and

$$\text{if} \quad \left\{ \begin{matrix} \sum_{j=1}^{n} a_{ij}x_j \leq b_i \\ \sum_{j=1}^{n} a_{ij}x_j \geq b_i \\ \sum_{j=1}^{n} a_{ij}x_j = b_i \end{matrix} \right\}, \quad \text{then} \quad \left\{ \begin{matrix} y_i \geq 0 \\ y_i \leq 0 \\ y_i \in \mathbb{R} \end{matrix} \right\}.$$

Problem 47.4. Apply the procedure of Problem 47.3 to show that the dual of the (general) linear program

$$\begin{aligned} &\text{maximize} \quad 3x_1 + 2x_2 + 5x_3 \\ &\text{subject to} \\ &\qquad 5x_1 + 3x_2 + x_3 = -8 \\ &\qquad 4x_1 + 2x_2 + 8x_3 \leq 23 \\ &\qquad 6x_1 + 7x_2 + 3x_3 \geq 1 \\ &\qquad x_1 \leq 4,\ x_3 \geq 0 \end{aligned}$$

is the (general) linear program:

$$\begin{aligned} &\text{minimize} \quad -8y_1 + 23y_2 - y_3 + 4y_4 \\ &\text{subject to} \\ &\qquad 5y_1 + 4y_2 - 6y_3 + y_4 = 3 \\ &\qquad 3y_1 + 2y_2 - 7y_3 = 2 \\ &\qquad y_1 + 8y_2 - 3y_3 \geq 5 \\ &\qquad y_2,\ y_3,\ y_4 \geq 0. \end{aligned}$$

Problem 47.5. (1) Prove that the dual of the (general) linear program

$$\begin{aligned} &\text{maximize} \quad cx \\ &\text{subject to} \quad Ax = b \text{ and } x \in \mathbb{R}^n \end{aligned}$$

is

$$\begin{aligned} &\text{minimize} \quad yb \\ &\text{subject to} \quad yA = c \text{ and } y \in \mathbb{R}^m. \end{aligned}$$

(2) Prove that the dual of the (general) linear program

$$\begin{aligned} &\text{maximize} \quad cx \\ &\text{subject to} \quad Ax \geq b \text{ and } x \geq 0 \end{aligned}$$