Problem 33.5. Induct on $m \geq 2$ to prove the canonical isomorphism

$$V^{\otimes m} \otimes V^{\otimes n} \cong V^{\otimes(m+n)}.$$

Use this isomorphism to show that $\cdot\colon V^{\otimes m} \times V^{\otimes n} \longrightarrow V^{\otimes(m+n)}$ defined as

$$(v_1 \otimes \cdots \otimes v_m) \cdot (w_1 \otimes \cdots \otimes w_n) = v_1 \otimes \cdots \otimes v_m \otimes w_1 \otimes \cdots \otimes w_n.$$

induces a multiplication on $T(V)$.
Hint. See Jacobson [99], Section 3.9, or Bertin [15], Chapter 4, Section 2.).

Problem 33.6. Prove Proposition 33.19.
Hint. See Knapp [104] (Appendix A, Proposition A.14) or Bertin [15] (Chapter 4, Theorem 2.4).

Problem 33.7. Given linear maps $f\colon E \to E'$ and $f'\colon E' \to E''$, show that

$$(f' \circ f) \odot (f' \circ f) = (f' \odot f') \circ (f \odot f).$$

Problem 33.8. Complete the proof of Proposition 33.28 for the case of an infinite dimensional vector space E.

Problem 33.9. Let I be a finite index set of cardinality p. Let m be a nonnegative integer. Show that the number of multisets over I with cardinality m is $\binom{p+m-1}{m}$.

Problem 33.10. Prove Proposition 33.29.

Problem 33.11. Using bases, show that the bilinear map at $(*)$ in Section 33.10 produces a nondegenerate pairing.

Problem 33.12. Let $\mathfrak{I}$ be the two-sided ideal generated by all tensors of the form $u \otimes v - v \otimes u \in V^{\otimes 2}$. Prove that $\mathrm{S}^m(V) \cong V^{\otimes m}/(\mathfrak{I} \cap V^{\otimes m})$.

Problem 33.13. Verify Equation $(*)$ of Section 33.11 for arbitrary nonnegative integers m and n.