The associated Dual Program $(D)$ is

$$\begin{aligned}
&\text{Minimize} \quad && 2y_1 + y_2 + 4y_3 \\
&\text{subject to} \quad && \begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix} \begin{pmatrix} 3 & 4 & -3 & 1 \\ 3 & -2 & 6 & -1 \\ 6 & 4 & 0 & 1 \end{pmatrix} \geq \begin{pmatrix} -1 & -3 & -3 & -1 \end{pmatrix}.
\end{aligned}$$

We initialize the primal-dual algorithm with the dual feasible point $y = (-1/3 \;\; 0 \;\; 0)$. Observe that only the first inequality of $(D)$ is actually an equality, and hence $J = \{1\}$. We form the Restricted Primal Program $(RP1)$

$$\begin{aligned}
&\text{Maximize} \quad && -(\xi_1 + \xi_2 + \xi_3) \\
&\text{subject to} \quad && \begin{pmatrix} 3 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \quad \text{and } x_1, \xi_1, \xi_2, \xi_3 \geq 0.
\end{aligned}$$

We now solve $(RP1)$ via the simplex algorithm. The initial tableau with $K = (2, 3, 4)$ and $J = \{1\}$ is

| | $x_1$ | $\xi_1$ | $\xi_2$ | $\xi_3$ |
|---|---|---|---|---|
| 7 | 12 | 0 | 0 | 0 |
| $\xi_1 = 2$ | 3 | 1 | 0 | 0 |
| $\xi_2 = 1$ | 3 | 0 | 1 | 0 |
| $\xi_3 = 4$ | 6 | 0 | 0 | 1 |

.

For $(RP1)$, $\hat{c} = (0, -1, -1, -1)$, $(x_1, \xi_1, \xi_2, \xi_3) = (0, 2, 1, 4)$, and the nonzero reduced cost is given by

$$0 - \begin{pmatrix} -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} = 12.$$

Since there is only one nonzero reduced cost, we must set $j^+ = 1$. Since $\min\{\xi_1/3, \xi_2/3, \xi_3/6\} = 1/3$, we see that $k^- = 3$ and $K = (2, 1, 4)$. Hence we pivot through the red circled 3 (namely we divide row 2 by 3, and then subtract $3\times$ (row 2) from row 1, $6\times$ (row 2) from row 3, and $12\times$ (row 2) from row 0), to obtain the tableau

| | $x_1$ | $\xi_1$ | $\xi_2$ | $\xi_3$ |
|---|---|---|---|---|
| 3 | 0 | 0 | $-4$ | 0 |
| $\xi_1 = 1$ | 0 | 1 | $-1$ | 0 |
| $x_1 = 1/3$ | 1 | 0 | $1/3$ | 0 |
| $\xi_3 = 2$ | 0 | 0 | $-2$ | 1 |

.

At this stage the simplex algorithm for $(RP1)$ terminates since there are no positive reduced costs. Since the upper left corner of the final tableau is not zero, we proceed with Step 4 of the primal dual algorithm and compute

$$z^* = (-1 \;\; -1 \;\; -1) - (0 \;\; -4 \;\; 0) = (-1 \;\; 3 \;\; -1),$$