Problem 56.4. Prove that the matrices

$$A = \begin{pmatrix} \mathbf{1}_m^\top & -\mathbf{1}_m^\top & 0_m^\top & 0_m^\top & 0 \\ \mathbf{1}_m^\top & \mathbf{1}_m^\top & 0_m^\top & 0_m^\top & 1 \\ I_m & 0_{m,m} & I_m & 0_{m,m} & 0_m \\ 0_{m,m} & I_m & 0_{m,m} & I_m & 0_m \end{pmatrix}, \quad A_2 = \begin{pmatrix} \mathbf{1}_m^\top & -\mathbf{1}_m^\top & 0_m^\top & 0_m^\top \\ \mathbf{1}_m^\top & \mathbf{1}_m^\top & 0_m^\top & 0_m^\top \\ I_m & 0_{m,m} & I_m & 0_{m,m} \\ 0_{m,m} & I_m & 0_{m,m} & I_m \end{pmatrix}$$

have rank $2m+2$.

Problem 56.5. Derive the version of ν-SV regression in which the linear penalty function $\sum_{i=1}^m (\xi_i + \xi_i')$ is replaced by the quadratic penalty function $\sum_{i=1}^m (\xi_i^2 + \xi_i'^2)$. Derive the dual program.

Problem 56.6. The linear penalty function $\sum_{i=1}^m (\xi_i + \xi_i')$ can be replaced by the quadratic penalty function $\sum_{i=1}^m (\xi_i^2 + \xi_i'^2)$. Prove that for an optimal solution we must have $\xi_i \geq 0$ and $\xi_i' \geq 0$, so we may omit the constraints $\xi_i \geq 0$ and $\xi_i' \geq 0$. We must also have $\gamma = 0$ so we may omit the variable γ as well. Prove that $\xi = (m/2C)\lambda$ and $\xi' = (m/2C)\mu$. This problem is very similar to the Soft Margin SVM (SVM_{s4}) discussed in Section 54.13.

Problem 56.7. Consider the version of ν-SV regression in Section 56.5. Prove that for any optimal solution with $w \neq 0$ and $\epsilon > 0$, if the inequalities $(p_f + q_f)/m < \nu < 1$ hold, then some point x_i is a support vector.

Problem 56.8. Prove that the matrix

$$A_3 = \begin{pmatrix} \mathbf{1}_m^\top & \mathbf{1}_m^\top & 0_m^\top & 0_m^\top \\ I_m & 0_{m,m} & I_m & 0_{m,m} \\ 0_{m,m} & I_m & 0_{m,m} & I_m \end{pmatrix}$$

has rank $2m+1$.

Problem 56.9. Consider the version of ν-SV regression in Section 56.5. Prove the following formulae: If $I_\lambda \neq \emptyset$, then

$$\epsilon = w^\top \biggl(\sum_{i \in I_\lambda} x_i \biggr) / |I_\lambda| + b - \biggl(\sum_{i \in I_\lambda} y_i \biggr) / |I_\lambda|,$$

and if $I_\mu \neq \emptyset$, then

$$\epsilon = -w^\top \biggl(\sum_{j \in I_\mu} x_j \biggr) / |I_\mu| - b + \biggl(\sum_{i \in I_\mu} y_i \biggr) / |I_\mu|.$$

Problem 56.10. Implement ν-Regression Version 2 described in Section 56.5. Run examples using both the original version and version 2 and compare the results.