(1) Let p_f be the number of points u_i such that $\lambda_i = K_s$, and let q_f the number of points v_j such that $\mu_j = K_s$. Then $p_f + q_f \leq (p+q)\nu$.

(2) Let p_m be the number of points u_i such that $\lambda_i > 0$, and let q_m the number of points v_j such that $\mu_j > 0$. Then $p_m + q_m \geq (p+q)\nu$. We have $p_m + q_m \geq 1$.

(3) If $p_f \geq 1$ or $q_f \geq 1$, then $\nu \geq 1/(p+q)$.

Proof. (1) Recall that for an optimal solution with $w \neq 0$ and $\eta > 0$ we have the equation

$$\sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \mu_j = \nu.$$

Since there are p_f points u_i such that $\lambda_i = K_s = 1/(p+q)$ and q_f points v_j such that $\mu_j = K_s = 1/(p+q)$, we have

$$\nu = \sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \mu_j \geq \frac{p_f + q_f}{p+q},$$

so

$$p_f + q_f \leq \nu(p+q).$$

(2) If

$$I_{\lambda>0} = \{i \in \{1, \ldots, p\} \mid \lambda_i > 0\} \quad \text{and} \quad p_m = |I_{\lambda>0}|$$

and

$$I_{\mu>0} = \{j \in \{1, \ldots, q\} \mid \mu_j > 0\} \quad \text{and} \quad q_m = |I_{\mu>0}|,$$

then

$$\nu = \sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \mu_j = \sum_{i \in I_{\lambda>0}} \lambda_i + \sum_{j \in I_{\mu>0}} \mu_j,$$

and since $\lambda_i, \mu_j \leq K_s = 1/(p+q)$, we have

$$\nu = \sum_{i \in I_{\lambda>0}} \lambda_i + \sum_{j \in I_{\mu>0}} \mu_j \leq \frac{p_m + q_m}{p+q},$$

which yields

$$p_m + q_m \geq \nu(p+q).$$

We already noted earlier that $p_m + q_m \geq 1$.

(3) This follows immediately from (1). $\square$