

The expression $Q(\alpha_1, \alpha_2, \dots, \alpha_{i+1})$ is called the *i-th difference quotient*. Then, we can compute the λ_i in terms of $\beta_1 = P(\alpha_1), \dots, \beta_{m+1} = P(\alpha_{m+1})$, using the inductive formulae for the $Q(\alpha_1, \dots, \alpha_i, X)$ given above, initializing the $Q(\alpha_i)$ such that $Q(\alpha_i) = \beta_i$.

The above method is called the method of *divided differences* and it is due to Newton.

An astute observation may be used to optimize the computation. Observe that if $P_i(X)$ is the polynomial of degree $\leq i$ taking the values $\beta_1, \dots, \beta_{i+1}$ at the points $\alpha_1, \dots, \alpha_{i+1}$, then the coefficient of X^i in $P_i(X)$ is $Q(\alpha_1, \alpha_2, \dots, \alpha_{i+1})$, which is the value of λ_i in the Newton interpolant

$$P_i(X) = \lambda_0 + \lambda_1(X - \alpha_1) + \lambda_2(X - \alpha_1)(X - \alpha_2) + \cdots + \lambda_i(X - \alpha_1)(X - \alpha_2) \cdots (X - \alpha_i).$$

As a consequence, $Q(\alpha_1, \alpha_2, \dots, \alpha_{i+1})$ does not depend on the specific ordering of the α_j and there are better ways of computing it. For example, $Q(\alpha_1, \alpha_2, \dots, \alpha_{i+1})$ can be computed using

$$Q(\alpha_1, \dots, \alpha_{i+1}) = \frac{Q(\alpha_2, \dots, \alpha_{i+1}) - Q(\alpha_1, \dots, \alpha_i)}{\alpha_{i+1} - \alpha_1}.$$

Then, the computation can be arranged into a triangular array reminiscent of Pascal's triangle, as follows:

Initially, $Q(\alpha_j) = \beta_j$, $1 \leq j \leq m+1$, and

$$\begin{array}{ccccccc}
Q(\alpha_1) & & & & & & \\
& Q(\alpha_1, \alpha_2) & & & & & \\
Q(\alpha_2) & & & Q(\alpha_1, \alpha_2, \alpha_3) & & & \\
& Q(\alpha_2, \alpha_3) & & & & \dots & \\
Q(\alpha_3) & & Q(\alpha_2, \alpha_3, \alpha_4) & & & & \\
& Q(\alpha_3, \alpha_4) & & \dots & & & \\
Q(\alpha_4) & \dots & & & & & \\
& \dots & & & & & \\
& & \dots & & & &
\end{array}$$

In this computation, each successive column is obtained by forming the difference quotients of the preceding column according to the formula

$$Q(\alpha_k, \dots, \alpha_{i+k}) = \frac{Q(\alpha_{k+1}, \dots, \alpha_{i+k}) - Q(\alpha_k, \dots, \alpha_{i+k-1})}{\alpha_{i+k} - \alpha_k}.$$

The λ_i are the elements of the descending diagonal.

Observe that if we performed the above computation starting with a polynomial $Q(X)$ of degree m , we could extend it by considering new given points α_{m+2} , α_{m+3} , etc. Then, from what we saw above, the $(m+1)$ th column consists of λ_m in the expression of $Q(X)$ as a Newton interpolant and the $(m+2)$ th column consists of zeros. Such divided differences are used in numerical analysis.