

 UNIVERSITY OF TECHNOLOGY - VNUHCM Faculty of Applied Science	Final Exams		Semester/Acad. year		2	2022-2023
			Date		May 27 th , 2023	
	Course title	Linear Algebra				
	Course ID	MT1007	Question sheet code	1121		
	Duration	100 minutes	Shift	09:00		
Instructions to students: - This is a closed book exam. Only your calculator is allowed. Total available score: 10. - At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 26 questions on 5 pages. Do not round between steps. Round your final answers to 4 decimal places.						
Student's full name:			Invigilator 1:			
Student Id:			Invigilator 2:			

new-en-long

Part I: Multiple choice (8 points, 80 minutes)

- Find m such that the following matrix is invertible: $A = \begin{pmatrix} -5 & -1 & 1 \\ 1 & -5 & 2 \\ -2 & -5 & m \end{pmatrix}$.
 (A) m does not exist. (B) $m = 0.8462$. (C) For all m . (D) m is not equal to 2.3462.
 (E) $m = -0.1538$.
- Find m such that the following system has unique solution $\begin{cases} -2x - 3y + z = 7, \\ -y + 2z = -1, \\ 4x - 2y + mz = 0. \end{cases}$
 (A) $m = 18$. (B) Does not exist m . (C) For all m . (D) m is different from 14. (E) Other answers are wrong.
- In \mathbb{R}^3 let $E = \{(0, -2, 1)^T, (3, 4, m)^T, (3, 0, 2)^T\}$ be a vector set. Find m such that E is not a basis of \mathbb{R}^3 .
 (A) $m = 4.5$. (B) m does not exist. (C) For all m . (D) $m = 0$. (E) $m = 2.5$.

Questions 4 through 7

A basis E for the nullspace of $A = \begin{pmatrix} -3 & 3 & -4 & 5 \\ -12 & 12 & -12 & 24 \end{pmatrix}$ has the form $\{(x, 1, y, 0), (z, 0, t, 1)\}$.

- Determine x, y, z, t .
 (A) $x = 1, y = 0, z = -2, t = -1$ (B) $x = -2, y = -1, z = 1, t = 5$ (C) Other answers are wrong.
 (D) $x = -2, y = -1, z = 0, t = 1$ (E) $x = -2, y = 0, z = -2, t = -5$
- Find u and v such that $w = (u, 2, v, 5)$ is in $\text{Null}(A)$. (A) $u = -7.5, v = -8$ (B) $u = -7, v = -4.5$
 (C) $u = -12, v = -1.5$ (D) Other answer are wrong. (E) $u = -8, v = -5$
- Let θ be a vector in $\text{Null}(A)$ such that $[\theta]_E = (5, 5)^T$. Find the vector θ .
 (A) $(75, 75, 80, 145)$ (B) $(-75, 75, -80, 145)$ (C) Other answer are wrong. (D) $(5, 5, 5, 5)$
 (E) $(-5, 5, -5, 5)$

7. Let F be another basis of $\text{Null}(A)$ and the transition matrix from F to E is $T_{F \rightarrow E} = [[f_1]_E [f_2]_E] = \begin{pmatrix} 1 & -4 \\ 5 & -19 \end{pmatrix}$. Find the coordinate vector of θ with respect to F .
- Ⓐ Other answer are wrong. Ⓑ $(-74.5, -20.5)^T$. Ⓒ $(-14.5, -70.5)^T$. Ⓓ $(-75, -20)^T$.
Ⓔ $(-15, -70)^T$.

8. In the vector space $P_2(\mathbb{R}) = \{p(x) = p_2x^2 + p_1x + p_0, \quad a, b, c \in \mathbb{R}\}$ define the inner product $(:, :) : P_2(\mathbb{R}) \times P_2(\mathbb{R}) \rightarrow \mathbb{R} : (p, q) = p_0q_0 + p_1q_1 + p_2q_2$. Evaluate the inner product of the following two vectors $f = 2 + 6x + 11x^2$, $g = -3 - 4x + 8x^2$.
- Ⓐ 3729. Ⓑ 1231. Ⓒ All other answers are wrong. Ⓓ 796. Ⓔ 451.

