(V) Two real eigenvalues α, β. The matrix Γ has the form

$$\Gamma = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 1 \\ 0 & 0 & \beta \end{pmatrix},$$

with $\alpha, \beta \in \mathbb{R}$ nonzero and distinct. The homography h, which is illustrated in Figure 26.28, has two fixed points P and Q. The line $\langle P, Q \rangle$ is invariant under h, and there is is another line Δ through Q invariant under h. The restriction of h to Δ is parabolic, and the restriction of h to $\langle P, Q \rangle$ is hyperbolic.

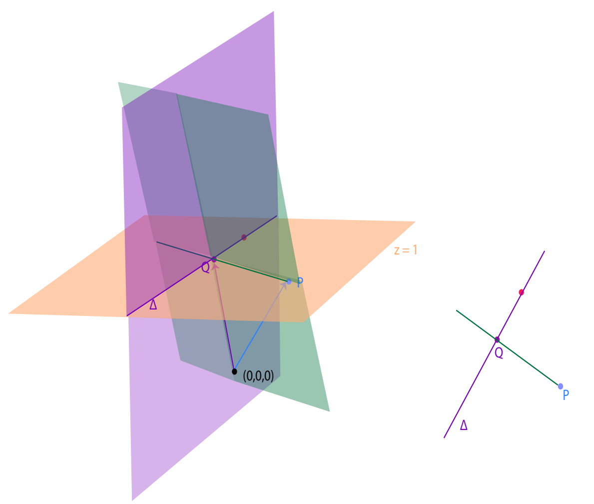

Figure 26.28: Case (V): The left figure is the hyperplane representation of $\mathbb{RP}^2$ and a homography with fixed points P, Q and invariant line Δ. Both the purple and green (linear) hyperplanes are invariant under the homography, but the invariance is not given by the identity map.

(VI) One real eigenvalue α. The matrix Γ has the form

$$\Gamma = \begin{pmatrix} \alpha & 1 & 0 \\ 0 & \alpha & 1 \\ 0 & 0 & \alpha \end{pmatrix},$$

with $\alpha \in \mathbb{R}$ nonzero. The homography h, which is illustrated in Figure 26.29, has one fixed point P, and a line Δ invariant under h containing P. The restriction of h to Δ is parabolic.

For the classification of the homographies of $\mathbb{CP}^2$, Case (II) becomes Case (I).