Well, it depends! Even in the case of lines (when $m = n = 1$), there are three possibilities: either the lines coincide, or they are parallel, or there is a single intersection point. In general, we expect mn intersection points, but some of these points may be missing because they are at infinity, because they coincide, or because they are imaginary.

What begins to transpire is that "points at infinity" cause trouble. They cause exceptions that invalidate geometric theorems (for example, consider the more general versions of the theorems of Pappus and Desargues), and make it difficult to classify geometric objects. Projective geometry is designed to deal with "points at infinity" and regular points in a uniform way, without making a distinction. Points at infinity are now just ordinary points, and many things become simpler. For example, the classification of conics and quadrics becomes simpler, and intersection theory becomes cleaner (although, to be honest, we need to consider complex projective spaces).

Technically, projective geometry can be defined axiomatically, or by building upon linear algebra. Historically, the axiomatic approach came first (see Veblen and Young [183, 184], Emil Artin [6], and Coxeter [45, 46, 43, 44]). Although very beautiful and elegant, we believe that it is a harder approach than the linear algebraic approach. In the linear algebraic approach, all notions are considered up to a scalar. For example, a projective point is really a line through the origin. In terms of coordinates, this corresponds to "homogenizing." For example, the homogeneous equation of a conic is

$$ax^2 + by^2 + cxy + dxz + eyz + fz^2 = 0.$$

Now, regular points are points of coordinates (x, y, z) with $z \neq 0$, and points at infinity are points of coordinates $(x, y, 0)$ (with x, y, z not all null, and up to a scalar). There is a useful model (interpretation) of plane projective geometry in terms of the central projection in $\mathbb{R}^3$ from the origin onto the plane $z = 1$. Another useful model is the spherical (or the half-spherical) model. In the spherical model, a projective point corresponds to a pair of antipodal points on the sphere.

As affine geometry is the study of properties invariant under affine bijections, projective geometry is the study of properties invariant under bijective projective maps. Roughly speaking, projective maps are linear maps up to a scalar. In analogy with our presentation of affine geometry, we will define projective spaces, projective subspaces, projective frames, and projective maps. The analogy will fade away when we define the projective completion of an affine space, and when we define duality.

One of the virtues of projective geometry is that it yields a very clean presentation of rational curves and rational surfaces. The general idea is that a plane rational curve is the projection of a simpler curve in a larger space, a polynomial curve in $\mathbb{R}^3$, onto the plane $z = 1$, as we now explain.

Polynomial curves are curves defined parametrically in terms of polynomials. More specifically, if $\mathcal{E}$ is an affine space of finite dimension $n \geq 2$ and $(a_0, (e_1, \ldots, e_n))$ is an affine frame