

and that  $L_B$  is an upper triangular matrix whose diagonal entries are

$$(\underbrace{\lambda_1, \dots, \lambda_1}_n, \dots, \underbrace{\lambda_n, \dots, \lambda_n}_n).$$

*Hint.* Figure out what are  $R_B(E_{ij}) = E_{ij}B$  and  $L_B(E_{ij}) = BE_{ij}$ .

(2) Use the fact that

$$L_A = L_P \circ L_B \circ L_P^{-1}, \quad R_A = R_P^{-1} \circ R_B \circ R_P,$$

to express  $\text{ad}_A = L_A - R_A$  in terms of  $L_B - R_B$ , and conclude that the eigenvalues of  $\text{ad}_A$  are  $\lambda_i - \lambda_j$ , for  $i = 1, \dots, n$ , and for  $j = n, \dots, 1$ .