Assignment 2: ARMA Processes and Seasonal Processes

Instructions: The assignment is to be handed in via DTU Learn "FeedbackFruits" latest at March 24th at 23:59. You are allowed to hand in in groups of 1 to 4 persons. You must hand in a single pdf file presenting the results using text, math, tables and plots, do not include code in the report!. Arrange the report in sections and subsections according to the questions in this document. Please indicate your student numbers on the report.

Be aware that the model parametrizations in the assignment and in your favorite software package may not be identical. You will study how the choice of coefficients (of the operator polynomials in ARMA processes) affects the structure of the process, through simulated data and empirical autocorrelation functions (ACFs). The assignment is to some extent specified in terms of R commands. Users of other software packages should find similar functions to answer the questions.

1 Stability

Let the process $\{X_t\}$ be an AR(2) given by

$$X_t + \phi_1 X_{t-1} + \phi_2 X_{t-2} = \epsilon_t$$

where $\{\epsilon_t\}$ is a white noise process with $\sigma_\varepsilon = 1$.

Answer the following:

1.1. Determine if the process is stationary for $\phi_1 = -0.7$ and $\phi_2 = -0.2$ by analysing the roots of the characteristic equation.

1.2. Is the process invertible?

1.3. Write the autocorrelation $\rho(k)$ for the AR(2) process as function of ϕ_1 and ϕ_2.

1.4. Plot the autocorrelation $\rho(k)$ up to $n_{\text{lag}} = 30$ for the coefficient values above.

2 Simulating seasonal processes

A process $\{Y_t\}$ is said to follow a multiplicative $(p, d, q) \times (P, D, Q)_s$ seasonal model if

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)\epsilon_t$$

where $\{\epsilon_t\}$ is a white noise process, and $\phi(B)$ and $\theta(B)$ are polynomials of order p and q, respectively. Furthermore, $\Phi(B^s)$ and $\Theta(B^s)$ are polynomials in B^s. All according to Definition 5.22 in the textbook.

Note: `arima.sim` does not have a seasonal module, so model formulations as standard ARIMA processes have to be made when using that function.

Simulate the following models. Plot the simulations and the associated autocorrelation functions (ACF and PACF). Comment on each result:

2.1. A $(1, 0, 0) \times (0, 0, 0)_{12}$ model with the parameter $\phi_1 = 0.6$.

2.2. A $(0, 0, 0) \times (1, 0, 0)_{12}$ model with the parameter $\Phi_1 = -0.9$.

2.3. A $(1, 0, 0) \times (0, 0, 1)_{12}$ model with the parameters $\phi_1 = 0.9$ and $\Theta_1 = -0.7$.

2.4. A $(1, 0, 0) \times (1, 0, 0)_{12}$ model with the parameters $\phi_1 = -0.6$ and $\Phi_1 = -0.8$.

2.5. A $(0, 0, 1) \times (0, 0, 1)_{12}$ model with the parameters $\theta_1 = 0.4$ and $\Theta_1 = -0.8$.

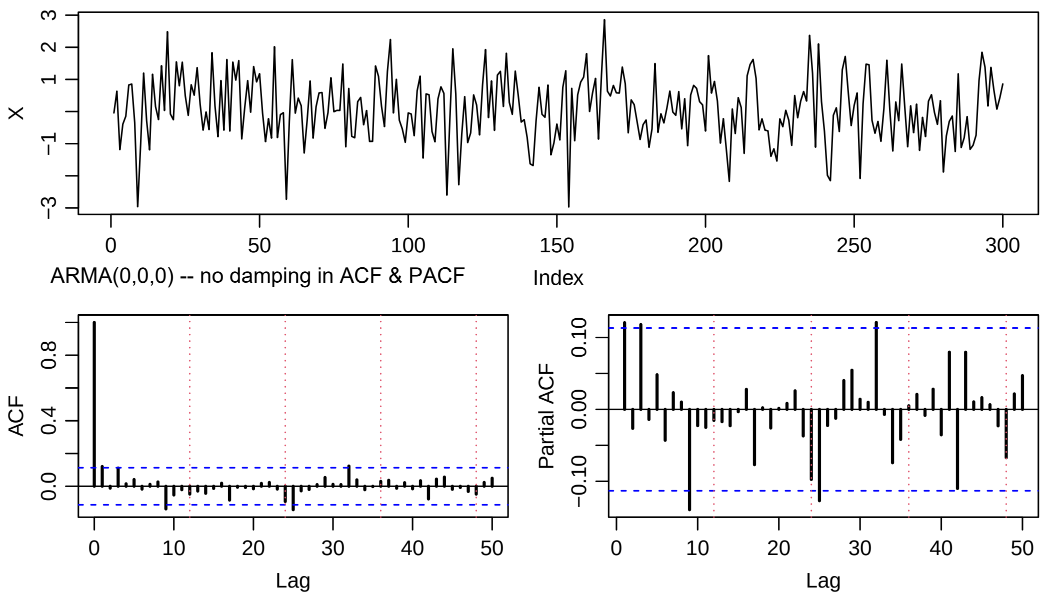
2.6. A $(0, 0, 1) \times (1, 0, 0)_{12}$ model with the parameters $\theta_1 = -0.4$ and $\Phi_1 = 0.7$.

