so these constraints are not satisfied unless $K_m \leq \min\{2pK_s, 2qK_s\}$, so we assume that $K_m \leq \min\{2pK_s, 2qK_s\}$. The equations in (†) also imply that there is some i_0 such that $\lambda_{i_0} > 0$ and some j_0 such that $\mu_{j_0} > 0$, and so $p_m \geq 1$ and $q_m \geq 1$.

For a finer classification of the points we find it convenient to define $\nu > 0$ such that

$$\nu = \frac{K_m}{(p+q)K_s},$$

so that the objective function $J(w, \epsilon, \xi, b, \eta)$ is given by

$$J(w, \epsilon, \xi, b, \eta) = \frac{1}{2}w^\top w + (p+q)K_s \left(-\nu\eta + \frac{1}{p+q}\begin{pmatrix}\epsilon^\top & \xi^\top\end{pmatrix} \mathbf{1}_{p+q}\right).$$

Observe that the condition $K_m \leq \min\{2pK_s, 2qK_s\}$ is equivalent to

$$\nu \leq \min\left\{\frac{2p}{p+q}, \frac{2q}{p+q}\right\} \leq 1.$$

Since we obtain an equivalent problem by rescaling by a common positive factor, theoretically it is convenient to normalize K_s as

$$K_s = \frac{1}{p+q},$$

in which case $K_m = \nu$. This method is called the *ν-support vector machine*. Actually, to program the method, it may be more convenient assume that K_s is arbitrary. This helps in avoiding λ_i and μ_j to become to small when $p+q$ is relatively large.

The equations (†) and the box inequalities

$$0 \leq \lambda_i \leq K_s, \quad 0 \leq \mu_j \leq K_s$$

also imply the following facts:

Proposition 54.1. *If Problem* $(\mathrm{SVM}_{s2'})$ *has an optimal solution with $w \neq 0$ and $\eta > 0$, then the following facts hold:*

(1) Let p_f be the number of points u_i such that $\lambda_i = K_s$, and let q_f the number of points v_j such that $\mu_j = K_s$. Then $p_f, q_f \leq \nu(p+q)/2$.

(2) Let p_m be the number of points u_i such that $\lambda_i > 0$, and let q_m the number of points v_j such that $\mu_j > 0$. Then $p_m, q_m \geq \nu(p+q)/2$. We have $p_m \geq 1$ and $q_m \geq 1$.

(3) If $p_f \geq 1$ or $q_f \geq 1$, then $\nu \geq 2/(p+q)$.