over the basis $(\overrightarrow{a_0a_1}, \ldots, \overrightarrow{a_0a_m})$ *in* $\overrightarrow{E}$*, then over the basis* $(\overrightarrow{a_0a_1}, \ldots, \overrightarrow{a_0a_m}, a_0)$ *in* $\widehat{E}$*, the coordinates of* v *are*

$$(v_1, \ldots, v_m, 0).$$

For any element $\langle a, \lambda\rangle$*, where* $\lambda \neq 0$*, if the barycentric coordinates of* a *w.r.t. the affine basis* $(a_0, \ldots, a_m)$ *in* E *are* $(\lambda_0, \ldots, \lambda_m)$ *with* $\lambda_0 + \cdots + \lambda_m = 1$*, then the coordinates of* $\langle a, \lambda\rangle$ *w.r.t. the basis* $(a_0, \ldots, a_m)$ *in* $\widehat{E}$ *are*

$$(\lambda\lambda_0, \ldots, \lambda\lambda_m).$$

If a vector $v \in \overrightarrow{E}$ *is expressed as*

$$v = v_1\overrightarrow{a_0a_1} + \cdots + v_m\overrightarrow{a_0a_m} = -(v_1 + \cdots + v_m)a_0 + v_1a_1 + \cdots + v_ma_m,$$

with respect to the affine basis $(a_0, \ldots, a_m)$ *in* E*, then its coordinates w.r.t. the basis* $(a_0, \ldots, a_m)$ *in* $\widehat{E}$ *are*

$$(-(v_1 + \cdots + v_m), v_1, \ldots, v_m).$$

Proof. We sketch parts of the proof, leaving the details as an exercise. Figure 25.2 shows the basis $(\overrightarrow{a_0a_1}, \overrightarrow{a_0a_2}, a_0)$ corresponding to the affine frame (a_0, a_1, a_2) in E.

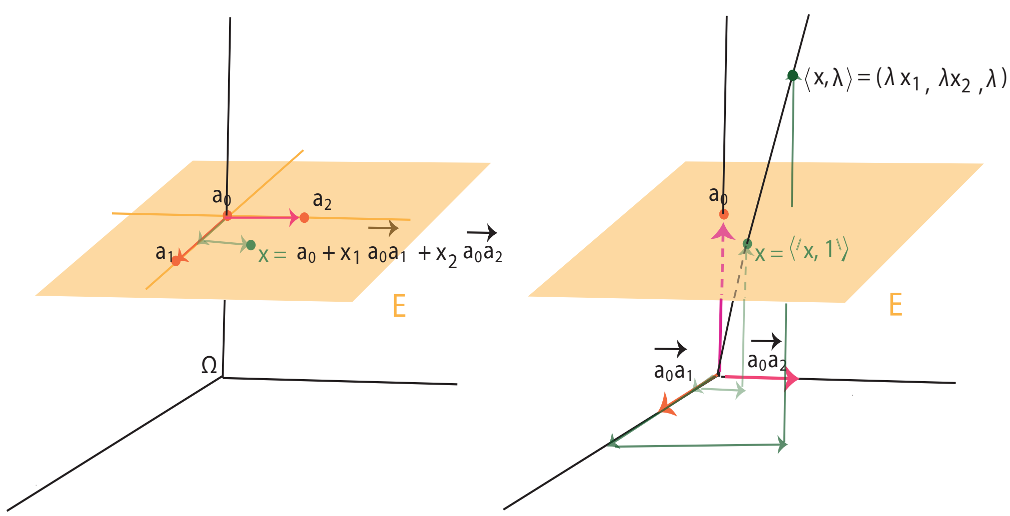

Figure 25.2: The affine frame (a_0, a_1, a_2) of E and the basis $(\overrightarrow{a_0a_1}, \overrightarrow{a_0a_2}, a_0)$ in $\widehat{E}$.

If we assume that we have a nontrivial linear combination

$$\lambda_1\overrightarrow{a_0a_1} \mathbin{\widehat{+}} \cdots \mathbin{\widehat{+}} \lambda_m\overrightarrow{a_0a_m} \mathbin{\widehat{+}} \mu a_0 = 0,$$

if $\mu \neq 0$, then we have

$$\lambda_1\overrightarrow{a_0a_1} \mathbin{\widehat{+}} \cdots \mathbin{\widehat{+}} \lambda_m\overrightarrow{a_0a_m} \mathbin{\widehat{+}} \mu a_0 = \langle a_0 + \mu^{-1}\lambda_1\overrightarrow{a_0a_1} + \cdots + \mu^{-1}\lambda_m\overrightarrow{a_0a_m}, \mu\rangle,$$