where there is some $i \notin E_\lambda$ and some $j \notin E_\mu$.

If our optimal solution does not have a blue support vector and a red support vector, then either $w^\top u_i - b > \eta$ for all $i \notin E_\lambda$ or $-w^\top v_j + b > \eta$ for all $j \notin E_\mu$.

Case 1. We have

$$\begin{aligned} w^\top u_i - b &> \eta & & i \notin E_\lambda \\ -w^\top v_j + b &\geq \eta & & j \notin E_\mu. \end{aligned}$$

There are two subcases.

Case 1a. Assume that there is some $j \notin E_\mu$ such that $-w^\top v_j + b = \eta$. Our strategy is to increase η and b by a small amount θ in such a way that some inequality becomes an equation for some $i \notin E_\lambda$. Geometrically, this amounts to raising the separating hyperplane $H_{w,b}$ and increasing the width of the slab, keeping the red margin hyperplane unchanged. See Figure 54.7.

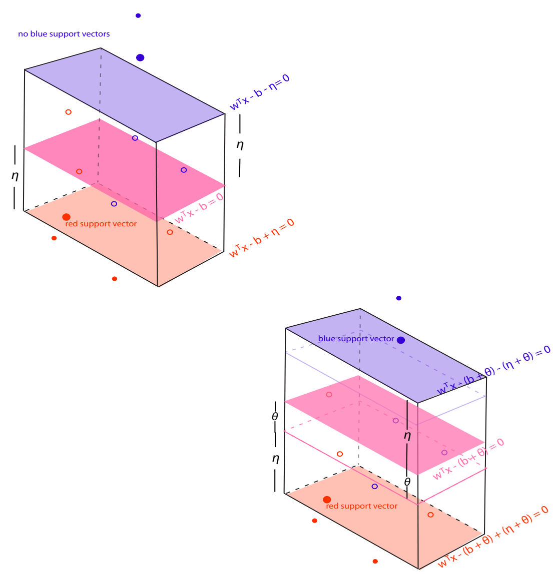

Figure 54.7: In this illustration points with errors are denoted by open circles. In the original, upper left configuration, there is no blue support vector. By raising the pink separating hyperplane and increasing the margin, we end up with a blue support vector.

Let us pick θ such that

$$\theta = (1/2)\min\{w^\top u_i - b - \eta \mid i \notin E_\lambda\}.$$