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Introduction to Microstrip Antennas

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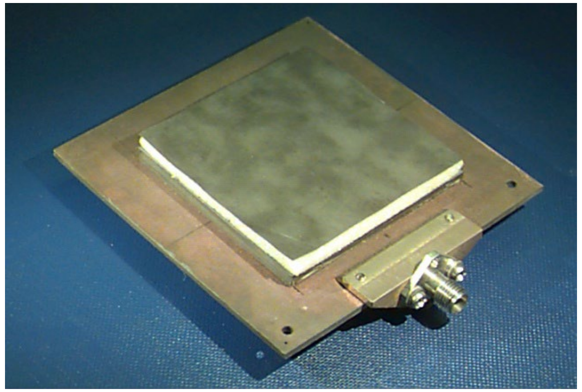
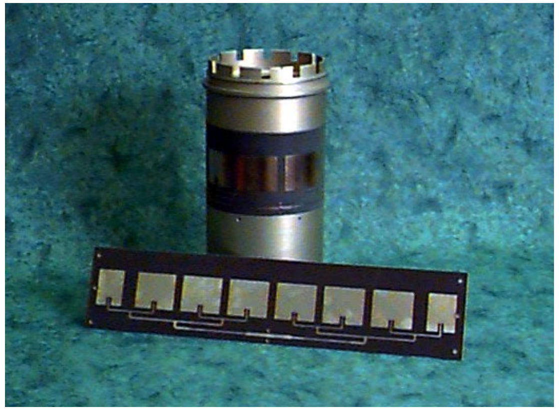
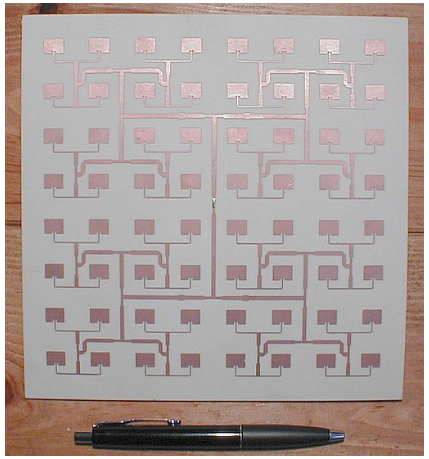
Updated 10/18/2024







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Purpose of Short Course

- Provide an introduction to microstrip antennas.
- Provide a physical and mathematical basis for understanding how microstrip antennas work.
- Provide a physical understanding of the basic physical properties of microstrip antennas.
- Provide an overview of some of the recent advances and trends in the area (but not an exhaustive survey – directed towards understanding the fundamental principles).

Additional Resources

- Some basic references are provided at the end of these viewgraphs.
- You are welcome to visit a website that goes along with a course at the University of Houston on microstrip antennas (PowerPoint viewgraphs from the course may be found there, along with the viewgraphs from this short course).

ECE 6345: Microstrip Antennas

http://courses.egr.uh.edu/ECE/ECE6345/

Note:
You are welcome to use anything that you find on this website, as long as you please acknowledge the source.

Outline

- ❖ Overview of microstrip antennas
- ❖ History of microstrip antennas
- ❖ Feeding methods
- ❖ Basic principles of operation
- ❖ General characteristics
- ❖ CAD formulas
- ❖ Input impedance
- ❖ Radiation pattern
- ❖ Circular polarization
- ❖ Circular patch
- ❖ Improving bandwidth
- ❖ Miniaturization
- ❖ Reducing surface waves and lateral radiation

Notation

c = speed of light in free space

λ_0 = wavelength of free space

k_0 = wavenumber of free space

k_1 = wavenumber of substrate

η_0 = intrinsic impedance of free space

η_1 = intrinsic impedance of substrate

ε_r = relative permtitivity (dielectric constant) of substrate

$\varepsilon_r^{\text{eff}}$ = effective relative permtitivity
(accouting for fringing of flux lines at edges)

$\varepsilon_{rc}^{\text{eff}}$ = complex effective relative permtitivity
(used in the cavity model to account for all losses)

$$c = 2.99792458 \times 10^{8} \; [\text{m/s}]$$

$$\lambda_0 = c / f$$

$$k_0 = \omega\sqrt{\mu_0 \varepsilon_0} = 2\pi / \lambda_0$$

$$k_1 = k_0 \sqrt{\varepsilon_r}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 376.7303 \; [\Omega]$$

$$\eta_1 = \eta_0 / \sqrt{\varepsilon_r}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\mu_0 = 4\pi \times 10^{7} \; [\text{H/m}]$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} \approx 8.854188 \times 10^{12} \; [\text{F/m}]$$

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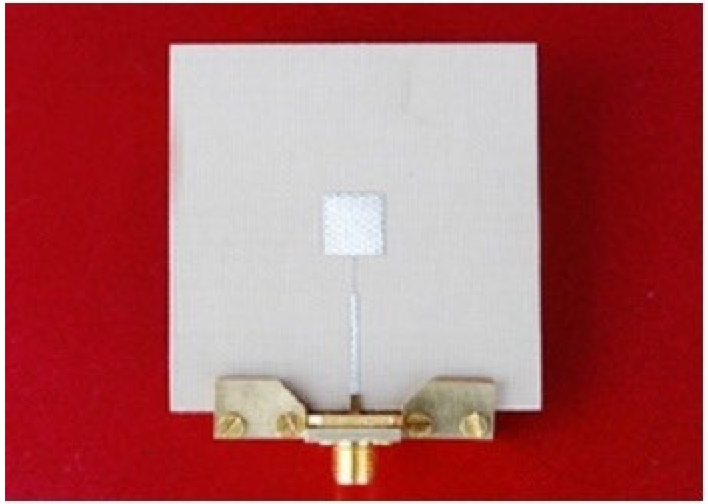
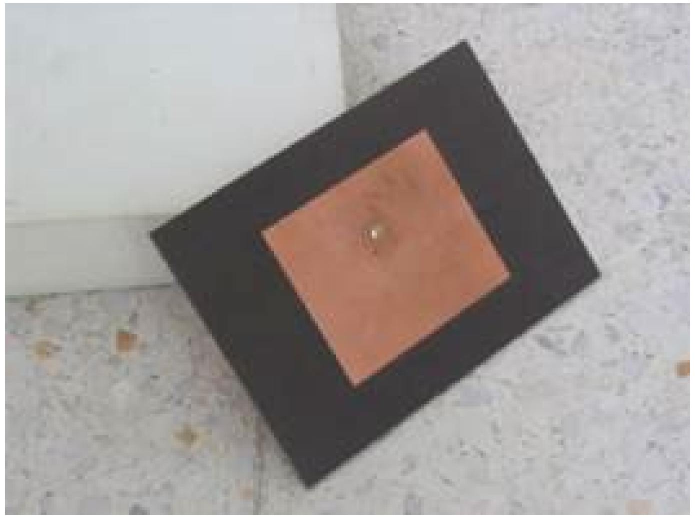
Overview of Microstrip Antennas

Also called "patch antennas"

- One of the most useful antennas at microwave frequencies ($f > 1$ GHz).
- It usually consists of a metal "patch" on top of a grounded dielectric substrate.
- The patch may be in a variety of shapes, but rectangular and circular are the most common.



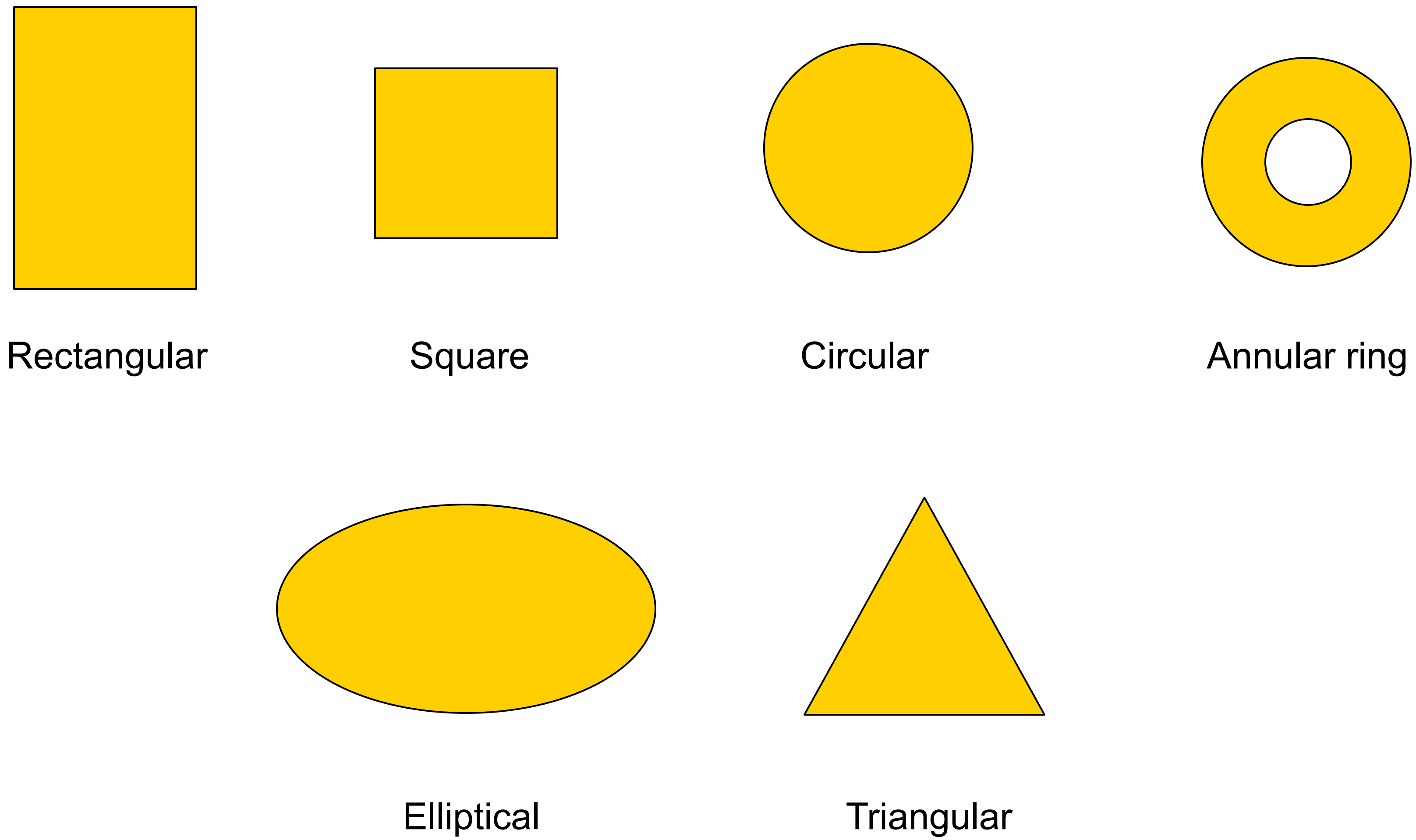
Microstrip line feed



Coax feed

Overview of Microstrip Antennas

Common Shapes



Overview of Microstrip Antennas

Advantages of Microstrip Antennas

- Low profile (can even be “conformal,” i.e. flexible to conform to a surface).
- Easy to fabricate (use etching and photolithography).
- Easy to feed (coaxial cable, microstrip line, etc.).
- Easy to incorporate with other microstrip circuit elements and integrate into systems.
- Patterns are somewhat hemispherical, with a moderate directivity (about 6-8 dB is typical).
- Easy to use in an array to increase the directivity.

Overview of Microstrip Antennas

Disadvantages of Microstrip Antennas

- Low bandwidth (but can be improved by a variety of techniques). Bandwidths of a few percent are typical. Bandwidth is roughly proportional to the substrate thickness and inversely proportional to the substrate permittivity.

- Efficiency may be lower than with other antennas. Efficiency is limited by conductor and dielectric losses*, and by surface-wave loss**.

- Only used at microwave frequencies and above (the substrate becomes too large at lower frequencies).

- Cannot handle extremely large amounts of power (dielectric breakdown).

* Conductor and dielectric losses become more severe for thinner substrates.

** Surface-wave losses become more severe for thicker substrates (unless air or foam is used).

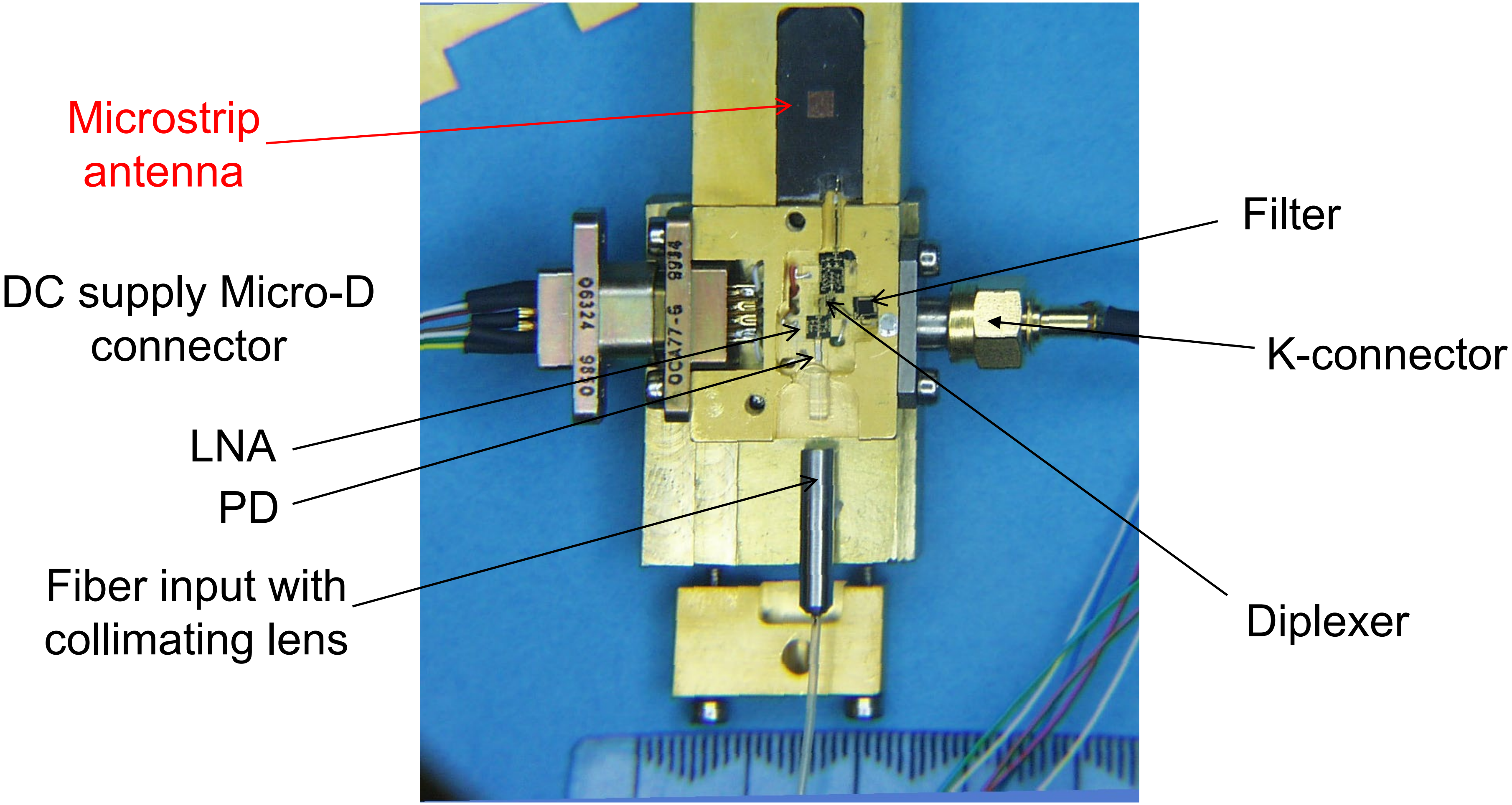
Overview of Microstrip Antennas

Applications

Applications include:

- Satellite communications
- Microwave communications
- Cell phone antennas
- GPS antennas

Overview of Microstrip Antennas

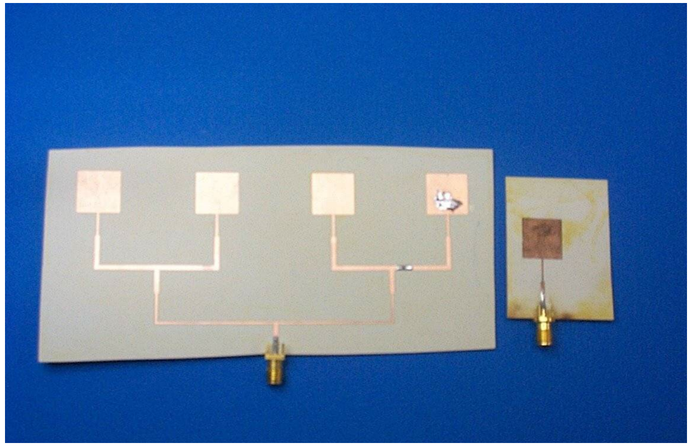


Microstrip Antenna Integrated into a System: HIC Antenna Base-Station for 28-43 GHz

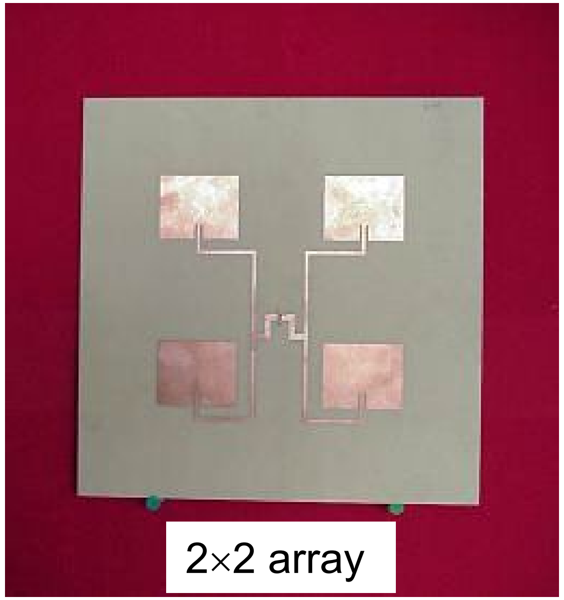
(Photo courtesy of Dr. Rodney B. Waterhouse)

Overview of Microstrip Antennas

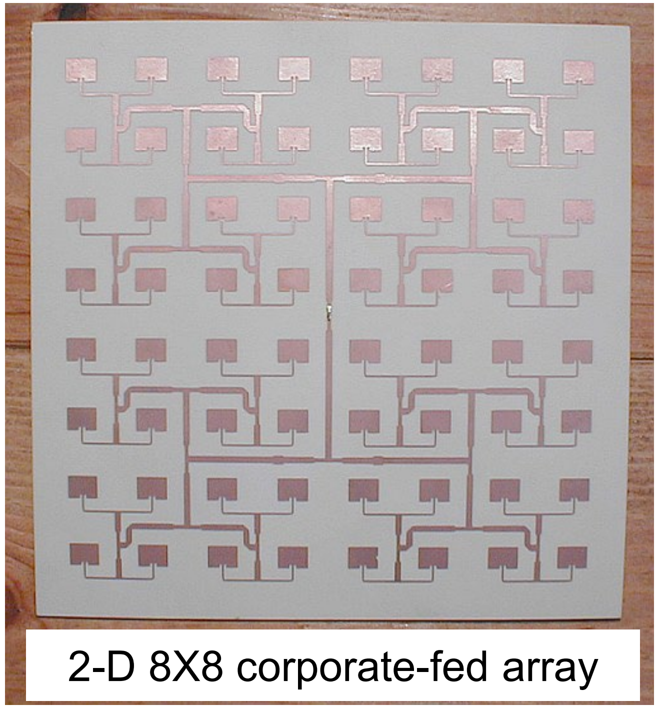
Arrays



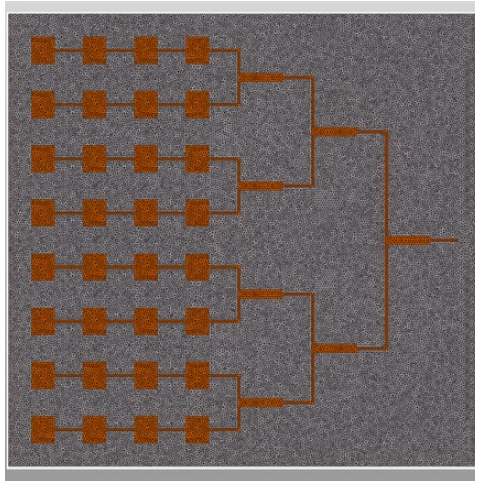
Linear array (1-D corporate feed)



2×2 array



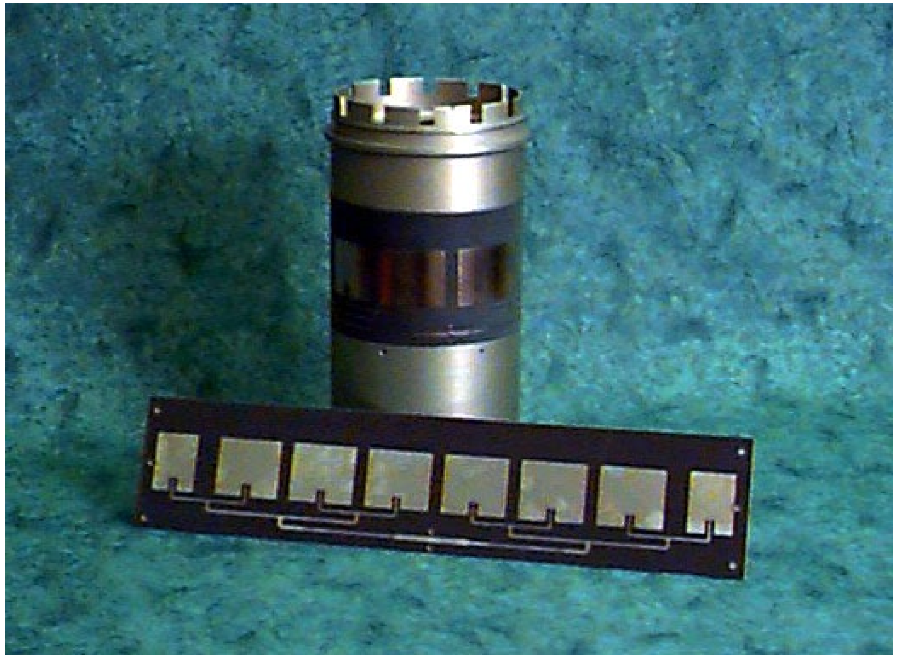
2-D 8X8 corporate-fed array



4 × 8 corporate-fed / series-fed array

Overview of Microstrip Antennas

Wraparound Array (conformal)



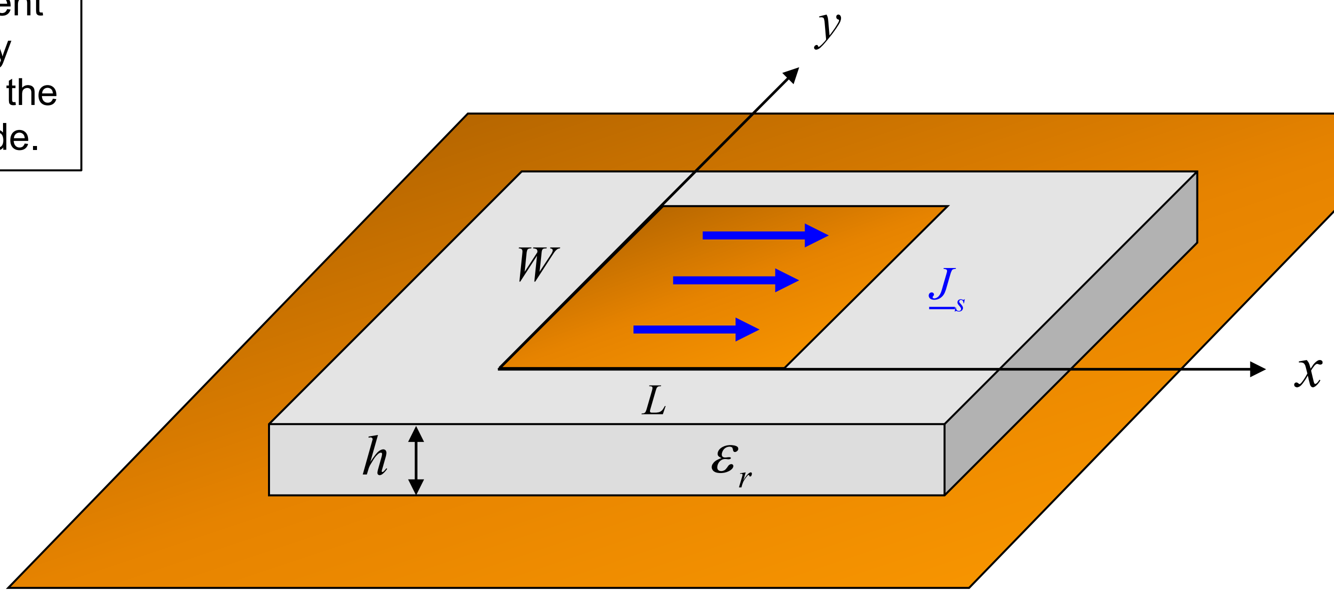
The substrate is so thin that it can be bent to “conform” to the surface.

(Photo courtesy of Dr. Rodney B. Waterhouse)

Overview of Microstrip Antennas

Rectangular patch

Note:
The fields and current are approximately independent of y for the dominant (1,0) mode.

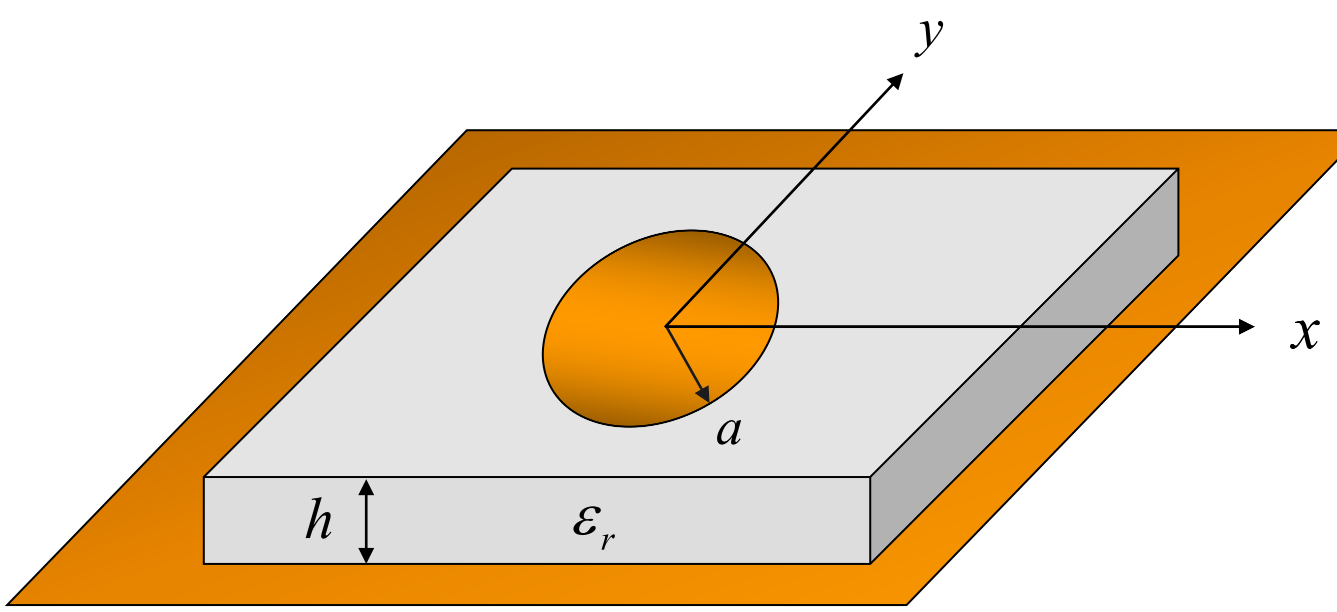


Note:
L is the resonant dimension (direction of current flow). The width W is usually chosen to be larger than L (to get higher bandwidth). However, usually $W < 2L$ (to avoid problems with the $(0,2)$ mode).

