defining the convex set

$$U = \left\{ \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \in \mathbb{R}_+^{p+q} \mid \mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu = (p+q)K_s\nu \right\}.$$

The proof is essentially the proof of 54.8 using the above SPD matrix and convex set. $\square$

The "kernelized" version of Problem (SVM_{s5}) is the following:

Soft margin kernel SVM (SVM_{s5}):

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\langle w, w\rangle + \frac{1}{2}b^2 - \nu\eta + K_s(\epsilon^\top\epsilon + \xi^\top\xi) \\ &\text{subject to} \\ &\qquad \langle w, \varphi(u_i)\rangle - b \geq \eta - \epsilon_i, \qquad i = 1, \ldots, p \\ &\qquad -\langle w, \varphi(v_j)\rangle + b \geq \eta - \xi_j, \qquad j = 1, \ldots, q, \end{aligned}$$

with $K_s = 1/(p+q)$.

Tracing through the derivation of the dual program, we obtain

Dual of the Soft margin kernel SVM (SVM_{s5}):

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(\mathbf{K} + \begin{pmatrix} \mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top \\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top \end{pmatrix} + \frac{1}{2K_s}I_{p+q}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\ &\text{subject to} \\ &\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = \nu \\ &\qquad \lambda_i \geq 0, \quad i = 1, \ldots, p \\ &\qquad \mu_j \geq 0, \quad j = 1, \ldots, q, \end{aligned}$$

where $\mathbf{K}$ is the kernel matrix of Section 54.1. Then w, b, and $f(x)$ are obtained exactly as in Section 54.13.

54.16 Solving SVM (SVM_{s5}) Using ADMM

In order to solve (SVM_5) using ADMM we need to write the matrix corresponding to the constraints in equational form,

$$\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = K_m,$$

with $K_m = (p+q)K_s\nu$. This is the $1 \times (p+q)$ matrix A given by

$$A = \begin{pmatrix}\mathbf{1}_p^\top & \mathbf{1}_q^\top\end{pmatrix}.$$