

two red points

$$v_1^\top = (v_{11}, v_{12}) \quad v_2^\top = (v_{21}, v_{22}),$$

and

$$w^\top = (w_1, w_2).$$

Step 1: Write the constraints in matrix form. Let

$$C = \begin{pmatrix} -u_{11} & -u_{12} & 1 \\ -u_{21} & -u_{22} & 1 \\ v_{11} & v_{12} & -1 \\ v_{21} & v_{22} & -1 \end{pmatrix} \quad d = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}. \quad (M)$$

The constraints become

$$C \begin{pmatrix} w \\ b \end{pmatrix} = \begin{pmatrix} -u_{11} & -u_{12} & 1 \\ -u_{21} & -u_{22} & 1 \\ v_{11} & v_{12} & -1 \\ v_{21} & v_{22} & -1 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ b \end{pmatrix} \leq \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}. \quad (C)$$

Step 2: Write the objective function in matrix form.

$$J(w_1, w_2, b) = \frac{1}{2} \begin{pmatrix} w_1 & w_2 & b \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ b \end{pmatrix}. \quad (O)$$

Step 3: Apply Proposition 50.7 to solve for w in terms of λ and μ . We obtain

$$\begin{pmatrix} w_1 \\ w_2 \\ 0 \end{pmatrix} + \begin{pmatrix} -u_{11} & -u_{21} & v_{11} & v_{21} \\ -u_{12} & -u_{22} & v_{12} & v_{22} \\ 1 & 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

i.e.

$$\nabla J_{(w,b)} + C^\top \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0_3, \quad \lambda^\top = (\lambda_1, \lambda_2), \quad \mu^\top = (\mu_1, \mu_2).$$

Then

$$\begin{pmatrix} w_1 \\ w_2 \\ 0 \end{pmatrix} = \begin{pmatrix} u_{11} & u_{21} & -v_{11} & -v_{21} \\ u_{12} & u_{22} & -v_{12} & -v_{22} \\ -1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \mu_1 \\ \mu_2 \end{pmatrix},$$

which implies

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \lambda_1 \begin{pmatrix} u_{11} \\ u_{12} \end{pmatrix} + \lambda_2 \begin{pmatrix} u_{21} \\ u_{22} \end{pmatrix} - \mu_1 \begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} - \mu_2 \begin{pmatrix} v_{21} \\ v_{22} \end{pmatrix} \quad (*_1)$$