Gander, Golub, and von Matt consider the following problem: Given an $(n+m)\times(n+m)$ real symmetric matrix A (with $n > 0$), an $(n+m)\times m$ matrix N with full rank, and a nonzero vector $t \in \mathbb{R}^m$ with $\left\|(N^\top)^+ t\right\| < 1$ (where $(N^\top)^+$ denotes the pseudo-inverse of $N^\top$),

$$\begin{array}{ll}\text{minimize} & x^\top A x\\ \text{subject to} & x^\top x = 1,\ N^\top x = t,\ x \in \mathbb{R}^{n+m}.\end{array}$$

The condition $\left\|(N^\top)^+ t\right\| < 1$ ensures that the problem has a solution and is not trivial. The authors begin by proving that the affine constraint $N^\top x = t$ can be eliminated. One way to do so is to use a QR decomposition of N. If

$$N = P\begin{pmatrix} R\\ 0\end{pmatrix},$$

where P is an orthogonal $(n+m)\times(n+m)$ matrix and R is an $m\times m$ invertible upper triangular matrix, then if we observe that

$$\begin{aligned} x^\top A x &= x^\top P P^\top A P P^\top x,\\ N^\top x &= (R^\top\ 0) P^\top x = t,\\ x^\top x &= x^\top P P^\top x = 1,\end{aligned}$$

and if we write

$$P^\top A P = \begin{pmatrix} B & \Gamma^\top\\ \Gamma & C\end{pmatrix},$$

where B is an $m\times m$ symmetric matrix, C is an $n\times n$ symmetric matrix, Γ is an $m\times n$ matrix, and

$$P^\top x = \begin{pmatrix} y\\ z\end{pmatrix},$$

with $y \in \mathbb{R}^m$ and $z \in \mathbb{R}^n$, we then get

$$\begin{aligned} x^\top A x &= y^\top B y + 2z^\top \Gamma y + z^\top C z,\\ R^\top y &= t,\\ y^\top y + z^\top z &= 1.\end{aligned}$$

Thus

$$y = (R^\top)^{-1} t,$$

and if we write

$$s^2 = 1 - y^\top y > 0$$

and

$$b = \Gamma y,$$