Problem 55.6. Implement in `Matlab` and test (**lasso3**) for various values of ρ and τ. Write a program to plot the coordinates of w as a function of τ. Compare the behavior of lasso with ridge regression (**RR6**$'$), (**RR3**b) (b penalized), and with least squares.

Problem 55.7. Check the details of the derivation of the dual of elastic net.

Problem 55.8. Write a `Matlab` program, solving the dual of elastic net; use inspiration from Section 56.3. Run tests to compare the behavior of ridge regression, lasso, and elastic net.

Problem 55.9. Prove that if an optimal solution exists for the elastic net method, then it is unique.

Problem 55.10. Prove that the matrix

$$P = \begin{pmatrix} I_n & -I_n & -X^\top & X^\top \\ -I_n & I_n & X^\top & -X^\top \\ -X & X & XX^\top + KI_m & -XX^\top - KI_m \\ X & -X & -XX^\top - KI_m & XX^\top + KI_m \end{pmatrix}$$

is almost positive definite, in the sense that

$$\begin{pmatrix} \beta_+^\top & \beta_-^\top & \mu_+^\top & \mu_-^\top \end{pmatrix} P \begin{pmatrix} \beta_+ \\ \beta_- \\ \mu_+ \\ \mu_- \end{pmatrix} = 0$$

if and only if $\beta_+ = \beta_-$ and $\mu_+ = \mu_-$, that is, $\beta = 0$ and $\mu = 0$.