Since w is given by the equation

$$w = -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix}$$

and since we just showed that $\lambda^\kappa = \kappa\lambda$, $\mu^\kappa = \kappa\mu$, we deduce that $w^\kappa = \kappa w$.

We showed earlier that η is given by the equation

$$(p+q)K_s \nu\eta = \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(X^\top X + \frac{1}{2K_s} I_{p+q} \right) \begin{pmatrix} \lambda \\ \mu \end{pmatrix}.$$

If we replace ν by $\kappa\nu$, since λ is replaced by $\kappa\lambda$ and μ by $\kappa\nu$, we see that $\eta^\kappa = \kappa\eta$. Finally, b is given by the equation

$$b = \frac{w^\top(u_{i_0} + v_{j_0}) + \epsilon_{i_0} - \xi_{j_0}}{2}$$

for and i_0 such that $\lambda_{i_0} > 0$ and any j_0 such that $\mu_{j_0} > 0$. If λ is replaced by $\kappa\lambda$ and μ by $\kappa\mu$, since $\epsilon = \lambda/(2K_s)$ and $\xi = \mu/(2K_s)$, we see that ϵ is replaced by $\kappa\epsilon$ and ξ by $\kappa\xi$, so $b^\kappa = \kappa b$.

Since $w^\kappa = \kappa w$ and $\eta^\kappa = \kappa\eta$ we obtain $\delta = \eta / \|w\| = \eta^\kappa / \|w^\kappa\| = \delta^\kappa$. Since $w^\kappa = \kappa w$, $\eta^\kappa = \kappa\eta$ and $b^\kappa = \kappa b$, the normalized equations of the hyperplanes $H_{w,b}$, $H_{w,b+\eta}$ and $H_{w,b-\eta}$ (obtained by dividing by $\|w\|$) are all identical, so the hyperplanes $H_{w,b}$, $H_{w,b+\eta}$ and $H_{w,b-\eta}$ are independent of ν. $\square$

The width of the slab is controlled by K. The larger K is the smaller is the width of the slab. Theoretically, since this method does not rely on support vectors to compute b, it cannot fail if a solution exists, but in practice the quadratic solver does not converge for values of K that are too large. However, the method handles very small values of K, which can yield slabs of excessive width.

The "kernelized" version of Problem (SVM$_{s4}$) is the following:

Soft margin kernel SVM (SVM$_{s4}$):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\langle w, w\rangle - \nu\eta + K_s(\epsilon^\top\epsilon + \xi^\top\xi) \\
&\text{subject to} \\
&\qquad \langle w, \varphi(u_i)\rangle - b \geq \eta - \epsilon_i, \qquad i = 1, \ldots, p \\
&\qquad -\langle w, \varphi(v_j)\rangle + b \geq \eta - \xi_j, \qquad j = 1, \ldots, q \\
&\qquad \eta \geq 0,
\end{aligned}$$

with $K_s = 1/(p+q)$.

By going over the derivation of the dual program, we obtain