# Chapter 54

# Soft Margin Support Vector Machines

In Sections 50.5 and 50.6 we considered the problem of separating two nonempty disjoint finite sets of $p$ *blue* points $\{u_i\}_{i=1}^p$ and $q$ *red* points $\{v_j\}_{j=1}^q$ in $\mathbb{R}^n$. The goal is to find a hyperplane $H$ of equation $w^\top x - b = 0$ (where $w \in \mathbb{R}^n$ is a nonzero vector and $b \in \mathbb{R}$), such that all the blue points $u_i$ are in one of the two open half-spaces determined by $H$, and all the red points $v_j$ are in the other open half-space determined by $H$; see Figure 54.1.

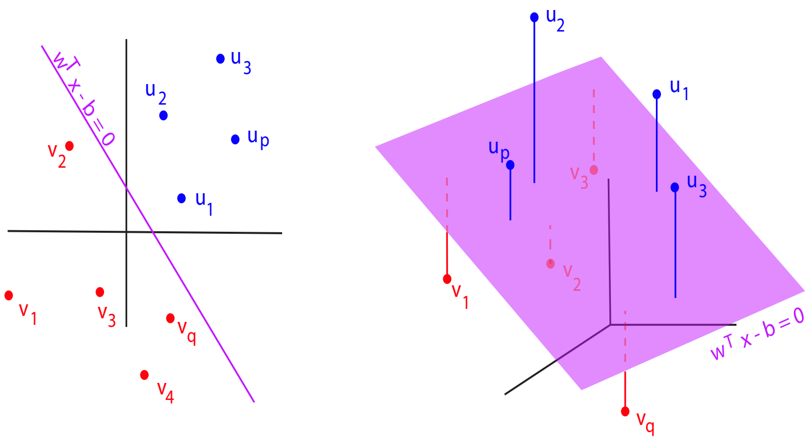


Figure 54.1: Two examples of the SVM separation problem. The left figure is SVM in $\mathbb{R}^2$, while the right figure is SVM in $\mathbb{R}^3$.

SVM picks a hyperplane which maximizes the minimum distance from these points to the hyperplane.

In this chapter we return to the problem of separating two disjoint sets of points, $\{u_i\}_{i=1}^p$ and $\{v_j\}_{j=1}^q$, but this time we do not assume that these two sets are separable. To cope with nonseparability, we allow points to invade the safety zone around the separating hyperplane, and even points on the wrong side of the hyperplane. Such a method is called *soft margin support vector machine*. We discuss variations of this method, including $\nu$-SV classification. In each case we present a careful derivation of the dual.