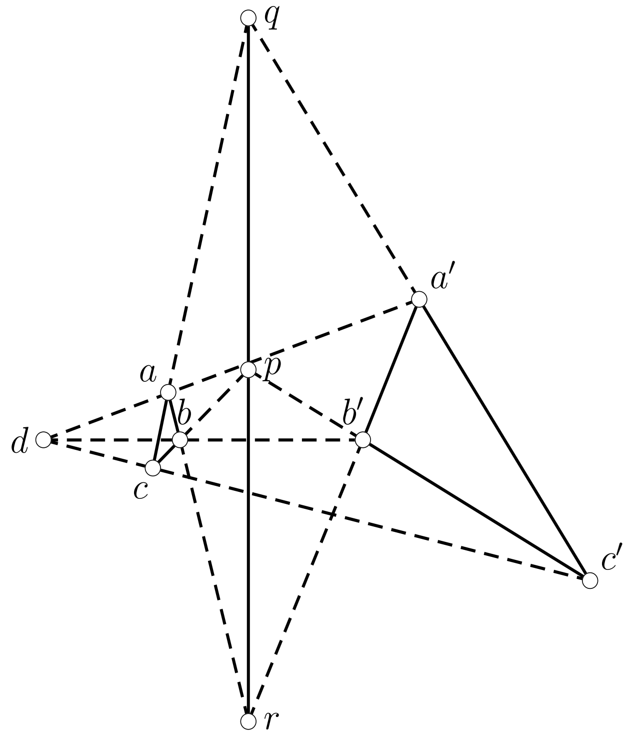

Figure 26.13: Desargues's theorem (projective version in the plane).

and (q_1, q_2, q_3) are linearly independent, and that if we write

$$p_4 = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3$$

and

$$q_4 = \lambda_1 q_1 + \lambda_2 q_2 + \lambda_3 q_3,$$

for some unique scalars $\alpha_1, \alpha_2, \alpha_3$ and $\lambda_1, \lambda_2, \lambda_3$, then $\alpha_i \neq 0$ and $\lambda_i \neq 0$ for $i = 1, 2, 3$. The problem is to find the 3×3 matrix (up to a scalar) representing the unique homography h mapping $[p_i]$ to $[q_i]$ for $i = 1, 2, 3, 4$.

We will use the *canonical basis* $\mathcal{E} = (e_1, e_2, e_3)$ of $\mathbb{R}^3$, with $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$, $e_3 = (0, 0, 1)$, and the bases $\mathcal{P} = (p_1, p_2, p_3)$ and $\mathcal{Q} = (q_1, q_2, q_3)$ of $\mathbb{R}^3$.

As a first step, it is convenient to express (q_1, q_2, q_3, q_4) over the basis $\mathcal{P} = (p_1, p_2, p_3)$, with $q_1 = (x_1, y_1, z_1)$, $q_2 = (x_2, y_2, z_2)$, $q_3 = (x_3, y_3, z_3)$, $q_4 = (x_4, y_4, z_4)$. Over the canonical basis $\mathcal{E}$, the points (p_1, p_2, p_3, p_4) are given by the coordinates $p_1 = (p_1^x, p_1^y, p_1^z)$, $p_2 = (p_2^x, p_2^y, p_2^z)$, $p_3 = (p_3^x, p_3^y, p_3^z)$, $p_4 = (p_4^x, p_4^y, p_4^z)$, and similarly, the points (q_1, q_2, q_3, q_4) are given by the coordinates $q_1 = (q_1^x, q_1^y, q_1^z)$, $q_2 = (q_2^x, q_2^y, q_2^z)$, $q_3 = (q_3^x, q_3^y, q_3^z)$, $q_4 = (q_4^x, q_4^y, q_4^z)$.