Program ν-SV Regression:

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}w^\top w + C\left(\nu\epsilon + \frac{1}{m}\sum_{i=1}^{m}(\xi_i + \xi_i')\right)\\
&\text{subject to}\\
&\qquad w^\top x_i + b - y_i \le \epsilon + \xi_i, \quad \xi_i \ge 0 \qquad i = 1, \ldots, m\\
&\quad -w^\top x_i - b + y_i \le \epsilon + \xi_i', \quad \xi_i' \ge 0 \qquad i = 1, \ldots, m\\
&\quad \epsilon \ge 0,
\end{aligned}$$

minimizing over the variables w, b, ϵ, ξ, and ξ'. The constraints are affine. The problem is to minimize ϵ and the errors ξ_i, ξ_i' so that the ℓ^1-error is "squeezed down" to zero as much as possible, in the sense that the right-hand side of the inequality

$$\sum_{i=1}^{m} |y_i - x_i^\top w - b| \le m\epsilon + \sum_{i=1}^{m}\xi_i + \sum_{i=1}^{m}\xi_i'$$

is as small as possible. As shown by Figure 56.2, the region associated with the constraint $w^\top x_i - z + b \le \epsilon$ contains the ϵ-slab. Similarly, as illustrated by Figure 56.3, the region associated with the constraint $w^\top x_i - z + b \ge -\epsilon$, equivalently $-w^\top x_i + z - b \le \epsilon$, also contains the ϵ-slab.

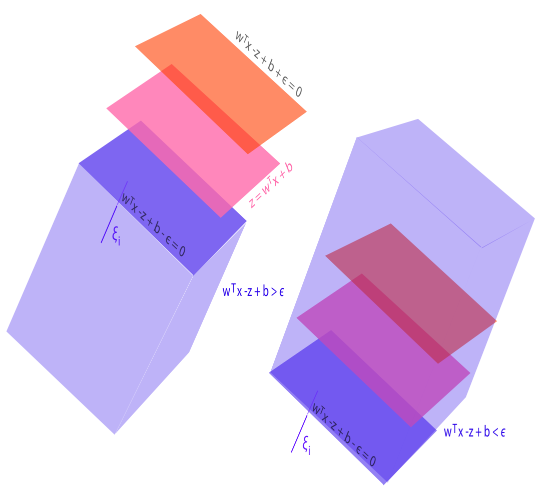

Figure 56.2: The two blue half spaces associated with the hyperplane $w^\top x_i - z + b = \epsilon$.

Observe that if we require $\epsilon = 0$, then the problem is equivalent to minimizing

$$\|y - Xw - b\mathbf{1}\|_1 + \frac{1}{2}w^\top w.$$