We have examples where there is a single support vector of type 1 and $\nu = 2q_f/(p+q)$, so the above method fails. Curiously, perturbing ν slightly yields a solution with some blue support vector of type 1 and some red support vector of type 1, and so we have not yet found an example where the above method succeeds with a single support vector of type 1. This suggests to conduct some perturbation analysis but it appears to be nontrivial.

Among its advantages, the support vector machinery is conducive to finding interesting statistical bounds in terms of the *VC dimension*, a notion invented by Vapnik and Chernovenkis. We will not go into this here and instead refer the reader to Vapnik [182] (especially, Chapter 4 and Chapters 9-13).

54.8 Solving SVM $(\mathrm{SVM}_{s2'})$ Using ADMM

In order to solve $(\mathrm{SVM}_{s2'})$ using ADMM we need to write the matrix corresponding to the constraints in equational form,

$$\begin{aligned}\sum_{i=1}^{p} \lambda_i - \sum_{j=1}^{q} \mu_j &= 0\\ \sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \mu_j - \gamma &= K_m\\ \lambda_i + \alpha_i &= K_s, \quad i = 1, \ldots, p\\ \mu_j + \beta_j &= K_s, \quad j = 1, \ldots, q,\end{aligned}$$

with $K_m = (p+q)K_s\nu$. This is the $(p+q+2)\times(2(p+q)+1)$ matrix A given by

$$A = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0_p^\top & 0_q^\top & 0\\ \mathbf{1}_p^\top & \mathbf{1}_q^\top & 0_p^\top & 0_q^\top & -1\\ I_p & 0_{p,q} & I_p & 0_{p,q} & 0_p\\ 0_{q,p} & I_q & 0_{q,p} & I_q & 0_q \end{pmatrix}.$$

Observe the remarkable analogy with the matrix arising in ν-regression in Section 56.3, except that $p = q = m$ and that -1 is replaced by $+1$. We leave it as an exercise to prove that A has rank $p+q+2$. The right-hand side is

$$c = \begin{pmatrix} 0\\ K_m\\ K_s\mathbf{1}_{p+q}\end{pmatrix}.$$

The symmetric positive semidefinite $(p+q)\times(p+q)$ matrix P defining the quadratic functional is

$$P = X^\top X, \quad \text{with} \quad X = \begin{pmatrix} -u_1 & \cdots & -u_p & v_1 & \cdots & v_q\end{pmatrix},$$