are the zeros of the equation

$$\lambda^2 - (17 + b^2 + c^2)\lambda + (4 - bc)^2 = 0.$$

(4) Prove that

$$\|A\|_N^2 = 17 + b^2 + c^2 + 2|4 - bc|.$$

Consider the cases where $4 - bc \geq 0$ and $4 - bc \leq 0$, and show that in both cases we must have $b^2 = c^2$. Then show that the minimum of $f(c, d) = 17 + b^2 + c^2 + 2|4 - bc|$ is achieved by $b = c$ with $-2 \leq b \leq 2$. Conclude that the matrices A completing B_0 that minimize $\|A\|_N$ are given by

$$A = \begin{pmatrix} 1 & b \\ b & 4 \end{pmatrix}, \qquad -2 \leq b \leq 2.$$

(5) Prove that the squares σ_1^2 and σ_2^2 of the singular values of

$$A = \begin{pmatrix} 1 & 2 \\ 3 & d \end{pmatrix}$$

are the zeros of the equation

$$\lambda^2 - (14 + d^2)\lambda + (6 - d)^2 = 0$$

(6) Prove that

$$\|A\|_N^2 = 14 + d^2 + 2|6 - d|.$$

Consider the cases where $6 - d \geq 0$ and $6 - d \leq 0$, and show that the minimum of $f(c, d) = 14 + d^2 + 2|6 - d|$ is achieved by $d = 1$. Conclude that the the matrix A completing C_0 that minimizes $\|A\|_N$ is given by

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}.$$

Problem 14.12. Prove Theorem 14.32 when E is a finite-dimensional Hermitian space.