The trace of the above matrix is $2\rho + \frac{\rho}{2} = \frac{5}{2}\rho > 0$, and the determinant is

$$\rho^2 - (\beta - \rho)^2 = \beta(2\rho - \beta).$$

This determinant is positive if $\rho > \beta/2$, in which case the matrix is symmetric positive definite. Minimizing $L_\rho(x, y, \lambda)$ with respect to x and y, we set the gradient of $L_\rho(x, y, \lambda)$ (with respect to x and y) to zero, and we obtain the equations:

$$\begin{aligned} 2\rho x + (\beta - \rho)y + \lambda &= 0 \\ 2(\beta - \rho)x + \rho y - \lambda &= 0. \end{aligned}$$

Since we are assuming that $\rho > \beta/2$, the solutions are

$$x = -\frac{\lambda}{2(2\rho - \beta)}, \quad y = \frac{\lambda}{(2\rho - \beta)}.$$

Thus the steps for the method of multipliers are

$$\begin{aligned} x^{k+1} &= -\frac{\lambda^k}{2(2\rho - \beta)} \\ y^{k+1} &= \frac{\lambda^k}{(2\rho - \beta)} \\ \lambda^{k+1} &= \lambda^k + \frac{\rho}{2(2\rho - \beta)} \begin{pmatrix} 2 & -1 \end{pmatrix} \begin{pmatrix} -\lambda^k \\ 2\lambda^k \end{pmatrix}, \end{aligned}$$

and the second step simplifies to

$$\lambda^{k+1} = \lambda^k + \frac{\rho}{2(2\rho - \beta)}(-4\lambda^k),$$

that is,

$$\lambda^{k+1} = -\frac{\beta}{2\rho - \beta}\lambda^k.$$

If we pick $\rho > \beta > 0$, which implies that $\rho > \beta/2$, then

$$\frac{\beta}{2\rho - \beta} < 1,$$

and the method converges for any intial value λ^0 to the solution

$$x = 0, \quad y = 0, \quad \lambda = 0.$$

Indeed, since the constraint $2x - y = 0$ holds, $2\beta xy = 4\beta x^2$, and the minimum of the function $x \mapsto 4\beta x^2$ is achieved for $x = 0$ (since $\beta > 0$).