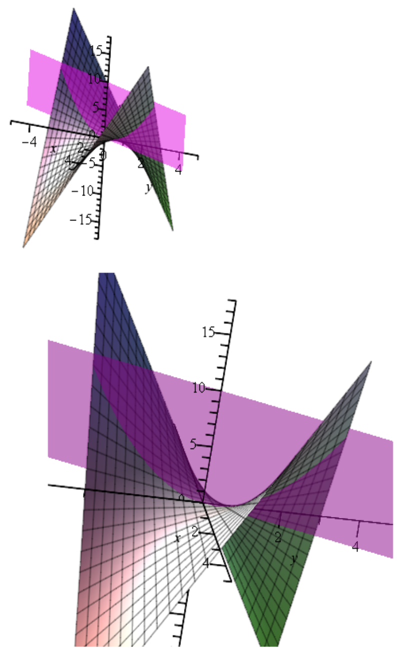

Figure 52.3: Two views of the graph of the saddle of $2xy$ ($\beta = 1$) intersected with the transparent magenta plane $2x - y = 0$. The solution to Example 52.3 is apex of the intersection curve, namely the point $(0, 0, 0)$.

Example 52.3. Consider the minimization problem

$$\begin{aligned} &\text{minimize} \quad 2\beta xy \\ &\text{subject to} \quad 2x - y = 0, \end{aligned}$$

with $\beta > 0$. See Figure 52.3.

The quadratic function

$$J(x, y) = 2\beta xy = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 0 & \beta \\ \beta & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

is not convex because the above matrix is not even positive semidefinite (the eigenvalues of the matrix are $-\beta$ and $+\beta$). The augmented Lagrangian is

$$\begin{aligned} L_\rho(x, y, \lambda) &= 2\beta xy + \lambda(2x - y) + (\rho/2)(2x - y)^2 \\ &= 2\rho x^2 + 2(\beta - \rho)xy + 2\lambda x - \lambda y + \frac{\rho}{2} y^2, \end{aligned}$$

which in matrix form is

$$L_\rho(x, y, \lambda) = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 2\rho & \beta - \rho \\ \beta - \rho & \dfrac{\rho}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2\lambda & -\lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$