Under the Standard Margin Hypothesis, there is some index i_0 such that $0 < \lambda_{i_0} < K$ and there is some index j_0 such that $0 < \mu_{j_0} < K$, and we obtain the value of b and δ as

$$\begin{aligned} b &= \frac{1}{2}(\langle w, \varphi(u_{i_0}\rangle + \langle w, \varphi(v_{j_0})\rangle) \\ \delta &= \frac{1}{2}(\langle w, \varphi(u_{i_0})\rangle - \langle w, \varphi(v_{j_0})\rangle). \end{aligned}$$

Using the above value for w, we obtain

$$b = \frac{\sum_{i=1}^{p} \lambda_i(\kappa(u_i, u_{i_0}) + \kappa(u_i, v_{j_0})) - \sum_{j=1}^{q} \mu_j(\kappa(v_j, u_{i_0}) + \kappa(v_j, v_{j_0}))}{2\left(\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} K \begin{pmatrix}\lambda \\ \mu\end{pmatrix}\right)^{1/2}}.$$

It follows that the classification function

$$f(x) = \mathrm{sgn}(\langle w, \varphi(x)\rangle - b)$$

is given by

$$\begin{aligned} f(x) = \mathrm{sgn}\bigg(&\sum_{i=1}^{p} \lambda_i(2\kappa(u_i, x) - \kappa(u_i, u_{i_0}) - \kappa(u_i, v_{j_0})) \\ &- \sum_{j=1}^{q} \mu_j(2\kappa(v_j, x) - \kappa(v_j, u_{i_0}) - \kappa(v_j, v_{j_0}))\bigg), \end{aligned}$$

which is solely expressed in terms of the kernel κ.

Kernel methods for SVM are discussed in Schölkopf and Smola [145] and Shawe–Taylor and Christianini [159].

54.2 Solving SVM (SVM_{s1}) Using ADMM

In order to solve (SVM_{s1}) using ADMM we need to write the matrix corresponding to the constraints in equational form,

$$\begin{aligned} \sum_{i=1}^{p} \lambda_i - \sum_{j=1}^{q} \mu_j &= 0 \\ \sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \mu_j &= 1 \\ \lambda_i + \alpha_i &= K, \quad i = 1, \ldots, p \\ \mu_j + \beta_j &= K, \quad j = 1, \ldots, q. \end{aligned}$$