are given, we obtain a triangular system that determines uniquely the $2m+2$ control points $b_0, \ldots, b_{2m+1}$.

Recall that $C^m([0,1])$ denotes the set of $C^m$ functions $f$ on $[0,1]$, which means that $f, f^{(1)}, \ldots, f^{(m)}$ exist are are continuous on $[0,1]$.

We define the vector space $V_N^m$ as the subspace of $C^m([0,1])$ consisting of all functions $f$ such that

1. $f(0) = f(1) = 0$.
2. The restriction of $f$ to $[x_i, x_{i+1}]$ is a polynomial of degree $2m+1$, for $i = 0, \ldots, N$.

Observe that the functions in $V_N^0$ are the piecewise affine functions $f$ with $f(0) = f(1) = 0$; an example is shown in Figure 19.2.

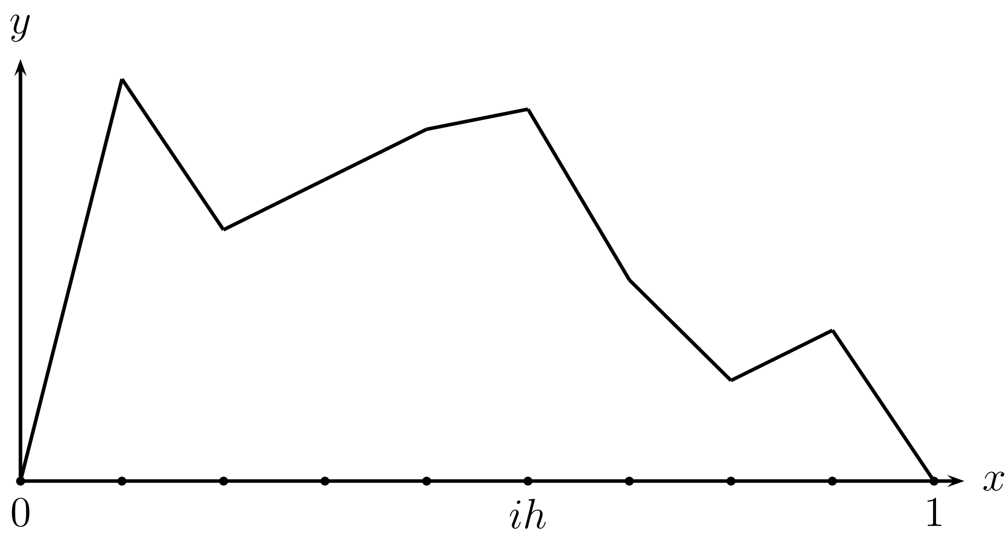


Figure 19.2: A piecewise affine function

This space has dimension $N$, and a basis consists of the "hat functions" $w_i$, where the only two nonflat parts of the graph of $w_i$ are the line segments from $(x_{i-1}, 0)$ to $(x_i, 1)$, and from $(x_i, 1)$ to $(x_{i+1}, 0)$, for $i = 1, \ldots, N$, see Figure 19.3.

The basis functions $w_i$ have a small support, which is good because in computing the integrals giving $a(w_i, w_j)$, we find that we get a tridiagonal matrix. They also have the nice property that every function $v \in V_N^0$ has the following expression on the basis $(w_i)$:

$$v(x) = \sum_{i=1}^{N} v(ih) w_i(x), \quad x \in [0,1].$$