We see immediately that the constraints

$$\begin{aligned} x + y - z &= 0 \\ x - y - z &= 0 \end{aligned}$$

imply that $z = x$ and $y = 0$. Then it is easy using calculus to find that the unique minimum is given by $(x, y, z) = (1, 0, 1)$. Running `qsolve1` on P_1, q_1, A_1, b_1 with $\rho = 10$, $tolr = tols = 10^{-12}$ and $iternum = 10000$, we find that after 83 iterations the primal and the dual residuals are less than 10^{-12}, and we get

$$(x, y, z) = (1.000000000000149, 0.000000000000000, 1.000000000000148).$$

Example 52.12. Consider the quadratic program for which

$$P_2 = \begin{pmatrix} 4 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{pmatrix} \qquad q_2 = -\begin{pmatrix} 4 \\ 4 \\ 4 \\ 4 \end{pmatrix}$$
$$A_2 = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & -1 & -1 & 0 \end{pmatrix} \qquad b_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Again, we see immediately that the constraints imply that $z = x$ and $y = 0$. Then it is easy using calculus to find that the unique minimum is given by $(x, y, z) = (28/31, 0, 28/31, 24/31)$. Running `qsolve1` on P_2, q_2, A_2, b_2 with $\rho = 10$, $tolr = tols = 10^{-12}$ and $iternum = 10000$, we find that after 95 iterations the primal and the dual residuals are less than 10^{-12}, and we get

$$(x, y, z, t) = (0.903225806451495, 0.000000000000000, 0.903225806451495, \\ 0.774193548387264),$$

which agrees with the answer found earlier up to 11 decimals.

As an illustration of the wide range of applications of ADMM we show in the next section how to solve the hard margin SVM (SVM_{h2}) discussed in Section 50.6.

52.7 Solving Hard Margin (SVM_{h2}) Using ADMM

Recall that we would like to solve the following optimization problem (see Section 50.6):

Hard margin SVM (SVM_{h2}):

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\|w\|^2 \\ &\text{subject to} \\ &\qquad w^\top u_i - b \geq 1 \qquad i = 1, \ldots, p \\ &\qquad -w^\top v_j + b \geq 1 \qquad j = 1, \ldots, q. \end{aligned}$$