Substituting the right hand side of (1) for x^{k+1} in (2) yields

$$z^{k+1} = \frac{\rho z^k}{\rho + 2} - \frac{1}{\rho + 2}. \tag{4}$$

Using (2), we obtain

$$2x^{k+1} - z^{k+1} = \frac{4x^{k+1}}{\rho + 2} - \frac{\lambda^k}{\rho + 2}, \tag{5}$$

and then using (3) we get

$$\lambda^{k+1} = \frac{2\lambda^k}{\rho + 2} + \frac{4\rho x^{k+1}}{\rho + 2}. \tag{6}$$

Substituting the right hand side of (1) for x^{k+1} in (6), we obtain

$$\lambda^{k+1} = \frac{2\rho z^k}{\rho + 2} - \frac{2}{\rho + 2}. \tag{7}$$

Equation (7) shows that z^k determines λ^{k+1}, and Equation (1) for $k+2$, along with Equation (4), shows that z^k also determines x^{k+2}. In particular, we find that

$$\begin{aligned} x^{k+2} &= \frac{1}{2}z^{k+1} - \frac{1}{2\rho}\lambda^{k+1} - \frac{1}{2\rho} \\ &= \frac{(\rho - 2)z^k}{2(\rho + 2)} - \frac{1}{\rho + 2}. \end{aligned}$$

Thus is suffices to find the limit of the sequence (z^k). Since we already know from Example 52.2 that this limit is $-1/2$, using (4), we write

$$z^{k+1} = -\frac{1}{2} + \frac{\rho z^k}{\rho + 2} - \frac{1}{\rho + 2} + \frac{1}{2} = -\frac{1}{2} + \frac{\rho}{\rho + 2}\left(\frac{1}{2} + z^k\right).$$

By induction, we deduce that

$$z^{k+1} = -\frac{1}{2} + \left(\frac{\rho}{\rho + 2}\right)^{k+1}\left(\frac{1}{2} + z^0\right),$$

and since $\rho > 0$, we have $\rho/(\rho + 2) < 1$, so the limit of the sequence (z^{k+1}) is indeed $-1/2$, and consequently the limit of (λ^{k+1}) is -1 and the limit of x^{k+2} is $-1/4$.

For ADMM to be practical, the x-minimization step and the z-minimization step have to be doable efficiently.

It is often convenient to write the ADMM updates in terms of the *scaled dual variable* $\mu = (1/\rho)\lambda$. If we define the *residual* as

$$r = Ax + bz - c,$$