On the other hand, when executing the dual simplex algorithm, we have $\overline{c}_j \leq 0$ for all $j \notin K$ (and $\overline{c}_k = 0$ for all $k \in K$), and the outgoing column k^- is determined by picking one of the row indices such that $u_k < 0$. The index j^+ of the incoming column is determined by looking at the maximum of the ratios $-\overline{c}_j/\gamma^j_{k^-}$ for which $\gamma^j_{k^-} < 0$ (along row k^-).

More details about the comparison between the simplex algorithm and the dual simplex algorithm can be found in Bertsimas and Tsitsiklis [21] and Papadimitriou and Steiglitz [134].

Here is an example of the the dual simplex method.

Example 47.2. Consider the following linear program in standard form:

$$\text{Maximize} \quad -4x_1 - 2x_2 - x_3$$

$$\text{subject to} \quad \begin{pmatrix} -1 & -1 & 2 & 1 & 0 & 0 \\ -4 & -2 & 1 & 0 & 1 & 0 \\ 1 & 1 & -4 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ 2 \end{pmatrix} \quad \text{and } x_1, x_2, x_3, x_4, x_5, x_6 \geq 0.$$

We initialize the dual simplex procedure with (u, K) where $u = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3 \\ -4 \\ 2 \end{pmatrix}$ and $K = (4, 5, 6)$.

The initial tableau, before explicitly calculating the reduced cost, is

0	$\overline{c}_1$	$\overline{c}_2$	$\overline{c}_3$	$\overline{c}_4$	$\overline{c}_5$	$\overline{c}_6$
$u_4 = -3$	-1	-1	2	1	0	0
$u_5 = -4$	-4	-2	1	0	1	0
$u_6 = 2$	1	1	-4	0	0	1

.

Since u has negative coordinates, Case (B) applies, and we will set $k^- = 4$. We must now determine whether Case (B1) or Case (B2) applies. This determination is accomplished by scanning the first three columns in the tableau and observing each column has a negative entry. Thus Case (B2) is applicable, and we need to determine the reduced costs. Observe that $c = (-4, -2, -1, 0, 0, 0)$, which in turn implies $c_{(4,5,6)} = (0, 0, 0)$. Equation $(*_2)$ implies