There is an injection of A into $\mathcal{P}_A(n)$ given by the map $a \mapsto a1$ (where 1 denotes $e_{(0,\ldots,0)}$). There is also an injection of $\mathbb{N}^{(n)}$ into $\mathcal{P}_A(n)$ given by the map $(h_1, \ldots, h_n) \mapsto e_{(h_1,\ldots,h_n)}$. Note that $e_{(h_1,\ldots,h_n)} e_{(k_1,\ldots,k_n)} = e_{(h_1+k_1,\ldots,h_n+k_n)}$. In order to alleviate the notation, let $X_1, \ldots, X_n$ be n distinct variables and denote $e_{(0,\ldots,0,1,0\ldots,0)}$, where 1 occurs in the position i, by X_i (where $1 \leq i \leq n$). With this convention, in view of $e_{(h_1,\ldots,h_n)} e_{(k_1,\ldots,k_n)} = e_{(h_1+k_1,\ldots,h_n+k_n)}$, the polynomial $e_{(k_1,\ldots,k_n)}$ is denoted by $X_1^{k_1} \cdots X_n^{k_n}$ (with $e_{(0,\ldots,0)} = X_1^0 \cdots X_n^0 = 1$) and it is called a *primitive monomial*. Then, P is also written as

$$P = \sum_{(k_1,\ldots,k_n) \in \mathbb{N}^{(n)}} a_{(k_1,\ldots,k_n)} X_1^{k_1} \cdots X_n^{k_n}.$$

We also denote $\mathcal{P}_A(n)$ by $A[X_1, \ldots, X_n]$. A polynomial $P \in A[X_1, \ldots, X_n]$ is also denoted by $P(X_1, \ldots, X_n)$.

As in the case $n = 1$, there is nothing special about the choice of $X_1, \ldots, X_n$ as variables (or indeterminates). It is just a convenience. After all, the construction of $\mathcal{P}_A(n)$ has nothing to do with $X_1, \ldots, X_n$.

Given a nonnull polynomial P of degree d, the nonnull coefficients $a_{(k_1,\ldots,k_n)} \neq 0$ such that $d = k_1 + \cdots + k_n$ are called the *leading coefficients of* P. A polynomial of the form $a_{(k_1,\ldots,k_n)} X_1^{k_1} \cdots X_n^{k_n}$ is called a *monomial*. Note that $\deg(a_{(k_1,\ldots,k_n)} X_1^{k_1} \cdots X_n^{k_n}) = k_1 + \cdots + k_n$. Given a polynomial

$$P = \sum_{(k_1,\ldots,k_n) \in \mathbb{N}^{(n)}} a_{(k_1,\ldots,k_n)} X_1^{k_1} \cdots X_n^{k_n},$$

a monomial $a_{(k_1,\ldots,k_n)} X_1^{k_1} \cdots X_n^{k_n}$ *occurs in the polynomial* P if $a_{(k_1,\ldots,k_n)} \neq 0$.

A polynomial

$$P = \sum_{(k_1,\ldots,k_n) \in \mathbb{N}^{(n)}} a_{(k_1,\ldots,k_n)} X_1^{k_1} \cdots X_n^{k_n}$$

is *homogeneous of degree* d if

$$\deg(X_1^{k_1} \cdots X_n^{k_n}) = d,$$

for every monomial $a_{(k_1,\ldots,k_n)} X_1^{k_1} \cdots X_n^{k_n}$ occurring in P. If P is a polynomial of total degree d, it is clear that P can be written uniquely as

$$P = P^{(0)} + P^{(1)} + \cdots + P^{(d)},$$

where $P^{(i)}$ is the sum of all monomials of degree i occurring in P, where $0 \leq i \leq d$.

It is easily verified that $A[X_1, \ldots, X_n]$ is a commutative ring, with multiplicative identity $1X_1^0 \cdots X_n^0 = 1$. It is also easily verified that $A[X]$ is a module. When A is a field, $A[X]$ is a vector space.

Even when A is just a ring, the family of polynomials

$$(X_1^{k_1} \cdots X_n^{k_n})_{(k_1,\ldots,k_n) \in \mathbb{N}^{(n)}}$$