Assignment 2

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1 Stability

1.1 Determine if the process is stationary

It is stable if the root of $\phi(z^{-1}) = 1 + \phi_1 z^{-1} + \phi_2 z^{-2}$ all lie within the unit circle. let's analyze this polynomial:

$$1 - 0.7/z - 0.2/z^2 = 0 \tag{1}$$
$$z^2 - 0.2z - 0.7 = 0 \tag{2}$$

thus: $\Delta = 0.2^2 - 4 * -0.7 = 2.8 + 0.04 = 2.84$ we find the root using this $r_1 = (1 - \sqrt{2.84})/2)$ and $r_2 = (1 + \sqrt{2.84})/2)$ after using a calc we find: that one of them is superior to 1, thus it is not a stationary process.

1.2 Is process invertible

An Arma process is always reversible.

1.3 Write autocorrelation

Using luke walker equation:

$$\begin{cases} \rho(1) + \phi_1 + \rho(1)\phi_2 = 0 \\ \rho(2) + \rho(1)\phi_1 + \phi_2 = 0 \end{cases} \tag{3}$$
$$\begin{cases} \rho(1) = -\phi_1/(1 + \phi_2) \\ \rho(2) = -\phi_2 - \rho(1)\phi_1 = -\phi_2 + \phi_1^2/(1 + \phi_2) = (\phi_1^2 - \phi_2 - \phi_2^2)/(1 + \phi_2) \end{cases} \tag{4}$$

and finally we know: $\rho(k) = -\phi_1\rho(k-1) - \phi_2\rho(k-2)$
Comment: This has a solution independent of the previous ρ values (need to solve for alpha and lambda using $\rho(1)$ and $\rho(2)$):

$$\rho(k) = r_1^k * \alpha + r_2^k * \lambda$$

1.4 Plot autocorrelation

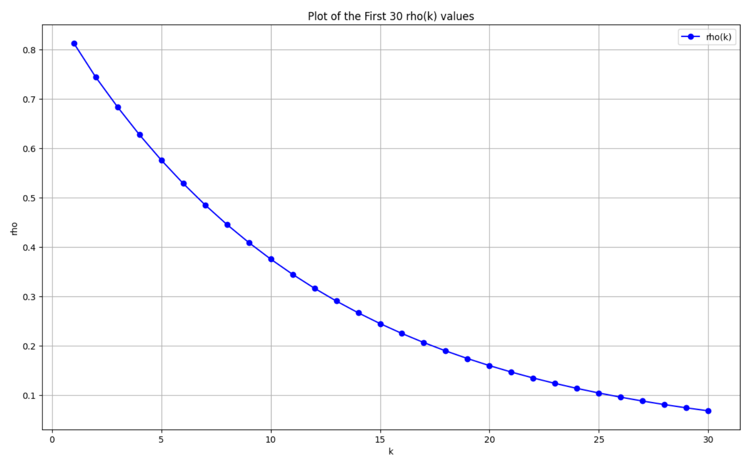
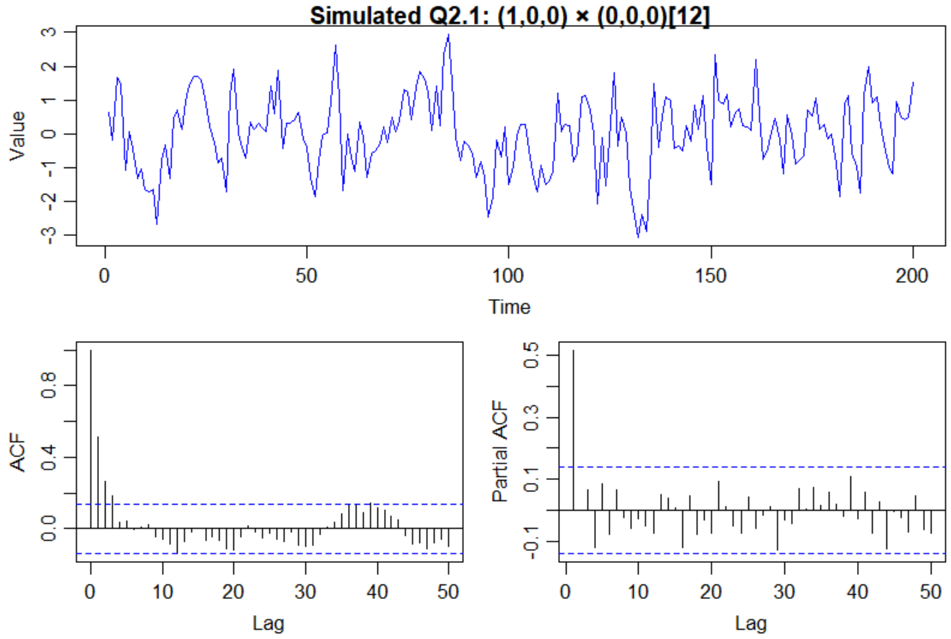


