The change in the objective function of the primal and dual program (which is the same, since $u_K = A_K^{-1}b$ and $y = c_K A_K^{-1}$ is chosen such that $cu = c_K u_K = yb$) is the same as in the simplex algorithm, namely

$$\theta^+\left(c^{j^+} - c_K\gamma_K^{j^+}\right).$$

We have $\theta^+ > 0$ and $c^{j^+} - c_K\gamma_K^{j^+} \leq 0$, so if $c^{j^+} - c_K\gamma_K^{j^+} < 0$, then the objective function of the dual program decreases strictly.

Case (B3). $\mu^+ = 0$.

The possibity that $\mu^+ = 0$, that is, $c^{j^+} - c_K\gamma_K^{j^+} = 0$, may arise. In this case, the objective function doesn't change. This is a case of degeneracy similar to the degeneracy that arises in the simplex algorithm. We still pick $j^+ \in N(\mu^+)$, but we need a pivot rule that prevents cycling. Such rules exist; see Bertsimas and Tsitsiklis [21] (Section 4.5) and Papadimitriou and Steiglitz [134] (Section 3.6).

The reader surely noticed that the dual simplex algorithm is very similar to the simplex algorithm, except that the simplex algorithm preserves the property that (u, K) is (primal) feasible, whereas the dual simplex algorithm preserves the property that $y = c_K A_K^{-1}$ is dual feasible. One might then wonder whether the dual simplex algorithm is equivalent to the simplex algorithm applied to the dual problem. This is indeed the case, there is a one-to-one correspondence between the dual simplex algorithm and the simplex algorithm applied to the dual problem in maximization form. This correspondence is described in Papadimitriou and Steiglitz [134] (Section 3.7).

The comparison between the simplex algorithm and the dual simplex algorithm is best illustrated if we use a description of these methods in terms of *(full) tableaux*.

Recall that a *(full) tableau* is an $(m+1)\times(n+1)$ matrix organized as follows:

$-c_K u_K$	$\overline{c}_1$	$\cdots$	$\overline{c}_j$	$\cdots$	$\overline{c}_n$
u_{k_1}	γ_1^1	$\cdots$	γ_1^j	$\cdots$	γ_1^n
$\vdots$	$\vdots$		$\vdots$		$\vdots$
u_{k_m}	γ_m^1	$\cdots$	γ_m^j	$\cdots$	γ_m^n

The top row contains the current value of the objective function and the reduced costs, the first column except for its top entry contain the components of the current basic solution u_K, and the remaining columns except for their top entry contain the vectors γ_K^j. Observe that the γ_K^j corresponding to indices j in K constitute a permutation of the identity matrix I_m. A tableau together with the new basis $K^+ = (K - \{k^-\}) \cup \{j^+\}$ contains all the data needed to compute the new u_{K^+}, the new $\gamma_{K^+}^j$, and the new reduced costs $\overline{c}_i - (\gamma_{k^-}^i/\gamma_{k^-}^{j^+})\overline{c}_{j^+}$.

When executing the simplex algorithm, we have $u_k \geq 0$ for all $k \in K$ (and $u_j = 0$ for all $j \notin K$), and the incoming column j^+ is determined by picking one of the column indices such that $\overline{c}_j > 0$. Then the index k^- of the leaving column is determined by looking at the minimum of the ratios $u_k/\gamma_k^{j^+}$ for which $\gamma_k^{j^+} > 0$ (along column j^+).