We are now reduced to *Case 1a* or *Case 2a*.

Case 2. We have

$$\begin{aligned} w^\top u_i - b &\geq \eta & i &\notin E_\lambda \\ -w^\top v_j + b &> \eta & j &\notin E_\mu. \end{aligned}$$

There are two subcases.

Case 2a. Assume that there is some $i \notin E_\lambda$ such that $w^\top u_i - b = \eta$. Our strategy is to increase η and decrease b by a small amount θ in such a way that some inequality becomes an equation for some $j \notin E_\mu$. Geometrically, this amounts to lowering the separating hyperplane $H_{w,b}$ and increasing the width of the slab, keeping the blue margin hyperplane unchanged. See Figure 54.10.

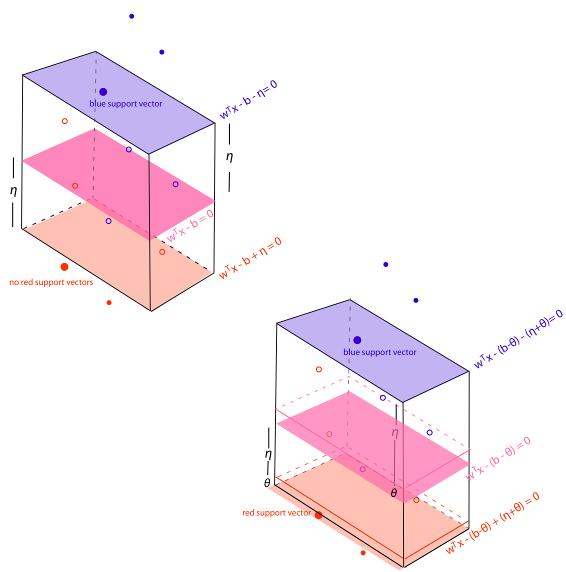

Figure 54.10: In this illustration points with errors are denoted by open circles. In the original, upper left configuration, there is no red support vector. By lowering the pink separating hyperplane and increasing the margin, we end up with a red support vector.

Let us pick θ such that

$$\theta = (1/2) \min\{-w^\top v_j + b - \eta \mid j \notin E_\mu\}.$$

Our hypotheses imply that $\theta > 0$. We can write

$$\begin{aligned} w^\top u_i - (b - \theta) &= \eta + \theta - \epsilon_i & \epsilon_i &> 0 & i &\in E_\lambda \\ -w^\top v_j + b - \theta &= \eta + \theta - (\xi_j + 2\theta) & \xi_j &> 0 & j &\in E_\mu \\ w^\top u_i - (b - \theta) &\geq \eta + \theta & & & i &\notin E_\lambda \\ -w^\top v_j + b - \theta &\geq \eta + \theta & & & j &\notin E_\mu. \end{aligned}$$