The above program is converted in ADMM form as follows:

$$\begin{aligned} &\text{minimize} && f(x) + g(z) \\ &\text{subject to} && x - z = 0, \end{aligned}$$

with

$$f(x) = \frac{1}{2} x^\top P x + q^\top x + r, \quad \mathrm{dom}(f) = \{x \in \mathbb{R}^n \mid Ax = b\},$$

and

$$g = I_{\mathbb{R}^n_+},$$

the indicator function of the positive orthant $\mathbb{R}^n_+$. In view of Example 52.8 and Example 52.10, the scaled form of ADMM consists of the following steps:

$$\begin{aligned} x^{k+1} &= \operatorname*{arg\,min}_x \left(f(x) + (\rho/2) \left\| x - z^k + u^k \right\|_2^2 \right) \\ z^{k+1} &= (x^{k+1} + u^k)_+ \\ u^{k+1} &= u^k + x^{k+1} - z^{k+1}. \end{aligned}$$

The x-update involves solving the KKT equations

$$\begin{pmatrix} P + \rho I & A^\top \\ A & 0 \end{pmatrix} \begin{pmatrix} x^{k+1} \\ y \end{pmatrix} = \begin{pmatrix} -q + \rho(z^k - u^k) \\ b \end{pmatrix}.$$

This is an important example because it provides one of the best methods for solving quadratic problems, in particular, the SVM problems discussed in Chapter 54.

(3) *Quadratic Programming, Version 2*. This problem is similar to the previous one, except that the variable $x \in \mathbb{R}^n$ is *not restricted* to be nonnegative. The problem is

$$\begin{aligned} &\text{minimize} && \frac{1}{2} x^\top P x + q^\top x + r \\ &\text{subject to} && Ax = b, \end{aligned}$$

where P is an $n \times n$ symmetric positive semidefinite matrix, $q \in \mathbb{R}^n$, $r \in \mathbb{R}$, and A is an $m \times n$ matrix of rank m.

The above program is converted in ADMM form as follows:

$$\begin{aligned} &\text{minimize} && f(x) + g(z) \\ &\text{subject to} && x - z = 0, \end{aligned}$$

with

$$f(x) = \frac{1}{2} x^\top P x + q^\top x + r, \quad \mathrm{dom}(f) = \{x \in \mathbb{R}^n \mid Ax = b\},$$