$W = 1.5L$ is typical.

Overview of Microstrip Antennas

Circular Patch



The location of the feed determines the direction of current flow and hence the polarization of the radiated field.

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History of Microstrip Antennas

- Invented by Bob Munson in 1972 Co-invented by John. Q. Howell in 1972.
- Became popular starting in the 1970s.

Note: Earlier work by Dechamps goes back to 1953, but this earlier work was on planar horn antennas.

Robert E. Munson
(1940-2015)

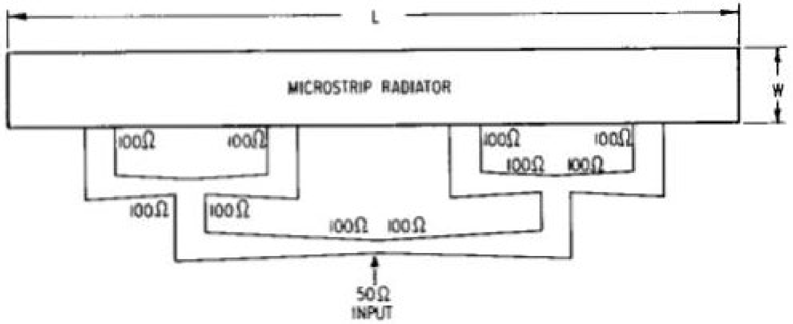
G. Deschamps and W. Sichak, "Microstrip Microwave Antennas," Proc. of Third Symp. on USAF Antenna Research and Development Program, October 18–22, 1953.

R. E. Munson, "Microstrip Phased Array Antennas," *Proc. of Twenty-Second Symp. on USAF Antenna Research and Development Program,* October 1972.

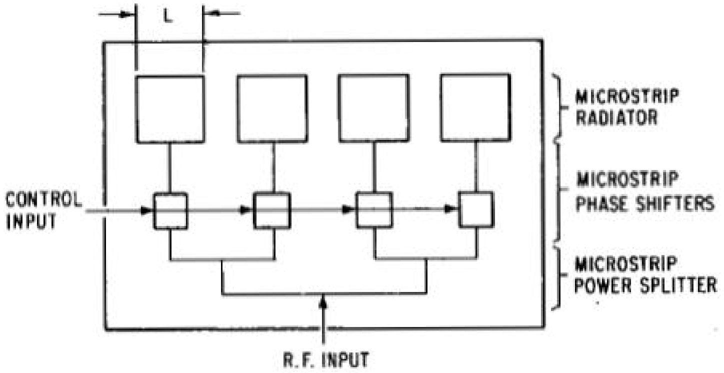
R. E. Munson, "Conformal Microstrip Antennas and Microstrip Phased Arrays," *IEEE Trans. Antennas Propagat*., vol. AP-22, no. 1 (January 1974): 74–78.

J. Q. Howell, "Microstrip Antennas," IEEE AP-S Intl. Symp. Digest, pp. 177–180, Dec. 1972.

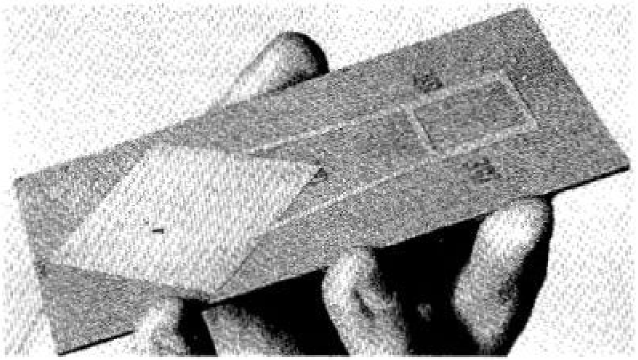
History of Microstrip Antennas



Flat “wraparound” microstrip antenna



Phased array of patch antennas



Circularly-polarized patch antenna

Note that the circularly-polarized patch was introduced here!

R. E. Munson, “Conformal microstrip antennas and microstrip phased arrays,” IEEE Trans. Antennas Propagat., vol. AP–22, no. 1, pp. 74-78, Jan. 1974.

History of Microstrip Antennas

MICROSTRIP ANTENNAS

John Q. Howell
NASA Langley Research Center

A simple yet efficient microwave antenna can be constructed on a dielectric substrate over a ground plane using microwave integrated circuit techniques.[1] The antenna protrudes above the ground plane only as far as the substrate thickness plus whatever protective coating is necessary. It does not extend behind the ground plane, and only holes for the coax feed through and perhaps mounting are needed in this surface. Both linearly and circularly polarized antennas can be made. In Fig. 1 is shown a linearly polarized antenna for L-band.

1. Bob Munson, Ball Brothers Research Corp., Boulder, Colo., private communication.

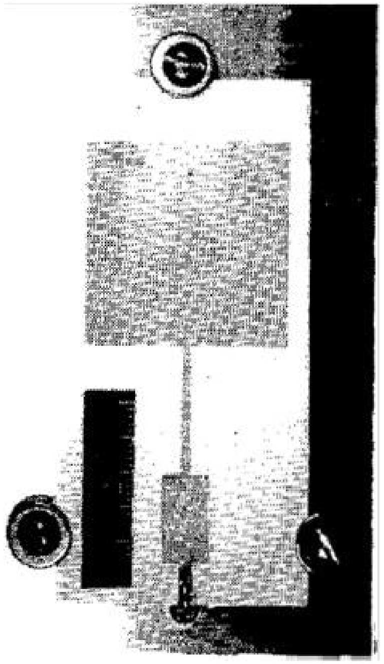


Figure 1

J. Q. Howell, "Microstrip antennas," IEEE AP-S Intl. Symp. Digest, pp. 177–180, Dec. 1972.

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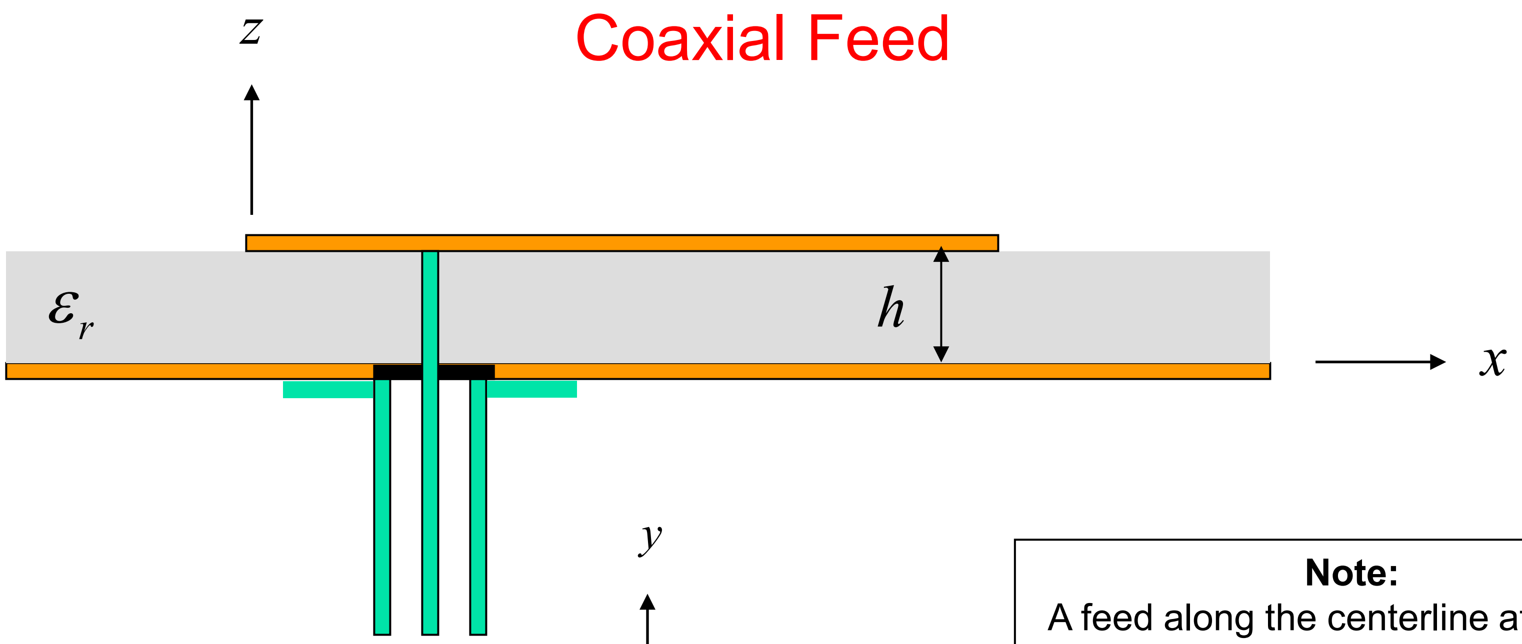
Feeding Methods

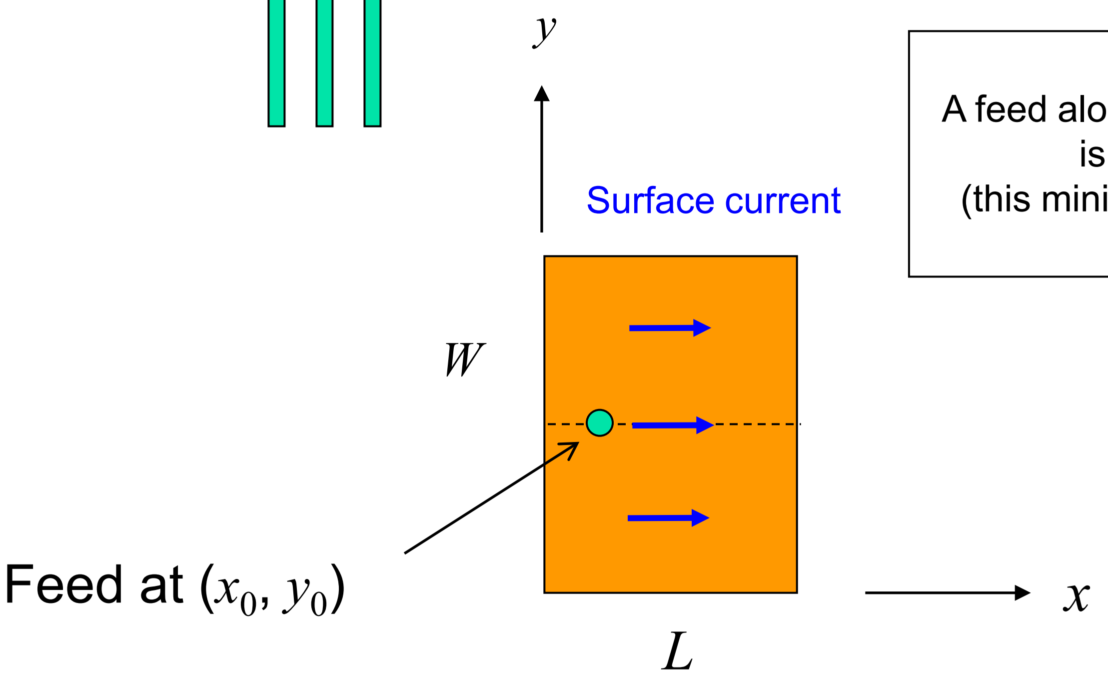
Some of the more common methods for feeding microstrip antennas are shown.

The feeding methods are illustrated for a rectangular patch, but the principles apply for circular and other shapes as well.

Feeding Methods

Coaxial Feed





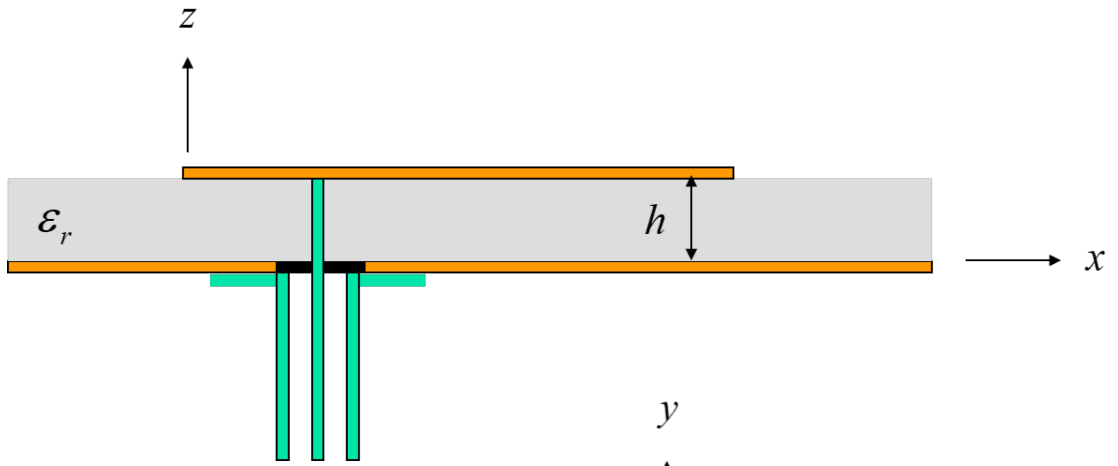
Note:
A feed along the centerline at $y = W/2$ is the most common (this minimizes higher-order modes and cross-pol).

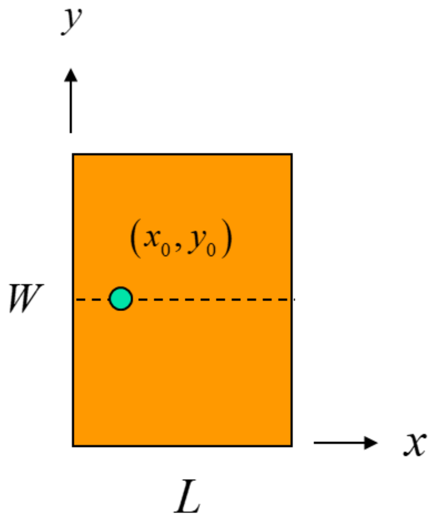
Feeding Methods

Coaxial Feed

$$R = R_{\text{edge}} \cos^2\left(\frac{\pi x_0}{L}\right)$$

(The resistance varies as the square of the modal field shape.)





Advantages:

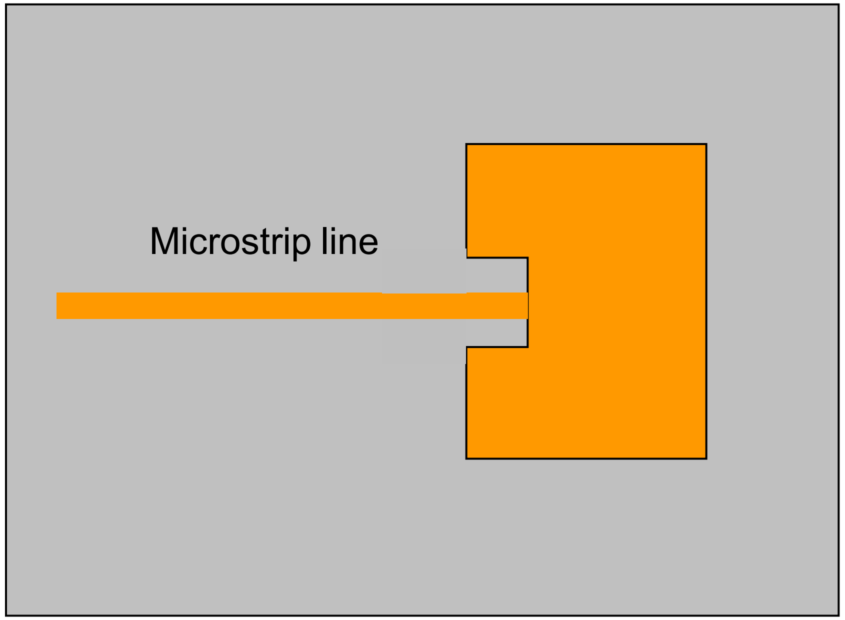
- Simple.
- Directly compatible with coaxial cables.
- Easy to obtain input match by adjusting feed position.

Disadvantages:

- Significant probe (feed) radiation for thicker substrates.
- Significant probe inductance for thicker substrates (limits bandwidth).
- Not easily compatible with arrays.

Feeding Methods

Inset Feed



Advantages:

- Simple.
- Allows for planar feeding.
- Easy to use with arrays.
- Easy to obtain input match.

Disadvantages:

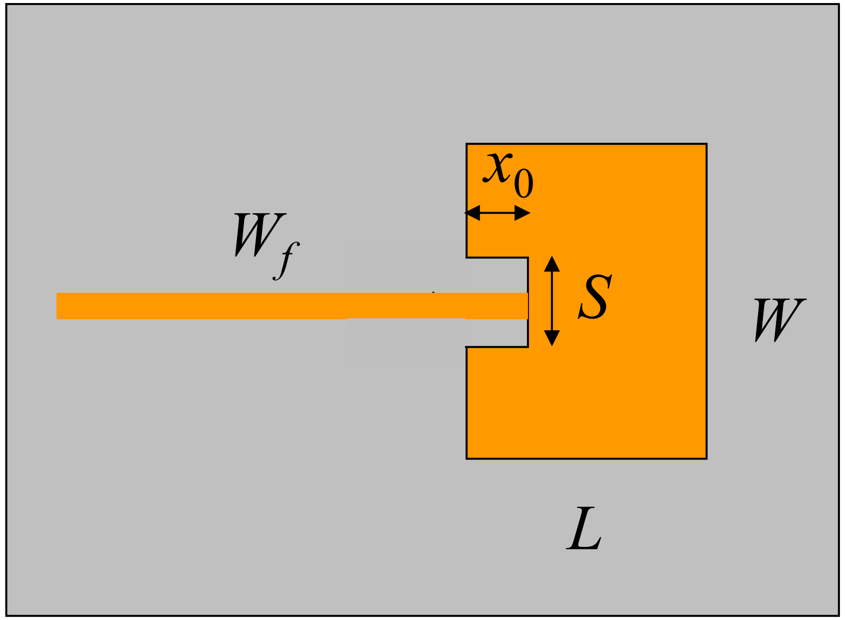
- Significant line radiation for thicker substrates.
- For deep notches, patch current and radiation pattern may show distortion.

Feeding Methods

Inset Feed

An investigation has shown that the resonant input resistance varies as:

$$R_{\text{in}} = A\cos^2\left(\frac{\pi}{2}\left(\frac{2x_0}{L} - B\right)\right)$$



Less accurate approximation:

$$R \approx R_{\text{edge}} \cos^2\left(\frac{\pi x_0}{L}\right)$$

The coefficients A and B depend on the notch width S but (to a good approximation) not on the line width W_f.

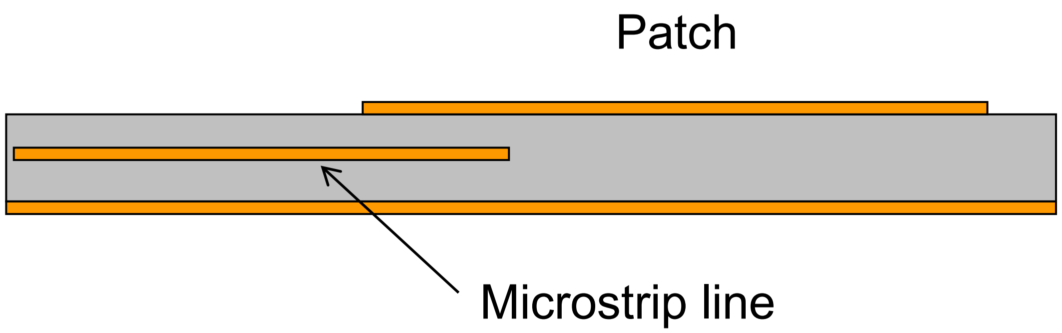
Y. Hu, D. R. Jackson, J. T. Williams, and S. A. Long, “Characterization of the Input Impedance of the Inset-Fed Rectangular Microstrip Antenna,” *IEEE Trans. Antennas and Propagation*, Vol. 56, No. 10, pp. 3314-3318, Oct. 2008.

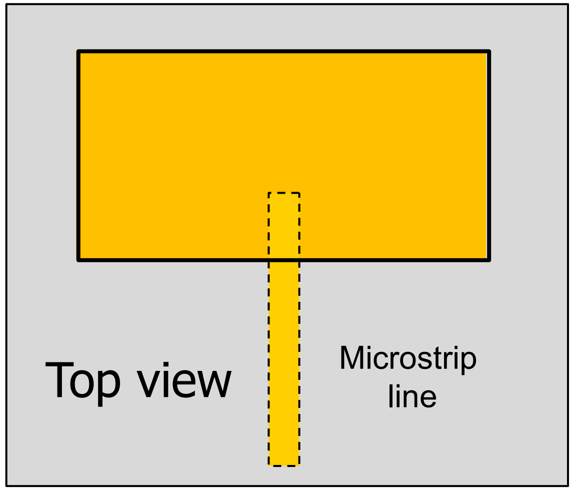
Feeding Methods

Proximity-Coupled Feed
(Electromagnetically-Coupled Feed)

Advantages:

- Allows for planar feeding.
- Less line radiation compared to microstrip feed (the line is closer to the ground plane).
- Can allow for higher bandwidth (no probe inductance, so substrate can be thicker).





Disadvantages:

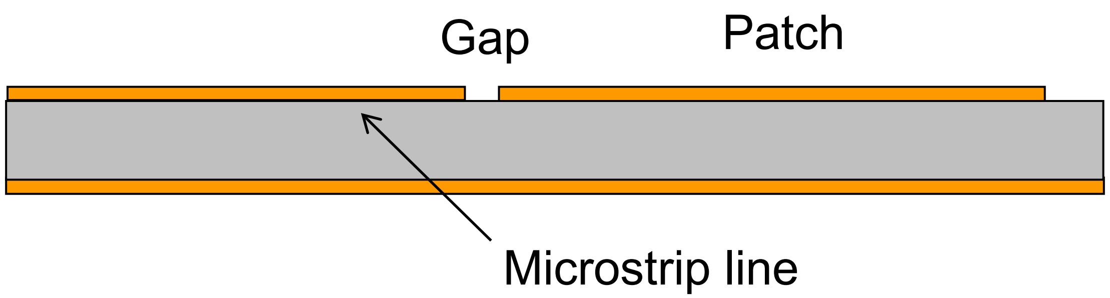
- Requires multilayer fabrication.
- Alignment is important for input match.

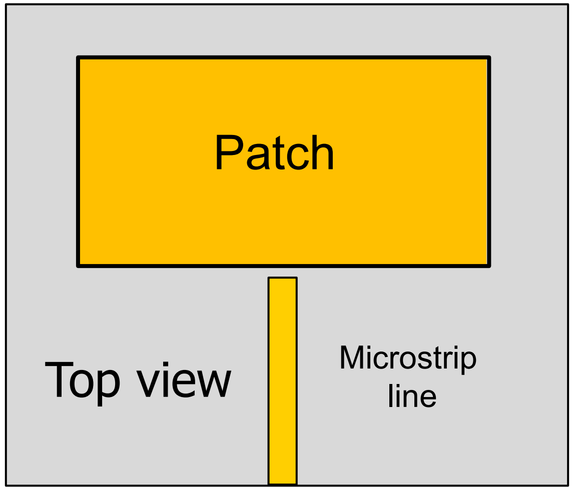
Feeding Methods

Gap-Coupled Feed

Advantages:

- Allows for planar feeding.
- Can allow for a match even with high edge impedances, where a notch might be too large (e.g., when using a high permittivity substrate).





Disadvantages:

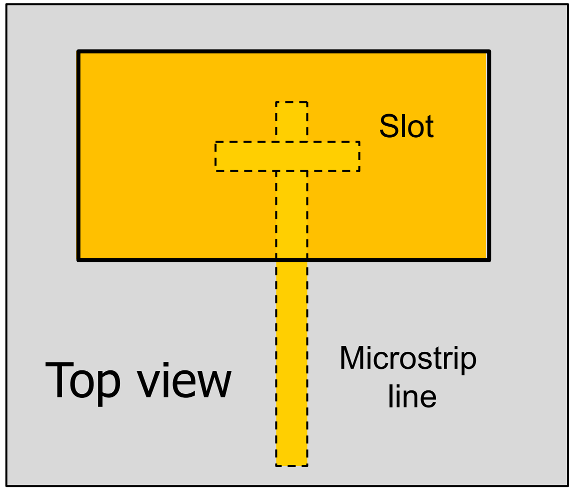
- Requires accurate gap fabrication.
- Requires full-wave design.

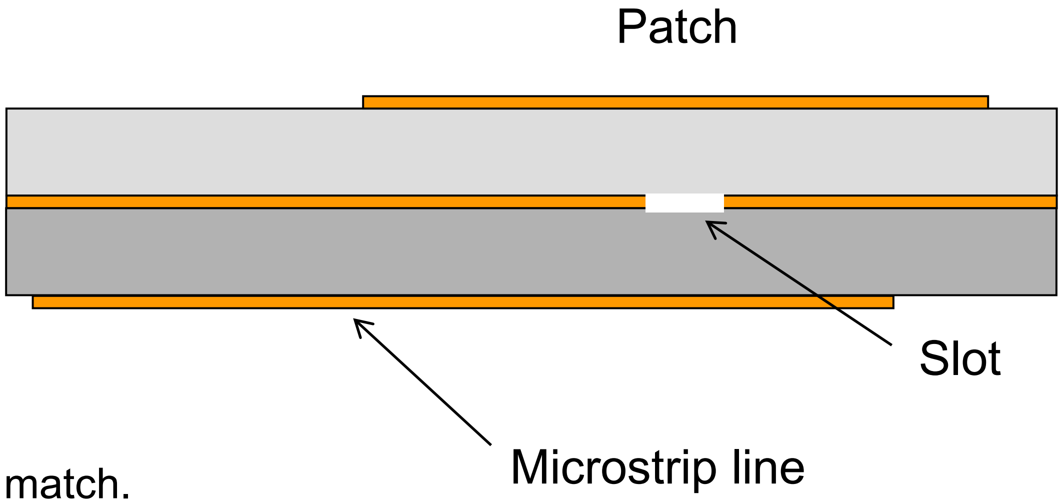
Feeding Methods

Aperture-Coupled Patch (ACP)

Advantages:

- Allows for planar feeding.
- Feed-line radiation is isolated from patch radiation.
- Higher bandwidth is possible, since probe inductance is eliminated (allowing for a thick substrate), and also a double-resonance can be created.
- Allows for use of different substrates to optimize antenna and feed-circuit performance.





Disadvantages:

- Requires multilayer fabrication.
- Alignment is important for input match.

