

and show that its action on u_k, v_k, x_k, y_k is the same as the affine action of Part (3).

Use **Matlab** to find the eigenvalues of the matrix

$$T = \begin{pmatrix} 2 & -2 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

You will be stunned!

Let N be the matrix given by

$$N = T - I.$$

Prove that

$$N^4 = 0.$$

Use this to prove that

$$T^k = I + kN + \frac{1}{2}k(k-1)N^2 + \frac{1}{6}k(k-1)(k-2)N^3,$$

for all $k \geq 0$.

(5) Prove that

$$\begin{pmatrix} u_k \\ v_k \\ x_k \\ y_k \\ 1 \end{pmatrix} = T^k \begin{pmatrix} -1 \\ -1 \\ -2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & -2 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}^k \begin{pmatrix} -1 \\ -1 \\ -2 \\ -1 \\ 1 \end{pmatrix},$$

for $k \geq 0$.

Prove that

$$T^k = \begin{pmatrix} k+1 & -k(k+1) & k & -k^2 & \frac{1}{6}(k-1)k(2k-7) \\ 0 & k+1 & 0 & k & -\frac{1}{2}(k-1)k \\ -k & k^2 & 1-k & (k-1)k & -\frac{1}{3}k((k-6)k+11) \\ 0 & -k & 0 & 1-k & \frac{1}{2}(k-3)k \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

and thus that

$$\begin{pmatrix} u_k \\ v_k \\ x_k \\ y_k \end{pmatrix} = \begin{pmatrix} \frac{1}{6}(2k^3 + 3k^2 - 5k - 6) \\ -\frac{1}{2}(k^2 + 3k + 2) \\ \frac{1}{3}(-k^3 + k - 6) \\ \frac{1}{2}(k^2 + k - 2) \end{pmatrix},$$