involving ϵ and ξ are missing from the Lagrangian and the effect is that the box constraints are missing; we simply have $\lambda_i \geq 0$ and $\mu_j \geq 0$.

We can use the dual program to solve the primal. Once $\lambda \geq 0, \mu \geq 0$ have been found, w is given by

$$w = -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \sum_{i=1}^{p} \lambda_i u_i - \sum_{j=1}^{q} \mu_j v_j.$$

To find b we use the complementary slackness conditions.

If the primal has a solution $w \neq 0$, then the equation

$$w = \sum_{i=1}^{p} \lambda_i u_i - \sum_{j=1}^{q} \mu_j v_j$$

implies that either there is some index i_0 such that $\lambda_{i_0} > 0$ or there is some index j_0 such that $\mu_{j_0} > 0$. The constraint

$$\sum_{i=1}^{p} \lambda_i - \sum_{j=1}^{q} \mu_j = 0$$

implies that there is some index i_0 such that $\lambda_{i_0} > 0$ and there is some index j_0 such that $\mu_{j_0} > 0$. However, a priori, nothing prevents the situation where $\lambda_i = K$ for all nonzero λ_i or $\mu_j = K$ for all nonzero μ_j. If this happens, we can rerun the optimization method with a larger value of K. Observe that the equation

$$\sum_{i=1}^{p} \lambda_i - \sum_{j=1}^{q} \mu_j = 0$$

implies that if there is some index i_0 such that $0 < \lambda_{i_0} < K$, then there is some index j_0 such that $0 < \mu_{j_0} < K$, and vice-versa. If the following mild hypothesis holds, then b can be found.

Standard Margin Hypothesis for (SVM_{s2}). There is some index i_0 such that $0 < \lambda_{i_0} < K$ and there is some index j_0 such that $0 < \mu_{j_0} < K$. This means that some u_{i_0} is a support vector of type 1 on the blue margin, and some v_{j_0} is a support vector of type 1 on the red margin.

If the **Standard Margin Hypothesis** for (SVM_{s2}) holds, then $\epsilon_{i_0} = 0$ and $\mu_{j_0} = 0$, and then we have the active equations

$$w^\top u_{i_0} - b = 1 \quad \text{and} \quad -w^\top v_{j_0} + b = 1,$$

and we obtain

$$b = \frac{1}{2}(w^\top u_{i_0} + w^\top v_{j_0}).$$