is any isotropic vector, since $y \in U_1^{\perp}$, by a previous remark, $y \in U_1$, so $y \in D \cap U_1$. But, $D \subseteq N$ with $N \cap (S_1 + S_2) = (0)$, and $D \cap (U + W) = (0)$, so $D \cap (U + S_1) = D \cap U_1 = (0)$, which yields $y = 0$. The statements about dimensions are easily obtained. $\square$

Finally, Theorem 29.33 yields the strong form of the Witt decomposition in which W is anistropic. Given any matrix $A \in \mathrm{M}_n(K)$, we say that A is *definite* if $x^{\top} A x \neq 0$ for all $x \in K^n$.

Theorem 29.34. *Let φ be an ϵ-Hermitian form on E which is nondegenerate and satisfies property (T).*

(1) Any two totally isotropic maximal spaces of finite dimension have the same dimension.

(2) For any totally isotropic maximal subspace U of finite dimension $r \geq 1$, there is another totally isotropic maximal subspace U' of dimension r such that $U \cap U' = (0)$, and $U \oplus U'$ is nondegenerate. Furthermore, if $D = (U \oplus U')^{\perp}$, then (U, U', D) is a Witt decomposition of E; that is, there are no nonzero isotropic vectors in D (D is anisotropic).

(3) If E has finite dimension $n \geq 1$ and there is some nonzero isotropic vector for φ (E is not anisotropic), then E has a nontrivial Witt decomposition (U, U', D) as in (2). There is a basis of E such that

(a) if φ is alternating ($\epsilon = -1$ and $\lambda = \overline{\lambda}$ for all $\lambda \in K$), then $n = 2m$ and φ is represented by a matrix of the form

$$\begin{pmatrix} 0 & I_m \\ -I_m & 0 \end{pmatrix}$$

(b) if φ is symmetric ($\epsilon = +1$ and $\lambda = \overline{\lambda}$ for all $\lambda \in K$), then φ is represented by a matrix of the form

$$\begin{pmatrix} 0 & I_r & 0 \\ I_r & 0 & 0 \\ 0 & 0 & P \end{pmatrix},$$

where either $n = 2r$ and P does not occur, or $n > 2r$ and P is a definite symmetric matrix.

(c) if φ is ϵ-Hermitian (the involutive automorphism $\lambda \mapsto \overline{\lambda}$ is not the identity), then φ is represented by a matrix of the form

$$\begin{pmatrix} 0 & I_r & 0 \\ \epsilon I_r & 0 & 0 \\ 0 & 0 & P \end{pmatrix},$$

where either $n = 2r$ and P does not occur, or $n > 2r$ and P is a definite matrix such that $P^* = \epsilon P$.