we have $x_1 - x_0 = -A_0^{-1}(x_0)(f(x_0))$, so by (1) and (3) and since $0 < \beta < 1$, we have

$$\|x_1 - x_0\| \leq M \|f(x_0)\| \leq r(1-\beta) \leq r,$$

establishing (a) and (b) for $k = 1$. We also have $f(x_0) = -A_0(x_0)(x_1 - x_0)$, so $-f(x_0) - A_0(x_0)(x_1 - x_0) = 0$ and thus

$$f(x_1) = f(x_1) - f(x_0) - A_0(x_0)(x_1 - x_0).$$

By the mean value theorem (Proposition 39.12) applied to the function $x \mapsto f(x) - A_0(x_0)(x)$, by (2), we get

$$\|f(x_1)\| \leq \sup_{x \in B} \|f'(x) - A_0(x_0)\| \|x_1 - x_0\| \leq \frac{\beta}{M} \|x_1 - x_0\|,$$

which is (c) for $k = 1$. We now establish the induction step.

Since by definition

$$x_k - x_{k-1} = -A_{k-1}^{-1}(x_\ell)(f(x_{k-1})), \quad 0 \leq \ell \leq k-1,$$

by (1) and the fact that by the induction hypothesis for (b), $x_\ell \in B$, we get

$$\|x_k - x_{k-1}\| \leq M \|f(x_{k-1})\|,$$

which proves (a) for k. As a consequence, since by the induction hypothesis for (c),

$$\|f(x_{k-1})\| \leq \frac{\beta}{M} \|x_{k-1} - x_{k-2}\|,$$

we get

$$\|x_k - x_{k-1}\| \leq M \|f(x_{k-1})\| \leq \beta \|x_{k-1} - x_{k-2}\|, \tag{$*_1$}$$

and by repeating this step,

$$\|x_k - x_{k-1}\| \leq \beta^{k-1} \|x_1 - x_0\|. \tag{$*_2$}$$

Using ($*_2$) and (3), we obtain

$$\begin{aligned}\|x_k - x_0\| &\leq \sum_{j=1}^{k} \|x_j - x_{j-1}\| \leq \left(\sum_{j=1}^{k} \beta^{j-1}\right) \|x_1 - x_0\| \\ &\leq \frac{\|x_1 - x_0\|}{1-\beta} \leq \frac{M}{1-\beta} \|f(x_0)\| \leq r,\end{aligned}$$

which proves that $x_k \in B$, which is (b) for k.

Since

$$x_k - x_{k-1} = -A_{k-1}^{-1}(x_\ell)(f(x_{k-1}))$$