of the ring $\bigwedge E$ on $\bigwedge E^*$ which makes $\bigwedge E^*$ into a right $\bigwedge E$-module.

Similarly, we have maps

$$\llcorner : \bigwedge^{p+q} E \times \bigwedge^{p} E^* \longrightarrow \bigwedge^{q} E$$

which in turn leads to the following dual formation of the right hook.

Definition 34.11. Let $u^* \in \bigwedge^p E^*$ and $z \in \bigwedge^{p+q} E$. We define $z \llcorner u^* \in \bigwedge^q$ to be the q-vector uniquely defined by

$$\langle u^* \wedge v^*, z\rangle = \langle v^*, z \llcorner u^*\rangle, \qquad \text{for all } v^* \in \bigwedge^q E^*.$$

We can prove that

$$z \llcorner (u^* \wedge v^*) = (z \llcorner u^*) \llcorner v^*,$$

so the family of operators $\llcorner_{p,q}$ defines a right action

$$\llcorner : \bigwedge E \times \bigwedge E^* \longrightarrow \bigwedge E$$

of the ring $\bigwedge E^*$ on $\bigwedge E$ which makes $\bigwedge E$ into a right $\bigwedge E^*$-module.

Since the left hook $\lrcorner : \bigwedge^p E \times \bigwedge^{p+q} E^* \longrightarrow \bigwedge^q E^*$ is defined by

$$\langle u \lrcorner z^*, v\rangle = \langle z^*, v \wedge u\rangle, \quad \text{for all } u \in \bigwedge^p E,\ v \in \bigwedge^q E \text{ and } z^* \in \bigwedge^{p+q} E^*,$$

the right hook

$$\llcorner : \bigwedge^{p+q} E^* \times \bigwedge^{p} E \longrightarrow \bigwedge^{q} E^*$$

by

$$\langle z^* \llcorner u, v\rangle = \langle z^*, u \wedge v\rangle, \quad \text{for all } u \in \bigwedge^p E,\ v \in \bigwedge^q E, \text{ and } z^* \in \bigwedge^{p+q} E^*,$$

and $v \wedge u = (-1)^{pq} u \wedge v$, we conclude that

$$z^* \llcorner u = (-1)^{pq}\, u \lrcorner z^*.$$

Similarly, since

$$\begin{aligned}
\langle v^* \wedge u^*, z\rangle &= \langle v^*, u^* \lrcorner z\rangle, \quad \text{for all } u^* \in \bigwedge^p E^*,\ v^* \in \bigwedge^q E^* \text{ and } z \in \bigwedge^{p+q} E\\
\langle u^* \wedge v^*, z\rangle &= \langle v^*, z \llcorner u^*\rangle, \quad \text{for all } u^* \in \bigwedge^p E^*,\ v^* \in \bigwedge^q E^*, \text{ and } z \in \bigwedge^{p+q} E,
\end{aligned}$$

and $v^* \wedge u^* = (-1)^{pq} u^* \wedge v^*$, we have

$$z \llcorner u^* = (-1)^{pq}\, u^* \lrcorner z.$$

We summarize the above facts in the following proposition.