positive semidefinite. Thus we are in a position to apply ADMM since the constraints are

$$\begin{aligned}\sum_{i=1}^m \lambda_i - \sum_{i=1}^m \mu_i &= 0\\ \sum_{i=1}^m \lambda_i + \sum_{i=1}^m \mu_i + \gamma &= C\nu\\ \lambda + \alpha = \frac{C}{m}, \quad \mu + \beta &= \frac{C}{m},\end{aligned}$$

namely affine. We need to check that the $(2m+2)\times(4m+1)$ matrix A corresponding to this system has rank $2m+2$. Let us clarify this point. The matrix A corresponding to the above equations is

$$A = \begin{pmatrix} \mathbf{1}_m^\top & -\mathbf{1}_m^\top & 0_m^\top & 0_m^\top & 0\\ \mathbf{1}_m^\top & \mathbf{1}_m^\top & 0_m^\top & 0_m^\top & 1\\ I_m & 0_{m,m} & I_m & 0_{m,m} & 0_m\\ 0_{m,m} & I_m & 0_{m,m} & I_m & 0_m \end{pmatrix}.$$

For example, for $m = 3$ we have the 8×13 matrix

$$\begin{pmatrix} 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

We leave it as an exercise to show that A has rank $2m+2$. Recall that

$$q = \begin{pmatrix} y\\ -y \end{pmatrix}$$

and we also define the vector c (of dimension $2m+2$) as

$$c = \begin{pmatrix} 0\\ C\nu\\ \frac{C}{m}\mathbf{1}_{2m} \end{pmatrix}.$$

The constraints are given by the system of affine equations $Ax = c$, where

$$x = \begin{pmatrix} \lambda^\top & \mu^\top & \alpha^\top & \beta^\top & \gamma \end{pmatrix}^\top.$$