Furthermore, prove that Cases (A) and (B), Cases (B1) and (B3), and Cases (B2) and (B3) are mutually exclusive, while Cases (B1) and (B2) are not.

Problem 46.3. Consider the linear program (due to E.M.L. Beale):

$$\begin{aligned} \text{maximize} \quad & (3/4)x_1 - 150x_2 + (1/50)x_3 - 6x_4 \\ \text{subject to} \quad & \\ & (1/4)x_1 - 60x_2 - (1/25)x_3 + 9x_4 \leq 0 \\ & (1/4)x_1 - 90x_2 - (1/50)x_3 + 3x_4 \leq 0 \\ & x_3 \leq 1 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0. \end{aligned}$$

(1) Convert the above program to standard form.

(2) Show that if we apply the simplex algorithm with the pivot rule which selects the column entering the basis as the column of smallest index, then the method cycles.

Problem 46.4. Read carefully the proof given by Chvatal that the lexicographic pivot rule and Bland's pivot rule prevent cycling; see Chvatal [40] (Chapter 3, pages 34-38).

Problem 46.5. Solve the following linear program (from Chvatal [40], Chapter 3, page 44) using the two-phase simplex algorithm:

$$\begin{aligned} \text{maximize} \quad & 3x_1 + x_2 \\ \text{subject to} \quad & \\ & x_1 - x_2 \leq -1 \\ & -x_1 - x_2 \leq -3 \\ & 2x_1 + x_2 \leq 4 \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

Problem 46.6. Solve the following linear program (from Chvatal [40], Chapter 3, page 44) using the two-phase simplex algorithm:

$$\begin{aligned} \text{maximize} \quad & 3x_1 + x_2 \\ \text{subject to} \quad & \\ & x_1 - x_2 \leq -1 \\ & -x_1 - x_2 \leq -3 \\ & 2x_1 + x_2 \leq 2 \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$