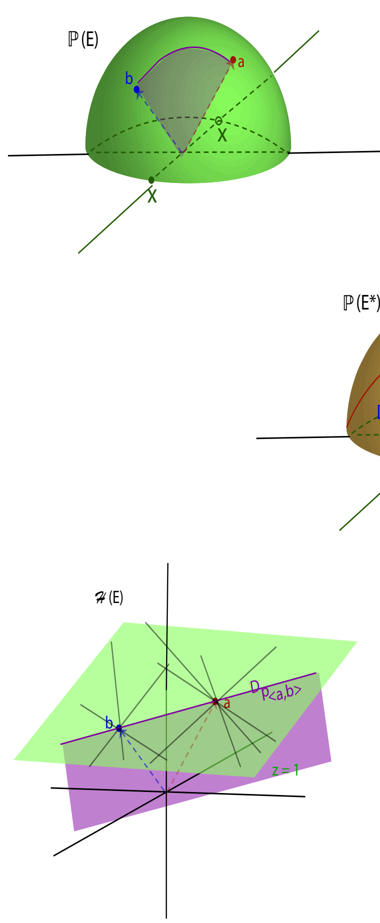

Figure 26.38: The duality between a line through two points in $\mathbf{P}(E)$ and a point incident to two lines in $\mathbf{P}(E^*)$.

which means that the cross-ratio of the d_i is independent of the line Δ (see Figure 26.39).

In fact, this cross-ratio is equal to $[D_1, D_2, D_3, D_4]$, as shown in the next proposition.

Proposition 26.27. *Let $P = \mathbf{P}(U)$ be a pencil of hyperplanes in $\mathcal{H}(E)$, and let $\Delta = \mathbf{P}(D)$ be any projective line such that $\Delta \notin H$ for all $H \in P$. The map $h\colon P \to \Delta$ defined such that $h(H) = H \cap \Delta$ for every hyperplane $H \in P$ is a projectivity. Furthermore, for any sequence (H_1, H_2, H_3, H_4) of hyperplanes in the pencil P, if H_1, H_2, H_3 are distinct and $d_i = \Delta \cap H_i$, then $[d_1, d_2, d_3, d_4] = [H_1, H_2, H_3, H_4]$.*

Proof. First, the map $h\colon P \to \Delta$ is well–defined, since in a projective space, every line $\Delta = \mathbf{P}(D)$ not contained in a hyperplane intersects this hyperplane in exactly one point. Since $P = \mathbf{P}(U)$ is a pencil of hyperplanes in $\mathcal{H}(E)$, U has dimension 2, and let φ and ψ be two nonnull linear forms in E^* that constitute a basis of U, and let $F = \varphi^{-1}(0)$ and