



Figure 40.5: Figures (a) and (b) are the graphs of real valued functions. Figure (a) is the graph of convex function since the blue line lies above the graph of f . Figure (b) shows the graph of a function which is not convex.

- Any intersection of half-spaces is convex.
- More generally, any intersection of convex sets is convex.

Example 40.5. Here are some common examples of convex and concave functions.

- Linear forms are convex functions (but not strictly convex).
- Any norm $\| \cdot \| : E \rightarrow \mathbb{R}_+$ is a convex function.
- The max function,

$$\max(x_1, \dots, x_n) = \max\{x_1, \dots, x_n\}$$

is convex on \mathbb{R}^n .

- The exponential $x \mapsto e^{cx}$ is strictly convex for any $c \neq 0$ ($c \in \mathbb{R}$).
- The logarithm function is concave on $\mathbb{R}_+ - \{0\}$.
- The *log-determinant function* $\log \det$ is concave on the set of symmetric positive definite matrices. This function plays an important role in convex optimization.

An excellent exposition of convexity and its applications to optimization can be found in Boyd [29].