

Program (Dlasso3):

$$\begin{aligned} & \text{minimize} \quad \frac{1}{2} \|y - \lambda\|_2^2 \\ & \text{subject to} \quad \|X^\top \lambda\|_\infty \leq \tau \\ & \quad \mathbf{1}_m^\top \lambda = 0, \end{aligned}$$

minimizing over $\lambda \in \mathbb{R}^m$.

Once $\lambda = \xi$ and w are determined, we obtain b using the equation

$$b\mathbf{1}_m = y - Xw - \xi,$$

and since $\mathbf{1}_m^\top \mathbf{1}_m = m$ and $\mathbf{1}_m^\top \xi = \mathbf{1}_m^\top \lambda = 0$, the above yields

$$b = \frac{1}{m} \mathbf{1}_m^\top y - \frac{1}{m} \mathbf{1}_m^\top Xw - \frac{1}{m} \mathbf{1}_m^\top \xi = \bar{y} - \sum_{j=1}^n \bar{X}^j w_j,$$

where \bar{y} is the mean of y and \bar{X}^j is the mean of the j th column of X .

The equation

$$b = \hat{b} + \bar{y} - \sum_{j=1}^n \bar{X}^j w_j = \hat{b} + \bar{y} - (\bar{X}^1 \dots \bar{X}^n)w,$$

can be used as in ridge regression, (see Section 55.2), to show that the Program (**lasso3**) is equivalent to applying lasso regression (**lasso2**) without an intercept term to the centered data, by replacing y by $\hat{y} = y - \bar{y}\mathbf{1}$ and X by $\hat{X} = X - \bar{X}$. Then b is given by

$$b = \bar{y} - (\bar{X}^1 \dots \bar{X}^n)\hat{w},$$

where \hat{w} is the solution given by (**lasso2**). This is the method described by Hastie, Tibshirani, and Wainwright [89] (Section 2.2).

Example 55.3. We can create a data set (X, y) where X a 100×5 matrix and y is a 100×1 vector using the following **Matlab** program in which the command **randn** creates an array of normally distributed numbers.

```
X = randn(100,5);
ww = [0; 2; 0; -3; 0];
y = X*ww + randn(100,1)*0.1;
```

The purpose of the third line is to add some small noise to the “output” $X * ww$. The first five rows of X are

$$\begin{pmatrix} -1.1658 & -0.0679 & -1.6118 & 0.3199 & 0.4400 \\ -1.1480 & -0.1952 & -0.0245 & -0.5583 & -0.6169 \\ 0.1049 & -0.2176 & -1.9488 & -0.3114 & 0.2748 \\ 0.7223 & -0.3031 & 1.0205 & -0.5700 & 0.6011 \\ 2.5855 & 0.0230 & 0.8617 & -1.0257 & 0.0923 \end{pmatrix},$$