

Next we work down the second column of A using previously calculated expressions for b_{21} and b_{31} to find that

$$\begin{aligned} a_{22} &= b_{21}^2 + b_{22}^2 & b_{22} &= (a_{22} - b_{21}^2)^{\frac{1}{2}} \\ a_{32} &= b_{21}b_{31} + b_{22}b_{32} & b_{32} &= \frac{a_{32} - b_{21}b_{31}}{b_{22}}. \end{aligned}$$

Finally, we use the third column of A and the previously calculated expressions for b_{31} and b_{32} to determine b_{33} as

$$a_{33} = b_{31}^2 + b_{32}^2 + b_{33}^2 \qquad b_{33} = (a_{33} - b_{31}^2 - b_{32}^2)^{\frac{1}{2}}.$$

For another example, if

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix},$$

we find that

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

We leave it as an exercise to find similar formulae (involving conjugation) to factor a complex Hermitian positive definite matrix A as $A = BB^*$. The following **Matlab** program implements the Cholesky factorization.

```
function B = Cholesky(A)
n = size(A,1);
B = zeros(n,n);
for j = 1:n-1;
    if j == 1
        B(1,1) = sqrt(A(1,1));
        for i = 2:n
            B(i,1) = A(i,1)/B(1,1);
        end
    else
        B(j,j) = sqrt(A(j,j) - B(j,1:j-1)*B(j,1:j-1)');
        for i = j+1:n
```