We have the inclusions $E_\lambda \subseteq K_\lambda$ and $E_\mu \subseteq K_\mu$. The difference between the first sets and the second sets is that the second sets may contain support vectors of type 2 such that $\lambda_i = K_s$ and $\epsilon_i = 0$, or $\mu_j = K_s$ and $\xi_j = 0$. We also have the equations $I_\lambda \cup (K_\lambda - E_\lambda) \cup E_\lambda = I_{\lambda>0}$ and $I_\mu \cup (K_\mu - E_\mu) \cup E_\mu = I_{\mu>0}$, and the inequalities $p_{sf} \leq p_f \leq p_m$ and $q_{sf} \leq q_f \leq q_m$.

The blue points u_i of index $i \in I_{\lambda>0}$ are classified as follows:

(1) If $i \in I_\lambda$, then u_i is a support vector of type 1 ($\lambda_i < K_s$).

(2) If $i \in K_\lambda - E_\lambda$, then u_i is a support vector of type 2 ($\lambda_i = K_s$).

(3) If $i \in E_\lambda$, then u_i strictly fails the margin, that is $\epsilon_i > 0$.

Similarly the red points v_j of index $j \in I_{\mu>0}$ are classified as follows:

(1) If $j \in I_\mu$, then v_j is a support vector of type 1 ($\mu_j < K_s$).

(2) If $j \in K_\mu - E_\mu$, then v_j is a support vector of type 2 ($\mu_j = K_s$).

(3) If $j \in E_\mu$, then v_j strictly fails the margin, that is $\xi_j > 0$.

Note that $p_m - p_f$ is the number of blue support vectors of type 1 and $q_m - q_f$ is the number of red support vectors of type 1. The remaining blue points u_i for which $\lambda_i = 0$ are either exceptional support vectors or they are (strictly) in the open half-space corresponding to the blue side. Similarly, the remaining red points v_j for which $\mu_j = 0$ are either exceptional support vectors or they are (strictly) in the open half-space corresponding to the red side.

It is shown in Section 54.8 how the dual program is solved using ADMM from Section 52.6. If the primal problem is solvable, this yields solutions for λ and μ. Once a solution for λ and μ is obtained, we have

$$w = -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \sum_{i=1}^{p} \lambda_i u_i - \sum_{j=1}^{q} \mu_j v_j.$$

As we said earlier, the hypotheses of Theorem 50.17(2) hold, so *if* the primal problem (SVM$_{s2'}$) has an optimal solution with $w \neq 0$, *then* the dual problem has a solution too, and the duality gap is zero. Therefore, for optimal solutions we have

$$L(w, \epsilon, \xi, b, \eta, \lambda, \mu, \alpha, \beta, \gamma) = G(\lambda, \mu, \alpha, \beta, \gamma),$$

which means that

$$\frac{1}{2} w^\top w - K_m \eta + K_s \left(\sum_{i=1}^{p} \epsilon_i + \sum_{j=1}^{q} \xi_j \right) = -\frac{1}{2} \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix},$$

and since

$$w = -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix},$$