Chapter 23

Applications of SVD and Pseudo-Inverses

> De tous les principes qu'on peut proposer pour cet objet, je pense qu'il n'en est pas de plus général, de plus exact, ni d'une application plus facile, que celui dont nous avons fait usage dans les recherches précédentes, et qui consiste à rendre *minimum* la somme des carrés des erreurs. Par ce moyen il s'établit entre les erreurs une sorte d'équilibre qui, empêchant les extrêmes de prévaloir, est très propre às faire connaitre l'état du système le plus proche de la vérité.
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> —**Legendre, 1805,** *Nouvelles Méthodes pour la détermination des Orbites des Comètes*

23.1 Least Squares Problems and the Pseudo-Inverse

This chapter presents several applications of SVD. The first one is the pseudo-inverse, which plays a crucial role in solving linear systems by the method of least squares. The second application is data compression. The third application is principal component analysis (PCA), whose purpose is to identify patterns in data and understand the variance–covariance structure of the data. The fourth application is the best affine approximation of a set of data, a problem closely related to PCA.

The method of least squares is a way of "solving" an overdetermined system of linear equations

$$Ax = b,$$

i.e., a system in which A is a rectangular $m \times n$ matrix with more equations than unknowns (when $m > n$). Historically, the method of least squares was used by Gauss and Legendre to solve problems in astronomy and geodesy. The method was first published by Legendre in 1805 in a paper on methods for determining the orbits of comets. However, Gauss had already used the method of least squares as early as 1801 to determine the orbit of the asteroid