# Chapter 19

# Variational Approximation of Boundary-Value Problems; Introduction to the Finite Elements Method

## 19.1 A One-Dimensional Problem: Bending of a Beam

Consider a beam of unit length supported at its ends in 0 and 1, stretched along its axis by a force $P$, and subjected to a transverse load $f(x)dx$ per element $dx$, as illustrated in Figure 19.1.

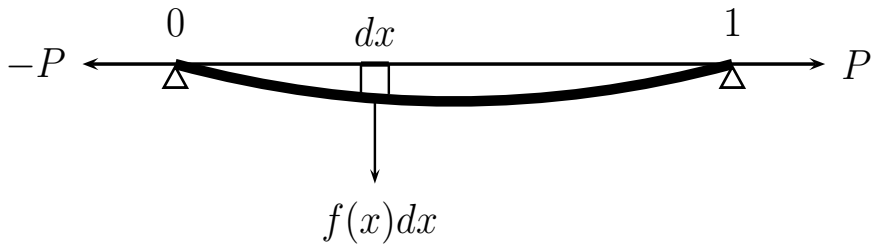


Figure 19.1: Vertical deflection of a beam

The bending moment $u(x)$ at the absissa $x$ is the solution of a boundary problem (BP) of the form

$$\begin{aligned} -u''(x) + c(x)u(x) &= f(x), \quad 0 < x < 1 \\ u(0) &= \alpha \\ u(1) &= \beta, \end{aligned}$$