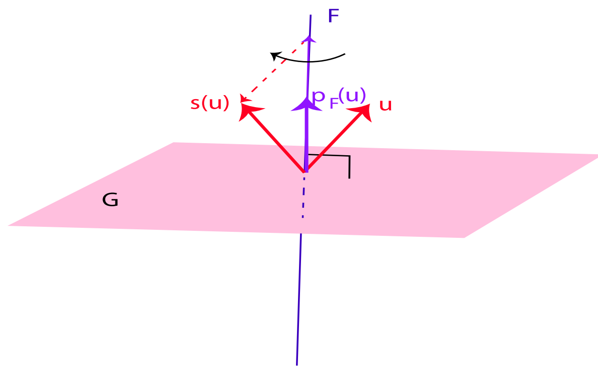

Figure 13.2: A flip in $\mathbb{R}^3$ is a rotation of π about the F axis.

for some $\lambda \in \mathbb{R}$, and we get

$$u \cdot w = \lambda \|w\|^2,$$

and thus

$$p_G(u) = \frac{(u \cdot w)}{\|w\|^2}\, w.$$

Since

$$s(u) = u - 2p_G(u),$$

we get

$$s(u) = u - 2\,\frac{(u \cdot w)}{\|w\|^2}\, w.$$

Since the above formula is important, we record it in the following proposition.

Proposition 13.1. *Let E be a finite-dimensional Euclidean space and let H be a hyperplane in E. For any nonzero vector w orthogonal to H, the hyperplane reflection s about H is given by*

$$s(u) = u - 2\,\frac{(u \cdot w)}{\|w\|^2}\, w, \quad u \in E.$$

Such reflections are represented by matrices called *Householder matrices*, which play an important role in numerical matrix analysis (see Kincaid and Cheney [102] or Ciarlet [41]).

Definition 13.3. A *Householder matrix* is a matrix of the form

$$H = I_n - 2\,\frac{WW^\top}{\|W\|^2} = I_n - 2\,\frac{WW^\top}{W^\top W},$$

where $W \in \mathbb{R}^n$ is a nonzero vector.