adjusted to determine the influence of this regularizing term. If the primal problem (SVM$_{s1}$) has an optimal solution $(w, \delta, b, \epsilon, \xi)$, we attempt to use the dual function $G$ to obtain it, but we will see that with this particular formulation of the problem, the constraint $w^\top w \leq 1$ causes troubles even though it is convex.

**Soft margin SVM** (SVM$_{s1}$):

$$\begin{aligned}
&\text{minimize} \quad -\delta + K\left(\sum_{i=1}^{p} \epsilon_i + \sum_{j=1}^{q} \xi_j\right) \\
&\text{subject to} \\
&\qquad w^\top u_i - b \geq \delta - \epsilon_i, \quad \epsilon_i \geq 0 \qquad i = 1, \ldots, p \\
&\qquad -w^\top v_j + b \geq \delta - \xi_j, \quad \xi_j \geq 0 \qquad j = 1, \ldots, q \\
&\qquad w^\top w \leq 1.
\end{aligned}$$

It is customary to write $\ell = p + q$. Figure 54.2 illustrates the correct margin half space associated with $w^\top x - b - \delta = 0$ while Figure 54.3 illustrates the correct margin half space associated with $w^\top x - b + \delta = 0$. Ideally, all the points should be contained in one of the two correct shifted margin regions described by affine constraints $w^\top u_i - b \geq \delta - \epsilon_i$, or $-w^\top v_j + b \geq \delta - \xi_j$.

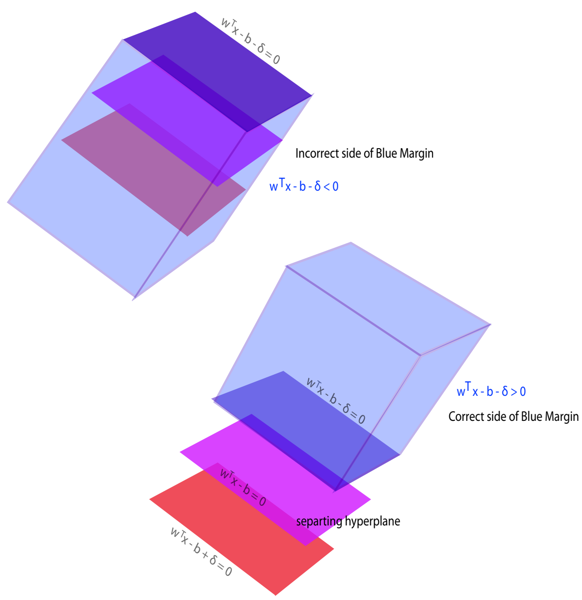


Figure 54.2: The blue margin half space associated with $w^\top x - b - \delta = 0$.

For this problem, the primal problem may have an optimal solution $(w, \delta, b, \epsilon, \xi)$ with $\|w\| = 1$ and $\delta > 0$, but if the sets of points are not linearly separable then an optimal solution of the dual may not yield $w$.