The classification of the points u_i and v_j in terms of the values of λ and μ and Definition 54.2 and Definition 54.3 are unchanged.

It is shown in Section 54.12 how the dual program is solved using ADMM from Section 52.6. If the primal problem is solvable, this yields solutions for λ and μ. Once a solution for λ and μ is obtained, we have

$$\begin{aligned} w &= -X\begin{pmatrix}\lambda \\ \mu\end{pmatrix} = \sum_{i=1}^{p}\lambda_i u_i - \sum_{j=1}^{q}\mu_j v_j \\ b &= -(\mathbf{1}_p^\top\lambda - \mathbf{1}_q^\top\mu) = -\sum_{i=1}^{p}\lambda_i + \sum_{j=1}^{q}\mu_j. \end{aligned}$$

We can compute η using duality. As we said earlier, the hypotheses of Theorem 50.17(2) hold, so *if* the primal problem (SVM_{s3}) has an optimal solution with $w \neq 0$, *then* the dual problem has a solution too, and the duality gap is zero. Therefore, for optimal solutions we have

$$L(w, \epsilon, \xi, b, \eta, \lambda, \mu, \alpha, \beta) = G(\lambda, \mu, \alpha, \beta),$$

which means that

$$\begin{aligned} &\frac{1}{2}w^\top w + \frac{b^2}{2} - (p+q)K_s\nu\eta + K_s\left(\sum_{i=1}^{p}\epsilon_i + \sum_{j=1}^{q}\xi_j\right) \\ &\qquad = -\frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(X^\top X + \begin{pmatrix}\mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top \\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix}. \end{aligned}$$

We can use the above equation to determine η.

Since

$$\frac{1}{2}w^\top w + \frac{b^2}{2} = \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(X^\top X + \begin{pmatrix}\mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top \\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix},$$

we get

$$(p+q)K_s\nu\eta = K_s\left(\sum_{i=1}^{p}\epsilon_i + \sum_{j=1}^{q}\xi_j\right) + \begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(X^\top X + \begin{pmatrix}\mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top \\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix}. \quad (*)$$

Since

$$X^\top X + \begin{pmatrix}\mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top \\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top\end{pmatrix}$$

is positive semidefinite, we confirm that $\eta \geq 0$.

Since nonzero ϵ_i and ξ_j may only occur for vectors u_i and v_j that fail the margin, namely $\lambda_i = K_s$, $\mu_j = K_s$, the corresponding constraints are active and we can solve for ϵ_i and ξ_j in