Figure 1: $\rho(k)$ plot

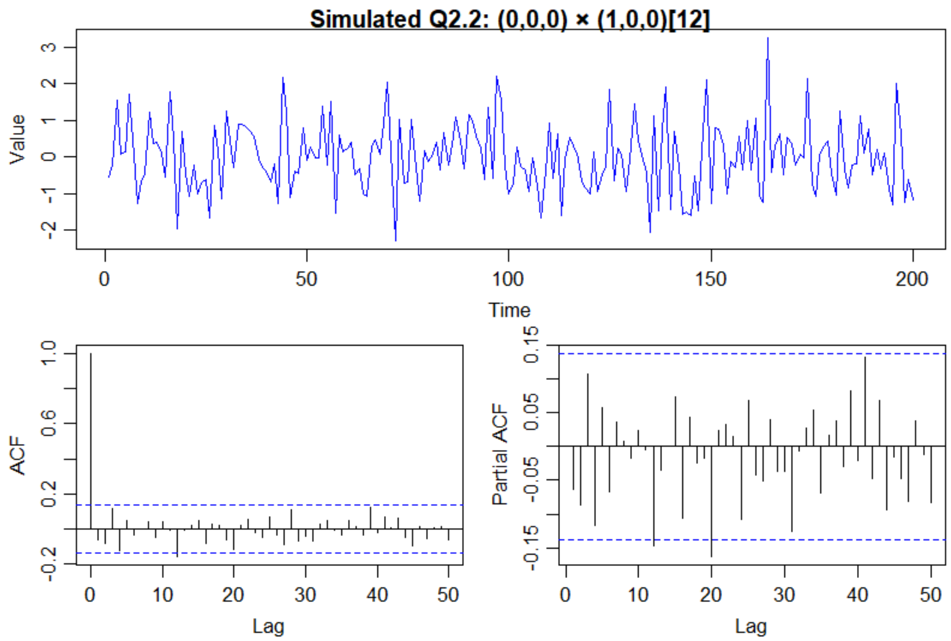
2 Simulating seasonal processes

2.1 $(1,0,0) \times (0,0,0)_{12}$ with $\phi_1 = 0.6$



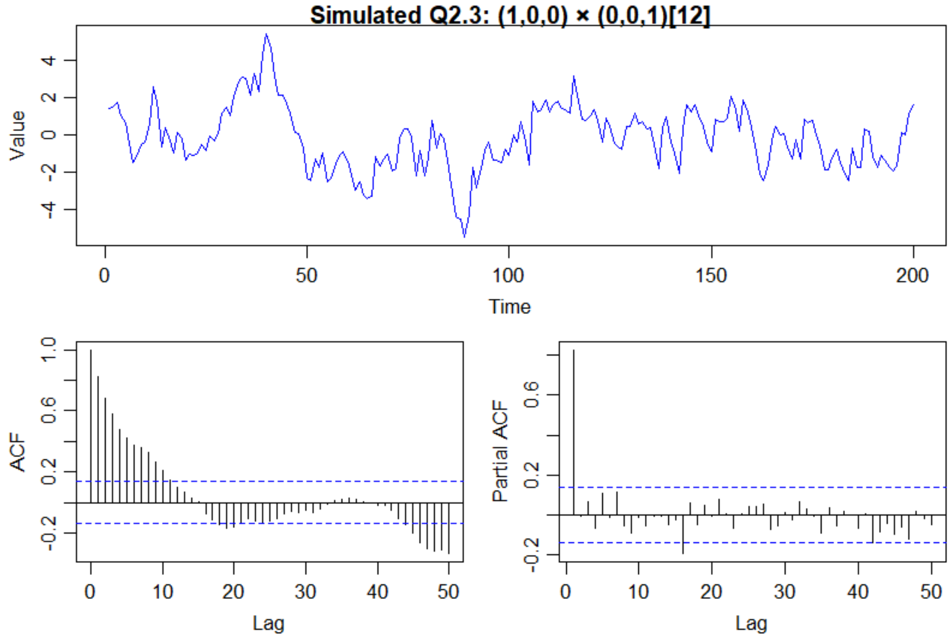
- ACF shows a significant positive spike at lag 1 due to the AR(1) component. This spike gradually decays exponentially towards zero in subsequent lags. The rate of decay depends on the value of ϕ_1; since $\phi_1 = 0.6$, the decay will be moderate.
