Dual of the Soft margin SVM (SVM_{s1}):

$$\begin{aligned}
&\text{minimize} \quad \begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} X^\top X \begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\
&\text{subject to} \\
&\qquad \sum_{i=1}^{p} \lambda_i - \sum_{j=1}^{q} \mu_j = 0 \\
&\qquad \sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \mu_j = 1 \\
&\qquad 0 \leq \lambda_i \leq K, \quad i = 1, \ldots, p \\
&\qquad 0 \leq \mu_j \leq K, \quad j = 1, \ldots, q.
\end{aligned}$$

The points u_i and v_j are naturally classified in terms of the values of λ_i and μ_j. The numbers of points in each category have a direct influence on the choice of the parameter K. Let us summarize some of the keys items from Definition 54.1.

The vectors u_i on the blue margin $H_{w,b+\delta}$ and the vectors v_j on the red margin $H_{w,b-\delta}$ are called *support vectors*. Support vectors correspond to vectors u_i for which $w^\top u_i - b - \delta = 0$ (which implies $\epsilon_i = 0$), and vectors v_j for which $w^\top v_j - b + \delta = 0$ (which implies $\xi_j = 0$). Support vectors u_i such that $0 < \lambda_i < K$ and support vectors v_j such that $0 < \mu_j < K$ are *support vectors of type 1*. Support vectors of type 1 play a special role so we denote the sets of indices associated with them by

$$\begin{aligned}
I_\lambda &= \{i \in \{1, \ldots, p\} \mid 0 < \lambda_i < K\} \\
I_\mu &= \{j \in \{1, \ldots, q\} \mid 0 < \mu_j < K\}.
\end{aligned}$$

We denote their cardinalities by $numsvl_1 = |I_\lambda|$ and $numsvm_1 = |I_\mu|$.

The vectors u_i for which $\lambda_i = K$ and the vectors v_j for which $\mu_j = K$ are said to *fail the margin*. The sets of indices associated with the vectors failing the margin are denoted by

$$\begin{aligned}
K_\lambda &= \{i \in \{1, \ldots, p\} \mid \lambda_i = K\} \\
K_\mu &= \{j \in \{1, \ldots, q\} \mid \mu_j = K\}.
\end{aligned}$$

We denote their cardinalities by $p_f = |K_\lambda|$ and $q_f = |K_\mu|$.

Vectors u_i such that $\lambda_i > 0$ and vectors v_j such that $\mu_j > 0$ are said to *have margin at most δ*. The sets of indices associated with these vectors are denoted by

$$\begin{aligned}
I_{\lambda>0} &= \{i \in \{1, \ldots, p\} \mid \lambda_i > 0\} \\
I_{\mu>0} &= \{j \in \{1, \ldots, q\} \mid \mu_j > 0\}.
\end{aligned}$$