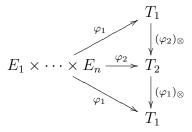
as illustrated by the following commutative diagram.

$$E_1 \times \cdots \times E_n \xrightarrow{\varphi_2} T_2$$

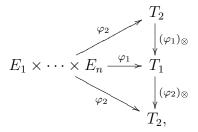
$$\downarrow^{(\varphi_1)_{\otimes}}$$

$$T_1$$

Putting these diagrams together, we obtain the commutative diagrams



and



which means that

$$\varphi_1 = (\varphi_1)_{\otimes} \circ (\varphi_2)_{\otimes} \circ \varphi_1$$
 and $\varphi_2 = (\varphi_2)_{\otimes} \circ (\varphi_1)_{\otimes} \circ \varphi_2$.

On the other hand, focusing on (T_1, φ_1) , we have a multilinear map $\varphi_1 : E_1 \times \cdots \times E_n \to T_1$, but the unique linear map $h : T_1 \to T_1$ with

$$\varphi_1 = h \circ \varphi_1$$

is h = id, as illustrated by the following commutative diagram

$$E_1 \times \cdots \times E_n \xrightarrow{\varphi_1} T_1$$

$$\downarrow_{\text{id}}$$

$$T_1,$$

and since $(\varphi_1)_{\otimes} \circ (\varphi_2)_{\otimes}$ is linear as a composition of linear maps, we must have

$$(\varphi_1)_{\otimes} \circ (\varphi_2)_{\otimes} = \mathrm{id}.$$