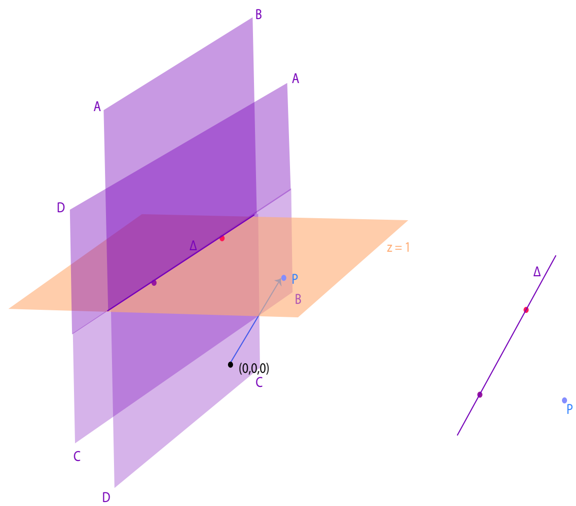

Figure 26.25: Case (II): The left figure is the hyperplane representation of $\mathbb{RP}^2$ and a homography with fixed point P and invariant line Δ. The purple (linear) hyperplane maps to itself under a rotation and rescaling.

(III) Two real eigenvalues α, β. The matrix Γ has the form

$$\Gamma = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \beta \end{pmatrix},$$

with $\alpha, \beta \in \mathbb{R}$ nonzero and distinct. The homography h, as illustrated in Figure 26.26, has one fixed point P, and a line Δ invariant under h and not containing P. The restriction of h to Δ is the identity. Every line through P is invariant under h and the restriction of h to this line is hyperbolic.

(IV) One real eigenvalue α. The matrix Γ has the form

$$\Gamma = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 1 \\ 0 & 0 & \alpha \end{pmatrix},$$

with $\alpha \in \mathbb{R}$ nonzero. As illustrated by Figure 26.27, the homography h has one fixed point P, and a line Δ invariant under h containing P. The restriction of h to Δ is the identity. Every line through P is invariant under h and the restriction of h to this line is parabolic.