

- Unreduced and reduced Hessenberg matrices.
- Deflation.
- Shift.
- Wilkinson shift.
- Double shift.
- Francis shift.
- Implicit shifting.
- Implicit  $Q$ -theorem.
- Arnoldi iteration.
- Breakdown of Arnoldi iteration.
- Krylov subspace.
- Rayleigh–Ritz method.
- Ritz values, Arnoldi estimates.
- Residual.
- GMRES
- Lanczos iteration.
- Power iteration.
- Inverse power iteration.
- Rayleigh ratio.

## 18.9 Problems

**Problem 18.1.** Prove Theorem 18.2; see Problem 13.7.

**Problem 18.2.** Prove that if a matrix  $A$  is Hermitian (or real symmetric), then any Hessenberg matrix  $H$  similar to  $A$  is Hermitian tridiagonal (real symmetric tridiagonal).

**Problem 18.3.** For any matrix (real or complex)  $A$ , if  $A = QR$  is a  $QR$ -decomposition of  $A$  using Householder reflections, prove that if  $A$  is upper Hessenberg then so is  $Q$ .