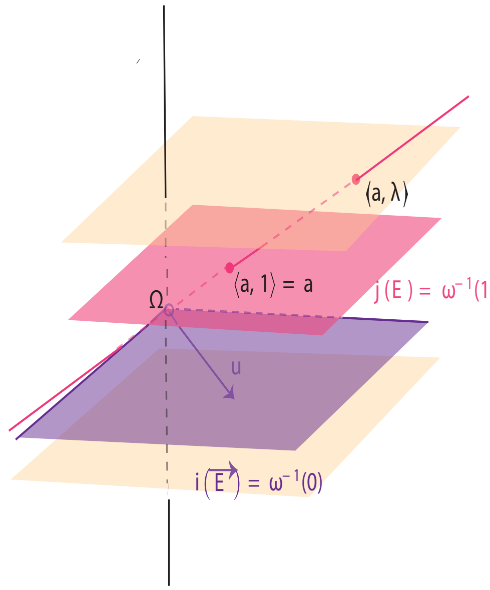

Figure 25.1: Embedding an affine space $\big(E, \overrightarrow{E}\big)$ into a vector space $\widehat{E}$.

Figure 25.1 illustrates the embedding of the affine space E into the vector space $\widehat{E}$, when E is an affine plane.

Note that $\widehat{E}$ is isomorphic to $\overrightarrow{E} \cup (E \times \mathbb{R}^*)$. Intuitively, we can think of $\widehat{E}$ as a stack of parallel hyperplanes, one for each λ, a little bit like an infinite stack of very thin pancakes! There are two privileged pancakes: one corresponding to E, for $\lambda = 1$, and one corresponding to $\overrightarrow{E}$, for $\lambda = 0$.

From now on, we will identify $j(E)$ and E, and $i\big(\overrightarrow{E}\big)$ and $\overrightarrow{E}$. We will also write λa instead of $\langle a, \lambda \rangle$, which we will call a *weighted point*, and write $1a$ just as a. When we want to be more precise, we may also write $\langle a, 1 \rangle$ as $\overline{a}$. In particular, when we consider the homogenized version $\widehat{\mathbb{A}}$ of the affine space $\mathbb{A}$ associated with the field $\mathbb{R}$ considered as an affine space, we write $\overline{\lambda}$ for $\langle \lambda, 1 \rangle$, when viewing λ as a point in both $\mathbb{A}$ and $\widehat{\mathbb{A}}$, and simply λ, when viewing λ as a vector in $\mathbb{R}$ and in $\widehat{\mathbb{A}}$. As an example, the expression $2 + 3$ denotes the real number 5, in $\mathbb{A}$, $(\overline{2} + \overline{3})/2$ denotes the midpoint of the segment $\big[\overline{2}, \overline{3}\big]$, which can be denoted by $\overline{2.5}$, and $\overline{2} + \overline{3}$ does not make sense in $\mathbb{A}$, since it is not a barycentric combination. However, in $\widehat{\mathbb{A}}$, the expression $\overline{2} + \overline{3}$ makes sense: It is the weighted point $\big\langle \overline{2.5}, 2 \big\rangle$.

Then, in view of the fact that

$$\langle a + u, 1 \rangle = \langle a, 1 \rangle \,\widehat{+}\, u,$$

and since we are identifying $a + u$ with $\langle a + u, 1 \rangle$ (under the injection j), in the simplified notation the above reads as $a + u = a \,\widehat{+}\, u$. Thus, we go one step further, and denote $a \,\widehat{+}\, u$