We can now compute the cross-ratio explicitly for any given basis (u, v) of D. Assume that a, b, c, d have homogeneous coordinates $[\lambda_1, \mu_1]$, $[\lambda_2, \mu_2]$, $[\lambda_3, \mu_3]$, and $[\lambda_4, \mu_4]$ over the projective frame induced by (u, v). Letting $w_i = \lambda_i u + \mu_i v$, we have $a = p(w_1)$, $b = p(w_2)$, $c = p(w_3)$, and $d = p(w_4)$. Since a and b are distinct, w_1 and w_2 are linearly independent, and we can write $w_3 = \alpha w_1 + \beta w_2$ and $w_4 = \gamma w_1 + \delta w_2$, which can also be written as

$$w_4 = \frac{\gamma}{\alpha}\,\alpha\, w_1 + \frac{\delta}{\beta}\,\beta\, w_2,$$

and by Proposition 26.21, $[a, b, c, d] = \big[\gamma/\alpha, \delta/\beta\big]$. However, since w_1 and w_2 are linearly independent, it is possible to solve for $\alpha, \beta, \gamma, \delta$ in terms of the homogeneous coordinates, obtaining expressions involving determinants:

$$\begin{aligned} \alpha &= \frac{\det(w_3, w_2)}{\det(w_1, w_2)}, & \beta &= \frac{\det(w_1, w_3)}{\det(w_1, w_2)}, \\ \gamma &= \frac{\det(w_4, w_2)}{\det(w_1, w_2)}, & \delta &= \frac{\det(w_1, w_4)}{\det(w_1, w_2)}, \end{aligned}$$

and thus, assuming that $d \neq a$, we get

$$[a, b, c, d] = \frac{\begin{vmatrix} \lambda_3 & \lambda_1 \\ \mu_3 & \mu_1 \end{vmatrix}}{\begin{vmatrix} \lambda_3 & \lambda_2 \\ \mu_3 & \mu_2 \end{vmatrix}} \Bigg/ \frac{\begin{vmatrix} \lambda_4 & \lambda_1 \\ \mu_4 & \mu_1 \end{vmatrix}}{\begin{vmatrix} \lambda_4 & \lambda_2 \\ \mu_4 & \mu_2 \end{vmatrix}}.$$

When $d = a$, we have $[a, b, c, d] = \infty$. In particular, if Δ is the projective completion of an affine line D, then $\mu_i = 1$, and we get

$$[a, b, c, d] = \frac{\lambda_3 - \lambda_1}{\lambda_3 - \lambda_2} \Bigg/ \frac{\lambda_4 - \lambda_1}{\lambda_4 - \lambda_2} = \frac{\overrightarrow{ca}}{\overrightarrow{cb}} \Bigg/ \frac{\overrightarrow{da}}{\overrightarrow{db}}.$$

When $d = \infty$, we get

$$[a, b, c, \infty] = \frac{\overrightarrow{ca}}{\overrightarrow{cb}},$$

which is just the usual ratio (although we defined it earlier as $-\mathrm{ratio}(a, c, b)$).

We briefly mention some of the properties of the cross-ratio. For example, the cross-ratio $[a, b, c, d]$ is invariant if any two elements and the complementary two elements are transposed, and letting $0^{-1} = \infty$ and $\infty^{-1} = 0$, we have

$$[a, b, c, d] = [b, a, c, d]^{-1} = [a, b, d, c]^{-1}$$

and

$$[a, b, c, d] = 1 - [a, c, b, d].$$