Chapter 44

Convex Sets, Cones, $\mathcal{H}$-Polyhedra

44.1 What is Linear Programming?

What is *linear programming*? At first glance, one might think that this is some style of computer programming. After all, there is imperative programming, functional programming, object-oriented programming, *etc.* The term linear programming is somewhat misleading, because it really refers to a method for *planning* with linear constraints, or more accurately, an *optimization method* where both the objective function and the constraints are linear.[1]

Linear programming was created in the late 1940's, one of the key players being George Dantzing, who invented the simplex algorithm. Kantorovitch also did some pioneering work on linear programming as early as 1939. The term *linear programming* has a military connotation because in the early 1950's it was used as a synonym for plans or schedules for training troops, logistical supply, resource allocation, *etc.* Unfortunately the term linear programming is well established and we are stuck with it.

Interestingly, even though originally most applications of linear programming were in the field of economics and industrial engineering, linear programming has become an important tool in theoretical computer science and in the theory of algorithms. Indeed, linear programming is often an effective tool for designing approximation algorithms to solve hard problems (typically NP-hard problems). Linear programming is also the "baby version" of convex programming, a very effective methodology which has received much attention in recent years.

Our goal is to present the mathematical underpinnings of linear programming, in particular the existence of an optimal solution if a linear program is feasible and bounded, and the duality theorem in linear programming, one of the deepest results in this field. The duality theorem in linear programming also has significant algorithmic implications but we do not discuss this here. We present the simplex algorithm, the dual simplex algorithm, and the primal dual algorithm. We also describe the tableau formalism for running the simplex

[1]Again, we witness another unfortunate abuse of terminology; the constraints are in fact *affine*.