Consider any two points μ and $\mu + \xi$ in $\mathbb{R}^m_+$. By definition of u_μ we have

$$L(u_\mu, \mu) \leq L(u_{\mu+\xi}, \mu),$$

which means that

$$J(u_\mu) + \sum_{i=1}^{m} \mu_i \varphi_i(u_\mu) \leq J(u_{\mu+\xi}) + \sum_{i=1}^{m} \mu_i \varphi_i(u_{\mu+\xi}), \tag{$*_1$}$$

and since $G(\mu) = L(u_\mu, \mu) = J(u_\mu) + \sum_{i=1}^m \mu_i \varphi_i(u_\mu)$ and $G(\mu + \xi) = L(u_{\mu+\xi}, \mu + \xi) = J(u_{\mu+\xi}) + \sum_{i=1}^m (\mu_i + \xi_i) \varphi_i(u_{\mu+\xi})$, we have

$$G(\mu + \xi) - G(\mu) = J(u_{\mu+\xi}) - J(u_\mu) + \sum_{i=1}^{m} (\mu_i + \xi_i) \varphi_i(u_{\mu+\xi}) - \sum_{i=1}^{m} \mu_i \varphi_i(u_\mu). \tag{$*_2$}$$

Since $(*_1)$ can be written as

$$0 \leq J(u_{\mu+\xi}) - J(u_\mu) + \sum_{i=1}^{m} \mu_i \varphi_i(u_{\mu+\xi}) - \sum_{i=1}^{m} \mu_i \varphi_i(u_\mu),$$

by adding $\sum_{i=1}^m \xi_i \varphi_i(u_{\mu+\xi})$ to both sides of the above inequality and using $(*_2)$ we get

$$\sum_{i=1}^{m} \xi_i \varphi_i(u_{\mu+\xi}) \leq G(\mu + \xi) - G(\mu). \tag{$*_3$}$$

By definition of $u_{\mu+\xi}$ we have

$$L(u_{\mu+\xi}, \mu + \xi) \leq L(u_\mu, \mu + \xi),$$

which means that

$$J(u_{\mu+\xi}) + \sum_{i=1}^{m} (\mu_i + \xi_i) \varphi_i(u_{\mu+\xi}) \leq J(u_\mu) + \sum_{i=1}^{m} (\mu_i + \xi_i) \varphi_i(u_\mu). \tag{$*_4$}$$

This can be written as

$$J(u_{\mu+\xi}) - J(u_\mu) + \sum_{i=1}^{m} (\mu_i + \xi_i) \varphi_i(u_{\mu+\xi}) - \sum_{i=1}^{m} (\mu_i + \xi_i) \varphi_i(u_\mu) \leq 0,$$

and by adding $\sum_{i=1}^m \xi_i \varphi_i(u_\mu)$ to both sides of the above inequality and using $(*_2)$ we get

$$G(\mu + \xi) - G(\mu) \leq \sum_{i=1}^{m} \xi_i \varphi_i(u_\mu). \tag{$*_5$}$$