and any sequence $K = (k_1, k_2, \cdots, k_m)$ of m elements with $k_i \in \{1, \ldots, n\}$, the matrix A_K denotes the $m \times m$ matrix whose ith column is the k_ith column of A, and similarly for any vector $u \in \mathbb{R}^n$ (resp. any linear form $c \in (\mathbb{R}^n)^*$), the vector $u_K \in \mathbb{R}^m$ (the linear form $c_K \in (\mathbb{R}^m)^*$) is the vector whose ith entry is the k_ith entry in u (resp. the linear whose ith entry is the k_ith entry in c).

For each nonbasic $j \notin K$, we have

$$A^j = \gamma^j_{k_1} A^{k_1} + \cdots + \gamma^j_{k_m} A^{k_m} = A_K \gamma^j_K,$$

so the vector γ^j_K is given by $\gamma^j_K = A_K^{-1} A^j$, that is, by solving the system

$$A_K \gamma^j_K = A^j. \tag{$*_\gamma$}$$

To be very precise, since the vector γ^j_K depends on K its components should be denoted by $(\gamma^j_K)_{k_i}$, but as we said before, to simplify notation we write $\gamma^j_{k_i}$ instead of $(\gamma^j_K)_{k_i}$.

In order to decide which case applies ((A), (B1), (B2), (B3)), we need to compute the numbers $c_j - \sum_{k \in K} \gamma^j_k c_k$ for all $j \notin K$. For this, observe that

$$c_j - \sum_{k \in K} \gamma^j_k c_k = c_j - c_K \gamma^j_K = c_j - c_K A_K^{-1} A^j.$$

If we write $\beta_K = c_K A_K^{-1}$, then

$$c_j - \sum_{k \in K} \gamma^j_k c_k = c_j - \beta_K A^j,$$

and we see that $\beta_K^\top \in \mathbb{R}^m$ is the solution of the system $\beta_K^\top = (A_K^{-1})^\top c_k^\top$, which means that $\beta_K^\top$ is the solution of the system

$$A_K^\top \beta_K^\top = c_K^\top. \tag{$*_\beta$}$$

Remark: Observe that since u is a basis feasible solution of (P), we have $u_j = 0$ for all $j \notin K$, so u is the solution of the equation $A_K u_K = b$. As a consequence, the value of the objective function for u is $cu = c_K u_K = c_K A_K^{-1} b$. This fact will play a crucial role in Section 47.2 to show that when the simplex algorithm terminates with an optimal solution of the Linear Program (P), then it also produces an optimal solution of the Dual Linear Program (D).

Assume that we have a basic feasible solution u, a basis K for u, and that we also have the matrix A_K as well its inverse A_K^{-1} (perhaps implicitly) and also the inverse $(A_K^\top)^{-1}$ of $A_K^\top$ (perhaps implicitly). Here is a description of an iteration step of the simplex algorithm, following almost exactly Chvatal (Chvatal [40], Chapter 7, Box 7.1).

An Iteration Step of the (Revised) Simplex Method