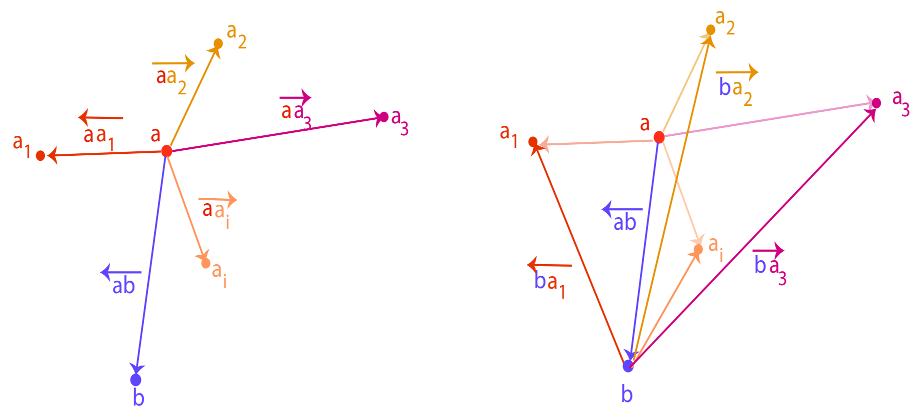

Figure 24.10: Part (1) of Proposition 24.1.

Thus, by Proposition 24.1, for any family of points $(a_i)_{i\in I}$ in E, for any family $(\lambda_i)_{i\in I}$ of scalars such that $\sum_{i\in I}\lambda_i = 1$, the point

$$x = a + \sum_{i\in I}\lambda_i \overrightarrow{aa_i}$$

is independent of the choice of the origin $a \in E$. This property motivates the following definition.

Definition 24.2. For any family of points $(a_i)_{i\in I}$ in E, for any family $(\lambda_i)_{i\in I}$ of scalars such that $\sum_{i\in I}\lambda_i = 1$, and for any $a \in E$, the point

$$a + \sum_{i\in I}\lambda_i \overrightarrow{aa_i}$$

(which is independent of $a \in E$, by Proposition 24.1) is called the *barycenter (or barycentric combination, or affine combination) of the points a_i assigned the weights λ_i*, and it is denoted by

$$\sum_{i\in I}\lambda_i a_i.$$

In dealing with barycenters, it is convenient to introduce the notion of a *weighted point*, which is just a pair (a, λ), where $a \in E$ is a point, and $\lambda \in \mathbb{R}$ is a scalar. Then, given a family of weighted points $((a_i, \lambda_i))_{i\in I}$, where $\sum_{i\in I}\lambda_i = 1$, we also say that the point $\sum_{i\in I}\lambda_i a_i$ is the *barycenter of the family of weighted points* $((a_i, \lambda_i))_{i\in I}$.

Note that the barycenter x of the family of weighted points $((a_i, \lambda_i))_{i\in I}$ is the unique point such that

$$\overrightarrow{ax} = \sum_{i\in I}\lambda_i \overrightarrow{aa_i} \quad \text{for every } a \in E,$$