positive definite, and since a solution (v, w) of the KKT-system should have $Av = b$, we also have $A^\top QAv = A^\top Qb$, so the KKT-system is equivalent to

$$\begin{pmatrix} P + A^\top QA & A^\top \\ A & 0 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} = \begin{pmatrix} -q + A^\top Qb \\ b \end{pmatrix},$$

and since $P + A^\top QA$ is symmetric positive definite, we can solve this system by elimination.

Another way to solve Problem (P) is to use variants of Newton's method as described in Section 49.9 dealing with equality constraints. Such methods are discussed extensively in Boyd and Vandenberghe [29] (Chapter 10, Sections 10.2-10.4).

There are two variants of this method:

(1) The first method, called *feasible start Newton method*, assumes that the starting point u_0 is feasible, which means that $Au_0 = b$. The Newton step d_{nt} is a feasible direction, which means that $Ad_{\mathrm{nt}} = 0$.

(2) The second method, called *infeasible start Newton method*, does *not* assume that the starting point u_0 is feasible, which means that $Au_0 = b$ may not hold. This method is a little more complicated than the other method.

We only briefly discuss the feasible start Newton method, leaving it to the reader to consult Boyd and Vandenberghe [29] (Chapter 10, Section 10.3) for a discussion of the infeasible start Newton method.

The Newton step d_{nt} is the solution of the linear system

$$\begin{pmatrix} \nabla^2 J(x) & A^\top \\ A & 0 \end{pmatrix} \begin{pmatrix} d_{\mathrm{nt}} \\ w \end{pmatrix} = \begin{pmatrix} -\nabla J_x \\ 0 \end{pmatrix}.$$

The Newton decrement $\lambda(x)$ is defined as in Section 49.9 as

$$\lambda(x) = (d_{\mathrm{nt}}^\top \nabla^2 J(x)\, d_{\mathrm{nt}})^{1/2} = \left((\nabla J_x)^\top (\nabla^2 J(x))^{-1} \nabla J_x\right)^{1/2}.$$

Newton's method with equality constraints (with feasible start) consists of the following steps: Given a starting point $u_0 \in \mathrm{dom}(J)$ with $Au_0 = b$, and a tolerance $\epsilon > 0$ do:

repeat

(1) Compute the Newton step and decrement
$d_{\mathrm{nt},k} = -(\nabla^2 J(u_k))^{-1} \nabla J_{u_k}$ and $\lambda(u_k)^2 = (\nabla J_{u_k})^\top (\nabla^2 J(u_k))^{-1} \nabla J_{u_k}$.

(2) Stopping criterion. **quit** if $\lambda(u_k)^2/2 \leq \epsilon$.

(3) Line Search. Perform an exact or backtracking line search to find ρ_k.

(4) Update. $u_{k+1} = u_k + \rho_k d_{\mathrm{nt},k}$.