CAP4630: Predicate Calculus Practice Problems

1. Express the following English language sentences into predicate calculus expressions.

   a) There exists a prime number which is not odd.

   b) All animals that swim are either fish or mammals.

   c) Ada does not eat any food which contains dairy.

   d) All students should do all homeworks or they fail.

2. Attempt to unify the following pair of expressions. Either show their most general unifiers or explain why they will not unify.

   a) p(X, Y) and p(a, Z).

   b) p(X, X) and p(a,b).

   c) ancestor(X, Y) and ancestor(bill, father(bill)).

   d) ancestor(X, father(X)) and ancestor(david, george).

   e) q(X) and $\neg$q(a).

   f) p(X, a, Y) and p(Z, Z, b).

3. Convert the following expressions into clause form.

   a) $(a \vee b) \rightarrow (c \wedge d)$

   b) $[((a \wedge b) \rightarrow c) \rightarrow d] \vee e$

   c) $\forall X \exists Y \ (a(X) \leftrightarrow b(Y))$

   d) $\forall X \forall Y [(f(X) = g(Y)) \rightarrow (\exists Z ((g(Y) = h(Z)) \rightarrow (f(X) = h(Z))))]$

4. Given the following knowledge base, (i) express these propositions in conjunctive normal form (clause form) and (ii) use resolution refutation to prove the proposition q. You must show each resolution step and for each you must identify the propositions involved.

    1. $p \rightarrow (q \vee r)$
    2. $p \vee r$
    3. $\neg r$

5. Given the following knowledge base, (i) express these propositions in conjunctive normal form (clause form) and (ii) use resolution refutation to prove the proposition r. You must show each resolution step and for each you must identify the propositions involved.

    1. $(p \vee r) \wedge (s \rightarrow q)$
    2. $\neg p \vee s$
    3. $\neg q$

6. Given the following knowledge base, (i) express these propositions in conjunctive normal form (clause form) and (ii) use resolution refutation to prove the proposition $\neg t$. You must show each resolution step and for each you must identify the propositions involved.

    1. $(q \rightarrow s) \rightarrow r$
    2. $\neg(q \wedge t)$
    3. r

7. I married a widow (let's call her W) who has a grown up daughter (call her D). My father (F), who visited us quite often, fell in love with my step-daughter and married her. Hence my father became my son-in-law and my step-daughter became my mother. Some months later, my wife gave birth to a son (S_1), who became the brother-in-law of my father, as well as my uncle. The wife of my father, that is, my step-daughter, also had a son (S_2).

    Using predicate calculus, create a set of expressions that represent the situation in the above story. Add additional expressions defining basic family relationships, as needed, and use resolution refutation on the entire system of expressions to prove the following conclusions:

    (a) My step-daughter us my mother (N.B. – you should treat a step-daughter as a daughter) and
    (b) S_1 is my father's brother-in-law.