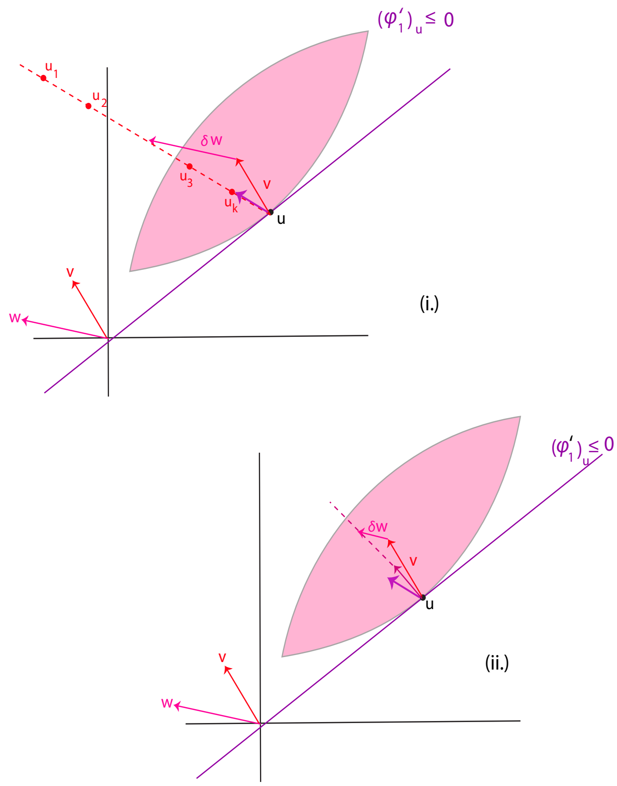

Figure 50.10: Let U be the pink lounge in $\mathbb{R}^2$. Let u satisfy the non-affine constraint $\varphi_1(u)$. Choose vectors v and w in the half space $(\varphi_1')_u \leq 0$. Figure (i.) approaches u along the line $u + t(\delta w + v)$ and shows that $v + \delta w \in C(u)$ for fixed δ. Figure (ii.) varies δ in order that the purple vectors approach v as $\delta \to \infty$.

50.3 The Karush–Kuhn–Tucker Conditions

If the domain U is defined by inequality constraints satisfying mild differentiability conditions and if the constraints at u are qualified, then there is a necessary condition for the function J to have a local minimum at $u \in U$ involving generalized Lagrange multipliers. The proof uses a version of Farkas lemma. In fact, the necessary condition stated next holds for infinite-dimensional vector spaces because there a version of Farkas lemma holding for *real* Hilbert spaces, but we will content ourselves with the version holding for finite dimensional normed vector spaces. For the more general version, see Theorem 48.12 (or Ciarlet [41], Chapter 9).

We will be using the following version of Farkas lemma.

Proposition 50.3. *(Farkas Lemma, Version I) Let A be a real $m \times n$ matrix and let $b \in \mathbb{R}^m$ be any vector. The linear system $Ax = b$ has no solution $x \geq 0$ iff there is some nonzero*