

Identification of Intense Vortex-induced Vibration on Bridge Cables Applying Dynamic Indicators and Probability-based Partitioning

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Abstract

Vortex-induced vibration (VIV) is one of the most common anomalous oscillations faced by cables on cable-stayed bridges, suspension bridges and arch bridges. Typically, cable VIV is a kind of abnormal vibration with large displacements and thus imposes detrimental implications on cables and even the whole bridge structure. Therefore, this study designs an efficient method to spontaneously recognize intense VIV and send instant warnings. While the root mean square of acceleration represents the intensity of vibration, another indicator deriving from derivation transform is designed to quantify the degree of single-mode vibration. Comparing with other equivalent coefficients, this indicator can be easily calculated and is free from the interference of end effect, thus enhance both the efficiency and accuracy. Subsequently, the two dynamic indicators, after several algebraic transformations, are utilized together to create a mixed feature (MF). Given a certain threshold, a probability-based partitioning strategy considering the distribution of MF is introduced to extract VIV samples from all the vibration samples, hence consolidating the foundation of multi-level intense VIV warning. The proposed method has been validated in VIV identification of cables on a large-span cable-stayed bridge. The dynamic features of cables both frequently and hardly influenced by VIV are compared and discussed.

Keywords: Bridge cable, Vortex-induced vibration, Derivation transform, Dynamic indicator, Probability-based partitioning, Instant warning.

1. Introduction

Known for its frequent occurrence in cable-stayed bridges and suspension bridges, vortex-induced vibration (VIV) usually brings about large cable displacement and follows irrevocable detrimental influences. Therefore, it is essential to conceive an efficacious method that can spontaneously recognize VIV and send instant warnings. Generated by vortices periodically separating from either side of a bluff body when fluid passes it, the force that causes VIV is represented by a non-dimensional number, i.e., the Strouhal number, which can be calculated by

$$St = \frac{fD}{U}, \quad (1)$$

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where St is the Strouhal number and f , D and U refer to the frequency of the airflow, the diameter of a bridge cable, and the speed of the airflow, respectively [1].

Traditionally, three major methods are frequently utilized to explore the characteristics of VIV, i.e., theoretical analyses, computational fluid dynamics simulations, and wind tunnel tests. Nevertheless, the mechanism underlying VIV is still uncovered due to the high nonlinearity of airflow and cable-fluid interaction. Nowadays, the prevalence of structural health monitoring (SHM) system offers a tremendous opportunity for the big-data-based study of cable vibration. Due to their convenience and relatively low price, accelerometers are widely deployed in SHM systems as the major data resources. The root mean square (RMS) of acceleration time history is then deemed as the prime indicator to quantify the cable vibration intensity. However, more information is desired to distinguish VIV from other anomalies with intense oscillations.

ref: why intense VIV is single-mode

Typically, researchers employed the Fourier transform to obtain either the frequency spectrum or power spectrum, which can be used to analyze the excited modes. The spectrum with a single high peak indicates the occurrence of VIV since this kind of vibration usually comprises only one major vibration frequency [2]. Recently, scholars presented several innovative methods to detect VIV from acceleration time history. Huang, Z. et al. [3] employs Random Decrement Method to deepen the difference between VIV and normal ambient vibration, thus making it more precise for VIV identification. To automatically sieve out the non-stationary section of the so-called abnormal vibration, i.e., the VIV, Zhao, H. et al. [4] takes advantage of Gaussian mixture modeling of the envelope of time history. After that, the stationary section is selected to extract modal parameters. Among all these methods, the novel algorithm based on the Hilbert transform (HT) is one of the most prevalent. A composite complex analytic signal, whose real part is the original signal while the imaginary part represents the HT of the original, is introduced into this method. The projection of such a signal on the complex plain reflects the constituent of the vibration, as Dan, D. and Li, H. [5] suggest, the more it resembles a hollow ring, the more remarkable its single-mode feature, and thus the more exact the occurrence of vortex-induced vibration. Although the method that employs HT is precise, it is somewhat time-consuming, since based on the Fast Fourier transform, whose time complexity is $O(n \log n)$. To enhance the time efficiency, this paper proposes a derivative indicator based on discrete numerical differentiation, which has an $O(n)$ time complexity, thus obviously lowering the calculative time and bringing about a dependable on-time warning.

