Step 2: The objective function in matrix form is given by

$$J(w,b) = \frac{1}{2}\begin{pmatrix} w^\top & b \end{pmatrix}\begin{pmatrix} I_n & 0_n \\ 0_n^\top & 0 \end{pmatrix}\begin{pmatrix} w \\ b \end{pmatrix}.$$

Note that the corresponding matrix is symmetric positive semidefinite, but it is *not* invertible. Thus, the function J is convex but not strictly convex. This will cause some minor trouble in finding the dual function of the problem.

Step 3: If we introduce the generalized Lagrange multipliers $\lambda \in \mathbb{R}^p$ and $\mu \in \mathbb{R}^q$, according to Proposition 50.7, the first KKT condition is

$$\nabla J_{(w,b)} + C^\top \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0_{n+1},$$

with $\lambda \geq 0, \mu \geq 0$. By the result of Example 39.5,

$$\nabla J_{(w,b)} = \begin{pmatrix} I_n & 0_n \\ 0_n^\top & 0 \end{pmatrix}\begin{pmatrix} w \\ b \end{pmatrix} = \begin{pmatrix} w \\ 0 \end{pmatrix},$$

so we get

$$\begin{pmatrix} w \\ 0 \end{pmatrix} = -C^\top \begin{pmatrix} \lambda \\ \mu \end{pmatrix},$$

that is,

$$\begin{pmatrix} w \\ 0 \end{pmatrix} = \begin{pmatrix} u_1 & \cdots & u_p & -v_1 & \cdots & -v_q \\ -1 & \cdots & -1 & 1 & \cdots & 1 \end{pmatrix}\begin{pmatrix} \lambda \\ \mu \end{pmatrix}.$$

Consequently,

$$w = \sum_{i=1}^{p} \lambda_i u_i - \sum_{j=1}^{q} \mu_j v_j, \tag{$*_1$}$$

and

$$\sum_{j=1}^{q} \mu_j - \sum_{i=1}^{p} \lambda_i = 0. \tag{$*_2$}$$

Step 4: Rewrite the constraint using $(*_1)$. Plugging the above expression for w into the constraints $C\begin{pmatrix} w \\ b \end{pmatrix} \leq d$ we get

$$\begin{pmatrix} -u_1^\top & 1 \\ \vdots & \vdots \\ -u_p^\top & 1 \\ v_1^\top & -1 \\ \vdots & \vdots \\ v_q^\top & -1 \end{pmatrix}\begin{pmatrix} u_1 & \cdots & u_p & -v_1 & \cdots & -v_q & 0_n \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}\begin{pmatrix} \lambda \\ \mu \\ b \end{pmatrix} \leq \begin{pmatrix} -1 \\ \vdots \\ -1 \end{pmatrix},$$