This is the $(p + q + 1) \times 2(p + q)$ matrix A given by

$$A = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\ I_p & 0_{p,q} & I_p & 0_{p,q} \\ 0_{q,p} & I_q & 0_{q,p} & I_q \end{pmatrix}.$$

We leave it as an exercise to prove that A has rank $p + q + 1$. The right-hand side is

$$c = \begin{pmatrix} 0 \\ K\mathbf{1}_{p+q} \end{pmatrix}.$$

The symmetric positive semidefinite $(p+q)\times(p+q)$ matrix P defining the quadratic functional is

$$P = X^\top X, \quad \text{with} \quad X = \begin{pmatrix} -u_1 & \cdots & -u_p & v_1 & \cdots & v_q \end{pmatrix},$$

and

$$q = -\mathbf{1}_{p+q}.$$

Since there are $2(p + q)$ Lagrange multipliers $(\lambda, \mu, \alpha, \beta)$, the $(p + q) \times (p + q)$ matrix $X^\top X$ must be augmented with zero's to make it a $2(p + q) \times 2(p + q)$ matrix P_a given by

$$P_a = \begin{pmatrix} X^\top X & 0_{p+q,p+q} \\ 0_{p+q,p+q} & 0_{p+q,p+q} \end{pmatrix},$$

and similarly q is augmented with zeros as the vector

$$q_a = \begin{pmatrix} -\mathbf{1}_{p+q} \\ 0_{p+q}. \end{pmatrix}$$

54.5 Soft Margin Support Vector Machines; $(\mathrm{SVM}_{s2'})$

In this section we consider a generalization of Problem (SVM_{s2}) for a version of the soft margin SVM coming from Problem (SVM_{h2}) by adding an extra degree of freedom, namely instead of the margin $\delta = 1/\left\|w\right\|$, we use the margin $\delta = \eta/\left\|w\right\|$ where η is some positive constant that we wish to maximize. To do so, we add a term $-K_m\eta$ to the objective function $(1/2)w^\top w$ as well as the "regularizing term" $K_s\left(\sum_{i=1}^p \epsilon_i + \sum_{j=1}^q \xi_j\right)$ whose purpose is to make ϵ and ξ sparse, where $K_m > 0$ (m refers to margin) and $K_s > 0$ (s refers to sparse) are fixed constants that can be adjusted to determine the influence of η and the regularizing term.