As we can see, C has more zero entries than A; it is a compressed version of A. We can further compress C by setting to 0 all entries of absolute value at most 0.5. Then we get

$$C_2 = \begin{pmatrix} 32.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & -4 & 4 & -4 \\ 0 & 0 & 0 & 0 & 4 & -4 & 4 & -4 \\ 0 & 0 & 0 & 0 & 27 & -25 & 23 & -21 \\ 0 & 0 & 0 & 0 & -11 & 9 & -7 & 5 \\ 0 & 0 & 0 & 0 & -5 & 7 & -9 & 11 \\ 0 & 0 & 0 & 0 & 21 & -23 & 25 & -27 \end{pmatrix}.$$

We find that the reconstructed image is

$$A_2 = \begin{pmatrix} 63.5 & 1.5 & 3.5 & 61.5 & 59.5 & 5.5 & 7.5 & 57.5 \\ 9.5 & 55.5 & 53.5 & 11.5 & 13.5 & 51.5 & 49.5 & 15.5 \\ 17.5 & 47.5 & 45.5 & 19.5 & 21.5 & 43.5 & 41.5 & 23.5 \\ 39.5 & 25.5 & 27.5 & 37.5 & 35.5 & 29.5 & 31.5 & 33.5 \\ 31.5 & 33.5 & 35.5 & 29.5 & 27.5 & 37.5 & 39.5 & 25.5 \\ 41.5 & 23.5 & 21.5 & 43.5 & 45.5 & 19.5 & 17.5 & 47.5 \\ 49.5 & 15.5 & 13.5 & 51.5 & 53.5 & 11.5 & 9.5 & 55.5 \\ 7.5 & 57.5 & 59.5 & 5.5 & 3.5 & 61.5 & 63.5 & 1.5 \end{pmatrix},$$

which is pretty close to the original image matrix A.

It turns out that `Matlab` has a wonderful command, `image(X)` (also `imagesc(X)`, which often does a better job), which displays the matrix X has an image in which each entry is shown as a little square whose gray level is proportional to the numerical value of that entry (lighter if the value is higher, darker if the value is closer to zero; negative values are treated as zero). The images corresponding to A and C are shown in Figure 5.10. The compressed images corresponding to A_2 and C_2 are shown in Figure 5.11. The compressed versions appear to be indistinguishable from the originals!

If we use the normalized matrices H_m and H_n, then the equations relating the image matrix A and its normalized Haar transform C are

$$\begin{aligned} C &= H_m^\top A H_n \\ A &= H_m C H_n^\top . \end{aligned}$$

The Haar transform can also be used to send large images progressively over the internet. Indeed, we can start sending the Haar coefficients of the matrix C starting from the coarsest coefficients (the first column from top down, then the second column, *etc.*), and at the receiving end we can start reconstructing the image as soon as we have received enough data.