

Then it is easy to see that we can compute b and η using the following averaging formulae:

$$b = w^\top \left(\left(\sum_{i \in I_\lambda} u_i \right) / |I_\lambda| + \left(\sum_{j \in I_\mu} v_j \right) / |I_\mu| \right) / 2$$

$$\eta = w^\top \left(\left(\sum_{i \in I_\lambda} u_i \right) / |I_\lambda| - \left(\sum_{j \in I_\mu} v_j \right) / |I_\mu| \right) / 2.$$

The “kernelized” version of Problem (SVM_{s2'}) is the following:

Soft margin kernel SVM (SVM_{s2'}):

$$\begin{aligned} & \text{minimize} \quad \frac{1}{2} \langle w, w \rangle - K_m \eta + K_s \begin{pmatrix} \epsilon^\top & \xi^\top \end{pmatrix} \mathbf{1}_{p+q} \\ & \text{subject to} \\ & \quad \langle w, \varphi(u_i) \rangle - b \geq \eta - \epsilon_i, \quad \epsilon_i \geq 0 \quad i = 1, \dots, p \\ & \quad -\langle w, \varphi(v_j) \rangle + b \geq \eta - \xi_j, \quad \xi_j \geq 0 \quad j = 1, \dots, q \\ & \quad \eta \geq 0. \end{aligned}$$

Tracing through the derivation of the dual program we obtain

Dual of the Soft margin kernel SVM (SVM_{s2'}):

$$\begin{aligned} & \text{minimize} \quad \frac{1}{2} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \mathbf{K} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\ & \text{subject to} \\ & \quad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0 \\ & \quad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq K_m \\ & \quad 0 \leq \lambda_i \leq K_s, \quad i = 1, \dots, p \\ & \quad 0 \leq \mu_j \leq K_s, \quad j = 1, \dots, q, \end{aligned}$$

where \mathbf{K} is the kernel matrix of Section 54.1.

As in Section 54.3, we obtain

$$w = \sum_{i=1}^p \lambda_i \varphi(u_i) - \sum_{j=1}^q \mu_j \varphi(v_j),$$