- *Gradient* of a function (grad $f(a)$, $\nabla f(a)$).

- *Mean value theorem.*

- $C^0$-*functions,* $C^1$-*functions.*

- The *implicit function theorem.*

- *Local homeomorphisms, local diffeomorphisms, diffeomorphisms.*

- The *inverse function theorem.*

- *Immersions, submersions.*

- Second-order and higher-order derivatives.

- *Schwarz's lemma.*

- *Hessian matrix.*

- $C^\infty$-*functions, smooth functions.*

- *Taylor–Young's formula.*

- Generalized mean value theorem.

- *Taylor–MacLaurin's formula.*

- *Taylor's formula with integral remainder.*

- *Faà di Bruno's formula.*

## 39.11   Problems

**Problem 39.1.** Let $f\colon \mathrm{M}_n(\mathbb{R}) \to \mathrm{M}_n(\mathbb{R})$ be the function defined on $n \times n$ matrices by

$$f(A) = A^2.$$

Prove that

$$\mathrm{D}f_A(H) = AH + HA,$$

for all $A, H \in \mathrm{M}_n(\mathbb{R})$.

**Problem 39.2.** Let $f\colon \mathrm{M}_n(\mathbb{R}) \to \mathrm{M}_n(\mathbb{R})$ be the function defined on $n \times n$ matrices by

$$f(A) = A^3.$$

Prove that

$$\mathrm{D}f_A(H) = A^2H + AHA + HA^2,$$

for all $A, H \in \mathrm{M}_n(\mathbb{R})$.