Let us go back to the linear program of Example 46.1 with objective function x_2 and where the matrix A and the vector b are given by

$$A = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 2 \end{pmatrix}.$$

Recall that $u_0 = (0, 0, 0, 2)$ is a degenerate basic feasible solution, and the objective function has the value 0. See Figure 46.2 for a planar picture of the $\mathcal{H}$-polyhedron associated with Example 46.1.

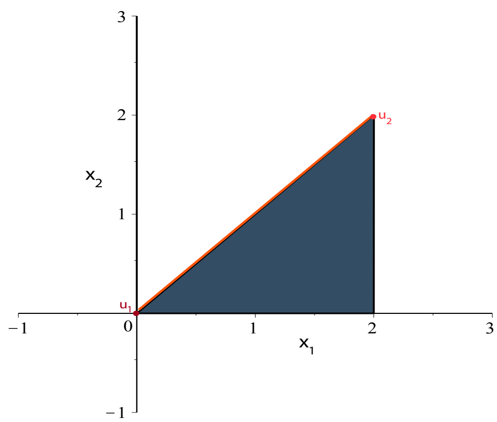

Figure 46.2: The planar $\mathcal{H}$-polyhedron associated with Example 46.1. The initial basic feasible solution is the origin. The simplex algorithm moves along the slanted orange line to the apex of the triangle.

Pick the basis (A^3, A^4). Then we have

$$\begin{aligned} A^1 &= -A^3 + A^4 \\ A^2 &= A^3, \end{aligned}$$

and we get

$$\begin{aligned} b &= 2A^4 - \theta A^1 + \theta A^1 \\ &= 2A^4 - \theta(-A^3 + A^4) + \theta A^1 \\ &= \theta A^1 + \theta A^3 + (2 - \theta)A^4, \end{aligned}$$

and

$$\begin{aligned} b &= 2A^4 - \theta A^2 + \theta A^2 \\ &= 2A^4 - \theta A^3 + \theta A^2 \\ &= \theta A^2 - \theta A^3 + 2A^4. \end{aligned}$$