CFA 一级知识框架图

Quantitative Analysis

专业来自101%的投入！

Framework

Study Session 2 Quantitative Methods(1)	R6 The Time Value of Money
	R7 Discounted Cash Flow Applications
	R8 Statistical Concepts and Market Returns
	R9 Probability Concepts
Study Session 3 Quantitative Methods(2)	R10 Common Probability Distributions
	R11 Sampling and Estimation
	R12 Hypothesis Testing
	R13 Technical Analysis

Reading 6

The Time Value of Money

Required Interest Rate on a Security	
概念	➢ Required rate of return ➢ Discount rate ➢ Opportunity cost
构成	➢ Nominal risk-free rate=real risk-free rate + expected inflation rate ➢ Required interest rate on a security=nominal risk-free rate + default risk premium + liquidity risk premium + maturity risk premium

EAR ★★	
计算	➢ $EAR = \left(1 + \frac{r}{m}\right)^{m} - 1$ ➢ Continuous compounding: $EAR = e^{r} - 1$
性质	➢ The more frequency of compounding, the larger the EAR; ➢ The largest EAR exists if it is continuously compounding.

Annuity	
计算	➢ Ordinary annuity ➢ Annuity due ● 计算器BGN模式 ● 从ordinary annuity推算 ➢ Perpetuity ● PV= A(PMT)/r

Reading 7

Discounted Cash Flow Applications

NPV&IRR

	NPV	IRR
计算 ★	$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_n}{(1+r)^n} = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t}$	$NPV = CF_0 + \frac{CF_1}{(1+IRR)^1} + \cdots + \frac{CF_n}{(1+IRR)^n} = \sum_{t=0}^{n} \frac{CF_t}{(1+IRR)^t} = 0$
性质	➢ Assume the project's cash flows will be reinvested at the cost of capital	➢ Assume the project's cash flows will be reinvested at the IRR ➢ Multiple or no solutions problem of the IRR calculation
应用	**Independent Projects**	**Mutually Exclusive Projects**
	➢ Accept it if NPV>0 ➢ Accept it if IRR>r (required rate of return)	➢ Choose the one with higher NPV ➢ Choose the one with higher IRR ➢ NPV和IRR 冲突，以NPV为准

TWRR & MWRR ★★

	TWRR	MWRR
概念	Measure the compound rate of growth	The IRR based on the cash flows related to the investment
计算	➢ Calculate HPRs of each year ➢ Calculate their geometric mean return	➢ Determine the timing of each CF ➢ Use financial calculator to compute IRR
性质	➢ 不会受到现金流流入流出的影响 ➢ 衡量现金流不可控的基金业绩更准确	➢ 会受到现金流影响 ➢ 衡量现金流可控的基金业绩更准确
	都是年化的收益率	

Convert among HPR, R_{BD}, EAR, R_{MM} and BEY ★

计算

- 折扣率 → $r_{BD} = \frac{F-P_0}{F} \times \frac{360}{t}$
- 收益率 → $HPY = \frac{P_1-P_0+D_1}{P_0}$

$$r_{MM} = HPY \times \frac{360}{t}$$

$$EAY = (1+HPY)^{\frac{365}{t}} - 1 \longrightarrow \left(1+\frac{BEY}{2}\right)^2 = 1+EAY$$

Reading 8

Statistical Concepts and Market Return

Measurement scales, fundamental concepts of statistics

Types of Measurement Scales ★

	Nominal Scales	Ordinal Scales	Interval Scales	Ratio Scales
概念	分类	排序 (>,<)	(>,<,+, -)	(>,<,+ ,-,*,/)
性质	只能求mode	mode、median	没有绝对零点	Most refined

