

Problem 3.6. Let A_2 be the following matrix:

$$A_2 = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 2 & 3 \\ -1 & 0 & 1 & -1 \\ -2 & -1 & 3 & 0 \end{pmatrix}.$$

Express the fourth column of A_2 as a linear combination of the first three columns of A_2 . Is the vector $x = (7, 14, -1, 2)$ a linear combination of the columns of A_2 ?

Problem 3.7. Let A_3 be the following matrix:

$$A_3 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}.$$

Prove that the columns of A_3 are linearly independent. Find the coordinates of the vector $x = (6, 9, 14)$ over the basis consisting of the column vectors of A_3 .

Problem 3.8. Let A_4 be the following matrix:

$$A_4 = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 2 & 3 \\ -1 & 0 & 1 & -1 \\ -2 & -1 & 4 & 0 \end{pmatrix}.$$

Prove that the columns of A_4 are linearly independent. Find the coordinates of the vector $x = (7, 14, -1, 2)$ over the basis consisting of the column vectors of A_4 .

Problem 3.9. Consider the following Haar matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{pmatrix}.$$

Prove that the columns of H are linearly independent.

Hint. Compute the product $H^T H$.

Problem 3.10. Consider the following Hadamard matrix

$$H_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

Prove that the columns of H_4 are linearly independent.

Hint. Compute the product $H_4^T H_4$.