2.7. Summarize your observations on the processes and the autocorrelation functions. Which conclusions can you draw on the general behavior of the autocorrelation function for seasonal processes?

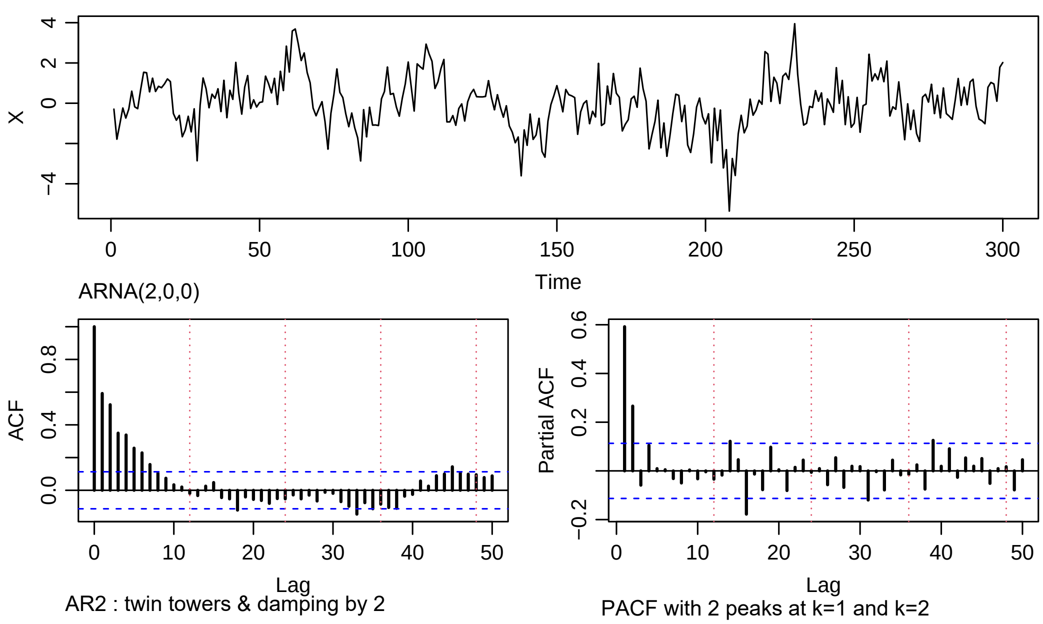
3 Identifying arma models

Below are plots of three simulated ARMA processes: time series plot, ACF and PACF. Guess the ARMA model structure for each of them, and give a short reasoning of your guess.

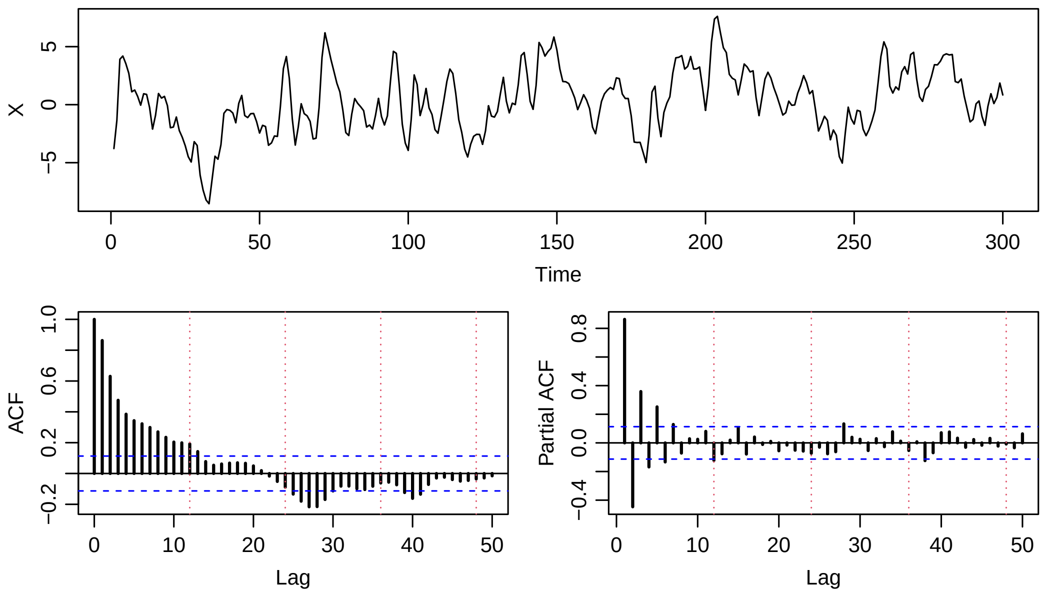
3.1. Process 1:



3.2. Process 2:



3.3. Process 3:



Steps:
- start with simple model
- plot ACF & PACF of residuals
- increase complexity