This is indeed the case, but a rigorous proof requires induction, and such a proof is surprisingly involved. Readers may accept without proof the fact that sums of the form $\sum_{i\in I} a_i$ are indeed well defined, and jump directly to Definition 3.3. For those who want to see the gory details, here we go.

First, we define sums $\sum_{i\in I} a_i$, where I is a finite sequence of distinct natural numbers, say $I = (i_1, \ldots, i_m)$. If $I = (i_1, \ldots, i_m)$ with $m \geq 2$, we denote the sequence $(i_2, \ldots, i_m)$ by $I - \{i_1\}$. We proceed by induction on the size m of I. Let

$$\sum_{i\in I} a_i = a_{i_1}, \quad \text{if } m = 1,$$
$$\sum_{i\in I} a_i = a_{i_1} + \Bigg(\sum_{i\in I-\{i_1\}} a_i\Bigg), \quad \text{if } m > 1.$$

For example, if $I = (1, 2, 3, 4)$, we have

$$\sum_{i\in I} a_i = a_1 + (a_2 + (a_3 + a_4)).$$

If the operation $+$ is not associative, the grouping of the terms matters. For instance, in general

$$a_1 + (a_2 + (a_3 + a_4)) \neq (a_1 + a_2) + (a_3 + a_4).$$

However, if the operation $+$ is associative, the sum $\sum_{i\in I} a_i$ should not depend on the grouping of the elements in I, as long as their order is preserved. For example, if $I = (1, 2, 3, 4, 5)$, $J_1 = (1, 2)$, and $J_2 = (3, 4, 5)$, we expect that

$$\sum_{i\in I} a_i = \Bigg(\sum_{j\in J_1} a_j\Bigg) + \Bigg(\sum_{j\in J_2} a_j\Bigg).$$

This indeed the case, as we have the following proposition.

Proposition 3.2. *Given any nonempty set A equipped with an associative binary operation $+\colon A \times A \to A$, for any nonempty finite sequence I of distinct natural numbers and for any partition of I into p nonempty sequences $I_{k_1}, \ldots, I_{k_p}$, for some nonempty sequence $K = (k_1, \ldots, k_p)$ of distinct natural numbers such that $k_i < k_j$ implies that $\alpha < \beta$ for all $\alpha \in I_{k_i}$ and all $\beta \in I_{k_j}$, for every sequence $(a_i)_{i\in I}$ of elements in A, we have*

$$\sum_{\alpha\in I} a_\alpha = \sum_{k\in K}\Bigg(\sum_{\alpha\in I_k} a_\alpha\Bigg).$$

Proof. We proceed by induction on the size n of I.

If $n = 1$, then we must have $p = 1$ and $I_{k_1} = I$, so the proposition holds trivially.