

Figure 56.9: In this illustration points within ϵ -tube are denoted by open circles. In the original, upper left configuration, there is no blue support vector and no red support vector. By decreasing the width of the slab, we end up with a blue support vector and reduce to Case 2a.

Our hypotheses imply that $0 < \theta < \epsilon$. We can write

$$w^{\top} x_i + b - y_i = \epsilon - \theta + \xi_i + \theta \qquad \qquad \xi_i > 0 \qquad \qquad i \in E_{\lambda}$$

$$-w^{\top} x_j - b + y_j = \epsilon - \theta + \xi'_j + \theta \qquad \qquad \xi'_j > 0 \qquad \qquad j \in E_{\mu}$$

$$w^{\top} x_i + b - y_i \le \epsilon - \theta \qquad \qquad i \notin (E_{\lambda} \cup E_{\mu})$$

$$-w^{\top} x_i - b + y_i \le \epsilon - \theta \qquad \qquad i \notin (E_{\lambda} \cup E_{\mu}),$$

and by the choice of θ , either

$$w^{\top} x_i + b - y_i = \epsilon - \theta$$
 for some $i \notin (E_{\lambda} \cup E_{\mu})$

or

$$-w^{\top}x_i - b + y_i = \epsilon - \theta$$
 for some $i \notin (E_{\lambda} \cup E_{\mu})$.

The new value of the objective function is

$$\omega(\theta) = \frac{1}{2} w^{\top} w + \nu(\epsilon - \theta) + \frac{1}{m} \left(\sum_{i \in E_{\lambda}} (\xi_i + \theta) + \sum_{j \in E_{\mu}} (\xi'_j + \theta) \right)$$
$$= \frac{1}{2} w^{\top} w + \nu \epsilon + \frac{1}{m} \left(\sum_{i \in E_{\lambda}} \xi_i + \sum_{j \in E_{\mu}} \xi'_j \right) - \left(\nu - \frac{p_{sf} + q_{sf}}{m} \right) \theta.$$