is the intersection of half spaces passing through the origin, so it is a convex set, and obviously it is a cone. If $I(u) = \emptyset$, then $C^*(u) = V$.

The special kinds of $\mathcal{H}$-polyhedra of the form $C^*(u)$ cut out by hyperplanes through the origin are called *$\mathcal{H}$-cones*. It can be shown that every $\mathcal{H}$-cone is a polyhedral cone (also called a *$\mathcal{V}$-cone*), and conversely. The proof is nontrivial; see Gallier [73] and Ziegler [195].

We will prove shortly that we always have the inclusion

$$C(u) \subseteq C^*(u).$$

However, the inclusion can be strict, as in Example 50.1. Indeed for $u = (0,0)$ we have $I(0,0) = \{1,2\}$ and since

$$(\varphi_1')_{(u_1,u_2)} = (-1 \;\; -1), \quad (\varphi_2')_{(u_1,u_2)} = (3u_1^2 + u_2^2 - 2u_1 \;\; 2u_1u_2 + 2u_2),$$

we have $(\varphi_2')_{(0,0)} = (0 \;\; 0)$, and thus $C^*(0) = \{(u_1,u_2) \in \mathbb{R}^2 \mid u_1 + u_2 \geq 0\}$ as illustrated in Figure 50.7.

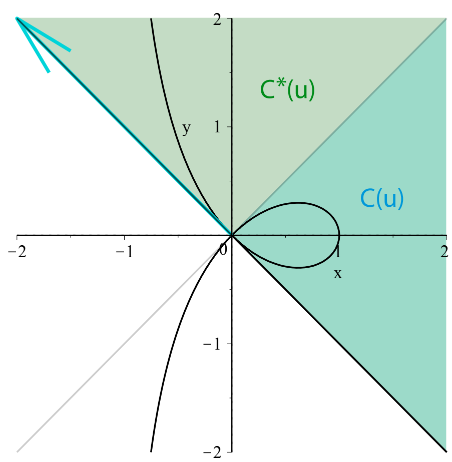

Figure 50.7: For $u = (0,0)$, $C^*(u)$ is the sea green half space given by $u_1 + u_2 \geq 0$. This half space strictly contains $C(u)$, namely the union of the turquoise triangular cone and the directional ray $(-1,1)$.

The conditions stated in the following definition are sufficient conditions that imply that $C(u) = C^*(u)$, as we will prove next.

Definition 50.5. For any $u \in U$, with

$$U = \{x \in \Omega \mid \varphi_i(x) \leq 0, \;\; 1 \leq i \leq m\},$$

if the functions φ_i are differentiable at u (in fact, we only this for $i \in I(u)$), we say that the constraints are *qualified* at u if the following conditions hold: