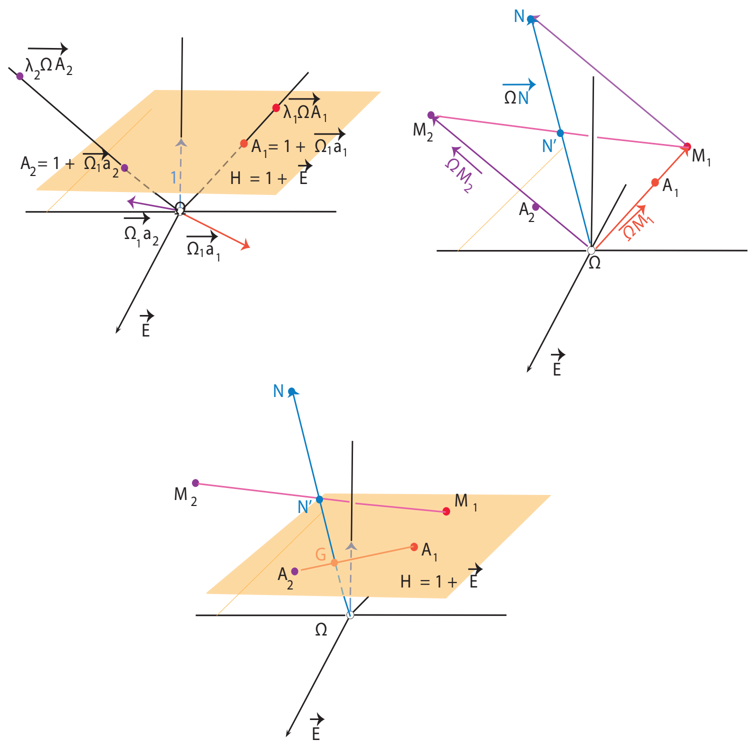

Figure 25.5: The geometric construction of $\widehat{\Omega}(\langle a_1, \lambda_1\rangle) + \widehat{\Omega}(\langle a_2, \lambda_2\rangle)$ for $\lambda_1 + \lambda_2 \neq 0$.

If $\lambda_1 + \lambda_2 = 0$, then $\langle a_1, \lambda_1\rangle \,\widehat{+}\, \langle a_2, \lambda_2\rangle$ is a vector determined as follows. Again, find the points M_1 and M_2 on the lines passing through the origin Ω of $\mathcal{F}$ and the points $A_1 = \widehat{\Omega}(a_1)$ and $A_2 = \widehat{\Omega}(a_2)$ in the hyperplane H, such that $\overrightarrow{\Omega M_1} = \lambda_1 \overrightarrow{\Omega A_1}$ and $\overrightarrow{\Omega M_2} = \lambda_2 \overrightarrow{\Omega A_2}$, and add the vectors $\overrightarrow{\Omega M_1}$ and $\overrightarrow{\Omega M_2}$, getting a point N such that $\overrightarrow{\Omega N} = \overrightarrow{\Omega M_1} + \overrightarrow{\Omega M_2}$. The desired vector is $\overrightarrow{\Omega N}$, which is parallel to the line A_1A_2. Equivalently, let N' be the middle of the segment M_1M_2, and the desired vector is $2\overrightarrow{\Omega N'}$. See Figure 25.6.

We can also give a geometric interpretation of $\langle a, \lambda\rangle + u$. Let $A = \widehat{\Omega}(a)$ in the hyperplane H, let D be the line determined by A and u, let M_1 be the point such that $\overrightarrow{\Omega M_1} = \lambda \overrightarrow{\Omega A}$, and let M_2 be the point such that $\overrightarrow{\Omega M_2} = u$, that is, $M_2 = \Omega + u$. By construction, the line D is in the hyperplane H, and it is parallel to $\overrightarrow{\Omega M_2}$, so that D, M_1, and M_2 are coplanar. Then, add the vectors $\overrightarrow{\Omega M_1}$ and $\overrightarrow{\Omega M_2}$, getting a point N such that $\overrightarrow{\Omega N} = \overrightarrow{\Omega M_1} + \overrightarrow{\Omega M_2}$, and let G be the intersection of the line determined by Ω and N with the line D. If $g = \widehat{\Omega}^{-1}(\overrightarrow{\Omega G})$, then, $\widehat{\Omega}^{-1}(\overrightarrow{\Omega N}) = \langle g, \lambda\rangle$. Equivalently, if N' is the middle of the segment M_1M_2, then G is the intersection of the line determined by Ω and N', with the line D; see Figure 25.7.

We now consider the universal property of $\widehat{E}$ mentioned at the beginning of this section.