Deduce that we may assume that the n possible values $(z_1)_k$ for z_1 are given by

$$(z_1)_k = e^{\frac{k\pi i}{n+1}}, \quad k = 1, \ldots, n,$$

and find

$$2\lambda_k = (z_1)_k + (z_1)_k^{-1}.$$

Show that an eigenvector $(y_1^{(k)}, \ldots, y_n^{(k)})$ associated with the eigenvalue λ_k is given by

$$y_j^{(k)} = \sin\left(\frac{kj\pi}{n+1}\right), \quad j = 1, \ldots, n.$$

(2) Find the spectral radius $\rho(J)$, $\rho(\mathcal{L}_1)$, and $\rho(\mathcal{L}_{\omega_0})$, as functions of $h = 1/(n+1)$.