(2) $(E \otimes F) \otimes G \cong E \otimes (F \otimes G) \cong E \otimes F \otimes G$

(3) $(E \oplus F) \otimes G \cong (E \otimes G) \oplus (F \otimes G)$

(4) $K \otimes E \cong E$

such that respectively

(a) $u \otimes v \mapsto v \otimes u$
(b) $(u \otimes v) \otimes w \mapsto u \otimes (v \otimes w) \mapsto u \otimes v \otimes w$
(c) $(u, v) \otimes w \mapsto (u \otimes w, v \otimes w)$
(d) $\lambda \otimes u \mapsto \lambda u$.

Proof. Except for (3), these isomorphisms are proved using the universal mapping property of tensor products.

(1) The map from $E \times F$ to $F \otimes E$ given by $(u, v) \mapsto v \otimes u$ is clearly bilinear, thus it induces a unique linear $\alpha\colon E \otimes F \to F \otimes E$ making the following diagram commute

$$\begin{tikzcd} E \times F \arrow[r, "\iota_\otimes"] \arrow[dr] & E \otimes F \arrow[d, "\alpha"] \\ & F \otimes E, \end{tikzcd}$$

such that

$$\alpha(u \otimes v) = v \otimes u, \quad \text{for all } u \in E \text{ and all } v \in F.$$

Similarly, the map from $F \times E$ to $E \otimes F$ given by $(v, u) \mapsto u \otimes v$ is clearly bilinear, thus it induces a unique linear $\beta\colon F \otimes E \to E \otimes F$ making the following diagram commute

$$\begin{tikzcd} F \times E \arrow[r, "\iota_\otimes"] \arrow[dr] & F \otimes E \arrow[d, "\beta"] \\ & E \otimes F, \end{tikzcd}$$

such that

$$\beta(v \otimes u) = u \otimes v, \quad \text{for all } u \in E \text{ and all } v \in F.$$

It is immediately verified that

$$(\beta \circ \alpha)(u \otimes v) = u \otimes v \quad \text{and} \quad (\alpha \circ \beta)(v \otimes u) = v \otimes u$$

for all $u \in E$ and all $v \in F$. Since the tensors of the form $u \otimes v$ span $E \otimes F$ and similarly the tensors of the form $v \otimes u$ span $F \otimes E$, the map $\beta \circ \alpha$ is actually the identity on $E \otimes F$, and similarly $\alpha \circ \beta$ is the identity on $F \otimes E$, so α and β are isomorphisms.