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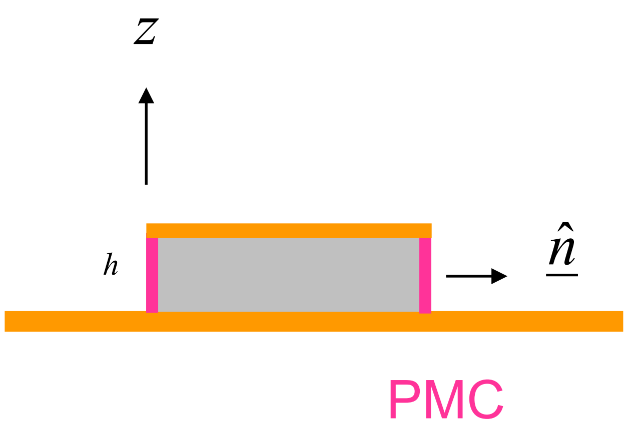
Basic Principles of Operation

- The basic principles are illustrated here for a rectangular patch, but the principles apply similarly for other patch shapes.

- We use the cavity model to explain the operation of the patch antenna.



Y. T. Lo, D. Solomon, and W. F. Richards, "Theory and Experiment on Microstrip Antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-27, no. 3 (March 1979): 137–145.

Basic Principles of Operation

Main Ideas:

- The patch acts approximately as a <u>resonant cavity</u> (with perfect electric conductor (PEC) walls on top and bottom, and perfect magnetic conductor (PMC) walls on the edges).

- Radiation is accounted for by using an effective loss tangent for the substrate.

- In a cavity, only certain modes are allowed to exist, at different resonance frequencies.

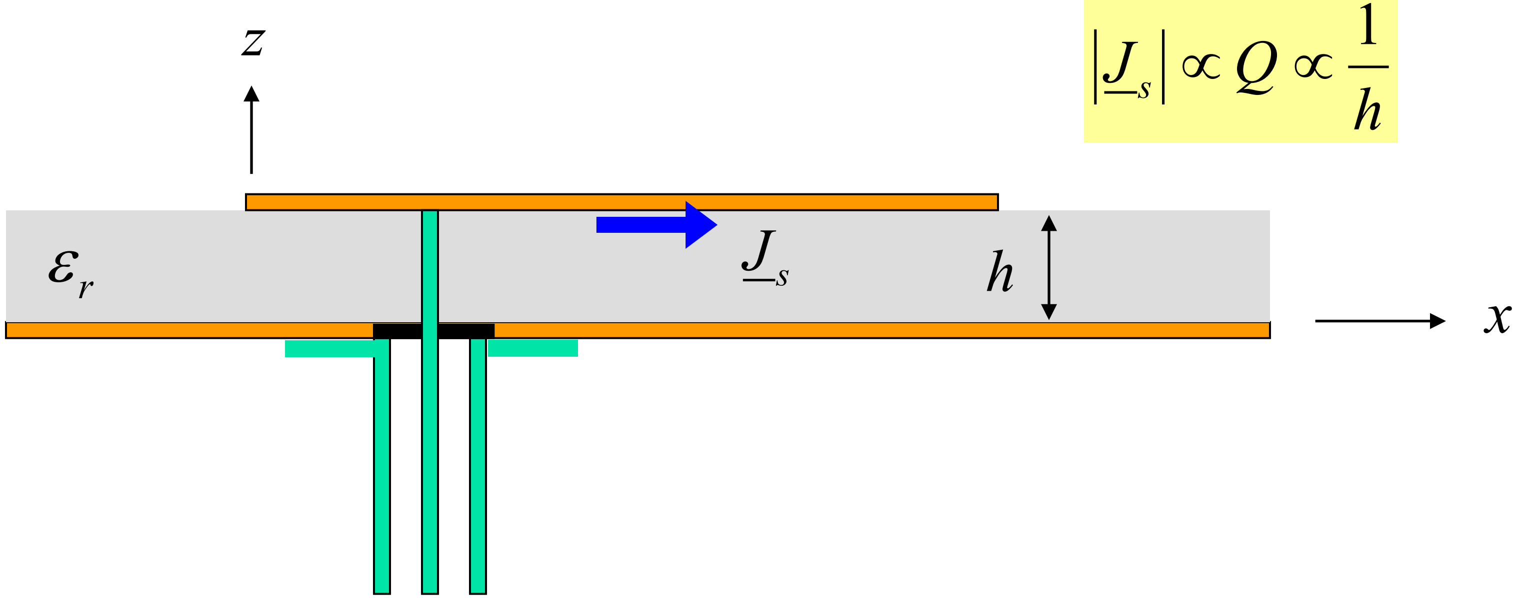
- If the antenna is excited at a resonance frequency, a strong field is set up inside the cavity, and a strong current on the (bottom) surface of the patch. This produces significant radiation (a good antenna).

Basic Principles of Operation

A microstrip antenna can radiate well, even with a thin substrate, because of resonance.

- As the substrate gets thinner the patch current radiates less, due to image cancellation (current and image are separated by $2h$).
- However, the Q of the resonant cavity mode also increases, making the patch currents stronger at resonance.
- These two effects cancel, allowing the patch to radiate well even for thin substrates (though the bandwidth decreases).

$$\left|\underline{J}_s\right| \propto Q \propto \frac{1}{h}$$



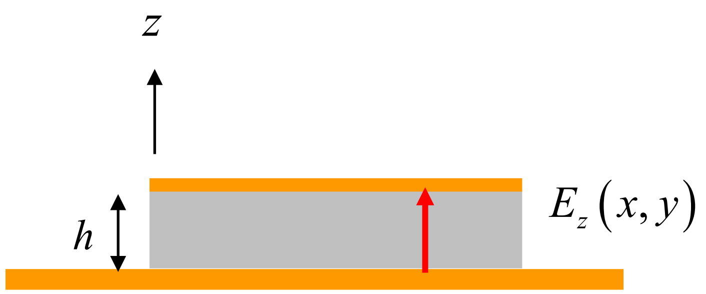
Basic Principles of Operation

Thin Substrate Approximation

On patch and ground plane: $\underline{E}_t = \underline{0} \Rightarrow \underline{E} = \underline{\hat{z}}\, E_z$

Inside the patch cavity, because of the thin substrate,
the electric field vector is approximately independent of z.

Hence $$\underline{E}(x, y, z) \approx \underline{\hat{z}}\, E_z(x, y)$$



Basic Principles of Operation

Thin Substrate Approximation

Magnetic field inside patch cavity:

$$
\begin{aligned}
\underline{H} &= -\frac{1}{j\omega\mu}\nabla\times\underline{E} \\
&= -\frac{1}{j\omega\mu}\nabla\times\left(\underline{\hat{z}}E_z\left(x,y\right)\right) \\
&= -\frac{1}{j\omega\mu}\left(-\underline{\hat{z}}\times\nabla E_z\left(x,y\right)\right)
\end{aligned}
$$

Useful vector identity:

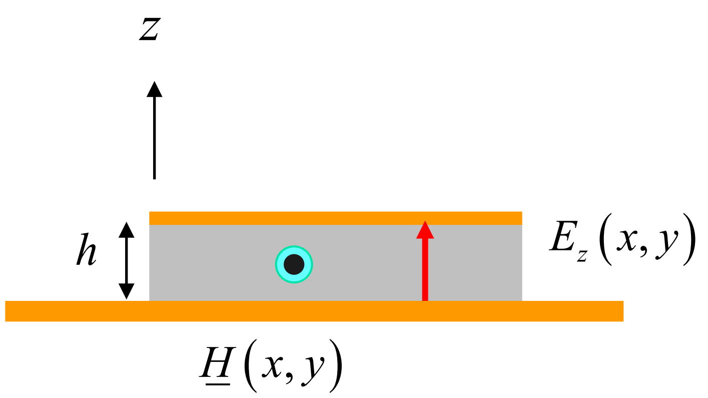
$$\nabla\times\left(\psi\underline{V}\right)=\psi\,\nabla\times\underline{V}+\nabla\psi\times\underline{V}$$

Basic Principles of Operation

Thin Substrate Approximation

$$\underline{H}(x,y)=\frac{1}{j\omega\mu}\left(\underline{\hat{z}}\times\nabla E_z(x,y)\right)$$

Note:
The magnetic field is purely horizontal.
(The mode is TM_z.)



Basic Principles of Operation

Magnetic-wall Approximation

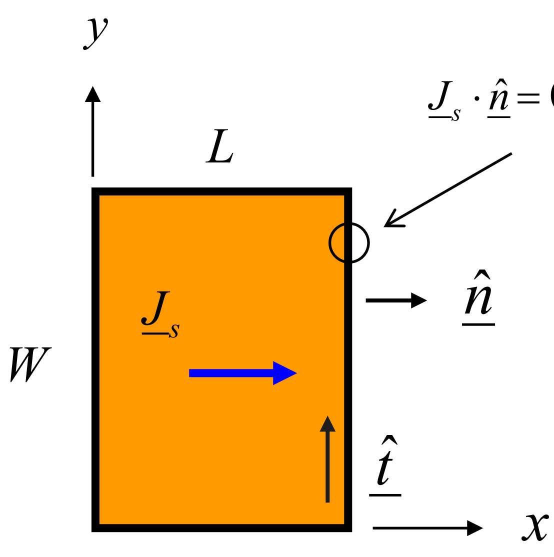
The patch edge acts as an approximate open circuit.

$$\underline{H}_t = \underline{0} \quad \text{(PMC)}$$

or

$$\hat{\underline{n}} \times \underline{H}(x, y) = \underline{0}$$

$\underline{J}_s \cdot \hat{\underline{n}} = 0$ (at edge)



$\underline{J}_s^b \cdot \hat{\underline{n}} \approx 0$ (at edge)

On the bottom surface of the patch:

$$\underline{J}_s^b = -\hat{\underline{z}} \times \underline{H}$$

$$\Rightarrow \quad \underline{H}_t^{\text{edge}} \approx \underline{0}$$

$\underline{H}_t^{\text{edge}} \approx \underline{0}$

Actual patch

PMC

PMC Model

Basic Principles of Operation

Magnetic-wall Approximation

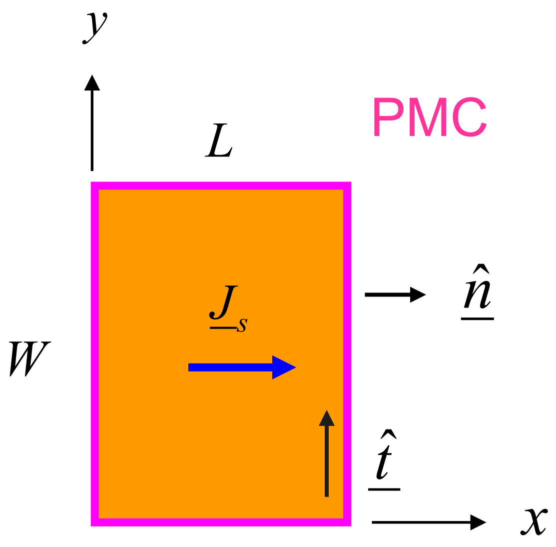
On PMC patch edge:

$$\underline{\hat{n}} \times \underline{H}(x, y) = \underline{0}$$

$$\underline{H}(x, y) = \frac{1}{j\omega\mu}\left(\underline{\hat{z}} \times \nabla E_z(x, y)\right)$$

Hence,

$$\underline{\hat{n}} \times \left(\underline{\hat{z}} \times \nabla E_z(x, y)\right) = \underline{0}$$



$$\underline{\hat{n}} \times \left(\underline{\hat{z}} \times \nabla E_z(x, y)\right) = \underline{\hat{z}}\left(\underline{\hat{n}} \cdot \nabla E_z(x, y)\right) - \nabla E_z(x, y)\cancel{\left(\underline{\hat{n}} \cdot \underline{\hat{z}}\right)}$$

$$\Rightarrow \quad \underline{\hat{z}}\left(\underline{\hat{n}} \cdot \nabla E_z(x, y)\right) = \underline{0}$$

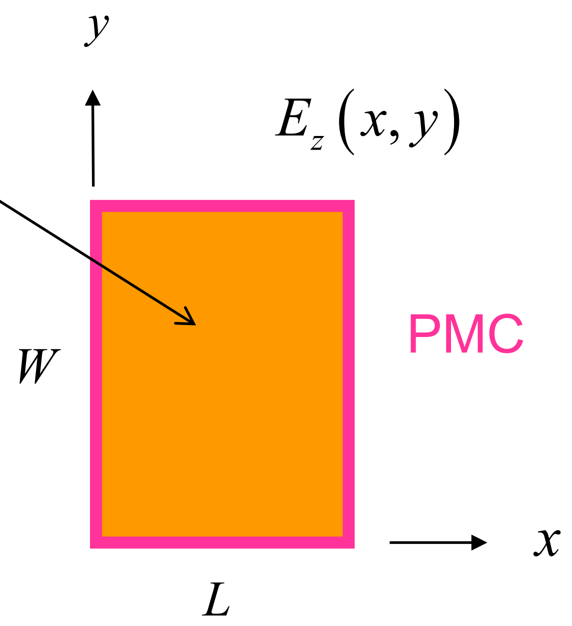
$$\boxed{\frac{\partial E_z}{\partial n} = 0}$$

(Neumann B.C.)

Basic Principles of Operation

Resonance Frequencies

$$\nabla^2 E_z + k^2 E_z = 0 \qquad k = k_1 = k_0\sqrt{\varepsilon_r}$$



From separation of variables:

$$E_z = \cos\left(\frac{m\pi x}{L}\right)\cos\left(\frac{n\pi y}{W}\right)$$

(TM$_{mn}$ mode)

We then have $\left[-\left(\frac{m\pi}{L}\right)^2 - \left(\frac{n\pi}{W}\right)^2 + k_1^2\right]E_z = 0$

Hence $\left[-\left(\frac{m\pi}{L}\right)^2 - \left(\frac{n\pi}{W}\right)^2 + k_1^2\right] = 0$

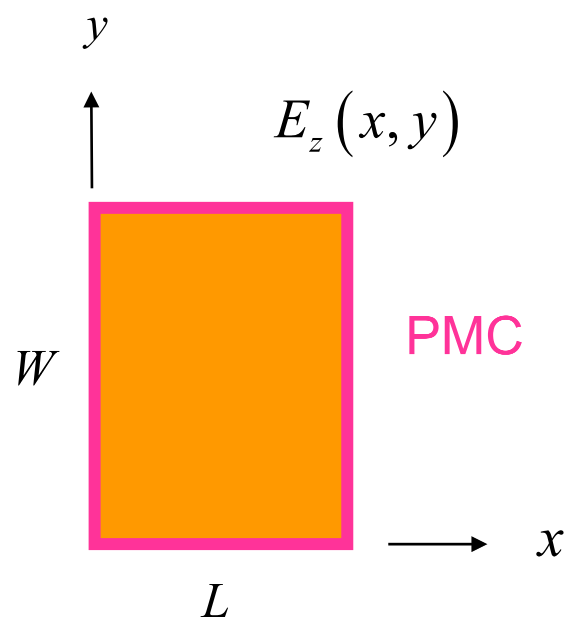
Note:
We ignore the loss tangent of the substrate for the calculation of the resonance frequencies.

Basic Principles of Operation

Resonance Frequencies

We thus have

$$k_1^2 = \left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2$$



Recall that

$$k_1 = k_0\sqrt{\varepsilon_r} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r}$$

$$\omega = 2\pi f$$

Assume $\mu_r = 1$

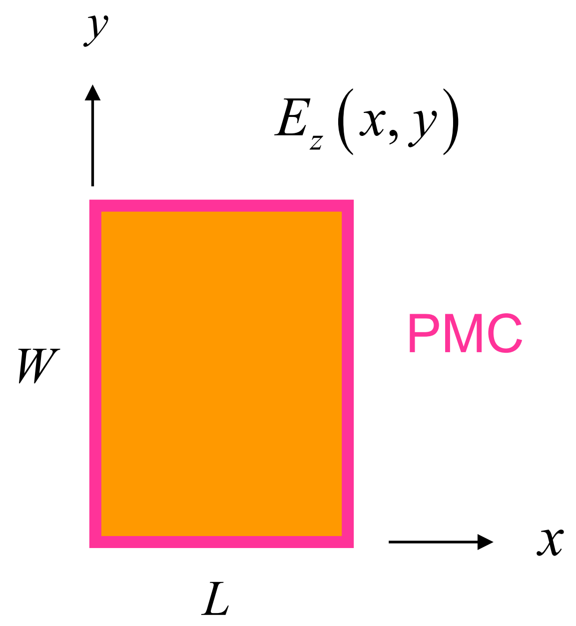
Hence

$$f = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2}$$

$$c = 1/\sqrt{\mu_0\varepsilon_0}$$

Basic Principles of Operation

Resonance Frequencies



Hence $f = f_{mn}$

(resonance frequency of (m,n) mode)

where

$$f_{mn} = \frac{c}{2\pi\sqrt{\varepsilon_r}} \sqrt{\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2}$$

Basic Principles of Operation

Dominant (1,0) mode

This structure operates as a "fat planar dipole."

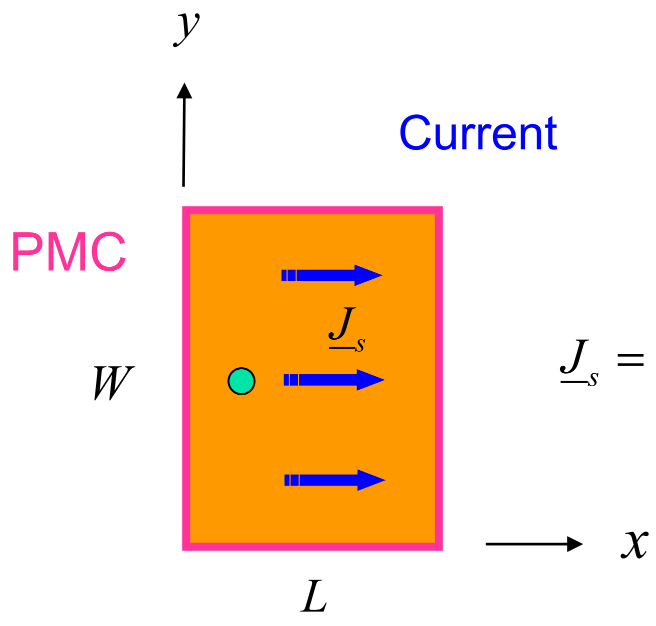
This mode is usually used because the radiation pattern has a broadside beam.

$$f_{10} = \frac{c}{2\sqrt{\varepsilon_r}}\left(\frac{1}{L}\right)$$

$$E_z = \cos\left(\frac{\pi x}{L}\right)$$

$$\underline{H}(x,y) = -\underline{\hat{y}}\left(\frac{1}{j\omega\mu_0}\right)\left(\frac{\pi}{L}\right)\sin\left(\frac{\pi x}{L}\right)$$

$$\underline{J}_s = \underline{\hat{x}}\left(\frac{-1}{j\omega\mu_0}\right)\left(\frac{\pi}{L}\right)\sin\left(\frac{\pi x}{L}\right)$$



$$\underline{J}_s = \underline{J}_s^b = -\underline{\hat{z}} \times \underline{H}$$

The current is maximum in the middle of the patch, when plotted along x.

The resonant length L is about 0.5 dielectric wavelengths in the x direction (see next slide).

Basic Principles of Operation

Resonance Frequency of Dominant (1,0) Mode

The resonance frequency is mainly controlled by the patch length L and the substrate permittivity.

Approximately, (assuming PMC walls):

$$k_1^2 = \left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2$$

This is equivalent to saying that the length L is one-half of a wavelength in the dielectric.

(1,0) mode: $k_1 L = \pi \Rightarrow L = \lambda_d / 2 = \dfrac{\lambda_0 / 2}{\sqrt{\varepsilon_r}}$

$k_1 = 2\pi / \lambda_d$

Comment:
A higher substrate permittivity allows for a smaller antenna (miniaturization), but with a lower bandwidth.

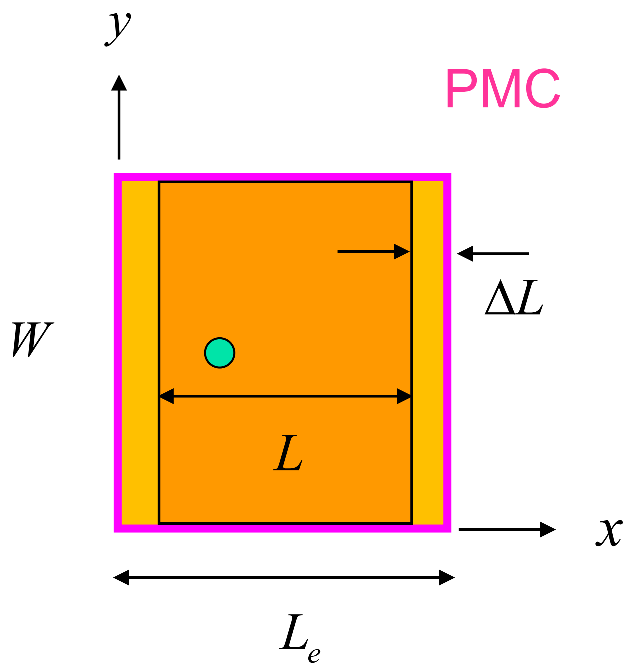
Basic Principles of Operation

Resonance Frequency of Dominant Mode

The resonance frequency calculation can be improved by adding a "fringing length extension" ΔL to each edge of the patch to get an "effective length" L_e .

$$L_e = L + 2\Delta L$$

$$f_{10} = \frac{c}{2\sqrt{\varepsilon_r}} \left(\frac{1}{L_e} \right)$$



Note: Some authors use *effective permittivity* in this equation.
(This would change the value of L_e.)

Basic Principles of Operation

Resonance Frequency of Dominant Mode

Hammerstad formula:

$$\Delta L / h = 0.412 \left[\frac{\left(\varepsilon_r^{\text{eff}} + 0.3\right)\left(\dfrac{W}{h} + 0.264\right)}{\left(\varepsilon_r^{\text{eff}} - 0.258\right)\left(\dfrac{W}{h} + 0.8\right)} \right]$$

$$\varepsilon_r^{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left[1 + 12\left(\frac{h}{W}\right)\right]^{-1/2}$$

(This formula is also used for a microstrip line.
(It treats the patch as a wide microstrip line of width W.)

Note:
Even though the Hammerstad formula involves an effective permittivity, we still use the *actual substrate permittivity* in the resonance frequency formula.

$$f_{10} = \frac{c}{2\sqrt{\varepsilon_r}}\left(\frac{1}{L + 2\Delta L}\right)$$

Basic Principles of Operation

Resonance Frequency of Dominant Mode

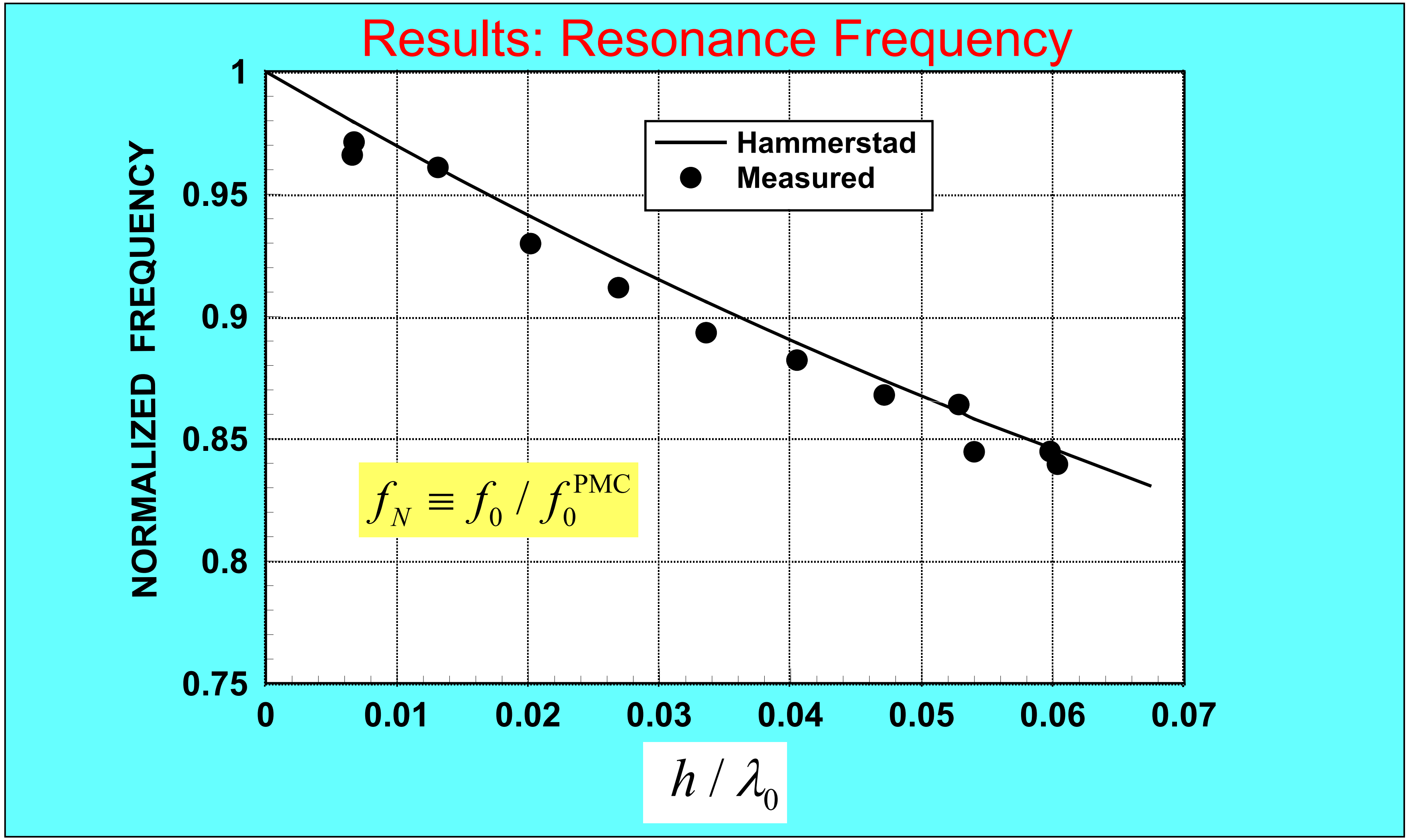
Note: $\Delta L \approx 0.5\ h$

This is a good “rule of thumb” to give a quick estimate.

Basic Principles of Operation

$\varepsilon_r = 2.2$

$W / L = 1.5$



The resonance frequency f_0 has been normalized by the zero-order value f_0^{PMC} (without fringing).

Outline

- Overview of microstrip antennas
- History of microstrip antennas
- Feeding methods
- Basic principles of operation
- General characteristics
- CAD formulas
- Input impedance
- Radiation pattern
- Circular polarization
- Circular patch
- Improving bandwidth
- Miniaturization
- Reducing surface waves and lateral radiation

General Characteristics

Bandwidth

- The bandwidth is directly proportional to the substrate thickness h.

- However, if h is greater than about $0.05\ \lambda_0$, the probe inductance (for a coaxial feed) becomes large enough so that matching is difficult – the bandwidth will decrease.

- The bandwidth is inversely proportional to ε_r (a low permittivity foam substrate gives a high bandwidth).

- The bandwidth of a rectangular patch is proportional to the patch width W (but we need to keep $W < 2L$; please see the next slide).

These properties will become clear after we see the CAD formulas.

General Characteristics

Width Restriction for a Rectangular Patch

$$W < 2L$$

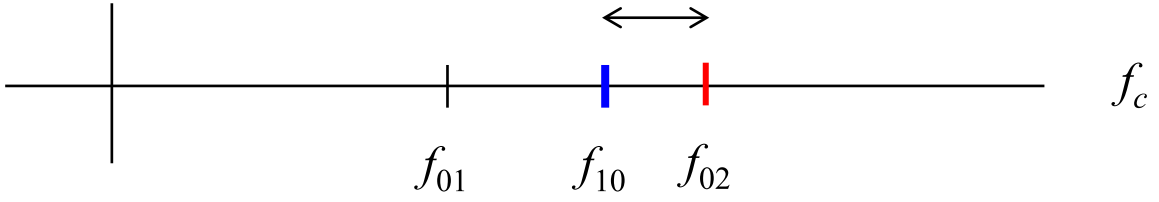
$$f_{mn} = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2}$$

$$f_{01} = \frac{c}{2\sqrt{\varepsilon_r}}\left(\frac{1}{W}\right)$$

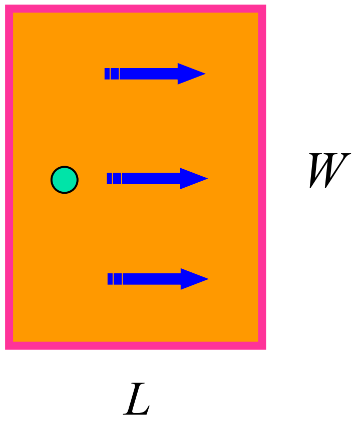
$$f_{10} = \frac{c}{2\sqrt{\varepsilon_r}}\left(\frac{1}{L}\right)$$

$$f_{02} = \frac{c}{2\sqrt{\varepsilon_r}}\left(\frac{2}{W}\right)$$

$$f_{02} - f_{10} = \frac{c}{\sqrt{\varepsilon_r}}\left(\frac{1}{W} - \frac{1}{2L}\right)$$



$W = 1.5\,L$ is typical.



General Characteristics

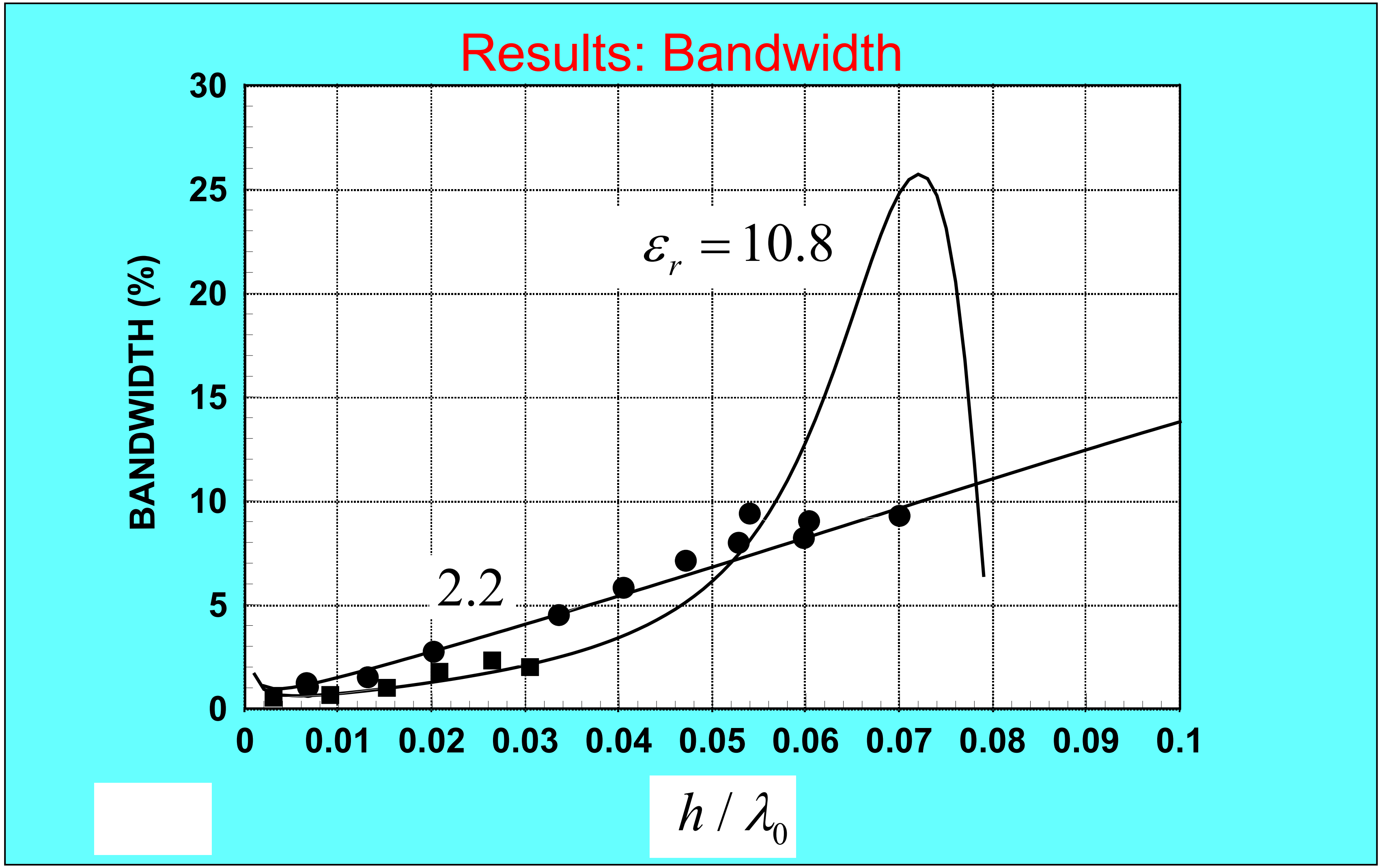
Some Bandwidth Observations

- For a typical substrate thickness ($h/\lambda_0 = 0.02$), and a typical substrate permittivity ($\varepsilon_r = 2.2$) the bandwidth is about 3%.
- By using a thick foam substrate, bandwidth of about 10% can be achieved.
- By using special feeding techniques (aperture coupling) and stacked patches, bandwidths of 100% have been achieved.

General Characteristics

$\varepsilon_r = 2.2$ or 10.8

$W / L = 1.5$



The discrete data points are measured values.
The solid curves are from a CAD formula (given later).

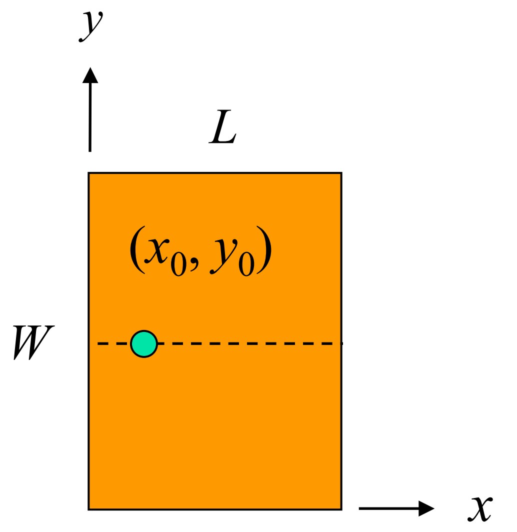
General Characteristics

Resonant Input Resistance

- The resonant input resistance is fairly independent of the substrate thickness h, unless h gets small (the variation is then mainly due to dielectric and conductor loss)*.

- The resonant input resistance is proportional to ε_r.

- The resonant input resistance is directly controlled by the x-axis location of the feed point (maximum at edges $x = 0$ or $x = L$, zero at center of patch).



* If there is no conductor or dielectric loss, then the input resonance at resonance approaches a constant as the substrate thickness tends to zero.

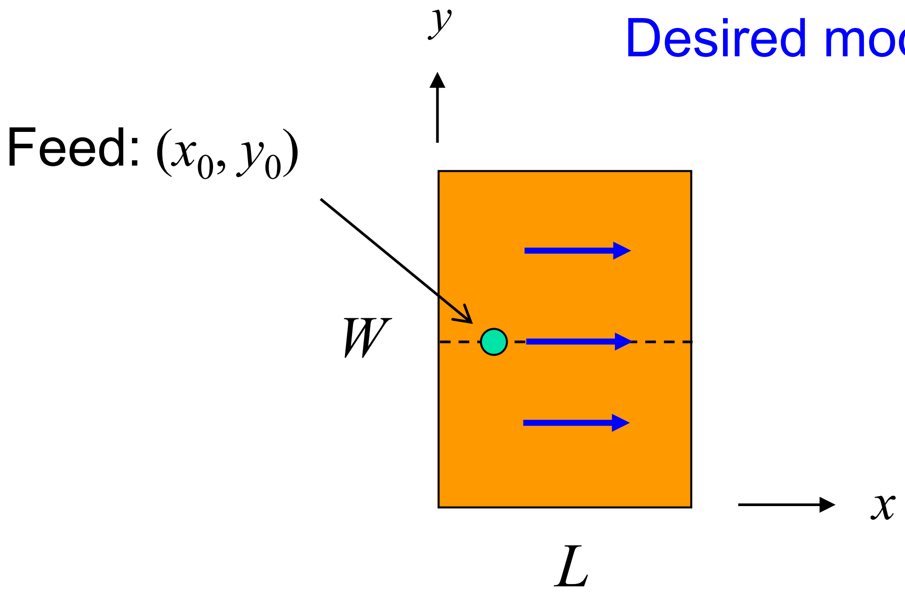
These properties will become clear after we see the CAD formulas.

General Characteristics

Resonant Input Resistance (cont.)

Note:
The patch is usually fed along the <u>centerline</u> ($y_0 = W/2$)
to maintain symmetry and thus minimize excitation of undesirable modes (which cause cross-pol).

Desired mode: (1,0)



The TM_{01} mode is not excited for a feed on the centerline.

$$E_z^{(1,0)} = \cos\left(\frac{\pi x}{L}\right)$$

$$E_z^{(0,1)} = \cos\left(\frac{\pi y}{W}\right)$$ (zero on centerline)

General Characteristics

Resonant Input Resistance (cont.)

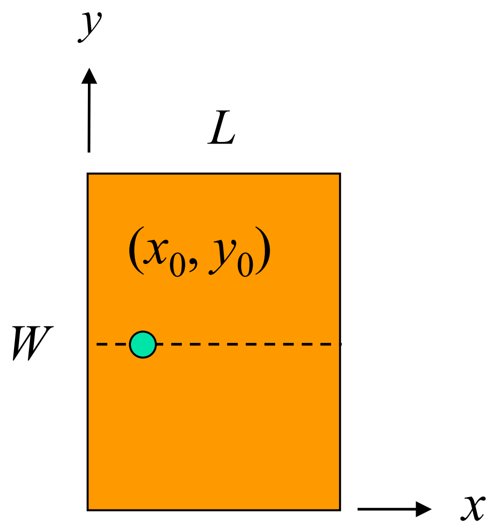
For a given mode, it can be shown that the resonant input resistance is proportional to the square of the cavity-mode field at the feed point.

This is seen from the cavity-model eigenfunction analysis (please see the reference).

$$\boxed{R_{\text{in}} \propto E_z^2\left(x_0, y_0\right)}$$

For $(1,0)$ mode:

$$R_{\text{in}} \propto \cos^2\left(\frac{\pi x_0}{L}\right)$$



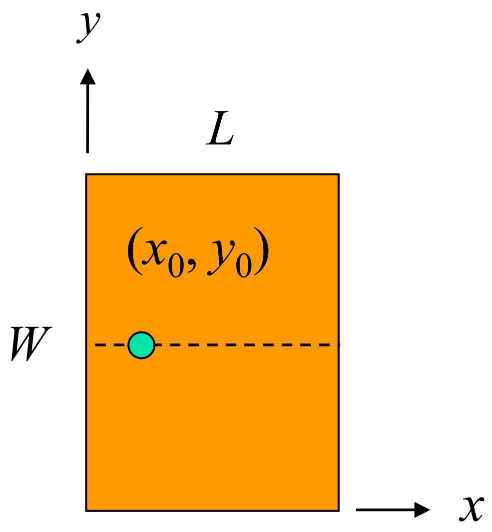
Y. T. Lo, D. Solomon, and W. F. Richards, "Theory and Experiment on Microstrip Antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-27, no. 3 (March 1979): 137–145.

General Characteristics

Resonant Input Resistance (cont.)

Hence, for (1,0) mode:

$$R_{\text{in}} = R_{\text{edge}} \cos^2\left(\frac{\pi x_0}{L}\right)$$

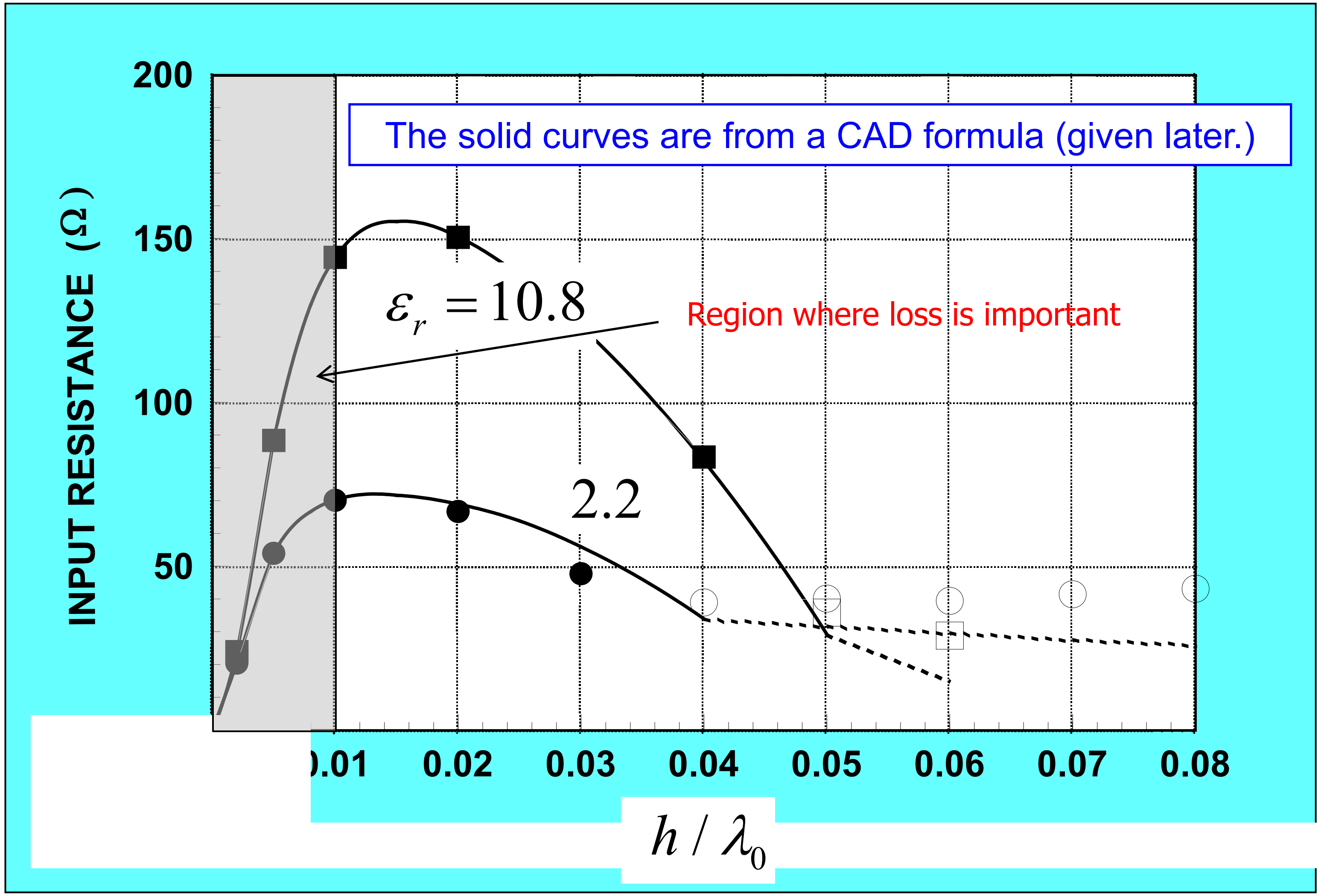


The value of R_{edge} depends strongly on the substrate permittivity (it is proportional to the permittivity).

For a typical patch, it is often in the range of 150-200 Ohms.

General Characteristics

Results: Resonant Input Resistance



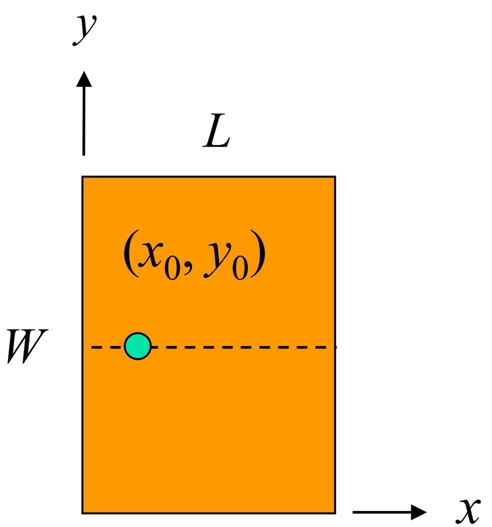
$\varepsilon_r = 2.2$ or 10.8

$W / L = 1.5$

$y_0 = W / 2$

$x_0 = L / 4$

(not fed on edge)



Note: The hollow symbols indicate that the probe reactance is too large for the patch to be resonant (input reactance is zero).

General Characteristics

Radiation Efficiency

- Radiation efficiency is the ratio of power radiated into space, to the total input power.

$$e_r = \frac{P_{\text{sp}}}{P_{\text{tot}}}$$

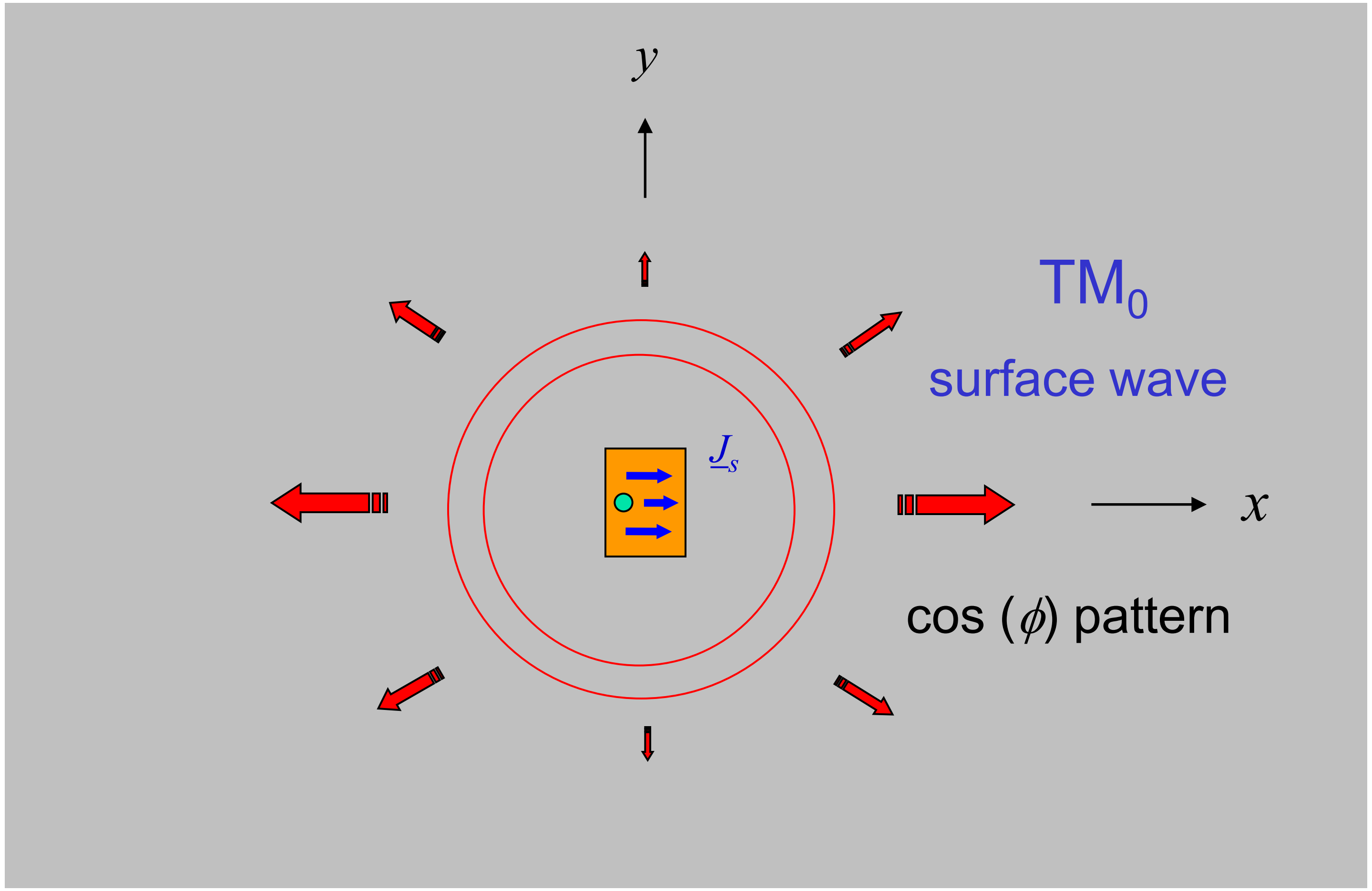
P_{sp} = power radiated into space $\quad$ P_{tot} = total input power

- The radiation efficiency is less than 100% due to:
  - Conductor loss
  - Dielectric loss
  - Surface-wave excitation*

*assuming the substrate is infinite

General Characteristics

Radiation Efficiency (cont.)



General Characteristics

Radiation Efficiency (cont.)

Hence,

$$e_r = \frac{P_{\text{sp}}}{P_{\text{tot}}} = \frac{P_{\text{sp}}}{P_{\text{sp}} + \left(P_c + P_d + P_{\text{sw}}\right)}$$

P_{sp} = power radiated into space

P_{tot} = total input power

P_c = power dissipated by conductors

P_d = power dissipated by dielectric

P_{sw} = power launched into surface wave*

* This assumes an infinite substrate or one with absorber at the edges, so the surface-wave power is treated as a lost power.

General Characteristics

Radiation Efficiency (cont.)

Some observations:

- Conductor and dielectric loss is more important for thinner substrates (the Q of the cavity is higher, and thus the resonance is more seriously affected by loss).
- Conductor loss increases with frequency (proportional to $f^{1/2}$) due to the skin effect. It can be very serious at millimeter-wave frequencies.
- Conductor loss is usually more important than dielectric loss for typical substrate thicknesses and loss tangents.

These properties will become clear after we see the CAD formulas.

$$R_s = \frac{1}{\sigma\delta} \quad , \quad \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

R_s is the surface resistance of the metal. The skin depth of the metal is δ.

$$\Rightarrow \quad R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} \propto \sqrt{f}$$

General Characteristics

Radiation Efficiency (cont.)

- Surface-wave power is more important for thicker substrates or for higher-substrate permittivities. (The surface-wave power can be minimized by using a thin substrate or a foam substrate.)

  - For a foam substrate, a high radiation efficiency is obtained by making the substrate thicker (minimizing the conductor and dielectric losses). There is no surface-wave power to worry about by making the substrate thicker. (There is still probe inductance to worry about.)

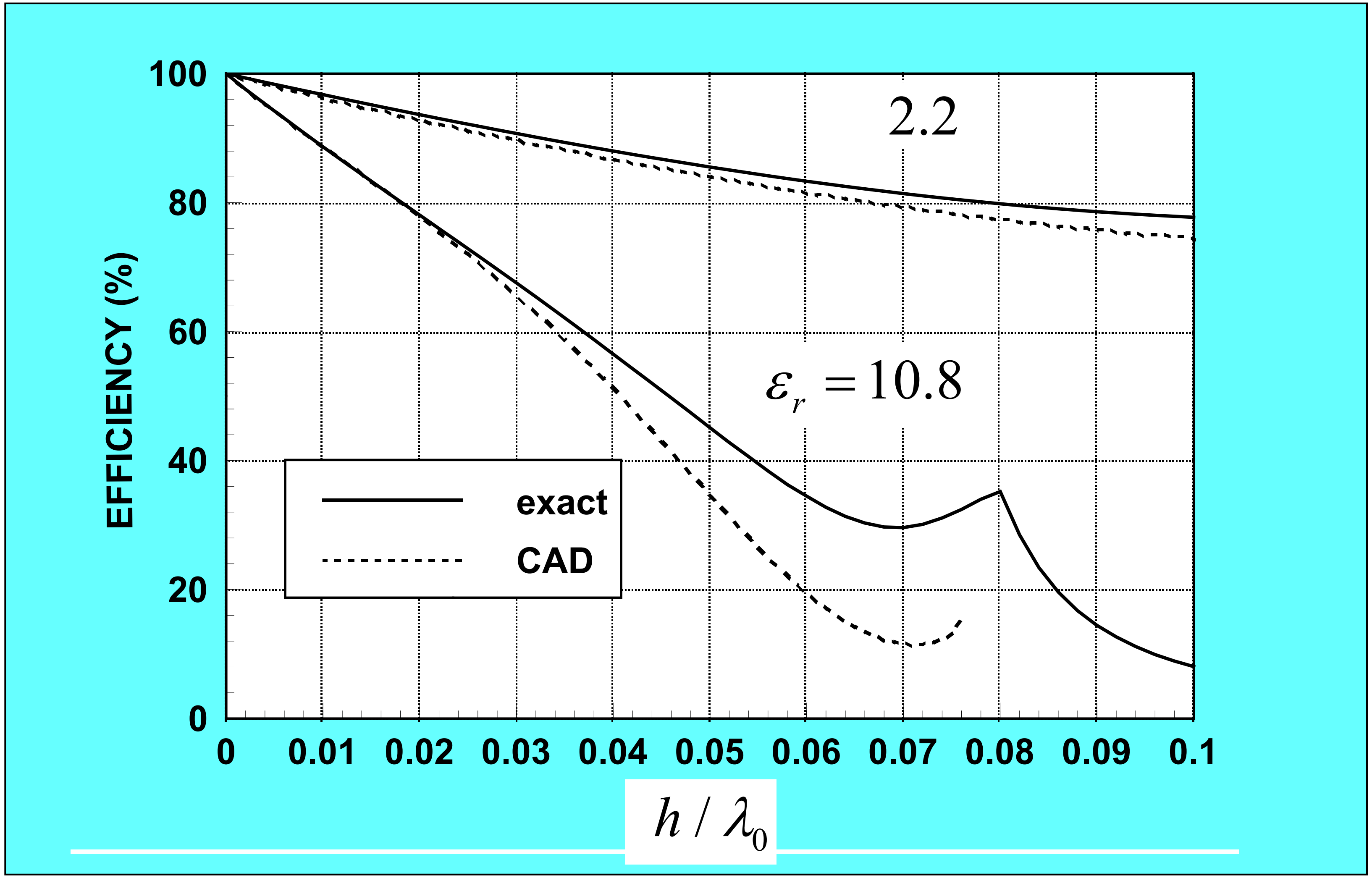
  - For a typical substrate such as $\varepsilon_r = 2.2$, the radiation efficiency is maximum for $h / \lambda_0 \approx 0.02$.

General Characteristics

Results: Efficiency (Conductor and dielectric losses are neglected.)

