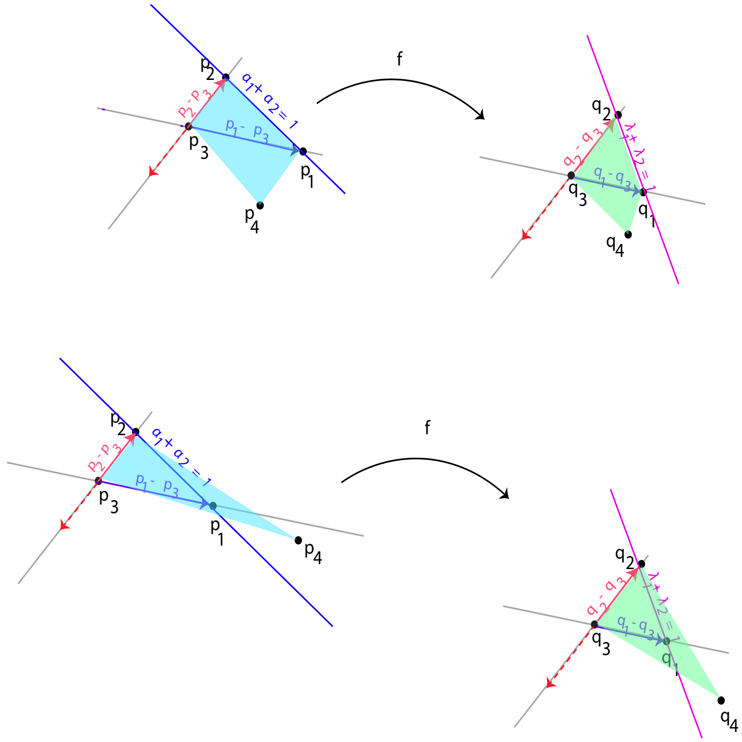

Figure 26.16: Case (3)

reduces to proving that if a projective transformation given by an invertible matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

does not have points at infinity on the line segment in $\mathbb{R}^2$ corresponding to the points of coordinates $(x, 1)$ with $0 \leq x \leq 1$, then the image of the line segment $[(0,1),(1,1)]$ is the line segment $[(b/d, 1), ((a+b)/(c+d), 1)]$ (or $[((a+b)/(c+d), 1), (b/d, 1)]$).

We have

$$\begin{aligned} \frac{ax+b}{cx+d} - \frac{b}{d} &= \frac{adx + bd - bcx - bd}{d(cx+d)} \\ &= \frac{(ad-bc)x}{d(cx+d)} \end{aligned}$$

and

$$\begin{aligned} \frac{ax+b}{cx+d} - \frac{a+b}{c+d} &= \frac{acx + bc + adx + bd - acx - ad - bcx - bd}{(c+d)(cx+d)} \\ &= \frac{(ad-bc)(x-1)}{(c+d)(cx+d)} \end{aligned}$$