

Our hypotheses imply that $\theta > 0$, and we have $\theta \leq \epsilon$, because $(1/2)(\epsilon - w^\top x_i - b + y_i) \leq \epsilon$ is equivalent to $\epsilon - w^\top x_i - b + y_i \leq 2\epsilon$ which is equivalent to $-w^\top x_i - b + y_i \leq \epsilon$, which holds for all $i \notin (E_\lambda \cup E_\mu)$ by hypothesis.

We can write

$$\begin{aligned} w^\top x_i + b + \theta - y_i &= \epsilon - \theta + \xi_i + 2\theta & \xi_i &> 0 & i &\in E_\lambda \\ -w^\top x_j - (b + \theta) + y_j &= \epsilon - \theta + \xi'_j & \xi'_j &> 0 & j &\in E_\mu \\ w^\top x_i + b + \theta - y_i &\leq \epsilon - \theta & & & i &\notin (E_\lambda \cup E_\mu) \\ -w^\top x_i - (b + \theta) + y_i &\leq \epsilon - \theta & & & i &\notin (E_\lambda \cup E_\mu). \end{aligned}$$

By hypothesis

$$-w^\top x_j - (b + \theta) + y_j = \epsilon - \theta \quad \text{for some } j \notin (E_\lambda \cup E_\mu)$$

and by the choice of θ ,

$$w^\top x_i + b + \theta - y_i = \epsilon - \theta \quad \text{for some } i \notin (E_\lambda \cup E_\mu).$$

The value of $C > 0$ is irrelevant in the following argument so we may assume that $C = 1$. The new value of the objective function is

$$\begin{aligned} \omega(\theta) &= \frac{1}{2}w^\top w + \nu(\epsilon - \theta) + \frac{1}{m} \left(\sum_{i \in E_\lambda} (\xi_i + 2\theta) + \sum_{j \in E_\mu} \xi'_j \right) \\ &= \frac{1}{2}w^\top w + \nu\epsilon + \frac{1}{m} \left(\sum_{i \in E_\lambda} \xi_i + \sum_{j \in E_\mu} \xi'_j \right) - \left(\nu - \frac{2p_{sf}}{m} \right) \theta. \end{aligned}$$

By Proposition 56.2 we have

$$\max \left\{ \frac{2p_f}{m}, \frac{2q_f}{m} \right\} \leq \nu$$

and $p_{sf} \leq p_f$ and $q_{sf} \leq q_f$, which implies that

$$\nu - \frac{2p_{sf}}{m} \geq 0, \tag{*_1}$$

and so $\omega(\theta) \leq \omega(0)$. If inequality $(*_1)$ is strict, then this contradicts the optimality of the original solution. Therefore, $\nu = 2p_{sf}/m$, $\omega(\theta) = \omega(0)$ and $(w, b + \theta, \epsilon - \theta, \xi + 2\theta, \xi')$ is an optimal solution such that

$$\begin{aligned} w^\top x_i + b + \theta - y_i &= \epsilon - \theta \\ -w^\top x_j - (b + \theta) + y_j &= \epsilon - \theta \end{aligned}$$

for some $i, j \notin (E_\lambda \cup E_\mu)$ with $i \neq j$.