1. "Good" triangulations must be found. This in itself is a vast research topic. Delaunay triangulations are good candidates.

2. "Good" spaces of functions must be found; typically piecewise polynomials and splines.

3. "Good" bases consisting of functions will small support must be found, so that integrals can be easily computed and sparse banded matrices arise.

We now consider boundary problems where the solution varies with time.

19.3 Time-Dependent Boundary Problems: The Wave Equation

Consider a homogeneous string (or rope) of constant cross-section, of length L, and stretched (in a vertical plane) between its two ends which are assumed to be fixed and located along the x-axis at $x = 0$ and at $x = L$.

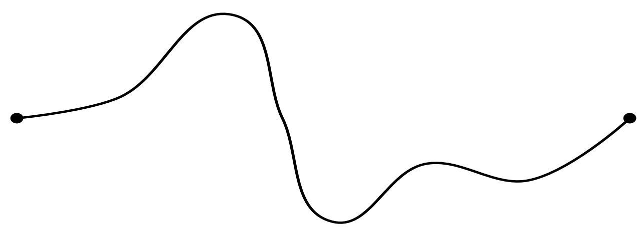

Figure 19.6: A vibrating string

The string is subjected to a transverse force $\tau f(x)dx$ per element of length dx (where τ is the tension of the string). We would like to investigate the small displacements of the string in the vertical plane, that is, how it vibrates.

Thus, we seek a function $u(x, t)$ defined for $t \geq 0$ and $x \in [0, L]$, such that $u(x, t)$ represents the vertical deformation of the string at the abscissa x and at time t.

It can be shown that u must satisfy the following PDE

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}(x, t) - \frac{\partial^2 u}{\partial x^2}(x, t) = f(x, t), \quad 0 < x < L,\ t > 0,$$

with $c = \sqrt{\tau/\rho}$, where ρ is the linear density of the string, known as the *one-dimensional wave equation*.