- Properties of the eigenvalues and eigenvectors of a normal linear map.
- The *complexification* of a real vector space, of a linear map, and of a Euclidean inner product.
- The eigenvalues of a self-adjoint map in a Hermitian space are *real*.
- The eigenvalues of a self-adjoint map in a Euclidean space are *real*.
- Every self-adjoint linear map on a Euclidean space has an orthonormal basis of eigenvectors.
- Every normal linear map on a Euclidean space can be block diagonalized (blocks of size at most 2×2) with respect to an orthonormal basis of eigenvectors.
- Every normal linear map on a Hermitian space can be diagonalized with respect to an orthonormal basis of eigenvectors.
- The spectral theorems for self-adjoint, skew-self-adjoint, and orthogonal linear maps (on a Euclidean space).
- The spectral theorems for normal, symmetric, skew-symmetric, and orthogonal (real) matrices.
- The spectral theorems for normal, Hermitian, skew-Hermitian, and unitary (complex) matrices.
- The *Rayleigh ratio* and the *Rayleigh–Ritz theorem*.
- *Interlacing inequalities* and the *Cauchy interlacing theorem*.
- The *Poincaré separation theorem*.
- The *Courant–Fischer theorem*.
- Inequalities involving perturbations of the eigenvalues of a symmetric matrix.
- The *Weyl inequalities*.

17.9 Problems

Problem 17.1. Prove that the structure $E_{\mathbb{C}}$ introduced in Definition 17.2 is indeed a complex vector space.