(2) *Basis pursuit.*

This is the following minimization problem:

$$\begin{aligned} &\text{minimize} && \|x\|_1 \\ &\text{subject to} && Ax = b, \end{aligned}$$

where $A$ is an $m \times n$ matrix of rank $m < n$, and $b \in \mathbb{R}^m$, $x \in \mathbb{R}^n$. The problem is to find a sparse solution to an underdetermined linear system, which means a solution $x$ with many zero coordinates. This problem plays a central role in compressed sensing and statistical signal processing.

Basis pursuit can be expressed in ADMM form as the problem

$$\begin{aligned} &\text{minimize} && I_C(x) + \|z\|_1 \\ &\text{subject to} && x - z = 0, \end{aligned}$$

with $C = \{x \in \mathbb{R}^n \mid Ax = b\}$. It is easy to see that the ADMM procedure (in scaled form) is

$$\begin{aligned} x^{k+1} &= \Pi_C(z^k - u^k) \\ z^{k+1} &= S_{1/\rho}(x^{k+1} + u^k) \\ u^{k+1} &= u^k + x^{k+1} - z^{k+1}, \end{aligned}$$

where $\Pi_C$ is the orthogonal projection onto the subspace $C$. In fact, it is not hard to show that

$$x^{k+1} = (I - A^\top(AA^\top)^{-1}A)(z^k - u^k) + A^\top(AA^\top)^{-1}b.$$

In some sense, an $\ell^1$-minimization problem is reduced to a sequence of $\ell^2$-norm problems. There are ways of improving the efficiency of the method; see Boyd et al. [28] (Section 6.2)

(3) *General $\ell^1$-regularized loss minimization.*

This is the following minimization problem:

$$\text{minimize} \quad l(x) + \tau \|x\|_1,$$

where $l$ is any proper closed and convex loss function, and $\tau > 0$. We convert the problem to the ADMM problem:

$$\begin{aligned} &\text{minimize} && l(x) + \tau \|z\|_1 \\ &\text{subject to} && x - z = 0. \end{aligned}$$