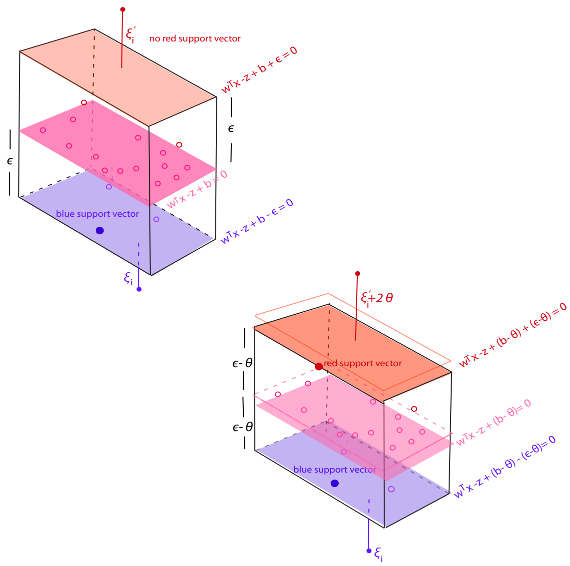

Figure 56.10: In this illustration points within the ϵ-tube are denoted by open circles. In the original, upper left configuration, there is no red support vector. By lowering the pink separating hyperplane and decreasing the width of the slab, we end up with a red support vector.

By hypothesis

$$w^\top x_i + (b - \theta) - y_i = \epsilon - \theta \quad \text{for some } i \notin (E_\lambda \cup E_\mu),$$

and by the choice of θ,

$$-w^\top x_j - (b - \theta) + y_j = \epsilon - \theta \quad \text{for some } j \notin (E_\lambda \cup E_\mu).$$

The new value of the objective function is

$$\begin{aligned} \omega(\theta) &= \frac{1}{2} w^\top w + \nu(\epsilon - \theta) + \frac{1}{m}\left(\sum_{i \in E_\lambda} \xi_i + \sum_{j \in E_\mu} (\xi_j' + 2\theta)\right) \\ &= \frac{1}{2} w^\top w + \nu\epsilon + \frac{1}{m}\left(\sum_{i \in E_\lambda} \xi_i + \sum_{j \in E_\mu} \xi_j'\right) - \left(\nu - \frac{2q_{sf}}{m}\right)\theta. \end{aligned}$$

The rest of the proof is similar except that $2p_{sf}/m$ is replaced by $2q_{sf}/m$. Observe that the exceptional case in which $\theta = \epsilon$ may arise. In this case all points (x_i, y_i) that are not errors (strictly outside the ϵ-slab) are on the blue margin hyperplane. This case can only arise if $\nu = 2q_{sf}/m$.

Case 2b. We have $w^\top x_i + b - y_i < \epsilon$ for all $i \notin (E_\lambda \cup E_\mu)$. Since we also assumed that $-w^\top x_i - b + y_i < \epsilon$ for all $i \notin (E_\lambda \cup E_\mu)$, Case 2b is identical to Case 1b and we are done. $\square$