If φ is bilinear, it is shown in E. Artin [6] (and in Jacobson [98]) that orthogonality is symmetric iff either φ is symmetric or φ is alternating ($\varphi(u, u) = 0$ for all $u \in E$).

If φ is sesquilinear, the answer is more complicated. In addition to the previous two cases, there is a third possibility:

$$\varphi(u, v) = \epsilon\overline{\varphi(v, u)} \quad \text{for all } u, v \in E,$$

where ϵ is some nonzero element in K. We say that φ is *ϵ-Hermitian*. Observe that

$$\varphi(u, u) = \epsilon\overline{\epsilon}\varphi(u, u),$$

so if φ is not alternating, then $\varphi(u, u) \neq 0$ for some u, and we must have $\epsilon\overline{\epsilon} = 1$. The most common cases are

1. $\epsilon = 1$, in which case φ is *Hermitian*, and

2. $\epsilon = -1$, in which case φ is *skew-Hermitian*.

If φ is alternating and K is not of characteristic 2, then equation $(*)$ from Section 29.2 implies that the automorphism $\lambda \mapsto \overline{\lambda}$ must be the identity if φ is nonzero. If so, φ is skew-symmetric, so $\epsilon = -1$.

In summary, if φ is either symmetric, alternating, or ϵ-Hermitian, then orthogonality is symmetric, and it makes sense to talk about *the* orthogonal subspace $U^\perp$ of U.

Observe that if φ is ϵ-Hermitian, then

$$r_\varphi = \epsilon l_\varphi.$$

This is because

$$\begin{aligned} l_\varphi(u)(y) &= \overline{\varphi(u, y)} \\ r_\varphi(u)(y) &= \varphi(y, u) \\ &= \epsilon\overline{\varphi(u, y)}, \end{aligned}$$

so $r_\varphi = \epsilon l_\varphi$.

If E and F are finite-dimensional with bases $(e_1, \ldots, e_m)$ and $(f_1, \ldots, f_n)$, and if φ is represented by the $n \times m$ matrix M, then φ is ϵ-Hermitian iff

$$M = \epsilon M^*,$$

where $M^* = (\overline{M})^\top$ (as usual). This captures the following kinds of familiar matrices:

1. Symmetric matrices ($\epsilon = 1$)

2. Skew-symmetric matrices ($\epsilon = -1$)