(4) Consider the dual space E^* of E, and let $(e_i^*)_{i\in\mathbb{N}}$ be the family of dual forms of the basis $(e_i)_{i\in N}$. Check that the family $(e_i^*)_{i\in\mathbb{N}}$ is linearly independent.

(5) Let $f \in E^*$ be the linear form defined by

$$f(e_i) = 1 \quad \text{for all } i \in \mathbb{N}.$$

Prove that f is not in the subspace spanned by the e_i^*. If F is the subspace of E^* spanned by the e_i^* and f, find F^0 and F^{00}, and prove that

$$F \neq F^{00}.$$