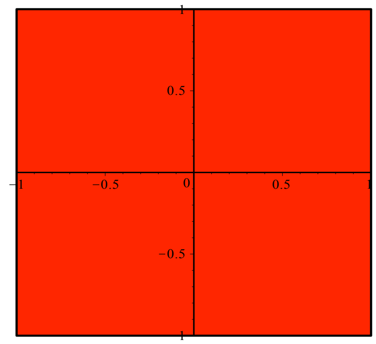

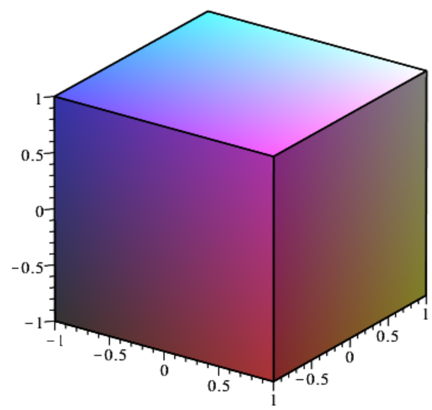

Figure 9.3: The top figure is $\{x \in \mathbb{R}^2 \mid \|x\|_\infty \leq 1\}$, while the bottom figure is $\{x \in \mathbb{R}^3 \mid \|x\|_\infty \leq 1\}$.

for all $x, y \in \mathbb{R}$ and all θ with $0 \leq \theta \leq 1$.

Since the case $\alpha\beta = 0$ is trivial, let us assume that $\alpha > 0$ and $\beta > 0$. If we replace θ by $1/p$, x by $p \log \alpha$ and y by $q \log \beta$, then we get

$$e^{\frac{1}{p} p \log \alpha + \frac{1}{q} q \log \beta} \leq \frac{1}{p} e^{p \log \alpha} + \frac{1}{q} e^{q \log \beta},$$

which simplifies to

$$\alpha\beta \leq \frac{\alpha^p}{p} + \frac{\beta^q}{q},$$

as claimed.

We will now prove that for any two vectors $u, v \in E$, (where E is of dimension n), we have

$$\sum_{i=1}^{n} |u_i v_i| \leq \|u\|_p \|v\|_q . \tag{$**$}$$