that $2(x-y)=0$, so the set V of solutions is given by

$$\begin{aligned} y &= x \\ z &= 0. \end{aligned}$$

This is a one dimensional subspace of $\mathbb{R}^3$. Geometrically, this is the line of equation $y=x$ in the plane $z=0$ as illustrated by Figure 11.1.

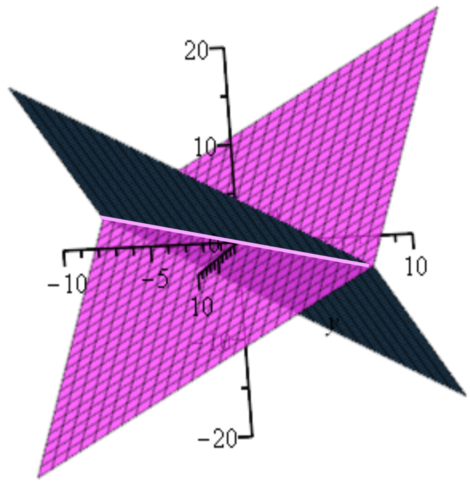

Figure 11.1: The intersection of the magenta plane $x-y+z=0$ with the blue-gray plane $x-y-z=0$ is the pink line $y=x$.

Now why did we say that the above equations are linear? Because as functions of (x,y,z), both maps $f_1\colon (x,y,z)\mapsto x-y+z$ and $f_2\colon (x,y,z)\mapsto x-y-z$ are linear. The set of all such linear functions from $\mathbb{R}^3$ to $\mathbb{R}$ is a vector space; we used this fact to form linear combinations of the "equations" f_1 and f_2. Observe that the dimension of the subspace V is 1. The ambient space has dimension $n=3$ and there are two "independent" equations f_1, f_2, so it appears that the dimension $\dim(V)$ of the subspace V defined by m independent equations is

$$\dim(V) = n-m,$$

which is indeed a general fact (proven in Theorem 11.4).

More generally, in $\mathbb{R}^n$, a linear equation is determined by an n-tuple $(a_1,\ldots,a_n)\in\mathbb{R}^n$, and the solutions of this linear equation are given by the n-tuples $(x_1,\ldots,x_n)\in\mathbb{R}^n$ such that

$$a_1x_1+\cdots+a_nx_n=0;$$

these solutions constitute the kernel of the linear map $(x_1,\ldots,x_n)\mapsto a_1x_1+\cdots+a_nx_n$. The above considerations assume that we are working in the canonical basis $(e_1,\ldots,e_n)$ of