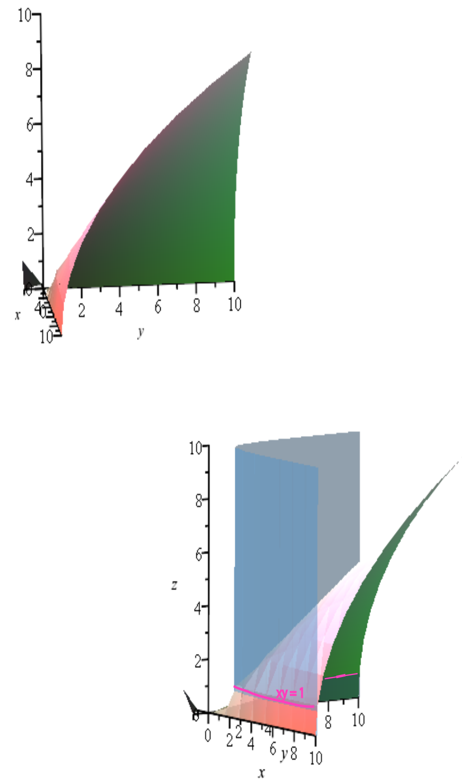

Figure 8.6: Two views of the surface $xy = z^2$ in $\mathbb{R}^3$. The intersection of the surface with a constant z plane results in a hyperbola. The region associated with the 2×2 symmetric positive definite matrices lies in "front" of the green side.

According to the above criterion, the two matrices on the left-hand side are real symmetric positive definite, but the matrix on the right-hand side is not even symmetric, and

$$\begin{pmatrix} -6 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2/\sqrt{2} \\ -1/\sqrt{2} & 5/\sqrt{2} \end{pmatrix} \begin{pmatrix} -6 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 & 1 \end{pmatrix} \begin{pmatrix} 2/\sqrt{2} \\ 11/\sqrt{2} \end{pmatrix} = -1/\sqrt{2},$$

even though its eigenvalues are both real and positive.

Next we show that a real symmetric positive definite matrix has a special LU-factorization of the form $A = BB^\top$, where B is a lower-triangular matrix whose diagonal elements are strictly positive. This is the *Cholesky factorization*.

First we note that a symmetric positive definite matrix satisfies the condition of Proposition 8.2.

Proposition 8.9. *If A is a real symmetric positive definite matrix, then $A(1 : k, 1 : k)$ is symmetric positive definite and thus invertible for $k = 1, \ldots, n$.*