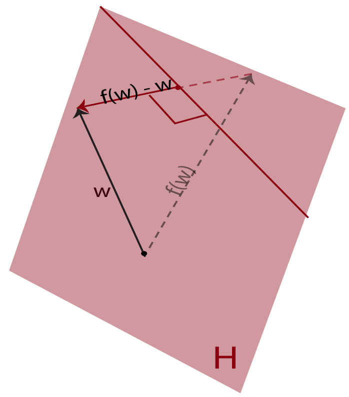

Figure 27.2: The construction of the hyperplane H for Case 2 of Theorem 27.1.

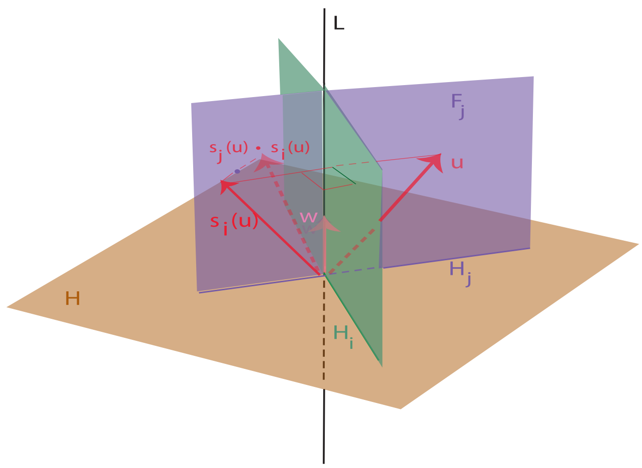

Figure 27.3: An isometry f as a composition of reflections, when 1 is an eigenvalue of f.

(4) It is natural to ask what is the minimal number of hyperplane reflections needed to obtain an isometry f. This has to do with the dimension of the eigenspace $\mathrm{Ker}\,(f - \mathrm{id})$ associated with the eigenvalue 1. We will prove later that every isometry is the composition of k hyperplane reflections, where

$$k = n - \dim(\mathrm{Ker}\,(f - \mathrm{id})),$$

and that this number is minimal (where $n = \dim(E)$).

When $n = 2$, a reflection is a reflection about a line, and Theorem 27.1 shows that every isometry in $\mathbf{O}(2)$ is either a reflection about a line or a rotation, and that every rotation is the product of two reflections about some lines. In general, since $\det(s) = -1$ for a reflection s, when $n \geq 3$ is odd, every rotation is the product of an even number less than or equal