(1) The first step is to pick some nonzero entry $a_{i\,1}$ in the first column of A. Such an entry must exist, since A is invertible (otherwise, the first column of A would be the zero vector, and the columns of A would not be linearly independent. Equivalently, we would have $\det(A) = 0$). The actual choice of such an element has some impact on the numerical stability of the method, but this will be examined later. For the time being, we assume that some arbitrary choice is made. This chosen element is called the *pivot* of the elimination step and is denoted π_1 (so, in this first step, $\pi_1 = a_{i\,1}$).

(2) Next we permute the row (i) corresponding to the pivot with the first row. Such a step is called *pivoting*. So after this permutation, the first element of the first row is nonzero.

(3) We now eliminate the variable x_1 from all rows except the first by adding suitable multiples of the first row to these rows. More precisely we add $-a_{i\,1}/\pi_1$ times the first row to the ith row for $i = 2, \ldots, n$. At the end of this step, all entries in the first column are zero except the first.

(4) Increment k by 1. If $k = n$, stop. Otherwise, $k < n$, and then iteratively repeat Steps (1), (2), (3) on the $(n-k+1) \times (n-k+1)$ subsystem obtained by deleting the first $k-1$ rows and $k-1$ columns from the current system.

If we let $A_1 = A$ and $A_k = (a^{(k)}_{i\,j})$ be the matrix obtained after $k-1$ elimination steps ($2 \leq k \leq n$), then the kth elimination step is applied to the matrix A_k of the form

$$A_k = \begin{pmatrix} a^{(k)}_{1\,1} & a^{(k)}_{1\,2} & \cdots & \cdots & \cdots & a^{(k)}_{1\,n} \\ 0 & a^{(k)}_{2\,2} & \cdots & \cdots & \cdots & a^{(k)}_{2\,n} \\ \vdots & \ddots & \ddots & \vdots & & \vdots \\ 0 & 0 & 0 & a^{(k)}_{k\,k} & \cdots & a^{(k)}_{k\,n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & a^{(k)}_{n\,k} & \cdots & a^{(k)}_{n\,n} \end{pmatrix}.$$

Actually, note that

$$a^{(k)}_{i\,j} = a^{(i)}_{i\,j}$$

for all i, j with $1 \leq i \leq k-2$ and $i \leq j \leq n$, since the first $k-1$ rows remain unchanged after the $(k-1)$th step.

We will prove later that $\det(A_k) = \pm \det(A)$. Consequently, A_k is invertible. The fact that A_k is invertible iff A is invertible can also be shown without determinants from the fact that there is some invertible matrix M_k such that $A_k = M_k A$, as we will see shortly.

Since A_k is invertible, some entry $a^{(k)}_{i\,k}$ with $k \leq i \leq n$ is nonzero. Otherwise, the last $n-k+1$ entries in the first k columns of A_k would be zero, and the first k columns of A_k would yield k vectors in $\mathbb{R}^{k-1}$. But then the first k columns of A_k would be linearly