Then if we denote the $2m$ dual variables by (y', y''), with $y', y'' \in (\mathbb{R}^m)^*$, the dual of the above program is

$$\begin{aligned} &\text{minimize} \quad y'b - y''b \\ &\text{subject to} \quad \begin{pmatrix} y' & y'' \end{pmatrix} \begin{pmatrix} A \\ -A \end{pmatrix} \geq c \text{ and } y', y'' \geq 0, \end{aligned}$$

where $y', y'' \in (\mathbb{R}^m)^*$, which is equivalent to

$$\begin{aligned} &\text{minimize} \quad (y' - y'')b \\ &\text{subject to} \quad (y' - y'')A \geq c \text{ and } y', y'' \geq 0, \end{aligned}$$

where $y', y'' \in (\mathbb{R}^m)^*$. If we write $y = y' - y''$, we find that the above linear program is equivalent to the following Linear Program (D):

$$\begin{aligned} &\text{minimize} \quad yb \\ &\text{subject to} \quad yA \geq c, \end{aligned}$$

where $y \in (\mathbb{R}^m)^*$. Observe that y is *not required* to be nonnegative; it is arbitrary.

Next we would like to know what is the version of Theorem 47.8 for a linear program already in standard form. This is very simple.

Theorem 47.12. *Consider the Linear Program (P2) in standard form*

$$\begin{aligned} &\text{maximize} \quad cx \\ &\text{subject to} \quad Ax = b \textit{ and } x \geq 0, \end{aligned}$$

and its Dual (D) given by

$$\begin{aligned} &\text{minimize} \quad yb \\ &\text{subject to} \quad yA \geq c, \end{aligned}$$

where $y \in (\mathbb{R}^m)^*$. If the simplex algorithm applied to the Linear Program $(P2)$ terminates with an optimal solution (u^*, K^*), where u^* is a basic feasible solution and K^* is a basis for u^*, then $y^* = c_{K^*} A_{K^*}^{-1}$ is an optimal solution for (D) such that $cu^* = y^* b$. Furthermore, if we assume that the simplex algorithm is started with a basic feasible solution (u_0, K_0) where $K_0 = (n-m+1, \ldots, n)$ (the indices of the last m columns of A) and $A_{(n-m+1,\ldots,n)} = I_m$ (the last m columns of A constitute the identity matrix I_m), then the optimal solution $y^* = c_{K^*} A_{K^*}^{-1}$ for (D) is given in terms of the reduced costs by

$$y^* = c_{(n-m+1,\ldots,n)} - (\overline{c}_{K^*})_{(n-m+1,\ldots,n)},$$

and the $m \times m$ matrix consisting of last m columns and the last m rows of the final tableau is $A_{K^*}^{-1}$.