(4) $\mu_i = C/m$. By $(\lambda\mu)$, $\lambda_i = 0$, and by $(*)$, $\xi_i = 0$. Thus we have

$$\begin{aligned} w^\top x_i + b - y_i &\leq \epsilon \\ -w^\top x_i - b + y_i &= \epsilon + \xi_i'. \end{aligned}$$

The second equation is equivalent to $w^\top x_i + b - y_i = -\epsilon - \xi_i'$, and since $\epsilon > 0$ and $\xi_i' \geq 0$, the first inequality it is trivially satisfied. If $\xi_i' = 0$, then x_i is on the red margin $H_{w,b+\epsilon}$, else x_i is an error and it lies in the open half-space bounded by the red margin $H_{w,b-\epsilon}$ and not containing the best fit hyperplane $H_{w,b}$ (it is outside of the ϵ-slab). See Figure 56.5.

(5) $\lambda_i = 0$ and $\mu_i = 0$. By $(*)$, $\xi_i = 0$ and $\xi_i' = 0$, so we have

$$\begin{aligned} w^\top x_i + b - y_i &\leq \epsilon \\ -w^\top x_i - b + y_i &\leq \epsilon, \end{aligned}$$

that is

$$-\epsilon \leq w^\top x_i + b - y_i \leq \epsilon.$$

If $w^\top x_i + b - y_i = \epsilon$, then x_i is on the blue margin, and if $w^\top x_i + b - y_i = -\epsilon$, then x_i is on the red margin. If $-\epsilon < w^\top x_i + b - y_i < \epsilon$, then x_i is strictly inside of the ϵ-slab (bounded by the blue margin and the red margin). See Figure 56.6.

The above classification shows that the point x_i is an error iff $\lambda_i = C/m$ and $\xi_i > 0$ or or $\mu_i = C/m$ and $\xi_i' > 0$.

As in the case of SVM (see Section 50.6) we define support vectors as follows.

Definition 56.1. A vector x_i such that either $w^\top x_i + b - y_i = \epsilon$ (which implies $\xi_i = 0$) or $-w^\top x_i - b + y_i = \epsilon$ (which implies $\xi_i' = 0$) is called a *support vector*. Support vectors x_i such that $0 < \lambda_i < C/m$ and support vectors x_j such that $0 < \mu_j < C/m$ are *support vectors of type 1*. Support vectors of type 1 play a special role so we denote the sets of indices associated with them by

$$\begin{aligned} I_\lambda &= \{i \in \{1, \ldots, m\} \mid 0 < \lambda_i < C/m\} \\ I_\mu &= \{j \in \{1, \ldots, m\} \mid 0 < \mu_j < C/m\}. \end{aligned}$$

We denote their cardinalities by $numsvl_1 = |I_\lambda|$ and $numsvm_1 = |I_\mu|$. Support vectors x_i such that $\lambda_i = C/m$ and support vectors x_j such that $\mu_j = C/m$ are *support vectors of type 2*. Support vectors for which $\lambda_i = \mu_i = 0$ are called *exceptional support vectors*.

The following definition also gives a useful classification criterion.

Definition 56.2. A point x_i such that either $\lambda_i = C/m$ or $\mu_i = C/m$ is said to *fail the margin*. The sets of indices associated with the vectors failing the margin are denoted by

$$\begin{aligned} K_\lambda &= \{i \in \{1, \ldots, m\} \mid \lambda_i = C/m\} \\ K_\mu &= \{j \in \{1, \ldots, m\} \mid \mu_j = C/m\}. \end{aligned}$$