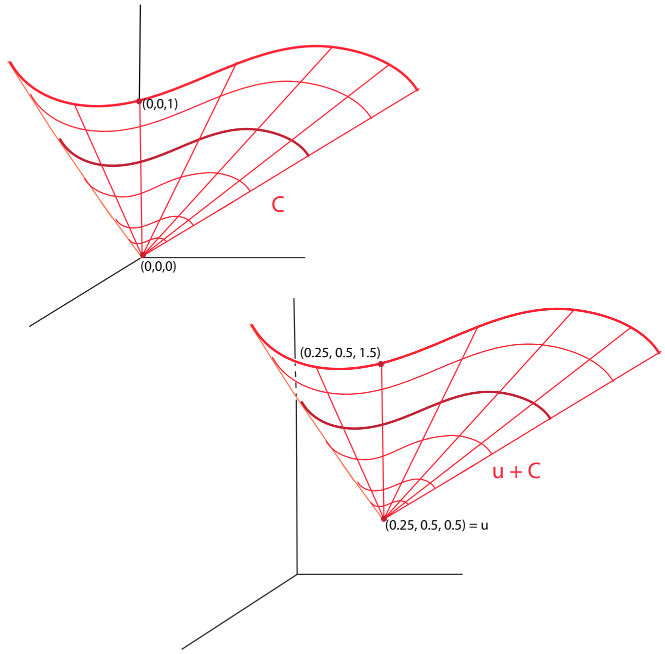

Figure 50.1: Let C be the cone determined by the bold orange curve through $(0,0,1)$ in the plane $z = 1$. Then $u + C$, where $u = (0.25, 0.5, 0.5)$, is the affine translate of C via the vector u.

Definition 50.2. Let V be a normed vector space and let U be a nonempty subset of V. For any point $u \in U$, the *cone $C(u)$ of feasible directions at u* is the union of $\{0\}$ and the set of all nonzero vectors $w \in V$ for which there exists some convergent sequence $(u_k)_{k \geq 0}$ of vectors such that

(1) $u_k \in U$ and $u_k \neq u$ for all $k \geq 0$, and $\lim_{k \mapsto \infty} u_k = u$.

(2) $\lim_{k \mapsto \infty} \dfrac{u_k - u}{\|u_k - u\|} = \dfrac{w}{\|w\|}$, with $w \neq 0$.

Condition (2) can also be expressed as follows: there is a sequence $(\delta_k)_{k \geq 0}$ of vectors $\delta_k \in V$ such that

$$u_k = u + \|u_k - u\| \frac{w}{\|w\|} + \|u_k - u\| \, \delta_k, \quad \lim_{k \mapsto \infty} \delta_k = 0, \;\; w \neq 0.$$

Figure 50.2 illustrates the construction of w in $C(u)$.

Clearly, the cone $C(u)$ of feasible directions at u is a cone with apex 0, and $u + C(u)$ is a cone with apex u. Obviously, it would be desirable to have conditions on U that imply that $C(u)$ is a convex cone. Such conditions will be given later on.

Observe that the cone $C(u)$ of feasible directions at u contains the velocity vectors at u of all curves γ in U through u. If $\gamma\colon (-1,1) \to U$ is such a curve with $\gamma(0) = u$, and if $\gamma'(u) \neq 0$