three lines D, D', D'' of equations

$$\begin{aligned} \alpha x + \beta y + \gamma z &= 0 \\ \alpha' x + \beta' y + \gamma' z &= 0 \\ \alpha'' x + \beta'' y + \gamma'' z &= 0 \end{aligned}$$

are concurrent iff

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha' & \beta' & \gamma' \\ \alpha'' & \beta'' & \gamma'' \end{vmatrix} = 0.$$

We can also find the equation of the unique line $D = \langle P, P' \rangle$ passing through two distinct points $P = (u \colon v \colon w)$ and $P' = (u' \colon v' \colon w')$ of a projective plane. This line is given by the equation

$$(vw' - v'w)x + (wu' - w'u) + (uv' - u'v)z = 0, \tag{††}$$

and since

$$\begin{pmatrix} u & v & w \\ u' & v' & w' \end{pmatrix}$$

has rank 2 because $P \neq P'$, at least one of the coordinates of the equation (††) is nonzero. Observe that the coefficients of the equation (††) correspond to the cross-product

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \times \begin{pmatrix} u' \\ v' \\ w'' \end{pmatrix}.$$

The equation of the line $D = \langle P, P' \rangle$ must be satisfied by the homogeneous coordinates of the points P and P'. Equation (††) can be written as

$$\begin{vmatrix} x & y & z \\ u & v & w \\ u' & v' & w' \end{vmatrix} = 0,$$

and a reasoning as in the case of the intersection of lines shows that the equation of the line passing through P and P' is given by equation (††).

Then, in a projective plane, three points $P = (u \colon v \colon w)$, $P' = (u' \colon v' \colon w')$ and $P'' = (u'' \colon v'' \colon w'')$ belong to a common line (are collinear) iff

$$\begin{vmatrix} u & v & w \\ u' & v' & w' \\ u'' & v'' & w'' \end{vmatrix} = 0.$$

More generally, in a projective space $\mathbf{P}(E)$ of dimension $n \geq 2$, if n points $P_1, \ldots, P_n$ are projectively independent and if P_i has homogeneous coordinates $(u_1^i \colon \cdots \colon u_{n+1}^i)$ (with