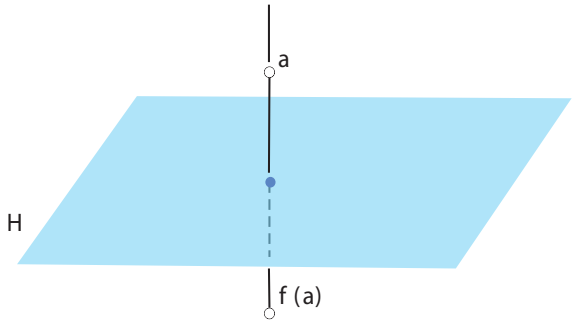

Figure 27.10: 3D improper affine rigid motion with a plane H of fixed points (reflection).

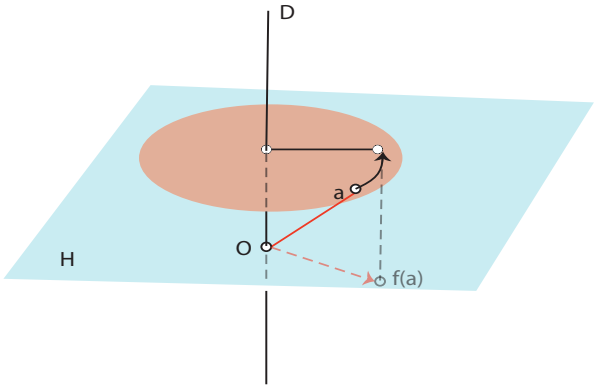

Figure 27.11: $3D$ improper affine rigid motion with a unique fixed point.

The third kind is the composition of an affine reflection about a plane followed by a nontrivial translation by a vector parallel to the direction of the plane of the reflection, as illustrated in Figure 27.13.

This last transformation is an improper affine isometry.

27.5 The Cartan–Dieudonné Theorem for Affine Isometries

The Cartan–Dieudonné theorem also holds for affine isometries, with a small twist due to translations. The reader is referred to Berger [11], Snapper and Troyer [162], or Tisseron [175] for a detailed treatment of the Cartan–Dieudonné theorem and its variants.

Theorem 27.11. *Let E be an affine Euclidean space of dimension $n \geq 1$. Every affine isometry $f \in \mathbf{Is}(E)$ that has a fixed point and is not the identity is the composition of at most n affine reflections. Every affine isometry $f \in \mathbf{Is}(E)$ that has no fixed point is the*