Soft margin kernel SVM (SVM_{s1}):

$$\begin{aligned}
&\text{minimize} \quad -\delta + K\left(\sum_{i=1}^{p}\epsilon_i + \sum_{j=1}^{q}\xi_j\right)\\
&\text{subject to}\\
&\qquad \langle w, \varphi(u_i)\rangle - b \geq \delta - \epsilon_i, \quad \epsilon_i \geq 0 \qquad i = 1, \ldots, p\\
&\qquad -\langle w, \varphi(v_j)\rangle + b \geq \delta - \xi_j, \quad \xi_j \geq 0 \qquad j = 1, \ldots, q\\
&\qquad \langle w, w\rangle \leq 1.
\end{aligned}$$

Tracing through the computation that led us to the dual program with u_i replaced by $\varphi(u_i)$ and v_j replaced by $\varphi(v_j)$, we find the following version of the dual program:

Dual of Soft margin kernel SVM (SVM_{s1}):

$$\begin{aligned}
&\text{minimize} \quad \begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} \mathbf{K} \begin{pmatrix}\lambda\\ \mu\end{pmatrix}\\
&\text{subject to}\\
&\qquad \sum_{i=1}^{p}\lambda_i - \sum_{j=1}^{q}\mu_j = 0\\
&\qquad \sum_{i=1}^{p}\lambda_i + \sum_{j=1}^{q}\mu_j = 1\\
&\qquad 0 \leq \lambda_i \leq K, \quad i = 1, \ldots, p\\
&\qquad 0 \leq \mu_j \leq K, \quad j = 1, \ldots, q,
\end{aligned}$$

where $\mathbf{K}$ is the $\ell \times \ell$ kernel symmetric matrix (with $\ell = p + q$) given by

$$\mathbf{K}_{ij} = \begin{cases} \kappa(u_i, u_j) & 1 \leq i \leq p,\ 1 \leq j \leq q\\ -\kappa(u_i, v_{j-p}) & 1 \leq i \leq p,\ p+1 \leq j \leq p+q\\ -\kappa(v_{i-p}, u_j) & p+1 \leq i \leq p+q,\ 1 \leq j \leq p\\ \kappa(v_{i-p}, v_{j-q}) & p+1 \leq i \leq p+q,\ p+1 \leq j \leq p+q. \end{cases}$$

We also find that

$$w = \frac{\displaystyle\sum_{i=1}^{p}\lambda_i\varphi(u_i) - \sum_{j=1}^{q}\mu_j\varphi(v_j)}{\left(\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} K \begin{pmatrix}\lambda\\ \mu\end{pmatrix}\right)^{1/2}}.$$