Proof. The proof of Theorem 47.8 applies with A instead of $\widehat{A}$, and we can show that

$$c_{K^*}A_{K^*}^{-1}A_{N^*} \geq c_{N^*},$$

and that $y^* = c_{K^*}A_{K^*}^{-1}$ satisfies, $cu^* = y^*b$, and

$$\begin{aligned} y^*A_{K^*} &= c_{K^*}A_{K^*}^{-1}A_{K^*} = c_{K^*}, \\ y^*A_{N^*} &= c_{K^*}A_{K^*}^{-1}A_{N^*} \geq c_{N^*}. \end{aligned}$$

Let P be the $n \times n$ permutation matrix defined so that

$$AP = \begin{pmatrix} A_{K^*} & A_{N^*} \end{pmatrix}.$$

Then we also have

$$cP = \begin{pmatrix} c_{K^*} & c_{N^*} \end{pmatrix},$$

and using the above equations and inequalities we obtain

$$y^* \begin{pmatrix} A_{K^*} & A_{N^*} \end{pmatrix} \geq \begin{pmatrix} c_{K^*} & c_{N^*} \end{pmatrix},$$

that is, $y^*AP \geq cP$, which is equivalent to

$$y^*A \geq c,$$

which shows that y^* is a feasible solution of (D) (remember, y^* is arbitrary so there is no need for the constraint $y^* \geq 0$).

The reduced costs are given by

$$(\overline{c}_{K^*})_i = c_i - c_{K^*}A_{K^*}^{-1}A^i,$$

and since for $j = n-m+1, \ldots, n$ the column A^j is the $(j+m-n)$th column of the identity matrix I_m, we have

$$(\overline{c}_{K^*})_j = c_j - (c_{K^*}A_{K^*}^{-1})_{j+m-n} \quad j = n-m+1, \ldots, n,$$

that is,

$$y^* = c_{(n-m+1,\ldots,n)} - (\overline{c}_{K^*})_{(n-m+1,\ldots,n)},$$

as claimed. Since the last m rows of the final tableau is obtained by multiplying $[u_0 \;\; A]$ by $A_{K^*}^{-1}$, and the last m columns of A constitute I_m, the last m rows and the last m columns of the final tableau constitute $A_{K^*}^{-1}$. $\square$

Let us now take a look at the complementary slackness conditions of Theorem 47.11. If we go back to the version of (P) given by

$$\begin{aligned} &\text{maximize} \quad cx \\ &\text{subject to} \quad \begin{pmatrix} A \\ -A \end{pmatrix} x \leq \begin{pmatrix} b \\ -b \end{pmatrix} \;\text{ and }\; x \geq 0, \end{aligned}$$