The considerations in this chapter reveal the need to find methods for finding the zeros of the derivative map

$$dJ\colon \Omega \to E',$$

where Ω is some open subset of a normed vector space E and E' is the space of all continuous linear forms on E (a subspace of E^*). Generalizations of *Newton's method* yield such methods and they are the object of the next chapter.

40.4 Summary

The main concepts and results of this chapter are listed below:

- Local minimum, local maximum, local extremum, strict local minimum, strict local maximum.
- Necessary condition for a local extremum involving the derivative; critical point.
- Local minimum with respect to a subset U, local maximum with respect to a subset U, local extremum with respect to a subset U.
- Constrained local extremum.
- Necessary condition for a constrained extremum.
- Necessary condition for a constrained extremum in terms of Lagrange multipliers.
- Lagrangian.
- Critical points of a Lagrangian.
- Necessary condition of an unconstrained local minimum involving the second-order derivative.
- Sufficient condition for a local minimum involving the second-order derivative.
- A sufficient condition involving nondegenerate critical points.
- Convex sets, convex functions, concave functions, strictly convex functions, strictly concave functions.
- Necessary condition for a local minimum on a convex set involving the derivative.
- Convexity of a function involving a condition on its first derivative.
- Convexity of a function involving a condition on its second derivative.
- Minima of convex functions on convex sets.