



Figure 9.1: The top figure is  $\{x \in \mathbb{R}^2 \mid \|x\|_1 \leq 1\}$ , while the bottom figure is  $\{x \in \mathbb{R}^3 \mid \|x\|_1 \leq 1\}$ .

There are other norms besides the  $\ell^p$ -norms. Here are some examples.

1. For  $E = \mathbb{R}^2$ ,

$$\|(u_1, u_2)\| = |u_1| + 2|u_2|.$$

See Figure 9.5.

2. For  $E = \mathbb{R}^2$ ,

$$\|(u_1, u_2)\| = ((u_1 + u_2)^2 + u_1^2)^{1/2}.$$

See Figure 9.6.

3. For  $E = \mathbb{C}^2$ ,

$$\|(u_1, u_2)\| = |u_1 + iu_2| + |u_1 - iu_2|.$$

The reader should check that they satisfy all the axioms of a norm.

Some work is required to show the triangle inequality for the  $\ell^p$ -norm.