Furthermore, since any individual indicator merely denotes either the intensity or the degree of single-mode, more than one indicator should be grouped together for the identification of intense VIV. He, M. et al. [6] present two indicators based on the power spectrum and HT analytical signal respectively. These two indicators are then employed to differentiate VIV and normal vibration based on a pre-set line that relies on practical experience. In addition to that, multifarious clustering algorithms are introduced to achieve unsupervised classification. He, M. et al. [7] claim that KMeans might be the best choice to separate different kinds of vibrations since both hierarchical and density-based clustering entail some hyper-parameters that could not be precisely determined. Li, S. et al. [8], however, use a novel clustering strategy deriving from the traditional density-based algorithm to detect the VIV in the beam of a suspension bridge. Nevertheless, KMeans applied by He, M. et al. [7] is extremely sensitive to outliers, while the method employed by Li, S. et al. [8] demands laborious analysis when determining the number of cluster centroids. To overcome these shortages, a method applying the DBSCAN clustering algorithm is used in discriminating different vibrations based on RMS and the derivative indicator, i.e., the hollow coefficient of the derivative analytical signal (HCD). SHM data of a long-span cable-stayed bridge are analyzed accordingly, and the result proves both the accuracy and efficiency of such a method.

The remaining sections are organized as follows. The methodology of VIV identification is explicated in Sec. 2, where two indicators together with a probability-based partitioning strategy are introduced. In Sec. 3, these methods are carried out to identify the VIV occurring in cables on a long-span cable-stayed bridge, whose SHM system has recorded the long-time acceleration time history of several cables. Finally, conclusions are drawn in Sec. 4.

2. Automated cable VIV identification method

2.1. Key indicators to represent vibration features

2.1.1. RMS to indicate the intensity of vibration

When VIV occurs on a cable, its vibration is usually much more intense than in normal circumstances. Therefore, the RMS of a piece of acceleration time history $\{a_i\}$ is applied to represent the intensity of cable vibration, which can be calculated by

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^N a_i^2}, \quad (2)$$

where N is the length of the time series. Generally, the larger the RMS, the more likely a certain kind of abnormal vibration is to occur.

2.1.2. Indicators to quantify the single-mode characteristic

The RMS, though a helpful indicator to suggest the presence of abnormal vibration, purveys no additional information to discriminate VIV from other large-amplitude abnormal vibrations. Therefore, the single-modal property of large-amplitude VIV is taken into consideration by applying HT, which is demonstrated by

$$y(t) = x(t) * \frac{1}{\pi t} = \int_{-\infty}^{+\infty} \frac{x(\tau)}{\pi(t - \tau)} dt, \quad (3)$$

where $y(t)$ and $x(t)$ represent the transformed signal and the original signal, respectively.

The Hilbert analytical signal is then defined as a complex signal

$$z(t) = x(t) + iy(t), \quad (4)$$

which consists of both the original signal $x(t)$ and the transformed signal $y(t)$.

It is mathematically proved that the HT of a sine function is its corresponding cosine function and vice versa. According to this feature, if a cable is under an ideal VIV situation, i.e., the original acceleration signal is a sine function, the projection of its Hilbert analytical signal in the complex plane will be a circle (see Fig. 1 (a), where all the physical quantities are normalized in advance). Practically, due to the broadband components of the force caused by the vortex and the influence of environmental noises, the projection of the analytical signal is a ring with an inner radius R_1 and outer radius R_2 , as shown in Fig. 1 (b).

