$I \times K$ matrix $C = (c_{ik})_{(i,k) \in I \times K}$ representing the linear map $g \circ f \colon E \to G$ with respect to the basis $(u_k)_{k \in K}$ of E and the basis $(w_i)_{i \in I}$ of G is given by

$$C = AB,$$

where for all $i \in I$ and all $k \in K$,

$$c_{ik} = \sum_{j \in J} a_{ij} b_{jk}.$$

Let E, F, G be three vector spaces expressed as direct sums

$$E = \bigoplus_{k=1}^{p} E_k, \quad F = \bigoplus_{j=1}^{n} F_j, \quad G = \bigoplus_{i=1}^{m} G_i,$$

and let $f \colon E \to F$ and $g \colon F \to G$ be two linear maps. Furthermore, assume that E has a finite basis $(u_t)_{t \in T}$, where T is the disjoint union $T = T_1 \cup \cdots \cup T_p$ of nonempty subsets T_k so that $(u_t)_{t \in T_k}$ is a basis of E_k, F has a finite basis $(v_s)_{s \in S}$, where S is the disjoint union $S = S_1 \cup \cdots \cup S_n$ of nonempty subsets S_j so that $(v_s)_{s \in S_j}$ is a basis of F_j, and G has a finite basis $(w_r)_{r \in R}$, where R is the disjoint union $R = R_1 \cup \cdots \cup R_m$ of nonempty subsets R_i so that $(w_r)_{r \in R_i}$ is a basis of G_i. Also let $M = |R|$, $N = |S|$, $P = |T|$, $r_i = |R_i|$, $s_j = |S_j|$, $t_k = |T_k|$, so that $M = \dim(G) = r_1 + \cdots + r_m$, $N = \dim(F) = s_1 + \cdots + s_n$, and $P = \dim(E) = t_1 + \cdots + t_p$.

Let B be the $N \times P$ matrix representing f with respect to the basis $(u_t)_{t \in T}$ of E and the basis $(v_s)_{s \in S}$ of F, let A be the $M \times N$ matrix representing g with respect to the basis $(v_s)_{s \in S}$ of F and the basis $(w_r)_{r \in R}$ of G, and let C be the $M \times P$ matrix representing $h = g \circ f$ with respect to the basis basis $(u_t)_{t \in T}$ of E and the basis $(w_r)_{r \in R}$ of G.

The matrix $[A]$ is an $m \times n$ block matrix of $r_i \times s_j$ matrices A_{ij} ($1 \leq i \leq m, 1 \leq j \leq n$), the matrix $[B]$ is an $n \times p$ block matrix of $s_j \times t_k$ matrices B_{jk} ($1 \leq j \leq n, 1 \leq k \leq p$), and the matrix $[C]$ is an $m \times p$ block matrix of $r_i \times t_k$ matrices C_{ik} ($1 \leq i \leq m, 1 \leq k \leq p$). Furthermore, to be precise, $A_{ij} = A_{R_i, S_j}$, $B_{jk} = B_{S_j, T_k}$, and $C_{ik} = C_{R_i, T_k}$.

Now recall that the matrix A_{R_i, S_j} represents the linear map $g_{ij} \colon F_j \to G_i$ with respect to the basis $(v_s)_{s \in S_j}$ of F_j and the basis $(w_r)_{r \in R_i}$ of G_i, the matrix B_{S_j, T_k} represents the linear map $f_{jk} \colon E_k \to F_j$ with respect to the basis $(u_t)_{t \in T_k}$ of E_k and the basis $(v_s)_{s \in S_j}$ of F_j, and the matrix C_{R_i, T_k} represents the linear map $h_{ik} \colon E_k \to G_i$ with respect to the basis $(u_t)_{t \in T_k}$ of E_k and the basis $(w_r)_{r \in R_i}$ of G_i.

By Proposition 6.12, h_{ik} is given by the formula

$$h_{ik} = \sum_{j=1}^{n} g_{ij} \circ f_{jk}, \quad 1 \leq i \leq m,\ 1 \leq k \leq p, \tag{$*_5$}$$