The augmented Lagrangian is

$$\begin{aligned} L_\rho(x, y, \lambda) &= y^2 + 2x + \lambda(2x - y) + (\rho/2)(2x - y)^2 \\ &= 2\rho x^2 - 2\rho xy + 2(1+\lambda)x - \lambda y + \left(1 + \frac{\rho}{2}\right) y^2, \end{aligned}$$

which in matrix form is

$$L_\rho(x, y, \lambda) = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 2\rho^2 & -\rho \\ -\rho & 1 + \dfrac{\rho}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2(1+\lambda) & -\lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

The trace of the above matrix is $1 + \frac{\rho}{2} + 2\rho^2 > 0$, and the determinant is

$$2\rho^2 \left(1 + \frac{\rho}{2}\right) - \rho^2 = \rho^2(1 + \rho) > 0,$$

since $\rho > 0$. Therefore, the above matrix is symmetric positive definite. Minimizing $L_\rho(x, y, \lambda)$ with respect to x and y, we set the gradient of $L_\rho(x, y, \lambda)$ (with respect to x and y) to zero, and we obtain the equations:

$$\begin{aligned} 2\rho x - \rho y + (1 + \lambda) &= 0 \\ -2\rho x + (2 + \rho)y - \lambda &= 0. \end{aligned}$$

The solution is

$$x = -\frac{1}{4} - \frac{1+\lambda}{2\rho}, \quad y = -\frac{1}{2}.$$

Thus the steps for the method of multipliers are

$$\begin{aligned} x^{k+1} &= -\frac{1}{4} - \frac{1 + \lambda^k}{2\rho} \\ y^{k+1} &= -\frac{1}{2} \\ \lambda^{k+1} &= \lambda^k + \rho \begin{pmatrix} 2 & -1 \end{pmatrix} \begin{pmatrix} -\frac{1}{4} - \frac{1+\lambda^k}{2\rho} \\ -\frac{1}{2} \end{pmatrix}, \end{aligned}$$

and the second step simplifies to

$$\lambda^{k+1} = -1.$$

Consequently, we see that the method converges after two steps for any initial value of λ^0, and we get

$$x = -\frac{1}{4} \quad y = -\frac{1}{2}, \quad \lambda = -1.$$

The method of multipliers also converges for functions J that are not even convex, as illustrated by the next example.