Using the above criterion, it is a good exercise to reprove that if $\dim(E) = n$, then every tensor in $\bigwedge^{n-1}(E)$ is decomposable. We already proved this fact as a corollary of Proposition 34.23.

Given any $z = \sum_I \lambda_I e_I \in \bigwedge^p E$ where $\dim(E) = n$, the family of scalars (λ_I) (with $I = \{i_1 < \cdots < i_p\} \subseteq \{1, \ldots, n\}$ listed in increasing order) is called the *Plücker coordinates* of z.The Grassmann-Plücker's equations give necessary and sufficient conditions for any nonzero z to be decomposable.

For example, when $\dim(E) = n = 4$ and $p = 2$, these equations reduce to the single equation

$$\lambda_{12}\lambda_{34} - \lambda_{13}\lambda_{24} + \lambda_{14}\lambda_{23} = 0.$$

However, it should be noted that the equations given by Proposition 34.29 are not independent in general.

We are now in the position to prove that the Grassmannian $G(p, n)$ can be embedded in the projective space $\mathbb{RP}^{\binom{n}{p}-1}$,

For any $n \geq 1$ and any k with $1 \leq p \leq n$, recall that the Grassmannian $G(p, n)$ is the set of all linear p-dimensional subspaces of $\mathbb{R}^n$ (also called *p-planes*). Any p-dimensional subspace U of $\mathbb{R}^n$ is spanned by p linearly independent vectors $u_1, \ldots, u_p$ in $\mathbb{R}^n$; write $U = \mathrm{span}(u_1, \ldots, u_k)$. By Proposition 34.8, $(u_1, \ldots, u_p)$ are linearly independent iff $u_1 \wedge \cdots \wedge u_p \neq 0$. If $(v_1, \ldots, v_p)$ are any other linearly independent vectors spanning U, then we have

$$v_j = \sum_{i=1}^{p} a_{ij} u_i, \quad 1 \leq j \leq p,$$

for some $a_{ij} \in \mathbb{R}$, and by Proposition 34.2

$$v_1 \wedge \cdots \wedge v_p = \det(A)\, u_1 \wedge \cdots \wedge u_p,$$

where $A = (a_{ij})$. As a consequence, we can define a map $i_G \colon G(p, n) \to \mathbb{RP}^{\binom{n}{p}-1}$ such that for any k-plane U, for any basis $(u_1, \ldots, u_p)$ of U,

$$i_G(U) = [u_1 \wedge \cdots \wedge u_p],$$

the point of $\mathbb{RP}^{\binom{n}{p}-1}$ given by the one-dimensional subspace of $\mathbb{R}^{\binom{n}{p}}$ spanned by $u_1 \wedge \cdots \wedge u_p$.

Proposition 34.30. *The map $i_G \colon G(p, n) \to \mathbb{RP}^{\binom{n}{p}-1}$ is injective.*

Proof. Let U and V be any two p-planes and assume that $i_G(U) = i_G(V)$. This means that there is a basis $(u_1, \ldots, u_p)$ of U and a basis $(v_1, \ldots, v_p)$ of V such that

$$v_1 \wedge \cdots \wedge v_p = c\, u_1 \wedge \cdots \wedge u_p$$