

Since

$$1A^3 + 3A^4 + 2A^5 = Au_0 = b,$$

for any $\theta \in \mathbb{R}$, we have

$$\begin{aligned} b &= 1A^3 + 3A^4 + 2A^5 - \theta A^1 + \theta A^1 \\ &= 1A^3 + 3A^4 + 2A^5 - \theta(-A^3 + A^4) + \theta A^1 \\ &= \theta A^1 + (1 + \theta)A^3 + (3 - \theta)A^4 + 2A^5, \end{aligned}$$

and

$$\begin{aligned} b &= 1A^3 + 3A^4 + 2A^5 - \theta A^2 + \theta A^2 \\ &= 1A^3 + 3A^4 + 2A^5 - \theta(A^3 + A^5) + \theta A^2 \\ &= \theta A^2 + (1 - \theta)A^3 + 3A^4 + (2 - \theta)A^5. \end{aligned}$$

In the first case, the vector $(\theta, 0, 1 + \theta, 3 - \theta, 2)$ is a feasible solution iff $0 \leq \theta \leq 3$, and the new value of the objective function is θ .

In the second case, the vector $(0, \theta, 1 - \theta, 3, 2 - \theta, 1)$ is a feasible solution iff $0 \leq \theta \leq 1$, and the new value of the objective function is also θ .

Consider the first case. It is natural to ask whether we can get another vertex and increase the objective function by setting to zero one of the coordinates of $(\theta, 0, 1 + \theta, 3 - \theta, 2)$, in this case the fourth one, by picking $\theta = 3$. This yields the feasible solution $(3, 0, 4, 0, 2)$, which corresponds to the basis (A^1, A^3, A^5) , and so is indeed a basic feasible solution, with an improved value of the objective function equal to 3. Note that A^4 *left* the basis (A^3, A^4, A^5) and A^1 *entered* the new basis (A^1, A^3, A^5) .

We can now express A^2 and A^4 in terms of the basis (A^1, A^3, A^5) , which is easy to do since we already have A^1 and A^2 in term of (A^3, A^4, A^5) , and A^1 and A^4 are swapped. Such a step is called a *pivoting step*. We obtain

$$\begin{aligned} A^2 &= A^3 + A^5 \\ A^4 &= A^1 + A^3. \end{aligned}$$

Then we repeat the process with $u_1 = (3, 0, 4, 0, 2)$ and the basis (A^1, A^3, A^5) . We have

$$\begin{aligned} b &= 3A^1 + 4A^3 + 2A^5 - \theta A^2 + \theta A^2 \\ &= 3A^1 + 4A^3 + 2A^5 - \theta(A^3 + A^5) + \theta A^2 \\ &= 3A^1 + \theta A^2 + (4 - \theta)A^3 + (2 - \theta)A^5, \end{aligned}$$

and

$$\begin{aligned} b &= 3A^1 + 4A^3 + 2A^5 - \theta A^4 + \theta A^4 \\ &= 3A^1 + 4A^3 + 2A^5 - \theta(A^1 + A^3) + \theta A^4 \\ &= (3 - \theta)A^1 + (4 - \theta)A^3 + \theta A^4 + 2A^5. \end{aligned}$$