so the principal directions are $u_1 = (0.9995, 0.0325)$ and $u_2 = (0.0325, -0.9995)$. Observe that u_1 is almost the direction of the x-axis, and u_2 is almost the opposite direction of the y-axis. We also find that the projections Y_1 and Y_2 along the principal directions are

$$VD = \begin{pmatrix} -51.5550 & 3.9249 \\ 9.8031 & -6.0843 \\ -76.5417 & 3.1116 \\ -2.0929 & -9.4731 \\ 33.4651 & 4.6912 \\ 25.5669 & 1.4325 \\ 53.3894 & 7.3408 \\ 13.2107 & 6.0330 \\ 6.4794 & 3.8128 \\ 15.0607 & -13.9174 \end{pmatrix}, \quad \text{with} \quad X - \mu = \begin{pmatrix} -51.4000 & -5.6000 \\ 9.6000 & 6.4000 \\ -76.4000 & -5.6000 \\ -2.4000 & 9.4000 \\ 33.6000 & -3.6000 \\ 25.6000 & -0.6000 \\ 53.6000 & -5.6000 \\ 13.4000 & -5.6000 \\ 6.6000 & -3.6000 \\ 14.6000 & 14.4000 \end{pmatrix}.$$

See Figures 23.4, 23.5, and 23.6.

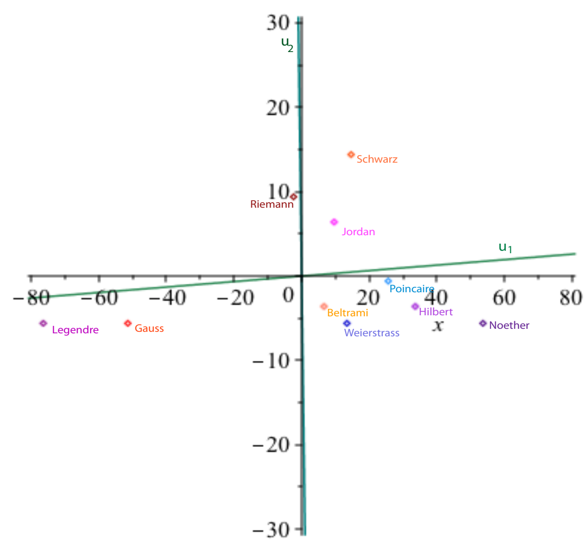

Figure 23.4: The centered data points of Example 23.9 and the two principal directions of Example 23.10.

We know from our study of SVD that $\sigma_1^2, \ldots, \sigma_d^2$ are the eigenvalues of the symmetric positive semidefinite matrix $(X - \mu)^\top (X - \mu)$ and that $u_1, \ldots, u_d$ are corresponding eigenvectors. Numerically, it is preferable to use SVD on $X - \mu$ rather than to compute explicitly $(X - \mu)^\top (X - \mu)$ and then diagonalize it. Indeed, the explicit computation of $A^\top A$ from a matrix A can be numerically quite unstable, and good SVD algorithms avoid computing $A^\top A$ explicitly.