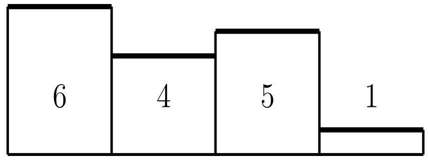

Figure 5.1: The original signal v.

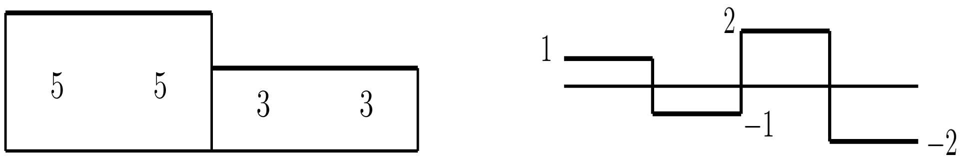

Figure 5.2: First averages and first half differences.

again we compute averages and half differences obtaining Figure 5.3. We get the coefficients $c_1 = 4$ and $c_2 = 1$. Note that the original signal v can be reconstructed from the two signals in Figure 5.2, and the signal on the left of Figure 5.2 can be reconstructed from the two signals in Figure 5.3. In particular, the data from Figure 5.2 gives us

$$\begin{aligned}
5 + 1 &= \frac{v_1 + v_2}{2} + \frac{v_1 - v_2}{2} = v_1 \\
5 - 1 &= \frac{v_1 + v_2}{2} - \frac{v_1 - v_2}{2} = v_2 \\
3 + 2 &= \frac{v_3 + v_4}{2} + \frac{v_3 - v_4}{2} = v_3 \\
3 - 2 &= \frac{v_3 + v_4}{2} - \frac{v_3 - v_4}{2} = v_4.
\end{aligned}$$

5.2 Haar Bases and Haar Matrices, Scaling Properties of Haar Wavelets

The method discussed in Section 5.1 can be generalized to signals of any length 2^n. The previous case corresponds to $n = 2$. Let us consider the case $n = 3$. The *Haar basis*