we have

$$E_j^k = I - \mathcal{E}_j^k,$$

and

$$\mathcal{E}_j^k = P_k \mathcal{E}_j^{k-1}, \quad 1 \leq j \leq n-2,\ j+1 \leq k \leq n-1,$$

where $P_k = I$ *or else* $P_k = P(k, i)$ *for some* i *such that* $k+1 \leq i \leq n$*; if* $P_k \neq I$*, this means that* $(E_j^k)^{-1}$ *is obtained from* $(E_j^{k-1})^{-1}$ *by permuting the entries on rows* i *and* k *in column* j*. Because the matrices* $(E_j^k)^{-1}$ *are all lower triangular, the matrix* L *is also lower triangular.*

In order to find L*, define lower triangular* $n \times n$ *matrices* Λ_k *of the form*

$$\Lambda_k = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 \\
\lambda_{21}^{(k)} & 0 & 0 & 0 & 0 & \vdots & \vdots & 0 \\
\lambda_{31}^{(k)} & \lambda_{32}^{(k)} & \ddots & 0 & 0 & \vdots & \vdots & 0 \\
\vdots & \vdots & \ddots & 0 & 0 & \vdots & \vdots & \vdots \\
\lambda_{k+11}^{(k)} & \lambda_{k+12}^{(k)} & \cdots & \lambda_{k+1k}^{(k)} & 0 & \cdots & \cdots & 0 \\
\lambda_{k+21}^{(k)} & \lambda_{k+22}^{(k)} & \cdots & \lambda_{k+2k}^{(k)} & 0 & \ddots & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\lambda_{n1}^{(k)} & \lambda_{n2}^{(k)} & \cdots & \lambda_{nk}^{(k)} & 0 & \cdots & \cdots & 0
\end{pmatrix}$$

to assemble the columns of L *iteratively as follows: let*

$$(-\ell_{k+1k}^{(k)}, \ldots, -\ell_{nk}^{(k)})$$

be the last $n-k$ *elements of the* k*th column of* E_k*, and define* Λ_k *inductively by setting*

$$\Lambda_1 = \begin{pmatrix}
0 & 0 & \cdots & 0 \\
\ell_{21}^{(1)} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\ell_{n1}^{(1)} & 0 & \cdots & 0
\end{pmatrix},$$

then for $k = 2, \ldots, n-1$*, define*

$$\Lambda_k' = P_k \Lambda_{k-1}, \tag{$\dagger_2$}$$