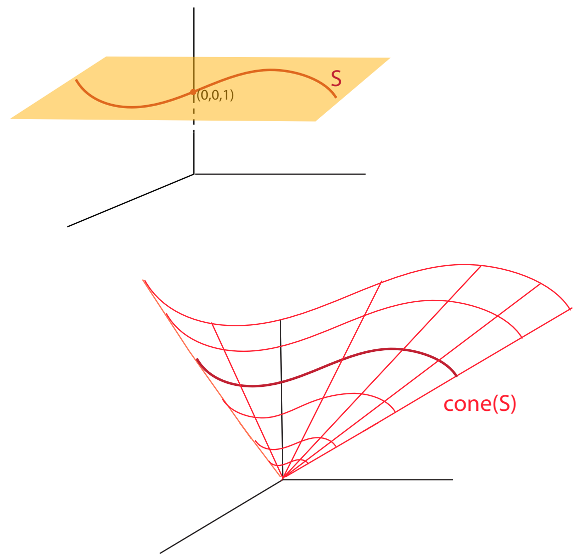

Figure 44.4: Let S be a planar curve in $z = 1$. The linear cone of S, consisting of all half rays connecting S to the origin, is not convex.

An unbounded $\mathcal{H}$-polyhedron is not equal to the convex hull of finite set of points. To obtain an equivalent notion we introduce the notion of a $\mathcal{V}$-polyhedron.

Definition 44.11. A *$\mathcal{V}$-polyhedron* is any convex subset $A \subseteq \mathbb{R}^n$ of the form

$$A = \mathrm{conv}(Y) + \mathrm{cone}(V) = \{a + v \mid a \in \mathrm{conv}(Y),\ v \in \mathrm{cone}(V)\},$$

where $Y \subseteq \mathbb{R}^n$ and $V \subseteq \mathbb{R}^n$ are *finite* (possibly empty).

When $V = \emptyset$ we simply have a *polytope*, and when $Y = \emptyset$ or $Y = \{0\}$, we simply have a cone.

It can be shown that every $\mathcal{H}$-polyhedron is a $\mathcal{V}$-polyhedron and conversely. This is one of the major theorems in the theory of polyhedra, and its proof is nontrivial. For a complete proof, see Gallier [73] and Ziegler [195].

Every polyhedral cone is closed. This is an important fact that is used in the proof of several other key results such as Proposition 45.1 and the Farkas–Minkowski proposition (Proposition 47.2).

Although it seems obvious that a polyhedral cone should be closed, a rigorous proof is not entirely trivial.

Indeed, the fact that a polyhedral cone is closed relies crucially on the fact that C is spanned by a *finite* number of vectors, because the cone generated by an infinite set may not be closed. For example, consider the closed disk $D \subseteq \mathbb{R}^2$ of center $(0, 1)$ and radius 1,