- PACF shows a significant spike at lag 1, reflecting the AR(1) component. After lag 1, PACF drops to zero, indicating that the AR(1) process is fully captured by the first lag.

2.2 $(0,0,0) \times (1,0,0)_{12}$ with $\Phi_1 = -0.9$



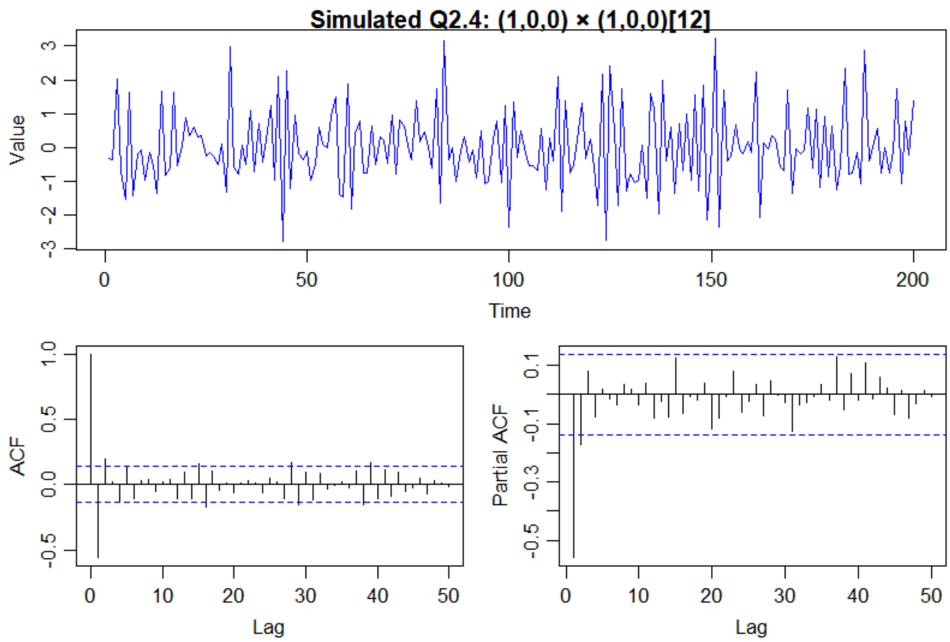
- The seasonal AR(1) process exhibits oscillations at a 12-period interval, with strong negative correlation causing alternating peaks and troughs.
- ACF shows a significant negative spike, confirming the seasonal structure.

