where $v \in \bigwedge^q E$. The transpose of $\wedge_R(u)$ yields a linear map

$$(\wedge_R(u))^\top \colon \left(\bigwedge^{p+q} E\right)^* \longrightarrow \left(\bigwedge^{q} E\right)^*,$$

which, using the isomorphisms $\left(\bigwedge^{p+q} E\right)^* \cong \bigwedge^{p+q} E^*$ and $\left(\bigwedge^{q} E\right)^* \cong \bigwedge^{q} E^*$, can be viewed as a map

$$(\wedge_R(u))^\top \colon \bigwedge^{p+q} E^* \longrightarrow \bigwedge^{q} E^*$$

given by

$$z^* \mapsto z^* \circ \wedge_R(u),$$

where $z^* \in \bigwedge^{p+q} E^*$. We denote $z^* \circ \wedge_R(u)$ by $u \lrcorner\, z^*$. In terms of our pairing, the adjoint $u \lrcorner$ of $\wedge_R(u)$ defined by

$$\langle u \lrcorner\, z^*, v\rangle = \langle z^*, \wedge_R(u)(v)\rangle;$$

this in turn leads to the following definition.

Definition 34.8. Let $u \in \bigwedge^p E$ and $z^* \in \bigwedge^{p+q} E^*$. We define $u \lrcorner\, z^* \in \bigwedge^q E^*$ to be q-vector uniquely determined by

$$\langle u \lrcorner\, z^*, v\rangle = \langle z^*, v \wedge u\rangle, \quad \text{for all } v \in \bigwedge^q E.$$

Remark: Note that to be precise the operator

$$\lrcorner \colon \bigwedge^p E \times \bigwedge^{p+q} E^* \longrightarrow \bigwedge^q E^*$$

depends of p, q, so we really defined a family of operators $\lrcorner_{p,q}$. This family of operators $\lrcorner_{p,q}$ induces a map

$$\lrcorner \colon \bigwedge E \times \bigwedge E^* \longrightarrow \bigwedge E^*,$$

with

$$\lrcorner_{p,q} \colon \bigwedge^p E \times \bigwedge^{p+q} E^* \longrightarrow \bigwedge^q E^*$$

as defined before. The common practice is to omit the subscripts of $\lrcorner$.

It is immediately verified that

$$(u \wedge v) \lrcorner\, z^* = u \lrcorner\, (v \lrcorner\, z^*),$$

for all $u \in \bigwedge^k E, v \in \bigwedge^{p-k} E$, $z^* \in \bigwedge^{p+q} E^*$ since

$$\langle (u \wedge v) \lrcorner\, z^*, w\rangle = \langle z^*, w \wedge u \wedge v\rangle = \langle v \lrcorner\, z^*, w \wedge u\rangle = \langle u \lrcorner\, (v \lrcorner\, z^*), w\rangle,$$