By definition $p_f = |K_\lambda|, q_f = |K_\mu|$. Since the equation

$$\sum_{i=1}^m \lambda_i + \sum_{j=1}^m \mu_j = C\nu$$

holds, by definition of K_λ and K_μ we have

$$(p_f + q_f)\frac{C}{m} = \sum_{i\in K_\lambda} \lambda_i + \sum_{j\in K_\mu} \mu_j \leq \sum_{i=1}^m \lambda_i + \sum_{j=1}^m \mu_j = C\nu,$$

which implies that

$$p_f + q_f \leq m\nu.$$

(2) Let $I_{\lambda>0}$ and $I_{\mu>0}$ be the sets of indices

$$\begin{aligned} I_{\lambda>0} &= \{i \in \{1,\ldots,m\} \mid \lambda_i > 0\} \\ I_{\mu>0} &= \{i \in \{1,\ldots,m\} \mid \mu_i > 0\}. \end{aligned}$$

By definition $p_m = |I_{\lambda>0}|, q_m = |I_{\mu>0}|$. We have

$$\sum_{i=1}^m \lambda_i + \sum_{j=1}^m \mu_j = \sum_{i\in I_{\lambda>0}} \lambda_i + \sum_{j\in I_{\mu>0}} \mu_j = C\nu.$$

Since $\lambda_i \leq C/m$ and $\mu_j \leq C/m$, we obtain

$$C\nu \leq (p_m + q_m)\frac{C}{m},$$

that is, $p_m + q_m \geq m\nu$.

(3) follows immediately from (1). $\square$

Proposition 56.7 yields the following bounds on ν:

$$\frac{p_f + q_f}{m} \leq \nu \leq \frac{p_m + q_m}{m}.$$

Again, the smaller ν is, the wider the ϵ-slab is, and the larger ν is, the narrower the ϵ-slab is.

Remark: It can be shown that for any optimal solution with $w \neq 0$ and $\epsilon > 0$, if the inequalities $(p_f + q_f)/m < \nu < 1$ hold, then some point x_i is a support vector. The proof is essentially Case 1b in the proof of Proposition 56.4. We leave the details as an exercise.