

2	-1	0	0	-2	0	0	-4
$u_2 = 1$	$-1/2$	1	0	$-1/2$	0	0	$1/2$
$u_3 = 3$	$3/2$	0	1	$3/2$	0	0	$-1/2$
$u_6 = 0$	$-3/2$	0	0	$-3/2$	0	1	$1/2$
$u_5 = 2$	1	0	0	0	1	0	0

Since all the reduced cost are  $\leq 0$ , we have reached an optimal solution, namely  $(0, 1, 3, 0, 2, 0, 0, 0)$ , with optimal value  $-2$ .

The progression of the simplex algorithm from one basic feasible solution to another corresponds to the visit of vertices of the polyhedron  $\mathcal{P}$  associated with the constraints of the linear program illustrated in Figure 46.4.

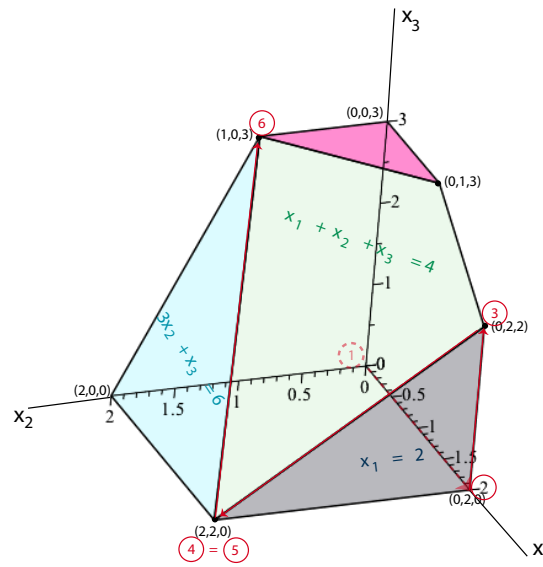


Figure 46.4: The polytope  $\mathcal{P}$  associated with the linear program optimized by the tableau method. The red arrowed path traces the progression of the simplex method from the origin to the vertex  $(0, 1, 3)$ .

As a final comment, if it is necessary to run Phase I of the simplex algorithm, in the event that the simplex algorithm terminates with an optimal solution  $(u^*, 0_m)$  and a basis  $K^*$  such that some  $u_i = 0$ , then the basis  $K^*$  contains indices of basic columns  $A^j$  corresponding to slack variables that need to be *driven out* of the basis. This is easy to achieve by performing a pivoting step involving some other column  $j^+$  corresponding to one of the original variables (not a slack variable) for which  $(\gamma_{K^*})_i^{j^+} \neq 0$ . In such a step, it doesn't matter whether  $(\gamma_{K^*})_i^{j^+} < 0$  or  $(\bar{c}_{K^*})_{j^+} \leq 0$ . If the original matrix  $A$  has no redundant equations, such a step