Step 2a.

If $m = 1$ go to Step 2b.

If $m > 1$, then there are two possibilities:

(i) M is of the form

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{m2} & \cdots & a_{mn} \end{pmatrix}.$$

If $n = 1$, stop; else go to Step 2b.

(ii) There is some nonzero entry a_{i1} $(i > 1)$ below a_{11} in the first column.

(a) If there is some entry a_{k1} in the first column such that a_{11} does not divide a_{k1}, then pick such an entry (say, with the smallest index i such that $\sigma(a_{i1})$ is minimal), and divide a_{k1} by a_{11}; that is, find b_k and b_{k1} such that

$$a_{k1} = a_{11}b_k + b_{k1}, \quad \text{with} \quad \sigma(b_{k1}) < \sigma(a_{11}).$$

Subtract b_k times row 1 from row k and permute row k and row 1, to obtain a matrix of the form

$$M = \begin{pmatrix} b_{k1} & b_{k2} & \cdots & b_{kn} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}.$$

Go back to Step 2a.

(b) If a_{11} divides every (nonzero) entry a_{i1} for $i \geq 2$, say $a_{i1} = a_{11}b_i$, then subtract b_i times row 1 from row i for $i = 2, \ldots, m$; go to Step 2b.

Observe that whenever we return to the beginning of Step 2a, we have $\sigma(b_{k1}) < \sigma(a_{11})$. Therefore, after a finite number of steps, we must exit Step 2a with a matrix in which all entries in column 1 but the first are zero and go to Step 2b.

Step 2b.

This step is reached only if $n > 1$ and if the only nonzero entry in the first column is a_{11}.

(a) If M is of the form

$$\begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

and $m = 1$ stop; else go to Step 3.