$\varepsilon_r = 2.2$ or 10.8

$W / L = 1.5$



The loss of efficiency here is due only to the surface wave.

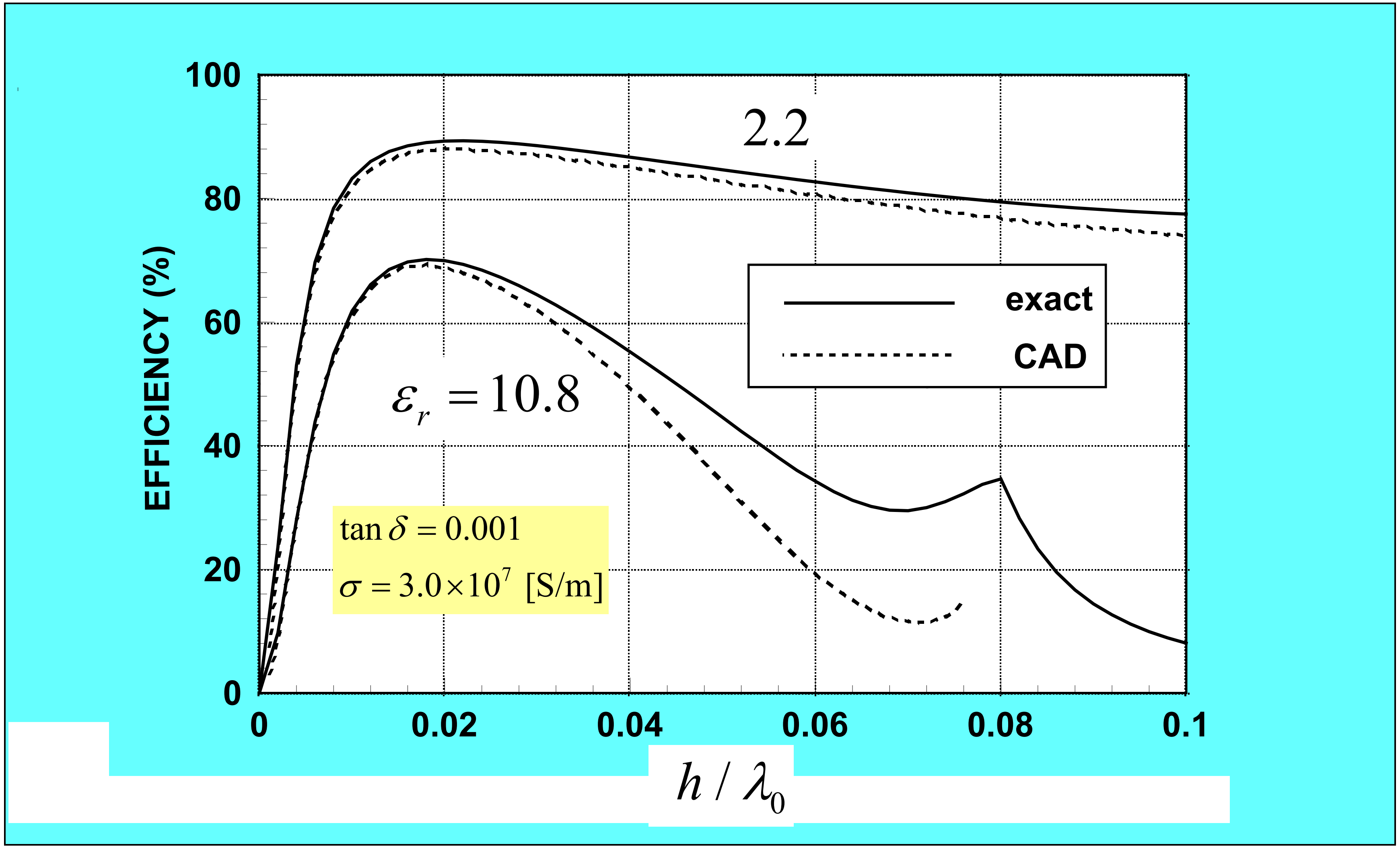
Note: CAD plot uses the Pozar formula (given later).

General Characteristics

Results: Efficiency (All losses are accounted for.)

$\varepsilon_r = 2.2$ or 10.8

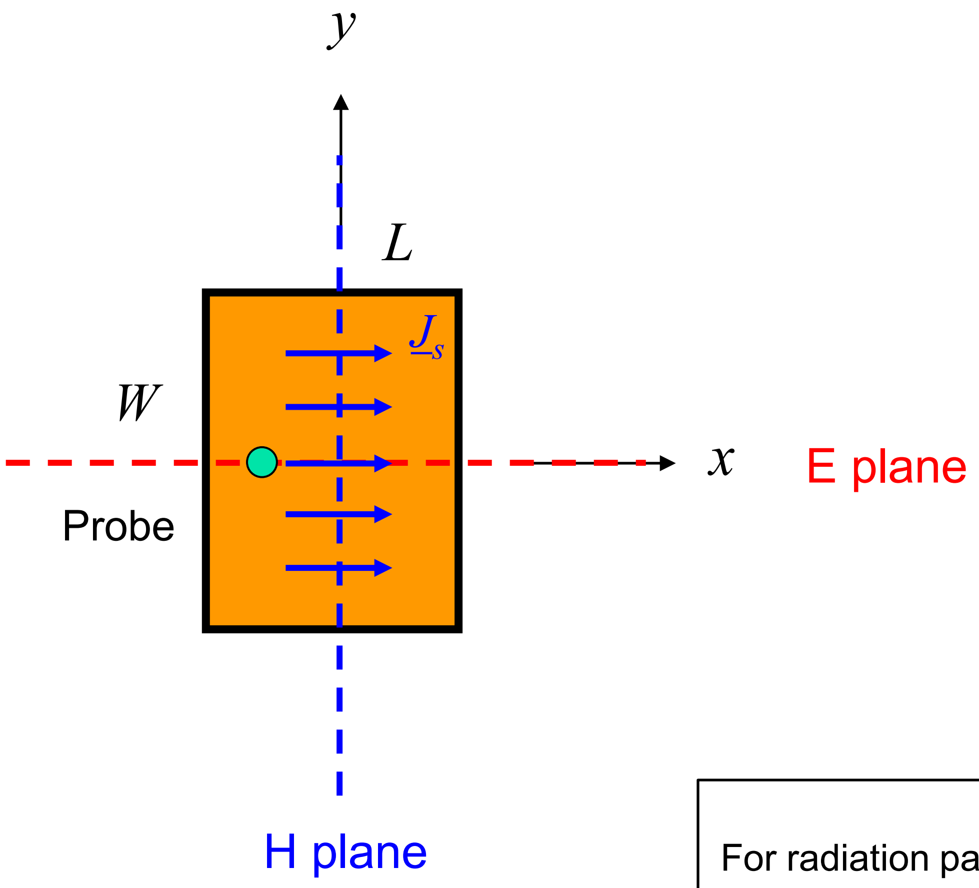
$W / L = 1.5$



Note: CAD plot uses the Pozar formula (given later).

General Characteristics

Radiation Pattern



E-plane: co-pol is E_θ

H-plane: co-pol is E_ϕ

Note:
For radiation patterns, it is usually more convenient to place the origin at the middle of the patch.
(This keeps the formulas as simple as possible.)

General Characteristics

Radiation Patterns (cont.)

Comments on radiation patterns:

- The E-plane pattern is typically broader than the H-plane pattern.

- The truncation of the ground plane will cause edge diffraction, which tends to degrade the pattern by introducing:

  - Rippling in the forward direction
  - Back-radiation

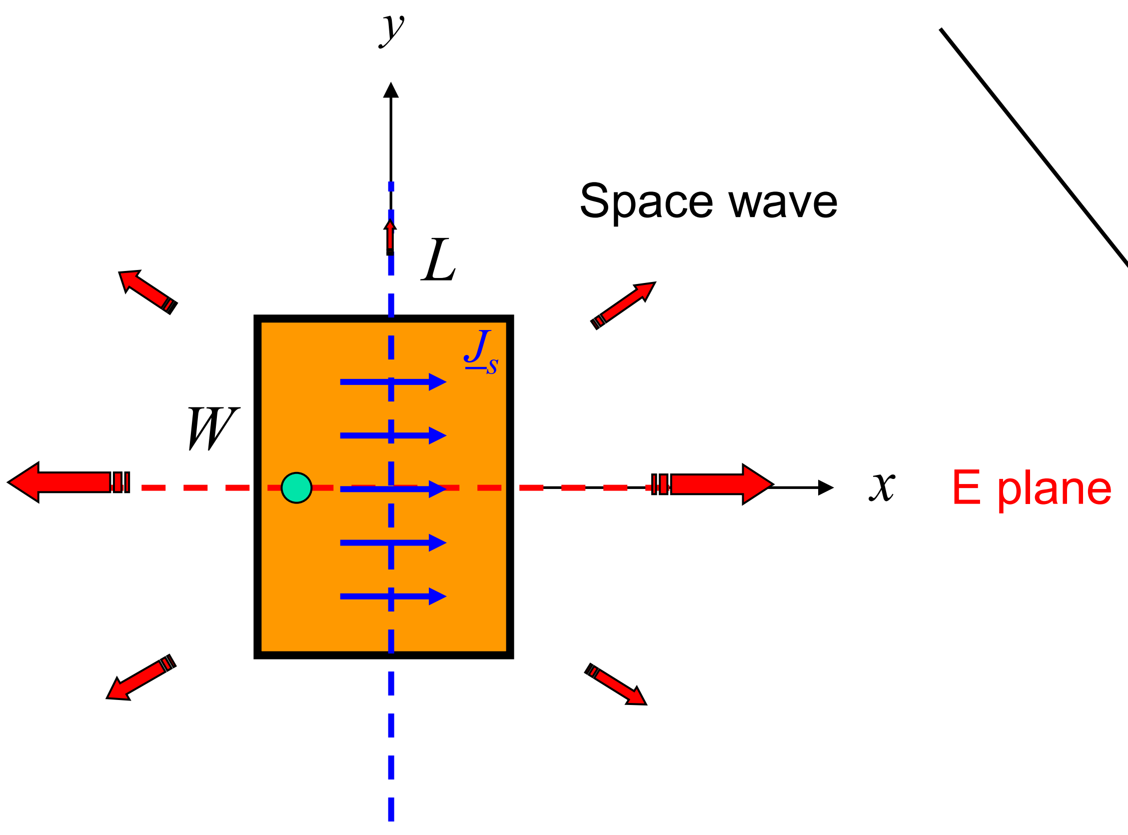
- Pattern distortion is more severe in the E-plane, due to the angle dependence of the vertically-polarized field E_θ that is radiated by the patch along the ground plane. (It varies as cos (ϕ)).

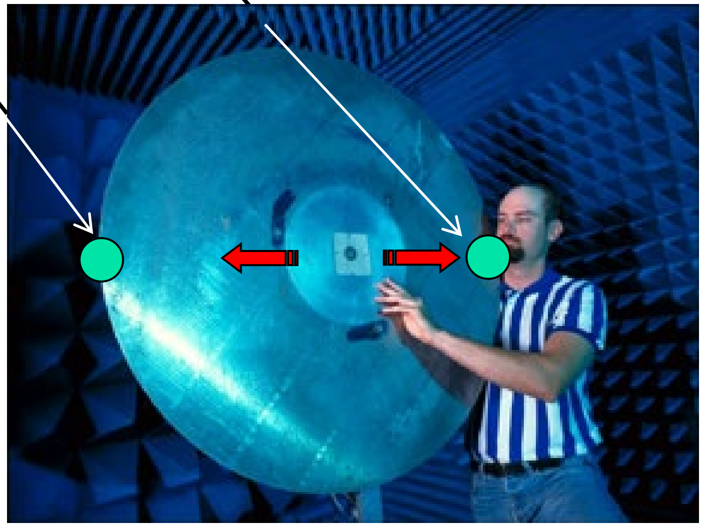
General Characteristics

Radiation Patterns

Edge diffraction is the most serious in the E plane.

E_θ varies as $\cos\phi$





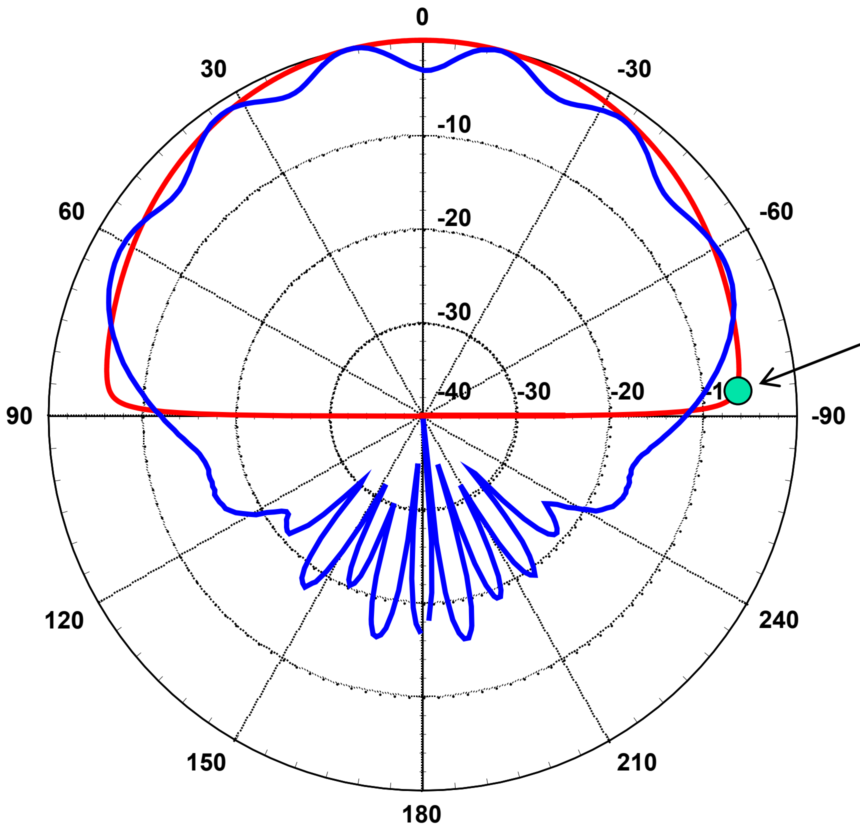
General Characteristics

Radiation Patterns

E-plane pattern

Red: infinite substrate and ground plane

Blue: 1 meter ground plane



Note:
The E-plane pattern “tucks in” and tends to zero at the horizon due to the presence of the infinite substrate. (It would not tuck in if the substrate were truncated at the patch edges.)

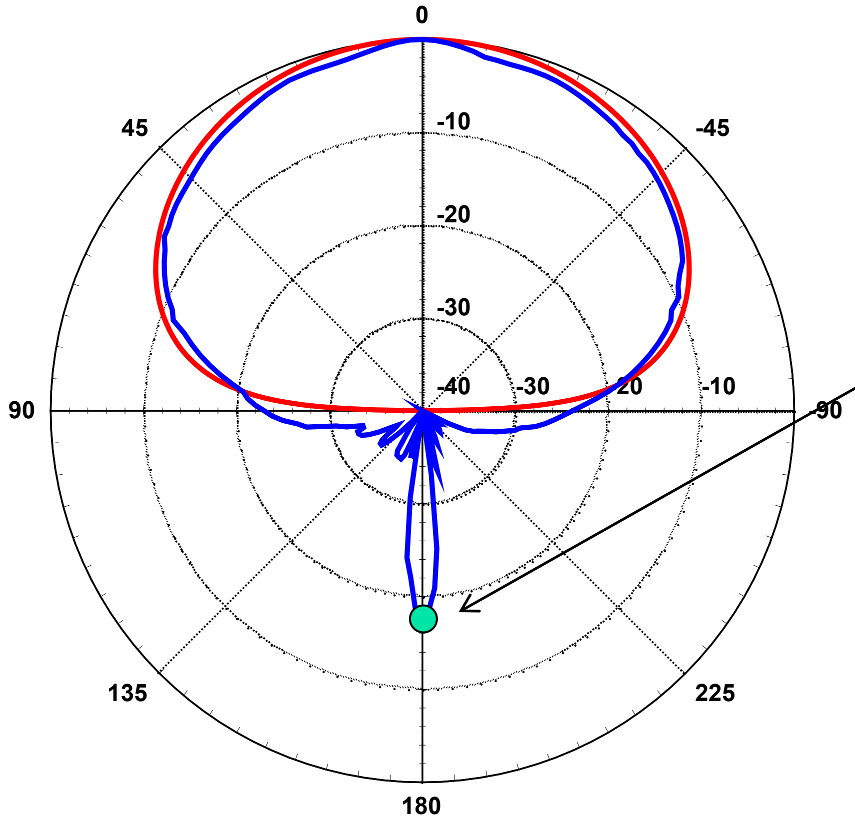
General Characteristics

Radiation Patterns

H-plane pattern

Red: infinite substrate and ground plane

Blue: 1 meter ground plane



Note:
E-plane diffraction only contributes to the H plane in the back boresight direction.

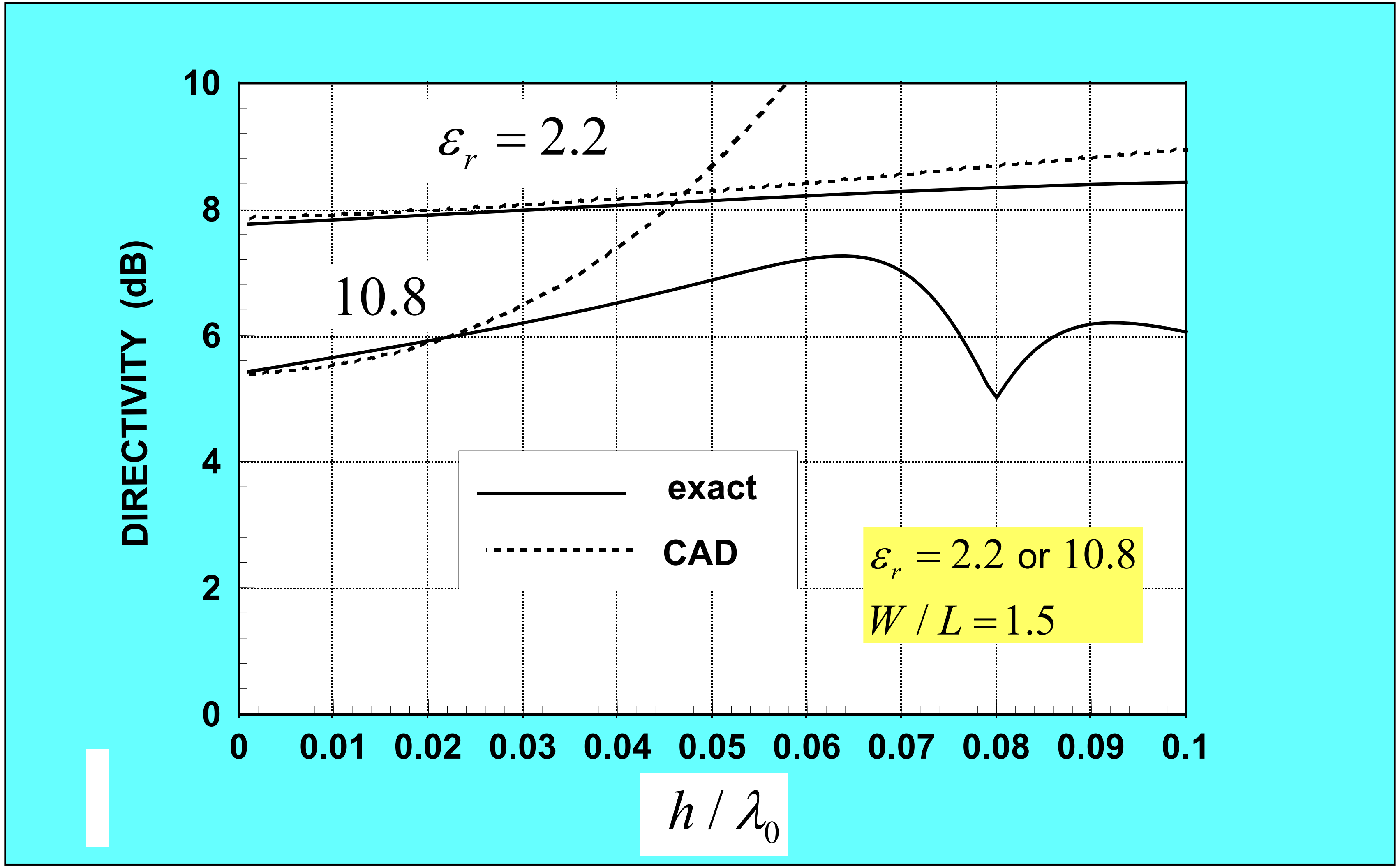
General Characteristics

Directivity

- The directivity is fairly insensitive to the substrate thickness.
- The directivity is higher for lower permittivity, because the patch is larger.

General Characteristics

Results: Directivity (relative to isotropic)



Outline

- Overview of microstrip antennas
- History of microstrip antennas
- Feeding methods
- Basic principles of operation
- General characteristics
- CAD formulas
- Input impedance
- Radiation pattern
- Circular polarization
- Circular patch
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- Reducing surface waves and lateral radiation

CAD Formulas

CAD formulas for the important properties of the *rectangular microstrip antenna* will be shown.

- Radiation efficiency
- Bandwidth (Q)
- Resonant input resistance
- Directivity

These formulas are derived (in ECE 6345) from making approximations to the exact expression for the radiated power, assuming a thin substrate.

- D. R. Jackson, "Microstrip Antennas," Chapter 7 of *Antenna Engineering Handbook*, 5th Ed., J. L. Volakis, Editor, McGraw Hill, 2019.
- D. R. Jackson, S. A. Long, J. T. Williams, and V. B. Davis, "Computer-Aided Design of Rectangular Microstrip Antennas," Ch. 5 of *Advances in Microstrip and Printed Antennas*, K. F. Lee and W. Chen, Eds., John Wiley, 1997.
- D. R. Jackson and N. G. Alexopoulos, "Simple Approximate Formulas for Input Resistance, Bandwidth, and Efficiency of a Resonant Rectangular Patch," *IEEE Trans. Antennas and Propagation*, Vol. 39, pp. 407-410, March 1991.

CAD Formulas

Radiation Efficiency

$$e_r = \frac{e_r^{\text{hed}}}{1 + e_r^{\text{hed}}\left[\ell_d + \left(\frac{R_s^{\text{ave}}}{\pi \eta_0}\right)\left(\frac{1}{h/\lambda_0}\right)\right]\left[\left(\frac{3}{16}\right)\left(\frac{\varepsilon_r}{p\, c_1}\right)\left(\frac{L}{W}\right)\left(\frac{1}{h/\lambda_0}\right)\right]}$$

Comment:
The efficiency becomes small as the substrate gets thin, if there is dielectric or conductor loss.

where

HED

$\ell_d = \tan\delta =$ loss tangent of substrate

$R_s =$ surface resistance of metal $= \dfrac{1}{\sigma\delta} = \sqrt{\dfrac{\omega\mu}{2\sigma}}$

$$R_s^{\text{ave}} = \left(R_s^{\text{patch}} + R_s^{\text{ground}}\right)/2$$

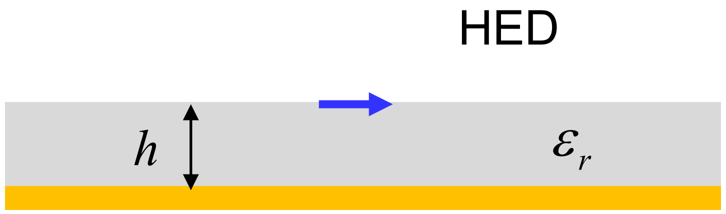
The surface-wave efficiency is approximated as that of the HED.

Note: "hed" refers to a unit-amplitude horizontal electric dipole (HED).

CAD Formulas

Radiation Efficiency (cont.)

$$e_r^{\text{hed}} = \frac{P_{\text{sp}}^{\text{hed}}}{P_{\text{sp}}^{\text{hed}} + P_{\text{sw}}^{\text{hed}}} = \frac{1}{1 + \dfrac{P_{\text{sw}}^{\text{hed}}}{P_{\text{sp}}^{\text{hed}}}}$$



where

$$P_{\text{sp}}^{\text{hed}} = \frac{1}{\lambda_0^2}(k_0 h)^2 \left(80\pi^2 c_1\right)$$

$$P_{\text{sw}}^{\text{hed}} = \frac{1}{\lambda_0^2}(k_0 h)^3 \left[60\pi^3 \left(1 - \frac{1}{\varepsilon_r}\right)^3\right]$$

Note: "hed" refers to a unit-amplitude horizontal electric dipole (HED).

Note: When we say "unit amplitude" here, we assume peak (not RMS) values.

CAD Formulas

Radiation Efficiency (cont.)

Hence, we have:

$$e_r^{\text{hed}} = \frac{1}{1+\frac{3}{4}\pi\left(k_0 h\right)\left(\frac{1}{c_1}\right)\left(1-\frac{1}{\varepsilon_r}\right)^3}$$

Physically, this term is the radiation efficiency of a horizontal electric dipole (HED) on top of the substrate.

CAD Formulas

Radiation Efficiency (cont.)

The constants are defined as follows:

$$c_1 = 1 - \frac{1}{\varepsilon_r} + \frac{2/5}{\varepsilon_r^2}$$

$$p = 1 + \frac{a_2}{10}\left(k_0 W\right)^2 + \left(a_2^2 + 2a_4\right)\left(\frac{3}{560}\right)\left(k_0 W\right)^4 + c_2\left(\frac{1}{5}\right)\left(k_0 L\right)^2 + a_2 c_2\left(\frac{1}{70}\right)\left(k_0 W\right)^2\left(k_0 L\right)^2$$

$$c_2 = -0.0914153$$

$$a_2 = -0.16605$$

$$a_4 = 0.00761$$

CAD Formulas

Improved formula for HED surface-wave power (due to Pozar)

$$P_{\mathrm{sw}}^{\mathrm{hed}} = \frac{\eta_0 k_0^2}{8} \frac{\varepsilon_r \left(x_0^2 - 1\right)^{3/2}}{\varepsilon_r \left(1 + x_1\right) + (k_0 h)\sqrt{x_0^2 - 1}\left(1 + \varepsilon_r^2 x_1\right)}$$

Note: x_0 in this formula is not the feed location!

$$x_1 = \frac{x_0^2 - 1}{\varepsilon_r - x_0^2} \qquad x_0 = 1 + \frac{-\varepsilon_r^2 + \alpha_0 \alpha_1 + \varepsilon_r \sqrt{\varepsilon_r^2 - 2\alpha_0 \alpha_1 + \alpha_0^2}}{\varepsilon_r^2 - \alpha_1^2}$$

$$\alpha_0 = s \tan\left[(k_0 h) s\right] \qquad \alpha_1 = -\frac{1}{s}\left[\tan\left[(k_0 h) s\right] + \frac{(k_0 h) s}{\cos^2\left[(k_0 h) s\right]}\right]$$

$$s = \sqrt{\varepsilon_r - 1}$$

Note: The above formula for the surface-wave power is different from that given in Pozar's paper by a factor of 2, since Pozar used RMS instead of peak values.

D. M. Pozar, "Rigorous Closed-Form Expressions for the Surface-Wave Loss of Printed Antennas," *Electronics Letters,* vol. 26, pp. 954-956, June 1990.

CAD Formulas

Bandwidth

$$\mathrm{BW} = \frac{1}{\sqrt{2}}\left[\ell_d + \left(\frac{R_s^{\mathrm{ave}}}{\pi\eta_0}\right)\left(\frac{1}{h/\lambda_0}\right) + \left(\frac{16}{3}\right)\left(\frac{p\,c_1}{\varepsilon_r}\right)\left(\frac{h}{\lambda_0}\right)\left(\frac{W}{L}\right)\left(\frac{1}{e_r^{\mathrm{hed}}}\right)\right]$$

$$\mathrm{BW} = \frac{1}{\sqrt{2}Q}$$

$\ell_d = \tan\delta =$ loss tangent of substrate

$R_s^{\mathrm{ave}} = \left(R_s^{\mathrm{patch}} + R_s^{\mathrm{ground}}\right)/2$

Comments:

For a lossless patch, the bandwidth is approximately proportional to the patch width and to the substrate thickness. It is inversely proportional to the substrate permittivity.

For very thin substrates the bandwidth will increase for a lossy patch, but as the expense of efficiency.

The (relative) BW is defined from the frequency limits f_1 and f_2 at which $\mathrm{SWR} = 2.0$.

$$\mathrm{BW} = \frac{f_2 - f_1}{f_0}$$

(multiply by 100 if you want to get % bandwidth)

CAD Formulas

Quality Factor Q

$$Q \equiv \omega_0 \frac{U_s}{P}$$

U_s = energy stored in patch cavity

P = power that is radiated and dissipated by patch

$$\frac{1}{Q} = \frac{P}{\omega_0 U_s}$$

$$P = P_d + P_c + P_{\mathrm{sp}} + P_{\mathrm{sw}}$$

$$\Rightarrow \quad \frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{\mathrm{sp}}} + \frac{1}{Q_{\mathrm{sw}}}$$

Note:
If there is no substrate outside the patch, then we can ignore the Q_{sw} term.

CAD Formulas

Q Components

$$Q_d = 1/\ell_d = 1/\tan\delta$$

$\ell_d = \tan\delta =$ loss tangent of substrate

$$Q_c = \left(\frac{\eta_0}{2}\right)\left[\frac{(k_0 h)}{R_s^{\text{ave}}}\right]$$

$$R_s^{\text{ave}} = \left(R_s^{\text{patch}} + R_s^{\text{ground}}\right)/2$$

Note:
The Q_c and Q_d formulas are the same for <u>any</u> shaped patch.

$$Q_{\text{sp}} \approx \frac{3}{16}\left(\frac{\varepsilon_r}{pc_1}\right)\left(\frac{L}{W}\right)\left(\frac{1}{h/\lambda_0}\right)$$

The constants p and c_1 were defined previously.

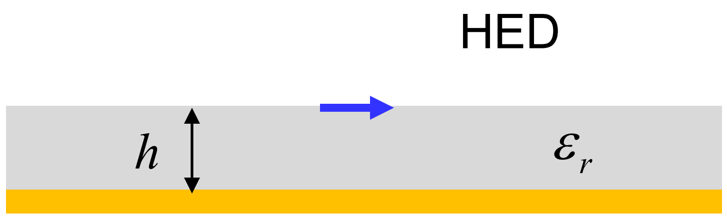
$$Q_{\text{sw}} = Q_{\text{sp}}\left(\frac{e_r^{\text{hed}}}{1-e_r^{\text{hed}}}\right)$$

$$e_r^{\text{hed}} = \frac{1}{1+\frac{3}{4}\pi\left(k_0 h\right)\left(\frac{1}{c_1}\right)\left(1-\frac{1}{\varepsilon_r}\right)^3}$$

Note: "hed" refers to a horizontal electric dipole (HED).

CAD Formulas

Relation Between Q_{sp} and Q_{sw}



Note:
"hed" refers to a horizontal electric dipole.

$$e_r^{\mathrm{hed}} = \frac{P_{\mathrm{sp}}}{P_{\mathrm{tot}}} = \frac{P_{\mathrm{sp}}}{P_{\mathrm{sp}} + P_{\mathrm{sw}}}$$

$$\Rightarrow \frac{1}{e_r^{\mathrm{hed}}} = \frac{P_{\mathrm{sp}} + P_{\mathrm{sw}}}{P_{\mathrm{sp}}} = 1 + \frac{P_{\mathrm{sw}}}{P_{\mathrm{sp}}}$$

$$\Rightarrow \frac{P_{\mathrm{sw}}}{P_{\mathrm{sp}}} = \frac{1}{e_r^{\mathrm{hed}}} - 1$$

$$\Rightarrow \frac{Q_{\mathrm{sp}}}{Q_{\mathrm{sw}}} = \left(\frac{1}{e_r^{\mathrm{hed}}} - 1\right) = \frac{1 - e_r^{\mathrm{hed}}}{e_r^{\mathrm{hed}}}$$

$$\Rightarrow \frac{Q_{\mathrm{sw}}}{Q_{\mathrm{sp}}} = \frac{e_r^{\mathrm{hed}}}{1 - e_r^{\mathrm{hed}}}$$

CAD Formulas

Resonant Input Resistance

Probe-feed Patch

$$R = R_{\text{in}}^{\max} = R_{\text{edge}} \cos^2\left(\frac{\pi x_0}{L}\right)$$

$$R_{\text{edge}} = \frac{\left(\dfrac{4\eta_0}{\pi}\right)\left(\dfrac{L}{W}\right)\left(\dfrac{h}{\lambda_0}\right)}{\ell_d + \left(\dfrac{R_s^{\text{ave}}}{\pi \eta_0}\right)\left(\dfrac{1}{h/\lambda_0}\right) + \left(\dfrac{16}{3}\right)\left(\dfrac{p\, c_1}{\varepsilon_r}\right)\left(\dfrac{W}{L}\right)\left(\dfrac{h}{\lambda_0}\right)\left(\dfrac{1}{e_r^{\text{hed}}}\right)}$$

Comments:

- For a lossless patch, the resonant resistance is approximately independent of the substrate thickness.
- For a lossy patch, it tends to zero as the substrate gets very thin.
- For a lossless patch it is inversely proportional to the square of the patch width, and it is proportional to the substrate permittivity.

CAD Formulas

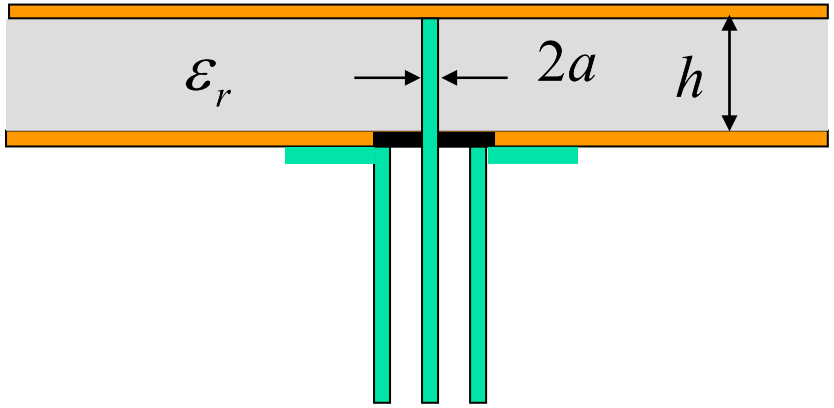
Approximate CAD formula for probe (feed) reactance (in Ohms)

a = probe radius $\qquad$ h = probe height

$$X_p = \frac{\eta_0}{2\pi}\left(k_0 h\right)\left[-\gamma + \ln\left(\frac{2}{\sqrt{\varepsilon_r}\left(k_0 a\right)}\right)\right]$$

This is based on an infinite parallel-plate model.

$$X_p = \omega L_p \qquad L_p \approx X_p\left(\omega_0\right)/\,\omega_0$$



$\gamma \doteq 0.577216$ (Euler's constant)

$$\eta_0 = \sqrt{\mu_0 / \varepsilon_0} = 376.7303\ \Omega$$

CAD Formulas

Observations:

- The feed (probe) reactance increases proportionally with substrate thickness h.
- The feed reactance increases for smaller probe radius.

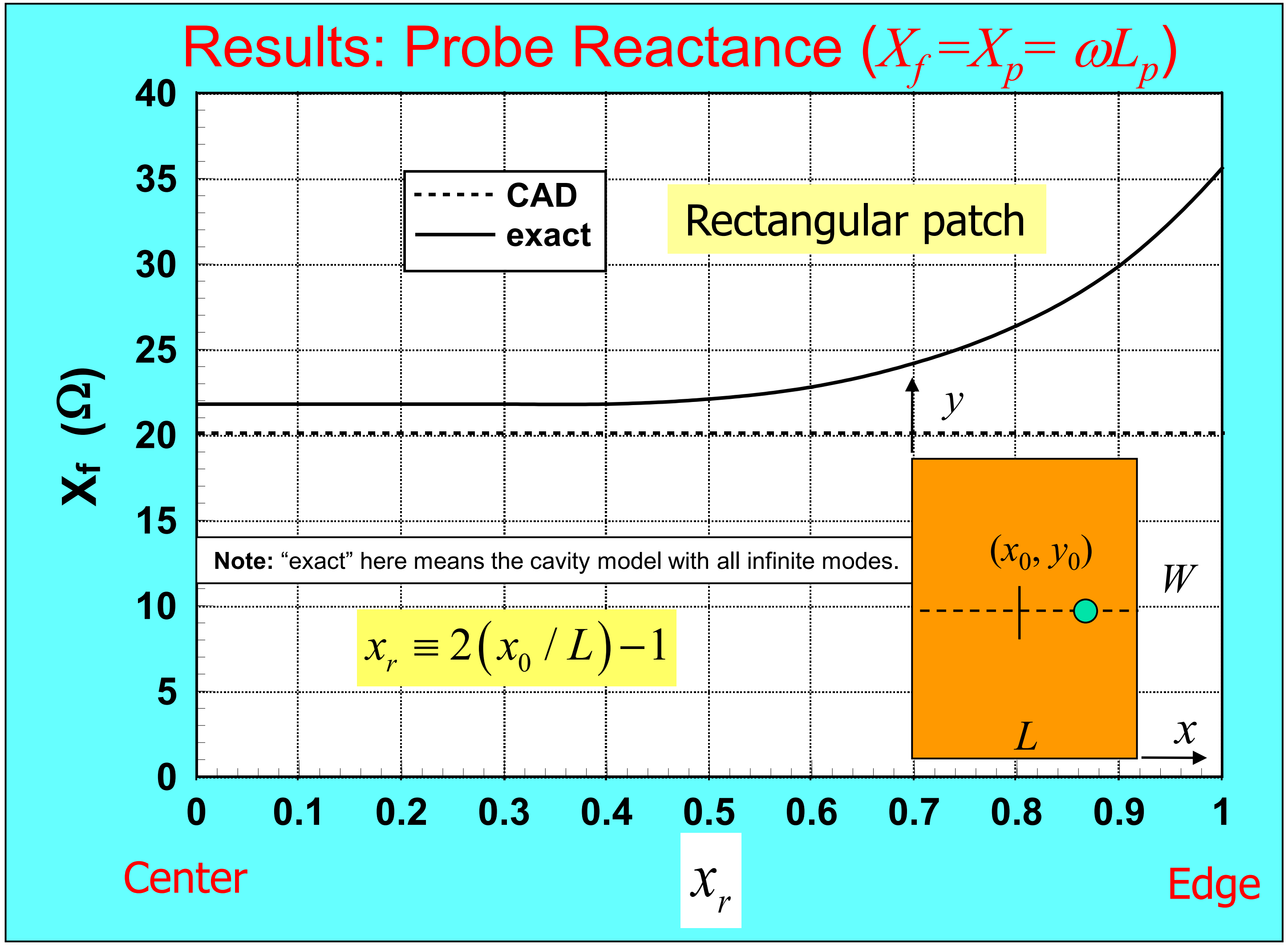
$$X_p = \frac{\eta_0}{2\pi}\left(k_0 h\right)\left[-\gamma + \ln\left(\frac{2}{\sqrt{\varepsilon_r}\left(k_0 a\right)}\right)\right]$$

Important point:

If the substrate gets too thick, the probe reactance will make it difficult to get an input match, and the bandwidth will suffer.

(Compensating techniques will be discussed later.)

CAD Formulas



$\varepsilon_r = 2.2$

$W / L = 1.5$

$h = 0.0254\lambda_0$

$a = 0.5\,\text{mm}$

The normalized feed location ratio x_r is zero at the center of the patch ($x = L/2$), and is 1.0 at the patch edge ($x = L$).

CAD Formulas

Directivity

$$D = \left(\frac{3}{pc_1}\right)\left[\frac{\varepsilon_r}{\varepsilon_r + \tan^2\left(k_1 h\right)}\right]\left(\operatorname{tanc}^2\left(k_1 h\right)\right)$$

$$k_1 = k_0\sqrt{\varepsilon_r}$$

where

$$\operatorname{tanc}\left(x\right) \equiv \tan\left(x\right)/x$$

The constants p and c_1 were defined previously.

CAD Formulas

Directivity (cont.)

For thin substrates:

$$D \approx \frac{3}{p\, c_1}$$

(The directivity is essentially independent of the substrate thickness.)

Outline

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- History of microstrip antennas
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- Basic principles of operation
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- CAD formulas
- Input impedance
- Radiation pattern
- Circular polarization
- Circular patch
- Improving bandwidth
- Miniaturization
- Reducing surface waves and lateral radiation

Input Impedance

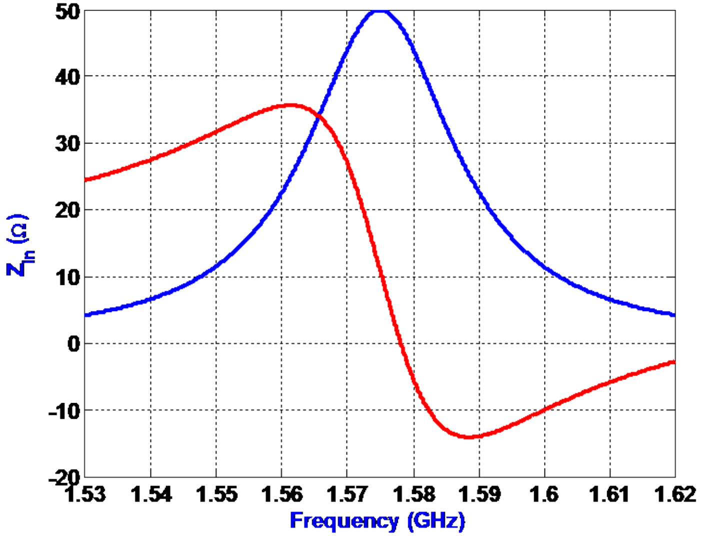
Various models have been proposed over the years for calculating the input impedance of a microstrip patch antenna.

- Transmission line model
  - The first model introduced
  - Very simple
- Cavity model (eigenfunction expansion)
  - Simple yet accurate for thin substrates
  - Gives physical insight into operation
- CAD circuit model
  - Extremely simple and almost as accurate as the cavity model
- Spectral-domain method
  - More challenging to implement
  - Accounts rigorously for both radiation and surface-wave excitation
- Commercial software
  - Very accurate
  - Can be time consuming

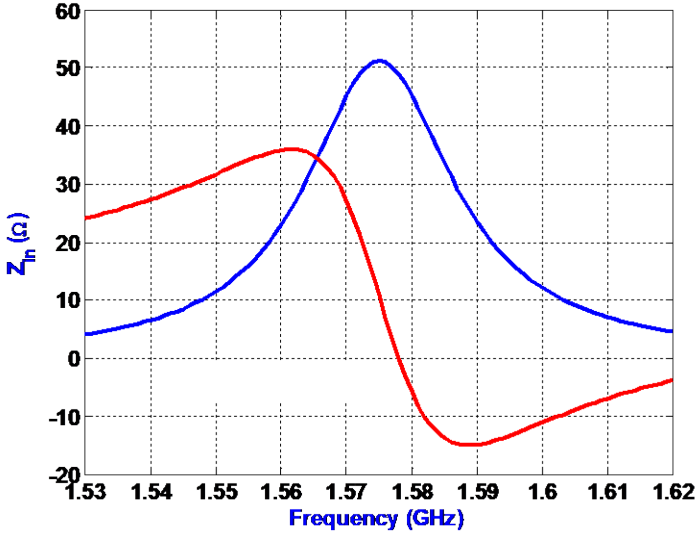
Input Impedance

Comparison of the Three Simplest Models

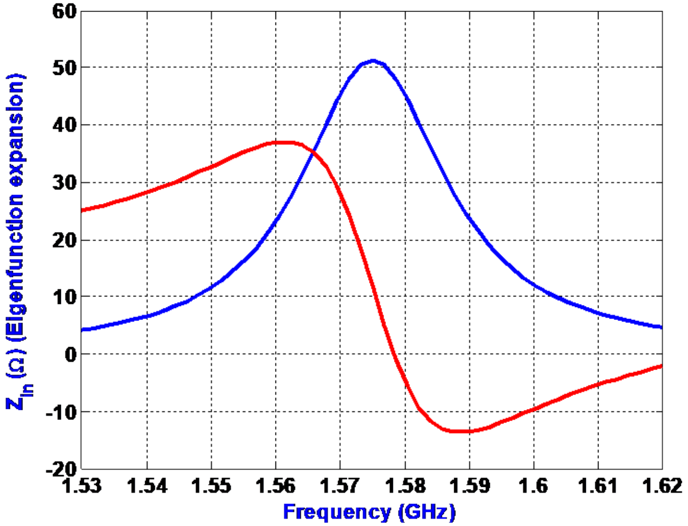
CAD Circuit model of patch



Transmission line model of patch



Cavity model (eigenfunction expansion) of patch



$\varepsilon_r = 2.2$ $\quad L = 6.255$ cm $\quad x_0 = 1.875$ cm

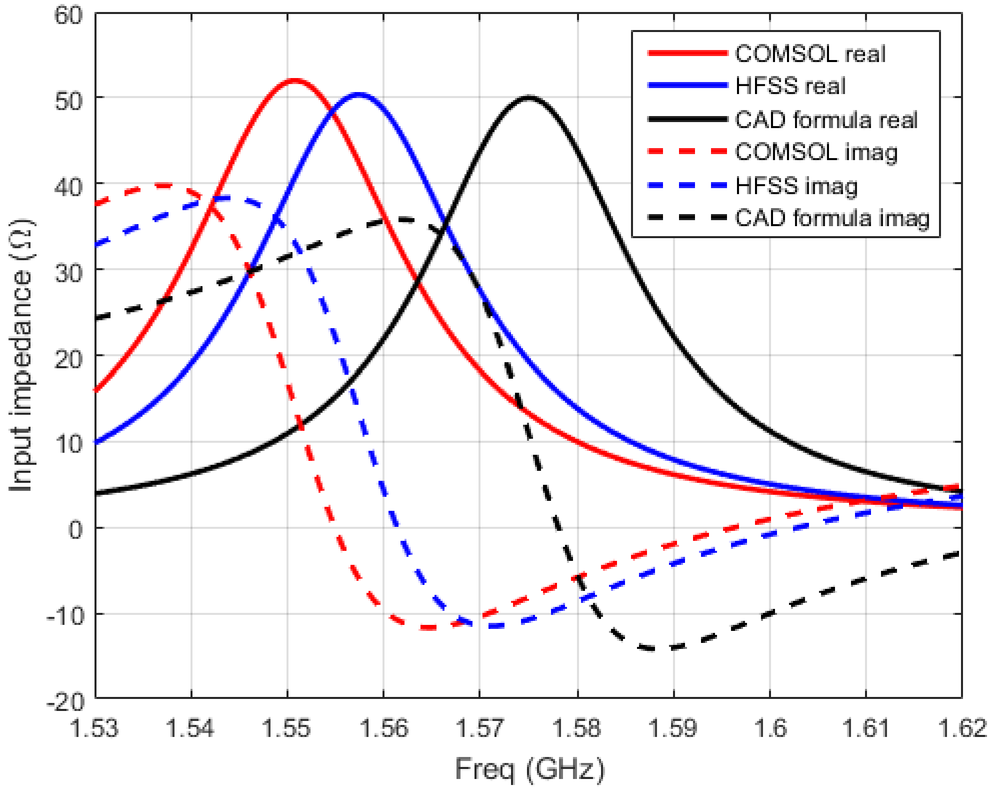
$\tan\delta = 0.001$ $\quad W / L = 1.5$ $\quad y_0 = W / 2$

$h = 1.524$ mm $\quad \sigma = 3.0 \times 10^7$ S/m $\quad a = 0.635$ mm

Results for a typical patch show that the first three methods agree very well, provided the correct Q is used and the probe inductance is accounted for.

Input Impedance

Comparison of CAD with Full-Wave



Results from full-wave analysis agree well with the simple CAD circuit model, except for a shift in resonance frequency.

$\varepsilon_r = 2.2$ $\qquad L = 6.255$ cm $\qquad x_0 = 1.875$ cm

$\tan\delta = 0.001$ $\qquad W / L = 1.5$ $\qquad y_0 = W / 2$

$h = 1.524$ mm $\qquad \sigma = 3.0 \times 10^7$ S/m $\qquad a = 0.635$ mm

Input Impedance

CAD Circuit Model for Input Impedance

The circuit model discussed next assumes a coaxial probe feed.
Other circuit models exist for other types of feeds.

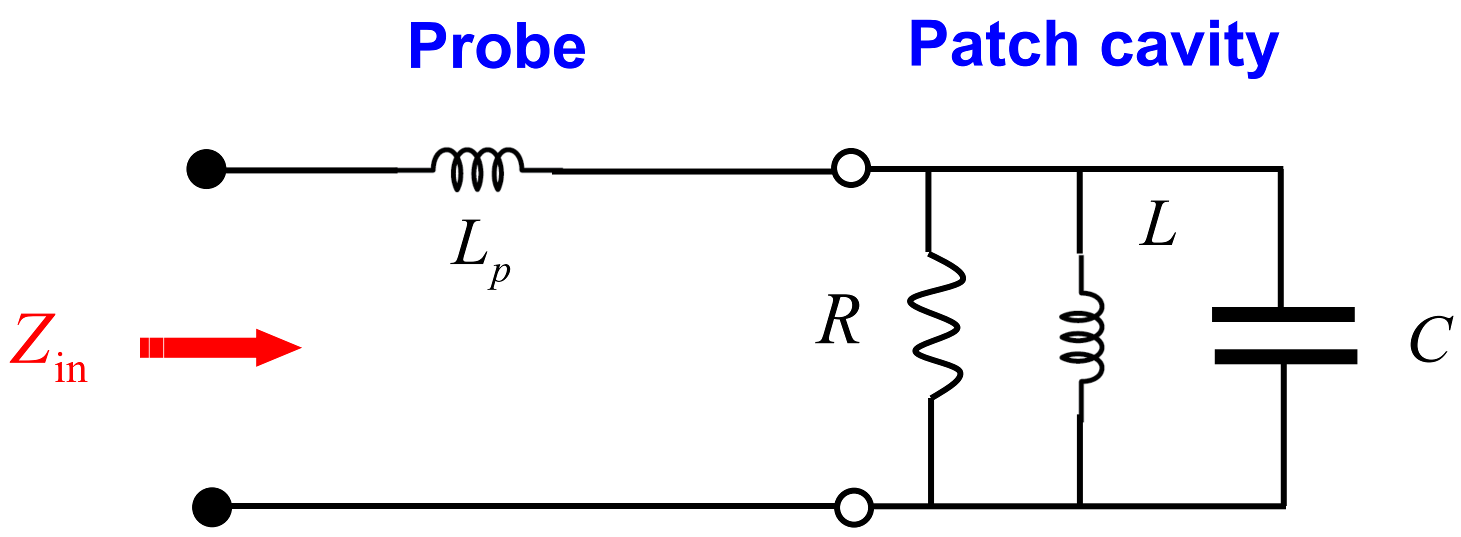
Note:
The mathematical justification of the CAD circuit model comes from a cavity-model eigenfunction analysis
(This analysis is in the ECE 6345 notes.)

Y. T. Lo, D. Solomon, and W. F. Richards, "Theory and Experiment on Microstrip Antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-27, no. 3, pp. 137–145, March 1979.

Input Impedance

Probe-fed Patch

- Near the resonance frequency, the patch cavity can be approximately modeled as a resonant RLC circuit.
- The resistance R accounts for radiation and losses.
- A probe inductance L_p is added in series, to account for the "probe inductance" of a coaxial probe feed.



Input Impedance

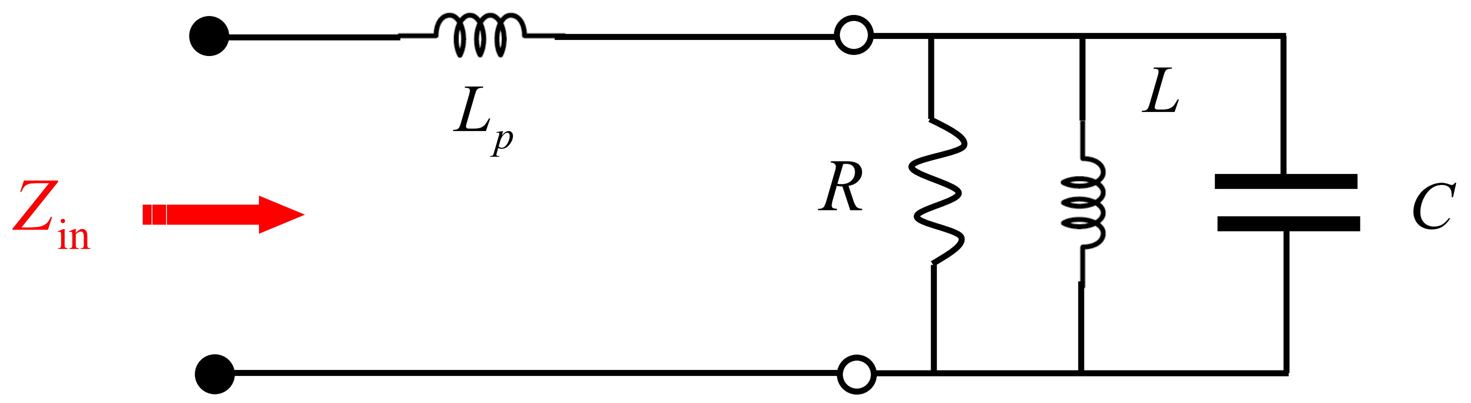
$$Z_{\text{in}} \approx j\omega L_p + \frac{R}{1 + jQ\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)}$$

$$Q = \frac{R}{\omega_0 L} \qquad \mathrm{BW} = \frac{1}{\sqrt{2}\,Q}$$

BW is defined here by $\mathrm{SWR} < 2.0$ when the RLC circuit is fed by a matched line ($Z_0 = R$).

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$$

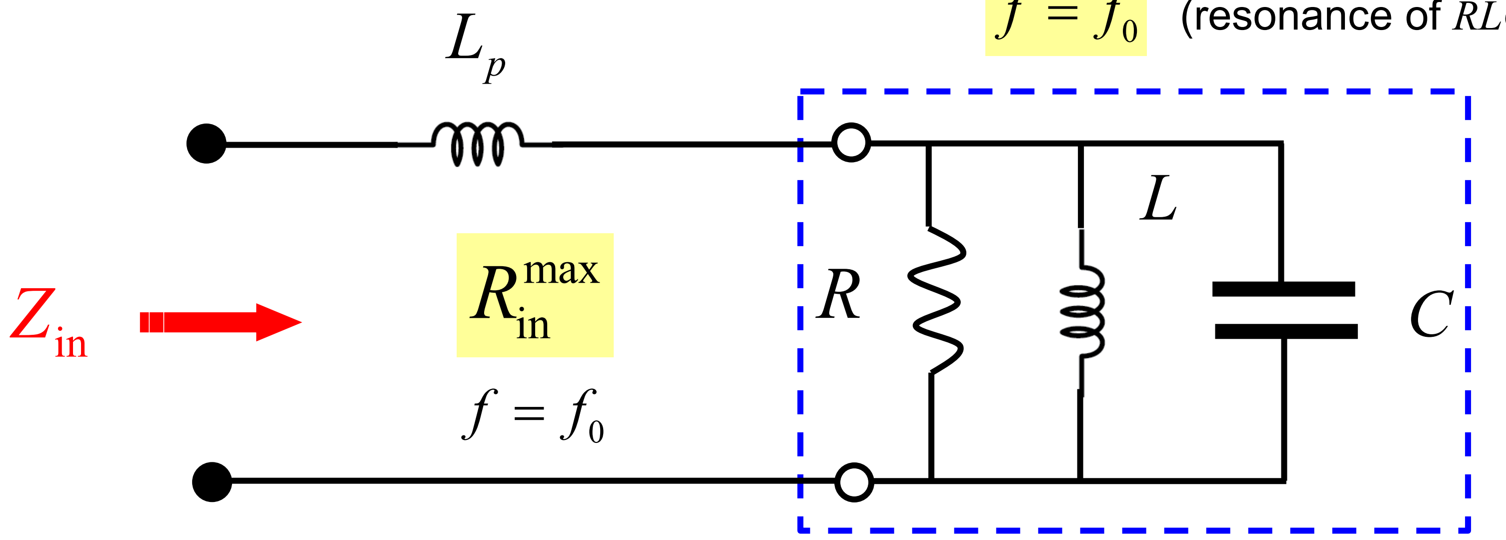


Input Impedance

$$R_{\text{in}} = \frac{R}{1+\left[Q\left(\dfrac{f}{f_0}-\dfrac{f_0}{f}\right)\right]^2} \quad \Rightarrow \quad R_{\text{in}}^{\max} = R_{\text{in}}\Big|_{f=f_0} = R$$

R is the input resistance at the resonance of the patch cavity
(the frequency that maximizes R_{in}).

$f = f_0$ (resonance of RLC circuit)



Input Impedance

$$Z_{\text{in}} \approx j\omega L_p + \frac{R}{1 + jQ\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)}$$

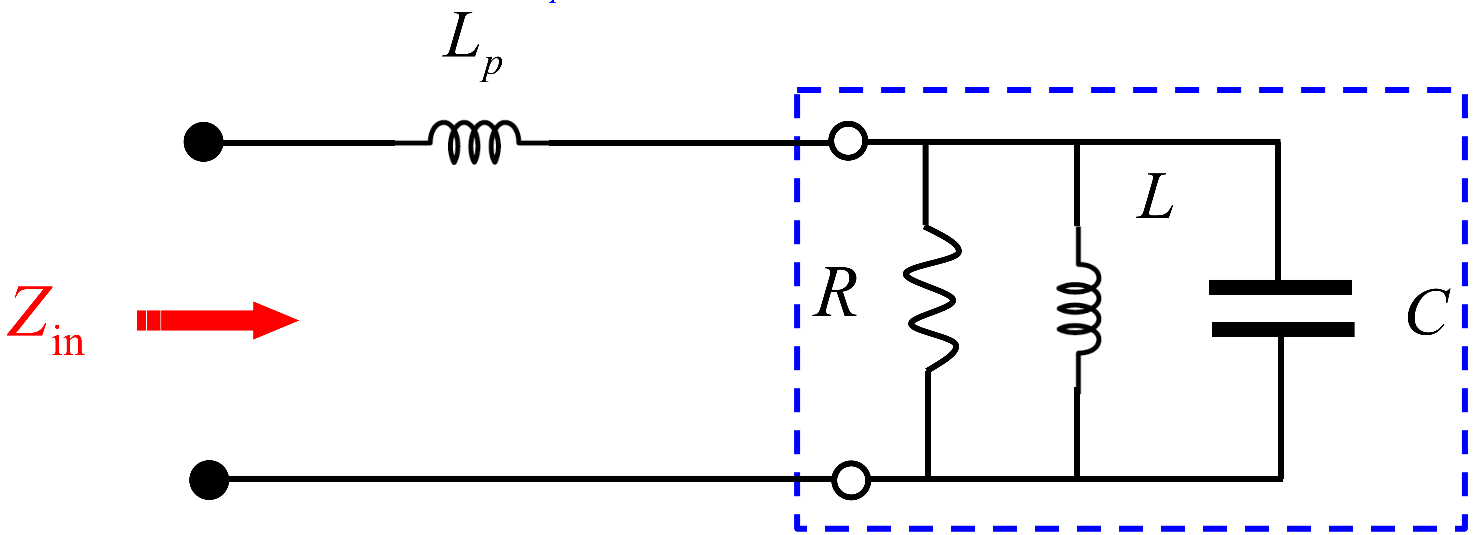
Note: From ω_0, Q, and R, we can get L and C if we wish.

$$Q = \frac{R}{\omega_0 L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

The input impedance at any f is determined once we know four parameters:

CAD formulas for all of these four parameters have been given earlier.

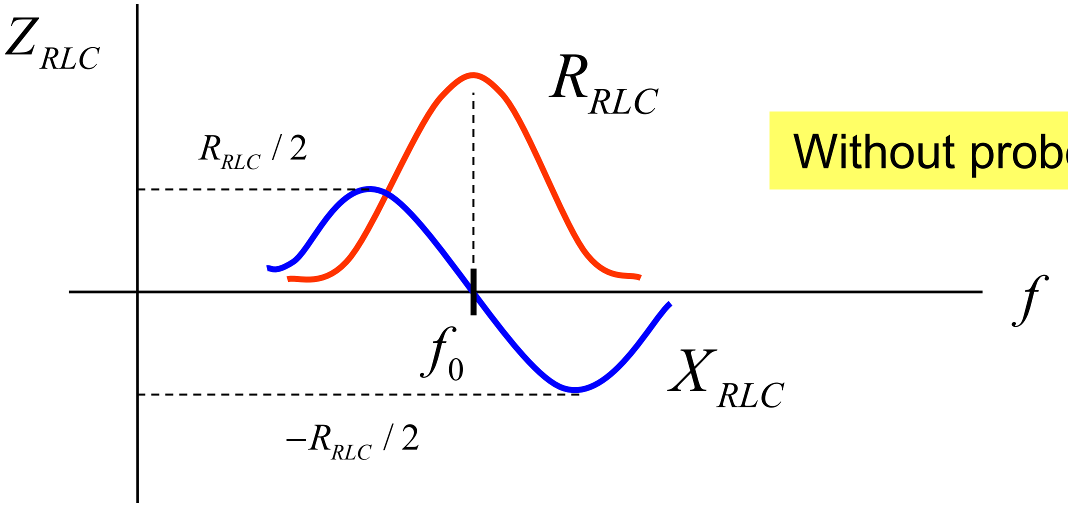
- f_0: the resonance frequency of the patch cavity
- R: the input resistance at the cavity resonance frequency f_0
- Q: the quality factor of the patch cavity
- L_p: the probe inductance



(R, f_0, Q)

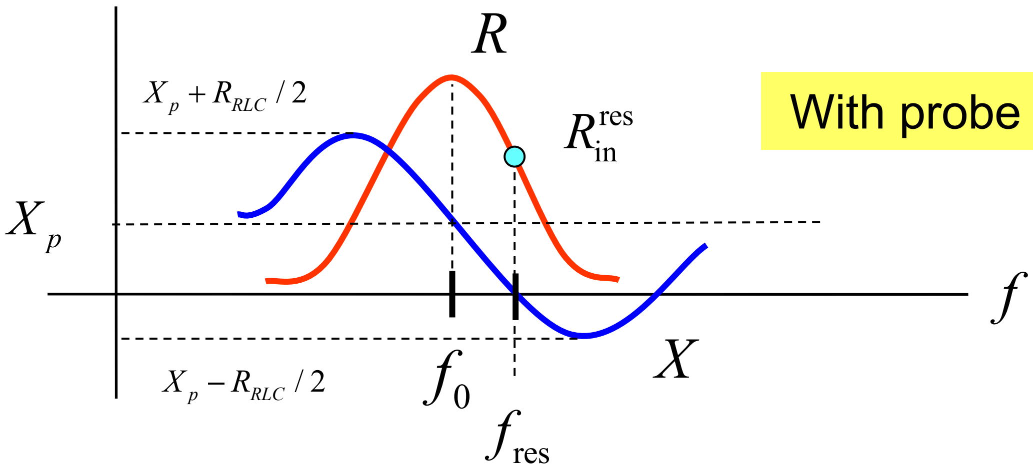
Input Impedance

Typical plot of input impedance



Without probe inductance

Note:
Without the probe reactance, the input reactance is zero at the peak of the input resistance curve.

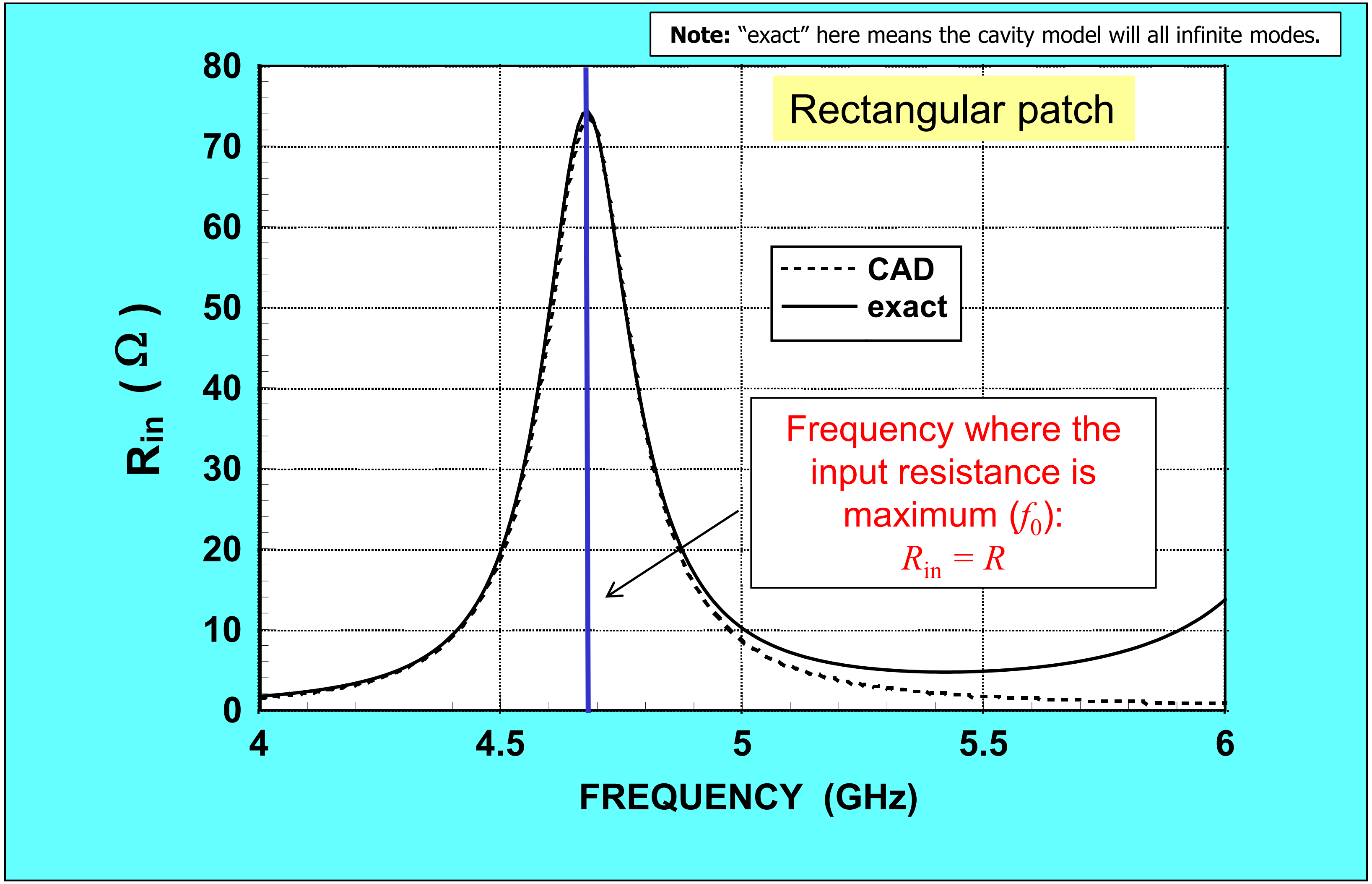


With probe inductance

Note:
The probe reactance can be found by averaging the input reactance values at the two peaks of the reactance curve.

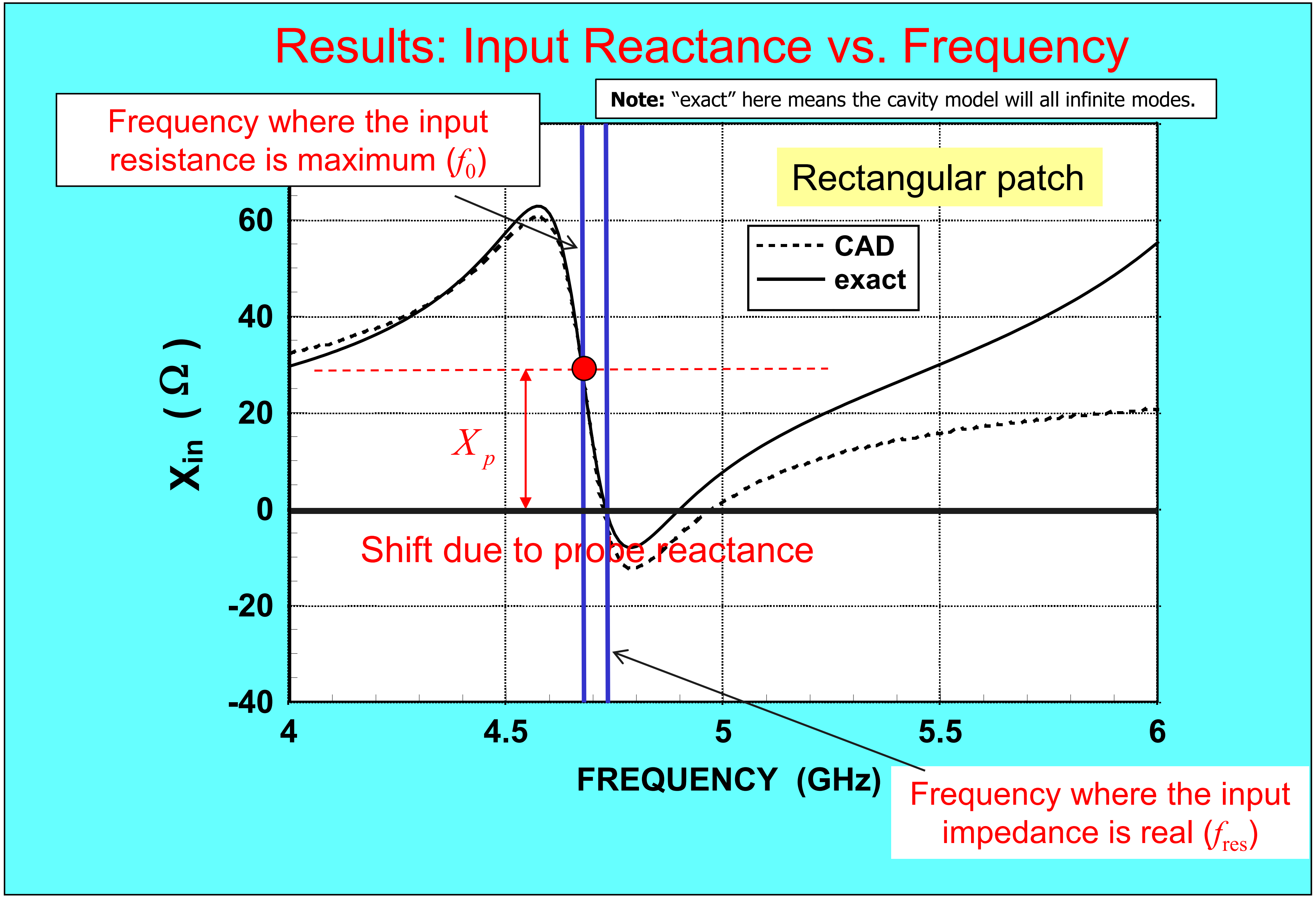
Input Impedance

Results: Input Resistance vs. Frequency



$\varepsilon_r = 2.2 \qquad h = 0.1524\,\text{cm} \qquad W/L = 1.5 \qquad L = 3.0\,\text{cm}$

Input Impedance



$\varepsilon_r = 2.2 \qquad h = 0.1524\,\text{cm} \qquad W/L = 1.5 \qquad L = 3.0\,\text{cm}$

Input Impedance

Optimization procedure to get exactly 50 Ω at the desired resonance frequency:

- Start with an initial design that ignores the probe reactance.
- Vary the length L first until you find the value that gives an input reactance of zero at the desired frequency.
- Then adjust the feed position x_0 to make the real part of the input impedance 50 Ω at this frequency.

Design Example

Design a probe-fed rectangular patch antenna on a substrate having a relative permittivity of 2.33 and a thickness of 62 mils (0.1575 cm). (This is Rogers RT Duroid 5870.) Choose an aspect ratio of $W / L = 1.5$. The patch should resonate at the operating frequency of 1.575 GHz (the GPS L1 frequency). Ignore the probe inductance in your design, but account for fringing at the patch edges when you determine the dimensions. At the operating frequency the input impedance should be $50\ \Omega$ (ignoring the probe inductance). Assume an SMA connector is used to feed the patch along the centerline (at $y = W / 2$), and that the inner conductor of the SMA connector has a radius of 0.635 mm. The copper patch and ground plane have a conductivity of $\sigma = 3.0 \times 10^7$ S/m and the dielectric substrate has a loss tangent of $\tan\delta = 0.001$.

1) Calculate the following:

- The final patch dimensions L and W (in cm)
- The feed location x_0 (distance of the feed from the closest patch edge, in cm)
- The bandwidth of the antenna (SWR < 2 definition, expressed in percent)
- The radiation efficiency of the antenna (accounting for conductor, dielectric, and surface-wave loss, and expressed in percent)
- The probe reactance X_p at the operating frequency (in Ω)
- The expected complex input impedance (in Ω) at the operating frequency, accounting for the probe inductance
- The directivity
- The gain

Design Example

Continued

2) Find (f_0, R, X_p, and Q) and plot the input impedance vs. frequency using the CAD circuit model.

3) Keep $W/L = 1.5$, but now vary the length L of the patch and the feed position x_0 until you find the value that makes the input impedance exactly $50 + j(0)\ \Omega$ at 1.575 GHz.

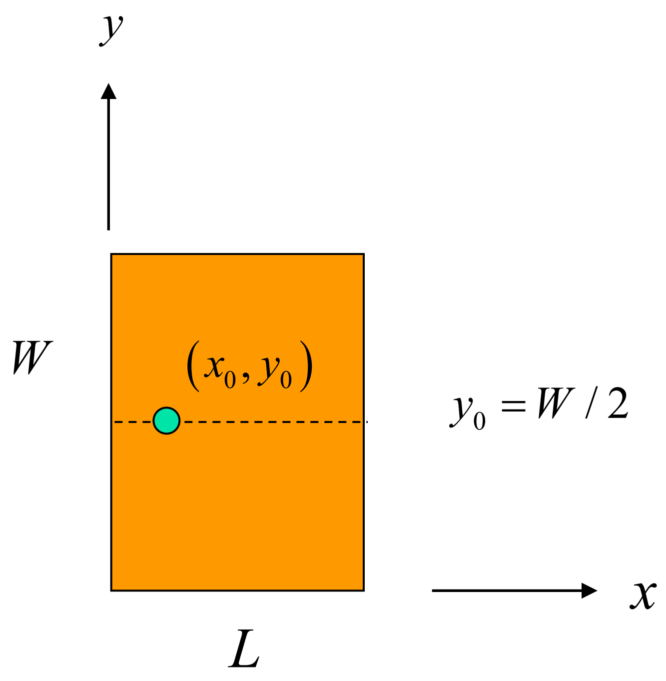
Design Example

Results

Part 1

Results from the CAD formulas:

1) $L = 6.071$ cm, $W = 9.106$ cm
2) $x_0 = 1.832$ cm
3) BW = 1.23%
4) $e_r = 82.9\%$
5) $X_p = 11.1\ \Omega$
6) $Z_{\text{in}} = 50.0 + j(11.1)\ \Omega$
7) $D = 5.85$ (7.67 dB)
8) $G = (D)(e_r) = 4.85$ (6.86 dB)



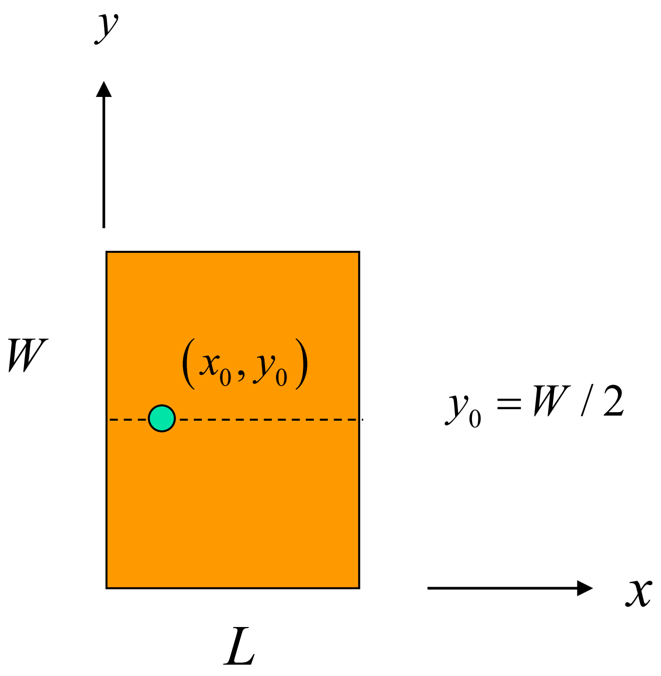
Design Example

Part 2

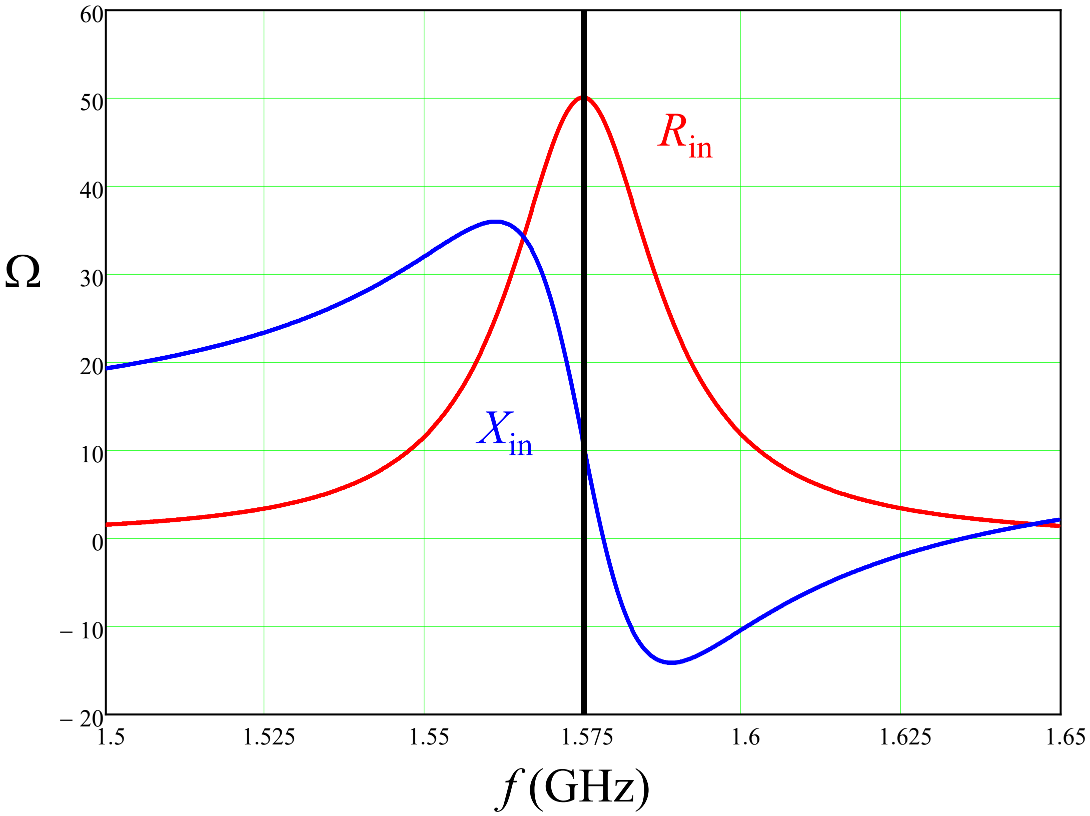
Results

Results from the CAD formulas:

$$f_0 = 1.575 \times 10^9 \text{ Hz}$$
$$R = 50\ \Omega$$
$$Q = 57.5$$
$$X_p = 11.1\ \Omega$$



$$Z_{\text{in}} \approx jX_p + \frac{R}{1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$



Design Example

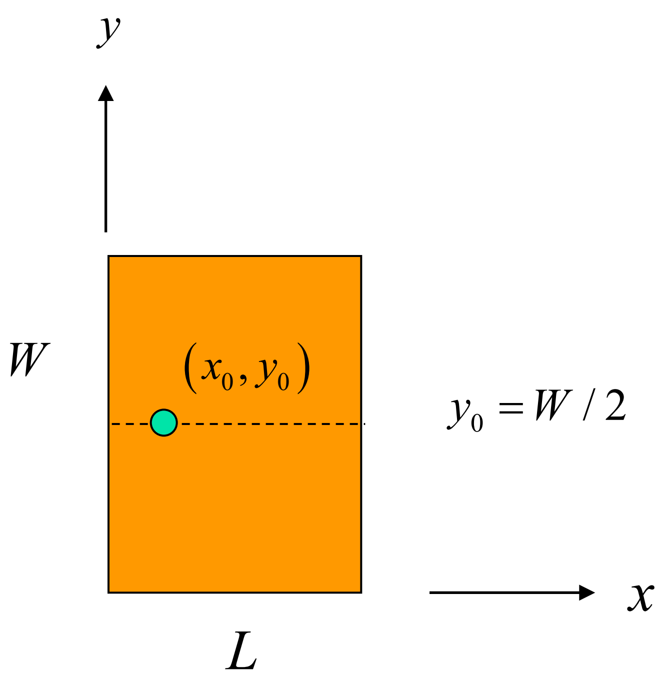
Part 3

After optimization:

