(1) Note that $\mathrm{Ker}\left(\overrightarrow{f} - \mathrm{id}\right) = \{0\}$ iff $\mathrm{Fix}(g)$ consists of a single element, which is the unique fixed point of f. However, even if f is not a translation, f may not have any fixed points. For example, this happens when E is the affine Euclidean plane and f is the composition of a reflection about a line composed with a nontrivial translation parallel to this line.

(2) The fact that E has finite dimension is used only to prove (b).

(3) It is easily checked that $\mathrm{Fix}(g)$ consists of the set of points x such that $\|\overrightarrow{xf(x)}\|$ is minimal.

In the affine Euclidean plane it is easy to see that the affine isometries (besides the identity) are classified as follows. An affine isometry f that has a fixed point is a rotation if it is a direct isometry; otherwise, it is an affine reflection about a line. If f has no fixed point, then it is either a nontrivial translation or the composition of an affine reflection about a line with a nontrivial translation parallel to this line.

In an affine space of dimension 3 it is easy to see that the affine isometries (besides the identity) are classified as follows. There are three kinds of affine isometries that have a fixed point. A proper affine isometry with a fixed point is a rotation around a line D (its set of fixed points), as illustrated in Figure 27.9.

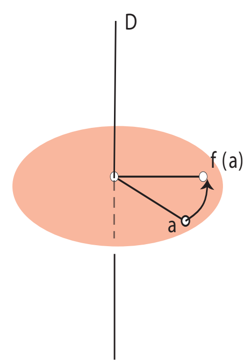

Figure 27.9: 3D proper affine rigid motion with line D of fixed points (rotation).

An improper affine isometry with a fixed point is either an affine reflection about a plane H (the set of fixed points) or the composition of a rotation followed by an affine reflection about a plane H orthogonal to the axis of rotation D, as illustrated in Figures 27.10 and 27.11. In the second case, there is a single fixed point $O = D \cap H$.

There are three types of affine isometries with no fixed point. The first kind is a nontrivial translation. The second kind is the composition of a rotation followed by a nontrivial translation parallel to the axis of rotation D. Such an affine rigid motion is proper, and is called a *screw motion*. A screw motion is illustrated in Figure 27.12.