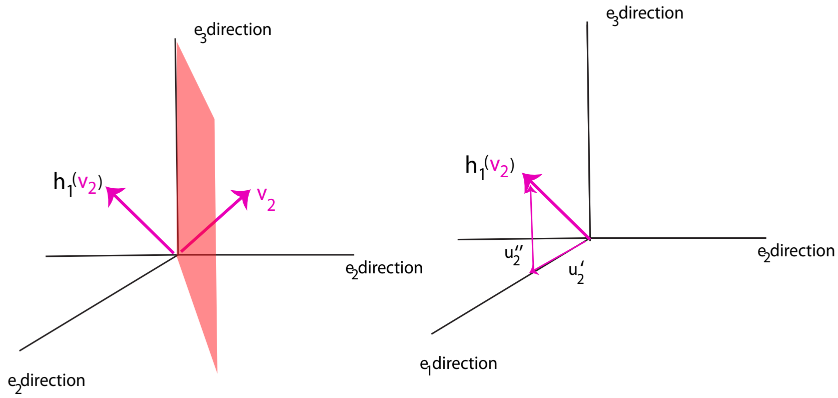

Figure 13.6: The construction of $u_2 = h_1(v_2)$ and its decomposition as $u_2 = u_2' + u_2''$.

Remarks:

(1) Since every h_i is a hyperplane reflection or the identity,

$$\rho = h_n \circ \cdots \circ h_2 \circ h_1$$

is an isometry.

(2) If we allow negative diagonal entries in R, the last isometry h_n may be omitted.

(3) Instead of picking $r_{k,k} = \|u_k''\|$, which means that

$$w_k = r_{k,k}\, e_k - u_k'',$$

where $1 \leq k \leq n$, it might be preferable to pick $r_{k,k} = -\|u_k''\|$ if this makes $\|w_k\|^2$ larger, in which case

$$w_k = r_{k,k}\, e_k + u_k''.$$

Indeed, since the definition of h_k involves division by $\|w_k\|^2$, it is desirable to avoid division by very small numbers.

(4) The method also applies to any m-tuple of vectors $(v_1, \ldots, v_m)$, with $m \leq n$. Then R is an upper triangular $m \times m$ matrix and Q is an $n \times m$ matrix with orthogonal columns ($Q^\top Q = I_m$). We leave the minor adjustments to the method as an exercise to the reader

Proposition 13.3 directly yields the QR-decomposition in terms of Householder transformations (see Strang [169, 170], Golub and Van Loan [80], Trefethen and Bau [176], Kincaid and Cheney [102], or Ciarlet [41]).