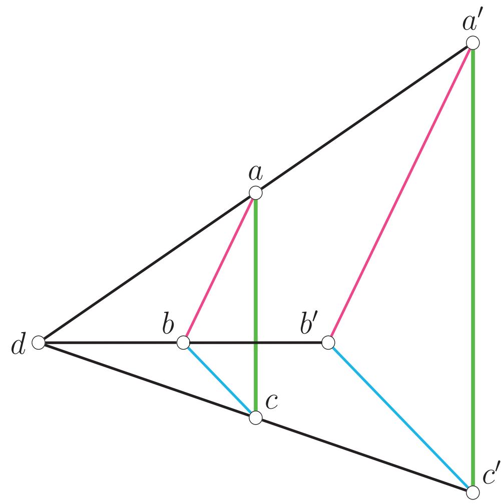

Figure 24.24: Desargues's theorem (affine version).

24.10 Affine Hyperplanes

We now consider affine forms and affine hyperplanes. In Section 24.5 we observed that the set L of solutions of an equation

$$ax + by = c$$

is an affine subspace of $\mathbb{A}^2$ of dimension 1, in fact, a line (provided that a and b are not both null). It would be equally easy to show that the set P of solutions of an equation

$$ax + by + cz = d$$

is an affine subspace of $\mathbb{A}^3$ of dimension 2, in fact, a plane (provided that a, b, c are not all null). More generally, the set H of solutions of an equation

$$\lambda_1 x_1 + \cdots + \lambda_m x_m = \mu$$

is an affine subspace of $\mathbb{A}^m$, and if $\lambda_1, \ldots, \lambda_m$ are not all null, it turns out that it is a subspace of dimension $m-1$ called a *hyperplane*.

We can interpret the equation

$$\lambda_1 x_1 + \cdots + \lambda_m x_m = \mu$$

in terms of the map $f\colon \mathbb{R}^m \to \mathbb{R}$ defined such that

$$f(x_1, \ldots, x_m) = \lambda_1 x_1 + \cdots + \lambda_m x_m - \mu$$

for all $(x_1, \ldots, x_m) \in \mathbb{R}^m$. It is immediately verified that this map is affine, and the set H of solutions of the equation

$$\lambda_1 x_1 + \cdots + \lambda_m x_m = \mu$$