The matrix Pa_3 is shown in Part (3) and Pa_4 is shown below:

$$Pa_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix}.$$

Find n elementary matrices $E_{i_k,j_k;\beta_k}$ such that

$$E_{i_n,j_n;\beta_n} \cdots E_{i_1,j_1;\beta_1} Pa_n = \begin{pmatrix} 1 & 0 \\ 0 & Pa_{n-1} \end{pmatrix}.$$

Use the above to prove that the inverse of Pa_n is the lower triangular matrix whose ith row is given by the signed binomial coefficients

$$(-1)^{i+j-2}\binom{i-1}{j-1} = (-1)^{i+j}\binom{i-1}{j-1},$$

with $1 \leq i \leq n+1$, $1 \leq j \leq n+1$. For example,

$$Pa_4^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ -1 & 3 & -3 & 1 & 0 \\ 1 & -4 & 6 & -4 & 1 \end{pmatrix}.$$

Hint. Given any $n \times n$ matrix A, multiplying A by the elementary matrix $E_{i,j;\beta}$ *on the right* yields the matrix $AE_{i,j;\beta}$ in which β times the ith column is added to the jth column.

Problem 8.14. (1) Implement the method for converting a rectangular matrix to reduced row echelon form in `Matlab`.

(2) Use the above method to find the inverse of an invertible $n \times n$ matrix A by applying it to the the $n \times 2n$ matrix $[A\, I]$ obtained by adding the n columns of the identity matrix to A.

(3) Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & \cdots & n \\ 2 & 3 & 4 & 5 & \cdots & n+1 \\ 3 & 4 & 5 & 6 & \cdots & n+2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ n & n+1 & n+2 & n+3 & \cdots & 2n-1 \end{pmatrix}.$$

Using your program, find the row reduced echelon form of A for $n = 4, \ldots, 20$.