

A clean way to handle the situation in which the denominator vanishes is to work in a projective space. Intuitively, this means viewing a rational curve in \mathbb{A}^n as some appropriate projection of a polynomial curve in \mathbb{A}^{n+1} , back onto \mathbb{A}^n .

Given an affine space \mathcal{E} , for any hyperplane H in \mathcal{E} and any point a_0 not in H , the *central projection* (or *conic projection*, or *perspective projection*) of center a_0 onto H , is the partial map p defined as follows: For every point x not in the hyperplane passing through a_0 and parallel to H , we define $p(x)$ as the intersection of the line defined by a_0 and x with the hyperplane H ; see Figure 26.1.

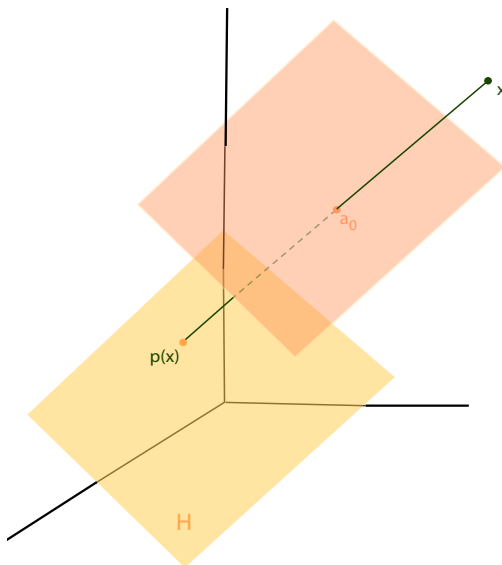


Figure 26.1: A central projection in \mathbb{A}^3 through a_0 onto the yellow hyperplane H . This central projection is not defined for any points in the peach hyperplane.

For example, we can view G as a rational curve in \mathbb{A}^3 given by

$$G_1(t) = a_0 + t^2e_1 + e_2 + te_3.$$

If we project this curve G_1 (in fact, a parabola in \mathbb{A}^3) using the central projection (perspective projection) of center a_0 onto the plane of equation $x_3 = 1$, we get the previous hyperbola; see Figure 26.2. For $t = 0$, the point $G_1(0) = a_0 + e_2$ in \mathbb{A}^3 is in the plane of equation $x_3 = 0$, and its projection is undefined. We can consider that $G_1(0) = a_0 + e_2$ in \mathbb{A}^3 is projected to infinity in the direction of e_2 in the plane $x_3 = 0$. In the setting of projective spaces, this direction corresponds rigorously to a point at infinity; see Figure 26.2.

Let us verify that the central projection used in the previous example has the desired effect. Let us assume that \mathcal{E} has dimension $n + 1$ and that $(a_0, (e_1, \dots, e_{n+1}))$ is an affine