of the affine subspace defining the affine flip f_{2k}. Finally, appealing to Lemma 27.9, since $\tau \in \overrightarrow{F_{2k}}^{\perp}$, the translation t_τ can be expressed as the composition $f'_{2k} \circ f'_{2k-1}$ of two affine flips f'_{2k-1} and f'_{2k} about the two parallel subspaces $\Omega + \overrightarrow{F_{2k}}$ and $\Omega + \tau/2 + \overrightarrow{F_{2k}}$, whose distance is $\|\tau\|/2$. However, since f'_{2k-1} and f_{2k} are both the identity on $\Omega + \overrightarrow{F_{2k}}$, we must have $f'_{2k-1} = f_{2k}$, and thus

$$\begin{aligned} f &= t_\tau \circ f_{2k} \circ f_{2k-1} \circ \cdots \circ f_1 \\ &= f'_{2k} \circ f'_{2k-1} \circ f_{2k} \circ f_{2k-1} \circ \cdots \circ f_1 \\ &= f'_{2k} \circ f_{2k-1} \circ \cdots \circ f_1, \end{aligned}$$

since $f'_{2k-1} = f_{2k}$ and $f'_{2k-1} \circ f_{2k} = f_{2k} \circ f_{2k} = \mathrm{id}$, since f_{2k} is an affine symmetry. $\square$

Remark: It is easy to prove that if f is a screw motion in $\mathbf{SE}(3)$, D its axis, θ is its angle of rotation, and τ the translation along the direction of D, then f is the composition of two affine flips about lines D_1 and D_2 orthogonal to D, at a distance $\|\tau\|/2$ and making an angle $\theta/2$.