from $(E^*)^n$ to $\mathrm{Hom}(\bigwedge^n(E), K)$, which extends to a linear map L from $\bigwedge^n(E^*)$ to $\mathrm{Hom}(\bigwedge^n(E), K)$ making the following diagram commute:

$$\begin{tikzcd}
(E^*)^n \arrow[r, "\iota_{\wedge^*}"] \arrow[dr] & \bigwedge^n(E^*) \arrow[d, "L"] \\
& \mathrm{Hom}(\bigwedge^n(E), K).
\end{tikzcd}$$

However, in view of the isomorphism

$$\mathrm{Hom}(U \otimes V, W) \cong \mathrm{Hom}(U, \mathrm{Hom}(V, W)),$$

with $U = \bigwedge^n(E^*)$, $V = \bigwedge^n(E)$ and $W = K$, we can view L as a linear map

$$L\colon \bigwedge^n(E^*) \otimes \bigwedge^n(E) \longrightarrow K,$$

which by Proposition 33.8 corresponds to a bilinear map

$$\langle -, - \rangle\colon \bigwedge^n(E^*) \times \bigwedge^n(E) \longrightarrow K. \tag{$*$}$$

This pairing is given explicitly in terms of generators by

$$\langle v_1^* \wedge \cdots \wedge v_n^*, u_1, \ldots, u_n \rangle = \det(v_j^*(u_i)).$$

Now this pairing in nondegenerate. This can be shown using bases. Given any basis $(e_1, \ldots, e_m)$ of E, for every basis element $e_{i_1}^* \wedge \cdots \wedge e_{i_n}^*$ of $\bigwedge^n(E^*)$ (with $1 \leq i_1 < \cdots < i_n \leq m$), we have

$$\langle e_{i_1}^* \wedge \cdots \wedge e_{i_n}^*, e_{j_1}, \ldots, e_{j_n} \rangle = \begin{cases} 1 & \text{if } (j_1, \ldots, j_n) = (i_1, \ldots, i_n) \\ 0 & \text{otherwise.} \end{cases}$$

We leave the details as an exercise to the reader. As a consequence we get the following canonical isomorphisms.

Proposition 34.10. *There is a canonical isomorphism*

$$(\bigwedge^n(E))^* \cong \bigwedge^n(E^*).$$

There is also a canonical isomorphism

$$\mu\colon \bigwedge^n(E^*) \cong \mathrm{Alt}^n(E; K)$$

which allows us to interpret alternating tensors over E^* as alternating multilinear maps.