we find the matrix

$$T = \begin{pmatrix} 5.8794 & 0.0015 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 \\ 0.0015 & 5.5321 & 0.0001 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0.0001 & 5.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0 & 0.0000 & 4.3473 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0 & 0 & 0 & 0.0000 & 3.6527 & 0.0000 & 0.0000 & -0.0000 \\ 0 & 0 & 0 & 0 & 0.0000 & 3.0000 & 0.0000 & -0.0000 \\ 0 & 0 & 0 & 0 & 0 & 0.0000 & 2.4679 & 0.0000 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0000 & 2.1206 \end{pmatrix}.$$

The diagonal entries match the eigenvalues found by running the `Matlab` function `eig(A)`.

If several eigenvalues have the same modulus, then the proof breaks down, we can no longer claim (†), namely that

$$\lim_{k \mapsto \infty} \Lambda^k L \Lambda^{-k} = I.$$

If we assume that P^{-1} has a suitable "block LU-factorization," it can be shown that the matrices A_{k+1} converge to a block upper-triangular matrix, where each block corresponds to eigenvalues having the same modulus. For example, if A is a 9×9 matrix with eigenvalues λ_i such that $|\lambda_1| = |\lambda_2| = |\lambda_3| > |\lambda_4| > |\lambda_5| = |\lambda_6| = |\lambda_7| = |\lambda_8| = |\lambda_9|$, then A_k converges to a block diagonal matrix (with three blocks, a 3×3 block, a 1×1 block, and a 5×5 block) of the form

$$\begin{pmatrix} \star & \star & \star & * & * & * & * & * & * \\ \star & \star & \star & * & * & * & * & * & * \\ \star & \star & \star & * & * & * & * & * & * \\ 0 & 0 & 0 & \star & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \star & \star & \star & \star & \star \\ 0 & 0 & 0 & 0 & \star & \star & \star & \star & \star \\ 0 & 0 & 0 & 0 & \star & \star & \star & \star & \star \\ 0 & 0 & 0 & 0 & \star & \star & \star & \star & \star \\ 0 & 0 & 0 & 0 & \star & \star & \star & \star & \star \end{pmatrix}.$$

See Ciarlet [41] (Chapter 6 Section 6.3) for more details.

Under the conditions of Theorem 18.1, in particular, if A is a symmetric (or Hermitian) positive definite matrix, the eigenvectors of A can be approximated. However, when A is not a symmetric matrix, since the upper triangular part of A_k does not necessarily converge, one has to be cautious that a rigorous justification is lacking.

Suppose we apply the QR algorithm to a matrix A satisfying the hypotheses of Theorem Theorem 18.1. For k large enough, $A_{k+1} = P_k^* A P_k$ is nearly upper triangular and the diagonal entries of A_{k+1} are all distinct, so we can consider that they are the eigenvalues of A_{k+1}, and thus of A. To avoid too many subscripts, write T for the upper triangular matrix