and

$$g = 1,$$

the constant function which is the indicator function of the convex set $C = \mathbb{R}^n$. In view of Example 52.8 and (1), since $\Pi_{\mathbb{R}^n}(x^{k+1} + u^k) = x^{k+1} + u^k$, the scaled form of ADMM consists of the following steps:

$$\begin{aligned}
x^{k+1} &= \arg\min_x \left(f(x) + (\rho/2) \left\| x - z^k + u^k \right\|_2^2 \right) \\
z^{k+1} &= x^{k+1} + u^k \\
u^{k+1} &= u^k + x^{k+1} - z^{k+1} = 0,
\end{aligned}$$

for all $k \geq 0$, so

$$\begin{aligned}
u^k &= 0 \\
z^{k+1} &= x^{k+1}
\end{aligned}$$

for all $k \geq 1$. Consequently we have

$$\begin{aligned}
x^{k+1} &= \arg\min_x \left(f(x) + (\rho/2) \left\| x - z^k + u^k \right\|_2^2 \right) \\
z^{k+1} &= x^{k+1} + u^k \\
u^1 &= 0,
\end{aligned}$$

for $k = 0, 1$, and for $k \geq 2$ we have $u^k = 0$ and $z^k = x^k$, with

$$x^{k+1} = \arg\min_x \left(f(x) + (\rho/2) \left\| x - x^k \right\|_2^2 \right).$$

As before, the x-update involves solving the KKT equations

$$\begin{pmatrix} P + \rho I & A^\top \\ A & 0 \end{pmatrix} \begin{pmatrix} x^{k+1} \\ y \end{pmatrix} = \begin{pmatrix} -q + \rho(z^k - u^k) \\ b \end{pmatrix},$$

with $u^k = 0$ if $k \geq 1$ and $z^k = x^k$ if $k \geq 2$.

We programmed the above method in `Matlab` as the function `qsolve1`, see Appendix B, Section B.1. Here are two examples.

Example 52.11. Consider the quadratic program for which

$$P_1 = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{pmatrix} \qquad q_1 = -\begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \qquad b_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$