Chapter 5

Haar Bases, Haar Wavelets, Hadamard Matrices

In this chapter, we discuss two types of matrices that have applications in computer science and engineering:

(1) Haar matrices and the corresponding Haar wavelets, a fundamental tool in signal processing and computer graphics.

2) Hadamard matrices which have applications in error correcting codes, signal processing, and low rank approximation.

5.1 Introduction to Signal Compression Using Haar Wavelets

We begin by considering *Haar wavelets* in $\mathbb{R}^4$. Wavelets play an important role in audio and video signal processing, especially for *compressing* long signals into much smaller ones that still retain enough information so that when they are played, we can't see or hear any difference.

Consider the four vectors w_1, w_2, w_3, w_4 given by

$$w_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad w_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \qquad w_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \qquad w_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}.$$

Note that these vectors are pairwise orthogonal, which means that their inner product is 0 (see Section 12.1, Example 12.1, and Section 12.2, Definition 12.2), so they are indeed linearly independent (see Proposition 12.4). Let $\mathcal{W} = \{w_1, w_2, w_3, w_4\}$ be the *Haar basis*, and let