whose effective domain is $\text{dom}(J)$ (since we are assuming that $\text{dom}(J) \subseteq \text{dom}(\varphi_i)$, $i = 1, \ldots, m$). Thus $h(x) = L(x, \lambda, \mu)$, but h is a *function only of* x, so we denote it differently to avoid confusion (also, technically, $L(x, \lambda, \mu)$ may take the value $-\infty$, but h does not). Since J and the φ_i are proper convex functions and the ψ_j are affine, the function h is a proper convex function.

A proof of a generalized version of Theorem 50.18 can be obtained by putting together Theorem 28.1, Theorem 28.2, and Theorem 28.3, in Rockafellar [138]. For the sake of completeness, we state these theorems. Here is Theorem 28.1.

Theorem 51.39. *(Theorem 28.1, Rockafellar) Let (P) be an ordinary convex program. Let $(\lambda, \mu) \in \mathbb{R}^m_+ \times \mathbb{R}^p$ be Lagrange multipliers such that the infimum of the function $h = J + \sum_{i=1}^m \lambda_i \varphi_i + \sum_{j=1}^p \mu_j \psi_j$ is finite and equal to the optimal value of J over U. Let D be the minimal set of h over $\mathbb{R}^n$, and let $I = \{i \in \{1, \ldots, m\} \mid \lambda_i = 0\}$. If D_0 is the subset of D consisting of vectors x such that*

$$\begin{aligned} &\varphi_i(x) \leq 0 && \text{for all } i \in I \\ &\varphi_i(x) = 0 && \text{for all } i \notin I \\ &\psi_j(x) = 0 && \text{for all } j = 1, \ldots, p, \end{aligned}$$

then D_0 is the set of minimizers of (P) over U.

And now here is Theorem 28.2.

Theorem 51.40. *(Theorem 28.2, Rockafellar) Let (P) be an ordinary convex program, and let $I \subseteq \{1, \ldots, m\}$ be the subset of indices of inequality constraints that are not affine. Assume that the optimal value of (P) is finite, and that (P) has at least one feasible solution $x \in$* **relint**$(\text{dom}(J))$ *such that*

$$\varphi_i(x) < 0 \quad \text{for all } i \in I.$$

Then there exist some Lagrange multipliers $(\lambda, \mu) \in \mathbb{R}^m_+ \times \mathbb{R}^p$ (not necessarily unique) such that

(a) The infimum of the function $h = J + \sum_{i=1}^m \lambda_i \varphi_i + \sum_{j=1}^p \mu_j \psi_j$ is finite and equal to the optimal value of J over U.

The hypotheses of Theorem 51.40 are qualification conditions on the constraints, essentially Slater's conditions from Definition 50.6.

Definition 51.21. Let (P) be an ordinary convex program, and let $I \subseteq \{1, \ldots, m\}$ be the subset of indices of inequality constraints that are not affine. The constraints are *qualified* is there is a feasible solution $x \in$ **relint**$(\text{dom}(J))$ such that

$$\varphi_i(x) < 0 \quad \text{for all } i \in I.$$

Finally, here is Theorem 28.3 from Rockafellar [138].