UNIVERSITY OF TECHNOLOGY - VNUHCM Faculty of Applied Science	**Final Exams**	Semester/Acad. year	2	2022-2023
		Date	May 27th, 2023	
	Course title	Linear Algebra		
	Course ID	MT1007	Question sheet code	1121
	Duration	100 minutes	Shift	09:00

Instructions to students:

- *This is a closed book exam. Only your calculator is allowed. Total available score: 10.*
- *At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 26 questions on 5 pages. Do not round between steps. Round your final answers to 4 decimal places.*

Student's full name:	Invigilator 1: .....
Student Id:	Invigilator 2: .....

new-en-long

Part I: Multiple choice (8 points, 80 minutes)

1. Find m such that the following matrix is invertible: $A = \begin{pmatrix} -5 & -1 & 1 \\ 1 & -5 & 2 \\ -2 & -5 & m \end{pmatrix}$.

   Ⓐ m does not exist. Ⓑ $m = 0.8462$. Ⓒ For all m. Ⓓ m is not equal to 2.3462. Ⓔ $m = -0.1538$.

2. Find m such that the following system has unique solution $\begin{cases} -2x - 3y + z = 7, \\ -y + 2z = -1, \\ 4x - 2y + mz = 0. \end{cases}$

   Ⓐ m = 18. Ⓑ Does not exist m. Ⓒ For all m. Ⓓ m is different from 14. Ⓔ Other answers are wrong.

3. In $\mathbb{R}^3$ let $E = \{(0, -2, 1)^T, (3, 4, m)^T, (3, 0, 2)^T\}$ be a vector set. Find m such that E is not a basis of $\mathbb{R}^3$.

   Ⓐ $m = 4.5$. Ⓑ m does not exist. Ⓒ For all m. Ⓓ $m = 0$. Ⓔ $m = 2.5$.

Questions 4 through 7

A basis E for the nullspace of $A = \begin{pmatrix} -3 & 3 & -4 & 5 \\ -12 & 12 & -12 & 24 \end{pmatrix}$ has the form $\{(x, 1, y, 0), (z, 0, t, 1)\}$.

4. Determine x, y, z, t.

   Ⓐ $x = 1, y = 0, z = -2, t = -1$ Ⓑ $x = -2, y = -1, z = 1, t = 5$ Ⓒ Other answers are wrong. Ⓓ $x = -2, y = -1, z = 0, t = 1$ Ⓔ $x = -2, y = 0, z = -2, t = -5$

5. Find u and v such that $w = (u, 2, v, 5)$ is in Null(A). Ⓐ u = -7.5, v = -8 Ⓑ u = -7, v = -4.5 Ⓒ u = -12, v = -1.5 Ⓓ Other answer are wrong. Ⓔ u = -8, v = -5

6. Let θ be a vector in Null(A) such that $[\theta]_E = (5, 5)^T$. Find the vector θ.

   Ⓐ $(75, 75, 80, 145)$ Ⓑ $(-75, 75, -80, 145)$ Ⓒ Other answer are wrong. Ⓓ $(5, 5, 5, 5)$ Ⓔ $(-5, 5, -5, 5)$

7. Let F be another basis of Null(A) and the transition matrix from F to E is $T_{F\to E} = [[f_1]_E[f_2]_E] = \begin{pmatrix} 1 & -4 \\ 5 & -19 \end{pmatrix}$. Find the coordinate vector of θ with respect to F.

Ⓐ Other answer are wrong. Ⓑ $(-74.5, -20.5)^T$. Ⓒ $(-14.5, -70.5)^T$. Ⓓ $(-75, -20)^T$. Ⓔ $(-15, -70)^T$.

8. In the vector space $P_2(\mathbb{R}) = \{p(x) = p_2x^2 + p_1x + p_0, \quad a, b, c \in \mathbb{R}\}$ define the inner product $(:,:) : P_2(\mathbb{R}) \times P_2(\mathbb{R}) \to \mathbb{R} : (: p, q :) = p_0q_0 + p_1q_1 + p_2q_2$. Evaluate the inner product of the following two vectors $f = 2 + 6x + 11x^2, \quad g = -3 - 4x + 8x^2$.

Ⓐ 3729. Ⓑ 1231. Ⓒ All other answers are wrong. Ⓓ 796. Ⓔ 451.

