Since the matrix $P$ is symmetric positive definite, the functional

$$F(\lambda, \mu) = -G(\lambda, \mu) = \frac{1}{2} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} P \begin{pmatrix} \lambda \\ \mu \end{pmatrix}$$

is strictly convex and $U$ is convex, so by Theorem 40.13(2,4), if it has a minimum, then it is unique. Consider the convex set

$$U^\kappa = \left\{ \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \in \mathbb{R}_+^{p+q} \;\middle|\; \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 0 \\ (p+q)K_s\kappa\nu \end{pmatrix} \right\}.$$

Observe that

$$\kappa U = \left\{ \begin{pmatrix} \kappa\lambda \\ \kappa\mu \end{pmatrix} \in \mathbb{R}_+^{p+q} \;\middle|\; \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top \end{pmatrix} \begin{pmatrix} \kappa\lambda \\ \kappa\mu \end{pmatrix} = \begin{pmatrix} 0 \\ (p+q)K_s\kappa\nu \end{pmatrix} \right\} = U^\kappa.$$

By Theorem 40.13(3), $(\lambda, \mu) \in U$ is a minimum of $F$ over $U$ iff

$$dF_{\lambda,\mu} \begin{pmatrix} \lambda' - \lambda \\ \mu' - \mu \end{pmatrix} \geq 0 \quad \text{for all} \quad \begin{pmatrix} \lambda' \\ \mu' \end{pmatrix} \in U.$$

Since

$$dF_{\lambda,\mu} \begin{pmatrix} \lambda' - \lambda \\ \mu' - \mu \end{pmatrix} = \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} P \begin{pmatrix} \lambda' - \lambda \\ \mu' - \mu \end{pmatrix}$$

the above conditions are equivalent to

$$\begin{aligned} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} P \begin{pmatrix} \lambda' - \lambda \\ \mu' - \mu \end{pmatrix} &\geq 0 \\ \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} &= \begin{pmatrix} 0 \\ (p+q)K_s\nu \end{pmatrix} \\ &\lambda, \lambda' \in \mathbb{R}_+^p, \; \mu, \mu' \in \mathbb{R}_+^q. \end{aligned}$$

Since $\kappa > 0$, by multiplying the above inequality by $\kappa^2$ and the equations by $\kappa$, the following conditions hold:

$$\begin{aligned} \begin{pmatrix} \kappa\lambda^\top & \kappa\mu^\top \end{pmatrix} P \begin{pmatrix} \kappa\lambda' - \kappa\lambda \\ \kappa\mu' - \kappa\mu \end{pmatrix} &\geq 0 \\ \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top \end{pmatrix} \begin{pmatrix} \kappa\lambda \\ \kappa\mu \end{pmatrix} &= \begin{pmatrix} 0 \\ (p+q)K_s\kappa\nu \end{pmatrix} \\ &\kappa\lambda, \kappa\lambda' \in \mathbb{R}_+^p, \; \kappa\mu, \kappa\mu' \in \mathbb{R}_+^q. \end{aligned}$$

By Theorem 40.13(3), $(\kappa\lambda, \kappa\mu) \in U^\kappa$ is a minimum of $F$ over $U^\kappa$, and because $F$ is strictly convex and $U^\kappa$ is convex, if $F$ has a minimum over $U^\kappa$, then $(\kappa\lambda, \kappa\mu) \in U^\kappa$ is the unique minimum. Therefore, $\lambda^\kappa = \kappa\lambda, \; \mu^\kappa = \kappa\mu$.