Problem 54.14. Implement Program (SVM_{s4}) in `Matlab`. You may adapt the programs given in Section B.2 and Section B.3.

Problem 54.15. Prove that the kernel version of Program (SVM_{s4}) is given by:

Dual of the Soft margin kernel SVM (SVM_{s4}):

$$\begin{aligned}
\text{minimize} \quad & \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(\mathbf{K} + \frac{p+q}{2} I_{p+q}\right)\begin{pmatrix}\lambda\\ \mu\end{pmatrix}\\
\text{subject to} \quad & \\
& \sum_{i=1}^{p}\lambda_i - \sum_{j=1}^{q}\mu_j = 0\\
& \sum_{i=1}^{p}\lambda_i + \sum_{j=1}^{q}\mu_j \geq \nu\\
& \lambda_i \geq 0, \quad i = 1, \ldots, p\\
& \mu_j \geq 0, \quad j = 1, \ldots, q,
\end{aligned}$$

where $\mathbf{K}$ is the kernel matrix of Section 54.1.

Problem 54.16. Implement Program (SVM_{s5}) in `Matlab`. You may adapt the programs given in Section B.2 and Section B.3.

Problem 54.17. Prove that the kernel version of Program (SVM_{s5}) is given by:

Dual of the Soft margin kernel SVM (SVM_{s5}):

$$\begin{aligned}
\text{minimize} \quad & \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(\mathbf{K} + \begin{pmatrix}\mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top\\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top\end{pmatrix} + \frac{p+q}{2} I_{p+q}\right)\begin{pmatrix}\lambda\\ \mu\end{pmatrix}\\
\text{subject to} \quad & \\
& \sum_{i=1}^{p}\lambda_i + \sum_{j=1}^{q}\mu_j = \nu\\
& \lambda_i \geq 0, \quad i = 1, \ldots, p\\
& \mu_j \geq 0, \quad j = 1, \ldots, q,
\end{aligned}$$

where $\mathbf{K}$ is the kernel matrix of Section 54.1.