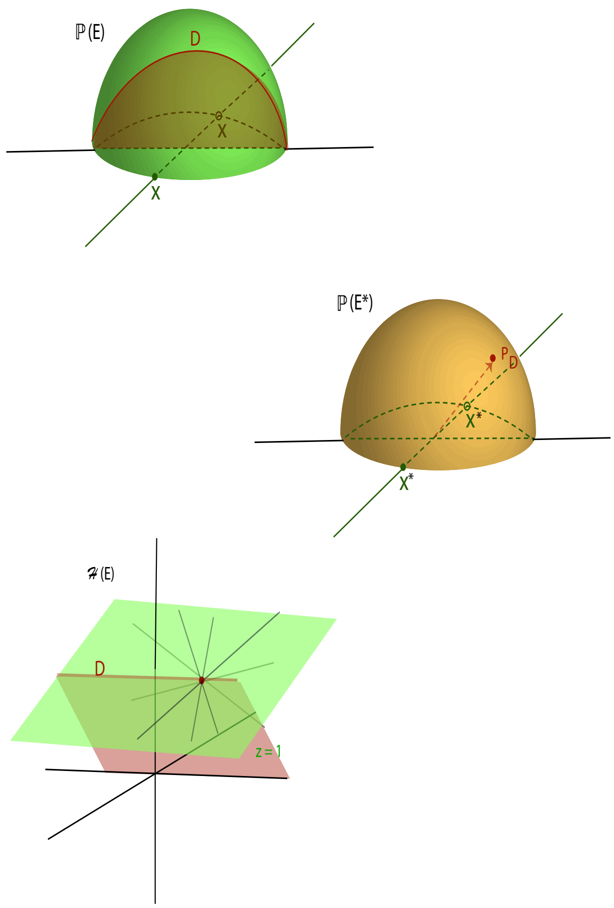

Figure 26.37: The duality between a line in $\mathbf{P}(E)$ and point in $\mathbf{P}(E^*)$. The point in $\mathbf{P}(E^*)$ is also represented by Line D in $\mathcal{H}(E)$.

26.13 Cross-Ratios of Hyperplanes

Given a pencil $P = \mathbf{P}(U)$ of hyperplanes in $\mathcal{H}(E)$, for any sequence (H_1, H_2, H_3, H_4) of hyperplanes in this pencil, if H_1, H_2, H_3 are distinct, we define the cross-ratio $[H_1, H_2, H_3, H_4]$ as the cross-ratio of the hyperplanes H_i considered as points on the projective line P in $\mathbf{P}(E^*)$. In particular, in a projective plane $\mathbf{P}(E)$, given any four concurrent lines D_1, D_2, D_3, D_4, where D_1, D_2, D_3 are distinct, for any two distinct lines Δ and Δ' not passing through the common intersection c of the lines D_i, letting $d_i = \Delta \cap D_i$, and $d_i' = \Delta' \cap D_i$, note that the projection of center c from Δ to Δ' maps each d_i to d_i'.

Since such a projection is a projectivity, and since projectivities between lines preserve cross-ratios, we have

$$[d_1, d_2, d_3, d_4] = [d_1', d_2', d_3', d_4'],$$