In the special case where $m = 1$, $\alpha_1 = 0$, and $\alpha_2 = 1$, we leave as an exercise to show that the Hermite polynomials are

$$\begin{aligned}
H_0^0 &= 2X^3 - 3X^2 + 1, \\
H_1^0 &= -2X^3 + 3X^2, \\
H_0^1 &= X^3 - 2X^2 + X, \\
H_1^1 &= X^3 - X^2.
\end{aligned}$$

As a consequence, the polynomial P of degree 3 such that $P(0) = x_0$, $P(1) = x_1$, $P'(0) = m_0$, and $P'(1) = m_1$, can be written as

$$P(X) = x_0(2X^3 - 3X^2 + 1) + m_0(X^3 - 2X^2 + X) + m_1(X^3 - X^2) + x_1(-2X^3 + 3X^2).$$

If we want the polynomial P of degree 3 such that $P(a) = x_0$, $P(b) = x_1$, $P'(a) = m_0$, and $P'(b) = m_1$, where $b \neq a$, then we have

$$P(X) = x_0(2t^3 - 3t^2 + 1) + (b-a)m_0(t^3 - 2t^2 + t) + (b-a)m_1(t^3 - t^2) + x_1(-2t^3 + 3t^2),$$

where

$$t = \frac{X-a}{b-a}.$$

Observe the presence of the extra factor $(b - a)$ in front of m_0 and m_1, the formula would be false otherwise!

We now consider the case where $n_1 = \ldots = n_{m+1} = 2$. Let us try

$$H_j^i(X) = (a^i(X - \alpha_j)^2 + b^i(X - \alpha_j) + c^i)L_j^3,$$

where $0 \leq i \leq 2$. Sparing the readers some (tedious) computations, we find:

$$\begin{aligned}
H_j^0(X) &= \Big(\big(6(\mathrm{D}L_j(\alpha_j))^2 - \frac{3}{2}\mathrm{D}^2 L_j(\alpha_j)\big)(X - \alpha_j)^2 - 3\mathrm{D}L_j(\alpha_j)(X - \alpha_j) + 1\Big)L_j^3(X), \\
H_j^1(X) &= \Big(9(\mathrm{D}L_j(\alpha_j))^2(X - \alpha_j)^2 - 3\mathrm{D}L_j(\alpha_j)(X - \alpha_j)\Big)L_j^3(X), \\
H_j^2(X) &= \frac{1}{2}(X - \alpha_j)^2 L_j^3(X).
\end{aligned}$$

Going back to the general problem, it seems to us that a kind of Newton interpolant will be more manageable. Let

$$\begin{aligned}
&P_0^0(X) = 1, \\
&P_j^0(X) = (X - \alpha_1)^{n_1+1} \cdots (X - \alpha_j)^{n_j+1}, \ \ 1 \leq j \leq m \\
&P_0^i(X) = (X - \alpha_1)^i (X - \alpha_2)^{n_2+1} \cdots (X - \alpha_{m+1})^{n_{m+1}+1}, \ \ 1 \leq i \leq n_1, \\
&P_j^i(X) = (X - \alpha_1)^{n_1+1} \cdots (X - \alpha_j)^{n_j+1} (X - \alpha_{j+1})^i (X - \alpha_{j+2})^{n_{j+2}+1} \cdots (X - \alpha_{m+1})^{n_{m+1}+1}, \\
&\qquad 1 \leq j \leq m-1, \ 1 \leq i \leq n_{j+1}, \\
&P_m^i(X) = (X - \alpha_1)^{n_1+1} \cdots (X - \alpha_m)^{n_m+1} (X - \alpha_{m+1})^i, \ \ 1 \leq i \leq n_{m+1},
\end{aligned}$$