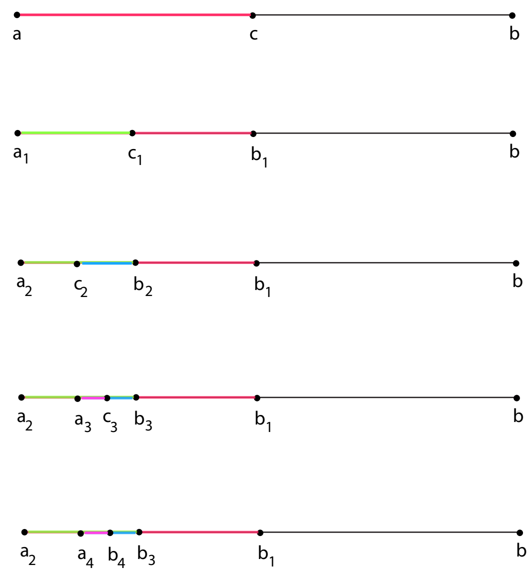

Figure 37.29: The first four stages of the nested interval construction utilized in the proof of Proposition 37.25.

The argument of Proposition 37.25 can be adapted to show that in $\mathbb{R}^m$, every closed set, $[a_1, b_1] \times \cdots \times [a_m, b_m]$, is compact. At every stage, we need to divide into 2^m subpieces instead of 2.

We next discuss some important properties of compact spaces. We begin with two separations axioms which only hold for Hausdorff spaces:

Proposition 37.26. *Given a topological Hausdorff space, E, for every compact subset, A, and every point, b, not in A, there exist disjoint open sets, U and V, such that $A \subseteq U$ and $b \in V$. See Figure 37.30. As a consequence, every compact subset is closed.*

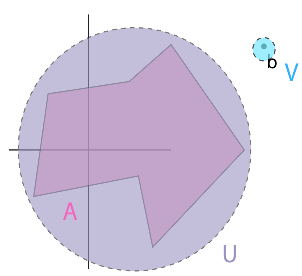

Figure 37.30: The compact set of $\mathbb{R}^2$, A, is separated by any point in its complement.