Prove that the columns and rows of $W_{n,k}$ are orthogonal, and use this to prove that the columns of W_n and the rows of W_n^{-1} are orthogonal. Are the rows of W_n orthogonal? Are the columns of W_n^{-1} orthogonal? Prove that

$$W_{n,k}^{-1} = \begin{pmatrix} \frac{1}{2}W_{k,k}^{\top} & 0_{2^k,2^n-2^k} \\ 0_{2^n-2^k,2^k} & I_{2^n-2^k} \end{pmatrix}.$$

Problem 5.3. Prove that if H is a Hadamard matrix of dimension n, then the block matrix of dimension $2n$,

$$\begin{pmatrix} H & H \\ H & -H \end{pmatrix},$$

is a Hadamard matrix.

Problem 5.4. Plot the graphs of the eight Walsh functions $\mathrm{Wal}(k,t)$ for $k = 0, 1, \ldots, 7$.

Problem 5.5. Describe a recursive algorithm to compute the product $H_{2^m}\, x$ of the Sylvester–Hadamard matrix H_{2^m} by a vector $x \in \mathbb{R}^{2^m}$ that uses m recursive calls.