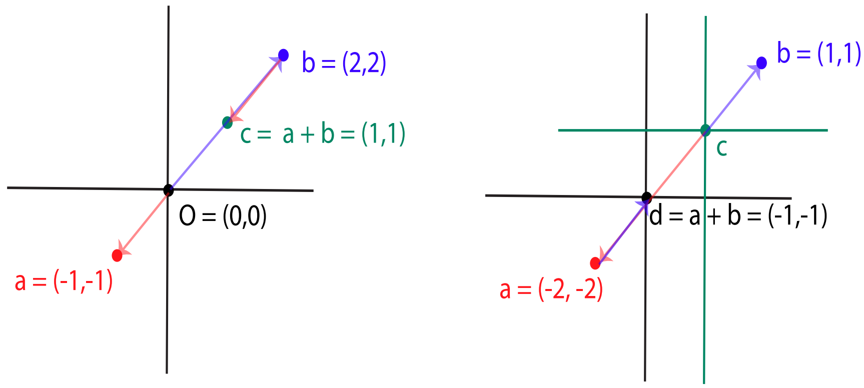

Figure 24.9: The example from the beginning of Section 24.4.

(2) If $\sum_{i\in I}\lambda_i = 0$*, then*

$$\sum_{i\in I}\lambda_i\overrightarrow{aa_i} = \sum_{i\in I}\lambda_i\overrightarrow{ba_i}.$$

Proof. (1) By Chasles's identity (see Section 24.3), we have

$$\begin{aligned} a + \sum_{i\in I}\lambda_i\overrightarrow{aa_i} &= a + \sum_{i\in I}\lambda_i(\overrightarrow{ab} + \overrightarrow{ba_i}) \\ &= a + \Big(\sum_{i\in I}\lambda_i\Big)\overrightarrow{ab} + \sum_{i\in I}\lambda_i\overrightarrow{ba_i} \\ &= a + \overrightarrow{ab} + \sum_{i\in I}\lambda_i\overrightarrow{ba_i} && \text{since } \textstyle\sum_{i\in I}\lambda_i = 1 \\ &= b + \sum_{i\in I}\lambda_i\overrightarrow{ba_i} && \text{since } b = a + \overrightarrow{ab}. \end{aligned}$$

An illustration of this calculation in $\mathbb{A}^2$ is provided by Figure 24.10.

(2) We also have

$$\begin{aligned} \sum_{i\in I}\lambda_i\overrightarrow{aa_i} &= \sum_{i\in I}\lambda_i(\overrightarrow{ab} + \overrightarrow{ba_i}) \\ &= \Big(\sum_{i\in I}\lambda_i\Big)\overrightarrow{ab} + \sum_{i\in I}\lambda_i\overrightarrow{ba_i} \\ &= \sum_{i\in I}\lambda_i\overrightarrow{ba_i}, \end{aligned}$$

since $\sum_{i\in I}\lambda_i = 0$. $\square$