



Figure 50.6: Let U be the light purple planar region which lies between the curves $y = x^2$ and $y^2 = x$. Figure (i.) illustrates the boundary point $(1, 1)$ given by the equalities $y - x^2 = 0$ and $y^2 - x = 0$. The affine translate of cone of feasible directions, $C(1, 1)$, is illustrated by the pink triangle whose sides are the tangent lines to the boundary curves. Figure (ii.) illustrates the boundary point $(1/4, 1/2)$ given by the equality $y^2 - x = 0$. The affine translate of $C(1/4, 1/2)$ is the lilac half space bounded by the tangent line to $y^2 = x$ through $(1/4, 1/2)$.

Definition 50.4. For any $u \in U$, with

$$U = \{x \in \Omega \mid \varphi_i(x) \leq 0, \ 1 \leq i \leq m\},$$

we define $I(u)$ as the set of indices

$$I(u) = \{i \in \{1, \dots, m\} \mid \varphi_i(u) = 0\}$$

where the constraints are active. We define the set $C^*(u)$ as

$$C^*(u) = \{v \in V \mid (\varphi'_i)_u(v) \leq 0, \ i \in I(u)\}.$$

Since each $(\varphi'_i)_u$ is a linear form, the subset

$$C^*(u) = \{v \in V \mid (\varphi'_i)_u(v) \leq 0, \ i \in I(u)\}$$