Every constraint $a_i x \geq b_i$ is replaced by the constraint $-a_i x \leq -b_i$. Every equality constraint $a_i x = b_i$ is replaced by the two constraints $a_i x \leq b_i$ and $-a_i x \leq -b_i$.

If there are n variables x_i, we create n new variables y_i and n new variables z_i and replace every variable x_i by $y_i - z_i$. We also add the $2n$ constraints $y_i \geq 0$ and $z_i \geq 0$. If the constraints are given by the inequalities $Ax \leq b$, we now have constraints given by

$$\begin{pmatrix} A & -A \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} \leq b, \quad y \geq 0,\ z \geq 0.$$

We replace the objective function cx by $cy - cz$.

Remark: We also showed that we can replace the inequality constraints $Ax \leq b$ by equality constraints $Ax = b$, by adding slack variables constrained to be nonnegative.

45.3 Summary

The main concepts and results of this chapter are listed below:

- Linear program.
- Objective function, constraints.
- Feasible solution.
- Bounded and unbounded linear programs.
- Optimal solution, optimum.
- Slack variables, linear program in standard form.
- Basic feasible solution.
- Basis of a variable.
- Basic, nonbasic index, basic, nonbasic variable.
- Vertex, face, edge, facet.

45.4 Problems

Problem 45.1. Convert the following program to standard form:

$$\begin{aligned} &\text{maximize} && x_1 + x_2 \\ &\text{subject to} && \\ & && x_2 - x_1 \leq 1 \\ & && x_1 + 6x_2 \leq 15 \\ & && -4x_1 + x_2 \geq 10. \end{aligned}$$