The new ingredient in the *relaxation method* is to incorporate part of the matrix D into N: we define M and N by

$$M = \frac{D}{\omega} - E$$

$$N = \frac{1-\omega}{\omega} D + F,$$

where $\omega \neq 0$ is a real parameter to be suitably chosen. Actually, we show in Section 10.4 that for the relaxation method to converge, we must have $\omega \in (0, 2)$. Note that the case $\omega = 1$ corresponds to the method of Gauss–Seidel.

If we assume that *all* diagonal entries of D are nonzero, the matrix M is invertible. The matrix B is denoted by $\mathcal{L}_\omega$ and called the *matrix of relaxation*, with

$$\mathcal{L}_\omega = \left(\frac{D}{\omega} - E\right)^{-1} \left(\frac{1-\omega}{\omega} D + F\right) = (D - \omega E)^{-1}((1-\omega)D + \omega F).$$

The number ω is called the *parameter of relaxation*.

When $\omega > 1$, the relaxation method is known as *successive overrelaxation*, abbreviated as *SOR*.

At first glance the relaxation matrix $\mathcal{L}_\omega$ seems a lot more complicated than the Gauss–Seidel matrix $\mathcal{L}_1$, but the iterative system associated with the relaxation method is very similar to the method of Gauss–Seidel, and is quite simple. Indeed, the system associated with the relaxation method is given by

$$\left(\frac{D}{\omega} - E\right) u_{k+1} = \left(\frac{1-\omega}{\omega} D + F\right) u_k + b,$$

which is equivalent to

$$(D - \omega E) u_{k+1} = ((1-\omega)D + \omega F) u_k + \omega b,$$

and can be written

$$D u_{k+1} = D u_k - \omega (D u_k - E u_{k+1} - F u_k - b).$$

Explicitly, this is the system

$$\begin{aligned}
a_{11} u_1^{k+1} &= a_{11} u_1^k - \omega(a_{11} u_1^k + a_{12} u_2^k + a_{13} u_3^k + \cdots + a_{1\,n-2} u_{n-2}^k + a_{1\,n-1} u_{n-1}^k + a_{1\,n} u_n^k - b_1) \\
a_{22} u_2^{k+1} &= a_{22} u_2^k - \omega(a_{21} u_1^{k+1} + a_{22} u_2^k + a_{23} u_3^k + \cdots + a_{2\,n-2} u_{n-2}^k + a_{2\,n-1} u_{n-1}^k + a_{2\,n} u_n^k - b_2) \\
&\vdots \\
a_{n\,n} u_n^{k+1} &= a_{n\,n} u_n^k - \omega(a_{n\,1} u_1^{k+1} + a_{n\,2} u_2^{k+1} + \cdots + a_{n\,n-2} u_{n-2}^{k+1} + a_{n\,n-1} u_{n-1}^{k+1} + a_{n\,n} u_n^k - b_n).
\end{aligned}$$

In `Matlab` one step of relaxation iteration is achieved by the following function: