One of the side-effects of this choice is that Bourbaki's version of Formula (4) of Proposition 34.18 (Bourbaki [25], Chapter III, page 168) is

$$\begin{aligned} e_H^* \lrcorner\, e_L &= 0 \quad \text{if } H \not\subseteq L \\ e_H^* \lrcorner\, e_L &= (-1)^{p(p-1)/2} \rho_{H,L-H} e_{L-H} \quad \text{if } H \subseteq L, \end{aligned}$$

where $|H| = p$ and $|L| = p + q$. This correspond to Formula (1) of Proposition 34.21 up to the sign factor $(-1)^{p(p-1)/2}$, which we find horribly confusing. Curiously, an older edition of Bourbaki (1958) uses the same pairing as Fulton and Harris [68]. The reason (and the advantage) for this change of sign convention is not clear to us.

We also have the following version of Proposition 34.19 for the right hook.

Proposition 34.22. *For the right hook*

$$\llcorner : \bigwedge^{q+1} E^* \times E \longrightarrow \bigwedge^{q} E^*,$$

for every $u \in E$, $x^* \in \bigwedge^r E^*$, and $y^* \in \bigwedge^{q+1-r} E^*$, we have

$$(x^* \wedge y^*) \llcorner u = (x^* \llcorner u) \wedge y^* + (-1)^r x^* \wedge (y^* \llcorner u).$$

Proof. A proof involving determinants can be found in Warner [186], Chapter 2. □

Thus, $\llcorner : \bigwedge^{q+1} E^* \times E \longrightarrow \bigwedge^q E^*$ is an anti-derivation. A similar formula holds for the the right hook $\llcorner : \bigwedge^{q+1} E \times E^* \longrightarrow \bigwedge^q E$, namely

$$(x \wedge y) \llcorner u^* = (x \llcorner u^*) \wedge y + (-1)^r x \wedge (y \llcorner u^*),$$

for every $u^* \in E$, $\in \bigwedge^r E$, and $y \in \bigwedge^{q+1-r} E$. This formula is used by Shafarevitch [158] to define a hook, but beware that Shafarevitch use the left hook notation $u^* \lrcorner\, x$ rather than the right hook notation. Shafarevitch uses the terminology *convolution*, which seems very unfortunate.

For $u \in E$, the right hook $z^* \llcorner u$ is also denoted $i(u)z^*$, and called *insertion operator* or *interior product*. This operator plays an important role in differential geometry.

Definition 34.12. Let $u \in E$ and $z^* \in \bigwedge^{n+1}(E^*)$. If we view z^* as an alternating multilinear map in $\mathrm{Alt}^{n+1}(E; K)$, then we define $i(u)z^* \in \mathrm{Alt}^n(E; K)$ as given by

$$(i(u)z^*)(v_1, \ldots, v_n) = z^*(u, v_1, \ldots, v_n).$$

Using the left hook $\lrcorner$ and the right hook $\llcorner$ we can define two linear maps $\gamma\colon \bigwedge^p E \to \bigwedge^{n-p} E^*$ and $\delta\colon \bigwedge^p E^* \to \bigwedge^{n-p} E$ as follows: