where $|\{1, \ldots, m\} - (E_\lambda \cup E_\mu)| \geq 2$.

If our optimal solution does not have a blue support vector and a red support vector, then either $w^\top x_i + b - y_i < \epsilon$ for all $i \notin (E_\lambda \cup E_\mu)$ or $-w^\top x_i - b + y_i < \epsilon$ for all $i \notin (E_\lambda \cup E_\mu)$.

*Case 1*. We have

$$\begin{aligned} w^\top x_i + b - y_i &< \epsilon & & i \notin (E_\lambda \cup E_\mu) \\ -w^\top x_i - b + y_i &\leq \epsilon & & i \notin (E_\lambda \cup E_\mu). \end{aligned}$$

There are two subcases.

*Case 1a*. Assume that there is some $j \notin (E_\lambda \cup E_\mu)$ such that $-w^\top x_j - b + y_j = \epsilon$. Our strategy is to decrease $\epsilon$ and increase $b$ by a small amount $\theta$ in such a way that some inequality $w^\top x_i + b - y_i < \epsilon$ becomes an equation for some $i \notin (E_\lambda \cup E_\mu)$. Geometrically, this amounts to raising the separating hyperplane $H_{w,b}$ and decreasing the width of the slab, keeping the red margin hyperplane unchanged. See Figure 56.7.

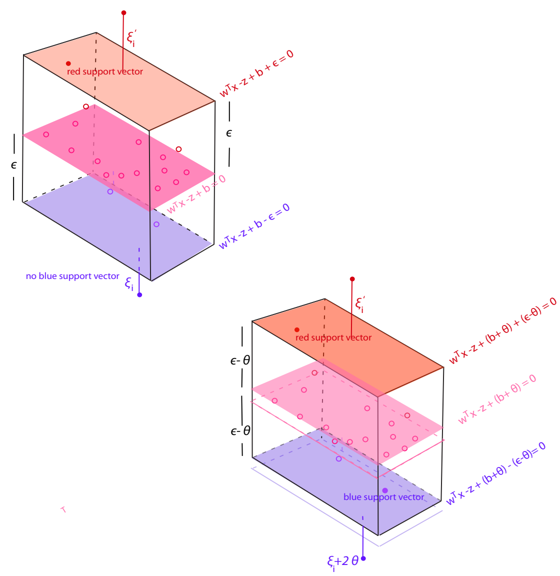


Figure 56.7: In this illustration points within the $\epsilon$-tube are denoted by open circles. In the original, upper left configuration, there is no blue support vector. By raising the pink separating hyperplane and decreasing the width of the slab, we end up with a blue support vector.

The inequalities imply that

$$-\epsilon \leq w^\top x_i + b - y_i < \epsilon.$$

Let us pick $\theta$ such that

$$\theta = (1/2) \min\{\epsilon - w^\top x_i - b + y_i \mid i \notin (E_\lambda \cup E_\mu)\}.$$