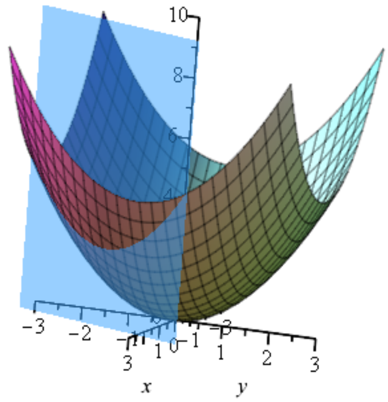

Figure 52.1: The graph of $J(x, y) = (1/2)(x^2 + y^2)$ is the parabolic surface while the graph of $2x - y = 5$ is the transparent blue plane. The solution to Example 52.1 is apex of the intersection curve, namely the point $(2, -1, \frac{5}{2})$.

Since

$$J(x, y) = \frac{1}{2}\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix},$$

we observe that $J(x, y)$ is a quadratic function determined by the positive definite matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and hence to calculate $G(\lambda)$, we must set $\nabla L_{x,y} = 0$. By calculating $\frac{\partial J}{\partial x} = 0$ and $\frac{\partial J}{\partial y} = 0$, we find that $x = -2\lambda$ and $y = \lambda$. Then $G(\lambda) = -5/2\lambda^2 - 5\lambda$, and we must calculate the maximum of $G(\lambda)$ with respect to $\lambda \in \mathbb{R}$. This means calculating $G'(\lambda) = 0$ and obtaining $\lambda = -1$ for the solution of $(x, y, \lambda) = (-2\lambda, \lambda, \lambda) = (2, -1, -1)$.

Instead of solving *directly* for λ in terms of (x, y), the method of dual assent begins with a *numerical* estimate for λ, namely λ^0, and forms the "numerical" Lagrangian

$$L(x, y, \lambda^0) = \frac{1}{2}(x^2 + y^2) + \lambda^0(2x - y - 5).$$

With this numerical value λ^0, we minimize $L(x, y, \lambda^0)$ with respect to (x, y). This calculation will be identical to that used to form $G(\lambda)$ above, and as such, we obtain the iterative step $(x^1, y^1) = (-2\lambda^0, \lambda^0)$. So if we replace λ^0 by λ^k, we have the first step of the dual ascent method, namely

$$u^{k+1} = \begin{pmatrix} x^{k+1} \\ y^{k+1} \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}\lambda^k.$$

The second step of the dual ascent method refines the numerical estimate of λ by calculating

$$\lambda^{k+1} = \lambda^k + \alpha^k\left(\begin{pmatrix} 2 & -1 \end{pmatrix}\begin{pmatrix} x^{k+1} \\ y^{k+1} \end{pmatrix} - 5\right).$$