by permuting rows i and k, *i.e.*,

$$P(i,k) = \begin{pmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 0 & & & & 1 & & \\ & & & 1 & & & & & \\ & & & & \ddots & & & & \\ & & & & & 1 & & & \\ & & 1 & & & & 0 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \end{pmatrix}.$$

For example, if $m = 3$,

$$P(1,3) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

then

$$P(1,3)B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & \cdots & \cdots & \cdots b_{1n} \\ b_{21} & b_{22} & \cdots & \cdots & \cdots b_{2n} \\ b_{31} & b_{32} & \cdots & \cdots & \cdots b_{3n} \end{pmatrix} = \begin{pmatrix} b_{31} & b_{32} & \cdots & \cdots & \cdots b_{3n} \\ b_{21} & b_{22} & \cdots & \cdots & \cdots b_{2n} \\ b_{11} & b_{12} & \cdots & \cdots & \cdots b_{1n} \end{pmatrix}.$$

Observe that $\det(P(i,k)) = -1$. Furthermore, $P(i,k)$ is *symmetric* ($P(i,k)^\top = P(i,k)$), and

$$P(i,k)^{-1} = P(i,k).$$

During the permutation Step (2), if row k and row i need to be permuted, the matrix A is multiplied on the left by the matrix P_k such that $P_k = P(i,k)$, else we set $P_k = I$.

Adding β times row j to row i (with $i \neq j$) is achieved by multiplying A on the left by the *elementary matrix*,

$$E_{i,j;\beta} = I + \beta e_{i\,j},$$

where

$$(e_{i\,j})_{k\,l} = \begin{cases} 1 & \text{if } k = i \text{ and } l = j \\ 0 & \text{if } k \neq i \text{ or } l \neq j, \end{cases}$$

i.e.,

$$E_{i,j;\beta} = \begin{pmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 1 & & & & & & \\ & & & 1 & & & & & \\ & & & & \ddots & & & & \\ & & & & & 1 & & & \\ & & \beta & & & & 1 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \end{pmatrix} \quad \text{or} \quad E_{i,j;\beta} = \begin{pmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 1 & & & & \beta & & \\ & & & 1 & & & & & \\ & & & & \ddots & & & & \\ & & & & & 1 & & & \\ & & & & & & 1 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \end{pmatrix},$$