and

$$
\begin{aligned}
c_{K^+} E(\gamma^j_K)^{-1} \gamma^i_K &= \begin{pmatrix} c_1 & \cdots & c_{\ell-1} & \dfrac{c_j}{\gamma^j_\ell} - \displaystyle\sum_{k=1, k\neq \ell}^m c_k \frac{\gamma^j_k}{\gamma^j_\ell} & c_{\ell+1} & \cdots & c_m \end{pmatrix} \begin{pmatrix} \gamma^i_1 \\ \vdots \\ \gamma^i_{\ell-1} \\ \gamma^i_\ell \\ \gamma^i_{\ell+1} \\ \vdots \\ \gamma^i_m \end{pmatrix} \\
&= \sum_{k=1, k\neq \ell}^m c_k \gamma^i_k + \frac{\gamma^i_\ell}{\gamma^j_\ell} \left(c_j - \sum_{k=1, k\neq \ell}^m c_k \gamma^j_k \right) \\
&= \sum_{k=1, k\neq \ell}^m c_k \gamma^i_k + \frac{\gamma^i_\ell}{\gamma^j_\ell} \left(c_j + c_\ell \gamma^j_\ell - \sum_{k=1}^m c_k \gamma^j_k \right) \\
&= \sum_{k=1}^m c_k \gamma^i_k + \frac{\gamma^i_\ell}{\gamma^j_\ell} \left(c_j - \sum_{k=1}^m c_k \gamma^j_k \right) \\
&= c_K \gamma^i_K + \frac{\gamma^i_\ell}{\gamma^j_\ell} (c_j - c_K \gamma^j_K),
\end{aligned}
$$

and thus

$$
c_i - c_{K^+} \gamma^i_{K^+} = c_i - c_{K^+} E(\gamma^j_K)^{-1} \gamma^i_K = c_i - c_K \gamma^i_K - \frac{\gamma^i_\ell}{\gamma^j_\ell} (c_j - c_K \gamma^j_K),
$$

as claimed. $\square$

Since $(\gamma^1_{k^-}, \ldots, \gamma^n_{k^-})$ is the ℓth row of Γ, we see that Proposition 46.2 shows that

$$
\overline{c}_{K^+} = \overline{c}_K - \frac{(\overline{c}_K)_{j^+}}{\gamma^{j^+}_{k^-}} \, \Gamma_\ell, \tag{$\dagger$}
$$

where Γ_ℓ denotes the ℓ-th row of Γ and $\gamma^{j^+}_{k^-}$ is the pivot. This means that $\overline{c}_{K^+}$ is obtained by the elementary row operations which consist of first normalizing the ℓth row by dividing it by the pivot $\gamma^{j^+}_{k^-}$, and then subtracting $(\overline{c}_K)_{j^+} \times$ the normalized Row ℓ from $\overline{c}_K$. *These are exactly the row operations that make the reduced cost* $(\overline{c}_K)_{j^+}$ *zero.*

Remark: It easy easy to show that we also have

$$
\overline{c}_{K^+} = c - c_{K^+} \Gamma^+.
$$