we have

$$\begin{aligned}
&\overline{f}((u_1, \ldots, u_i + v_i, \ldots, u_n) - (u_1, \ldots, u_i, \ldots, u_n) - (u_1, \ldots, v_i, \ldots, u_n))\\
&\qquad = f(u_1, \ldots, u_i + v_i, \ldots, u_n) - f(u_1, \ldots, u_i, \ldots, u_n) - f(u_1, \ldots, v_i, \ldots, u_n)\\
&\qquad = f(u_1, \ldots, u_i, \ldots, u_n) + f(u_1, \ldots, v_i, \ldots, u_n) - f(u_1, \ldots, u_i, \ldots, u_n)\\
&\qquad\quad - f(u_1, \ldots, v_i, \ldots, u_n)\\
&\qquad = 0.
\end{aligned}$$

But then, $\overline{f}\colon M \to F$ factors through M/N, which means that there is a unique linear map $h\colon M/N \to F$ such that $\overline{f} = h \circ \pi$ making the following diagram commute

$$\begin{tikzcd}
M \arrow[r, "\pi"] \arrow[dr, "\overline{f}"'] & M/N \arrow[d, "h"] \\
& F,
\end{tikzcd}$$

by defining $h([z]) = \overline{f}(z)$ for every $z \in M$, where $[z]$ denotes the equivalence class in M/N of $z \in M$. Indeed, the fact that $\overline{f}$ vanishes on N insures that h is well defined on M/N, and it is clearly linear by definition. Since $f = \overline{f} \circ \iota$, from the equation $\overline{f} = h \circ \pi$, by composing on the right with ι, we obtain

$$f = \overline{f} \circ \iota = h \circ \pi \circ \iota = h \circ \varphi,$$

as in the following commutative diagram.

$$\begin{tikzcd}
& K^{(E_1 \times \cdots \times E_n)} \arrow[dr, "\pi"] \arrow[dd, "\overline{f}"] & \\
E_1 \times \cdots \times E_n \arrow[ur, "\iota"] \arrow[dr, "f"'] & & K^{(E_1 \times \cdots \times E_n)}/N \arrow[dl, "h"] \\
& F &
\end{tikzcd}$$

We now prove the uniqueness of h. For any linear map $f_\otimes\colon E_1 \otimes \cdots \otimes E_n \to F$ such that $f = f_\otimes \circ \varphi$, since the vectors $u_1 \otimes \cdots \otimes u_n$ generate $E_1 \otimes \cdots \otimes E_n$ and since $\varphi(u_1, \ldots, u_n) = u_1 \otimes \cdots \otimes u_n$, the map $f_\otimes$ is uniquely defined by

$$f_\otimes(u_1 \otimes \cdots \otimes u_n) = f(u_1, \ldots, u_n).$$

Since $f = h \circ \varphi$, the map h is unique, and we let $f_\otimes = h$. $\square$

The map φ from $E_1 \times \cdots \times E_n$ to $E_1 \otimes \cdots \otimes E_n$ is often denoted by $\iota_\otimes$, so that

$$\iota_\otimes(u_1, \ldots, u_n) = u_1 \otimes \cdots \otimes u_n.$$