The inverse of the isomorphism $\flat\colon \overline{E} \to E^*$ is denoted by $\sharp\colon E^* \to \overline{E}$.

As a corollary of the isomorphism $\flat\colon \overline{E} \to E^*$ we have the following result.

Proposition 14.7. *If E is a Hermitian space of finite dimension, then every linear form $f \in E^*$ corresponds to a unique $v \in E$, such that*

$$f(u) = u \cdot v, \quad \text{for every } u \in E.$$

In particular, if f is not the zero form, the kernel of f, which is a hyperplane H, is precisely the set of vectors that are orthogonal to v.

Remarks:

1. The "musical map" $\flat\colon \overline{E} \to E^*$ is not surjective when E has infinite dimension. This result can be salvaged by restricting our attention to continuous linear maps and by assuming that the vector space E is a *Hilbert space*.

2. *Dirac's "bra-ket" notation.* Dirac invented a notation widely used in quantum mechanics for denoting the linear form $\varphi_u = \flat(u)$ associated to the vector $u \in E$ *via* the duality induced by a Hermitian inner product. Dirac's proposal is to denote the vectors u in E by $|u\rangle$, and call them *kets*; the notation $|u\rangle$ is pronounced "ket u." Given two kets (vectors) $|u\rangle$ and $|v\rangle$, their inner product is denoted by

$$\langle u|v\rangle$$

(instead of $|u\rangle \cdot |v\rangle$). The notation $\langle u|v\rangle$ for the inner product of $|u\rangle$ and $|v\rangle$ anticipates duality. Indeed, we define the dual (usually called adjoint) *bra u* of ket u, denoted by $\langle u|$, as the linear form whose value on any ket v is given by the inner product, so

$$\langle u|(|v\rangle) = \langle u|v\rangle.$$

Thus, bra $u = \langle u|$ is Dirac's notation for our $\flat(u)$. Since the map $\flat$ is semi-linear, we have

$$\langle \lambda u| = \overline{\lambda}\langle u|.$$

Using the bra-ket notation, given an orthonormal basis $(|u_1\rangle, \ldots, |u_n\rangle)$, ket v (a vector) is written as

$$|v\rangle = \sum_{i=1}^{n} \langle v|u_i\rangle |u_i\rangle,$$

and the corresponding linear form bra v is written as

$$\langle v| = \sum_{i=1}^{n} \overline{\langle v|u_i\rangle} \langle u_i| = \sum_{i=1}^{n} \langle u_i|v\rangle \langle u_i|$$

over the dual basis $(\langle u_1|, \ldots, \langle u_n|)$. As cute as it looks, we do not recommend using the Dirac notation.