When $m = 1$ and $n \geq 2$, check that

$$(f \wedge g)(u_1, \ldots, u_{m+1}) = \sum_{i=1}^{m+1} (-1)^{i-1} f(u_i) g(u_1, \ldots, \widehat{u_i}, \ldots, u_{m+1}),$$

where the hat over the argument u_i means that it should be omitted.

Here is another explicit example. Suppose $m = 2$ and $n = 1$. Given $v_1^*, v_2^*, v_3^* \in E^*$, the multiplication structure on $\bigwedge(E^*)$ implies that $(v_1^* \wedge v_2^*) \cdot v_3^* = v_1^* \wedge v_2^* \wedge v_3^* \in \bigwedge^3(E^*)$. Furthermore, for $u_1, u_2, u_3, \in E$,

$$\begin{aligned}
\mu_3(v_1^* \wedge v_2^* \wedge v_3^*)(u_1, u_2, u_3) &= \sum_{\sigma \in \mathfrak{S}_3} \mathrm{sgn}(\sigma) v_{\sigma(1)}^*(u_1) v_{\sigma(2)}^*(u_2) v_{\sigma(3)}^*(u_3) \\
&= v_1^*(u_1) v_2^*(u_2) v_3^*(u_3) - v_1^*(u_1) v_3^*(u_2) v_2^*(u_3) \\
&\quad - v_2^*(u_1) v_1^*(u_2) v_3^*(u_3) + v_2^*(u_1) v_3^*(u_2) v_1^*(u_3) \\
&\quad + v_3^*(u_1) v_1^*(u_2) v_2^*(u_3) - v_3^*(u_1) v_2^*(u_2) v_1^*(u_3).
\end{aligned}$$

Now the $(2, 1)$- shuffles of $\{1, 2, 3\}$ are the following three permutations, namely

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}.$$

If $f \cong \mu_2(v_1^* \wedge v_2^*)$ and $g \cong \mu_1(v_3^*)$, then $(**)$ implies that

$$\begin{aligned}
(f \cdot g)(u_1, u_2, u_3) &= \sum_{\sigma \in \mathrm{shuffle}(2,1)} \mathrm{sgn}(\sigma) f(u_{\sigma(1)}, u_{\sigma(2)}) g(u_{\sigma(3)}) \\
&= f(u_1, u_2) g(u_3) - f(u_1, u_3) g(u_2) + f(u_2, u_3) g(u_1) \\
&= \mu_2(v_1^* \wedge v_2^*)(u_1, u_2) \mu_1(v_3^*)(u_3) - \mu_2(v_1^* \wedge v_2^*)(u_1, u_3) \mu_1(v_3^*)(u_2) \\
&\quad + \mu_2(v_1^* \wedge v_2^*)(u_2, u_3) \mu_1(v_3^*)(u_1) \\
&= (v_1^*(u_1) v_2^*(u_2) - v_2^*(u_1) v_1^*(u_2)) v_3^*(u_3) \\
&\quad - (v_1^*(u_1) v_2^*(u_3) - v_2^*(u_1) v_1^*(u_3)) v_3^*(u_2) \\
&\quad + (v_1^*(u_2) v_2^*(u_3) - v_2^*(u_2) v_1^*(u_3)) v_3^*(u_1) \\
&= \mu_3(v_1^* \wedge v_2^* \wedge v_3^*)(u_1, u_2, u_3).
\end{aligned}$$

As a result of all this, the direct sum

$$\mathrm{Alt}(E) = \bigoplus_{n \geq 0} \mathrm{Alt}^n(E; K)$$

is an algebra under the above multiplication, and this algebra is isomorphic to $\bigwedge(E^*)$. For the record we state