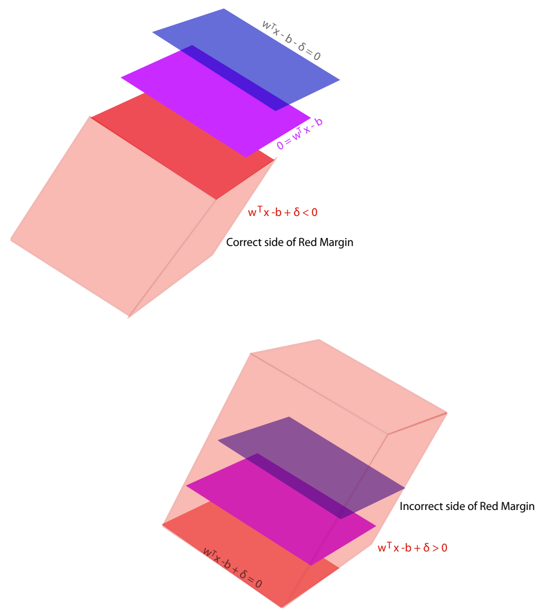

Figure 54.3: The red margin half space associated with $w^\top x - b + \delta = 0$.

The objective function of our problem is affine and the only nonaffine constraint $w^\top w \leq 1$ is convex. This constraint is qualified because for any $w \neq 0$ such that $w^\top w < 1$ and for any $\delta > 0$ and any b we can pick ϵ and ξ large enough so that the constraints are satisfied. Consequently, by Theorem 50.17(2) *if* the primal problem (SVM_{s1}) has an optimal solution, *then* the dual problem has a solution too, and the duality gap is zero.

Unfortunately this does not imply that an optimal solution of the dual yields an optimal solution of the primal because the hypotheses of Theorem 50.17(1) fail to hold. In general, there may not be a unique vector $(w, \epsilon, \xi, b, \delta)$ such that

$$\inf_{w,\epsilon,\xi,b,\delta} L(w, \epsilon, \xi, b, \delta, \lambda, \mu, \alpha, \beta, \gamma) = G(\lambda, \mu, \alpha, \beta, \gamma).$$

If the sets $\{u_i\}$ and $\{v_j\}$ are *not* linearly separable, then the dual problem may have a solution for which $\gamma = 0$,

$$\sum_{i=1}^{p} \lambda_i = \sum_{j=1}^{q} \mu_j = \frac{1}{2},$$

and

$$\sum_{i=1}^{p} \lambda_i u_i = \sum_{j=1}^{q} \mu_j v_j,$$

so that the dual function $G(\lambda, \mu, \alpha, \beta, \gamma)$, which is a *partial function*, is defined and has the value $G(\lambda, \mu, \alpha, \beta, 0) = 0$. Such a pair (λ, μ) corresponds to the coefficients of two convex