

```

function u = GaussSeidel3(A,b,u)
    n = size(A,1);
    for i = 1:n
        u(i,1) = u(i,1) + (-A(i,:)*u + b(i))/A(i,i);
    end
end

```

It is remarkable that the only difference with `Jacobi2` is that the same variable u is used on both sides of the assignment. In order to run m iteration steps, run the following function:

```

function u = GaussSeidel1(A,b,u0,m)
    u = u0;
    for j = 1:m
        u = GaussSeidel3(A,b,u);
    end
end

```

Example 10.2. Consider the same linear system

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 25 \\ -24 \\ 21 \\ -15 \end{pmatrix}$$

as in Example 10.1, whose solution is

$$x_1 = 11, x_2 = -3, x_3 = 7, x_4 = -4.$$

After 10 Gauss–Seidel iterations, we find the approximate solution

$$x_1 = 10.9966, x_2 = -3.0044, x_3 = 6.9964, x_4 = -4.0018.$$

After 20 iterations, we find the approximate solution

$$x_1 = 11.0000, x_2 = -3.0001, x_3 = 6.9999, x_4 = -4.0000.$$

After 25 iterations, we find the approximate solution

$$x_1 = 11.0000, x_2 = -3.0000, x_3 = 7.0000, x_4 = -4.0000,$$

correct up to at least four decimals. We observe that for this example, Gauss–Seidel’s method converges about twice as fast as Jacobi’s method. It will be shown in Proposition 10.8 that for a tridiagonal matrix, the spectral radius of the Gauss–Seidel matrix \mathcal{L}_1 is given by

$$\rho(\mathcal{L}_1) = (\rho(J))^2,$$

so our observation is consistent with the theory.