Note that

$$E_2 \prod E_1 = \{ \{ \langle 2, v \rangle, \langle 1, u \rangle \} \mid v \in E_2, \ u \in E_1 \} = E_1 \prod E_2.$$

Thus, every member $\{\langle 1, u \rangle, \langle 2, v \rangle\}$ of $E_1 \coprod E_2$ can be viewed as an *unordered pair* consisting of the two vectors u and v, tagged with the index 1 and 2, respectively.

Remark: In fact, $E_1 \coprod E_2$ is just the product $\prod_{i \in \{1,2\}} E_i$ of the family $(E_i)_{i \in \{1,2\}}$.



This is not to be confused with the cartesian product $E_1 \times E_2$. The vector space $E_1 \times E_2$ is the set of all ordered pairs $\langle u, v \rangle$, where $u \in E_1$, and $v \in E_2$, with addition and multiplication by a scalar defined such that

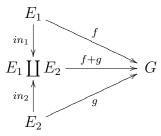
$$\langle u_1, v_1 \rangle + \langle u_2, v_2 \rangle = \langle u_1 + u_2, v_1 + v_2 \rangle,$$

$$\lambda \langle u, v \rangle = \langle \lambda u, \lambda v \rangle.$$

There is a bijection between $\prod_{i \in \{1,2\}} E_i$ and $E_1 \times E_2$, but as we just saw, elements of $\prod_{i \in \{1,2\}} E_i$ are certain sets. The product $E_1 \times \cdots \times E_n$ of any number of vector spaces can also be defined. We will do this shortly.

The following property holds.

Proposition 6.1. Given any two vector spaces, E_1 and E_2 , the set $E_1 \coprod E_2$ is a vector space. For every pair of linear maps, $f: E_1 \to G$ and $g: E_2 \to G$, there is a unique linear map, $f+g: E_1 \coprod E_2 \to G$, such that $(f+g) \circ in_1 = f$ and $(f+g) \circ in_2 = g$, as in the following diagram:



Proof. Define

$$(f+g)(\{\langle 1, u \rangle, \langle 2, v \rangle\}) = f(u) + g(v),$$

for every $u \in E_1$ and $v \in E_2$. It is immediately verified that f + g is the unique linear map with the required properties.

We already noted that $E_1 \coprod E_2$ is in bijection with $E_1 \times E_2$. If we define the *projections* $\pi_1 \colon E_1 \coprod E_2 \to E_1$ and $\pi_2 \colon E_1 \coprod E_2 \to E_2$, such that

$$\pi_1(\{\langle 1, u \rangle, \langle 2, v \rangle\}) = u,$$

and

$$\pi_2(\{\langle 1, u \rangle, \langle 2, v \rangle\}) = v,$$

we have the following proposition.