such that each block D_j *is either* 1, -1, *or a two-dimensional matrix of the form*

$$D_j = \begin{pmatrix} \cos\theta_j & -\sin\theta_j \\ \sin\theta_j & \cos\theta_j \end{pmatrix}$$

where $0 < \theta_j < \pi$. *In particular, the eigenvalues of* A *are of the form* $\cos\theta_j \pm i\sin\theta_j$, 1, *or* -1.

Theorem 17.21 can be used to show that the exponential map $\exp\colon \mathfrak{so}(n) \to \mathbf{SO}(n)$ is surjective; see Gallier [72].

We now consider complex matrices.

Definition 17.4. Given a complex $m \times n$ matrix A, the *transpose* $A^\top$ *of* A is the $n \times m$ matrix $A^\top = \left(a^\top_{i\,j}\right)$ defined such that

$$a^\top_{i\,j} = a_{j\,i}$$

for all i, j, $1 \leq i \leq m$, $1 \leq j \leq n$. The *conjugate* $\overline{A}$ *of* A is the $m \times n$ matrix $\overline{A} = (b_{i\,j})$ defined such that

$$b_{i\,j} = \overline{a_{i\,j}}$$

for all i, j, $1 \leq i \leq m$, $1 \leq j \leq n$. Given an $m \times n$ complex matrix A, the *adjoint* A^* *of* A is the matrix defined such that

$$A^* = \overline{(A^\top)} = (\overline{A})^\top.$$

A complex $n \times n$ matrix A is

- *normal* if
$$AA^* = A^*A,$$

- *Hermitian* if
$$A^* = A,$$

- *skew-Hermitian* if
$$A^* = -A,$$

- *unitary* if
$$AA^* = A^*A = I_n.$$