Since there are $2(p+q)$ Lagrange multipliers the $(p+q)\times(p+q)$ matrix $X^\top X$ must be augmented with zero's to make it a $2(p+q)\times 2(p+q)$ matrix P_{2a} given by

$$P_{2a} = \begin{pmatrix} X^\top X & 0_{p+q,p+q} \\ 0_{p+q,p+q} & 0_{p+q,p+q} \end{pmatrix},$$

and similarly q is augmented with zeros as the vector $q_{2a} = 0_{2(p+q)}$.

The `Matlab` programs implementing the above method are given in Appendix B, Section B.2. We ran our program on two sets of 30 points each generated at random using the following code which calls the function `runSVMs2pbv3`:

```
rho = 10;
u16 = 10.1*randn(2,30)+7 ;
v16 = -10.1*randn(2,30)-7;
[~,~,~,~,~,~,w3] = runSVMs2pbv3(0.37,rho,u16,v16,1/60)
```

We picked $K = 1/60$ and various values of ν starting with $\nu = 0.37$, which appears to be the smallest value for which the method converges; see Figure 54.11.

In this example, $p_f = 10, q_f = 11, p_m = 12, q_m = 12$. The quadratic solver converged after 8121 steps to reach primal and dual residuals smaller than 10^{-10}.

Reducing ν below $\nu = 0.37$ has the effect that p_f, q_f, p_m, q_m decrease but the following situation arises. Shrinking η a little bit has the effect that $p_f = 9, q_f = 10, p_m = 10, q_m = 11$. Then $\max\{p_f, q_f\} = \min\{p_m, q_m\} = 10$, so the only possible value for ν is $\nu = 20/60 = 1/3 = 0.3333333\cdots$. When we run our program with $\nu = 1/3$, it returns a value of η less than 10^{-13} and a value of w whose components are also less than 10^{-13}. This is probably due to numerical precision. Values of ν less than $1/3$ cause the same problem. It appears that the geometry of the problem constrains the values of p_f, q_f, p_m, q_m in such a way that it has no solution other than $w = 0$ and $\eta = 0$.

Figure 54.12 shows the result of running the program with $\nu = 0.51$. We have $p_f = 15, q_f = 16, p_m = 16, q_m = 16$. Interestingly, for $\nu = 0.5$, we run into the singular situation where there is only one support vector and $\nu = 2p_f/(p+q)$.