The proof is very similar to the proof of the corresponding formula in Section 56.5. By Theorem 56.5, there is some suppor vector x_i, say

$$w^\top x_{i_0} + b - y_{i_0} = \epsilon \quad \text{or} \quad - w^\top x_{j_0} - b + y_{j_0} = \epsilon.$$

Then we find an equation expressing ϵ in terms of λ, μ and w, provided that $\nu \neq 2p_f/m$ and $\nu \neq 2q_f/m$. The proof is analogous to the proof of Proposition 54.4 and is left as an exercise. $\square$

56.3 Solving ν-Regression Using ADMM

The quadratic functional $F(\lambda, \mu)$ occurring in the dual program given by

$$F(\lambda, \mu) = \frac{1}{2} \sum_{i,j=1}^{m} (\lambda_i - \mu_i)(\lambda_j - \mu_j) x_i^\top x_j + \sum_{i=1}^{m} (\lambda_i - \mu_i) y_i$$

is not of the form $\frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} P \begin{pmatrix}\lambda \\ \mu\end{pmatrix} + q^\top \begin{pmatrix}\lambda \\ \mu\end{pmatrix}$, but it can be converted in such a form using a trick. First, if we let $\mathbf{K}$ be the $m \times m$ symmetric matrix $\mathbf{K} = XX^\top = (x_i^\top x_j)$, then we have

$$F(\lambda, \mu) = \frac{1}{2}(\lambda^\top - \mu^\top)\mathbf{K}(\lambda - \mu) + y^\top \lambda - y^\top \mu.$$

Consequently, if we define the $2m \times 2m$ symmetric matrix P by

$$P = \begin{pmatrix} XX^\top & -XX^\top \\ -XX^\top & XX^\top \end{pmatrix} = \begin{pmatrix} \mathbf{K} & -\mathbf{K} \\ -\mathbf{K} & \mathbf{K} \end{pmatrix}$$

and the $2m \times 1$ matrix q by

$$q = \begin{pmatrix} y \\ -y \end{pmatrix},$$

it is easy to check that

$$F(\lambda, \mu) = \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} P \begin{pmatrix}\lambda \\ \mu\end{pmatrix} + q^\top \begin{pmatrix}\lambda \\ \mu\end{pmatrix} = \frac{1}{2}\lambda^\top \mathbf{K}\lambda + \frac{1}{2}\mu^\top \mathbf{K}\mu - \lambda^\top \mathbf{K}\mu + y^\top\lambda - y^\top\mu. \quad (*_q)$$

Since

$$\frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} P \begin{pmatrix}\lambda \\ \mu\end{pmatrix} = \frac{1}{2}(\lambda^\top - \mu^\top)\mathbf{K}(\lambda - \mu)$$

and the matrix $\mathbf{K} = XX^\top$ is symmetric positive semidefinite, the matrix P is also symmetric