Question 9 through 11: In $\mathbb{R}^3$ we define the weighted inner product $\langle ., . \rangle$ that is given by the formula

$$\langle x, y \rangle = 3x_1y_1 + 6x_2y_2 + 4x_3y_3.$$

Let X be the subspace of $\mathbb{R}_3$ that is spanned by the basis

$$S = \{u = (5, -4, 40), v = (-2, 2, -18)\}.$$

9. One basis of the orthogonal complement of X in $\mathbb{R}_3$ is

Ⓐ $\{(-5.3333, 3.3333, 1)\}$. Ⓑ $\{(5, -4, 40), (-2, 2, -18)\}$. Ⓒ $\{(1, 0, 4), (0, 1, -5)\}$. Ⓓ All other answers are wrong. Ⓔ $\{(3.3333, -5.3333, 1)\}$.

10. The projection of $z = (2, -5, 2)$ onto X is

Ⓐ $(8.4786, -5.2991, -1.5897)$. Ⓑ $(2, -5, 2)$. Ⓒ $(-5.3333, 3.3333, 1)$. Ⓓ All other answers are wrong. Ⓔ $(4.2393, -2.6496, -0.7949)$.

11. The distance from z to X is

Ⓐ 0.7949. Ⓑ All other answers are wrong Ⓒ 1.5897. Ⓓ 9.9279. Ⓔ 19.8558.

Question 12 through 13:

Let $A = \begin{bmatrix} 3 & 6 & 1 & -4 \\ 1 & 2 & 2 & -8 \\ 3 & 6 & 3 & -12 \end{bmatrix}$ and T be a linear transformation defined by $T(x) = Ax^T$.

12. Find the first component of the image vector $T(x)$ where $x = (-2, -4, 1, 5)^T$.

Ⓐ -46. Ⓑ -45. Ⓒ All other answers are wrong. Ⓓ -49. Ⓔ -51.

13. Which of the following sets is a basis of the range of T?

Ⓐ $\{(-2, 1, 0, 0), (0, 0, 4, 1)\}$. Ⓑ $\{(3, 1, 3), (1, 2, 3)\}$. Ⓒ $\{(3, 1, 3), (1, 2, 3), (-4, -8, -12)\}$. Ⓓ $\{(1, 2, 0, 0), (0, 0, 1, -4)\}$. Ⓔ All other answer are wrong.

Question 14 through 17:

In $\mathbb{R}^2$ with the standard inner product let d be the line $x - 2y = 0$.

14. Find the matrix of the reflection transformation f about the line d.

Ⓐ $\begin{pmatrix} 0.8 & 0.4 \\ 0.4 & 0.2 \end{pmatrix}$. Ⓑ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Ⓒ All other answers are wrong. Ⓓ $\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$. Ⓔ $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$.

15. Find the reflection of the triangle with vertices A(-4, 1), B(3, 2), C(-3, 7) about the line d.

Ⓐ $A'(-1.6, -3.8), B'(3.4, 1.2), C'(3.8, -6.6)$. Ⓑ All other answers are wrong.
Ⓒ $A'(3.4, 1.2), B'(3.8, -6.6), C'(-1.6, -3.8)$. Ⓓ $A'(1.6, -3.8), B'(-3.4, 1.2), C'(-3.8, -6.6)$.
Ⓔ $A'(-3.4, 1.2), B'(= 3.8, -6.6), C'(1.6, -3.8)$.

16. Find the matrix of the rotation transformation g about the origin by $\pi/4$ degree clockwise.

Ⓐ $\begin{pmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{pmatrix}$. Ⓑ $\begin{pmatrix} -0.2929 & -0.2929 \\ 0.7929 & 2.2071 \end{pmatrix}$. Ⓒ $\begin{pmatrix} -0.2929 & -0.7071 \\ -0.2929 & 2.2071 \end{pmatrix}$.
Ⓓ All other answers are wrong. Ⓔ $\begin{pmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}$.

17. Find the matrix of the composite transformation $f \circ g$.

Ⓐ All other answers are wrong. Ⓑ $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ Ⓒ $\begin{pmatrix} -0.1414 & 0.9899 \\ 0.9899 & 0.1414 \end{pmatrix}$ Ⓓ $\begin{pmatrix} 0.8586 & 3.9899 \\ 3.9899 & 0.1414 \end{pmatrix}$
Ⓔ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

18. Let $E = \{(570, 221)^T, (165, 64)^T\}$ and $F = \{(-546, -151)^T, (-159, -44)^T\}$ be two bases of $\mathbb{R}^2$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(x, y) = (-3x + 3y, -5y)$. Find the matrix A_{EF} of T with respect to the bases E and F.

