

Observe that the above homography may map some of the affine points p_1, p_2, p_3, p_4 (which are not “points at infinity”) to arbitrary points in \mathbb{RP}^2 , which may be points at infinity (in which case $q_i^z = 0$). The generalization to any dimension $n \geq 2$ is immediate.

We define the basis $\mathcal{E}^a = (e_1^a, e_2^a, e_3^a)$, with $e_1^a = (1, 0, 1)$, $e_2^a = (0, 1, 1)$, $e_3^a = (0, 0, 1)$, and call it the *affine canonical basis* (of \mathbb{R}^2). We also define e_4^a as $e_4^a = (1, 1, 1)$.

In the special case where (p_1, p_2, p_3, p_4) is the canonical square $(e_1^a, e_2^a, e_3^a, e_4^a)$, since

$$e_4^a = e_1^a + e_2^a - e_3^a,$$

we have $\alpha_1 = 1, \alpha_2 = 1$, and $\alpha_3 = -1$, so

$$\mathcal{B}_{\mathcal{P}} = \mathcal{B}_{\mathcal{E}^a} = P \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

where P is the change of basis matrix from the canonical basis $\mathcal{E} = (e_1, e_2, e_3)$ to the affine basis $\mathcal{E}^a = (e_1^a, e_2^a, e_3^a)$. We have

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix},$$

and its inverse is

$$P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}.$$

In this case,

$$\mathcal{B}_{\mathcal{E}^a} = \begin{pmatrix} \alpha_1 p_1^x & \alpha_2 p_2^x & \alpha_3 p_3^x \\ \alpha_1 p_1^y & \alpha_2 p_2^y & \alpha_3 p_3^y \\ \alpha_1 & \alpha_2 & \alpha_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix},$$

and since

$$\mathcal{B}_{\mathcal{E}^a}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} = \mathcal{B}_{\mathcal{E}^a},$$

we obtain

$$A_{\mathcal{E}} = \begin{pmatrix} q_1^x & q_2^x & q_3^x \\ q_1^y & q_2^y & q_3^y \\ q_1^z & q_2^z & q_3^z \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & -\lambda_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix},$$

that is,

$$A_{\mathcal{E}} = \begin{pmatrix} q_1^x & q_2^x & q_3^x \\ q_1^y & q_2^y & q_3^y \\ q_1^z & q_2^z & q_3^z \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}.$$