

(2) Fix some  $w \in G$ . The map

$$(u, v) \mapsto u \otimes v \otimes w$$

from  $E \times F$  to  $E \otimes F \otimes G$  is bilinear, and thus there is a linear map  $f_w: E \otimes F \rightarrow E \otimes F \otimes G$  making the following diagram commute

$$\begin{array}{ccc} E \times F & \xrightarrow{\iota_\otimes} & E \otimes F \\ & \searrow & \downarrow f_w \\ & & E \otimes F \otimes G, \end{array}$$

with  $f_w(u \otimes v) = u \otimes v \otimes w$ .

Next consider the map

$$(z, w) \mapsto f_w(z),$$

from  $(E \otimes F) \times G$  into  $E \otimes F \otimes G$ . It is easily seen to be bilinear, and thus it induces a linear map  $f: (E \otimes F) \otimes G \rightarrow E \otimes F \otimes G$  making the following diagram commute

$$\begin{array}{ccc} (E \otimes F) \times G & \xrightarrow{\iota_\otimes} & (E \otimes F) \otimes G \\ & \searrow & \downarrow f \\ & & E \otimes F \otimes G, \end{array}$$

with  $f((u \otimes v) \otimes w) = u \otimes v \otimes w$ .

Also consider the map

$$(u, v, w) \mapsto (u \otimes v) \otimes w$$

from  $E \times F \times G$  to  $(E \otimes F) \otimes G$ . It is trilinear, and thus there is a linear map  $g: E \otimes F \otimes G \rightarrow (E \otimes F) \otimes G$  making the following diagram commute

$$\begin{array}{ccc} E \times F \times G & \xrightarrow{\iota_\otimes} & E \otimes F \otimes G \\ & \searrow & \downarrow g \\ & & (E \otimes F) \otimes G, \end{array}$$

with  $g(u \otimes v \otimes w) = (u \otimes v) \otimes w$ . Clearly,  $f \circ g$  and  $g \circ f$  are identity maps, and thus  $f$  and  $g$  are isomorphisms. The other case is similar.

(3) Given a fixed vector space  $G$ , for any two vector spaces  $M$  and  $N$  and every linear map  $f: M \rightarrow N$ , let  $\tau_G(f) = f \otimes \text{id}_G$  be the unique linear map making the following diagram commute.

$$\begin{array}{ccc} M \times G & \xrightarrow{\iota_{M \otimes}} & M \otimes G \\ f \times \text{id}_G \downarrow & & \downarrow f \otimes \text{id}_G \\ N \times G & \xrightarrow{\iota_{N \otimes}} & N \otimes G \end{array}$$