

Figure 26.36: The duality between a point in $\mathbf{P}(E)$ and a line in $\mathbf{P}(E^*)$. The line in $\mathbf{P}(E^*)$ is also represented by the pencil of lines through a in $\mathcal{H}(E)$.

- A point a in $\mathbf{P}(E)$ maps to the hyperplane H_a in $\mathbf{P}(E^*)$ (the linear system of hyperplanes in $\mathcal{H}(E)$ containing a, also denoted by a^*).
- A hyperplane H in $\mathbf{P}(E)$ maps to the point p_H in $\mathbf{P}(E^*)$ (represented by the hyperplane H in $\mathcal{H}(E)$).

To conclude our quick tour of projective geometry, we establish a connection between the cross-ratio of hyperplanes in a pencil of hyperplanes with the cross-ratio of the intersection points of any line not contained in any hyperplane in this pencil with four hyperplanes in this pencil.