

In practice, the applicability of the penalty-function method is limited by the difficulty to construct effectively “good” functions  $\psi$ , for example, differentiable ones. Note that in the above example the function  $\psi$  is not differentiable. A better penalty function is

$$\psi(v) = \sum_{i=1}^m (\max\{\varphi_i(v), 0\})^2.$$

Another way to deal with constrained optimization problems is to use *duality*. This approach is investigated in Chapter 50.

## 49.13 Summary

The main concepts and results of this chapter are listed below:

- Minimization, minimizer.
- Coercive functions.
- Minima of quadratic functionals.
- The theorem of Lions and Stampacchia.
- Lax–Milgram’s theorem.
- Elliptic functionals.
- Descent direction, exact line search, backtracking line search.
- Method of relaxation.
- Gradient descent.
- Gradient descent method with fixed stepsize parameter.
- Gradient descent method with variable stepsize parameter.
- Steepest descent method for the Euclidean norm.
- Gradient descent method with backtracking line search.
- Normalized steepest descent direction.
- Unnormalized steepest descent direction.
- Steepest descent method (with respect to the norm  $\|\cdot\|$ ).
- Momentum term.