2.3 $(1,0,0)\times(0,0,1)_{12}$ with $\phi_1 = 0.9$ and $\Theta_1 = -0.7$



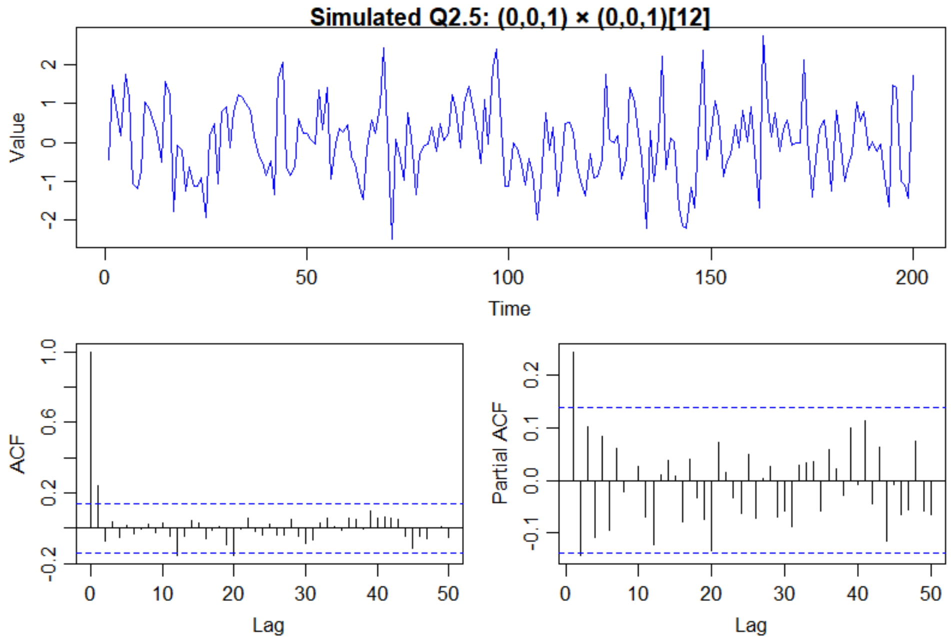
- ACF shows a slow decay due to the AR(1) component, with a sine function suggesting seasonal MA.
- PACF shows a significant spike at lag 1 (from AR(1)), with limited influence at seasonal lags.

2.4 $(1,0,0)\times(1,0,0)_{12}$ with $\phi_1 = -0.6$ and $\Phi_1 = -0.8$



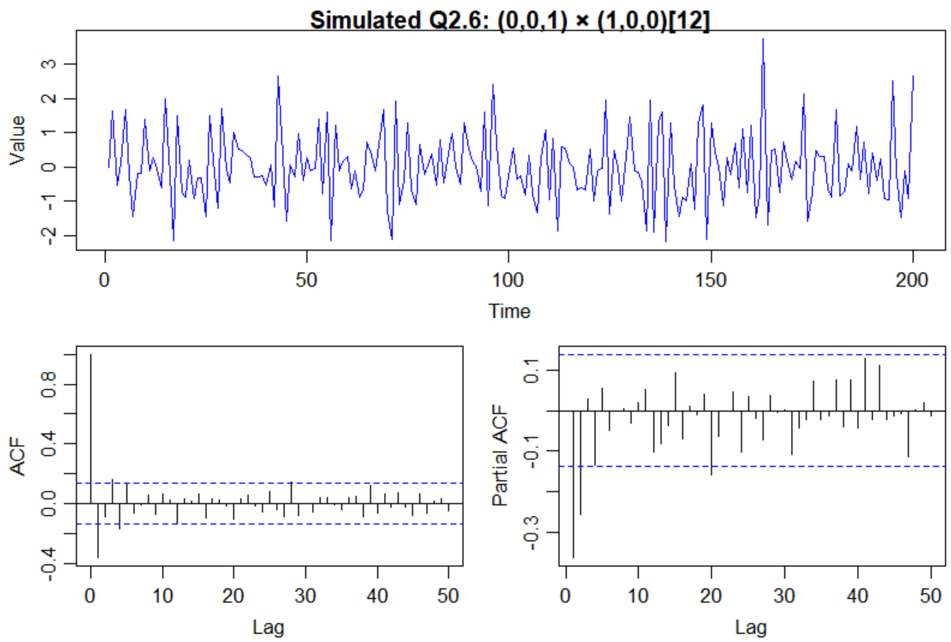
- The process exhibits both short-term and seasonal dependencies, with oscillations caused by the seasonal AR(1) term.
- ACF shows a mix of gradual decay from the AR(1) component and periodic oscillations due to the seasonal AR(1).
- PACF has a significant negative spike at lag 1 (from AR(1)) and a smaller decay at seasonal lags, confirming the influence of both short-term and seasonal AR effects.

