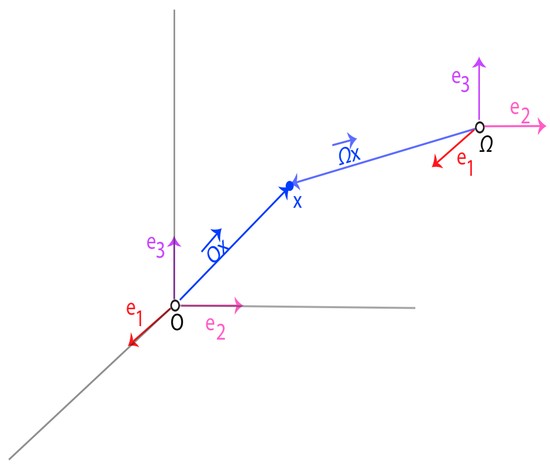

Figure 24.2: The two position vectors for the point x.

to the basis (e_1, e_2, e_3), for any two scalars λ, μ, we can define the linear combination $\lambda u + \mu v$ as the vector of coordinates

$$(\lambda u_1 + \mu v_1, \lambda u_2 + \mu v_2, \lambda u_3 + \mu v_3).$$

If we choose a different basis (e_1', e_2', e_3') and if the matrix P expressing the vectors (e_1', e_2', e_3') over the basis (e_1, e_2, e_3) is

$$P = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix},$$

which means that the columns of P are the coordinates of the e_j' over the basis (e_1, e_2, e_3), since

$$u_1 e_1 + u_2 e_2 + u_3 e_3 = u_1' e_1' + u_2' e_2' + u_3' e_3'$$

and

$$v_1 e_1 + v_2 e_2 + v_3 e_3 = v_1' e_1' + v_2' e_2' + v_3' e_3',$$

it is easy to see that the coordinates (u_1, u_2, u_3) and (v_1, v_2, v_3) of u and v with respect to the basis (e_1, e_2, e_3) are given in terms of the coordinates (u_1', u_2', u_3') and (v_1', v_2', v_3') of u and v with respect to the basis (e_1', e_2', e_3') by the matrix equations

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = P \begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = P \begin{pmatrix} v_1' \\ v_2' \\ v_3' \end{pmatrix}.$$

From the above, we get