The advantage of the above formula is that it gives an explicit remainder. We now examine briefly the situation where E is of finite dimension n, and $(a_0, (e_1, \ldots, e_n))$ is a frame for E. In this case, we get a more explicit expression for the expression

$$\sum_{i=0}^{k=m} \frac{1}{k!} \mathrm{D}^k f(a)(h^k)$$

involved in all versions of Taylor's formula, where by convention, $\mathrm{D}^0 f(a)(h^0) = f(a)$. If $h = h_1 e_1 + \cdots + h_n e_n$, then we have

$$\sum_{k=0}^{k=m} \frac{1}{k!} \mathrm{D}^k f(a)(h^k) = \sum_{k_1+\cdots+k_n \leq m} \frac{h_1^{k_1} \cdots h_n^{k_n}}{k_1! \cdots k_n!} \left(\frac{\partial}{\partial x_1}\right)^{k_1} \cdots \left(\frac{\partial}{\partial x_n}\right)^{k_n} f(a),$$

which, using the abbreviated notation introduced at the end of Section 39.6, can also be written as

$$\sum_{k=0}^{k=m} \frac{1}{k!} \mathrm{D}^k f(a)(h^k) = \sum_{|\alpha| \leq m} \frac{h^\alpha}{\alpha!} \partial^\alpha f(a).$$

The advantange of the above notation is that it is the same as the notation used when $n = 1$, i.e., when $E = \mathbb{R}$ (or $E = \mathbb{C}$). Indeed, in this case, the Taylor–MacLaurin formula reads as:

$$f(a + h) = f(a) + \frac{h}{1!} \mathrm{D}^1 f(a) + \cdots + \frac{h^m}{m!} \mathrm{D}^m f(a) + \frac{h^{m+1}}{(m+1)!} \mathrm{D}^{m+1} f(a + \theta h),$$

for some $\theta \in \mathbb{R}$, with $0 < \theta < 1$, where $\mathrm{D}^k f(a)$ is the value of the k-th derivative of f at a (and thus, as we have already said several times, this is the kth-order vector derivative, which is just a scalar, since $F = \mathbb{R}$).

In the above formula, the assumptions are that $f\colon [a, a + h] \to \mathbb{R}$ is a C^m-function on $[a, a + h]$, and that $\mathrm{D}^{m+1} f(x)$ exists for every $x \in (a, a + h)$.

Taylor's formula is useful to study the local properties of curves and surfaces. In the case of a curve, we consider a function $f\colon [r, s] \to F$ from a closed interval $[r, s]$ of $\mathbb{R}$ to some affine space F, the derivatives $\mathrm{D}^k f(a)(h^k)$ correspond to vectors $h^k \mathrm{D}^k f(a)$, where $\mathrm{D}^k f(a)$ is the kth vector derivative of f at a (which is really $\mathrm{D}^k f(a)(1, \ldots, 1)$), and for any $a \in (r, s)$, Theorem 39.23 yields the following formula:

$$f(a + h) = f(a) + \frac{h}{1!} \mathrm{D}^1 f(a) + \cdots + \frac{h^m}{m!} \mathrm{D}^m f(a) + h^m \epsilon(h),$$

for any h such that $a + h \in (r, s)$, and where $\lim_{h \to 0,\, h \neq 0} \epsilon(h) = 0$.

In the case of functions $f\colon \mathbb{R}^n \to \mathbb{R}$, it is convenient to have formulae for the Taylor–Young formula and the Taylor–MacLaurin formula in terms of the gradient and the Hessian.