10.6 Summary

The main concepts and results of this chapter are listed below:

- Iterative methods. Splitting A as $A = M - N$.
- *Convergence of a sequence of vectors or matrices*.
- A criterion for the convergence of the sequence (B^k) of powers of a matrix B to zero in terms of the spectral radius $\rho(B)$.
- A characterization of the spectral radius $\rho(B)$ as the limit of the sequence $(\|B^k\|^{1/k})$.
- A criterion of the convergence of iterative methods.
- Asymptotic behavior of iterative methods.
- Splitting A as $A = D - E - F$, and the methods of *Jacobi*, *Gauss–Seidel*, and *relaxation* (and *SOR*).
- The *Jacobi matrix*, $J = D^{-1}(E + F)$.
- The *Gauss–Seidel matrix*, $\mathcal{L}_1 = (D - E)^{-1}F$.
- The *matrix of relaxation*, $\mathcal{L}_\omega = (D - \omega E)^{-1}((1 - \omega)D + \omega F)$.
- Convergence of iterative methods: a general result when $A = M - N$ is Hermitian positive definite.
- A sufficient condition for the convergence of the methods of Jacobi, Gauss–Seidel, and relaxation. The *Ostrowski-Reich theorem*: A is Hermitian positive definite and $\omega \in (0, 2)$.
- A necessary condition for the convergence of the methods of Jacobi , Gauss–Seidel, and relaxation: $\omega \in (0, 2)$.
- The case of tridiagonal matrices (possibly by blocks). Simultaneous convergence or divergence of Jacobi's method and Gauss–Seidel's method, and comparison of the spectral radii of $\rho(J)$ and $\rho(\mathcal{L}_1)$: $\rho(\mathcal{L}_1) = (\rho(J))^2$.
- The case of tridiagonal Hermitian positive definite matrices (possibly by blocks). The methods of Jacobi, Gauss–Seidel, and relaxation, all converge.
- In the above case, there is a unique optimal relaxation parameter for which $\rho(\mathcal{L}_{\omega_0}) < \rho(\mathcal{L}_1) = (\rho(J))^2 < \rho(J)$ (if $\rho(J) \neq 0$).