Geometrically, (λ, μ) corresponds to the coefficients of two convex combinations

$$\sum_{i=1}^{p} 2\lambda_i u_i = \sum_{j=1}^{q} 2\mu_j v_j$$

which correspond to the *same point* in the intersection of the convex hulls $\mathrm{conv}(u_1, \ldots, u_p)$ and $\mathrm{conv}(v_1, \ldots, v_q)$ iff the sets $\{u_i\}$ and $\{v_j\}$ are *not linearly separable*. If the sets $\{u_i\}$ and $\{v_j\}$ are *linearly separable*, then the convex hulls $\mathrm{conv}(u_1, \ldots, u_p)$ and $\mathrm{conv}(v_1, \ldots, v_q)$ are disjoint, which implies that $\gamma > 0$.

Let us now assume that $\gamma > 0$. Plugging back w from equation $(*_w)$ into the Lagrangian, after simplifications we get

$$\begin{aligned} G(\lambda, \mu, \alpha, \beta, \gamma) &= -\frac{1}{2\gamma} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + \frac{\gamma}{4\gamma^2} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} - \gamma \\ &= -\frac{1}{4\gamma} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} - \gamma, \end{aligned}$$

so if $\gamma > 0$ the dual function is independent of α, β and is given by

$$G(\lambda, \mu, \alpha, \beta, \gamma) = \begin{cases} -\frac{1}{4\gamma} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} - \gamma & \text{if} \begin{cases} \sum_{i=1}^p \lambda_i = \sum_{j=1}^q \mu_j = \frac{1}{2} \\ 0 \leq \lambda_i \leq K, \ i = 1, \ldots, p \\ 0 \leq \mu_j \leq K, \ j = 1, \ldots, q \end{cases} \\ -\infty & \text{otherwise.} \end{cases}$$

Since $X^\top X$ is symmetric positive semidefinite and $\gamma \geq 0$, obviously

$$G(\lambda, \mu, \alpha, \beta, \gamma) \leq 0$$

for all $\gamma > 0$.

The dual program is given by

$$\begin{aligned} &\text{maximize} \quad -\frac{1}{4\gamma} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} - \gamma \quad \text{if } \gamma > 0 \\ &\qquad\qquad\quad 0 \quad \text{if} \quad \gamma = 0 \\ &\text{subject to} \\ &\qquad \sum_{i=1}^{p} \lambda_i - \sum_{j=1}^{q} \mu_j = 0 \\ &\qquad \sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \mu_j = 1 \\ &\qquad 0 \leq \lambda_i \leq K, \quad i = 1, \ldots, p \\ &\qquad 0 \leq \mu_j \leq K, \quad j = 1, \ldots, q. \end{aligned}$$