Since there are $p+q+1$ Lagrange multipliers (λ, μ, γ), the $(p+q) \times (p+q)$ matrix P must be augmented with zero's to make it a $(p+q+1) \times (p+q+1)$ matrix P_a given by

$$P_a = \begin{pmatrix} X^\top X & 0_{p+q} \\ 0_{p+q}^\top & 0 \end{pmatrix},$$

and similarly q is augmented with zeros as the vector $q_a = 0_{p+q+1}$.

As in Section 54.8, since $\eta \geq 0$ for an optimal solution, we can drop the constraint $\eta \geq 0$ from the primal problem. In this case, there are $p+q$ Lagrange multipliers (λ, μ). It is easy to see that the objective function of the dual is unchanged and the set of constraints is

$$\begin{aligned} \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j &= 0 \\ \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j &= K_m, \end{aligned}$$

with $K_m = (p+q)K_s\nu$. The matrix corresponding to the above equations is the $2 \times (p+q)$ matrix A_2 given by

$$A_2 = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top \end{pmatrix}.$$

We leave it as an exercise to prove that A_2 has rank 2. The right-hand side is

$$c_2 = \begin{pmatrix} 0 \\ K_m \end{pmatrix}.$$

The symmetric positive semidefinite $(p+q)\times(p+q)$ matrix P defining the quadratic functional is

$$P = X^\top X + \frac{1}{2K_s} I_{p+q}, \quad \text{with} \quad X = \begin{pmatrix} -u_1 & \cdots & -u_p & v_1 & \cdots & v_q \end{pmatrix},$$

and

$$q = 0_{p+q}.$$

Since there are $p+q$ Lagrange multipliers (λ, μ), the $(p+q) \times (p+q)$ matrix P need not be augmented with zero's, so $P_{2a} = P$ and similarly $q_{2a} = 0_{p+q}$.

We ran our `Matlab` implementation of the above version of (SVM_{s4}) on the data set of Section 54.12. Since the value of ν is irrelevant, we picked $\nu = 1$. First we ran our program with $K = 190$; see Figure 54.22. We have $p_m = 23$ and $q_m = 18$. The program does not converge for $K \geq 200$.