



Figure 54.8: In this illustration points with errors are denoted by open circles. In the original, upper left configuration, there is no blue support vector and no red support vector. By increasing the margin, we end up with a red support vector and reduce to Case 1a.

We have

$$\begin{aligned}
 w^\top u_i - b &= \eta - \epsilon_i & \epsilon_i > 0 & & i \in E_\lambda \\
 -w^\top v_j + b &= \eta - \xi_j & \xi_j > 0 & & j \in E_\mu \\
 w^\top u_i - b &> \eta & & & i \notin E_\lambda \\
 -w^\top v_j + b &> \eta & & & j \notin E_\mu.
 \end{aligned}$$

Let us pick  $\theta$  such that

$$\theta = \min\{w^\top u_i - b - \eta, -w^\top v_j + b - \eta \mid i \notin E_\lambda, j \notin E_\mu\}.$$

Our hypotheses imply that  $\theta > 0$ . We can write

$$\begin{aligned}
 w^\top u_i - b &= \eta + \theta - (\epsilon_i + \theta) & \epsilon_i > 0 & & i \in E_\lambda \\
 -w^\top v_j + b &= \eta + \theta - (\xi_j + \theta) & \xi_j > 0 & & j \in E_\mu \\
 w^\top u_i - b &\geq \eta + \theta & & & i \notin E_\lambda \\
 -w^\top v_j + b &\geq \eta + \theta & & & j \notin E_\mu,
 \end{aligned}$$

and by the choice of  $\theta$ , either

$$w^\top u_i - b = \eta + \theta \quad \text{for some } i \notin E_\lambda$$