An affine subspace is also called a *flat* by some authors. According to Definition 24.3, the empty set is trivially an affine subspace, and every intersection of affine subspaces is an affine subspace.

As an example, consider the subset $U$ of $\mathbb{R}^2$ defined by

$$U = \left\{(x, y) \in \mathbb{R}^2 \mid ax + by = c\right\},$$

i.e., the set of solutions of the equation

$$ax + by = c,$$

where it is assumed that $a \neq 0$ or $b \neq 0$. Given any $m$ points $(x_i, y_i) \in U$ and any $m$ scalars $\lambda_i$ such that $\lambda_1 + \cdots + \lambda_m = 1$, we claim that

$$\sum_{i=1}^{m} \lambda_i (x_i, y_i) \in U.$$

Indeed, $(x_i, y_i) \in U$ means that

$$ax_i + by_i = c,$$

and if we multiply both sides of this equation by $\lambda_i$ and add up the resulting $m$ equations, we get

$$\sum_{i=1}^{m} (\lambda_i a x_i + \lambda_i b y_i) = \sum_{i=1}^{m} \lambda_i c,$$

and since $\lambda_1 + \cdots + \lambda_m = 1$, we get

$$a \left(\sum_{i=1}^{m} \lambda_i x_i\right) + b \left(\sum_{i=1}^{m} \lambda_i y_i\right) = \left(\sum_{i=1}^{m} \lambda_i\right) c = c,$$

which shows that

$$\left(\sum_{i=1}^{m} \lambda_i x_i, \sum_{i=1}^{m} \lambda_i y_i\right) = \sum_{i=1}^{m} \lambda_i (x_i, y_i) \in U.$$

Thus, $U$ is an affine subspace of $\mathbb{A}^2$. In fact, it is just a usual line in $\mathbb{A}^2$.

It turns out that $U$ is closely related to the subset of $\mathbb{R}^2$ defined by

$$\overrightarrow{U} = \left\{(x, y) \in \mathbb{R}^2 \mid ax + by = 0\right\},$$

i.e., the set of solutions of the homogeneous equation

$$ax + by = 0$$