Problem 54.6. Prove that the kernel version of Program (SVM_{s2}) is given by:

Dual of Soft margin kernel SVM (SVM_{s2}):

$$
\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\mathbf{K}\begin{pmatrix}\lambda \\ \mu\end{pmatrix} - \begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix}\mathbf{1}_{p+q} \\
&\text{subject to} \\
&\qquad \sum_{i=1}^{p}\lambda_i - \sum_{j=1}^{q}\mu_j = 0 \\
&\qquad 0 \leq \lambda_i \leq K, \quad i = 1, \ldots, p \\
&\qquad 0 \leq \mu_j \leq K, \quad j = 1, \ldots, q,
\end{aligned}
$$

where $\mathbf{K}$ is the $\ell \times \ell$ kernel symmetric matrix (with $\ell = p + q$) given at the end of Section 54.1.

Problem 54.7. Prove that the matrix

$$
A = \begin{pmatrix}
\mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\
I_p & 0_{p,q} & I_p & 0_{p,q} \\
0_{q,p} & I_q & 0_{q,p} & I_q
\end{pmatrix}
$$

has rank $p + q + 1$.

Problem 54.8. Prove that the matrices

$$
A = \begin{pmatrix}
\mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0_p^\top & 0_q^\top & 0 \\
\mathbf{1}_p^\top & \mathbf{1}_q^\top & 0_p^\top & 0_q^\top & -1 \\
I_p & 0_{p,q} & I_p & 0_{p,q} & 0_p \\
0_{q,p} & I_q & 0_{q,p} & I_q & 0_q
\end{pmatrix}
\quad \text{and} \quad
A_2 = \begin{pmatrix}
\mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\
\mathbf{1}_p^\top & \mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\
I_p & 0_{p,q} & I_p & 0_{p,q} \\
0_{q,p} & I_q & 0_{q,p} & I_q
\end{pmatrix}
$$

have rank $p + q + 2$.

Problem 54.9. Prove that the kernel version of Program ($\mathrm{SVM}_{s2'}$) is given by: