Thus we get the following:

Theorem 15.10. *For any complex $n \times n$ matrix A, all the eigenvalues of A belong to the intersection of the Gershgorin domains $G(A) \cap G(A^\top)$. See Figure 15.3. Furthermore the following properties hold:*

(1) If A is strictly column diagonally dominant, that is

$$|a_{i\,i}| > \sum_{i=1,\, i \neq j}^{n} |a_{i\,j}|, \quad \textit{for } j = 1, \ldots, n,$$

then A is invertible.

(2) If A is strictly column diagonally dominant, and if $a_{i\,i} > 0$ for $i = 1, \ldots, n$, then every eigenvalue of A has a strictly positive real part.

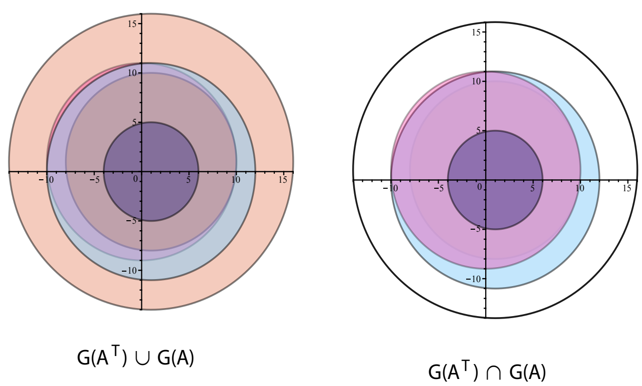

Figure 15.3: Let A be the 3×3 matrix specified at the end of Definition 15.5. The colored region in the second figure is $G(A) \cap G(A^\top)$.

There are refinements of Gershgorin's theorem and eigenvalue location results involving other domains besides discs; for more on this subject, see Horn and Johnson [95], Sections 6.1 and 6.2.

Remark: Neither strict row diagonal dominance nor strict column diagonal dominance are necessary for invertibility. Also, if we relax all strict inequalities to inequalities, then row diagonal dominance (or column diagonal dominance) is not a sufficient condition for invertibility.