This version of the SVM problem was first discussed in Schölkopf, Smola, Williamson, and Bartlett [147] under the name of ν-*SVC*, and also used in Schölkopf, Platt, Shawe–Taylor, and Smola [146].

In order for the problem to have a solution we must pick K_m and K_s so that

$$K_m \leq \min\{2pK_s, 2qK_s\}.$$

It is shown in Section 54.5 that the dual program is

Dual of the Basic Soft margin ν-SVM Problem ($\mathrm{SVM}_{s2'}$):

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\begin{pmatrix}\lambda^\top & \mu^\top\end{pmatrix} X^\top X \begin{pmatrix}\lambda \\ \mu\end{pmatrix} \\
&\text{subject to} \\
&\qquad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0 \\
&\qquad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq K_m \\
&\qquad 0 \leq \lambda_i \leq K_s, \quad i = 1, \ldots, p \\
&\qquad 0 \leq \mu_j \leq K_s, \quad j = 1, \ldots, q.
\end{aligned}$$

If the primal problem has an optimal solution with $w \neq 0$, then using the fact that the duality gap is zero we can show that $\eta \geq 0$. Thus constraint $\eta \geq 0$ could be omitted. As in the previous case w is given by

$$w = -X\begin{pmatrix}\lambda \\ \mu\end{pmatrix} = \sum_{i=1}^p \lambda_i u_i - \sum_{j=1}^q \mu_j v_j,$$

but b and η are not determined by the dual.

If we drop the constraint $\eta \geq 0$, then the inequality

$$\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j \geq K_m$$

is replaced by the equation

$$\sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = K_m.$$

It convenient to define $\nu > 0$ such that

$$\nu = \frac{K_m}{(p+q)K_s},$$