Parameter & Sample Statistic

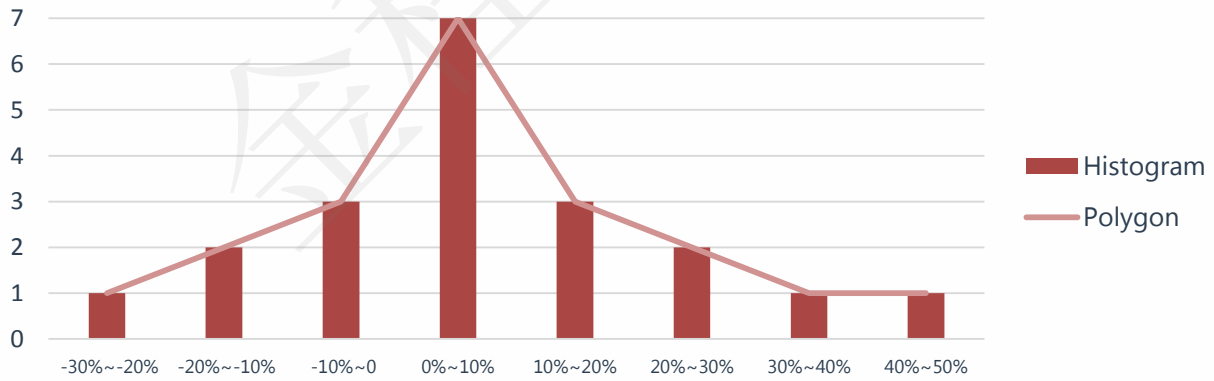
	Parameter	Sample Statistic
概念	A measure used to describe a characteristic of a population is referred to as a parameter.	A sample statistic is used to measure a characteristic of a sample.

Measurement scales, fundamental concepts of statistics

概念

Interval Relative	Absolute Frequency	Relative Frequency	Cumulative Absolute Frequency	Cumulative Relative Frequency
-10 - -5	3	0.97%	3	0.97%
-5 - 0	35	11.29%	38	12.26%
0 - 5	176	56.77%	214	69.03%
5 - 10	74	23.87%	288	92.90%
10 - 15	22	7.10%	310	100%
Total	**310**	**100%**		

图像



Measures of central tendency, quantiles ★

	Mean	Quantiles
概念	(Weighted) Arithmetic mean/ Geometric mean/ Harmonic mean	Quartile/ Quintile/ Deciles/ Percentile
性质	Harmonic≤ Geometric≤ Arithmetic	e.g. The third quartile> Mean
应用	➢ 预测未来业绩用arithmetic mean ➢ 衡量历史业绩 or 关注ending value用geometric mean	各分位数可先转换为percentile后确定 $L_y=(n+1)y/100$

MAD, variance & standard deviation ★

	MAD	Variance & Standard Deviation	
计算	$MAD=\frac{\sum_{i=1}^{n}\lvert X_i-\bar{X}\rvert}{n}$	For population	For sample
		$\sigma^2=\frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}$	$S^2=\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$

Chebyshev's inequality, CV & Sharpe ratio ★★

Chebyshev's inequality

概念

- Regardless of the shape of the distribution
- $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2}$

	CV	Sharpe Ratio
公式	Standard deviation (risk) per unit of sample mean $$CV = \frac{S_X}{\bar{X}}$$	Excess return per unit of risk $$Sharpe\ ratio = \frac{R_P - R_f}{\sigma_P}$$
性质	➢ Scale-free ➢ Relative dispersion	➢ 越大越好

Skewness, kurtosis

Skewness (掌握性质) ★★	**Positive skewed**	➢ Right fat tail ➢ Mode<median<mean ➢ Frequent small losses and a few extreme gains (mean=0时) ➢ prefer positive skewness	**Negative skewed** ➢ Left fat tail ➢ Mode>median> mean ➢ Frequent small gains and a few extreme losses (mean=0时)
Kurtosis (掌握性质) ★	**Leptokurtic**	➢ Sample kurtosis>3, Excess kurtosis>0 ➢ 尖峰肥尾: more frequent extremely large deviations from the mean than a normal distribution ● 假设与normal distribution有相同的离散程度 ● Investors dislike this distribution	
	Platykurtic	Sample kurtosis<3, Excess kurtosis<0	
	Normal	Sample kurtosis=3, Excess kurtosis=0	

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Probability Concepts

Basic concepts of probability, odds for ★

Basic Concepts of Probability	概念	➢ Event ● Mutually exclusive events — can not both happen at the same time. ● Exhaustive events — include all possible outcomes.	
	性质	➢ $0 \le P(E) \le 1$ ➢ $P(E_1)+P(E_2)+\ldots\ldots+P(E_n)=1$ ● $E_1\ldots\ldots E_n$: mutually exclusive and exhaustive	
Classification of the Probability	概念	➢ Empirical probability: 分析过去，得到未来 ➢ Priori probability: 分析过去，得到过去 ➢ Subjective probability: 主观	
Odds for	计算	**Odds for an event**	**Odds against an event**
		P(E)/(1-P(E))	(1-P(E))/P(E)

