so b and ϵ can be computed. In particular,

$$\begin{aligned} b &= \frac{1}{2}\left(y_{i_0} + y_{j_0} - w^\top(x_{i_0} + x_{j_0})\right) \\ \epsilon &= \frac{1}{2}\left(y_{j_0} - y_{i_0} + w^\top(x_{i_0} - x_{j_0})\right). \end{aligned}$$

The function $f(x) = w^\top x + b$ (often called *regression estimate*) is given by

$$f(x) = \sum_{i=1}^m (\mu_i - \lambda_i) x_i^\top x + b.$$

In practice, due to numerical inaccurracy, it is complicated to write a computer program that will select two *distinct indices* as above. It is preferable to compute the list I_λ of indices i such that $0 < \lambda_i < C/m$ and the list I_μ of indices j such that $0 < \mu_j < C/m$. Then it is easy to see that

$$\begin{aligned} b &= \left(\left(\sum_{i_0 \in I_\lambda} y_{i_0}\right)/|I_\lambda| + \left(\sum_{j_0 \in I_\mu} y_{j_0}\right)/|I_\mu| - w^\top\left(\left(\sum_{i_0 \in I_\lambda} x_{i_0}\right)/|I_\lambda| + \left(\sum_{j_0 \in I_\mu} x_{j_0}\right)/|I_\mu|\right)\right)/2 \\ \epsilon &= \left(\left(\sum_{j_0 \in I_\mu} y_{j_0}\right)/|I_\mu| - \left(\sum_{i_0 \in I_\lambda} y_{i_0}\right)/|I_\lambda| + w^\top\left(\left(\sum_{i_0 \in I_\lambda} x_{i_0}\right)/|I_\lambda| - \left(\sum_{j_0 \in I_\mu} x_{j_0}\right)/|I_\mu|\right)\right)/2. \end{aligned}$$

These formulae are numerically a lot more stable, but we still have to be cautious to set suitable tolerance factors to decide whether $\lambda_i > 0$ and $\lambda_i < C/m$ (and similarly for μ_i).

The following result gives sufficient conditions for expressing ϵ in terms of a single support vector.

Proposition 56.6. *For every optimal solution $(w, b, \epsilon, \xi, \xi')$ with $w \neq 0$ and $\epsilon > 0$, if*

$$\max\left\{\frac{2p_{sf}}{m}, \frac{2q_{sf}}{m}\right\} < \nu < (m-1)/m,$$

then ϵ and b are determined from a solution (λ, μ) of the dual in terms of a single support vector.

Proof sketch. If we express that the duality gap is zero we obtain the following equation expressing ϵ in terms of b:

$$\begin{aligned} C\left(\nu - \frac{p_f + q_f}{m}\right)\epsilon = &- \begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} P \begin{pmatrix}\lambda \\ \mu\end{pmatrix} - \begin{pmatrix}y^\top & -y^\top\end{pmatrix}\begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\ &- \frac{C}{m}\left(w^\top\left(\sum_{i \in K_\lambda} x_i - \sum_{j \in K_\mu} x_j\right) - \sum_{i \in K_\lambda} y_i + \sum_{j \in K_\mu} y_j + (p_f - q_f)b\right). \end{aligned}$$