

with respect to

$$\mu_1 + \mu_2 - \lambda_1 - \lambda_2 = 0. \quad (*_2)$$

**Step 4:** Rewrite the constraints at (C) using  $(*_1)$ . In particular  $C \begin{pmatrix} w \\ b \end{pmatrix} \leq d$  becomes

$$\begin{pmatrix} -u_{11} & -u_{12} & 1 \\ -u_{21} & -u_{22} & 1 \\ v_{11} & v_{12} & -1 \\ v_{21} & v_{22} & -1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{21} & -v_{11} & -v_{21} & 0 \\ u_{12} & u_{22} & -v_{12} & -v_{22} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \mu_1 \\ \mu_2 \\ b \end{pmatrix} \leq \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}.$$

Rewriting the previous equation in “block” format gives us

$$-\begin{pmatrix} -u_{11} & -u_{12} \\ -u_{21} & -u_{22} \\ v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix} \begin{pmatrix} -u_{11} & -u_{21} & v_{11} & v_{21} \\ -u_{12} & -u_{22} & v_{12} & v_{22} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \mu_1 \\ \mu_2 \end{pmatrix} + b \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \leq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

which with the definition

$$X = \begin{pmatrix} -u_{11} & -u_{21} & v_{11} & v_{21} \\ -u_{12} & -u_{22} & v_{12} & v_{22} \end{pmatrix}$$

yields

$$-X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + b \begin{pmatrix} \mathbf{1}_2 \\ -\mathbf{1}_2 \end{pmatrix} + \mathbf{1}_4 \leq \mathbf{0}_4. \quad (*_3)$$

Let us now consider the general case.

**Step 1:** Write the constraints in matrix form. First we rewrite the constraints as

$$\begin{aligned} -u_i^\top w + b &\leq -1 & i = 1, \dots, p \\ v_j^\top w - b &\leq -1 & j = 1, \dots, q, \end{aligned}$$

and we get the  $(p+q) \times (n+1)$  matrix  $C$  and the vector  $d \in \mathbb{R}^{p+q}$  given by

$$C = \begin{pmatrix} -u_1^\top & 1 \\ \vdots & \vdots \\ -u_p^\top & 1 \\ v_1^\top & -1 \\ \vdots & \vdots \\ v_q^\top & -1 \end{pmatrix}, \quad d = \begin{pmatrix} -1 \\ \vdots \\ -1 \end{pmatrix},$$

so the set of inequality constraints is

$$C \begin{pmatrix} w \\ b \end{pmatrix} \leq d.$$