- *Matrices*
- *Column vectors*, *row vectors*.
- *Matrix operations*: addition, scalar multiplication, multiplication.
- The vector space $\mathrm{M}_{m,n}(K)$ of $m \times n$ matrices over the field K; The ring $\mathrm{M}_n(K)$ of $n \times n$ matrices over the field K.
- The notion of a *linear map*.
- The *image* $\mathrm{Im}\, f$ (or *range*) of a linear map f.
- The *kernel* $\mathrm{Ker}\, f$ (or *nullspace*) of a linear map f.
- The *rank* $\mathrm{rk}(f)$ of a linear map f.
- The image and the kernel of a linear map are subspaces. A linear map is injective iff its kernel is the trivial space (0) (Proposition 3.17).
- The *unique homomorphic extension property* of linear maps with respect to bases (Proposition 3.18).
- *Quotient spaces*.
- The vector space of linear maps $\mathrm{Hom}_K(E, F)$.
- Linear forms (covectors) and the *dual space* E^*.
- Coordinate forms.
- The existence of *dual bases* (in finite dimension).

3.11 Problems

Problem 3.1. Let H be the set of 3×3 upper triangular matrices given by

$$H = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}.$$

(1) Prove that H with the binary operation of matrix multiplication is a group; find explicitly the inverse of every matrix in H. Is H abelian (commutative)?

(2) Given two groups G_1 and G_2, recall that a *homomorphism* if a function $\varphi\colon G_1 \to G_2$ such that

$$\varphi(ab) = \varphi(a)\varphi(b), \quad a, b \in G_1.$$