Dual of the Soft margin SVM (SVM_{s4}):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\left(X^\top X + \frac{1}{2K_s} I_{p+q}\right)\begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\
&\text{subject to} \\
&\qquad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0 \\
&\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq \nu \\
&\qquad \lambda_i \geq 0, \quad i = 1, \ldots, p \\
&\qquad \mu_j \geq 0, \quad j = 1, \ldots, q.
\end{aligned}$$

The above program is similar to the program that was obtained for Problem $(\mathrm{SVM}_{s2'})$ but the matrix $X^\top X$ is replaced by the matrix $X^\top X + (1/2K_s)I_{p+q}$, which is positive definite since $K_s > 0$, and also the inequalities $\lambda_i \leq K_s$ and $\mu_j \leq K_s$ no longer hold.

It is shown in Section 54.14 how the dual program is solved using ADMM from Section 52.6. If the primal problem is solvable, this yields solutions for λ and μ. We obtain w from λ and μ, as in Problem $(\mathrm{SVM}_{s2'})$; namely,

$$w = -X\begin{pmatrix}\lambda \\ \mu\end{pmatrix} = \sum_{i=1}^p \lambda_i u_i - \sum_{j=1}^q \mu_j v_j.$$

Since the variables ϵ_i and ξ_j are not restricted to be nonnegative we no longer have complementary slackness conditions involving them, but we know that

$$\epsilon = \frac{\lambda}{2K_s}, \quad \xi = \frac{\mu}{2K_s}.$$

Also since the constraints

$$\sum_{i=1}^p \lambda_i \geq \frac{\nu}{2} \quad \text{and} \quad \sum_{j=1}^q \mu_j \geq \frac{\nu}{2}$$

imply that there is some i_0 such that $\lambda_{i_0} > 0$ and some j_0 such that $\mu_{j_0} > 0$, we have $\epsilon_{i_0} > 0$ and $\xi_{j_0} > 0$, which means that at least two points are misclassified, so Problem (SVM_{s4}) should only be used when the sets $\{u_i\}$ and $\{v_j\}$ are *not* linearly separable.

Because $\epsilon_i = \lambda_i/(2K_s)$, $\xi_j = \mu_j/(2K_s)$, and there is no upper bound K_s on λ_i and μ_j, the classification of the points is simpler than in the previous cases.

(1) If $\lambda_i = 0$, then $\epsilon_i = 0$ and the inequality $w^\top u_i - b - \eta \geq 0$ holds. If equality holds then u_i is a support vector on the blue margin (the hyperplane $H_{w,b+\eta}$). Otherwise u_i is