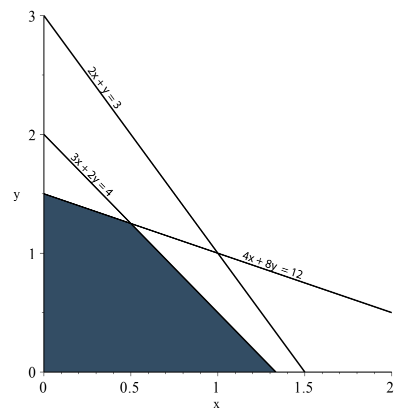

Figure 47.3: The $\mathcal{H}$-polytope for the linear program of Example 47.1. Note $x_1 \to x$ and $x_2 \to y$.

the action of y on the *columns* of A. This is the sense in which (D) is the *dual* (P). In most presentations, the fact that (P) and (D) perform a search for a solution in spaces that are dual to each other is obscured by excessive use of transposition.

To convert the Dual Program (D) to a standard maximization problem we change the objective function yb to $-b^\top y^\top$ and the inequality $yA \geq c$ to $-A^\top y^\top \leq -c^\top$. The Dual Linear Program (D) is now stated as (D')

$$\begin{aligned} &\text{maximize} && -b^\top y^\top \\ &\text{subject to} && -A^\top y^\top \leq -c^\top \;\text{ and }\; y^\top \geq 0, \end{aligned}$$

where $y \in (\mathbb{R}^m)^*$. Observe that the dual in maximization form (D'') of the Dual Program (D') gives back the Primal Program (P).

The above discussion established the following inequality known as *weak duality*.

Proposition 47.6. *(Weak Duality) Given any Linear Program (P)*

$$\begin{aligned} &\text{maximize} && cx \\ &\text{subject to} && Ax \leq b \;\text{ and }\; x \geq 0, \end{aligned}$$

with A an $m \times n$ matrix, for any feasible solution $x \in \mathbb{R}^n$ of the Primal Problem (P) and every feasible solution $y \in (\mathbb{R}^m)^*$ of the Dual Problem (D), we have

$$cx \leq yb.$$

Definition 47.3. We say that the Dual Linear Program (D) is *bounded below* if $\{yb \mid y^\top \in \mathcal{P}(-A^\top, -c^\top)\}$ is bounded below.