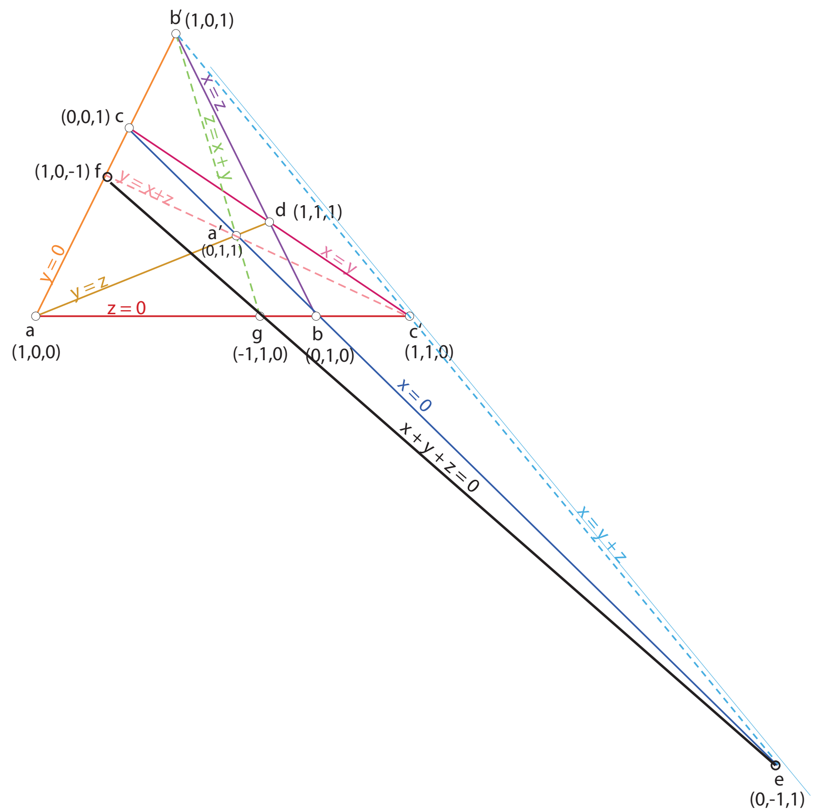

Figure 26.9: A projective frame (a, b, c, d).

are $y = z$, $x = z$, and $x = y$. The equations of the lines $\langle a', b'\rangle$, $\langle a', c'\rangle$, $\langle b', c'\rangle$ are $z = x + y$, $y = x + z$, and $x = y + z$.

If we let e be the intersection of $\langle b, c\rangle$ and $\langle b', c'\rangle$, f be the intersection of $\langle a, c\rangle$ and $\langle a', c'\rangle$, and g be the intersection of $\langle a, b\rangle$ and $\langle a', b'\rangle$, then it easily seen that e, f, g have homogeneous coordinates $(0, -1, 1)$, $(1, 0, -1)$, and $(-1, 1, 0)$. For example, since the equation of the line $\langle b, c\rangle$ is $x = 0$ and the equation of the line $\langle b', c'\rangle$ is $x = y + z$, for $x = 0$, we get $z = -y$, which correspond to the homogeneous coordinates $(0, -1, 1)$ for e.

The coordinates of the points e, f, g satisfy the equation $x + y + z = 0$, which shows that they are collinear.

As pointed out in Coxeter [45] (Proposition 2.41), this is a special case of the projective version of Desargues's theorem (Proposition 26.7) applied to the triangles (a, b, c) and (a', b', c'). Indeed, by construction, the lines $\langle a, a'\rangle$, $\langle b, b'\rangle$, and $\langle c, c'\rangle$ intersect in the common point d. The line containing the points e, f, g is called the *polar line (or fundamental line)* of d with respect to the triangle (a, b, c) (see Pedoe [136]). The diagram also shows the intersection g of $\langle a, b\rangle$ and $\langle a', b'\rangle$.

The projective space of circles provides a nice illustration of homogeneous coordinates.