



Figure 56.6: The closed ϵ -tube associated with zero multiplier classification, namely $\lambda_i = 0$ and $\mu_i = 0$.

(3) If $p_f \geq 1$ or $q_f \geq 1$, then $\nu \geq 2/m$.

Proof. (1) Recall that for an optimal solution with $w \neq 0$ and $\epsilon > 0$ we have $\gamma = 0$, so we have the equations

$$\sum_{i=1}^m \lambda_i = \frac{C\nu}{2} \quad \text{and} \quad \sum_{j=1}^m \mu_j = \frac{C\nu}{2}.$$

If there are p_f points such that $\lambda_i = C/m$, then

$$\frac{C\nu}{2} = \sum_{i=1}^m \lambda_i \geq p_f \frac{C}{m},$$

so

$$p_f \leq \frac{m\nu}{2}.$$

A similar reasoning applies if there are q_f points such that $\mu_i = C/m$, and we get

$$q_f \leq \frac{m\nu}{2}.$$

(2) If $I_{\lambda>0} = \{i \in \{1, \dots, m\} \mid \lambda_i > 0\}$ and $p_m = |I_{\lambda>0}|$, then

$$\frac{C\nu}{2} = \sum_{i=1}^m \lambda_i = \sum_{i \in I_{\lambda>0}} \lambda_i,$$

and since $\lambda_i \leq C/m$, we have

$$\frac{C\nu}{2} = \sum_{i \in I_{\lambda>0}} \lambda_i \leq p_m \frac{C}{m},$$