Newton's method requires that the KKT-matrix be invertible. Under some mild assumptions, Newton's method (with feasible start) converges; see Boyd and Vandenberghe [29] (Chapter 10, Section 10.2.4).

We now give an example illustrating Proposition 50.7, the *Support Vector Machine* (abbreviated as *SVM*).

50.5 Hard Margin Support Vector Machine; Version I

In this section we describe the following *classification problem*, or perhaps more accurately, *separation problem* (into two classes). Suppose we have two nonempty disjoint finite sets of p *blue* points $\{u_i\}_{i=1}^p$ and q *red* points $\{v_j\}_{j=1}^q$ in $\mathbb{R}^n$ (for simplicity, you may assume that these points are in the plane, that is, $n = 2$). Our goal is to find a hyperplane H of equation $w^\top x - b = 0$ (where $w \in \mathbb{R}^n$ is a nonzero vector and $b \in \mathbb{R}$), such that all the blue points u_i are in one of the two open half-spaces determined by H, and all the red points v_j are in the other open half-space determined by H; see Figure 50.11.

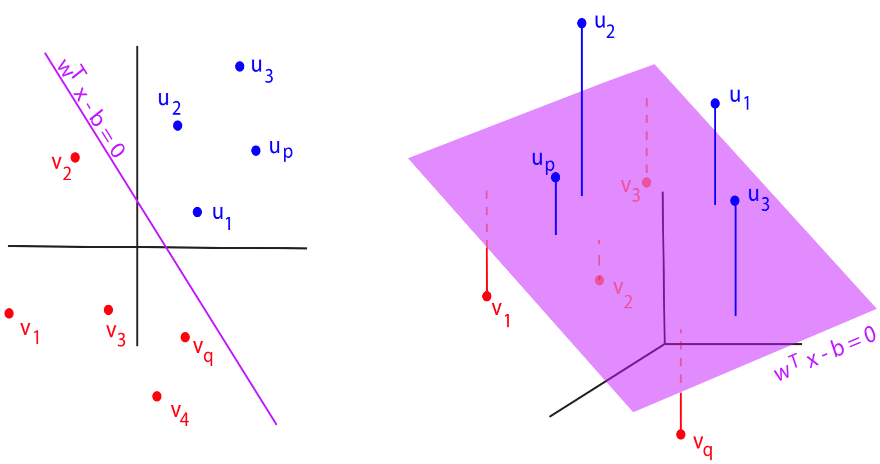

Figure 50.11: Two examples of the SVM separation problem. The left figure is SVM in $\mathbb{R}^2$, while the right figure is SVM in $\mathbb{R}^3$.

Without loss of generality, we may assume that

$$\begin{aligned} w^\top u_i - b &> 0 && \text{for } i = 1, \ldots, p \\ w^\top v_j - b &< 0 && \text{for } j = 1, \ldots, q. \end{aligned}$$

Of course, separating the blue and the red points may be impossible, as we see in Figure 50.12 for four points where the line segments (u_1, u_2) and (v_1, v_2) intersect. If a hyperplane separating the two subsets of blue and red points exists, we say that they are *linearly separable*.