The diagonal entries of $B_1\Delta_1$ are $(B_1)^2_{ii}$ and similarly the diagonal entries of $B_2\Delta_2$ are $(B_2)^2_{ii}$, so the above equation implies that

$$(B_1)^2_{ii} = (B_2)^2_{ii}, \quad i = 1, \ldots, n.$$

Since the diagonal entries of both B_1 and B_2 are assumed to be positive, we must have

$$(B_1)_{ii} = (B_2)_{ii}, \quad i = 1, \ldots, n;$$

that is, $\Delta_1 = \Delta_2$, and since both are invertible, we conclude from $(*)$ that $B_1 = B_2$.

Theorem 8.10 also holds for complex Hermitian positive definite matrices. In this case, we have $A = BB^*$ for some unique lower triangular matrix B with positive diagonal entries.

The proof of Theorem 8.10 immediately yields an algorithm to compute B from A by solving for a lower triangular matrix B such that $A = BB^\top$ (where both A and B are real matrices). For $j = 1, \ldots, n$,

$$b_{j\,j} = \left(a_{j\,j} - \sum_{k=1}^{j-1} b_{j\,k}^2\right)^{1/2},$$

and for $i = j+1, \ldots, n$ (and $j = 1, \ldots, n-1$)

$$b_{i\,j} = \left(a_{i\,j} - \sum_{k=1}^{j-1} b_{i\,k} b_{j\,k}\right) / b_{j\,j}.$$

The above formulae are used to compute the jth column of B from top-down, using the first $j-1$ columns of B previously computed, and the matrix A. In the case of $n = 3$, $A = BB^\top$ yields

$$\begin{aligned}
\begin{pmatrix} a_{11} & a_{12} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} &= \begin{pmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} b_{11} & b_{21} & b_{31} \\ 0 & b_{22} & b_{32} \\ 0 & 0 & b_{33} \end{pmatrix} \\
&= \begin{pmatrix} b_{11}^2 & b_{11}b_{21} & b_{11}b_{31} \\ b_{11}b_{21} & b_{21}^2 + b_{22}^2 & b_{21}b_{31} + b_{22}b_{32} \\ b_{11}b_{31} & b_{21}b_{31} + b_{22}b_{32} & b_{31}^2 + b_{32}^2 + b_{33}^2 \end{pmatrix}.
\end{aligned}$$

We work down the first column of A, compare entries, and discover that

$$\begin{aligned}
a_{11} &= b_{11}^2 & b_{11} &= \sqrt{a_{11}} \\
a_{21} &= b_{11}b_{21} & b_{21} &= \frac{a_{21}}{b_{11}} \\
a_{31} &= b_{11}b_{31} & b_{31} &= \frac{a_{31}}{b_{11}}.
\end{aligned}$$