

$$-E = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ a_{21} & 0 & 0 & \cdots & 0 & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ a_{n-11} & a_{n-12} & a_{n-13} & \ddots & 0 & 0 \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn-1} & 0 \end{pmatrix}, \quad -F = \begin{pmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1n-1} & a_{1n} \\ 0 & 0 & a_{23} & \cdots & a_{2n-1} & a_{2n} \\ 0 & 0 & 0 & \ddots & a_{3n-1} & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & a_{n-1n} \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}.$$

In *Jacobi's method*, we assume that *all* diagonal entries in A are nonzero, and we pick

$$\begin{aligned} M &= D \\ N &= E + F, \end{aligned}$$

so that by (*),

$$B = M^{-1}N = D^{-1}(E + F) = I - D^{-1}A.$$

As a matter of notation, we let

$$J = I - D^{-1}A = D^{-1}(E + F),$$

which is called *Jacobi's matrix*. The corresponding method, *Jacobi's iterative method*, computes the sequence (u_k) using the recurrence

$$u_{k+1} = D^{-1}(E + F)u_k + D^{-1}b, \quad k \geq 0.$$

In practice, we iteratively solve the systems

$$Du_{k+1} = (E + F)u_k + b, \quad k \geq 0.$$

If we write $u_k = (u_1^k, \dots, u_n^k)$, we solve iteratively the following system:

$$\begin{array}{rclclclcl} a_{11}u_1^{k+1} & = & & -a_{12}u_2^k & -a_{13}u_3^k & \cdots & -a_{1n}u_n^k & + b_1 \\ a_{22}u_2^{k+1} & = & -a_{21}u_1^k & & -a_{23}u_3^k & \cdots & -a_{2n}u_n^k & + b_2 \\ \vdots & \vdots & \vdots & & & & & \\ a_{n-1n-1}u_{n-1}^{k+1} & = & -a_{n-11}u_1^k & \cdots & -a_{n-1n-2}u_{n-2}^k & & -a_{n-1n}u_n^k & + b_{n-1} \\ a_{nn}u_n^{k+1} & = & -a_{n1}u_1^k & -a_{n2}u_2^k & \cdots & -a_{nn-1}u_{n-1}^k & & + b_n \end{array}.$$

In Matlab one step of Jacobi iteration is achieved by the following function:

```
function v = Jacobi2(A,b,u)
    n = size(A,1);
    v = zeros(n,1);
    for i = 1:n
        v(i,1) = u(i,1) + (-A(i,:)*u + b(i))/A(i,i);
    end
end
```