

The margin is $\delta = 1/\|w\|$. The separating hyperplane $H_{w,b}$ is the hyperplane of equation $w^\top x - b = 0$, and the margin hyperplanes are the hyperplanes $H_{w,b+1}$ (the blue hyperplane) of equation $w^\top x - b - 1 = 0$ and $H_{w,b-1}$ (the red hyperplane) of equation $w^\top x - b + 1 = 0$. The dual program derived in Section 50.10 is the following program:

Dual of the Hard margin SVM (SVM_{h2}):

$$\begin{aligned} & \text{minimize} \quad \frac{1}{2} (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} - (\lambda^\top \quad \mu^\top) \mathbf{1}_{p+q} \\ & \text{subject to} \\ & \quad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0 \\ & \quad \lambda \geq 0, \mu \geq 0, \end{aligned}$$

where X is the $n \times (p+q)$ matrix given by

$$X = \begin{pmatrix} -u_1 & \cdots & -u_p & v_1 & \cdots & v_q \end{pmatrix}.$$

Then w is determined as follows:

$$w = -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \sum_{i=1}^p \lambda_i u_i - \sum_{j=1}^q \mu_j v_j.$$

To solve the dual using ADMM we need to determine the matrices P, q, A and c as in Section 52.6(2). We renamed b as c to avoid a clash since b is already used. We have

$$P = X^\top X, \quad q = -\mathbf{1}_{p+q},$$

and since the only constraint is

$$\sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0,$$

the matrix A is the $1 \times (p+q)$ row vector

$$A = (\mathbf{1}_p^\top \quad -\mathbf{1}_q^\top),$$

and the right-hand side c is the scalar

$$c = 0.$$

Obviously the matrix A has rank 1. We obtain b using any i_0 such that $\lambda_{i_0} > 0$ and any j_0 such that $\mu_{j_0} > 0$. Since the corresponding constraints are active, we have

$$w^\top u_{i_0} - b = 1, \quad -w^\top v_{j_0} + b = 1,$$