solves part of the exercises and copies the rest from the others (which we do not recommend, of course!). It is assumed that each student solves his part of the homework at home, and that the solutions are communicated by phone. The problem is to minimize the number of phone calls. An obvious but expensive method is for each student to call each of the other seven students. A much better method is to imagine that the eight students are the vertices of a cube, say with coordinates from $\{0,1\}^3$. There are three types of edges:

1. Those parallel to the z-axis, called *type* 1;
2. Those parallel to the y-axis, called *type* 2;
3. Those parallel to the x-axis, called *type* 3.

The communication can proceed in three rounds as follows: All nodes connected by type 1 edges exchange solutions; all nodes connected by type 2 edges exchange solutions; and finally all nodes connected by type 3 edges exchange solutions.

It is easy to see that everybody has all the answers at the end of the three rounds. Furthermore, each student is involved only in three calls (making a call or receiving it), and the total number of calls is twelve.

In the general case, N nodes would like to exchange information in such a way that eventually every node has all the information. A good way to to this is to construct certain finite projective spaces, as explained in Beutelspacher and Rosenbaum [22]. We pick q to be an integer (for instance, a prime number) such that there is a finite projective space of any dimension over the finite field of order q. Then, we pick d such that

$$q^{d-1} < N \leq q^d.$$

Since q is prime, there is a projective space $\mathbf{P}(K^{d+1})$ of dimension d over the finite field K of order q, and letting $\mathcal{H}$ be the hyperplane at infinity in $\mathbf{P}(K^{d+1})$, we pick a frame $P_1, \ldots, P_d$ in $\mathcal{H}$. It turns out that the affine space $\mathcal{A} = \mathbf{P}(K^{d+1}) - \mathcal{H}$ has q^d points. Then the communication nodes can be identified with points in the affine space $\mathcal{A}$. Assuming for simplicity that $N = q^d$, the algorithm proceeds in d rounds. During round i, each node $Q \in \mathcal{A}$ sends the information it has received to all nodes in $\mathcal{A}$ on the line QP_i.

It can be shown that at the end of the d rounds, each node has the total information, and that the total number of transactions is at most

$$(q-1)\log_q(N)N.$$

Other applications of projective spaces to communication systems with switches are described in Chapter 2, Section 8, of Beutelspacher and Rosenbaum [22]. Applications to error-correcting codes are described in Chapter 5 of the same book. Introducing even the most elementary notions of coding theory would take too much space. Let us simply say that the existence of certain types of good codes called *linear* $[n, n-r]$*-codes with minimum*