★★ Calculation rules for probabilities

概念：两个事件	**Mutually Exclusive**	P(AB)=P(A|B)=P(B|A)=0
	Independent	➢ P(AB)=P(A) ×P(B) ➢ If exclusive , must not independence ➢ Independence →ρ=0, 反之不成立
计算：两个法则	**Multiplication Rule**	P(AB)=P(A|B)×P(B)=P(B|A)×P(A)
	Addition Rule	P(A or B)=P(A)+P(B)-P(AB)

Covariance & Correlation ★★

	Covariance	Correlation
计算	Cov(X, Y)= E[(X-E(X))(Y-E(Y))]	$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$
性质	➢ How one random variable moves with another random variable ➢ The covariance of X with itself is equal to the variance of X ➢ Covariance ranges from negative infinity to positive infinity	➢ Correlation measures the linear relationship between two random variables ➢ Correlation has no units, ranges from -1 to +1,standardization of covariance ➢ If ρ=0,this doesn't indicates independence

Expected Value, Variance and Standard Deviation

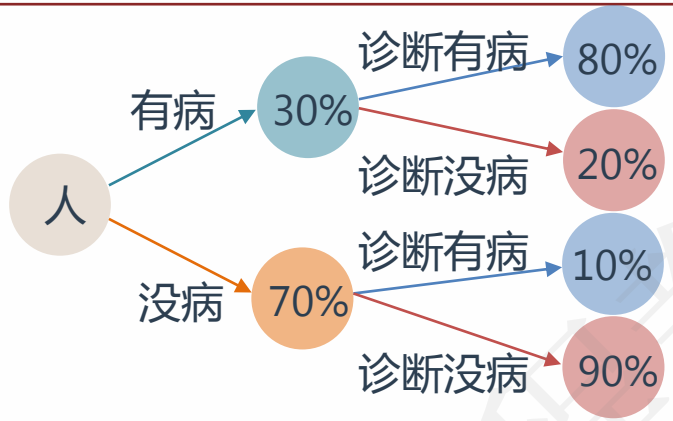
计算	Expected Return	$E(R_P) = \sum_{i=1}^{n} \omega_i E(R_i)$ $\sum_{i=1}^{n} \omega_i = 1$	
	Variance	两个资产做组合 $\sigma_p^2 = \omega_1^2\sigma_1^2+\omega_2^2\sigma_2^2+2\omega_1\omega_2\sigma_1\sigma_2\rho_{1,2}$	
性质	Variance	两个资产做组合	n个资产做组合
		➢ $\rho = 1$, $\sigma_p = \omega_1\sigma_1+\omega_2\sigma_2$, σ_p^2最大 ➢ $\rho = -1$, $\sigma_p = \lvert\omega_1\sigma_1 - \omega_2\sigma_2\rvert$, σ_p^2最小, ➢ ρ 减小, σ_p^2减小	➢ σ_p^2 的影响因素 ➢ n增加, $Cov_{i,j}$ 对σ_p^2影响更大

Bayes' Formula (计算，用二叉树图形，不用记公式)

计算

$$P(A|B) = \frac{P(B|A)}{P(B)} \times P(A)$$

图形 — 二叉树



注意：把非条件概率画在第一支

Counting Problems（了解）

计算

Factorial	Combination	Permutation
n!	nCr（用计算器算）	nPr （用计算器算）

Multiplication Rule	Labeling (or Multinomial)
$n_1 \times n_2 \times \cdots \times n_k$	$\frac{n!}{n_1! \times n_2! \times \cdots \times n_k!}$

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Common Probability Distributions

Probability Distributions		
	Discrete	**Continuous**
概念	The number of outcomes is counted.	The number of outcomes is infinite.
性质	Measurable and positive	P(x)=0 even though x can occur
注意：Cumulative probability function F(x)=P(X≤x)		

