There are various ways to prove this. One way is to use the Bernstein basis, because the kth derivative of a polynomial is given by a formula in terms of its control points. For example, for $m = 1$, every degree 3 polynomial can be written as

$$P(x) = (1-x)^3\, b_0 + 3(1-x)^2 x\, b_1 + 3(1-x)x^2\, b_2 + x^3\, b_3,$$

with $b_0, b_1, b_2, b_3 \in \mathbb{R}$, and we showed that

$$\begin{aligned} P'(0) &= 3(b_1 - b_0) \\ P'(1) &= 3(b_3 - b_2). \end{aligned}$$

Given $P(0)$ and $P(1)$, we determine b_0 and b_3, and from $P'(0)$ and $P'(1)$, we determine b_1 and b_2.

In general, for a polynomial of degree m written as

$$P(x) = \sum_{j=0}^{m} b_j B_j^m(x)$$

in terms of the Bernstein basis $(B_0^m(x), \ldots, B_m^m(x))$ with

$$B_j^m(x) = \binom{m}{j}(1-x)^{m-j}x^j,$$

it can be shown that the kth derivative of P at zero is given by

$$P^{(k)}(0) = m(m-1)\cdots(m-k+1)\left(\sum_{i=0}^{k}\binom{k}{i}(-1)^{k-i}\, b_i\right),$$

and there is a similar formula for $P^{(k)}(1)$.

Actually, we need to use the Bernstein basis of polynomials $B_k^m[r,s]$, where

$$B_j^m[r,s](x) = \binom{m}{j}\left(\frac{s-x}{s-r}\right)^{m-j}\left(\frac{x-r}{s-r}\right)^{j},$$

with $r < s$, in which case

$$P^{(k)}(0) = \frac{m(m-1)\cdots(m-k+1)}{(s-r)^k}\left(\sum_{i=0}^{k}\binom{k}{i}(-1)^{k-i}\, b_i\right),$$

with a similar formula for $P^{(k)}(1)$. In our case, we set $r = x_i, s = x_{i+1}$.

Now, if the $2m+2$ values

$$P(0), P^{(1)}(0), \ldots, P^{(m)}(0), P(1), P^{(1)}(1), \ldots, P^{(m)}(1)$$