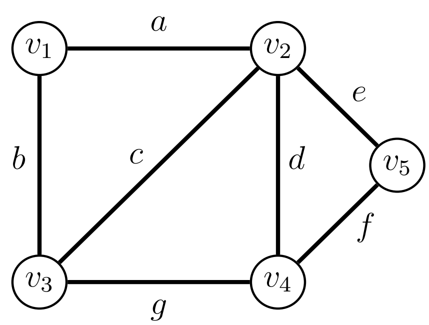

Figure 20.3: The undirected graph G_2.

in $\mathbb{R}^E$. This point of view is often useful. For example, the incidence matrix B can be interpreted as a linear map from $\mathbb{R}^E$ to $\mathbb{R}^V$, the *boundary map*, and $B^\top$ can be interpreted as a linear map from $\mathbb{R}^V$ to $\mathbb{R}^E$, the *coboundary map*.

Remark: Some authors adopt the opposite convention of sign in defining the incidence matrix, which means that their incidence matrix is $-B$.

Undirected graphs are obtained from directed graphs by forgetting the orientation of the edges.

Definition 20.5. A *graph* (or *undirected graph*) is a pair $G = (V, E)$, where $V = \{v_1, \ldots, v_m\}$ is a set of *nodes* or *vertices*, and E is a set of two-element subsets of V (that is, subsets $\{u, v\}$, with $u, v \in V$ and $u \neq v$), called *edges*.

Remark: Since an edge is a set $\{u, v\}$, we have $u \neq v$, so self-loops are not allowed. Also, for every set of nodes $\{u, v\}$, there is at most one edge between u and v. As in the case of directed graphs, such graphs are sometimes called *simple graphs*.

An example of a graph is shown in Figure 20.3.

Definition 20.6. For every node $v \in V$, the *degree* $d(v)$ of v is the number of edges incident to v:

$$d(v) = |\{u \in V \mid \{u, v\} \in E\}|.$$

The degree matrix $D(G)$ (or simply, D) is defined as in Definition 20.2.

Definition 20.7. Given a (undirected) graph $G = (V, E)$, for any two nodes $u, v \in V$, a *path from u to v* is a sequence of nodes $(v_0, v_1, \ldots, v_k)$ such that $v_0 = u$, $v_k = v$, and $\{v_i, v_{i+1}\}$ is an edge in E for all i with $0 \leq i \leq k - 1$. The integer k is the *length* of the path. A path is *closed* if $u = v$. The graph G is *connected* if for any two distinct nodes $u, v \in V$, there is a path from u to v.