Let us take a closer look at the inequalities $z^*A \geq 0$. For $j \in J$, since z^* is an optimal solution of (DRP), we know that $z^*A_J \geq 0$, so if $z^*A^j \geq 0$ for all $j \in N$, then $(P2)$ is not feasible.

Otherwise, there is some $j \in N = \{1, \ldots, n\} - J$ such that

$$z^*A^j < 0,$$

and then since by the definition of N we have $yA^j > c_j$ for all $j \in N$, if we pick θ such that

$$0 < \theta \leq \frac{yA^j - c_j}{-z^*A^j} \quad j \in N,\ z^*A^j < 0,$$

then we decrease the objective function $y(\theta)b = yb + \theta z^*b$ of (D) (since $z^*b < 0$). Therefore we pick the best θ, namely

$$\theta^+ = \min\left\{ \frac{yA^j - c_j}{-z^*A^j} \;\middle|\; j \notin J,\ z^*A^j < 0 \right\} > 0. \tag{$*_4$}$$

Next we update y to $y^+ = y(\theta^+) = y + \theta^+ z^*$, we create the new restricted primal with the new subset

$$J^+ = \{j \in \{1, \ldots, n\} \mid y^+A^j = c_j\},$$

and repeat the process.

Here are the steps of the primal-dual algorithm.

Step 1. Find some feasible solution y of the Dual Program (D). We will show later that this is always possible.

Step 2. Compute

$$J^+ = \{j \in \{1, \ldots, n\} \mid yA^j = c_j\}.$$

Step 3. Set $J = J^+$ and solve the Problem (RP) using the simplex algorithm, starting from the optimal solution determined during the previous round, obtaining the optimal solution (x_J^*, ξ^*) with the basis K^*.

Step 4.

If $\xi^* = 0$, then stop with an optimal solution u^* for (P) such that $u_J^* = x_J^*$ and the other components of u^* are zero.

Else let

$$z^* = -\mathbf{1}_m^\top - (\overline{c}_{K^*})_{(p+1,\ldots,p+m)},$$

be the optimal solution of (DRP) corresponding to (x_J^*, ξ^*) and the basis K^*.

If $z^*A^j \geq 0$ for all $j \notin J$, then stop; the Program (P) has no feasible solution.