whenever $w \in \bigwedge^q E$. This means that

$$\lrcorner\colon \bigwedge E \times \bigwedge E^* \longrightarrow \bigwedge E^*$$

is a left action of the (noncommutative) ring $\bigwedge E$ with multiplication $\wedge$ on $\bigwedge E^*$, which makes $\bigwedge E^*$ into a left $\bigwedge E$-module.

By interchanging E and E^* and using the isomorphism

$$\left(\bigwedge^k F\right)^* \cong \bigwedge^k F^*,$$

we can also define some maps

$$\lrcorner\colon \bigwedge^p E^* \times \bigwedge^{p+q} E \longrightarrow \bigwedge^q E,$$

and make the following definition.

Definition 34.9. Let $u^* \in \bigwedge^p E^*$, and $z \in \bigwedge^{p+q} E$. We define $u^* \lrcorner\, z \in \bigwedge^q$ as the q-vector uniquely defined by

$$\langle v^* \wedge u^*, z\rangle = \langle v^*, u^* \lrcorner\, z\rangle, \quad \text{for all } v^* \in \bigwedge^q E^*.$$

As for the previous version, we have a family of operators $\lrcorner_{p,q}$ which define an operator

$$\lrcorner\colon \bigwedge E^* \times \bigwedge E \longrightarrow \bigwedge E.$$

We easily verify that

$$(u^* \wedge v^*) \lrcorner\, z = u^* \lrcorner\, (v^* \lrcorner\, z),$$

whenever $u^* \in \bigwedge^k E^*$, $v^* \in \bigwedge^{p-k} E^*$, and $z \in \bigwedge^{p+q} E$; so this version of $\lrcorner$ is a left action of the ring $\bigwedge E^*$ on $\bigwedge E$ which makes $\bigwedge E$ into a left $\bigwedge E^*$-module.

In order to proceed any further we need some combinatorial properties of the basis of $\bigwedge^p E$ constructed from a basis $(e_1, \ldots, e_n)$ of E. Recall that for any (nonempty) subset $I \subseteq \{1, \ldots, n\}$, we let

$$e_I = e_{i_1} \wedge \cdots \wedge e_{i_p},$$

where $I = \{i_1, \ldots, i_p\}$ with $i_1 < \cdots < i_p$. We also let $e_\emptyset = 1$.

Given any two nonempty subsets $H, L \subseteq \{1, \ldots, n\}$ both listed in increasing order, say $H = \{h_1 < \ldots < h_p\}$ and $L = \{\ell_1 < \ldots < \ell_q\}$, if H and L are disjoint, let $H \cup L$ be union of H and L considered as the ordered sequence

$$(h_1, \ldots, h_p, \ell_1, \ldots, \ell_q).$$

Then let

$$\rho_{H,L} = \begin{cases} 0 & \text{if } H \cap L \neq \emptyset, \\ (-1)^\nu & \text{if } H \cap L = \emptyset, \end{cases}$$