

More explicitly,  $C$  is the following matrix:

$$C = \begin{pmatrix} -u_1^\top & -1 & \cdots & 0 & 0 & \cdots & 0 & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -u_p^\top & 0 & \cdots & -1 & 0 & \cdots & 0 & 1 & 1 \\ v_1^\top & 0 & \cdots & 0 & -1 & \cdots & 0 & -1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ v_q^\top & 0 & \cdots & 0 & 0 & \cdots & -1 & -1 & 1 \\ 0 & -1 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & -1 & 0 & 0 \end{pmatrix}.$$

The objective function is given by

$$J(w, \epsilon, \xi, b, \delta) = -\delta + K \begin{pmatrix} \epsilon^\top & \xi^\top \end{pmatrix} \mathbf{1}_{p+q}.$$

The Lagrangian  $L(w, \epsilon, \xi, b, \delta, \lambda, \mu, \alpha, \beta, \gamma)$  with  $\lambda, \alpha \in \mathbb{R}_+^p$ ,  $\mu, \beta \in \mathbb{R}_+^q$ , and  $\gamma \in \mathbb{R}^+$  is given by

$$\begin{aligned} L(w, \epsilon, \xi, b, \delta, \lambda, \mu, \alpha, \beta, \gamma) &= -\delta + K \begin{pmatrix} \epsilon^\top & \xi^\top \end{pmatrix} \mathbf{1}_{p+q} \\ &\quad + \begin{pmatrix} w^\top & \epsilon^\top & \xi^\top & b & \delta \end{pmatrix} C^\top \begin{pmatrix} \lambda \\ \mu \\ \alpha \\ \beta \end{pmatrix} + \gamma(w^\top w - 1). \end{aligned}$$

Since

$$\begin{aligned} \begin{pmatrix} w^\top & \epsilon^\top & \xi^\top & b & \delta \end{pmatrix} C^\top \begin{pmatrix} \lambda \\ \mu \\ \alpha \\ \beta \end{pmatrix} &= w^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} - \epsilon^\top (\lambda + \alpha) - \xi^\top (\mu + \beta) + b(\mathbf{1}_p^\top \lambda - \mathbf{1}_q^\top \mu) \\ &\quad + \delta(\mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu), \end{aligned}$$

the Lagrangian can be written as

$$\begin{aligned} L(w, \epsilon, \xi, b, \delta, \lambda, \mu, \alpha, \beta, \gamma) &= -\delta + K(\epsilon^\top \mathbf{1}_p + \xi^\top \mathbf{1}_q) + w^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + \gamma(w^\top w - 1) \\ &\quad - \epsilon^\top (\lambda + \alpha) - \xi^\top (\mu + \beta) + b(\mathbf{1}_p^\top \lambda - \mathbf{1}_q^\top \mu) + \delta(\mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu) \\ &= (\mathbf{1}_p^\top \lambda + \mathbf{1}_q^\top \mu - 1)\delta + w^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + \gamma(w^\top w - 1) \\ &\quad + \epsilon^\top (K\mathbf{1}_p - (\lambda + \alpha)) + \xi^\top (K\mathbf{1}_q - (\mu + \beta)) + b(\mathbf{1}_p^\top \lambda - \mathbf{1}_q^\top \mu). \end{aligned}$$