The *Gram–Schmidt orthonormalization procedure* also applies to Hermitian spaces of finite dimension, without any changes from the Euclidean case!

Proposition 14.12. *Given a nontrivial Hermitian space E of finite dimension $n \geq 1$, from any basis $(e_1, \ldots, e_n)$ for E we can construct an orthonormal basis $(u_1, \ldots, u_n)$ for E with the property that for every k, $1 \leq k \leq n$, the families $(e_1, \ldots, e_k)$ and $(u_1, \ldots, u_k)$ generate the same subspace.*

Remark: The remarks made after Proposition 12.10 also apply here, except that in the QR-decomposition, Q is a unitary matrix.

As a consequence of Proposition 12.9 (or Proposition 14.12), given any Hermitian space of finite dimension n, if $(e_1, \ldots, e_n)$ is an orthonormal basis for E, then for any two vectors $u = u_1e_1 + \cdots + u_ne_n$ and $v = v_1e_1 + \cdots + v_ne_n$, the Hermitian product $u \cdot v$ is expressed as

$$u \cdot v = (u_1e_1 + \cdots + u_ne_n) \cdot (v_1e_1 + \cdots + v_ne_n) = \sum_{i=1}^{n} u_i\overline{v_i},$$

and the norm $\|u\|$ as

$$\|u\| = \|u_1e_1 + \cdots + u_ne_n\| = \left(\sum_{i=1}^{n} |u_i|^2\right)^{1/2}.$$

The fact that a Hermitian space always has an orthonormal basis implies that any Gram matrix G can be written as

$$G = Q^*Q,$$

for some invertible matrix Q. Indeed, we know that in a change of basis matrix, a Gram matrix G becomes $G' = P^*GP$. If the basis corresponding to G' is orthonormal, then $G' = I$, so $G = (P^{-1})^*P^{-1}$.

Proposition 12.11 also holds unchanged.

Proposition 14.13. *Given any nontrivial Hermitian space E of finite dimension $n \geq 1$, for any subspace F of dimension k, the orthogonal complement $F^\perp$ of F has dimension $n - k$, and $E = F \oplus F^\perp$. Furthermore, we have $F^{\perp\perp} = F$.*

14.3 Linear Isometries (Also Called Unitary Transformations)

In this section we consider linear maps between Hermitian spaces that preserve the Hermitian norm. All definitions given for Euclidean spaces in Section 12.5 extend to Hermitian spaces,