Ⓐ Other answers are wrong. Ⓑ $\begin{pmatrix} -546 & -159 \\ -151 & -44 \end{pmatrix}$ Ⓒ $\begin{pmatrix} -765 & 1200 \\ -299 & 469 \end{pmatrix}$ Ⓓ $\begin{pmatrix} 570 & 165 \\ 221 & 64 \end{pmatrix}$
Ⓔ $\begin{pmatrix} 819 & -1284 \\ 229 & -359 \end{pmatrix}$

19. Let $A = \begin{pmatrix} -6 & 4 \\ -1 & -1 \end{pmatrix}$. Find all eigenvalues of A.

Ⓐ All other answers are wrong. Ⓑ $\lambda_1 = -5, \lambda_2 = -2$ Ⓒ $\lambda_1 = -8, \lambda_2 = 2$ Ⓓ $\lambda_1 = -4, \lambda_2 = -4$ Ⓔ $\lambda_1 = -9, \lambda_2 = 1$

Questions 20 through 22 Suppose that $u = (-5, 3, -2, -5)^T$ is an eigenvector of the matrix A corresponding to $\lambda = 3$, and $v = (4, 2, 1, 1)^T$ is an eigenvector of A corresponding to $\lambda = 2$. Given that A is invertible.

20. Find the first component of the vector $A^5 \times u$.

Ⓐ -1218.5 Ⓑ -1210 Ⓒ -1215 Ⓓ -1214 Ⓔ Other answer are wrong.

21. Find the first component of the vector $A \times (4u + 2v)$.

Ⓐ -44 Ⓑ -43 Ⓒ Other answer are wrong. Ⓓ -39 Ⓔ -40

22. Find the second component of the vector $A^{-1} \times (6u + 3v)$.

Ⓐ Other answer are wrong. Ⓑ 8.5 Ⓒ 14 Ⓓ 4.5 Ⓔ 9

23. Let $A_{n\times n} \in M_n(\mathbb{R})$ be a matrix. λ is an eigenvalue of A corresponding to the eigenvector x. Choose the CORRECT statement.

Ⓐ x is a nonzero vector. Ⓑ $\sqrt{\lambda}$ is an eigenvalue of A for all λ Ⓒ It is impossible to tell anything. Ⓓ λ is different from 0. Ⓔ λ is an eigenvalue of A^m, for all $m \in \mathbb{N}, m > 1$.

Question 24 through 25:

On the plane OXY, given the points $A(-2, -3)$, $B(1, 4)$ and $C(m, n)$.

24. Find the coordinates m, n of C such that we can construct a parabola passing through A,B,C.

Ⓐ All other answers are wrong. Ⓑ $(-0.5, 0.5)$ Ⓒ $(-1.7, -2.3)$ Ⓓ $(4.3, 7.3)$ Ⓔ $(-0.8, -0.2)$

25. With the point C found in the question 24, find the parabola.

Ⓐ $-40/189x^2 + 395/189x + 395/189$ Ⓑ $-40/189x^2 + 401/189x + 395/189$ Ⓒ $-40/189x^2 + 395/189x + 1/2$ Ⓓ All other answers are wrong. Ⓔ $401/189x^2 + 401/189x + 395/189$

Part II: Essay (2 points, 20 minutes)

26. In a survey investigating changes in housing patterns in one urban area, it was found that 35% of the population lived in single-family dwellings and 65% in multiple housing of some kind. Five years later, in a follow-up survey, of those who had been living in , 73% still did so, but 27% had moved to multiple-family dwellings. Of those in multiple-family housing, 60% were still living in that type of housing, while 40% had moved to single-family dwellings. Assume these trends continue.

a. What is the transition matrix? (The 2 states should be in this order: single-family dwellings, multiple housing)

b. What are the eigenvalues and corresponding eigenvectors of the transition matrix?

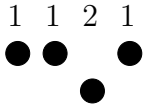
c. Find the proportion of population in each class after 10 years.

d. Using the diagonalization method to find the percent of the population can be expected in after many years.

Answers Sheet

Question sheet code 1121:

1 D. 2 D. 3 D. 4 A. 5 E. 6 E. 7 D. 8 B. 9 A. 10 E. 11 D. 12 D. 13 B. 14 E. 15 A. 16 A. 17 C. 18 E. 19 B. 20 C. 21 A. 22 E. 23 A. 24 D. 25 B.



MT1007

Linear Algebra

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