2.5 $(0, 0, 1) \times (0, 0, 1)_{12}$ with $\theta_1 = 0.4$ and $\Theta_1 = -0.8$



- ACF shows a significant spike at lag 1 due to the MA(1) component. This spike gradually decays to zero in subsequent lags.
- PACF shows a significant spike at lag 1, similar to the ACF, but it will drop to zero immediately after lag 1

2.6 $(0, 0, 1) \times (1, 0, 0)_{12}$ with $\theta_1 = -0.4$ and $\Phi_1 = 0.7$



- ACF shows a significant negative spike at lag 1 due to the MA(1) component. This spike will gradually decay to zero in subsequent lags.
- PACF shows a significant spike at lag 1, similar to the ACF, but it will drop to zero immediately after lag 1.

2.7 Summary

Impact of AR and MA Components:

- AR terms ϕ cause slow decay in the ACF.
- MA terms θ create a sharp cutoff in the ACF.
- Higher values of AR and MA parameters lead to stronger autocorrelation.

Pattern in ACF and PACF:

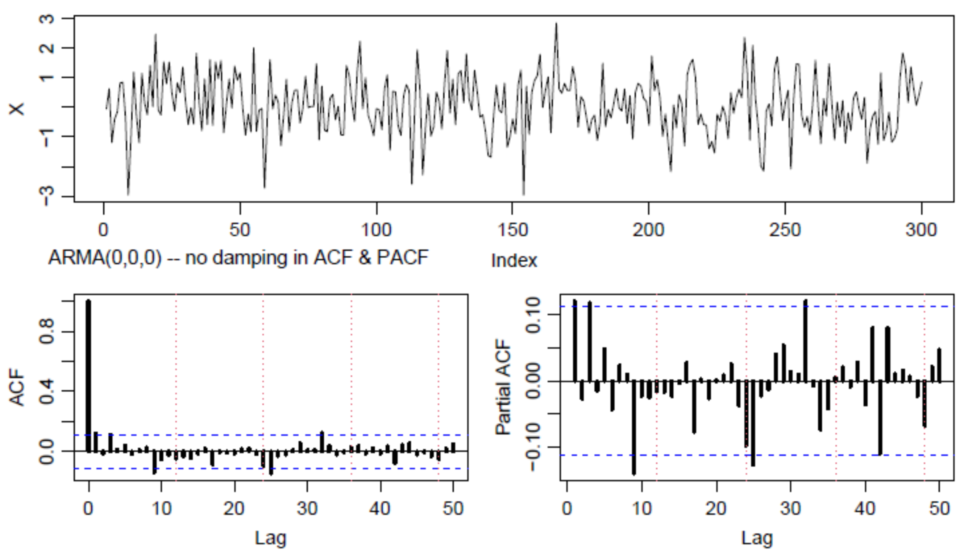
- Seasonality in the data is typically reflected in the ACF plot by significant auto-correlations at lags that are multiples of the seasonal period(s).
- A seasonal AR component Φ generally causes a damping exponential in the ACF at the seasonal lags. The parameter values in Φ determine the rate and direction of this decay.
- A seasonal MA component Θ typically results in non-zero auto-correlations at first then a cut-off.
- PACF can also be useful in identifying seasonal AR components, showing significant spikes at seasonal lags and then trailing off or cutting off depending on the model

Conclusion:

- Seasonal AR terms Φ cause a gradual decay in ACF at seasonal lags, indicating prolonged seasonal effects.
- Seasonal MA terms Θ cause a sharp cutoff in ACF at seasonal lags, leading to a short-lived seasonal influence.
- The combination of both AR and MA components results in a mix of damped oscillations and sharp drops in autocorrelation.
- Seasonal differencing (D), though not explored in this simulation, would typically remove seasonal trends, leading to a stationary series.

