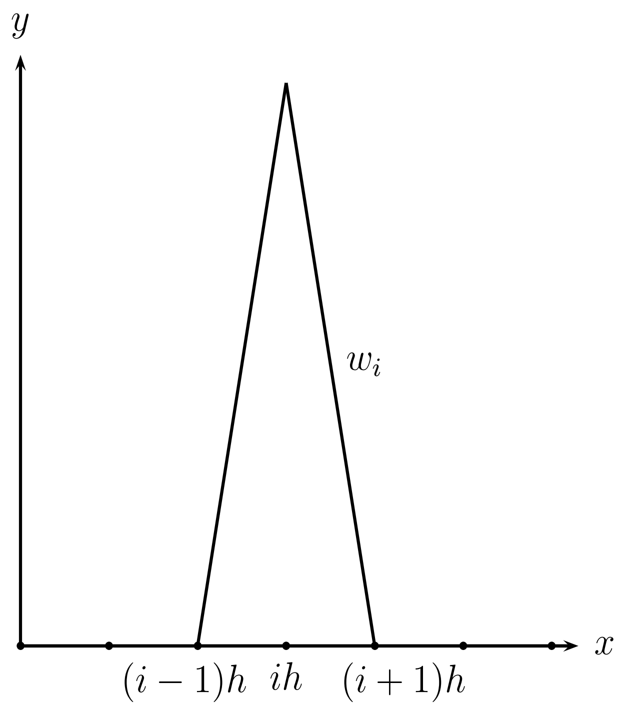

Figure 19.3: A basis "hat function"

In general, it it not hard to see that V_N^m has dimension $mN + 2(m-1)$.

Going back to our problem (the bending of a beam), assuming that c and f are constant functions, it is not hard to show that the linear system $(*)$ becomes

$$\frac{1}{h}\begin{pmatrix} 2+\frac{2c}{3}h^2 & -1+\frac{c}{6}h^2 & & & \\ -1+\frac{c}{6}h^2 & 2+\frac{2c}{3}h^2 & -1+\frac{c}{6}h^2 & & \\ \ddots & \ddots & \ddots & & \\ & -1+\frac{c}{6}h^2 & 2+\frac{2c}{3}h^2 & -1+\frac{c}{6}h^2 & \\ & & & -1+\frac{c}{6}h^2 & 2+\frac{2c}{3}h^2 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-1} \\ u_N \end{pmatrix} = h\begin{pmatrix} f \\ f \\ \vdots \\ f \\ f \end{pmatrix}.$$

We can also find a basis of $2N+2$ cubic functions for V_N^1 consisting of functions with small support. This basis consists of the N functions w_i^0 and of the $N+2$ functions w_i^1