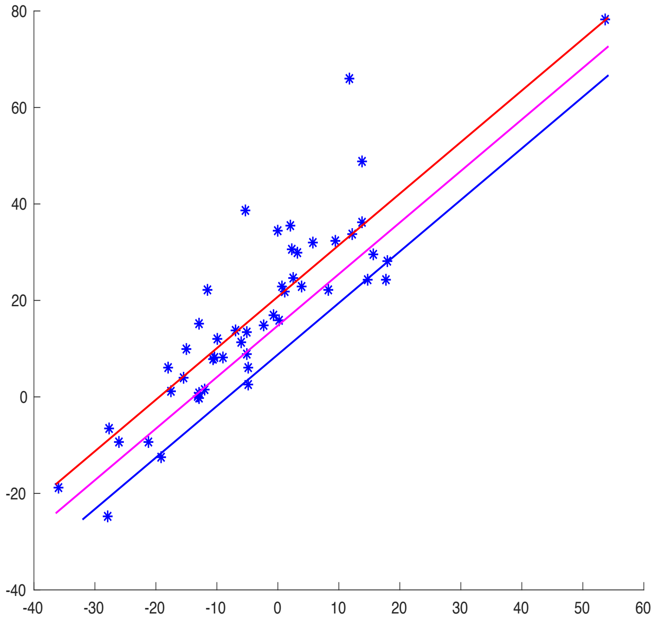

Figure 56.16: Running ν-SV regression version 2 on a set of 50 points; $\nu = 0.5$.

56.7 Problems

Problem 56.1. Prove that if ν-SV regression succeeds and yields $w, b, \epsilon > 0$, then ϵ-SV regression with the same C and the same value of ϵ also succeeds and returns the same pair (w, b).

Problem 56.2. Prove the formulae

$$b = \left(\left(\sum_{i_0 \in I_\lambda} y_{i_0}\right)/|I_\lambda| + \left(\sum_{j_0 \in I_\mu} y_{j_0}\right)/|I_\mu| - w^\top\left(\left(\sum_{i_0 \in I_\lambda} x_{i_0}\right)/|I_\lambda| + \left(\sum_{j_0 \in I_\mu} x_{j_0}\right)/|I_\mu|\right)\right)/2$$

$$\epsilon = \left(\left(\sum_{j_0 \in I_\mu} y_{j_0}\right)/|I_\mu| - \left(\sum_{i_0 \in I_\lambda} y_{i_0}\right)/|I_\lambda| + w^\top\left(\left(\sum_{i_0 \in I_\lambda} x_{i_0}\right)/|I_\lambda| - \left(\sum_{j_0 \in I_\mu} x_{j_0}\right)/|I_\mu|\right)\right)/2$$

stated just before Proposition 56.6.

Problem 56.3. Give the details of the proof of Proposition 56.6. In particular, prove that

$$\begin{aligned} C\left(\nu - \frac{p_f + q_f}{m}\right)\epsilon = &-\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} P \begin{pmatrix}\lambda \\ \mu\end{pmatrix} - \begin{pmatrix}y^\top & -y^\top\end{pmatrix}\begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\ &- \frac{C}{m}\left(w^\top\left(\sum_{i \in K_\lambda} x_i - \sum_{j \in K_\mu} x_j\right) - \sum_{i \in K_\lambda} y_i + \sum_{j \in K_\mu} y_j + (p_f - q_f)b\right). \end{aligned}$$