Question 9 through 11: In \mathbb{R}^3 we define the weighted inner product $\langle \cdot, \cdot \rangle$ that is given by the formula

$$\langle x, y \rangle = 3x_1y_1 + 6x_2y_2 + 4x_3y_3.$$

Let X be the subspace of \mathbb{R}_3 that is spanned by the basis

$$S = \{u = (5, -4, 40), v = (-2, 2, -18)\}.$$

9. One basis of the orthogonal complement of X in \mathbb{R}_3 is
- Ⓐ $\{(-5.3333, 3.3333, 1)\}$. Ⓑ $\{(5, -4, 40), (-2, 2, -18)\}$. Ⓒ $\{(1, 0, 4), (0, 1, -5)\}$. Ⓓ All other answers are wrong. Ⓔ $\{(3.3333, -5.3333, 1)\}$.
10. The projection of $z = (2, -5, 2)$ onto X is
- Ⓐ $(8.4786, -5.2991, -1.5897)$. Ⓑ $(2, -5, 2)$. Ⓒ $(-5.3333, 3.3333, 1)$. Ⓓ All other answers are wrong. Ⓔ $(4.2393, -2.6496, -0.7949)$.
11. The distance from z to X is
- Ⓐ 0.7949. Ⓑ All other answers are wrong Ⓒ 1.5897. Ⓓ 9.9279. Ⓔ 19.8558.

Question 12 through 13:

Let $A = \begin{bmatrix} 3 & 6 & 1 & -4 \\ 1 & 2 & 2 & -8 \\ 3 & 6 & 3 & -12 \end{bmatrix}$ and T be a linear transformation defined by $T(x) = Ax^T$.

12. Find the first component of the image vector $T(x)$ where $x = (-2, -4, 1, 5)^T$.
- Ⓐ -46. Ⓑ -45. Ⓒ All other answers are wrong. Ⓓ -49. Ⓔ -51.
13. Which of the following sets is a basis of the range of T ?
- Ⓐ $\{(-2, 1, 0, 0), (0, 0, 4, 1)\}$. Ⓑ $\{(3, 1, 3), (1, 2, 3)\}$. Ⓒ $\{(3, 1, 3), (1, 2, 3), (-4, -8, -12)\}$.
Ⓓ $\{(1, 2, 0, 0), (0, 0, 1, -4)\}$. Ⓔ All other answer are wrong.

Question 14 through 17:

In \mathbb{R}^2 with the standard inner product let d be the line $x - 2y = 0$.

14. Find the matrix of the reflection transformation f about the line d .

Ⓐ $\begin{pmatrix} 0.8 & 0.4 \\ 0.4 & 0.2 \end{pmatrix}$. Ⓑ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Ⓒ All other answers are wrong. Ⓓ $\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$. Ⓔ $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$.

15. Find the reflection of the triangle with vertices $A(-4, 1)$, $B(3, 2)$, $C(-3, 7)$ about the line d .
 (A) $A'(-1.6, -3.8)$, $B'(3.4, 1.2)$, $C'(3.8, -6.6)$. (B) All other answers are wrong.
 (C) $A'(3.4, 1.2)$, $B'(3.8, -6.6)$, $C'(-1.6, -3.8)$. (D) $A'(1.6, -3.8)$, $B'(-3.4, 1.2)$, $C'(-3.8, -6.6)$.
 (E) $A'(-3.4, 1.2)$, $B'(3.8, -6.6)$, $C'(1.6, -3.8)$.
16. Find the matrix of the rotation transformation g about the origin by $\pi/4$ degree clockwise.
 (A) $\begin{pmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{pmatrix}$. (B) $\begin{pmatrix} -0.2929 & -0.2929 \\ 0.7929 & 2.2071 \end{pmatrix}$. (C) $\begin{pmatrix} -0.2929 & -0.7071 \\ -0.2929 & 2.2071 \end{pmatrix}$.
 (D) All other answers are wrong. (E) $\begin{pmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}$.
17. Find the matrix of the composite transformation $f \circ g$.
 (A) All other answers are wrong. (B) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} -0.1414 & 0.9899 \\ 0.9899 & 0.1414 \end{pmatrix}$ (D) $\begin{pmatrix} 0.8586 & 3.9899 \\ 3.9899 & 0.1414 \end{pmatrix}$
 (E) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
18. Let $E = \{(570, 221)^T, (165, 64)^T\}$ and $F = \{(-546, -151)^T, (-159, -44)^T\}$ be two bases of \mathbb{R}^2 and $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation defined by $T(x, y) = (-3x + 3y, -5y)$. Find the matrix A_{EF} of T with respect to the bases E and F .
 (A) Other answers are wrong. (B) $\begin{pmatrix} -546 & -159 \\ -151 & -44 \end{pmatrix}$ (C) $\begin{pmatrix} -765 & 1200 \\ -299 & 469 \end{pmatrix}$ (D) $\begin{pmatrix} 570 & 165 \\ 221 & 64 \end{pmatrix}$
 (E) $\begin{pmatrix} 819 & -1284 \\ 229 & -359 \end{pmatrix}$
19. Let $A = \begin{pmatrix} -6 & 4 \\ -1 & -1 \end{pmatrix}$. Find all eigenvalues of A .
 (A) All other answers are wrong. (B) $\lambda_1 = -5, \lambda_2 = -2$ (C) $\lambda_1 = -8, \lambda_2 = 2$ (D) $\lambda_1 = -4, \lambda_2 = -4$ (E) $\lambda_1 = -9, \lambda_2 = 1$

