then

$$Q = E_1^{-1} E_2^{-1} \cdots E_k^{-1}.$$

Now, row operations operate on the left and column operations operate on the right, so the product $E_1^{-1} E_2^{-1} \cdots E_k^{-1}$ can be computed from left to right as a sequence of column operations.

Let us review the meaning of the elementary row and column operations $P(i, k)$, $E_{i,j;\beta}$, and $E_{i,\lambda}$.

1. As a row operation, $P(i, k)$ permutes row i and row k.
2. As a column operation, $P(i, k)$ permutes column i and column k.
3. The inverse of $P(i, k)$ is $P(i, k)$ itself.
4. As a row operation, $E_{i,j;\beta}$ adds β times row j to row i.
5. As a column operation, $E_{i,j;\beta}$ adds β times column i to column j (note the switch in the indices).
6. The inverse of $E_{i,j;\beta}$ is $E_{i,j;-\beta}$.
7. As a row operation, $E_{i,\lambda}$ multiplies row i by λ.
8. As a column operation, $E_{i,\lambda}$ multiplies column i by λ.
9. The inverse of $E_{i,\lambda}$ is $E_{i,\lambda^{-1}}$.

Given a square matrix A (over K), the row and column operations applied to $XI - A$ in converting it to its Smith normal form may involve coefficients that are polynomials and it is necessary to explain what is the action of an operation $E_{i,j;\beta}$ in this case. If the coefficient β in $E_{i,j;\beta}$ is a polynomial over K, as a row operation, the action of $E_{i,j;\beta}$ on a matrix X is to multiply the jth row of M by the matrix $\beta(A)$ obtained by substituting the matrix A for X and then to add the resulting vector to row i. Similarly, as a column operation, the action of $E_{i,j;\beta}$ on a matrix X is to multiply the ith column of M by the matrix $\beta(A)$ obtained by substituting the matrix A for X and then to add the resulting vector to column j. An algorithm to compute the rational canonical form of a matrix can now be given. We apply the elementary column operations E_i^{-1} for $i = 1, \ldots k$, starting with the identity matrix.

Algorithm for Converting an $n \times n$ matrix to Rational Canonical Form

While applying elementary row and column operations to compute the Smith normal form D of $XI - A$, keep track of the row operations and perform the following steps:

1. Let $P' = I_n$, and for every elementary row operation E do the following:

    (a) If $E = P(i, k)$, permute column i and column k of P'.