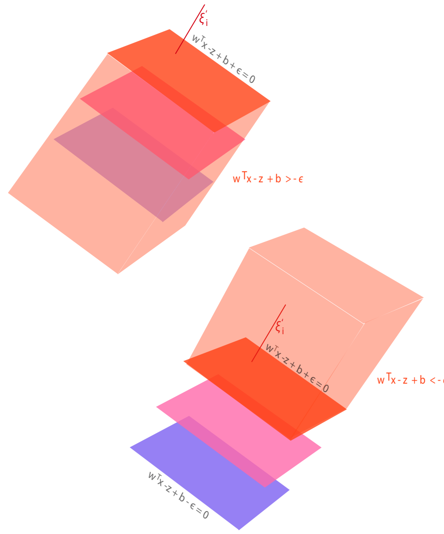

Figure 56.3: The two red half spaces associated with the hyperplane $w^\top x_i - z + b = -\epsilon$.

Thus it appears that the above problem is the version of Program (**RR3**) (see Section 55.2) in which the ℓ^2-norm of $y - Xw - b\mathbf{1}$ is replaced by its ℓ^1-norm. This a sort of "dual" of lasso (see Section 55.5) where $(1/2)w^\top w = (1/2)\|w\|_2^2$ is replaced by $\tau \|w\|_1$, and $\|y - Xw - b\mathbf{1}\|_1$ is replaced by $\|y - Xw - b\mathbf{1}\|_2^2$.

Proposition 56.1. *For any optimal solution, the equations*

$$\xi_i \xi_i' = 0, \quad i = 1, \ldots, m \tag{$\xi\xi'$}$$

hold. If $\epsilon > 0$, then the equations

$$\begin{aligned} w^\top x_i + b - y_i &= \epsilon + \xi_i \\ -w^\top x_i - b + y_i &= \epsilon + \xi_i' \end{aligned}$$

cannot hold simultaneously.

Proof. For an optimal solution we have

$$-\epsilon - \xi_i' \leq w^\top x_i + b - y_i \leq \epsilon + \xi_i.$$

If $w^\top x_i + b - y_i \geq 0$, then $\xi_i' = 0$ since the inequality

$$-\epsilon - \xi_i' \leq w^\top x_i + b - y_i$$

is trivially satisfied (because $\epsilon, \xi_i' \geq 0$), and if $w^\top x_i + b - y_i \leq 0$, then similarly $\xi_i = 0$. See Figure 56.4.