Since the map $u \mapsto \langle u, v\rangle$ (with v fixed) is linear, its derivative is

$$d\langle -, v\rangle_u(x) = \langle x, v\rangle. \tag{d_2}$$

The derivative of the Lagrangian

$$L(\xi, w, \lambda) = \xi^\top \xi + K\langle w, w\rangle - \sum_{i=1}^m \lambda_i \langle \varphi(x_i), w\rangle - \xi^\top \lambda + \lambda^\top y$$

with respect to ξ and w is

$$dL_{\xi,w}\big(\widetilde{\xi}, \widetilde{w}\big) = 2(\widetilde{\xi})^\top \xi - (\widetilde{\xi})^\top \lambda + \left\langle 2Kw - \sum_{i=1}^m \lambda_i \varphi(x_i), \widetilde{w} \right\rangle,$$

where we used (d_1) to calculate the derivative of $\xi^\top \xi + K\langle w, w\rangle$ and (d_2) to calculate the derivative of $-\sum_{i=1}^m \lambda_i \langle \varphi(x_i), w\rangle - \xi^\top \lambda$. We have $dL_{\xi,w}\big(\widetilde{\xi}, \widetilde{w}\big) = 0$ for all $\widetilde{\xi}$ and $\widetilde{w}$ iff

$$\begin{aligned} 2Kw &= \sum_{i=1}^m \lambda_i \varphi(x_i) \\ \lambda &= 2\xi. \end{aligned}$$

Again we define $\xi = K\alpha$, so we have $\lambda = 2K\alpha$, and

$$w = \sum_{i=1}^m \alpha_i \varphi(x_i).$$

Plugging back into the Lagrangian we get

$$\begin{aligned} G(\alpha) &= K^2 \alpha^\top \alpha + K \sum_{i,j=1}^m \alpha_i \alpha_j \langle \varphi(x_i), \varphi(x_j)\rangle - 2K \sum_{i,j=1}^m \alpha_i \alpha_j \langle \varphi(x_i), \varphi(x_j)\rangle \\ &\quad - 2K^2 \alpha^\top \alpha + 2K\alpha^\top y \\ &= -K^2 \alpha^\top \alpha - K \sum_{i,j=1}^m \alpha_i \alpha_j \langle \varphi(x_i), \varphi(x_j)\rangle + 2K\alpha^\top y. \end{aligned}$$

If $\mathbf{G}$ is the matrix given by $\mathbf{G}_{ij} = \langle \varphi(x_i), \varphi(x_j)\rangle$, then we have

$$G(\alpha) = -K\alpha^\top (\mathbf{G} + KI_m)\alpha + 2K\alpha^\top y.$$

The function G is strictly concave, so by Theorem 40.13(4) it has a maximum for

$$\alpha = (\mathbf{G} + KI_m)^{-1} y,$$

as claimed earlier.