where A_{11}, A_{12} are 2×2, 2×1; A_{21}, A_{22} are 1×2, 1×1; and A_{31}, A_{32} are 3×2, 3×1, and $[B]$ be the 2×3 block matrix

$$[B] = \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} \ \\ \ \end{bmatrix} & \begin{bmatrix} \ & \ \\ \ & \ \end{bmatrix} & \begin{bmatrix} \ & \ & \ \\ \ & \ & \ \end{bmatrix} \\ \begin{bmatrix} \ \end{bmatrix} & \begin{bmatrix} \ & \ \end{bmatrix} & \begin{bmatrix} \ & \ & \ \end{bmatrix} \end{pmatrix},$$

where B_{11}, B_{12}, B_{13} are 2×1, 2×2, 2×3; and B_{21}, B_{22}, B_{23} are 1×1, 1×2, 1×3. Then $[C] = [A][B]$ is the 3×3 block matrix

$$[C] = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} \ \\ \ \end{bmatrix} & \begin{bmatrix} \ & \ \\ \ & \ \end{bmatrix} & \begin{bmatrix} \ & \ & \ \\ \ & \ & \ \end{bmatrix} \\ \begin{bmatrix} \ \end{bmatrix} & \begin{bmatrix} \ & \ \end{bmatrix} & \begin{bmatrix} \ & \ & \ \end{bmatrix} \\ \begin{bmatrix} \ \\ \ \\ \ \end{bmatrix} & \begin{bmatrix} \ & \ \\ \ & \ \\ \ & \ \end{bmatrix} & \begin{bmatrix} \ & \ & \ \\ \ & \ & \ \\ \ & \ & \ \end{bmatrix} \end{pmatrix},$$

where C_{11}, C_{12}, C_{13} are 2×1, 2×2, 2×3; C_{21}, C_{22}, C_{23} are 1×1, 1×2, 1×3; and C_{31}, C_{32}, C_{33} are 3×1, 3×2, 3×3. For example,

$$C_{32} = A_{31}B_{12} + A_{32}B_{22}.$$

Example 6.5. This example illustrates some of the subtleties having to do with the partitioning of the index sets. Consider the 1×3 matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix}$$

and the 3×2 matrix

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}.$$

Consider the partition of the index set $R = \{1\}$ given by $R_1 = \{1\}$; of the index set $S = \{1, 2, 3\}$ given by $S_1 = \{1, 3\}$, $S_2 = \{2\}$; and of the index set $T = \{1, 2\}$ given by $T_1 = \{2\}$, $T_2 = \{1\}$. The corresponding block matrices are the 1×2 block matrix

$$[A] = \begin{pmatrix} A_{\{1\},\{1,3\}} & A_{\{1\},\{2\}} \end{pmatrix} = \left(\begin{bmatrix} a_{11} & a_{13} \end{bmatrix} \quad \begin{bmatrix} a_{12} \end{bmatrix} \right),$$

and the 2×2 block matrix

$$[B] = \begin{pmatrix} B_{\{1,3\},\{2\}} & B_{\{1,3\},\{1\}} \\ B_{\{2\},\{2\}} & B_{\{2\},\{1\}} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} b_{12} \\ b_{32} \end{bmatrix} & \begin{bmatrix} b_{11} \\ b_{31} \end{bmatrix} \\ \begin{bmatrix} b_{22} \end{bmatrix} & \begin{bmatrix} b_{21} \end{bmatrix} \end{pmatrix}.$$