Questions 20 through 22 Suppose that $u = (-5, 3, -2, -5)^T$ is an eigenvector of the matrix A corresponding to $\lambda = 3$, and $v = (4, 2, 1, 1)^T$ is an eigenvector of A corresponding to $\lambda = 2$. Given that A is invertible.

20. Find the first component of the vector $A^5 \times u$.
 (A) -1218.5 (B) -1210 (C) -1215 (D) -1214 (E) Other answer are wrong.
21. Find the first component of the vector $A \times (4u + 2v)$.
 (A) -44 (B) -43 (C) Other answer are wrong. (D) -39 (E) -40
22. Find the second component of the vector $A^{-1} \times (6u + 3v)$.
 (A) Other answer are wrong. (B) 8.5 (C) 14 (D) 4.5 (E) 9
23. Let $A_{n \times n} \in M_n(\mathbb{R})$ be a matrix. λ is an eigenvalue of A corresponding to the eigenvector x . Choose the CORRECT statement.
 (A) x is a nonzero vector. (B) $\sqrt{\lambda}$ is an eigenvalue of A for all λ (C) It is impossible to tell anything. (D) λ is different from 0. (E) λ is an eigenvalue of A^m , for all $m \in \mathbb{N}, m > 1$.

Question 24 through 25:

On the plane OXY, given the points $A(-2, -3)$, $B(1, 4)$ and $C(m, n)$.

24. Find the coordinates m, n of C such that we can construct a parabola passing through A,B,C.
 Ⓐ All other answers are wrong. Ⓑ $(-0.5, 0.5)$ Ⓒ $(-1.7, -2.3)$ Ⓓ $(4.3, 7.3)$
 Ⓔ $(-0.8, -0.2)$
25. With the point C found in the question 24, find the parabola.
 Ⓐ $-40/189x^2 + 395/189x + 395/189$ Ⓑ $-40/189x^2 + 401/189x + 395/189$ Ⓒ $-40/189x^2 + 395/189x + 1/2$ Ⓓ All other answers are wrong. Ⓔ $401/189x^2 + 401/189x + 395/189$

Part II: Essay (2 points, 20 minutes)

26. In a survey investigating changes in housing patterns in one urban area, it was found that 35% of the population lived in single-family dwellings and 65% in multiple housing of some kind. Five years later, in a follow-up survey, of those who had been living in , 73% still did so, but 27% had moved to multiple-family dwellings. Of those in multiple-family housing, 60% were still living in that type of housing, while 40% had moved to single-family dwellings. Assume these trends continue.
- What is the transition matrix? (The 2 states should be in this order: single-family dwellings, multiple housing)
 - What are the eigenvalues and corresponding eigenvectors of the transition matrix?
 - Find the proportion of population in each class after 10 years.
 - Using the diagonalization method to find the percent of the population can be expected in after many years.

Answers Sheet

Question sheet code 1121:

1 D. 2 D. 3 D. 4 A. 5 E. 6 E. 7 D. 8 B. 9 A. 10 E. 11 D. 12 D. 13 B. 14 E. 15 A. 16 A.
17 C. 18 E. 19 B. 20 C. 21 A. 22 E. 23 A. 24 D. 25 B.

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Linear Algebra
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