Dual Program kernel ν-SV Regression:

$$
\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\sum_{i,j=1}^{m}(\lambda_i - \mu_i)(\lambda_j - \mu_j)\kappa(x_i, x_j) + \sum_{i=1}^{m}(\lambda_i - \mu_i)y_i \\
&\text{subject to} \\
&\qquad \sum_{i=1}^{m}\lambda_i - \sum_{i=1}^{m}\mu_i = 0 \\
&\qquad \sum_{i=1}^{m}\lambda_i + \sum_{i=1}^{m}\mu_i \leq C\nu \\
&\qquad 0 \leq \lambda_i \leq \frac{C}{m}, \quad 0 \leq \mu_i \leq \frac{C}{m}, \quad i = 1, \ldots, m,
\end{aligned}
$$

minimizing over α and μ.

Everything we said before also applies to the kernel ν-SV regression method, except that x_i is replaced by $\varphi(x_i)$ and that the inner product $\langle -, - \rangle$ must be used, and we have the formulae

$$
\begin{aligned}
w &= \sum_{i=1}^{m}(\mu_i - \lambda_i)\varphi(x_i) \\
b &= \frac{1}{2}\left(y_{i_0} + y_{j_0} - \sum_{i=1}^{m}(\mu_i - \lambda_i)(\kappa(x_i, x_{i_0}) + \kappa(x_i, x_{j_0}))\right) \\
f(x) &= \sum_{i=1}^{m}(\mu_i - \lambda_i)\kappa(x_i, x) + b,
\end{aligned}
$$

expressions that only involve κ.

Remark: There is a variant of ν-SV regression obtained by setting $\nu = 0$ and holding $\epsilon > 0$ fixed. This method is called *ϵ-SV regression* or *(linear) ϵ-insensitive SV regression*. The corresponding optimization program is

Program ϵ-SV Regression:

$$
\begin{aligned}
&\text{minimize} \quad \frac{1}{2}w^\top w + \frac{C}{m}\sum_{i=1}^{m}(\xi_i + \xi_i') \\
&\text{subject to} \\
&\qquad w^\top x_i + b - y_i \leq \epsilon + \xi_i, \quad \xi_i \geq 0 \qquad i = 1, \ldots, m \\
&\qquad -w^\top x_i - b + y_i \leq \epsilon + \xi_i', \quad \xi_i' \geq 0 \qquad i = 1, \ldots, m,
\end{aligned}
$$

minimizing over the variables w, b, ξ, and ξ', holding ϵ fixed.

It is easy to see that the dual program is