We now show that the update of a tableau can be performed using elementary row operations identical to the operations used during the reduction of a matrix to row reduced echelon form (rref).

If $K = (k_1, \ldots, k_m)$, j^+ is the index of the incoming basis vector, $k^- = k_\ell$ is the index of the column leaving the basis, and if $K^+ = (k_1, \ldots, k_{\ell-1}, j^+, k_{\ell+1}, \ldots, k_m)$, since $A_{K^+} = A_K E(\gamma_K^{j^+})$, the new columns $\gamma_{K^+}^j$ are computed in terms of the old columns γ_K^j using $(*_\gamma)$ and the equations

$$\gamma_{K^+}^j = A_{K^+}^{-1} A^j = E(\gamma_K^{j^+})^{-1} A_K^{-1} A^j = E(\gamma_K^{j^+})^{-1} \gamma_K^j.$$

Consequently, the matrix Γ^+ is given in terms of Γ by

$$\Gamma^+ = E(\gamma_K^{j^+})^{-1} \Gamma.$$

But the matrix $E(\gamma_K^{j^+})^{-1}$ is of the form

$$E(\gamma_K^{j^+})^{-1} = \begin{pmatrix} 1 & & & -(\gamma_{k^-}^{j^+})^{-1} \gamma_{k_1}^{j^+} & & & \\ & \ddots & & \vdots & & & \\ & & 1 & -(\gamma_{k^-}^{j^+})^{-1} \gamma_{k_{\ell-1}}^{j^+} & & & \\ & & & (\gamma_{k^-}^{j^+})^{-1} & & & \\ & & & -(\gamma_{k^-}^{j^+})^{-1} \gamma_{k_{\ell+1}}^{j^+} & 1 & & \\ & & & \vdots & & \ddots & \\ & & & -(\gamma_{k^-}^{j^+})^{-1} \gamma_{k_m}^{j^+} & & & 1 \end{pmatrix},$$

with the column involving the γs in the ℓth column, and Γ^+ is obtained by applying the following elementary row operations to Γ:

1. Multiply Row ℓ by $1/\gamma_{k^-}^{j^+}$ (the inverse of the pivot) to make the entry on Row ℓ and Column j^+ equal to 1.

2. Subtract $\gamma_{k_i}^{j^+} \times$ (the normalized) Row ℓ from Row i, for $i = 1, \ldots, \ell - 1, \ell + 1, \ldots, m$.

These are *exactly* the elementary row operations that reduce the ℓth column $\gamma_K^{j^+}$ of Γ to the ℓth column of the identity matrix I_m. Thus, this step is identical to the sequence of steps that the procedure to convert a matrix to row reduced echelon from executes on the ℓth column of the matrix. The only difference is the criterion for the choice of the pivot.

Since the new basic solution u_{K^+} is given by $u_{K^+} = A_{K^+}^{-1} b$, we have

$$u_{K^+} = E(\gamma_K^{j^+})^{-1} A_K^{-1} b = E(\gamma_K^{j^+})^{-1} u_K.$$

This means that u_+ is obtained from u_K by applying exactly the *same* elementary row operations that were applied to Γ. Consequently, just as in the procedure for reducing a