

Since there are  $4m + 1$  Lagrange multipliers  $(\lambda, \mu, \alpha, \beta, \gamma)$ , we need to pad the  $2m \times 2m$  matrix  $P$  with zeros to make it into a  $(4m + 1) \times (4m + 1)$  matrix

$$P_a = \begin{pmatrix} P & 0_{2m, 2m+1} \\ 0_{2m+1, 2m} & 0_{2m+1, 2m+1} \end{pmatrix}.$$

Similarly, we pad  $q$  with zeros to make it a vector  $q_a$  of dimension  $4m + 1$ ,

$$q_a = \begin{pmatrix} q \\ 0_{2m+1} \end{pmatrix}.$$

In order to solve our dual program, we apply ADMM to the quadratic functional

$$\frac{1}{2}x^\top P_a x + q_a^\top x,$$

subject to the constraints

$$Ax = c, \quad x \geq 0,$$

with  $P_a, q_a, A, b$  and  $x$ , as above.

Since for an optimal solution with  $\epsilon > 0$  we must have  $\gamma = 0$  (from the KKT conditions), we can solve the dual problem with the following set of constraints only involving the Lagrange multipliers  $(\lambda, \mu, \alpha, \beta)$ ,

$$\begin{aligned} \sum_{i=1}^m \lambda_i - \sum_{i=1}^m \mu_i &= 0 \\ \sum_{i=1}^m \lambda_i + \sum_{i=1}^m \mu_i &= C\nu \\ \lambda + \alpha &= \frac{C}{m}, \quad \mu + \beta = \frac{C}{m}, \end{aligned}$$

which corresponds to the  $(2m + 2) \times 4m$   $A_2$  given by

$$A_2 = \begin{pmatrix} \mathbf{1}_m^\top & -\mathbf{1}_m^\top & 0_m^\top & 0_m^\top \\ \mathbf{1}_m^\top & \mathbf{1}_m^\top & 0_m^\top & 0_m^\top \\ I_m & 0_{m,m} & I_m & 0_{m,m} \\ 0_{m,m} & I_m & 0_{m,m} & I_m \end{pmatrix}.$$

We leave it as an exercise to show that  $A_2$  has rank  $2m + 2$ . We define the vector  $c_2$  (of dimension  $2m + 2$ ) as

$$c_2 = c = \begin{pmatrix} 0 \\ C\nu \\ \frac{C}{m}\mathbf{1}_{2m} \end{pmatrix}.$$