To quantify the single-modal feature of VIV, the HCH is defined by

$$HCH = \frac{R_1}{R_2}, \quad (5)$$

where R_1 and R_2 represent the inner radius and outer radius displayed in Fig. 1 (b).

It is understandable that the more the HCH is closer to one, the more similar the ring is to a circle, and thus the more obvious the emergence of single modal vibration is.

The analytical signal applying the signal transformed by HT as the imaginary part is indeed useful. Despite that, the long time it spent due to the $O(n \log n)$ time complexity of HT makes it difficult for in-time detection and warning. To cope with this predicament, the Derivative Transform (DT) is defined as

$$\begin{cases} \hat{y}(t) = \frac{dx(t)}{dt}, \\ y(t) = \frac{\hat{y}(t)}{\max(\hat{y}(t))}, \end{cases} \quad (6)$$

which is designed to substitute the HT, while the composite complex signal can still be represented by Eq. (4).

Overall, such a substitution reduces the time complexity to $O(n)$, which descends from the calculative expense of the numerical differentiation of a time series. The feasibility of this replacement relies on the fact that the derivative of a sine function is a cosine function and vice versa. Moreover, one additional matter worth attention is that the new signal derived from the derivative transform should be normalized one more time since this process introduces a multiplier ω , which is the circular frequency of a sine or cosine function.

Similarly, the projection of the derivative analytical signal of the sine function in a complex plain is shown in Fig 2., where Fig. 2 (a) and Fig. 2 (b) represent the ideal condition and noise-influenced condition, respectively. HCD is correspondingly defined as

$$HCD = \frac{R_1}{R_2}, \quad (7)$$

whose variables have identical meanings to those in Eq. (5).

merits: $O(n)$, no end effect, circular queue

2.2. Sample classification based on probability distribution and outlier threshold

why lower convex function? flowchart!

2.2.1. Typical distribution of RMS and HCD

2.2.2. Logarithm transformation of RMS and HCD to extend data distribution

2.2.3. Mixed Feature of TRMS and THCD

1. PCD if cannot PCD, usually not VIV 2. distribution

2.2.4. Sample partitioning according to the distribution of the mixed feature

2.2.5. Probability-based boundary curve in the RMS-HCD plain

lots of math...

2.3. Multi-level VIV warning applying RMS and HCD

2.3.1. VIV warning with several probability thresholds

2.3.2. VIV warning considering the abstract values of RMS and HCD

3. Application to a long-span cable-stayed bridge

3.1. Description of the Tongling Yangtze River Bridge and its SHM system

The methods and algorithms proposed in Sec. 2 are applied for cable VIV identification of the Tongling Yangtze River (TYR) Bridge. Locating in Tongling City, Anhui province, China, the TYR bridge is a long-

span cable-stayed bridge, whose cables intermittently suffer from VIV. Therefore, this research might help the bridge owner to detect VIV and send instant warnings.

One cable in the TYR Bridge, which is attached to an accelerometer named ACC-C01-01, is applied to explicate the whole process. The original data provided by SHM system is the time history of acceleration within an hour, whose file type is MATLAB's mat file. First of all, the one-hour continuous time history is divided into several pieces by a time window with a length of 10 minutes and a slide step of 5 minutes, and the RMS of each time interval is calculated. Then, the Fast Fourier Transform follows to secure the frequency spectrum of acceleration. Furthermore, both the HT and the derivative transform are performed respectively to get the HCH and HCD. Finally, all the indicators, i.e., the RMS, HCH, and HCD are considered together by KMeans and DBSCAN to classify VIV and normal vibration.

3.2. Sample acquisition and feature analysis

3.2.1. Data Processing and parameter setting

3.2.2. Statistical analysis to attest the equivalence of HCH and HCD

Fig. 4 indicates not only that the projections of the two analytical signals during VIV are distinct from that during non-VIV conditions, but that the two are almost identical when dealing with the same period of a signal. On the one hand, projections during the non-VIV are similar solid circles. On the other hand, those during the VIV are similar hollow rings with approximately equal inner radii and outer radii.

