Dual of the Basic Quadratic Soft margin ν-SVM Problem (SVM$_{s4}$):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(X^\top X + \frac{1}{2K}I_{p+q}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix}\\
&\text{subject to}\\
&\qquad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0\\
&\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq \nu\\
&\qquad \lambda_i \geq 0, \quad i = 1, \ldots, p\\
&\qquad \mu_j \geq 0, \quad j = 1, \ldots, q.
\end{aligned}$$

The above program is similar to the program that was obtained for Problem (SVM$_{s2'}$) but the matrix $X^\top X$ is replaced by the matrix $X^\top X + (1/2K)I_{p+q}$, which is positive definite since $K > 0$, and also the inequalities $\lambda_i \leq K$ and $\mu_j \leq K$ no longer hold. If the constraint $\eta \geq 0$ is dropped, then the inequality

$$\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq \nu$$

is replaced by the equation

$$\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = \nu.$$

We obtain w from λ and μ, and γ, as in Problem (SVM$_{s2'}$); namely,

$$w = -X\begin{pmatrix}\lambda \\ \mu\end{pmatrix} = \sum_{i=1}^p \lambda_i u_i - \sum_{j=1}^q \mu_j v_j$$

and η is given by

$$(p+q)K_s\nu\eta = \begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(X^\top X + \frac{1}{2K_s}I_{p+q}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix}.$$

The constraints imply that there is some i_o such that $\lambda_{i_0} > 0$ *and* some j_0 such that $\mu_{j_0} > 0$, which means that at least two points are misclassified, so Problem (SVM$_{s4}$) should only be used when the sets $\{u_i\}$ and $\{v_j\}$ are *not* linearly separable. We can solve for b using the active constraints corresponding to any i_0 such that $\lambda_{i_0} > 0$ and any j_0 such that $\mu_{j_0} > 0$. To improve numerical stability we average over the sets of indices I_λ and I_μ.