where $c \in \mathbb{R}^n$.

We would like to find necessary conditions for f_μ to have a maximum on

$$U = \{x \in \mathbb{R}^n_{++} \mid Ax = b\},$$

or equivalently to solve the following problem:

$$\begin{aligned} &\text{maximize} && f_\mu(x) \\ &\text{subject to} && \\ & && Ax = b \\ & && x > 0. \end{aligned}$$

Since maximizing f_μ is equivalent to minimizing $-f_\mu$, by Proposition 50.9, if x is an optimal of the above problem then there is some $y \in \mathbb{R}^m$ such that

$$-\nabla f_\mu(x) + A^\top y = 0.$$

Since

$$\nabla f_\mu(x) = \begin{pmatrix} c_1 + \frac{\mu}{x_1} \\ \vdots \\ c_n + \frac{\mu}{x_n} \end{pmatrix},$$

we obtain the equation

$$c + \mu \begin{pmatrix} \frac{1}{x_1} \\ \vdots \\ \frac{1}{x_n} \end{pmatrix} = A^\top y.$$

To obtain a more convenient formulation, we define $s \in \mathbb{R}^n_{++}$ such that

$$s = \mu \begin{pmatrix} \frac{1}{x_1} \\ \vdots \\ \frac{1}{x_n} \end{pmatrix}$$

which implies that

$$\begin{pmatrix} s_1 x_1 & \cdots & s_n x_n \end{pmatrix} = \mu \mathbf{1}_n^\top,$$

and we obtain the following necessary conditions for f_μ to have a maximum:

$$\begin{aligned} Ax &= b \\ A^\top y - s &= c \\ \begin{pmatrix} s_1 x_1 & \cdots & s_n x_n \end{pmatrix} &= \mu \mathbf{1}_n^\top \\ s, x &> 0. \end{aligned}$$