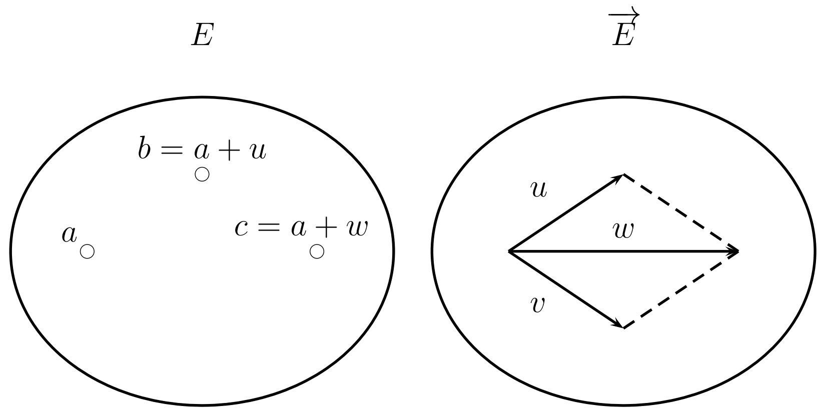

Figure 24.4: Intuitive picture of an affine space.

Conditions (A1) and (A2) say that the (abelian) group $\overrightarrow{E}$ acts on E, and Condition (A3) says that $\overrightarrow{E}$ acts transitively and faithfully on E. Note that

$$\overrightarrow{a(a+v)} = v$$

for all $a \in E$ and all $v \in \overrightarrow{E}$, since $\overrightarrow{a(a+v)}$ is the unique vector such that $a+v = a+\overrightarrow{a(a+v)}$. Thus, $b = a + v$ is equivalent to $\overrightarrow{ab} = v$. Figure 24.4 gives an intuitive picture of an affine space. It is natural to think of all vectors as having the same origin, the null vector.

The axioms defining an affine space $\langle E, \overrightarrow{E}, + \rangle$ can be interpreted intuitively as saying that E and $\overrightarrow{E}$ are two different ways of looking at the same object, but wearing different sets of glasses, the second set of glasses depending on the choice of an "origin" in E. Indeed, we can choose to look at the points in E, forgetting that every pair (a, b) of points defines a unique vector $\overrightarrow{ab}$ in $\overrightarrow{E}$, or we can choose to look at the vectors u in $\overrightarrow{E}$, forgetting the points in E. Furthermore, if we also pick any point a in E, a point that can be viewed as an *origin* in E, then we can recover all the points in E as the translated points $a + u$ for all $u \in \overrightarrow{E}$. This can be formalized by defining two maps between E and $\overrightarrow{E}$.

For every $a \in E$, consider the mapping from $\overrightarrow{E}$ to E given by

$$u \mapsto a + u,$$

where $u \in \overrightarrow{E}$, and consider the mapping from E to $\overrightarrow{E}$ given by

$$b \mapsto \overrightarrow{ab},$$

where $b \in E$. The composition of the first mapping with the second is

$$u \mapsto a + u \mapsto \overrightarrow{a(a+u)},$$