

Also, if $\gamma = 0$, then $X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0$.

Maximizing with respect to $\gamma > 0$ by setting $\frac{\partial}{\partial \gamma} G(\lambda, \mu, \alpha, \beta, \gamma) = 0$ yields

$$\gamma^2 = \frac{1}{4} (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix},$$

so we obtain

$$G(\lambda, \mu) = - \left((\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \right)^{1/2}.$$

Finally, since $G(\lambda, \mu) = 0$ and $X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0$ if $\gamma = 0$, the dual program is equivalent to the following minimization program:

Dual of Soft margin SVM (SVM_{s1}):

$$\begin{aligned} & \text{minimize} \quad (\lambda^\top \quad \mu^\top) X^\top X \begin{pmatrix} \lambda \\ \mu \end{pmatrix} \\ & \text{subject to} \\ & \quad \sum_{i=1}^p \lambda_i - \sum_{j=1}^q \mu_j = 0 \\ & \quad \sum_{i=1}^p \lambda_i + \sum_{j=1}^q \mu_j = 1 \\ & \quad 0 \leq \lambda_i \leq K, \quad i = 1, \dots, p \\ & \quad 0 \leq \mu_j \leq K, \quad j = 1, \dots, q. \end{aligned}$$

Observe that the constraints imply that K must be chosen so that

$$K \geq \max \left\{ \frac{1}{2p}, \frac{1}{2q} \right\}.$$

If $(w, \delta, b, \epsilon, \xi)$ is an optimal solution of Problem (SVM_{s1}), then the complementary slackness conditions yield a classification of the points u_i and v_j in terms of the values of λ and μ . Indeed, we have $\epsilon_i \alpha_i = 0$ for $i = 1, \dots, p$ and $\xi_j \beta_j = 0$ for $j = 1, \dots, q$. Also, if $\lambda_i > 0$, then corresponding constraint is active, and similarly if $\mu_j > 0$. Since $\lambda_i + \alpha_i = K$, it follows that $\epsilon_i \alpha_i = 0$ iff $\epsilon_i (K - \lambda_i) = 0$, and since $\mu_j + \beta_j = K$, we have $\xi_j \beta_j = 0$ iff $\xi_j (K - \mu_j) = 0$. Thus if $\epsilon_i > 0$, then $\lambda_i = K$, and if $\xi_j > 0$, then $\mu_j = K$. Consequently, if $\lambda_i < K$, then $\epsilon_i = 0$ and u_i is correctly classified, and similarly if $\mu_j < K$, then $\xi_j = 0$ and v_j is correctly classified. We have the following classification: