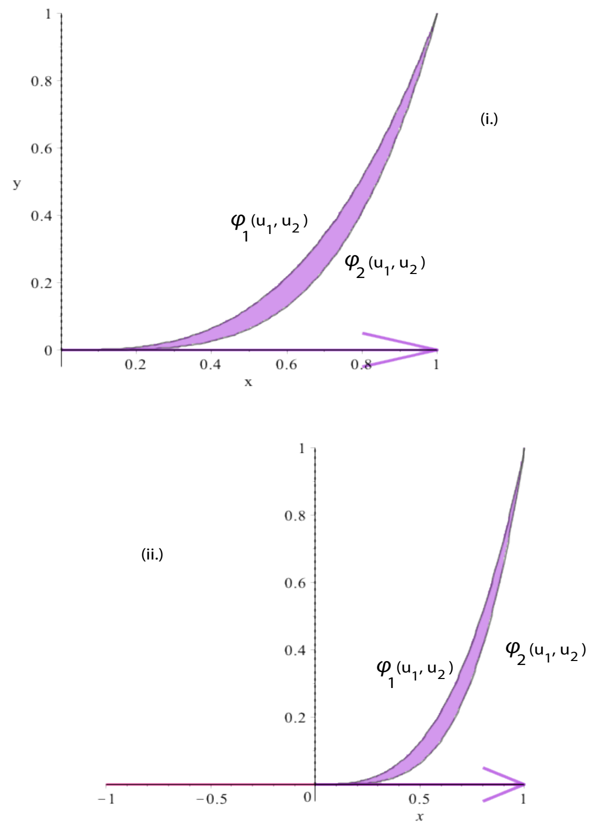

Figure 50.8: Figures (i.) and (ii.) illustrate the purple moon shaped region associated with Example 50.2. Figure (i.) also illustrates $C(0)$, the cone of feasible directions, while Figure (ii.) illustrates the strict containment of $C(0)$ in $C^*(0)$.

(2) *If the constraints are qualified at u (and the functions φ_i are continuous at u for all $i \notin I(u)$ if we only assume φ_i differentiable at u for all $i \in I(u)$), then*

$$C(u) = C^*(u).$$

Proof. (1) For every $i \in I(u)$, since $\varphi_i(v) \leq 0$ for all $v \in U$ and $\varphi_i(u) = 0$, the function $-\varphi_i$ has a local minimum at u with respect to U, so by Proposition 50.1(2), we have

$$(-\varphi_i')_u(v) \geq 0 \quad \text{for all } v \in C(u),$$

which is equivalent to $(\varphi_i')_u(v) \leq 0$ for all $v \in C(u)$ and for all $i \in I(u)$, that is, $u \in C^*(u)$.

(2)(a) First, let us assume that φ_i is affine for every $i \in I(u)$. Recall that φ_i must be given by $\varphi_i(v) = h_i(v) + c_i$ for all $v \in V$, where h_i is a linear form and $c_i \in \mathbb{R}$. Since the derivative of a linear map at any point is itself,

$$(\varphi_i')_u(v) = h_i(v) \quad \text{for all } v \in V.$$