

so we only need to find  $N + 1$  points  $d_0, \dots, d_N$ .

It turns out that the  $C^2$ -continuity constraints on the  $N$  Bézier curves yield only  $N - 1$  equations, so  $d_0$  and  $d_N$  can be chosen arbitrarily. In practice,  $d_0$  and  $d_N$  are chosen according to various *end conditions*, such as prescribed velocities at  $x_0$  and  $x_N$ . For the time being, we will assume that  $d_0$  and  $d_N$  are given.

Figure 8.4 illustrates an interpolation problem involving  $N + 1 = 7 + 1 = 8$  data points. The control points  $d_0$  and  $d_7$  were chosen arbitrarily.

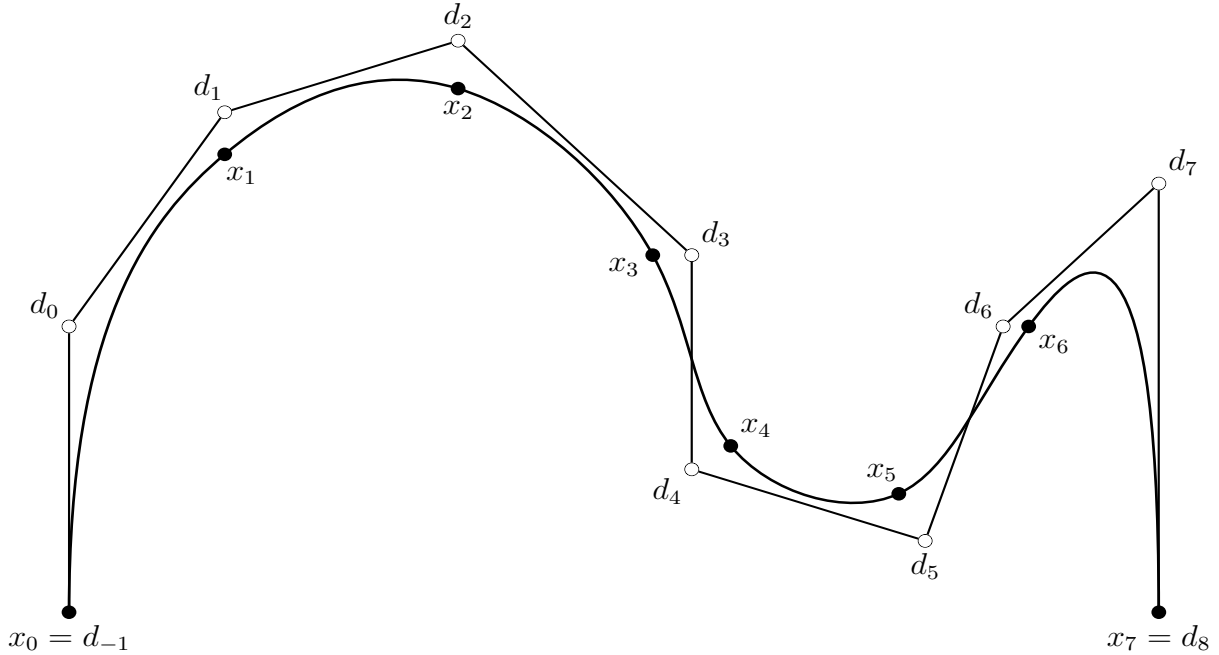


Figure 8.4: A  $C^2$  cubic interpolation spline curve passing through the points  $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7$ .

It can be shown that  $d_1, \dots, d_{N-1}$  are given by the linear system

$$\begin{pmatrix} \frac{7}{2} & 1 & & & \\ 1 & 4 & 1 & & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & & 1 & 4 & 1 \\ & & & 1 & \frac{7}{2} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_{N-2} \\ d_{N-1} \end{pmatrix} = \begin{pmatrix} 6x_1 - \frac{3}{2}d_0 \\ 6x_2 \\ \vdots \\ 6x_{N-2} \\ 6x_{N-1} - \frac{3}{2}d_N \end{pmatrix}.$$

We will show later that the above matrix is invertible because it is strictly diagonally dominant.