(b) There is some vector, $c \in \mathbb{R}^d$, so that $c^\top V > 0$, which means that $c^\top v_j > 0$, for $j = 1, \ldots, n$.

**Problem 47.2.** Check that the dual in maximization form $(D'')$ of the Dual Program $(D')$ (which is the dual of $(P)$ in maximization form),

$$\begin{aligned} &\text{maximize} && -b^\top y^\top \\ &\text{subject to} && -A^\top y^\top \leq -c^\top \text{ and } y^\top \geq 0, \end{aligned}$$

where $y \in (\mathbb{R}^m)^*$, gives back the Primal Program $(P)$.

**Problem 47.3.** In a General Linear Program $(P)$ with $n$ primal variables $x_1, \ldots, x_n$ and objective function $\sum_{j=1}^n c_j x_j$ (to be maximized), the $m$ constraints are of the form

$$\begin{aligned} \sum_{j=1}^n a_{ij} x_j &\leq b_i, \\ \sum_{j=1}^n a_{ij} x_j &\geq b_i, \\ \sum_{j=1}^n a_{ij} x_j &= b_i, \end{aligned}$$

for $i = 1, \ldots, m$, and the variables $x_j$ satisfy an inequality of the form

$$\begin{aligned} x_j &\geq 0, \\ x_j &\leq 0, \\ x_j &\in \mathbb{R}, \end{aligned}$$

for $j = 1, \ldots, n$. If $y_1, \ldots, y_m$ are the dual variables, show that the dual program of the linear program in standard form equivalent to $(P)$ is equivalent to the linear program whose objective function is $\sum_{i=1}^m y_i b_i$ (to be minimized) and whose constraints are determined as follows:

$$\text{if} \quad \begin{Bmatrix} x_j \geq 0 \\ x_j \leq 0 \\ x_j \in \mathbb{R} \end{Bmatrix}, \quad \text{then} \quad \begin{Bmatrix} \sum_{i=1}^m a_{ij} y_i \geq c_j \\ \sum_{i=1}^m a_{ij} y_i \leq c_j \\ \sum_{i=1}^m a_{ij} y_i = c_j \end{Bmatrix},$$