Remarks:

(1) If we use the vector $v + e^{-i\theta}u$ instead of $v - e^{-i\theta}u$, we get $s(u) = -e^{i\theta}v$.

(2) Certain authors, such as Kincaid and Cheney [102] and Ciarlet [41], use the vector $u + e^{i\theta}v$ instead of the vector $v + e^{-i\theta}u$. The effect of this choice is that they also get $s(u) = -e^{i\theta}v$.

(3) If $v = \|u\| e_1$, where e_1 is a basis vector, $u \cdot e_1 = a_1$, where a_1 is just the coefficient of u over the basis vector e_1. Then, since $u \cdot e_1 = e^{i\theta}|a_1|$, the choice of the plus sign in the vector $\|u\| e_1 + e^{-i\theta}u$ has the effect that the coefficient of this vector over e_1 is $\|u\| + |a_1|$, and no cancellations takes place, which is preferable for numerical stability (we need to divide by the square norm of this vector).

We now show that the QR-decomposition in terms of (complex) Householder matrices holds for complex matrices. We need the version of Proposition 14.19 and a trick at the end of the argument, but the proof is basically unchanged.

Proposition 14.20. *Let E be a nontrivial Hermitian space of dimension n. Given any orthonormal basis $(e_1, \ldots, e_n)$, for any n-tuple of vectors $(v_1, \ldots, v_n)$, there is a sequence of $n-1$ isometries $h_1, \ldots, h_{n-1}$, such that h_i is a (standard) hyperplane reflection or the identity, and if $(r_1, \ldots, r_n)$ are the vectors given by*

$$r_j = h_{n-1} \circ \cdots \circ h_2 \circ h_1(v_j), \quad 1 \leq j \leq n,$$

then every r_j is a linear combination of the vectors $(e_1, \ldots, e_j)$, $(1 \leq j \leq n)$. Equivalently, the matrix R whose columns are the components of the r_j over the basis $(e_1, \ldots, e_n)$ is an upper triangular matrix. Furthermore, if we allow one more isometry h_n of the form

$$h_n = \rho_{e_n, \varphi_n} \circ \cdots \circ \rho_{e_1, \varphi_1}$$

after $h_1, \ldots, h_{n-1}$, we can ensure that the diagonal entries of R are nonnegative.

Proof. The proof is very similar to the proof of Proposition 13.3, but it needs to be modified a little bit since Proposition 14.19 is weaker than Proposition 13.2. We explain how to modify the induction step, leaving the base case and the rest of the proof as an exercise.

As in the proof of Proposition 13.3, the vectors $(e_1, \ldots, e_k)$ form a basis for the subspace denoted as U_k', the vectors $(e_{k+1}, \ldots, e_n)$ form a basis for the subspace denoted as U_k'', the subspaces U_k' and U_k'' are orthogonal, and $E = U_k' \oplus U_k''$. Let

$$u_{k+1} = h_k \circ \cdots \circ h_2 \circ h_1(v_{k+1}).$$

We can write

$$u_{k+1} = u_{k+1}' + u_{k+1}'',$$