

In the first case, the point $(3, \theta, 4 - \theta, 0, 2 - \theta)$ is a feasible solution iff $0 \leq \theta \leq 2$, and the new value of the objective function is $3 + \theta$. In the second case, the point $(3 - \theta, 0, 4 - \theta, \theta, 2)$ is a feasible solution iff $0 \leq \theta \leq 3$, and the new value of the objective function is $3 - \theta$. To increase the objective function, we must choose the first case and we pick $\theta = 2$. Then we get the feasible solution $u_2 = (3, 2, 2, 0, 0)$, which corresponds to the basis (A^1, A^2, A^3) , and thus is a basic feasible solution. The new value of the objective function is 5.

Next we express A^4 and A^5 in terms of the basis (A^1, A^2, A^3) . Again this is easy to do since we just swapped A^5 and A^2 (a pivoting step), and we get

$$\begin{aligned} A^5 &= A^2 - A^3 \\ A^4 &= A^1 + A^3. \end{aligned}$$

We repeat the process with $u_2 = (3, 2, 2, 0, 0)$ and the basis (A^1, A^2, A^3) . We have

$$\begin{aligned} b &= 3A^1 + 2A^2 + 2A^3 - \theta A^4 + \theta A^5 \\ &= 3A^1 + 2A^2 + 2A^3 - \theta(A^1 + A^3) + \theta A^4 \\ &= (3 - \theta)A^1 + 2A^2 + (2 - \theta)A^3 + \theta A^4, \end{aligned}$$

and

$$\begin{aligned} b &= 3A^1 + 2A^2 + 2A^3 - \theta A^5 + \theta A^5 \\ &= 3A^1 + 2A^2 + 2A^3 - \theta(A^2 - A^3) + \theta A^5 \\ &= 3A^1 + (2 - \theta)A^2 + (2 + \theta)A^3 + \theta A^5. \end{aligned}$$

In the first case, the point $(3 - \theta, 2, 2 - \theta, \theta, 0)$ is a feasible solution iff $0 \leq \theta \leq 2$, and the value of the objective function is $5 - \theta$. In the second case, the point $(3, 2 - \theta, 2 + \theta, 0, \theta)$ is a feasible solution iff $0 \leq \theta \leq 2$, and the value of the objective function is also $5 - \theta$. Since we must have $\theta \geq 0$ to have a feasible solution, there is no way to increase the objective function. In this situation, it turns out that we have reached an optimal solution, in our case $u_2 = (3, 2, 2, 0, 0)$, with the maximum of the objective function equal to 5.

We could also have applied the simplex algorithm to the vertex $u_0 = (0, 0, 1, 3, 2)$ and to the vector $(0, \theta, 1 - \theta, 3, 2 - \theta, 1)$, which is a feasible solution iff $0 \leq \theta \leq 1$, with new value of the objective function θ . By picking $\theta = 1$, we obtain the feasible solution $(0, 1, 0, 3, 1)$, corresponding to the basis (A^2, A^4, A^5) , which is indeed a vertex. The new value of the objective function is 1. Then we express A^1 and A^3 in terms the basis (A^2, A^4, A^5) obtaining

$$\begin{aligned} A^1 &= A^4 - A^3 \\ A^3 &= A^2 - A^5, \end{aligned}$$

and repeat the process with $(0, 1, 0, 3, 1)$ and the basis (A^2, A^4, A^5) . After three more steps we will reach the optimal solution $u_2 = (3, 2, 2, 0, 0)$.