3. The cosets of $\mathbf{SO}(n)$ in $\mathbf{GL}^+(n, \mathbb{R})$ are the sets of matrices
$$A\,\mathbf{SO}(n) = \{AQ \mid Q \in \mathbf{SO}(n)\},\ A \in \mathbf{GL}^+(n, \mathbb{R}).$$
It can be shown (using the polar form for matrices) that there is a bijection between the cosets of $\mathbf{SO}(n)$ in $\mathbf{GL}^+(n, \mathbb{R})$ and the set of $n \times n$ symmetric, positive, definite matrices; these are the symmetric matrices whose eigenvalues are strictly positive.

4. The cosets of $\mathbf{SO}(2)$ in $\mathbf{SO}(3)$ are the sets of matrices
$$Q\,\mathbf{SO}(2) = \{QR \mid R \in \mathbf{SO}(2)\},\ Q \in \mathbf{SO}(3).$$
The group $\mathbf{SO}(3)$ moves the points on the sphere S^2 in $\mathbb{R}^3$, namely for any $x \in S^2$,
$$x \mapsto Qx \quad \text{for any rotation } Q \in \mathbf{SO}(3).$$
Here,
$$S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}.$$
Let $N = (0, 0, 1)$ be the north pole on the sphere S^2. Then it is not hard to show that $\mathbf{SO}(2)$ is precisely the subgroup of $\mathbf{SO}(3)$ that leaves N fixed. As a consequence, all rotations QR in the coset $Q\,\mathbf{SO}(2)$ map N to the same point $QN \in S^2$, and it can be shown that there is a bijection between the cosets of $\mathbf{SO}(2)$ in $\mathbf{SO}(3)$ and the points on S^2. The surjectivity of this map has to do with the fact that the action of $\mathbf{SO}(3)$ on S^2 is transitive, which means that for any point $x \in S^2$, there is some rotation $Q \in \mathbf{SO}(3)$ such that $QN = x$.

It is tempting to define a multiplication operation on left cosets (or right cosets) by setting
$$(g_1H)(g_2H) = (g_1g_2)H,$$
but this operation is not well defined in general, unless the subgroup H possesses a special property. In Example 2.3, it is possible to define multiplication of cosets in (1), but it is not possible in (2) and (3).

The property of the subgroup H that allows defining a multiplication operation on left cosets is typical of the kernels of group homomorphisms, so we are led to the following definition.

Definition 2.7. Given any two groups G and G', a function $\varphi\colon G \to G'$ is a *homomorphism* iff
$$\varphi(g_1g_2) = \varphi(g_1)\varphi(g_2), \quad \text{for all } g_1, g_2 \in G.$$

Taking $g_1 = g_2 = e$ (in G), we see that
$$\varphi(e) = e',$$
and taking $g_1 = g$ and $g_2 = g^{-1}$, we see that
$$\varphi(g^{-1}) = (\varphi(g))^{-1}.$$