Uniform and binominal random variable

	Discrete Uniform	Continuous Uniform
Uniform Random Variables	A discrete uniform random variable is one for which the probabilities for all possible outcomes for a discrete random variable are equal.	It is defined over a range that spans between some lower limit, a, and upper limit, b, which serve as the parameters of the distribution.
	例：X={1, 2, 3, 4}, P(X=1)= P(X=2) = P(X=3)= P(X=4)= 1/4	➢ $P(X<a \text{ or } X>b) = 0$ ➢ $P(x_1 \le X \le x_2)=(x_2-x_1)/(b-a)$
Binomial Random Variables ★★	**Bernoulli Random Variable**	**Binomial Random Variable**
	做1次实验，只有两个结果	做n次实验，每次实验只有两个结果
	➢ P(X=1)=p , P(X=0)=1-p ➢ Expectation =p ➢ Variance=p(1-p)	➢ $P(x)=P(X=x)={}_nC_x p^x(1-p)^{n-x}$ ➢ Expectation =np ➢ Variance = np(1-p)

Normal distribution, the confidence intervals ★★

Key Properties of Normal Distribution	➢ Symmetrical distribution: skewness=0, kurtosis = 3 ➢ A linear combination of normally distributed random variables is also normally distributed ➢ 取值区间: (-∞,+∞)
The Confidence Intervals	**Normal Distribution**
	➢ 68% confidence interval: [μ-σ, μ+σ] ➢ 90% confidence interval: [μ-1.65σ, μ+1.65σ] ➢ 95% confidence interval: [μ-1.96σ, μ+1.96σ] ➢ 99% confidence interval: [μ-2.58σ, μ+2.58σ]

Standard Normal Distribution ★★	
概念	The normal distribution with $\mu = 0$ and $\sigma^2 = 1$, $Z \sim N(0,1)$
性质	➢ Standardization: If $X \sim N(\mu,\sigma^2)$, then $Z = (X-\mu)/\sigma \sim N(0,1)$ ➢ $P(Z>z) = 1 - F(z) = F(-z)$

Roy's Safety-First Criterion ★★	
概念	➢ **Threshold level return:** minimum return required, R_L ➢ **Shortfall risk:** $P(R_p < R_L)$
计算	**Safety-first Ratio** = $SFR = [E(R_p) - R_L]/\sigma_p$
性质	Maximize SFR ↔ Minimize $P(R_p < R_L)$

Lognormal Distributions ★★	
概念	If lnX is normal, then X is lognormal
性质	➢ Right skewed; ➢ x > 0
应用	Lognormal → the price of asset; Normal → the return of asset

Monte Carlo Simulation and Historical Simulation (了解)		
	Monte Carlo Simulation	**Historical Simulation**
概念	Based on its assumed distributions, to produce a distribution of possible security values	Selected historical data to generate a distribution
性质	➢ It is fairly complex and will assume a parameter distribution ➢ It is not an analytic method but a statistical one	➢ The past can't indicate the future ➢ It cannot address the sort of "what if" questions that Monte Carlo simulation can

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Sampling and Estimation

Sampling, Time-Series and Cross-Sectional Data		
Sampling		
概念 — 抽样方法	➢ Simple random sampling / Systematic sampling ➢ Stratified random sampling: divide the population into smaller groups	
计算	Sampling error of the mean= sample mean- population mean	
性质	The sample statistic itself is a random variable and has a probability distribution.	
Time-Series and Cross-Sectional Data		
概念	**Time-Series Data**	**Cross-Sectional Data**
	Data taken over a period of time	Data taken at a single point of time

Central Limit Theorem	
概念	n≥30且总体均值、方差已知 → Sample mean~ N(μ,σ^2/n)
计算	Standard error = $\sigma/\sqrt{n}$ or $s/\sqrt{n}$

Desirable Properties of an Estimator ★★		
概念	**Unbiased**	Expected value of the estimator is equal to the parameter that are trying to estimate
	Efficient	The unbiased estimator has the smallest variance
	Consistent	The accuracy increases as sample size increases

Biases	
概念	➢ Data-mining bias ➢ Sample selection bias ➢ Survivorship bias ➢ Look-ahead bias ➢ Time-period bias

Point Estimate and a Confidence Interval Estimate ★★	
概念	➢ Point Estimate（点估计） ➢ Confidence interval estimate（区间估计） ● Level of significance (α) ● Degree of Confidence (1－α)
计算	➢ 区间估计: $\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ or $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$

