and

$$q = 0_{p+q}.$$

Since there are $2(p+q)+1$ Lagrange multipliers $(\lambda, \mu, \alpha, \beta, \gamma)$, the $(p+q) \times (p+q)$ matrix $X^\top X$ must be augmented with zero's to make it a $(2(p+q)+1) \times (2(p+q)+1)$ matrix P_a given by

$$P_a = \begin{pmatrix} X^\top X & 0_{p+q,p+q+1} \\ 0_{p+q+1,p+q} & 0_{p+q+1,p+q+1} \end{pmatrix},$$

and similarly q is augmented with zeros as the vector $q_a = 0_{2(p+q)+1}$.

As we mentioned in Section 54.5, since $\eta \geq 0$ for an optimal solution, we can drop the constraint $\eta \geq 0$ from the primal problem. In this case there are $2(p+q)$ Lagrange multipliers $(\lambda, \mu, \alpha, \beta)$. It is easy to see that the objective function of the dual is unchanged and the set of constraints is

$$\begin{aligned} \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j &= 0 \\ \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j &= K_m \\ \lambda_i + \alpha_i = K_s, &\quad i = 1, \ldots, p \\ \mu_j + \beta_j = K_s, &\quad j = 1, \ldots, q, \end{aligned}$$

with $K_m = (p+q)K_s\nu$. The constraint matrix corresponding to this system of equations is the $(p+q+2) \times 2(p+q)$ matrix A_2 given by

$$A_2 = \begin{pmatrix} \mathbf{1}_p^\top & -\mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\ \mathbf{1}_p^\top & \mathbf{1}_q^\top & 0_p^\top & 0_q^\top \\ I_p & 0_{p,q} & I_p & 0_{p,q} \\ 0_{q,p} & I_q & 0_{q,p} & I_q \end{pmatrix}.$$

We leave it as an exercise to prove that A_2 has rank $p+q+2$. The right-hand side is

$$c_2 = \begin{pmatrix} 0 \\ K_m \\ K_s \mathbf{1}_{p+q} \end{pmatrix}.$$

The symmetric positive semidefinite $(p+q) \times (p+q)$ matrix P defining the quadratic functional is

$$P = X^\top X, \quad \text{with} \quad X = \begin{pmatrix} -u_1 & \cdots & -u_p & v_1 & \cdots & v_q \end{pmatrix},$$

and

$$q = 0_{p+q}.$$