

the dual of the reformulated problem can be expressed as

$$\begin{aligned} & \text{maximize} && b^\top \mu - \log \left(\sum_{i=1}^n \mu_i \log \mu_i \right) \\ & \text{subject to} && \mathbf{1}^\top \mu = 1 \\ & && A^\top \mu = 0 \\ & && \mu \geq 0, \end{aligned}$$

an entropy maximization problem.

Example 50.13. Similarly the unconstrained norm minimization problem

$$\text{minimize} \quad \|Ax - b\|,$$

where $\|\cdot\|$ is any norm on \mathbb{R}^m , has a dual function which is a constant, and is not useful. This problem can be reformulated as

$$\begin{aligned} & \text{minimize} && \|y\| \\ & \text{subject to} && Ax - b = y. \end{aligned}$$

By Example 50.8(6), the conjugate of the norm is given by

$$\|y\|^* = \begin{cases} 0 & \text{if } \|y\|^D \leq 1 \\ +\infty & \text{otherwise,} \end{cases}$$

so the dual of the reformulated program is:

$$\begin{aligned} & \text{maximize} && b^\top \mu \\ & \text{subject to} && \|\mu\|^D \leq 1 \\ & && A^\top \mu = 0. \end{aligned}$$

Here is now an example of (2), replacing the objective function with an increasing function of the the original function.

Example 50.14. The norm minimization of Example 50.13 can be reformulated as

$$\begin{aligned} & \text{minimize} && \frac{1}{2} \|y\|^2 \\ & \text{subject to} && Ax - b = y. \end{aligned}$$