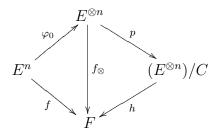
However, since f is symmetric, we have $f_{\otimes}(z) = 0$ for every $z \in C$. Thus, we get an induced linear map $h: (E^{\otimes n})/C \to F$ making the following diagram commute.



If we define $h([z]) = f_{\otimes}(z)$ for every $z \in E^{\otimes n}$, where [z] is the equivalence class in $(E^{\otimes n})/C$ of $z \in E^{\otimes n}$, the above diagram shows that $f = h \circ p \circ \varphi_0 = h \circ \varphi$. We now prove the uniqueness of h. For any linear map $f_{\odot} \colon (E^{\otimes n})/C \to F$ such that $f = f_{\odot} \circ \varphi$, since $\varphi(u_1, \ldots, u_n) = u_1 \odot \cdots \odot u_n$ and the vectors $u_1 \odot \cdots \odot u_n$ generate $(E^{\otimes n})/C$, the map f_{\odot} is uniquely defined by

$$f_{\odot}(u_1 \odot \cdots \odot u_n) = f(u_1, \dots, u_n).$$

Since $f = h \circ \varphi$, the map h is unique, and we let $f_{\odot} = h$. Thus, $S^{n}(E) = (E^{\otimes n})/C$ and φ constitute a symmetric n-th tensor power of E.

The map φ from E^n to $S^n(E)$ is often denoted ι_{\odot} , so that

$$\iota_{\odot}(u_1,\ldots,u_n)=u_1\odot\cdots\odot u_n.$$

Again, the actual construction is not important. What is important is that the symmetric *n*-th power has the universal mapping property with respect to symmetric multilinear maps.

Remark: The notation \odot for the commutative multiplication of symmetric tensor powers is not standard. Another notation commonly used is \cdot . We often abbreviate "symmetric tensor power" as "symmetric power." The symmetric power $S^n(E)$ is also denoted $\operatorname{Sym}^n E$ but we prefer to use the notation Sym to denote spaces of symmetric multilinear maps. To be consistent with the use of \odot , we could have used the notation $\bigodot^n E$. Clearly, $S^1(E) \cong E$ and it is convenient to set $S^0(E) = K$.

The fact that the map $\varphi \colon E^n \to \mathbf{S}^n(E)$ is symmetric and multilinear can also be expressed as follows:

$$u_{1} \odot \cdots \odot (v_{i} + w_{i}) \odot \cdots \odot u_{n} = (u_{1} \odot \cdots \odot v_{i} \odot \cdots \odot u_{n}) + (u_{1} \odot \cdots \odot w_{i} \odot \cdots \odot u_{n}),$$

$$u_{1} \odot \cdots \odot (\lambda u_{i}) \odot \cdots \odot u_{n} = \lambda (u_{1} \odot \cdots \odot u_{i} \odot \cdots \odot u_{n}),$$

$$u_{\sigma(1)} \odot \cdots \odot u_{\sigma(n)} = u_{1} \odot \cdots \odot u_{n},$$

for all permutations $\sigma \in \mathfrak{S}_n$.

The last identity shows that the "operation" \odot is commutative. This allows us to view the symmetric tensor $u_1 \odot \cdots \odot u_n$ as an object called a multiset.