signs of the entries whose indices belong to *pivlist* are the flipped signs of the 6 entries in the column U corresponding to the jth index in *freelist*. We can now read off from BK the 4×4 matrices that form a basis of V: every column of BK corresponds to a matrix whose rows have been concatenated. We get the following 10 matrices:

$$M_1 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad M_3 = \begin{pmatrix} 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$M_4 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad M_5 = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad M_6 = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$M_7 = \begin{pmatrix} -2 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad M_8 = \begin{pmatrix} -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad M_9 = \begin{pmatrix} -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$M_{10} = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Recall that a *magic square* is a square matrix that satisfies the two conditions about the sum of the entries in each row and in each column to be the same number, and also the additional two constraints that the main descending and the main ascending diagonals add up to this common number. Furthermore, the entries are also required to be positive integers. For $n = 4$, the additional two equations are

$$a_{22} + a_{33} + a_{44} - a_{12} - a_{13} - a_{14} = 0$$
$$a_{41} + a_{32} + a_{23} - a_{11} - a_{12} - a_{13} = 0,$$

and the 8 equations stating that a matrix is a magic square are linearly independent. Again, by running row elimination, we get a basis of the "generalized magic squares" whose entries are not restricted to be positive integers. We find a basis of 8 matrices. For $n = 3$, we find a basis of 3 matrices.

A magic square is said to be *normal* if its entries are precisely the integers $1, 2 \ldots, n^2$. Then since the sum of these entries is

$$1 + 2 + 3 + \cdots + n^2 = \frac{n^2(n^2 + 1)}{2},$$