For this reason, we can restrict ourselves to open or closed balls of center 0.

Examples of normed vector spaces were given in Example 9.1. We repeat the most important examples.

**Example 37.3.** Let $E = \mathbb{R}^n$ (or $E = \mathbb{C}^n$). There are three standard norms. For every $(x_1, \ldots, x_n) \in E$, we have the norm $\|x\|_1$, defined such that,

$$\|x\|_1 = |x_1| + \cdots + |x_n|,$$

we have the *Euclidean norm* $\|x\|_2$, defined such that,

$$\|x\|_2 = \left(|x_1|^2 + \cdots + |x_n|^2\right)^{\frac{1}{2}},$$

and the *sup*-norm $\|x\|_\infty$, defined such that,

$$\|x\|_\infty = \max\{|x_i| \mid 1 \leq i \leq n\}.$$

More generally, we define the $\ell_p$-*norm* (for $p \geq 1$) by

$$\|x\|_p = (|x_1|^p + \cdots + |x_n|^p)^{1/p}.$$

We proved in Proposition 9.1 that the $\ell_p$-norms are indeed norms. The closed unit balls centered at $(0,0)$ for $\| \|_1$, $\| \|_2$, and $\| \|_\infty$, along with the containment relationships, are shown in Figures 37.1 and 37.2. Figures 37.3 and 37.4 illustrate the situation in $\mathbb{R}^3$.

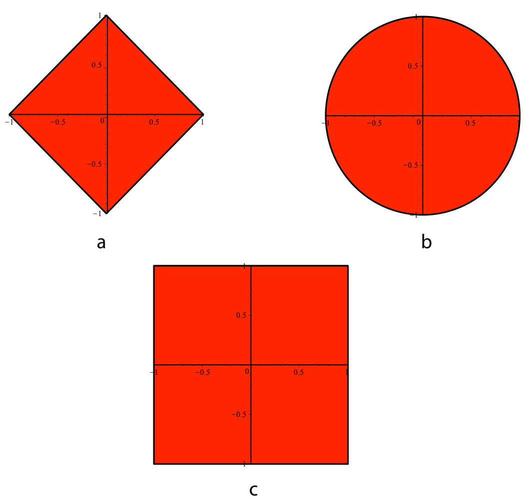


Figure 37.1: Figure $(a)$ shows the diamond shaped closed ball associated with $\| \|_1$. Figure $(b)$ shows the closed unit disk associated with $\| \|_2$, while Figure $(c)$ illustrates the closed unit ball associated with $\| \|_\infty$.