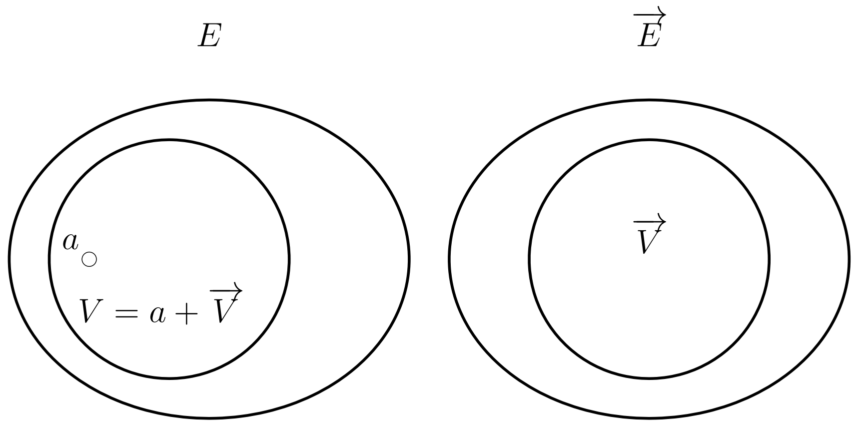


Figure 24.13: An affine subspace $V$ and its direction $\overrightarrow{V}$.

**Proposition 24.2.** *Let $\langle E, \overrightarrow{E}, +\rangle$ be an affine space.*

*(1) A nonempty subset $V$ of $E$ is an affine subspace iff for every point $a \in V$, the set*

$$\overrightarrow{V_a} = \{\overrightarrow{ax} \mid x \in V\}$$

*is a subspace of $\overrightarrow{E}$. Consequently, $V = a + \overrightarrow{V_a}$. Furthermore,*

$$\overrightarrow{V} = \{\overrightarrow{xy} \mid x, y \in V\}$$

*is a subspace of $\overrightarrow{E}$ and $\overrightarrow{V_a} = \overrightarrow{V}$ for all $a \in E$. Thus, $V = a + \overrightarrow{V}$.*

*(2) For any subspace $\overrightarrow{V}$ of $\overrightarrow{E}$ and for any $a \in E$, the set $V = a + \overrightarrow{V}$ is an affine subspace.*

*Proof.* The proof is straightforward, and is omitted. It is also given in Gallier [70]. □

In particular, when $E$ is the natural affine space associated with a vector space $\overrightarrow{E}$, Proposition 24.2 shows that every affine subspace of $E$ is of the form $u + \overrightarrow{U}$, for a subspace $\overrightarrow{U}$ of $\overrightarrow{E}$. The subspaces of $\overrightarrow{E}$ are the affine subspaces of $E$ that contain 0.

The subspace $\overrightarrow{V}$ associated with an affine subspace $V$ is called the *direction of $V$*. It is also clear that the map $+ \colon V \times \overrightarrow{V} \to V$ induced by $+ \colon E \times \overrightarrow{E} \to E$ confers to $\langle V, \overrightarrow{V}, +\rangle$ an affine structure. Figure 24.13 illustrates the notion of affine subspace.

By the dimension of the subspace $V$, we mean the dimension of $\overrightarrow{V}$.

An affine subspace of dimension 1 is called a *line*, and an affine subspace of dimension 2 is called a *plane*.