Also, given a vector

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},$$

we define the *additive inverse* $-x$ of x (pronounced minus x) as

$$-x = \begin{pmatrix} -x_1 \\ -x_2 \\ -x_3 \end{pmatrix}.$$

Observe that $-x = (-1)x$, the scalar multiplication of x by -1.

The set of all vectors with three components is denoted by $\mathbb{R}^{3\times 1}$. The reason for using the notation $\mathbb{R}^{3\times 1}$ rather than the more conventional notation $\mathbb{R}^3$ is that the elements of $\mathbb{R}^{3\times 1}$ are *column vectors*; they consist of three rows and a single column, which explains the superscript 3×1. On the other hand, $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ consists of all triples of the form (x_1, x_2, x_3), with $x_1, x_2, x_3 \in \mathbb{R}$, and these are *row vectors*. However, there is an obvious bijection between $\mathbb{R}^{3\times 1}$ and $\mathbb{R}^3$ and they are usually identified. For the sake of clarity, in this introduction, we will denote the set of column vectors with n components by $\mathbb{R}^{n\times 1}$.

An expression such as

$$x_1 u + x_2 v + x_3 w$$

where u, v, w are vectors and the x_is are scalars (in $\mathbb{R}$) is called a *linear combination*. Using this notion, the problem of solving our linear system

$$x_1 u + x_2 v + x_3 w = b.$$

is equivalent to *determining whether b can be expressed as a linear combination of u, v, w.*

Now if the vectors u, v, w are *linearly independent*, which means that there is *no* triple $(x_1, x_2, x_3) \neq (0, 0, 0)$ such that

$$x_1 u + x_2 v + x_3 w = 0_3,$$

it can be shown that *every* vector in $\mathbb{R}^{3\times 1}$ can be written as a linear combination of u, v, w. Here, 0_3 is the *zero vector*

$$0_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

It is customary to abuse notation and to write 0 instead of 0_3. This rarely causes a problem because in most cases, whether 0 denotes the scalar zero or the zero vector can be inferred from the context.

In fact, every vector $z \in \mathbb{R}^{3\times 1}$ can be written *in a unique way* as a linear combination

$$z = x_1 u + x_2 v + x_3 w.$$