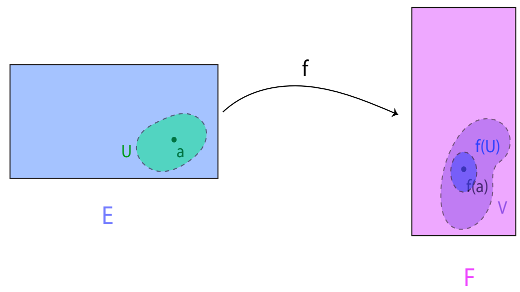

Figure 37.16: A schematic illustration of Definition 37.16.

containing a, and $f^{-1}(N)$ is a neighborhood of a. Conversely, if $f^{-1}(N)$ is a neighborhood of a whenever N is any neighborhood of $f(a)$, it is immediate that f is continuous at a. See Figure 37.17.

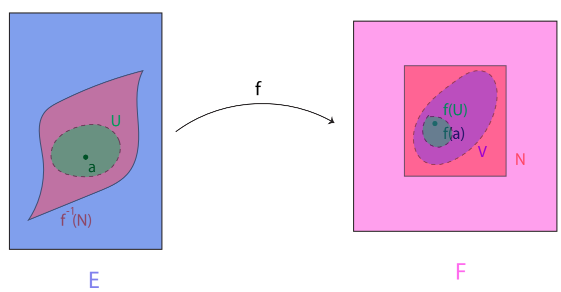

Figure 37.17: A schematic illustration of the neighborhood condition.

It is easy to see that Definition 37.16 is equivalent to the following statements.

Proposition 37.9. *Let $(E, \mathcal{O}_E)$ and $(F, \mathcal{O}_F)$ be topological spaces, and let $f\colon E \to F$ be a function. For every $a \in E$, the function f is continuous at $a \in E$ iff for every neighborhood N of $f(a) \in F$, then $f^{-1}(N)$ is a neighborhood of a. The function f is continuous on E iff $f^{-1}(V)$ is an open set in $\mathcal{O}_E$ for every open set $V \in \mathcal{O}_F$.*

If E and F are metric spaces defined by metrics d_E and d_F, we can show easily that f is continuous at a iff

for every $\epsilon > 0$, there is some $\eta > 0$, such that, for every $x \in E$,

$$\text{if } d_E(a, x) \leq \eta, \text{ then } d_F(f(a), f(x)) \leq \epsilon.$$

Similarly, if E and F are normed vector spaces defined by norms $\| \|_E$ and $\| \|_F$, we can show easily that f is continuous at a iff