can also be written as

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

which shows that it is the composition of a rotation of angle $\pi/3$, followed by a stretch (by a factor of 2 along the x-axis, and by a factor of $\frac{1}{2}$ along the y-axis), followed by a translation. It is easy to show that this affine map has a unique fixed point. On the other hand, the affine map

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} 8/5 & -6/5 \\ 3/10 & 2/5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

has no fixed point, even though

$$\begin{pmatrix} 8/5 & -6/5 \\ 3/10 & 2/5 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 4/5 & -3/5 \\ 3/5 & 4/5 \end{pmatrix},$$

and the second matrix is a rotation of angle θ such that $\cos\theta = \frac{4}{5}$ and $\sin\theta = \frac{3}{5}$.

There is a useful trick to convert the equation $y = Ax + b$ into what looks like a linear equation. The trick is to consider an $(n+1) \times (n+1)$ matrix. We add 1 as the $(n+1)$th component to the vectors x, y, and b, and form the $(n+1) \times (n+1)$ matrix

$$\begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix}$$

so that $y = Ax + b$ is equivalent to

$$\begin{pmatrix} y \\ 1 \end{pmatrix} = \begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ 1 \end{pmatrix}.$$

This trick is very useful in kinematics and dynamics, where A is a rotation matrix. Such affine maps are called *rigid motions*.

If $f\colon E \to E'$ is a bijective affine map, given any three collinear points a, b, c in E, with $a \neq b$, where, say, $c = (1-\lambda)a + \lambda b$, since f preserves barycenters, we have $f(c) = (1-\lambda)f(a) + \lambda f(b)$, which shows that $f(a), f(b), f(c)$ are collinear in E'. There is a converse to this property, which is simpler to state when the ground field is $K = \mathbb{R}$. The converse states that given any bijective function $f\colon E \to E'$ between two real affine spaces of the same dimension $n \geq 2$, if f maps any three collinear points to collinear points, then f is affine. The proof is rather long (see Berger [11] or Samuel [142]).

Given three collinear points a, b, c, where $a \neq c$, we have $b = (1-\beta)a + \beta c$ for some unique β, and we define the *ratio of the sequence* a, b, c, as

$$\text{ratio}(a, b, c) = \frac{\beta}{(1-\beta)} = \frac{\overrightarrow{ab}}{\overrightarrow{bc}},$$