Dual of the Soft margin kernel SVM ($\mathrm{SVM}_{s2'}$):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\mathbf{K}\begin{pmatrix}\lambda \\ \mu\end{pmatrix}\\
&\text{subject to}\\
&\qquad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0\\
&\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq K_m\\
&\qquad 0 \leq \lambda_i \leq K_s, \quad i = 1, \ldots, p\\
&\qquad 0 \leq \mu_j \leq K_s, \quad j = 1, \ldots, q,
\end{aligned}$$

where $\mathbf{K}$ is the kernel matrix of Section 54.1.

Problem 54.10. Prove the formulae determining b in terms of η stated just before Theorem 54.8.

Problem 54.11. Prove that the matrix

$$A = \begin{pmatrix} \mathbf{1}_p^\top & \mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\ I_p & 0_{p,q} & I_p & 0_{p,q} \\ 0_{q,p} & I_q & 0_{q,p} & I_q \end{pmatrix}$$

has rank $p + q + 1$.

Problem 54.12. Prove that the kernel version of Program (SVM_{s3}) is given by:

Dual of the Soft margin kernel SVM (SVM_{s3}):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(\mathbf{K} + \begin{pmatrix}\mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top \\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top\end{pmatrix}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix}\\
&\text{subject to}\\
&\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = \nu\\
&\qquad 0 \leq \lambda_i \leq K_s, \quad i = 1, \ldots, p\\
&\qquad 0 \leq \mu_j \leq K_s, \quad j = 1, \ldots, q,
\end{aligned}$$

where $\mathbf{K}$ is the kernel matrix of Section 54.1.

Problem 54.13. Prove that the matrices

$$A = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0 \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top & -1 \end{pmatrix} \quad \text{and} \quad A_2 = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top \end{pmatrix}$$

have rank 2.