such objects as algebraic curves, algebraic surfaces, and algebraic varieties. The point of view where a complex object such as a curve or a surface is treated as a point in a (projective) space is actually very fruitful and is one of the themes of algebraic geometry (see Fulton [66] or Harris [87]).

(2) When $\dim(E) = 1$, we have $\dim(\mathbf{P}(E)) = 0$. When $E = \{0\}$, we have $\mathbf{P}(E) = \emptyset$. By convention, we give it the dimension -1.

We denote the projective space $\mathbf{P}(K^{n+1})$ by $\mathbb{P}^n_K$. When $K = \mathbb{R}$, we also denote $\mathbb{P}^n_{\mathbb{R}}$ by $\mathbb{RP}^n$, and when $K = \mathbb{C}$, we denote $\mathbb{P}^n_{\mathbb{C}}$ by $\mathbb{CP}^n$. The projective space $\mathbb{P}^0_K$ is a (projective) point. The projective space $\mathbb{P}^1_K$ is called a *projective line*. The projective space $\mathbb{P}^2_K$ is called a *projective plane*.

The projective space $\mathbf{P}(E)$ can be visualized in the following way. For simplicity, assume that $E = \mathbb{R}^{n+1}$, and thus $\mathbf{P}(E) = \mathbb{RP}^n$ (the same reasoning applies to $E = K^{n+1}$, where K is any field).

Let H be the affine hyperplane consisting of all points $(x_1, \ldots, x_{n+1})$ such that $x_{n+1} = 1$. Every nonzero vector u in E determines a line D passing through the origin, and this line intersects the hyperplane H in a unique point a, unless D is parallel to H. When D is parallel to H, the line corresponding to the equivalence class of u can be thought of as a point at infinity, often denoted by u_∞. Thus, the projective space $\mathbf{P}(E)$ can be viewed as the set of points in the hyperplane H, together with points at infinity associated with lines in the hyperplane H_∞ of equation $x_{n+1} = 0$. We will come back to this point of view when we consider the projective completion of an affine space. Figure 26.3 illustrates the above representation of the projective space for $E = \mathbb{R}^2$ and $E = \mathbb{R}^3$.

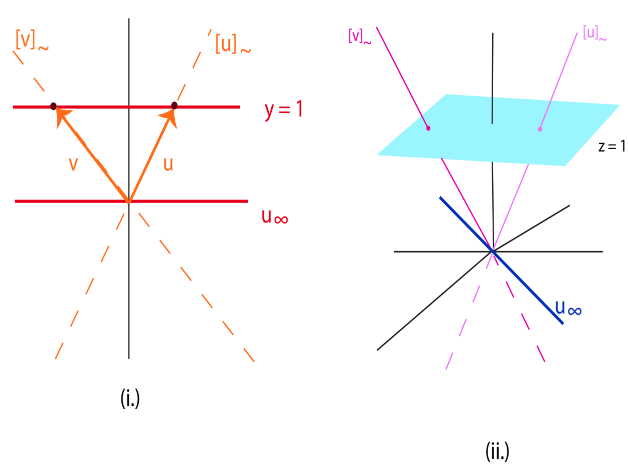

Figure 26.3: The hyperplane model representations of $\mathbb{RP}^1$ and $\mathbb{RP}^2$.