(5) **Quadratic Soft margin ν-SVM Problem** (SVM_{s5}). This is the variant of Problem (SVM_{s4}) in which we add the term $(1/2)b^2$ to the objective function. We also drop the constraint $\eta \geq 0$ which is redundant. We have the following optimization problem:

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}w^\top w + \frac{1}{2}b^2 + (p+q)K_s\left(-\nu\eta + \frac{1}{p+q}(\epsilon^\top \epsilon + \xi^\top \xi)\right) \\
&\text{subject to} \\
&\qquad w^\top u_i - b \geq \eta - \epsilon_i, \qquad i = 1, \ldots, p \\
&\qquad - w^\top v_j + b \geq \eta - \xi_j, \qquad j = 1, \ldots, q,
\end{aligned}$$

where ν and K_s are two given positive constants. As we saw earlier, it is convenient to pick $K_s = 1/(p+q)$. When writing a computer program, it is preferable to assume that K_s is arbitrary. In this case ν must be replaced by $(p+q)K_s\nu$ in all the formulae.

One of the advantages of this methods is that ϵ is determined by λ, ξ is determined by μ (as in (SVM_{s4})), and both η and b determined by λ and μ. We can omit the constraint $\eta \geq 0$, because for an optimal solution it can be shown using duality that $\eta \geq 0$. For K_s and ν fixed, if Program (SVM_{s5}) has an optimal solution, then it is unique; see Theorem 54.9.

A drawback of Program (SVM_{s5}) is that for fixed K_s, the quantity $\delta = \eta/\left\|w\right\|$ and the hyperplanes $H_{w,b}, H_{w,b+\eta}$ and $H_{w,b-\eta}$ are *independent* of ν. This is shown in Theorem 54.9. Thus this method is less flexible than ($\mathrm{SVM}_{s2'}$) and (SVM_{s3}).

It is shown in Section 54.15 that the dual of Program (SVM_{s5}) is given by

Dual of the Quadratic Soft margin ν-SVM Problem (SVM_{s5}):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(X^\top X + \begin{pmatrix}\mathbf{1}_p\mathbf{1}_p^\top & -\mathbf{1}_p\mathbf{1}_q^\top \\ -\mathbf{1}_q\mathbf{1}_p^\top & \mathbf{1}_q\mathbf{1}_q^\top\end{pmatrix} + \frac{1}{2K}I_{p+q}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\
&\text{subject to} \\
&\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = \nu \\
&\qquad \lambda_i \geq 0, \quad i = 1, \ldots, p \\
&\qquad \mu_j \geq 0, \quad j = 1, \ldots, q.
\end{aligned}$$