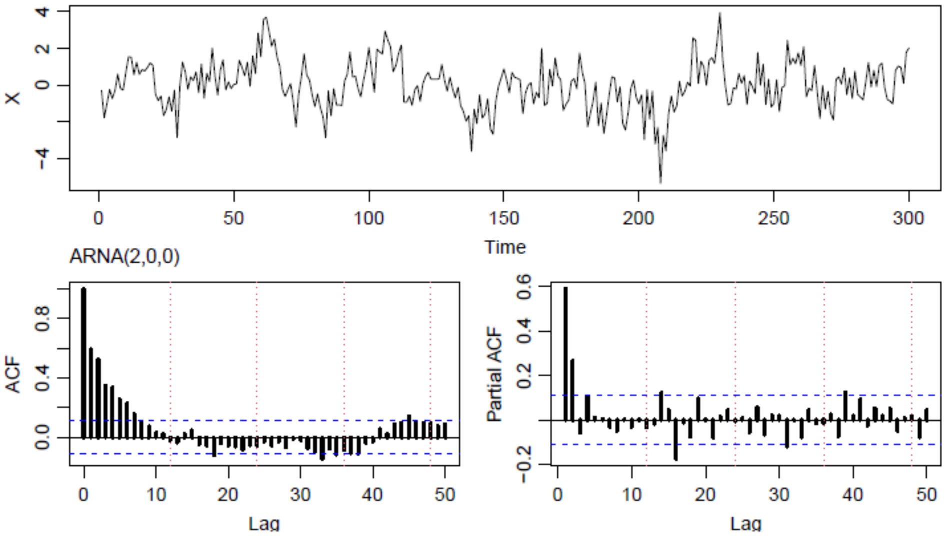
3 Identifying ARMA models

3.1 Process 1



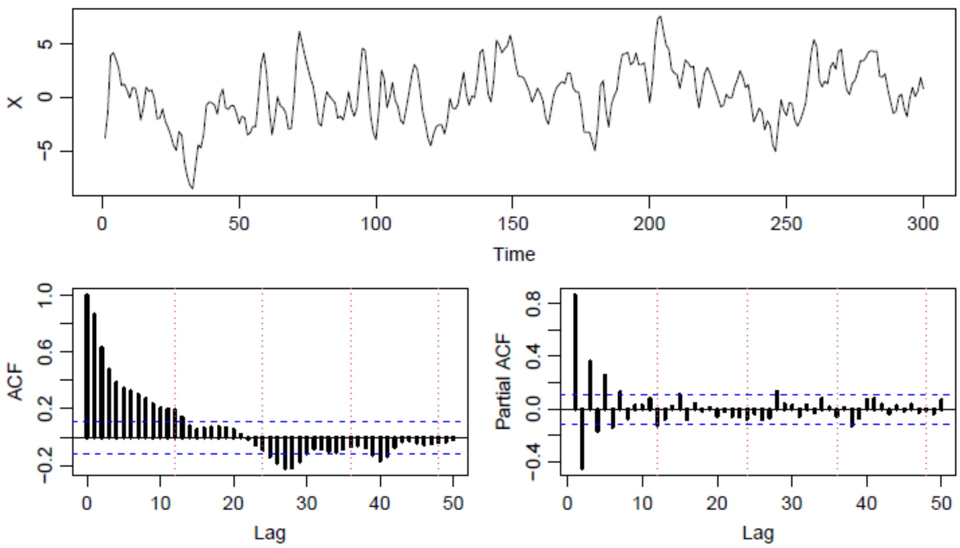
ARMA(0,0): There are no visible correlation in both ACF and PACF, suggesting no AR and MA terms. All values are within the blue confidence bands, meaning the correlations are not statistically significant. This suggest that it is a comepletly random, white noise.

3.2 Process 2



ARMA(2,0): A damped exponential decay can be seen in the ACF, suggesting an AR process. While no decaying trend is seen in the PACF, suggesting no MA process, two spikes can are present, suggesting a AR(2) process.

3.3 Process 3



ARMA(1,1): A damped exponential decay can be observed in both the ACF and PACF, suggesting both a AR and MA term.