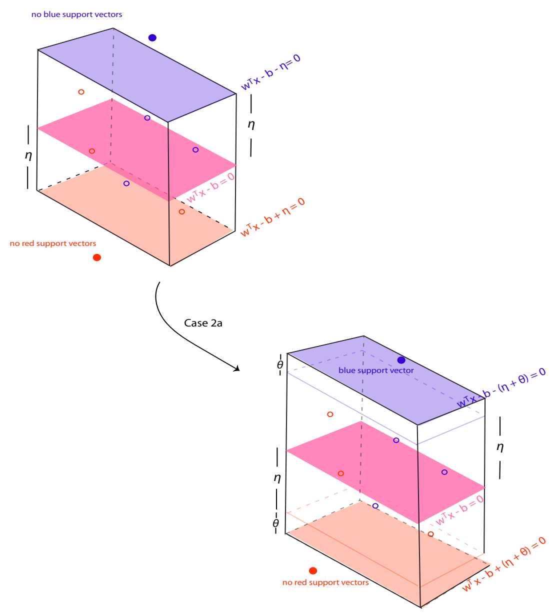

Figure 54.9: In this illustration points with errors are denoted by open circles. In the original, upper left configuration, there is no blue support vector and no red support vector. By increasing the margin, we end up with a blue support vector and reduce to Case 2a.

or

$$-w^\top v_j + b = \eta + \theta \quad \text{for some } j \notin E_\mu.$$

The new value of the objective function is

$$\begin{aligned}
\omega(\theta) &= \frac{1}{2}w^\top w - \nu(\eta + \theta) + \frac{1}{p+q}\left(\sum_{i\in E_\lambda}(\epsilon_i + \theta) + \sum_{j\in E_\mu}(\xi_j + \theta)\right) \\
&= \frac{1}{2}w^\top w - \nu\eta + \frac{1}{p+q}\left(\sum_{i\in E_\lambda}\epsilon_i + \sum_{j\in E_\mu}\xi_j\right) - \left(\nu - \frac{p_{sf} + q_{sf}}{p+q}\right)\theta.
\end{aligned}$$

Since $\max\{2p_f/(p+q), 2q_f/(p+q)\} \le \nu$ implies that $(p_f + q_f)/(p+q) \le \nu$ and $p_{sf} \le p_f$, $q_{sf} \le q_f$, we have

$$\nu - \frac{p_{sf} + q_{sf}}{p+q} \ge 0, \tag{$*_2$}$$

and so $\omega(\theta) \le \omega(0)$. If inequality $(*_2)$ is strict, then this contradicts the optimality of the original solution. Therefore, $\nu = (p_{sf} + q_{sf})/(p+q)$, $\omega(\theta) = \omega(0)$ and $(w, b, \eta+\theta, \epsilon+\theta, \xi+\theta)$ is an optimal solution such that either

$$w^\top u_i - b = \eta + \theta \quad \text{for some } i \notin E_\lambda$$

or

$$-w^\top v_j + b = \eta + \theta \quad \text{for some } j \notin E_\mu.$$