Observe that the exceptional case in which $\theta = \epsilon$ may arise. In this case all points (x_i, y_i) that are not errors (strictly outside the ϵ -slab) are on the red margin hyperplane. This case can only arise if $\nu = 2p_{sf}/m$.

Case 1b. We have $-w^{\top}x_i - b + y_i < \epsilon$ for all $i \notin (E_{\lambda} \cup E_{\mu})$. Our strategy is to decrease ϵ and increase the errors by a small θ in such a way that some inequality becomes an equation for some $i \notin (E_{\lambda} \cup E_{\mu})$. Geometrically, this corresponds to decreasing the width of the slab, keeping the separating hyperplane unchanged. See Figures 56.8 and 56.9. Then we are reduced to Case 1a or Case 2a.

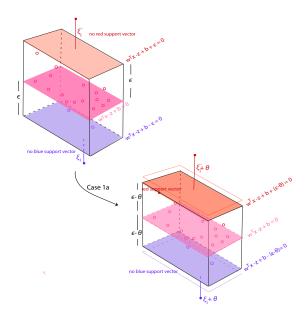


Figure 56.8: In this illustration points within the ϵ -tube are denoted by open circles. In the original, upper left configuration, there is no blue support vector and no red support vector. By decreasing the width of the slab, we end up with a red support vector and reduce to Case 1a.

We have

$$w^{\top} x_i + b - y_i = \epsilon + \xi_i \qquad \qquad \xi_i > 0 \qquad \qquad i \in E_{\lambda}$$

$$-w^{\top} x_j - b + y_j = \epsilon + \xi'_j \qquad \qquad \xi'_j > 0 \qquad \qquad j \in E_{\mu}$$

$$w^{\top} x_i + b - y_i < \epsilon \qquad \qquad i \notin (E_{\lambda} \cup E_{\mu})$$

$$-w^{\top} x_i - b + y_i < \epsilon \qquad \qquad i \notin (E_{\lambda} \cup E_{\mu})$$

Let us pick θ such that

$$\theta = \min\{\epsilon - (w^{\top}x_i + b - y_i), \ \epsilon + w^{\top}x_i + b - y_i \mid i \notin (E_{\lambda} \cup E_{\mu})\},\$$