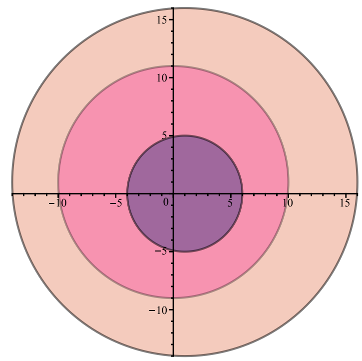

Figure 15.1: Let A be the 3×3 matrix specified at the end of Definition 15.5. For this particular A, we find that $R'_1(A) = 5$, $R'_2(A) = 10$, and $R'_3(A) = 15$. The blue/purple disk is $|z - 1| \leq 5$, the pink disk is $|z - i| \leq 10$, the peach disk is $|z - 1 - i| \leq 15$, and $G(A)$ is the union of these three disks.

(1) If A is strictly row diagonally dominant, that is

$$|a_{i\,i}| > \sum_{j=1,\, j\neq i}^{n} |a_{i\,j}|, \quad \textit{for } i = 1, \ldots, n,$$

then A is invertible.

(2) If A is strictly row diagonally dominant, and if $a_{i\,i} > 0$ for $i = 1, \ldots, n$, then every eigenvalue of A has a strictly positive real part.

Proof. Let λ be any eigenvalue of A and let u be a corresponding eigenvector (recall that we must have $u \neq 0$). Let k be an index such that

$$|u_k| = \max_{1 \leq i \leq n} |u_i|.$$

Since $Au = \lambda u$, we have

$$(\lambda - a_{k\,k})u_k = \sum_{\substack{j=1 \\ j\neq k}}^{n} a_{k\,j} u_j,$$

which implies that

$$|\lambda - a_{k\,k}||u_k| \leq \sum_{\substack{j=1 \\ j\neq k}}^{n} |a_{k\,j}||u_j| \leq |u_k| \sum_{\substack{j=1 \\ j\neq k}}^{n} |a_{k\,j}|.$$

Since $u \neq 0$ and $|u_k| = \max_{1\leq i\leq n} |u_i|$, we must have $|u_k| \neq 0$, and it follows that

$$|\lambda - a_{k\,k}| \leq \sum_{\substack{j=1 \\ j\neq k}}^{n} |a_{k\,j}| = R'_k(A),$$