To further confirm the identity of the derivative transform and HT when recognizing VIV, Fig. 3 plots all the samples into an HCD-HCH coordinate system, where all the scatter points almost distribute in the line $HCD = HCH$, which means these two indicators offer the same information to detect the single modality. Therefore, it is reasonable to replace the complicated HT with the much more efficient derivative transform. Another matter that demands attention is that all the normal vibration samples have an almost zero-value HCD and an almost zero-value HCH, making themselves concentrated in the coordinate origin and covered by some VIV sample points.

3.3. Illustration of typical VIV and non-VIV

As mentioned above, each one-hour continuous time history is sliced by a ten-minute time window. It is noteworthy that since the slide step of this time window is five minutes, there is a five-minute overlapping region between each two contiguous time windows. As a result, each one-hour raw data offers eleven samples.

The discrepancy between VIV and normal vibration is remarkable. Fig. 4 shows the ten-minute time history, frequency spectrum, and projections of both the Hilbert analytical signal proposed by Dan, D. et al. [5] and the derivative analytical signal. It is noticeable that the RMS of VIV is much bigger than that in normal vibration. Moreover, the frequency spectrum of acceleration during VIV almost merely comprise one eigenfrequency, while that during normal condition contains multiple modal components. What's more noticeable, both the projections of the Hilbert analytical signal and derivative analytical signal display a hollow ring and solid circle in VIV and non-VIV circumstance, respectively. In a nutshell, the VIV manifests a strong attribution of large amplitude and approximately unimodal vibration.

3.4. Recognition of intense VIV on different cables

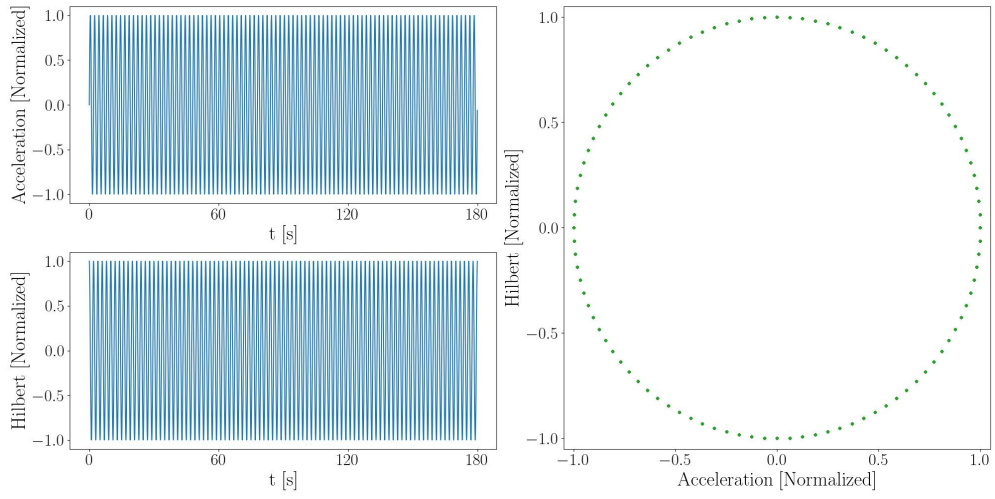
3.4.1. Cables with massive intense vibration samples

