with $d = \det(A)$, and where S is a product of elementary matrices of the form $E_{k,\ell;\beta}$.

In particular, every matrix in $\mathbf{SL}(n)$ (the group of invertible $n \times n$ matrices A with $\det(A) = +1$) can be written as a product of elementary matrices of the form $E_{k,\ell;\beta}$. Prove that at most $n(n+1) - 2$ such transformations are needed.

(3) Prove that every matrix in $\mathbf{SL}(n)$ can be written as a product of at most $(n-1)(\max\{n, 3\} + 1)$ elementary matrices of the form $E_{k,\ell;\beta}$.

Problem 8.12. A matrix A is called *strictly column diagonally dominant* iff

$$|a_{j\,j}| > \sum_{i=1,\, i\neq j}^{n} |a_{i\,j}|, \quad \text{for } j = 1, \ldots, n.$$

Prove that if A is strictly column diagonally dominant, then Gaussian elimination with partial pivoting does not require pivoting, and A is invertible.

Problem 8.13. (1) Find a lower triangular matrix E such that

$$E \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix}.$$

(2) What is the effect of the product (on the left) with

$$E_{4,3;-1}E_{3,2;-1}E_{4,3;-1}E_{2,1;-1}E_{3,2;-1}E_{4,3;-1}$$

on the matrix

$$Pa_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{pmatrix}.$$

(3) Find the inverse of the matrix Pa_3.

(4) Consider the $(n+1) \times (n+1)$ Pascal matrix Pa_n whose ith row is given by the binomial coefficients

$$\binom{i-1}{j-1},$$

with $1 \leq i \leq n+1$, $1 \leq j \leq n+1$, and with the usual convention that

$$\binom{0}{0} = 1, \quad \binom{i}{j} = 0 \quad \text{if} \quad j > i.$$