where A is an $m \times n$ matrix, F is a $p \times n$ matrix, and either A has rank n or F has rank n. This problem is converted to the ADMM problem

$$\begin{aligned} &\text{minimize} && \|Ax - b\|_2^2 + \tau \|z\|_1 \\ &\text{subject to} && Fx - z = 0, \end{aligned}$$

and the corresponding ADMM procedure (in scaled form) is

$$\begin{aligned} x^{k+1} &= (A^\top A + \rho F^\top F)^{-1}(A^\top b + \rho F^\top (z^k - u^k)) \\ z^{k+1} &= S_{\tau/\rho}(Fx^{k+1} + u^k) \\ u^{k+1} &= u^k + Fx^{k+1} - z^{k+1}. \end{aligned}$$

(6) *Group Lasso.*

This a generalization of (3). Here we assume that x is split as $x = (x_1, \ldots, x_N)$, with $x_i \in \mathbb{R}^{n_i}$ and $n_1 + \cdots + x_N = n$, and the regularizing term $\|x\|_1$ is replaced by $\sum_{i=1}^N \|x_i\|_2$. When $n_i = 1$, this reduces to (3). The z-update of the ADMM procedure needs to modified. We define the soft thresholding operator $\mathcal{S}_c \colon \mathbb{R}^m \to \mathbb{R}^m$ given by

$$\mathcal{S}_c(v) = \left(1 - \frac{c}{\|v\|_2}\right)_+ v,$$

with $\mathcal{S}_c(0) = 0$. Then the z-update consists of the N updates

$$z_i^{k+1} = \mathcal{S}_{\tau/\rho}(x_i^{k+1} + u^k), \quad i = 1, \ldots, N.$$

The method can be extended to deal with overlapping groups; see Boyd et al. [28] (Section 6.4).

There are many more applications of ADMM discussed in Boyd et al. [28], including consensus and sharing. See also Strang [171] for a brief overview.

52.9 Summary

The main concepts and results of this chapter are listed below:

- Dual ascent.
- Augmented Lagrangian.
- Penalty parameter.
- Method of multipliers.
- ADMM (alternating direction method of multipliers).