Our hypotheses imply that $\theta > 0$. We can write

$$\begin{aligned}
w^\top u_i - (b+\theta) &= \eta + \theta - (\epsilon_i + 2\theta) & \epsilon_i > 0 && i \in E_\lambda\\
-w^\top v_j + b + \theta &= \eta + \theta - \xi_j & \xi_j > 0 && j \in E_\mu\\
w^\top u_i - (b+\theta) &\geq \eta + \theta & && i \notin E_\lambda\\
-w^\top v_j + b + \theta &\geq \eta + \theta & && j \notin E_\mu.
\end{aligned}$$

By hypothesis

$$-w^\top v_j + b + \theta = \eta + \theta \quad \text{for some } j \notin E_\mu,$$

and by the choice of θ,

$$w^\top u_i - (b+\theta) = \eta + \theta \quad \text{for some } i \notin E_\lambda.$$

The new value of the objective function is

$$\begin{aligned}
\omega(\theta) &= \frac{1}{2} w^\top w - \nu(\eta + \theta) + \frac{1}{p+q}\biggl(\sum_{i \in E_\lambda} (\epsilon_i + 2\theta) + \sum_{j \in E_\mu} \xi_j\biggr)\\
&= \frac{1}{2} w^\top w - \nu\eta + \frac{1}{p+q}\biggl(\sum_{i \in E_\lambda} \epsilon_i + \sum_{j \in E_\mu} \xi_j\biggr) - \left(\nu - \frac{2p_{sf}}{p+q}\right)\theta.
\end{aligned}$$

By Proposition 54.1 we have

$$\max\left\{\frac{2p_f}{p+q}, \frac{2q_f}{p+q}\right\} \leq \nu$$

and $p_{sf} \leq p_f$ and $q_{sf} \leq q_f$, which implies that

$$\nu - \frac{2p_{sf}}{p+q} \geq 0, \tag{$*_1$}$$

and so $\omega(\theta) \leq \omega(0)$. If inequality $(*_1)$ is strict, then this contradicts the optimality of the original solution. Therefore, $\nu = 2p_{sf}/(p+q)$, $\omega(\theta) = \omega(0)$, and $(w, b+\theta, \eta+\theta, \epsilon + 2\theta, \xi)$ is an optimal solution such that

$$\begin{aligned}
w^\top u_i - (b+\theta) &= \eta + \theta\\
-w^\top v_j + b + \theta &= \eta + \theta
\end{aligned}$$

for some $i \notin E_\lambda$ and some $j \notin E_\mu$.

Case 1b. We have $-w^\top v_j + b > \eta$ for all $j \notin E_\mu$. Our strategy is to increase η and the errors by a small θ in such a way that some inequality becomes an equation for some $i \notin E_\lambda$ or for some $j \notin E_\mu$. Geometrically, this corresponds to increasing the width of the slab, keeping the separating hyperplane unchanged. See Figures 54.8 and 54.9. Then we are reduced to *Case 1a* or *Case 2a*.