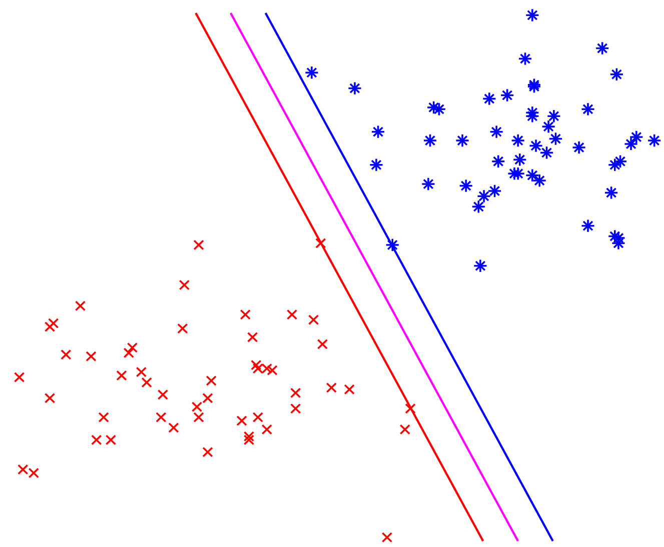

Figure 52.5: An example of hard margin SVM.

which is equivalent to minimizing

$$h(x) = x^2 + 2(c|x| - xv)$$

over x. If $x \geq 0$, then

$$h(x) = x^2 + 2(cx - xv) = x^2 + 2(c - v)x = (x - (v - c))^2 - (v - c)^2.$$

If $v - c > 0$, that is, $v > c$, since $x \geq 0$, the function $x \mapsto (x - (v - c))^2$ has a minimum for $x = v - c > 0$, else if $v - c \leq 0$, then the function $x \mapsto (x - (v - c))^2$ has a minimum for $x = 0$.

If $x \leq 0$, then

$$h(x) = x^2 + 2(-cx - xv) = x^2 - 2(c + v)x = (x - (v + c))^2 - (v + c)^2.$$

if $v + c < 0$, that is, $v < -c$, since $x \leq 0$, the function $x \mapsto (x - (v + c))^2$ has a minimum for $x = v + c$, else if $v + c \geq 0$, then the function $x \mapsto (x - (v + c))^2$ has a minimum for $x = 0$.

In summary, $\inf_x h(x)$ is the function of v given by

$$S_c(v) = \begin{cases} v - c & \text{if } v > c \\ 0 & \text{if } |v| \leq c \\ v + c & \text{if } v < -c. \end{cases}$$

The function S_c is known as a *soft thresholding operator*. The graph of S_c shown in Figure 52.6.