Theorem 17.18. *For every normal matrix A there is an orthogonal matrix P and a block diagonal matrix D such that $A = PD\,P^\top$, where D is of the form*

$$D = \begin{pmatrix} D_1 & & \dots & \\ & D_2 & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ & & \dots & D_p \end{pmatrix}$$

such that each block D_j is either a one-dimensional matrix (i.e., a real scalar) or a two-dimensional matrix of the form

$$D_j = \begin{pmatrix} \lambda_j & -\mu_j \\ \mu_j & \lambda_j \end{pmatrix},$$

where $\lambda_j, \mu_j \in \mathbb{R}$, with $\mu_j > 0$.

Theorem 17.19. *For every symmetric matrix A there is an orthogonal matrix P and a diagonal matrix D such that $A = PD\,P^\top$, where D is of the form*

$$D = \begin{pmatrix} \lambda_1 & & \dots & \\ & \lambda_2 & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ & & \dots & \lambda_n \end{pmatrix},$$

where $\lambda_i \in \mathbb{R}$.

Theorem 17.20. *For every skew-symmetric matrix A there is an orthogonal matrix P and a block diagonal matrix D such that $A = PD\,P^\top$, where D is of the form*

$$D = \begin{pmatrix} D_1 & & \dots & \\ & D_2 & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ & & \dots & D_p \end{pmatrix}$$

such that each block D_j is either 0 or a two-dimensional matrix of the form

$$D_j = \begin{pmatrix} 0 & -\mu_j \\ \mu_j & 0 \end{pmatrix},$$

where $\mu_j \in \mathbb{R}$, with $\mu_j > 0$. In particular, the eigenvalues of A are pure imaginary of the form $\pm i\mu_j$, or 0.

Theorem 17.21. *For every orthogonal matrix A there is an orthogonal matrix P and a block diagonal matrix D such that $A = PD\,P^\top$, where D is of the form*

$$D = \begin{pmatrix} D_1 & & \dots & \\ & D_2 & \dots & \\ \vdots & \vdots & \ddots & \vdots \\ & & \dots & D_p \end{pmatrix}$$