3.4.2. Cables with almost no intense vibration samples

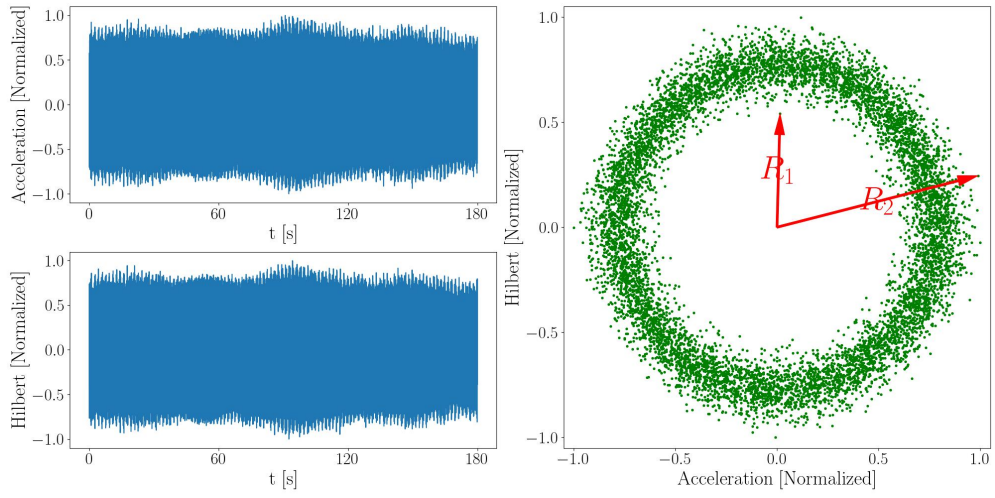
3.5. Parametric analysis

3.5.1. Order coefficient in the logarithm transformation

3.5.2. Thresholds for sample classification

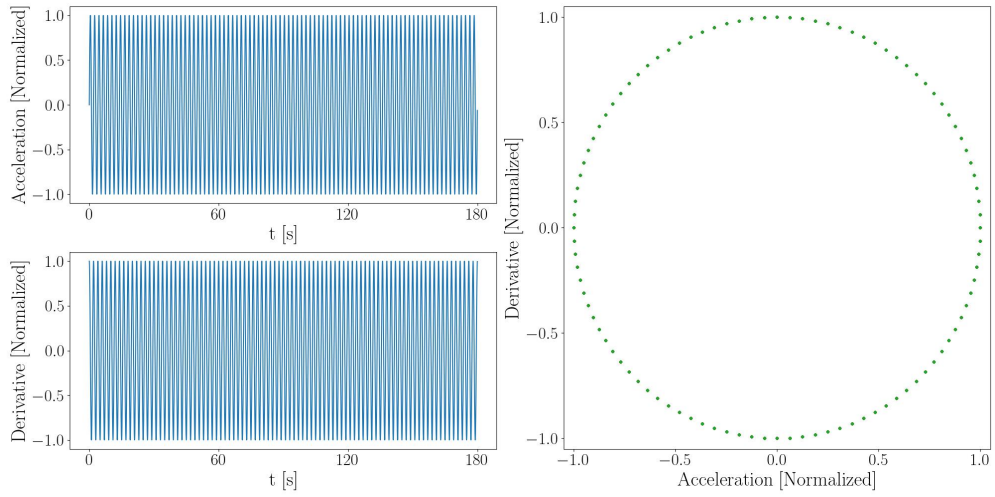


(a) Ideal sine function

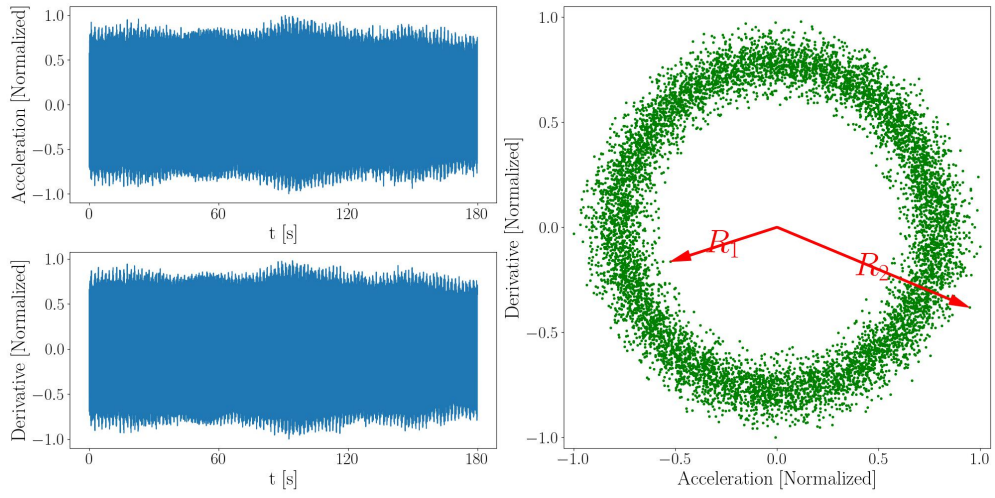


(b) sine function with noises

Fig. 1. Projection of the Hilbert analytical signal of a sine function in a complex plain.



(a) Ideal sine function



(b) sine function with noises

Fig. 2. Projection of the Derivative analytical signal of the sine function in a complex plain.

