since $\delta_k = b_k\delta_{k-1} - a_k c_{k-1}\delta_{k-2}$. □

It follows that there is a simple method to solve a linear system $Ax = d$ where A is tridiagonal (and $\delta_k \neq 0$ for $k = 1, \ldots, n$). For this, it is convenient to "squeeze" the diagonal matrix Δ defined such that $\Delta_{k\,k} = \delta_k/\delta_{k-1}$ into the factorization so that $A = (L\Delta)(\Delta^{-1}U)$, and if we let

$$z_1 = \frac{c_1}{b_1}, \quad z_k = c_k\frac{\delta_{k-1}}{\delta_k}, \quad 2 \leq k \leq n-1, \quad z_n = \frac{\delta_n}{\delta_{n-1}} = b_n - a_n z_{n-1},$$

$A = (L\Delta)(\Delta^{-1}U)$ is written as

$$A = \begin{pmatrix} \frac{c_1}{z_1} & & & & & \\ a_2 & \frac{c_2}{z_2} & & & & \\ & a_3 & \frac{c_3}{z_3} & & & \\ & & \ddots & \ddots & & \\ & & & a_{n-1} & \frac{c_{n-1}}{z_{n-1}} & \\ & & & & a_n & z_n \end{pmatrix} \begin{pmatrix} 1 & z_1 & & & & & \\ & 1 & z_2 & & & & \\ & & 1 & z_3 & & & \\ & & & \ddots & \ddots & & \\ & & & & 1 & z_{n-2} & \\ & & & & & 1 & z_{n-1} \\ & & & & & & 1 \end{pmatrix}.$$

As a consequence, the system $Ax = d$ can be solved by constructing three sequences: First, the sequence

$$z_1 = \frac{c_1}{b_1}, \quad z_k = \frac{c_k}{b_k - a_k z_{k-1}}, \quad k = 2, \ldots, n-1, \quad z_n = b_n - a_n z_{n-1},$$

corresponding to the recurrence $\delta_k = b_k\delta_{k-1} - a_k c_{k-1}\delta_{k-2}$ and obtained by dividing both sides of this equation by δ_{k-1}, next

$$w_1 = \frac{d_1}{b_1}, \quad w_k = \frac{d_k - a_k w_{k-1}}{b_k - a_k z_{k-1}}, \quad k = 2, \ldots, n,$$

corresponding to solving the system $L\Delta w = d$, and finally

$$x_n = w_n, \quad x_k = w_k - z_k x_{k+1}, \quad k = n-1, n-2, \ldots, 1,$$

corresponding to solving the system $\Delta^{-1}Ux = w$.

Remark: It can be verified that this requires $3(n-1)$ additions, $3(n-1)$ multiplications, and $2n$ divisions, a total of $8n - 6$ operations, which is much less that the $O(2n^3/3)$ required by Gaussian elimination in general.

We now consider the special case of symmetric positive definite matrices (SPD matrices).