To find the dual function $G(\lambda, \mu, \alpha, \beta, \gamma)$ we minimize $L(w, \epsilon, \xi, b, \delta, \lambda, \mu, \alpha, \beta, \gamma)$ with respect to w, ϵ, ξ, b, and δ. Since the Lagrangian is convex and $(w, \epsilon, \xi, b, \delta) \in \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \times \mathbb{R} \times \mathbb{R}$, a convex open set, by Theorem 40.13, the Lagrangian has a minimum in $(w, \epsilon, \xi, b, \delta)$ iff $\nabla L_{w,\epsilon,\xi,b,\delta} = 0$, so we compute the gradient with respect to $w, \epsilon, \xi, b, \delta$, and we get

$$\nabla L_{w,\epsilon,\xi,b,\delta} = \begin{pmatrix} X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + 2\gamma w \\ K\mathbf{1}_p - (\lambda + \alpha) \\ K\mathbf{1}_q - (\mu + \beta) \\ \mathbf{1}_p^\top \lambda - \mathbf{1}_q^\top \mu \\ \mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu - 1 \end{pmatrix}.$$

By setting $\nabla L_{w,\epsilon,\xi,b,\delta} = 0$ we get the equations

$$\begin{aligned} 2\gamma w &= -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\ \lambda + \alpha &= K\mathbf{1}_p \\ \mu + \beta &= K\mathbf{1}_q \\ \mathbf{1}_p^\top \lambda &= \mathbf{1}_q^\top \mu \\ \mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu &= 1. \end{aligned} \tag{$*_w$}$$

The second and third equations are equivalent to the inequalities

$$0 \leq \lambda_i, \mu_j \leq K, \quad i = 1, \ldots, p,\ j = 1, \ldots, q,$$

often called *box constraints*, and the fourth and fifth equations yield

$$\mathbf{1}_p^\top \lambda = \mathbf{1}_q^\top \mu = \frac{1}{2}.$$

First let us consider the singular case $\gamma = 0$. In this case, $(*_w)$ implies that

$$X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0,$$

and the term $\gamma(w^\top w - 1)$ is missing from the Lagrangian, which in view of the other four equations above reduces to

$$L(w, \epsilon, \xi, b, \delta, \lambda, \mu, \alpha, \beta, 0) = w^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0.$$

In summary, we proved that if $\gamma = 0$, then

$$G(\lambda, \mu, \alpha, \beta, 0) = \begin{cases} 0 & \text{if } \begin{cases} \sum_{i=1}^p \lambda_i = \sum_{j=1}^q \mu_j = \frac{1}{2} \\ 0 \leq \lambda_i \leq K,\ i = 1, \ldots, p \\ 0 \leq \mu_j \leq K,\ j = 1, \ldots, q \end{cases} \\ -\infty & \text{otherwise} \\ & \text{and } \sum_{i=1}^p \lambda_i u_i - \sum_{j=1}^q \mu_j v_j = 0. \end{cases}$$