so we get

$$\epsilon = -\left(\frac{m}{C}\left(\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(P + \begin{pmatrix}\mathbf{1}_m\mathbf{1}_m^\top & -\mathbf{1}_m\mathbf{1}_m^\top \\ -\mathbf{1}_m\mathbf{1}_m^\top & \mathbf{1}_m\mathbf{1}_m^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} + \begin{pmatrix}y^\top & -y^\top\end{pmatrix}\begin{pmatrix}\lambda \\ \mu\end{pmatrix}\right)\right.$$
$$\left. + w^\top\left(\sum_{i\in K_\lambda} x_i - \sum_{j\in K_\mu} x_j\right) - \sum_{i\in K_\lambda} y_i + \sum_{j\in K_\mu} y_j + (p_f - q_f)b\right) \Big/ (m\nu - p_f - q_f).$$

Using the equations

$$\begin{aligned} w &= X^\top(\mu - \lambda) \\ b &= -(\mathbf{1}_m^\top\lambda - \mathbf{1}_m^\top\mu) = \begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\begin{pmatrix}-\mathbf{1}_m \\ \mathbf{1}_m\end{pmatrix}, \end{aligned}$$

we see that ϵ is determined by λ and μ provided that $(p_f + q_f)/m < \nu$.

By Proposition 56.7(1),

$$\frac{p_f + q_f}{m} \leq \nu,$$

therefore the condition $(p_f + q_f)/m < \nu$ is very natural.

We have implemented this method in `Matlab`, and we have observed that for some examples the choice of ν caused the equation $\nu(p_f + q_f) = m$ to hold. In such cases, running the program again with a slightly perturbed value of ν always succeeded.

The other observation we made is that b tends to be smaller and ϵ tends to be bigger in ν-SV Regression Version 2, so the fit is actually not as good as in ν-SV Regression without penalizing b. Figure 56.16 shows the result of running our program on the data set of Section 56.3. Compare with Figure 56.13.

56.6 Summary

The main concepts and results of this chapter are listed below:

- ν-support vector regression (ν-SV regression).
- Regression estimate.
- Kernel ν-SV regression.
- ϵ-SV regression, ϵ-insensitive SV regression,
- ν-SV regression Version 2; penalizing b.