Remark: The above observation also applies to infinite families $(a_i)_{i\in I}$ of points in E and families $(u_i)_{i\in I-\{0\}}$ of vectors in $\overrightarrow{E}$, provided that the index set I contains 0.

When $(\overrightarrow{a_0a_1}, \ldots, \overrightarrow{a_0a_m})$ is a basis of $\overrightarrow{E}$ then, for every $x \in E$, since $x = a_0 + \overrightarrow{a_0x}$, there is a unique family $(x_1, \ldots, x_m)$ of scalars such that

$$x = a_0 + x_1\overrightarrow{a_0a_1} + \cdots + x_m\overrightarrow{a_0a_m}.$$

The scalars $(x_1, \ldots, x_m)$ may be considered as coordinates with respect to $(a_0, (\overrightarrow{a_0a_1}, \ldots, \overrightarrow{a_0a_m}))$. Since

$$x = a_0 + \sum_{i=1}^{m} x_i\overrightarrow{a_0a_i} \quad \text{iff} \quad x = \left(1 - \sum_{i=1}^{m} x_i\right) a_0 + \sum_{i=1}^{m} x_i a_i,$$

$x \in E$ can also be expressed uniquely as

$$x = \sum_{i=0}^{m} \lambda_i a_i$$

with $\sum_{i=0}^{m} \lambda_i = 1$, and where $\lambda_0 = 1 - \sum_{i=1}^{m} x_i$, and $\lambda_i = x_i$ for $1 \leq i \leq m$. The scalars $(\lambda_0, \ldots, \lambda_m)$ are also certain kinds of coordinates with respect to $(a_0, \ldots, a_m)$. All this is summarized in the following definition.

Definition 24.5. Given an affine space $\langle E, \overrightarrow{E}, +\rangle$, an *affine frame with origin* a_0 is a family $(a_0, \ldots, a_m)$ of $m + 1$ points in E such that the list of vectors $(\overrightarrow{a_0a_1}, \ldots, \overrightarrow{a_0a_m})$ is a basis of $\overrightarrow{E}$. The pair $(a_0, (\overrightarrow{a_0a_1}, \ldots, \overrightarrow{a_0a_m}))$ is also called an *affine frame with origin* a_0. Then, every $x \in E$ can be expressed as

$$x = a_0 + x_1\overrightarrow{a_0a_1} + \cdots + x_m\overrightarrow{a_0a_m}$$

for a unique family $(x_1, \ldots, x_m)$ of scalars, called the *coordinates of x w.r.t. the affine frame* $(a_0, (\overrightarrow{a_0a_1}, \ldots, \overrightarrow{a_0a_m}))$. Furthermore, every $x \in E$ can be written as

$$x = \lambda_0 a_0 + \cdots + \lambda_m a_m$$

for some unique family $(\lambda_0, \ldots, \lambda_m)$ of scalars such that $\lambda_0 + \cdots + \lambda_m = 1$ called the *barycentric coordinates of x with respect to the affine frame* $(a_0, \ldots, a_m)$. See Figure 24.15.

The coordinates $(x_1, \ldots, x_m)$ and the barycentric coordinates $(\lambda_0, \ldots, \lambda_m)$ are related by the equations $\lambda_0 = 1 - \sum_{i=1}^{m} x_i$ and $\lambda_i = x_i$, for $1 \leq i \leq m$. An affine frame is called an *affine basis* by some authors. A family $(a_i)_{i\in I}$ of points in E is *affinely dependent* if it is not affinely independent. We can also characterize affinely dependent families as follows.