The potential "bad" behavior of a basic feasible solution is recorded in the following definition.

Definition 46.1. Given a Linear Program (P) in standard form where the constraints are given by $Ax = b$ and $x \geq 0$, with A an $m \times n$ matrix of rank m, a basic feasible solution x is *degenerate* if $|J_>(x)| < m$, otherwise it is *nondegenerate*.

The origin 0_n, if it is a basic feasible solution, is degenerate. For a less trivial example, $x = (0, 0, 0, 2)$ is a degenerate basic feasible solution of the following linear program in which $m = 2$ and $n = 4$.

Example 46.1.

$$
\begin{aligned}
\text{maximize} \quad & x_2 \\
\text{subject to} \quad & \\
& -x_1 + x_2 + x_3 = 0 \\
& \qquad x_1 + x_4 = 2 \\
& x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0.
\end{aligned}
$$

The matrix A and the vector b are given by

$$A = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 2 \end{pmatrix},$$

and if $x = (0, 0, 0, 2)$, then $J_>(x) = \{4\}$. There are two ways of forming a set of two linearly independent columns of A containing the fourth column.

Given a basic feasible solution x associated with a subset K of size m, since the columns of the matrix A_K are linearly independent, by abuse of language we call the columns of A_K a *basis* of x.

If u is a vertex of (P), that is, a basic feasible solution of (P) associated with a basis K (of size m), in "normal mode," the simplex algorithm tries to move along an edge from the vertex u to an adjacent vertex v (with $u, v \in \mathcal{P}(A, b) \subseteq \mathbb{R}^n$) corresponding to a basic feasible solution whose basis is obtained by replacing one of the basic vectors A^k with $k \in K$ by another nonbasic vector A^j for some $j \notin K$, in such a way that the value of the objective function is increased.

Let us demonstrate this process on an example.