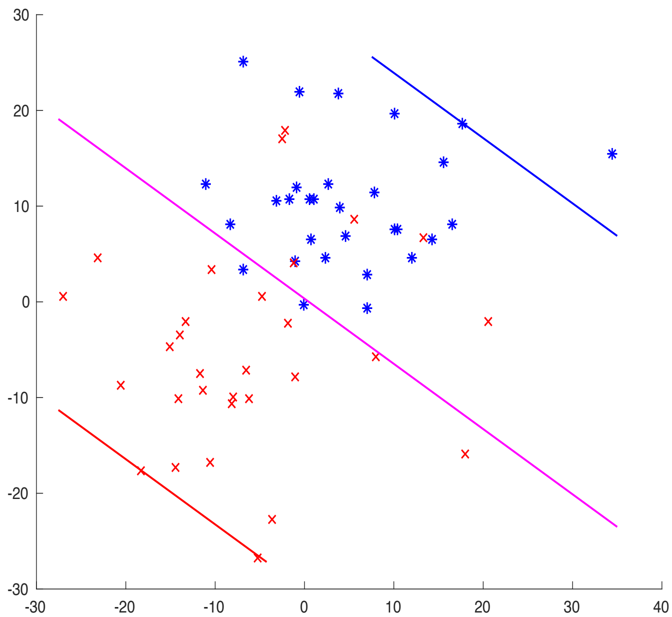

Figure 54.14: Running (SVM$_{s2'}$) on two sets of 30 points; $\nu = 0.95$.

Soft margin SVM (SVM$_{s3}$):

$$
\begin{aligned}
&\text{minimize} \quad \frac{1}{2}w^\top w + \frac{1}{2}b^2 + (p+q)K_s\left(-\nu\eta + \frac{1}{p+q}\begin{pmatrix}\epsilon^\top & \xi^\top\end{pmatrix}\mathbf{1}_{p+q}\right)\\
&\text{subject to}\\
&\qquad w^\top u_i - b \geq \eta - \epsilon_i, \quad \epsilon_i \geq 0 \qquad i = 1, \ldots, p\\
&\qquad -w^\top v_j + b \geq \eta - \xi_j, \quad \xi_j \geq 0 \qquad j = 1, \ldots, q.
\end{aligned}
$$

To simplify the presentation we assume that $K_s = 1/(p+q)$. When writing a computer program it is more convenient to assume that K_s is arbitrary. In this case, ν needs to be replaced by $(p+q)K_s\nu$ in all the formulae.

The Lagrangian $L(w, \epsilon, \xi, b, \eta, \lambda, \mu, \alpha, \beta)$ with $\lambda, \alpha \in \mathbb{R}^p_+$, $\mu, \beta \in \mathbb{R}^q_+$ is given by

$$
\begin{aligned}
L(w, \epsilon, \xi, b, \eta, \lambda, \mu, \alpha, \beta) &= \frac{1}{2}w^\top w + w^\top X\begin{pmatrix}\lambda\\ \mu\end{pmatrix} + \frac{b^2}{2} - \nu\eta + K_s(\epsilon^\top \mathbf{1}_p + \xi^\top \mathbf{1}_q) - \epsilon^\top(\lambda + \alpha)\\
&\quad - \xi^\top(\mu + \beta) + b(\mathbf{1}_p^\top\lambda - \mathbf{1}_q^\top\mu) + \eta(\mathbf{1}_p^\top\lambda + \mathbf{1}_q^\top\mu)\\
&= \frac{1}{2}w^\top w + w^\top X\begin{pmatrix}\lambda\\ \mu\end{pmatrix} + \frac{b^2}{2} + b(\mathbf{1}_p^\top\lambda - \mathbf{1}_q^\top\mu) + \eta(\mathbf{1}_p^\top\lambda + \mathbf{1}_q^\top\mu - \nu)\\
&\quad + \epsilon^\top(K_s\mathbf{1}_p - (\lambda + \alpha)) + \xi^\top(K_s\mathbf{1}_q - (\mu + \beta)).
\end{aligned}
$$