Proof. The second isomorphism follows from the canonical isomorphism $(\bigwedge^n(E))^* \cong \bigwedge^n(E^*)$ and the canonical isomorphism $(\bigwedge^n(E))^* \cong \mathrm{Alt}^n(E;K)$ given by Proposition 34.5. $\square$

Remarks:

1. The isomorphism $\mu\colon \bigwedge^n(E^*) \cong \mathrm{Alt}^n(E;K)$ discussed above can be described explicitly as the linear extension of the map given by
$$\mu(v_1^* \wedge \cdots \wedge v_n^*)(u_1, \ldots, u_n) = \det(v_j^*(u_i)).$$

2. The canonical isomorphism of Proposition 34.10 holds under more general conditions. Namely, that K is a commutative ring with identity and that E is a finitely-generated projective K-module (see Definition 35.7). See Bourbaki, [25] (Chapter III, §11, Section 5, Proposition 7).

3. Variants of our isomorphism μ are found in the literature. For example, there is a version μ', where
$$\mu' = \frac{1}{n!}\mu,$$
with the factor $\frac{1}{n!}$ added in front of the determinant. Each version has its its own merits and inconveniences. Morita [129] uses μ' because it is more convenient than μ when dealing with characteristic classes. On the other hand, μ' may not be defined for a field with positive characteristic, and when using μ', some extra factor is needed in defining the wedge operation of alternating multilinear forms (see Section 34.5) and for exterior differentiation. The version μ is the one adopted by Warner [186], Knapp [104], Fulton and Harris [68], and Cartan [34, 35].

If $f\colon E \to F$ is any linear map, by transposition we get a linear map $f^\top\colon F^* \to E^*$ given by
$$f^\top(v^*) = v^* \circ f, \qquad v^* \in F^*.$$
Consequently, we have
$$f^\top(v^*)(u) = v^*(f(u)), \qquad \text{for all } u \in E \text{ and all } v^* \in F^*.$$
For any $p \geq 1$, the map
$$(u_1, \ldots, u_p) \mapsto f(u_1) \wedge \cdots \wedge f(u_p)$$
from E^p to $\bigwedge^p F$ is multilinear alternating, so it induces a unique linear map $\bigwedge^p f\colon \bigwedge^p E \to \bigwedge^p F$ making the following diagram commute
$$\begin{tikzcd} E^p \arrow[r, "\iota_\wedge"] \arrow[dr] & \bigwedge^p E \arrow[d, "\bigwedge^p f"] \\ & \bigwedge^p F, \end{tikzcd}$$