



Figure 56.4: The two pink open half spaces associated with the hyperplane $w^\top x_i - z + b = 0$. Note, $\xi_i > 0$ is outside of the half space $w^\top x_i - z + b - \epsilon < 0$, and $\xi'_i > 0$ is outside of the half space $w^\top x_i - z + b + \epsilon > 0$.

Observe that the equations

$$\begin{aligned} w^\top x_i + b - y_i &= \epsilon + \xi_i \\ -w^\top x_i - b + y_i &= \epsilon + \xi'_i \end{aligned}$$

can only hold simultaneously if

$$\epsilon + \xi_i = -\epsilon - \xi'_i,$$

that is,

$$2\epsilon + \xi_i + \xi'_i = 0,$$

and since $\epsilon, \xi_i, \xi'_i \geq 0$, this can happen only if $\epsilon = \xi_i = \xi'_i = 0$, and then

$$w^\top x_i + b = y_i.$$

In particular, if $\epsilon > 0$, then the equations

$$\begin{aligned} w^\top x_i + b - y_i &= \epsilon + \xi_i \\ -w^\top x_i - b + y_i &= \epsilon + \xi'_i \end{aligned}$$

cannot hold simultaneously. □