The generalization to any dimension $n \geq 2$ is immediate.

Finally, we consider the special case where the points $([p_1], [p_2], [p_3], [p_4])$ and the points $[([q_1], [q_2], [q_3], [q_4])$ belong to the affine patch of $\mathbb{RP}^2$ corresponding to the plane H of equation $z = 1$. In this case, we may also identify $[q_i]$ with q_i, which has coordinates $(q_i^x, q_i^y, 1)$ with respect to the canonical basis. Then, the barycentric coordinates $\lambda_1, \lambda_2, \lambda_3$ of q_4 are solutions of the systems

$$\begin{pmatrix} q_1^x & q_2^x & q_3^x \\ q_1^y & q_2^y & q_3^y \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} q_4^x \\ q_4^y \\ 1 \end{pmatrix}.$$

By Proposition 26.12 we obtain the following result.

Proposition 26.13. *With respect to the canonical basis $\mathcal{E} = (e_1, e_2, e_3)$, the matrix $A_{\mathcal{E}}$ of the unique homography h of $\mathbb{RP}^2$ mapping (p_1, p_2, p_4, p_4) to (q_1, q_2, q_3, q_4), all points of the affine plane $z = 1$, is given by*

$$A_{\mathcal{E}} = \begin{pmatrix} q_1^x & q_2^x & q_3^x \\ q_1^y & q_2^y & q_3^y \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{\lambda_1}{\alpha_1} & 0 & 0 \\ 0 & \frac{\lambda_2}{\alpha_2} & 0 \\ 0 & 0 & \frac{\lambda_3}{\alpha_3} \end{pmatrix} \begin{pmatrix} p_1^x & p_2^x & p_3^x \\ p_1^y & p_2^y & p_3^y \\ 1 & 1 & 1 \end{pmatrix}^{-1}.$$

If

$$A_{\mathcal{E}} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},$$

the transformed point of a point $(x, y, 1)$ in the affine plane $z = 1$,

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11}x + a_{12}y + a_{13} \\ a_{21}x + a_{22}y + a_{23} \\ a_{31}x + a_{32}y + a_{33} \end{pmatrix},$$

is not a point at infinity iff $a_{31}x + a_{32}y + a_{33} \neq 0$, in which case it corresponds to the point in the affine plane $z = 1$ of coordinates

$$\begin{pmatrix} \dfrac{x'}{z'} \\[2ex] \dfrac{y'}{z'} \\[2ex] 1 \end{pmatrix} = \begin{pmatrix} \dfrac{a_{11}x + a_{12}y + a_{13}}{a_{31}x + a_{32}y + a_{33}} \\[2ex] \dfrac{a_{21}x + a_{22}y + a_{23}}{a_{31}x + a_{32}y + a_{33}} \\[2ex] 1 \end{pmatrix}.$$

The generalization to any dimension $n \geq 2$ is immediate.

Let us go back to the situation where the the points (p_1, p_2, p_3, p_4) and (q_1, q_2, q_3, q_4) are in the affine patch $z = 1$, and where the matrix of our linear map is expressed with