If we use a $QR$ decomposition of $C$, by permuting the columns of $C$ to make sure that the first $r$ columns of $C$ are linearly independent (where $r = \mathrm{rank}(C)$), we may assume that

$$C = Q^\top \begin{pmatrix} R & S \\ 0 & 0 \end{pmatrix} \Pi,$$

where $Q$ is an $n \times n$ orthogonal matrix, $R$ is an $r \times r$ invertible upper triangular matrix, $S$ is an $r \times (m-r)$ matrix, and $\Pi$ is a permutation matrix ($C$ has rank $r$). Then if we let

$$x = Q^\top \begin{pmatrix} y \\ z \end{pmatrix},$$

where $y \in \mathbb{R}^r$ and $z \in \mathbb{R}^{n-r}$, then $C^\top x = 0$ becomes

$$C^\top x = \Pi^\top \begin{pmatrix} R^\top & 0 \\ S^\top & 0 \end{pmatrix} Qx = \Pi^\top \begin{pmatrix} R^\top & 0 \\ S^\top & 0 \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = 0,$$

which implies $y = 0$, and every solution of $C^\top x = 0$ is of the form

$$x = Q^\top \begin{pmatrix} 0 \\ z \end{pmatrix}.$$

Our original problem becomes

$$\begin{aligned} &\text{minimize} && \frac{1}{2}(y^\top \; z^\top) QAQ^\top \begin{pmatrix} y \\ z \end{pmatrix} + (y^\top \; z^\top) Qb \\ &\text{subject to} && y = 0, \; y \in \mathbb{R}^r, \; z \in \mathbb{R}^{n-r}. \end{aligned}$$

Thus, the constraint $C^\top x = 0$ has been simplified to $y = 0$, and if we write

$$QAQ^\top = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix},$$

where $G_{11}$ is an $r \times r$ matrix and $G_{22}$ is an $(n-r) \times (n-r)$ matrix and

$$Qb = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad b_1 \in \mathbb{R}^r, \; b_2 \in \mathbb{R}^{n-r},$$

our problem becomes

$$\text{minimize} \; \frac{1}{2} z^\top G_{22} z + z^\top b_2, \quad z \in \mathbb{R}^{n-r},$$

the problem solved in Proposition 42.5.

Constraints of the form $C^\top x = t$ (where $t \neq 0$) can be handled in a similar fashion. In this case, we may assume that $C$ is an $n \times m$ matrix with full rank (so that $m \leq n$) and $t \in \mathbb{R}^m$. Then we use a $QR$-decomposition of the form

$$C = P \begin{pmatrix} R \\ 0 \end{pmatrix},$$