To be very clear, $\inf_{v \in U} J(v)$ denotes the *greatest lower bound* of the set of real numbers $\{J(u) \mid u \in U\}$. To make sure that we are on firm grounds, let us review the notions of greatest lower bound and least upper bound of a set of real numbers.

Let X be any nonempty subset of $\mathbb{R}$. The set $LB(X)$ of *lower bounds* of X is defined as

$$LB(X) = \{b \in \mathbb{R} \mid b \leq x \text{ for all } x \in X\}.$$

If the set X is not bounded below, which means that for every $r \in \mathbb{R}$ there is some $x \in X$ such that $x < r$, then $LB(X)$ is empty. Otherwise, if $LB(X)$ is nonempty, since it is bounded above by every element of X, by a fundamental property of the real numbers, the set $LB(X)$ has a greatest element denoted $\inf X$. The real number $\inf X$ is thus the *greatest lower bound* of X. In general, $\inf X$ does not belong to X, but if it does, then it is the least element of X.

If $LB(X) = \emptyset$, then X is *unbounded below* and $\inf X$ is undefined. In this case (with an abuse of notation), we write

$$\inf X = -\infty.$$

By convention, when $X = \emptyset$ we set

$$\inf \emptyset = +\infty.$$

For example, if $X = \{x \in \mathbb{R} \mid x \leq 0\}$, then $LB(X) = \emptyset$. On the other hand, if $X = \{1/n \mid n \in \mathbb{N} - \{0\}\}$, then $LB(X) = \{x \in \mathbb{R} \mid x \leq 0\}$ and $\inf X = 0$, which is not in X.

Similarly, the set $UB(X)$ of *upper bounds* of X is given by

$$UB(X) = \{u \in \mathbb{R} \mid x \leq u \text{ for all } x \in X\}.$$

If X is not bounded above, then $UB(X) = \emptyset$. Otherwise, if $UB(X) \neq \emptyset$, then it has least element denoted $\sup X$. Thus $\sup X$ is the *least upper bound* of X. If $\sup X \in X$, then it is the greatest element of X. If $UB(X) = \emptyset$, then

$$\sup X = +\infty.$$

By convention, when $X = \emptyset$ we set

$$\sup \emptyset = -\infty.$$

For example, if $X = \{x \in \mathbb{R} \mid x \geq 0\}$, then $UB(X) = \emptyset$. On the other hand, if $X = \{1 - 1/n \mid n \in \mathbb{N} - \{0\}\}$, then $UB(X) = \{x \in \mathbb{R} \mid x \geq 1\}$ and $\sup X = 1$, which is not in X.

The element $\inf_{v \in U} J(v)$ is just $\inf\{J(v) \mid v \in U\}$. The notation J^* is often used to denote $\inf_{v \in U} J(v)$. If the function J is not bounded below, which means that for every $r \in \mathbb{R}$, there is some $u \in U$ such that $J(u) < r$, then

$$\inf_{v \in U} J(v) = -\infty,$$