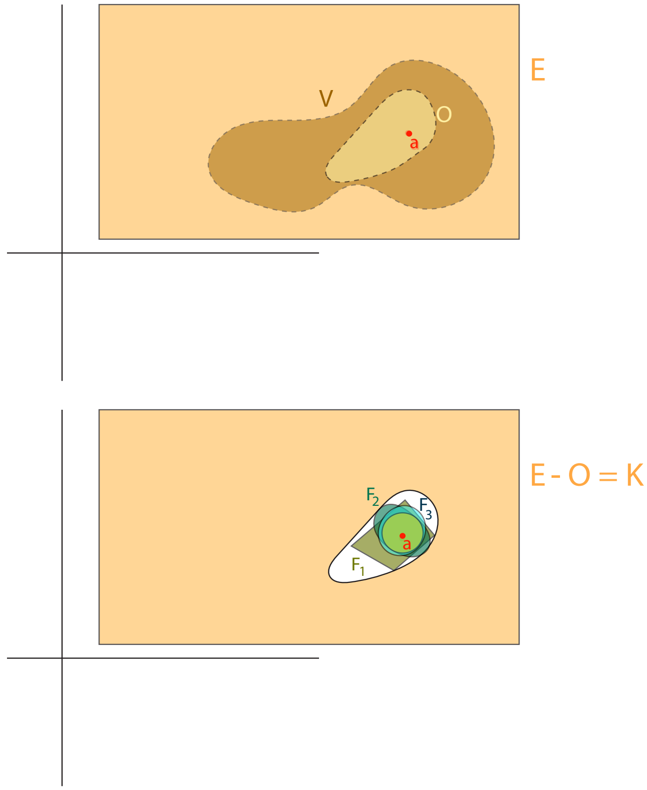

Figure 37.32: An illustration of the proof of Proposition 37.28. Both E and A are closed squares in $\mathbb{R}^2$. Note that an open cover of A, namely the green circles, when combined with the yellow square annulus $E - A$ covers all of the yellow square E.

Putting Proposition 37.27 and Proposition 37.28 together, we note that if X is compact, then for every pair of disjoint closed sets A and B, there exist disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 37.30. A topological space E is *normal* if every one-point set is closed, and for every pair of disjoint closed sets A and B, there exist disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$. A topological space E is *regular* if every one-point set is closed, and for every point $a \in E$ and every closed subset B of E, if $a \notin B$, then there exist disjoint open sets U and V such that $a \in U$ and $B \subseteq V$.

It is clear that a normal space is regular, and a regular space is Hausdorff. There are examples of Hausdorff spaces that are not regular, and of regular spaces that are not normal.

We just observed that a compact space is normal. An important property of metrizable spaces is that they are normal.

Proposition 37.29. *Every metrizable space E is normal.*