Student's t-distribution and Degrees of Freedom ★★	
性质	➢ Degrees of freedom (df): n-1 ➢ Symmetrical: skewness = 0 ➢ Less peaked than a normal distribution ("fatter tails"), kurtosis< 3 ➢ As the degrees of freedom gets larger, the shape of t-distribution approaches standard normal distribution ➢ 相同的significance level下，t分布对应的confidence level更宽

Confidence Interval for a Population Mean

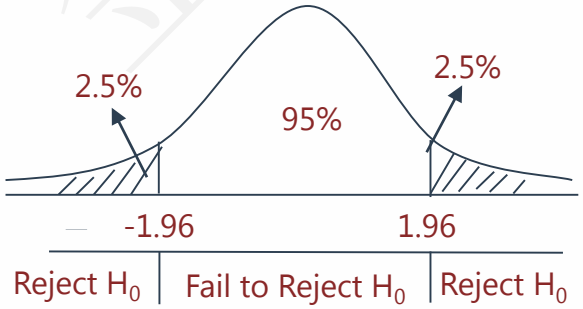
性质	选择哪一个分布？	➢ 方差已知用z, 方差未知用t, 非正态总体小样本不可估计 ➢ 如果n≥30, 都可以用z	
应用	When sampling from a	**Test Statistic**	
		Small Sample (n<30)	**Large Sample (n≥30)**
	Normal distribution with known variance	z-statistic	z-statistic
	Normal distribution with unknown variance	t-statistic	t/z-statistic
	Nonnormal distribution with known variance	not available	z-statistic
	Nonnormal distribution with unknown variance	not available	t/z-statistic

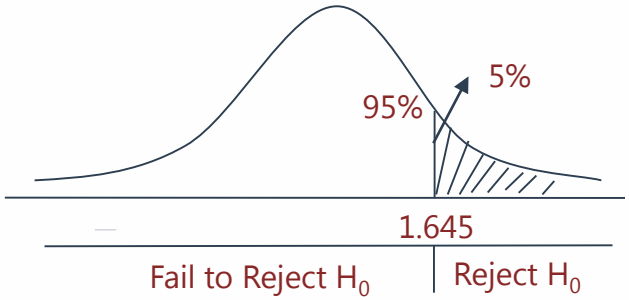
Reading 12

Hypothesis Testing

Introduction to the Steps of Hypothesis Testing

Define Hypothesis	➢ Null hypothesis (H_0) ● Two-tailed: H_0: $\mu=\mu_0$, H_a: $\mu\neq\mu_0$ ● One-tailed: H_0: $\mu\leq\mu_0$, H_a: $\mu>\mu_0$ or H_0: $\mu\geq\mu_0$, H_a: $\mu<\mu_0$
Identify Test Statistic	Test Statistic is calculated with the sample date.
Find Critical Value	➢ Given one or two tailed assumption, critical value is determined solely by the α. ➢ Critical value is found in the table.
Formulate a Decision Rule	➢ Critical value method (Find Reject region) ● Reject H_0, if |test statistic|> critical value ● Fail to reject H_0, if |test statistic|< critical value





Test Population Mean

	Population Distribution	Population Variance	H_0	Test Statistic	Critical Value
Single Mean	Normal or Sample size is large	Known	$\mu=\mu_0$	$z=\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$	N(0,1)
		Unknown	$\mu=\mu_0$	$t=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$	t(n-1)
Mean Differences	Normal, Independent	Unknown& Assumed equal	$\mu_1-\mu_2=0$	t	t
		Unknown& Not assumed equal	$\mu_1-\mu_2=0$	t	t
	Normal, Dependent (paired comparison test)	Unknown	$\mu_d=0$	$t=\bar{d}/s_{\bar{d}}$	t(n-1)

Test Population Mean (Unknown Variance)		
Population Distribution	**n<30**	**n≥30**
Normal	t-test	t/z-test
Non-normal	Not available	t/z-test

Test Population Variance				
	Population Distribution	**H_0**	**Test Statistic**	**Critical Value**
Single Variance	Normal	$\sigma^2=\sigma_0^2$	$\chi^2=\frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
Equality of Two Variances	Normal, Independent	$\sigma_1^2=\sigma_2^2$	$F=S_1^2/S_2^2$	$F(n_1-1,n_2-1)$

P-Value	
概念	The p-value is the smallest level of significance at which the null hypothesis can be rejected.
性质	P-value ↓, easier to reject H_0
应用	➢ P-value< α: reject H_0 ➢ P-value> α: do not reject H_0

