Step 1. Compute the numbers $c_j - \sum_{k\in K} \gamma_k^j c_k = c_j - \beta_K A^j$ for all $j \notin K$, and for this, compute $\beta_K^\top$ as the solution of the system

$$A_K^\top \beta_K^\top = c_K^\top.$$

If $c_j - \beta_K A^j \leq 0$ for all $j \notin K$, stop and return the optimal solution u (Case (A)).

Step 2. If Case (B) arises, use a pivot rule to determine which index $j^+ \notin K$ should enter the new basis K^+ (the condition $c_{j^+} - \beta_K A^{j^+} > 0$ should hold).

Step 3. Compute $\max_{k\in K} \gamma_k^{j^+}$. For this, solve the linear system

$$A_K \gamma_K^{j^+} = A^{j^+}.$$

Step 4. If $\max_{k\in K} \gamma_k^{j^+} \leq 0$, then stop and report that Linear Program (P) is unbounded (Case (B1)).

Step 5. If $\max_{k\in K} \gamma_k^{j^+} > 0$, use the ratios $u_k/\gamma_k^{j^+}$ for all $k \in K$ such that $\gamma_k^{j^+} > 0$ to compute θ^{j^+}, and use a pivot rule to determine which index $k^- \in K$ such that $\theta^{j^+} = u_{k^-}/\gamma_{k^-}^{j+}$ should leave K (Case (B2)).

If $\max_{k\in K} \gamma_k^{j^+} = 0$, then use a pivot rule to determine which index k^- for which $\gamma_{k^-}^{j^+} > 0$ should leave the basis K (Case (B3)).

Step 6. Update u, K, and A_K, to u^+ and K^+, and A_{K^+}. During this step, given the basis K specified by the sequence $K = (k_1, \ldots, k_\ell, \ldots, k_m)$, with $k^- = k_\ell$, then K^+ is the sequence obtained by replacing k_ℓ by the incoming index j^+, so $K^+ = (k_1, \ldots, j^+, \ldots, k_m)$ with j^+ in the ℓth slot.

The vector u is easily updated. To compute A_{K^+} from A_K we take advantage of the fact that A_K and A_{K^+} only differ by a *single column*, namely the ℓth column A^{j^+}, which is given by the linear combination

$$A^{j^+} = A_K \gamma_K^{j^+}.$$

To simplify notation, denote $\gamma_K^{j^+}$ by γ, and recall that $k^- = k_\ell$. If $K = (k_1, \ldots, k_m)$, then $A_K = [A^{k_1} \cdots A^{k^-} \cdots A^{i_m}]$, and since A_{K^+} is the result of replacing the ℓth column A^{k^-} of A_K by the column A^{j^+}, we have

$$A_{K^+} = [A^{k_1} \cdots A^{j^+} \cdots A^{i_m}] = [A^{k_1} \cdots A_K\gamma \cdots A^{i_m}] = A_K E(\gamma),$$

where $E(\gamma)$ is the following invertible matrix obtained from the identity matrix I_m by replacing its ℓth column by γ:

$$E(\gamma) = \begin{pmatrix} 1 & & & \gamma_1 & & & \\ & \ddots & & \vdots & & & \\ & & 1 & \gamma_{\ell-1} & & & \\ & & & \gamma_\ell & & & \\ & & & \gamma_{\ell+1} & 1 & & \\ & & & \vdots & & \ddots & \\ & & & \gamma_m & & & 1 \end{pmatrix}.$$