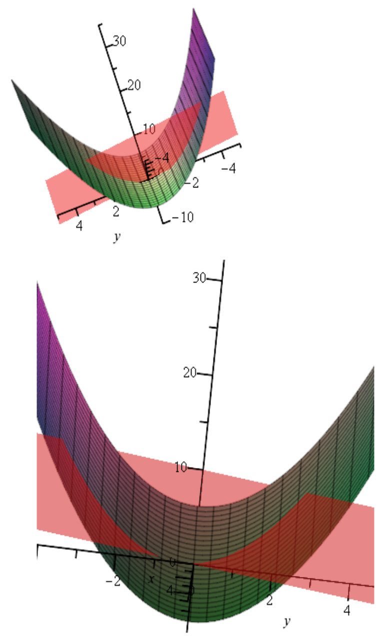

Figure 52.2: Two views of the graph of $y^2 + 2x$ intersected with the transparent red plane $2x - y = 0$. The solution to Example 52.2 is apex of the intersection curve, namely the point $(-\frac{1}{4}, -\frac{1}{2}, -\frac{15}{16})$.

See Figure 52.2.

The quadratic function

$$J(x,y) = y^2 + 2x = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

is convex but not strictly convex. Since $y = 2x$, the problem is equivalent to minimizing $y^2 + 2x = 4x^2 + 2x$, whose minimum is achieved for $x = -1/4$ (since setting the derivative of the function $x \mapsto 4x^2 + 2$ yields $8x + 2 = 0$). Thus, the unique minimum of our problem is achieved for $(x = -1/4, y = -1/2)$. The Lagrangian of our problem is

$$L(x,y,\lambda) = y^2 + 2x + \lambda(2x - y).$$

If we apply the dual ascent method, minimization of $L(x,y,\lambda)$ with respect to x and y holding λ constant yields the equations

$$\begin{aligned} 2 + 2\lambda &= 0 \\ 2y - \lambda &= 0, \end{aligned}$$

obtained by setting the gradient of L (with respect to x and y) to zero. If $\lambda \neq -1$, the problem has no solution. Indeed, if $\lambda \neq -1$, minimizing $L(x,y,\lambda) = y^2 + 2x + \lambda(2x - y)$ with respect to x and y yields $-\infty$.