We can use the dual program to solve the primal. Once $\lambda \geq 0, \mu \geq 0$ have been found, w is given by

$$w = -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \sum_{i=1}^{p} \lambda_i u_i - \sum_{j=1}^{q} \mu_j v_j,$$

but b is not determined by the dual.

The complementary slackness conditions imply that if $\epsilon_i > 0$, then $\lambda_i = K$, and if $\xi_j > 0$, then $\mu_j = K$. Consequently, if $\lambda_i < K$, then $\epsilon_i = 0$ and u_i is correctly classified, and similarly if $\mu_j < K$, then $\xi_j = 0$ and v_j is correctly classified.

A priori nothing prevents the situation where $\lambda_i = K$ for all nonzero λ_i or $\mu_j = K$ for all nonzero μ_j. If this happens, we can rerun the optimization method with a larger value of K. If the following mild hypothesis holds then b can be found.

Standard Margin Hypothesis for (SVM_{s2}). There is some support vector u_{i_0} of type 1 on the blue margin, and some support vector v_{j_0} of type 1 on the red margin.

If the **Standard Margin Hypothesis** for (SVM_{s2}) holds then $\epsilon_{i_0} = 0$ and $\mu_{j_0} = 0$, and then we have the active equations

$$w^\top u_{i_0} - b = 1 \quad \text{and} \quad -w^\top v_{j_0} + b = 1,$$

and we obtain

$$b = \frac{1}{2} w^\top (u_{i_0} + v_{j_0}).$$

(2) **Basic Soft margin ν-SVM Problem** $(\mathrm{SVM}_{s2'})$.

This a generalization of Problem (SVM_{s2}) for a version of the soft margin SVM coming from Problem (SVM_{h2}), obtained by adding an extra degree of freedom, namely instead of the margin $\delta = 1/\|w\|$, we use the margin $\delta = \eta/\|w\|$ where η is some positive constant that we wish to maximize. To do so, we add a term $-K_m\eta$ to the objective function. We have the following optimization problem:

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2} w^\top w - K_m \eta + K_s \begin{pmatrix} \epsilon^\top & \xi^\top \end{pmatrix} \mathbf{1}_{p+q} \\ &\text{subject to} \\ &\qquad w^\top u_i - b \geq \eta - \epsilon_i, \quad \epsilon_i \geq 0 \qquad i = 1, \ldots, p \\ &\qquad -w^\top v_j + b \geq \eta - \xi_j, \quad \xi_j \geq 0 \qquad j = 1, \ldots, q \\ &\qquad \eta \geq 0, \end{aligned}$$

where $K_m > 0$ and $K_s > 0$ are fixed constants that can be adjusted to determine the influence of η and the regularizing term.