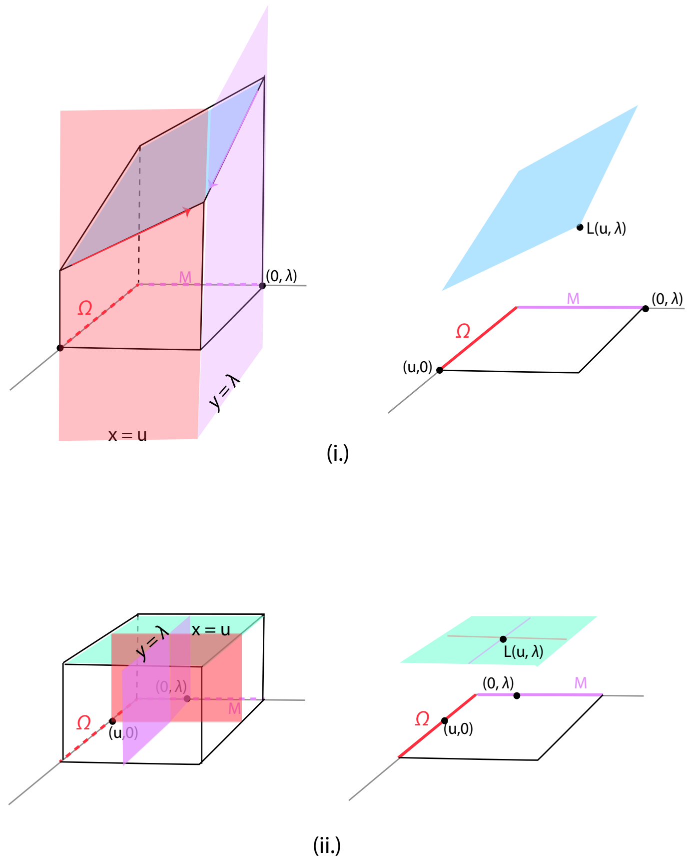

Figure 50.18: Let $\Omega = \{[t, 0, 0] \mid 0 \leq t \leq 1\}$ and $M = \{[0, t, 0] \mid 0 \leq t \leq 1\}$. In Figure (*i.*), $L(u, \lambda)$ is the blue slanted quadrilateral whose forward vertex is a saddle point. In Figure (*ii.*), $L(u, \lambda)$ is the planar green rectangle composed entirely of saddle points.

Pick any $w \in \Omega$ and any $\rho \in M$. By definition of inf (the greatest lower bound) and sup (the least upper bound), we have

$$\inf_{v \in \Omega} L(v, \rho) \leq L(w, \rho) \leq \sup_{\mu \in M} L(w, \mu).$$

The cases where $\inf_{v \in \Omega} L(v, \rho) = -\infty$ or where $\sup_{\mu \in M} L(w, \mu) = +\infty$ may arise, but this is not a problem. Since

$$\inf_{v \in \Omega} L(v, \rho) \leq \sup_{\mu \in M} L(w, \mu)$$

and the right-hand side is independent of ρ, it is an upper bound of the left-hand side for all ρ, so

$$\sup_{\mu \in M} \inf_{v \in \Omega} L(v, \mu) \leq \sup_{\mu \in M} L(w, \mu).$$