Problem 54.2. Prove the averaging formulae

$$b = w^\top \left(\left(\sum_{i \in I_\lambda} u_i \right) / |I_\lambda| + \left(\sum_{j \in I_\mu} v_j \right) / |I_\mu| \right) / 2$$
$$\delta = w^\top \left(\left(\sum_{i \in I_\lambda} u_i \right) / |I_\lambda| - \left(\sum_{j \in I_\mu} v_j \right) / |I_\mu| \right) / 2$$

stated at the end of Section 54.1.

Problem 54.3. Prove that the matrix

$$A = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\ I_p & 0_{p,q} & I_p & 0_{p,q} \\ 0_{q,p} & I_q & 0_{q,p} & I_q \end{pmatrix}$$

has rank $p + q + 2$.

Problem 54.4. Prove that the dual program of the kernel version of (SVM_{s1}) is given by:

Dual of Soft margin kernel SVM (SVM_{s1}):

$$\begin{aligned} &\text{minimize} \quad \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \mathbf{K} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\ &\text{subject to} \\ &\qquad \sum_{i=1}^p \lambda_i = \sum_{j=1}^q \mu_j = \frac{1}{2} \\ &\qquad 0 \leq \lambda_i \leq K, \quad i = 1, \ldots, p \\ &\qquad 0 \leq \mu_j \leq K, \quad j = 1, \ldots, q, \end{aligned}$$

where $\mathbf{K}$ is the $\ell \times \ell$ kernel symmetric matrix (with $\ell = p + q$) given by

$$\mathbf{K}_{ij} = \begin{cases} \kappa(u_i, u_j) & 1 \leq i \leq p,\ 1 \leq j \leq q \\ -\kappa(u_i, v_{j-p}) & 1 \leq i \leq p,\ p+1 \leq j \leq p+q \\ -\kappa(v_{i-p}, u_j) & p+1 \leq i \leq p+q,\ 1 \leq j \leq p \\ \kappa(v_{i-p}, v_{j-q}) & p+1 \leq i \leq p+q,\ p+1 \leq j \leq p+q. \end{cases}$$

Problem 54.5. Prove the averaging formula

$$b = w^\top \left(\left(\sum_{i \in I_\lambda} u_i \right) / |I_\lambda| + \left(\sum_{j \in I_\mu} v_j \right) / |I_\mu| \right) / 2$$

stated in Section 54.3.