Type I Error and Type II Error			
概念		**H_0 is true**	**H_0 is false**
	Do not reject H_0	Correct decision	Incorrect Decision Type II error
	Reject H_0	Incorrect decision Significance level= P(Type I error)	Correct decision Power of test=1- P(Type II error)
性质	➢ Type I error ↑ → Type II error ↓,with other conditions unchanged. ➢ Increase the Sample Size → Type I error & Type II error ↓		

Parametric and Nonparametric Tests（了解）		
概念	**Parametric tests**	Specific to population parameters
	Nonparametric tests	Nonparametric tests are used: ➢ The assumptions that support a parametric test are not met ➢ When data are given in ranks (ordinal measurement scale) rather than values ➢ The hypothesis does not involve the parameters of the distribution

Reading 13

Technical Analysis

Applications of Technical Analysis, Underlying Assumptions （特点）

原理	➢ Prices are determined by the interaction of supply and demand; ➢ Only participants who actually trade affect prices; ➢ Price and volume reflect the collective behavior of buyers and sellers.
假设	➢ Investor behavior is reflected in trends& patterns that tend to repeat; ➢ Efficient markets hypothesis does not hold.
区别	**Fundamentalists:** Prices react quickly to changing stock values; **Technicians:** The reaction is slow.
优缺点	➢ **Advantages** ● Actual price and volume data are observable; ● Technical analysis itself is objective, while much of the data used in fundamental analysis is subject to assumptions or restatements; ● It can be applied to the prices of assets that do not produce future cash flows, such as commodities; ● It doesn't have the risk of financial statement fraud. ➢ **Disadvantage** ● Illiquid markets 不能用 ● Markets that are subject to large outside manipulation 不能用

<table>
<tr><th colspan="4">Types of Charts, Uses of Trend</th></tr>
<tr><td colspan="4">Types of Charts</td></tr>
<tr><td>概念</td><td colspan="3">➢ Line charts
➢ Bar charts
➢ Candlestick charts
➢ Point and figure charts</td></tr>
<tr><td colspan="4">Uses of Trend</td></tr>
<tr><td rowspan="4">概念</td><td rowspan="2">Trend</td><td>Uptrend</td><td>Downtrend</td></tr>
<tr><td>Prices are consistently reaching higher highs and retracting to higher lows. (Demand>Supply)</td><td>Prices are consistently reaching lower lows and retracting to lower highs. (Demand<Supply)</td></tr>
<tr><td rowspan="2">Trend Line</td><td>Uptrend Line</td><td>Downtrend Line</td></tr>
<tr><td>Connect the increasing lows in prices</td><td>Connect the decreasing highs in prices</td></tr>
</table>

Common Chart Patterns

概念 ★		For Uptrend	For Downtrend
	Reversal Patterns	➢ Head-and-shoulders pattern ➢ Double top ➢ Triple top	➢ Inverse head-and-shoulders pattern ➢ Double bottom ➢ Triple bottom
	Continuation Patterns	➢ Triangles ➢ Rectangles ➢ Flags and pennants	

Technical Analysis Indicators

概念 ★

Price-based	Momentum Oscillators
➢ Moving average lines ➢ Bollinger bands	➢ Rate of change oscillator ➢ Relative strength index (RSI) ➢ Moving average convergence/ divergence (MACD) ➢ Stochastic oscillator
Sentiment	**Flow of Funds**
➢ Put/call ratio ➢ Volatility index (VIX) ➢ Margin debt ➢ Short interest ratio	➢ Short-term trading index ➢ Margin debt ➢ Mutual fund cash position ➢ New equity issuance

Technical Analysis Theory

概念	**Cycle Theory**	➢ 4-year presidential cycles: related to election years in the USA ➢ Decennial patterns: 10-year cycles ➢ Kondratieff wave: 18-year cycles, 54-year cycles
	Elliott wave Theory	➢ Based on the belief that financial market prices can be described by an interconnected sets of cycles ● Waves: chart patterns related to Elliott wave theory ● Fibonacci ratios: the sizes of these waves are thought to correspond with Fibonacci ratios
	Intermarket Analysis	➢ Analysis of the interrelationships among the market values of major asset classes, such as stocks, bonds, commodities and currencies ➢ Also useful for comparing the relative performance of equity market sectors or industries and of various international market