49.5. Geometrically this procedure corresponds to intersecting the plane $-8x + 12y = -8$ with the ellipsoid $J(x, y) = \frac{3}{2}x^2 + 2xy + 3y^2 - 2x + 8y$ to form the parabolic curve $f(x) = 25/6x^2 - 2/3x - 4$, and then locating the x-coordinate of its apex which occurs when $f'(x) = 0$, i.e when $x = 2/25$; see Figure 49.6. After 31 iterations, this procedure stabi-

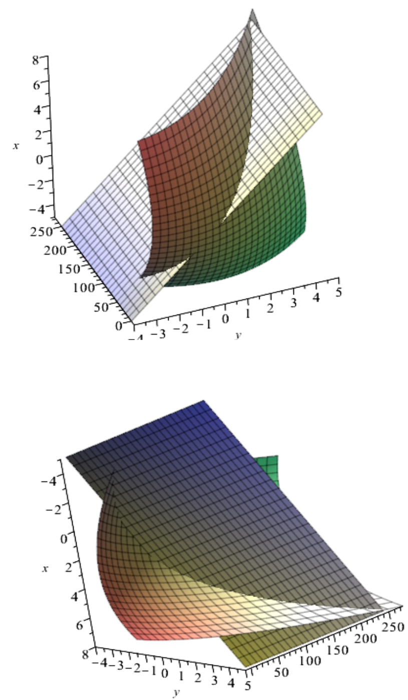

Figure 49.6: Two views of the intersection between the plane $-8x + 12y = -8$ and the ellipsoid $J(x, y) = \frac{3}{2}x^2 + 2xy + 3y^2 - 2x + 8y$. The point u_{k+1} is the minimum of the parabolic intersection.

lizes to point $(2, -2)$, which as we know, is the unique minimum of the quadratic ellipsoid $J(x, y) = \frac{3}{2}x^2 + 2xy + 3y^2 - 2x + 8y$.

A proof of the convergence of the gradient method with backtracking line search, under the hypothesis that J is strictly convex, is given in Boyd and Vandenberghe[29] (Section 9.3.1). More details on this method and the steepest descent method for the Euclidean norm can also be found in Boyd and Vandenberghe [29] (Section 9.3).

49.7 Convergence of Gradient Descent with Variable Stepsize

We now give a sufficient condition for the gradient method with variable stepsize parameter to converge. In addition to requiring J to be an elliptic functional, we add a Lipschitz