Ceres, and he published a paper about it in 1810 after the discovery of the asteroid Pallas. Incidentally, it is in that same paper that Gaussian elimination using pivots is introduced.

The reason why more equations than unknowns arise in such problems is that repeated measurements are taken to minimize errors. This produces an overdetermined and often inconsistent system of linear equations. For example, Gauss solved a system of eleven equations in six unknowns to determine the orbit of the asteroid Pallas.

Example 23.1. As a concrete illustration, suppose that we observe the motion of a small object, assimilated to a point, in the plane. From our observations, we suspect that this point moves along a straight line, say of equation $y = cx + d$. Suppose that we observed the moving point at three different locations (x_1, y_1), (x_2, y_2), and (x_3, y_3). Then we should have

$$\begin{aligned} d + cx_1 &= y_1, \\ d + cx_2 &= y_2, \\ d + cx_3 &= y_3. \end{aligned}$$

If there were no errors in our measurements, these equations would be compatible, and c and d would be determined by only two of the equations. However, in the presence of errors, the system may be inconsistent. Yet we would like to find c and d!

The idea of the method of least squares is to determine (c, d) such that it minimizes the sum of the squares of the errors, namely,

$$(d + cx_1 - y_1)^2 + (d + cx_2 - y_2)^2 + (d + cx_3 - y_3)^2.$$

See Figure 23.1.

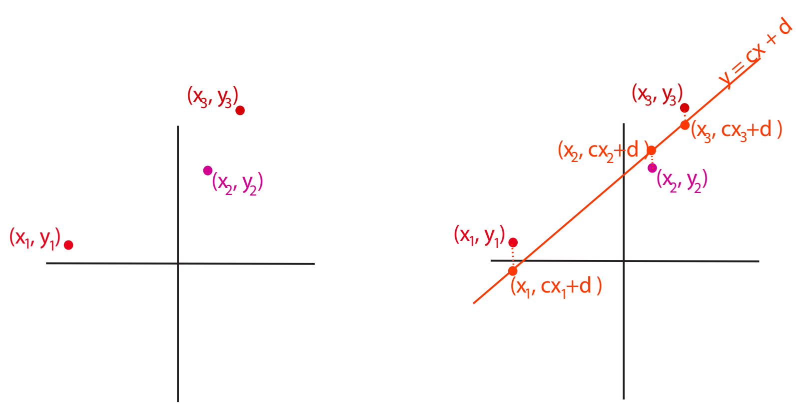

Figure 23.1: Given three points (x_1, y_1), (x_2, y_2), (x_3, y_3), we want to determine the line $y = cx + d$ which minimizes the lengths of the dashed vertical lines.