Even though the matrix D is an $m \times n$ rectangular matrix, since its only nonzero entries are on the descending diagonal, we still say that D is a diagonal matrix.

The `Matlab` command for computing an SVD $A = VDU^\top$ of a matrix A is `[V, D, U] = svd(A)`. Beware that `Matlab` uses the convention that the SVD of a matrix A is written as $A = UDV^\top$, and so the call for this version of the SVD is `[U, D, V] = svd(A)`.

If we view A as the representation of a linear map $f\colon E \to F$, where $\dim(E) = n$ and $\dim(F) = m$, the proof of Theorem 22.7 shows that there are two orthonormal bases $(u_1, \ldots, u_n)$ and $(v_1, \ldots, v_m)$ for E and F, respectively, where $(u_1, \ldots, u_n)$ are eigenvectors of $f^* \circ f$ and $(v_1, \ldots, v_m)$ are eigenvectors of $f \circ f^*$. Furthermore, $(u_1, \ldots, u_r)$ is an orthonormal basis of $\mathrm{Im}\, f^*$, $(u_{r+1}, \ldots, u_n)$ is an orthonormal basis of $\mathrm{Ker}\, f$, $(v_1, \ldots, v_r)$ is an orthonormal basis of $\mathrm{Im}\, f$, and $(v_{r+1}, \ldots, v_m)$ is an orthonormal basis of $\mathrm{Ker}\, f^*$.

The SVD of matrices can be used to define the pseudo-inverse of a rectangular matrix; we will do so in Chapter 23. The reader may also consult Strang [170], Demmel [48], Trefethen and Bau [176], and Golub and Van Loan [80].

One of the spectral theorems states that a symmetric matrix can be diagonalized by an orthogonal matrix. There are several numerical methods to compute the eigenvalues of a symmetric matrix A. One method consists in *tridiagonalizing* A, which means that there exists some orthogonal matrix P and some symmetric tridiagonal matrix T such that $A = PTP^\top$. In fact, this can be done using Householder transformations; see Theorem 18.2. It is then possible to compute the eigenvalues of T using a bisection method based on Sturm sequences. One can also use Jacobi's method. For details, see Golub and Van Loan [80], Chapter 8, Demmel [48], Trefethen and Bau [176], Lecture 26, Ciarlet [41], and Chapter 18. Computing the SVD of a matrix A is more involved. Most methods begin by finding orthogonal matrices U and V and a *bidiagonal* matrix B such that $A = VBU^\top$; see Problem 13.8 and Problem 22.3. This can also be done using Householder transformations. Observe that $B^\top B$ is symmetric tridiagonal. Thus, in principle, the previous method to diagonalize a symmetric tridiagonal matrix can be applied. However, it is unwise to compute $B^\top B$ explicitly, and more subtle methods are used for this last step; the matrix of Problem 22.1 can be used, and see Problem 22.3. Again, see Golub and Van Loan [80], Chapter 8, Demmel [48], and Trefethen and Bau [176], Lecture 31.

The polar form has applications in continuum mechanics. Indeed, in any deformation it is important to separate stretching from rotation. This is exactly what QS achieves. The orthogonal part Q corresponds to rotation (perhaps with an additional reflection), and the symmetric matrix S to stretching (or compression). The real eigenvalues $\sigma_1, \ldots, \sigma_r$ of S are the stretch factors (or compression factors) (see Marsden and Hughes [120]). The fact that S can be diagonalized by an orthogonal matrix corresponds to a natural choice of axes, the principal axes.

The SVD has applications to data compression, for instance in image processing. The idea is to retain only singular values whose magnitudes are significant enough. The SVD