so if let X be the $n \times (p+q)$ matrix given by

$$X = \begin{pmatrix} -u_1 & \cdots & -u_p & v_1 & \cdots & v_q \end{pmatrix},$$

we obtain

$$w = -X \begin{pmatrix} \lambda \\ \mu \end{pmatrix}, \tag{$*_1'$}$$

and the above inequalities are written in matrix form as

$$\begin{pmatrix} X^\top & \begin{matrix} \mathbf{1}_p \\ -\mathbf{1}_q \end{matrix} \end{pmatrix} \begin{pmatrix} -X & 0_n \\ 0_{p+q}^\top & 1 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \\ b \end{pmatrix} \leq -\mathbf{1}_{p+q};$$

that is,

$$-X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + b \begin{pmatrix} \mathbf{1}_p \\ -\mathbf{1}_q \end{pmatrix} + \mathbf{1}_{p+q} \leq 0_{p+q}. \tag{$*_3$}$$

Equivalently, the ith inequality is

$$-\sum_{j=1}^{p} u_i^\top u_j \lambda_j + \sum_{k=1}^{q} u_i^\top v_k \mu_k + b + 1 \leq 0 \qquad i = 1, \ldots, p,$$

and the $(p+j)$th inequality is

$$\sum_{i=1}^{p} v_j^\top u_i \lambda_i - \sum_{k=1}^{q} v_j^\top v_k \mu_k - b + 1 \leq 0 \qquad j = 1, \ldots, q.$$

We also have $\lambda \geq 0, \mu \geq 0$. Furthermore, if the ith inequality is inactive, then $\lambda_i = 0$, and if the $(p+j)$th inequality is inactive, then $\mu_j = 0$. Since the constraints are affine and since J is convex, if we can find $\lambda \geq 0$, $\mu \geq 0$, and b such that the inequalities in $(*_3)$ are satisfied, and $\lambda_i = 0$ and $\mu_j = 0$ when the corresponding constraint is inactive, then by Proposition 50.7 we have an optimum solution.

Remark: The second KKT condition can be written as

$$\begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \left(-X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + b \begin{pmatrix} \mathbf{1}_p \\ -\mathbf{1}_q \end{pmatrix} + \mathbf{1}_{p+q} \right) = 0;$$

that is,

$$-\begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + b \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \begin{pmatrix} \mathbf{1}_p \\ -\mathbf{1}_q \end{pmatrix} + \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} \mathbf{1}_{p+q} = 0.$$

Since $(*_2)$ says that $\sum_{i=1}^p \lambda_i = \sum_{j=1}^q \mu_j$, the second term is zero, and by $(*_1')$ we get

$$w^\top w = \begin{pmatrix} \lambda^\top & \mu^\top \end{pmatrix} X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \mu_j.$$