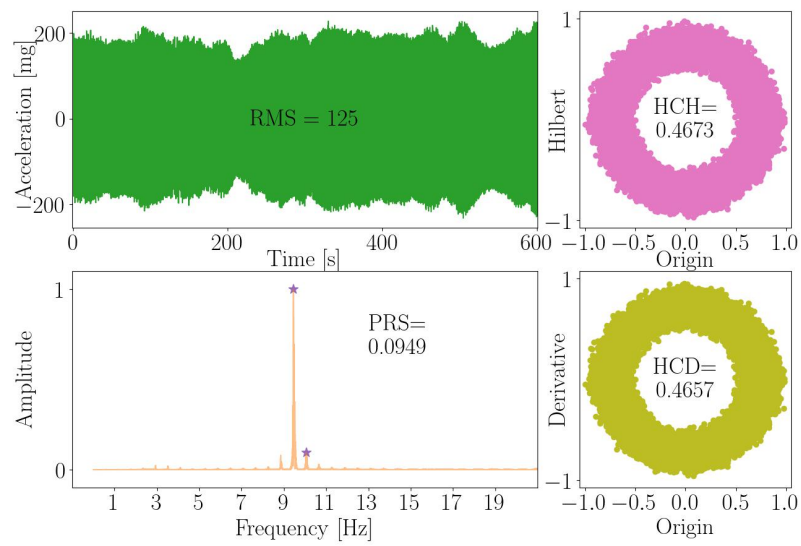
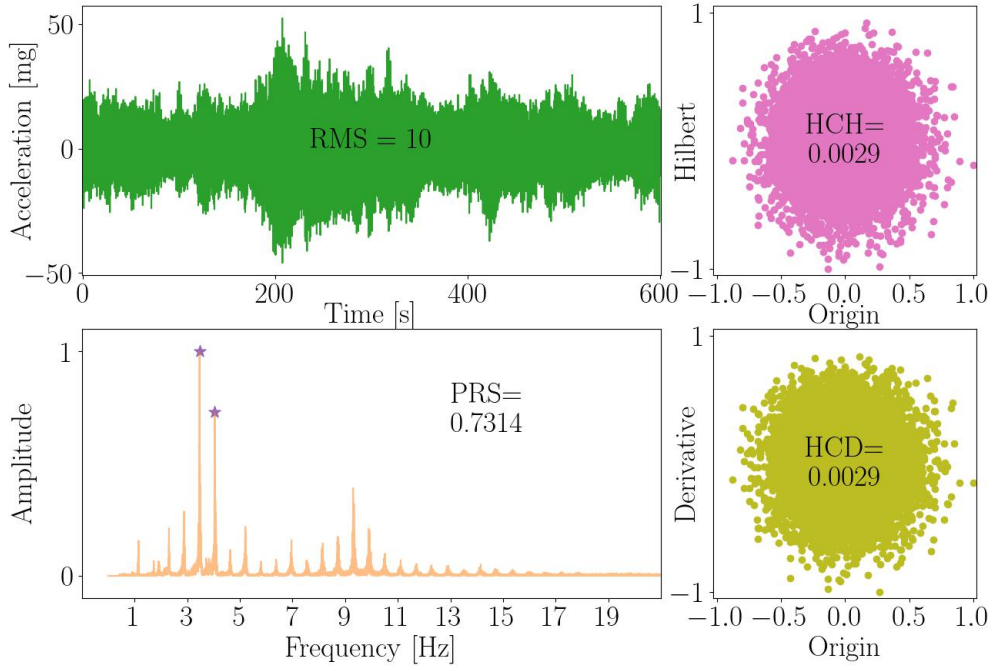
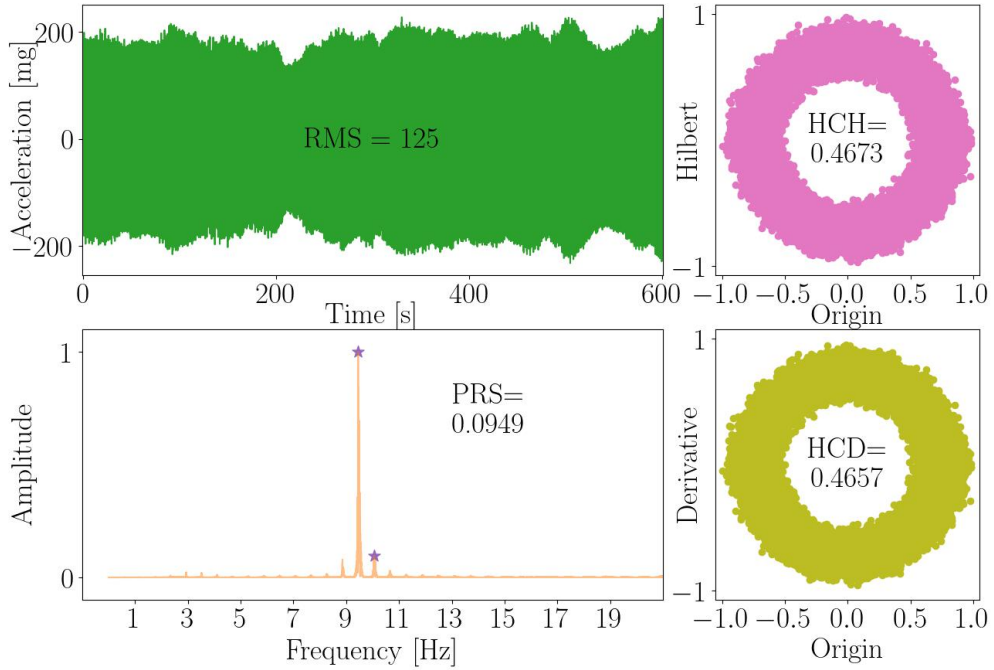


Fig. 3. Scatter diagram to compare HCH and HCD.



(a) Normal ambient vibration



(b) VIV

Fig. 4. Time history, frequency spectrum, projections of the two analytical signals in an hour.

4. Conclusions

To detect the occurrence of VIV in bridge cable, two key indicators, i.e., the RMS and HCH, is utilized to extract certain feature from acceleration time history. Drawn in the HCD-RMS coordinate system, the vibration sample points can be divided into two classes by two clustering algorithms automatically. The proposed methods make it possible to achieve real-time warning of VIV in the future and the following conclusions can be drawn.

- (1) RMS and HCH demonstrate high accuracy to quantify the intensity and mono-modality of a period of vibration. While offering nearly the same information compared with the HCH, the HCD is much more efficient since it is based on the derivative transform, whose time complexity is $O(n)$.
- (2) Probability-based classification is much more precise compared with that depending on the KMeans.
- (3) The method is based on past data and thus highly contingent on the specific environment.

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