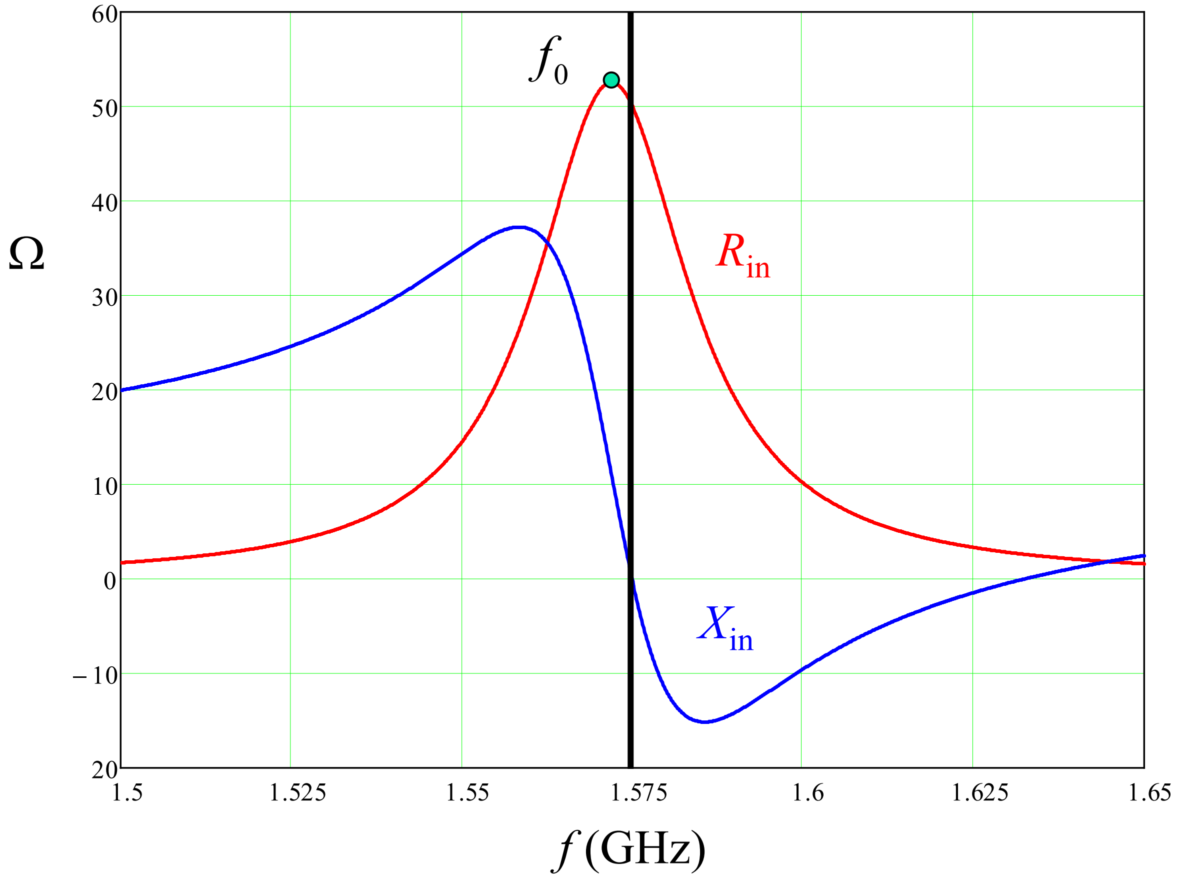
$$\boxed{\begin{aligned} L &= 6.083 \text{ cm} \\ x_0 &= 1.800 \text{ cm} \end{aligned}}$$

$$Z_{\text{in}} = 50 + j(0)\ \Omega$$

At 1.575 GHz



Results



Note: $f_0 < 1.575 \text{ GHz}, \ R > 50\,\Omega$

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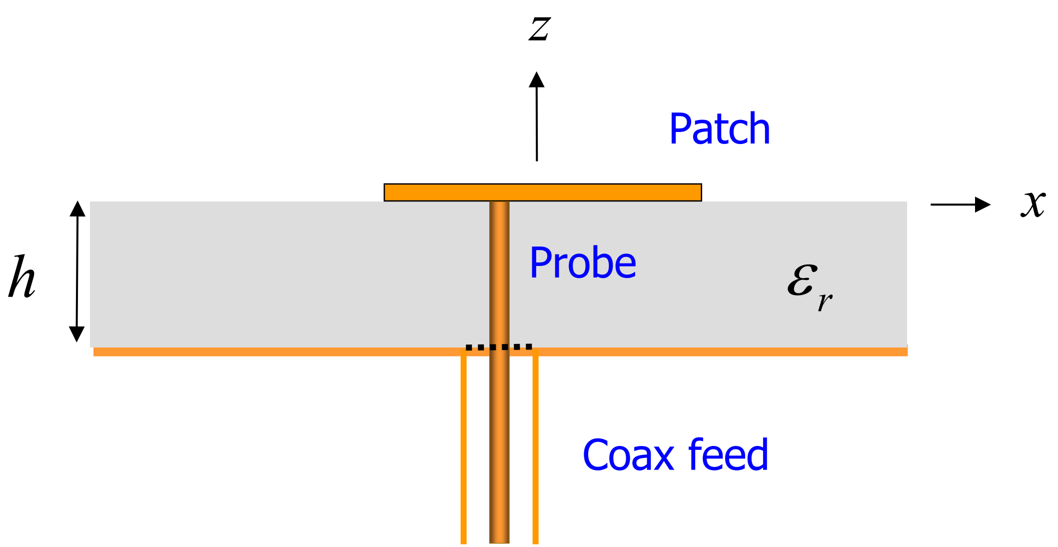
Radiation Pattern

There are two models often used for calculating the radiation pattern of a patch antenna:

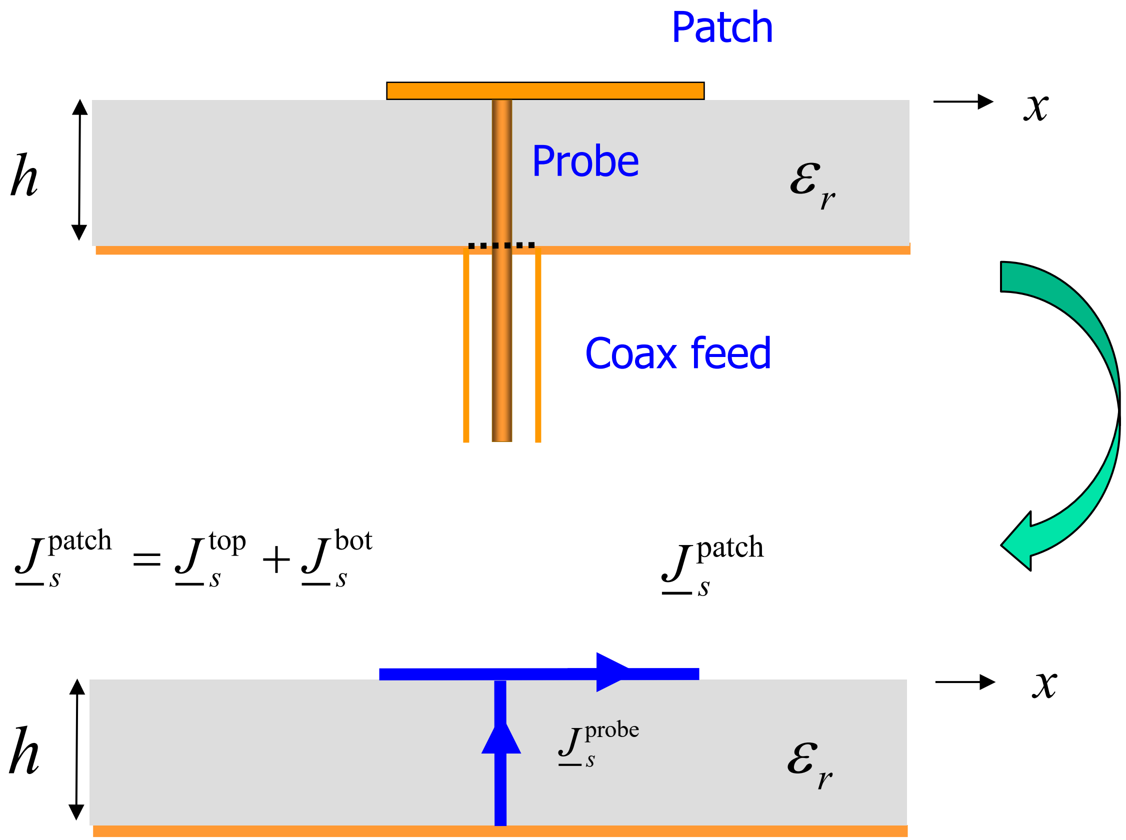
- Electric current model
- Magnetic current model

Note: The origin is placed at the center of the patch, at the top of the substrate, for the pattern calculations.



Radiation Pattern

Electric current model: We use the physical electric currents flowing on the patch (and feed).



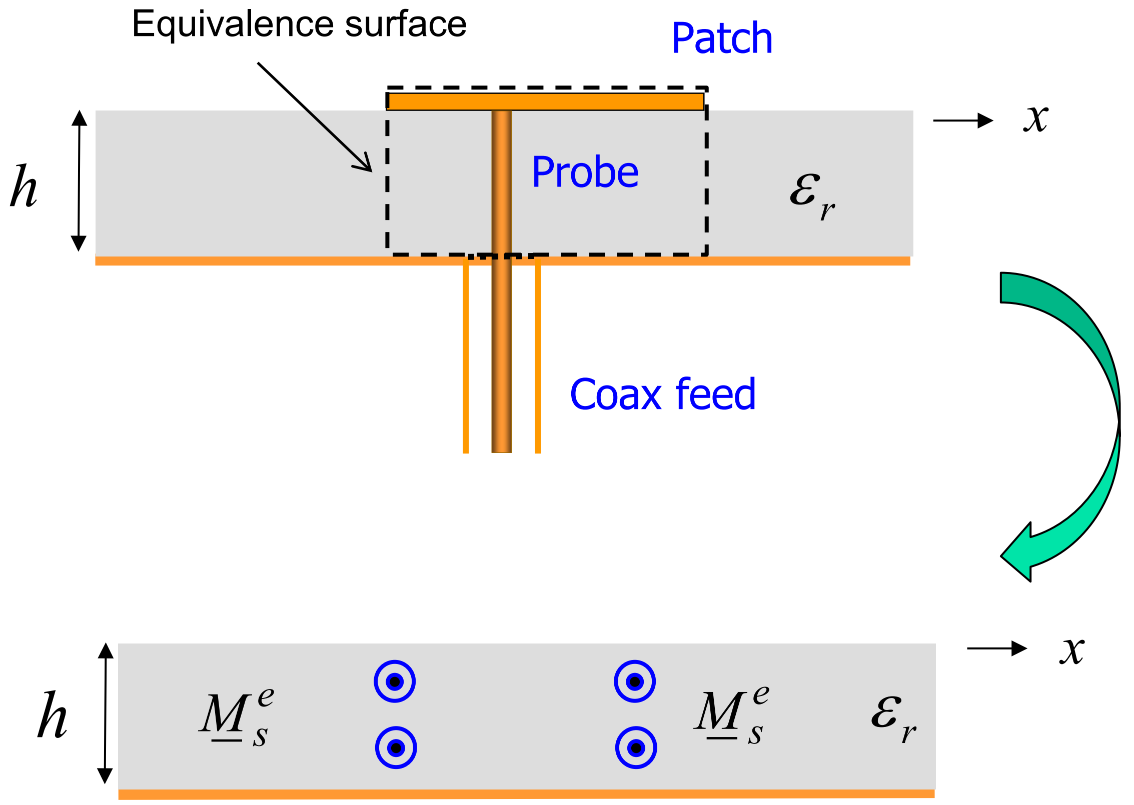
Radiation Pattern

Magnetic current model: We apply the *equivalence principle* and invoke the (approximate) PMC condition at the edges.

$$\underline{J}_s^e = \underline{\hat{n}} \times \underline{H}$$

$$\underline{M}_s^e = -\underline{\hat{n}} \times \underline{E}$$

The equivalent electric surface current is *approximately* zero on the top surface (weak fields) and the sides (PMC). We can ignore it on the ground plane (it does not radiate).



$$\underline{M}_s^e = -\underline{\hat{n}} \times \underline{E}$$

Radiation Pattern

Theorem

The electric and magnetic current models yield identical patterns at the resonance frequency of the cavity mode.

Assumption:

The electric and magnetic current models are based on the fields of a single cavity mode, corresponding to an ideal cavity with PMC walls.

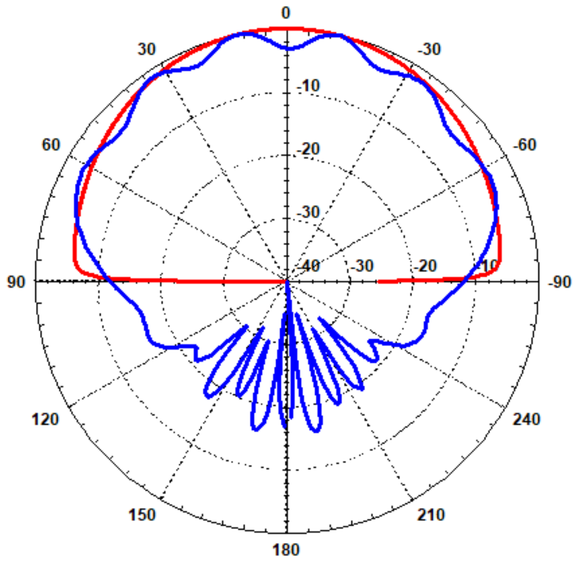
D. R. Jackson and J. T. Williams, "A Comparison of CAD Models for Radiation from Rectangular Microstrip Patches," *Intl. Journal of Microwave and Millimeter-Wave Computer Aided Design*, vol. 1, no. 2, pp. 236-248, April 1991.

Radiation Pattern

Comments on the Substrate Effects

- It is most convenient to assume an infinite substrate (in order to obtain a closed-form expression for the far-field pattern).
- Reciprocity can be used to calculate the far-field pattern of electric or magnetic current sources inside of an infinite layered structure (infinite horizontally).
- When an infinite substrate is assumed, the far-field pattern always goes to zero at the horizon.

E Plane



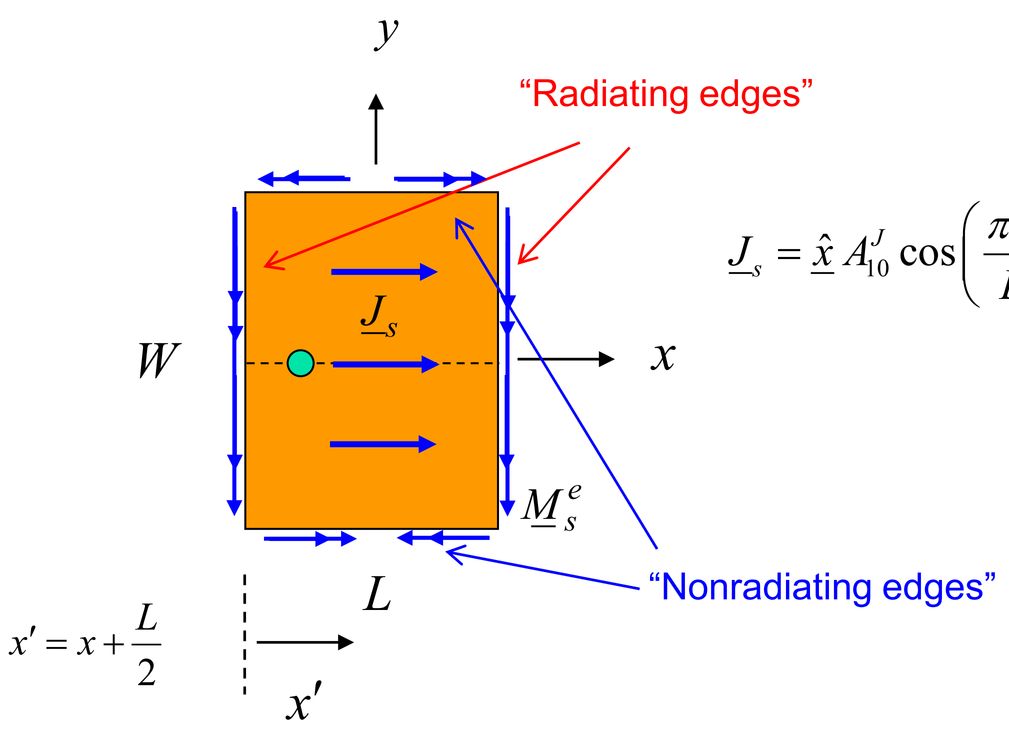
Red: infinite substrate and ground plane
Blue: one meter diameter circular ground plane

D. R. Jackson and J. T. Williams, "A Comparison of CAD Models for Radiation from Rectangular Microstrip Patches," *Intl. Journal of Microwave and Millimeter-Wave Computer Aided Design*, vol. 1, no. 2, pp. 236-248, April 1991.

Radiation Pattern

Comments on the Two Models

- For the rectangular patch, the electric current model is the simplest one since there is only one electric surface current on the patch (as opposed to magnetic surface current on the four edges).
- For the rectangular patch, the magnetic current model allows us to classify the “radiating” and “nonradiating” edges.
- For the circular patch, the magnetic current model is the simplest one (discussed later).



$$\underline{J}_s = \underline{\hat{x}}\, A_{10}^J \cos\left(\frac{\pi x}{L}\right)$$

$$\underline{M}_s^e = -\underline{\hat{n}} \times \underline{E}$$

$$E_z = \cos\left(\frac{\pi x'}{L}\right) = -\sin\left(\frac{\pi x}{L}\right)$$

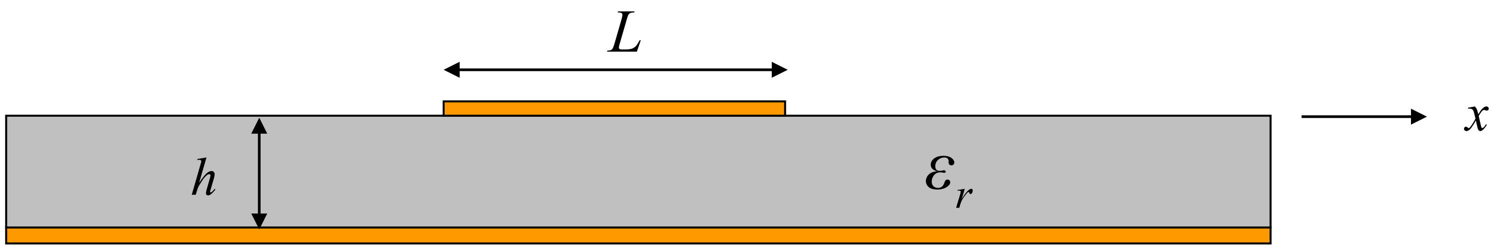
Note:
On the nonradiating edges, the magnetic currents are in opposite directions across the centerline ($x = 0$).

Note:
In the magnetic current model, the magnetic currents on the <u>nonradiating</u> edges to not contribute to the E and H plane patterns.

Radiation Pattern

Rectangular Patch Pattern Formula

(The formula is based on the electric current model.)

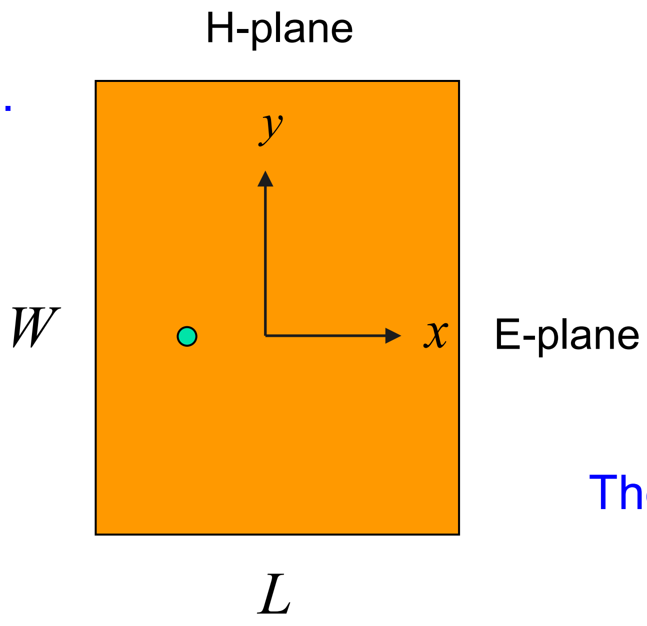


Infinite ground plane and substrate

The origin is at the <u>center</u> of the patch.

(1,0) mode

$$\underline{J}_s = \underline{\hat{x}} \cos\left(\frac{\pi x}{L}\right)$$



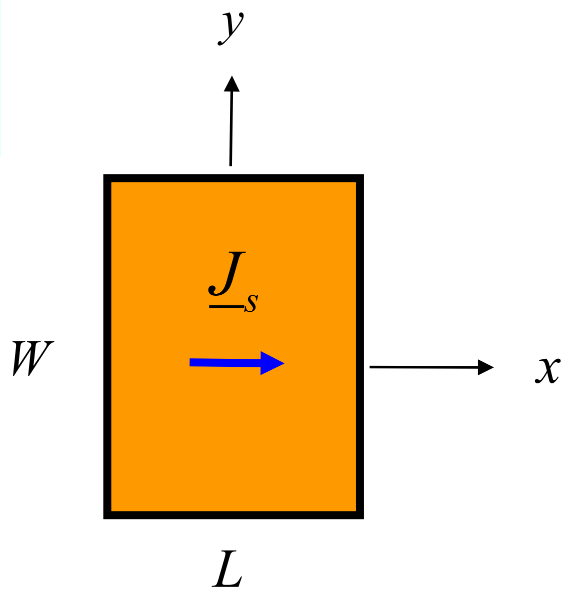
Note:
The surface current density is assumed to be unity at the patch center, for simplicity.

The probe is on the x axis.

Radiation Pattern

The far-field pattern can be determined by reciprocity.

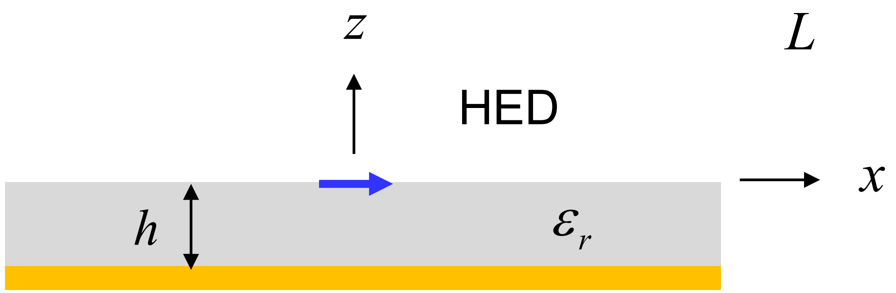
$$E_i(r,\theta,\phi) = E_i^{\text{hex}}(r,\theta,\phi)\left(\frac{\pi WL}{2}\right)\left[\frac{\sin\left(\frac{k_y W}{2}\right)}{\frac{k_y W}{2}}\right]\left[\frac{\cos\left(\frac{k_x L}{2}\right)}{\left(\frac{\pi}{2}\right)^2-\left(\frac{k_x L}{2}\right)^2}\right]$$



$$i = \theta \text{ or } \phi$$

$$k_x = k_0 \sin\theta \cos\phi \qquad k_y = k_0 \sin\theta \sin\phi$$

The “hex” pattern is for a unit-amplitude horizontal electric dipole (HED) in the x direction, sitting on top of the substrate, at $(0,0,0)$.



D. R. Jackson and J. T. Williams, “A Comparison of CAD Models for Radiation from Rectangular Microstrip Patches,” *Intl. Journal of Microwave and Millimeter-Wave Computer Aided Design*, vol. 1, no. 2, pp. 236-248, April 1991.

Radiation Pattern

$$E_\phi^{\mathrm{hex}}(r,\theta,\phi) = -E_0 \sin\phi \; F(\theta)$$

$$E_\theta^{\mathrm{hex}}(r,\theta,\phi) = E_0 \cos\phi \; G(\theta)$$

where

$$E_0 = \left(\frac{-j\omega\mu_0}{4\pi r}\right) e^{-jk_0 r}$$

$$F(\theta) = 1 + \Gamma^{\mathrm{TE}}(\theta) = \frac{2\tan\left(k_0 h N(\theta)\right)}{\tan\left(k_0 h N(\theta)\right) - j N(\theta)\sec\theta}$$

$$N(\theta) = \sqrt{\varepsilon_r - \sin^2(\theta)}$$

$$G(\theta) = \cos\theta\left(1 + \Gamma^{\mathrm{TM}}(\theta)\right) = \frac{2\tan\left(k_0 h N(\theta)\right)\cos\theta}{\tan\left(k_0 h N(\theta)\right) - j\dfrac{\varepsilon_r}{N(\theta)}\cos\theta}$$

Note: To account for lossy substrate, use $\varepsilon_r \to \varepsilon_{rc} = \varepsilon_r\left(1 - j\tan\delta\right)$

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- ❖ Circular patch
- ❖ Improving bandwidth
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- ❖ Reducing surface waves and lateral radiation

Circular Polarization

Three main techniques:

1) Single feed with "nearly degenerate" eigenmodes (compact design, but small CP bandwidth).

2) Dual feed with delay line or 90^{o} hybrid phase shifter (broader CP bandwidth but uses more space).

3) Synchronous subarray technique (produces high-quality CP due to cancellation effect, but requires even more space).

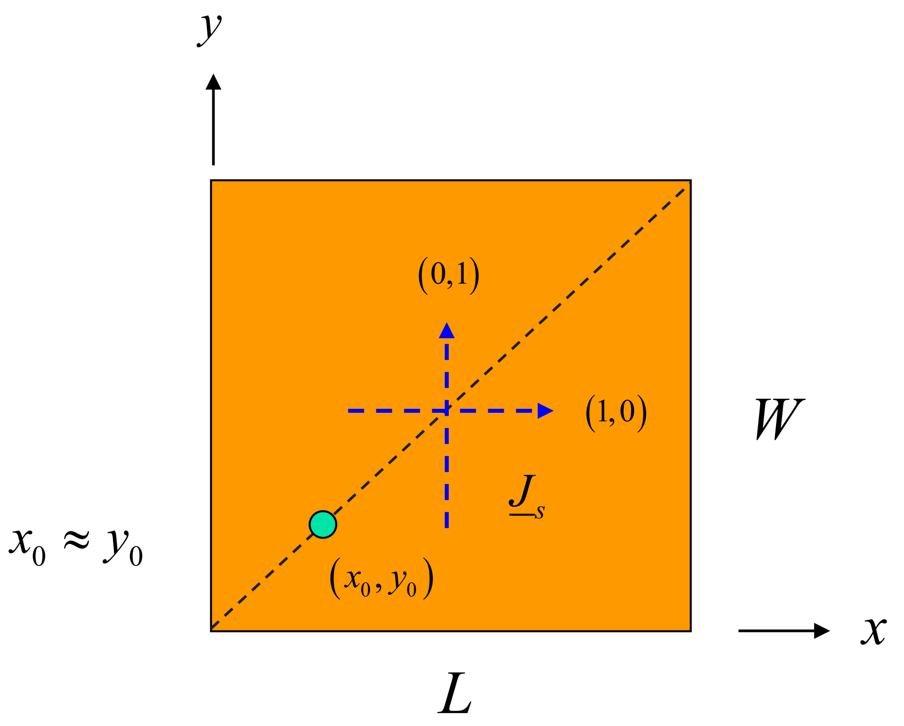
The techniques will be illustrated with a rectangular patch.

Circular Polarization

Single-Feed Method

The feed is on the diagonal. The patch is nearly (but not exactly) square.

$$L \approx W$$



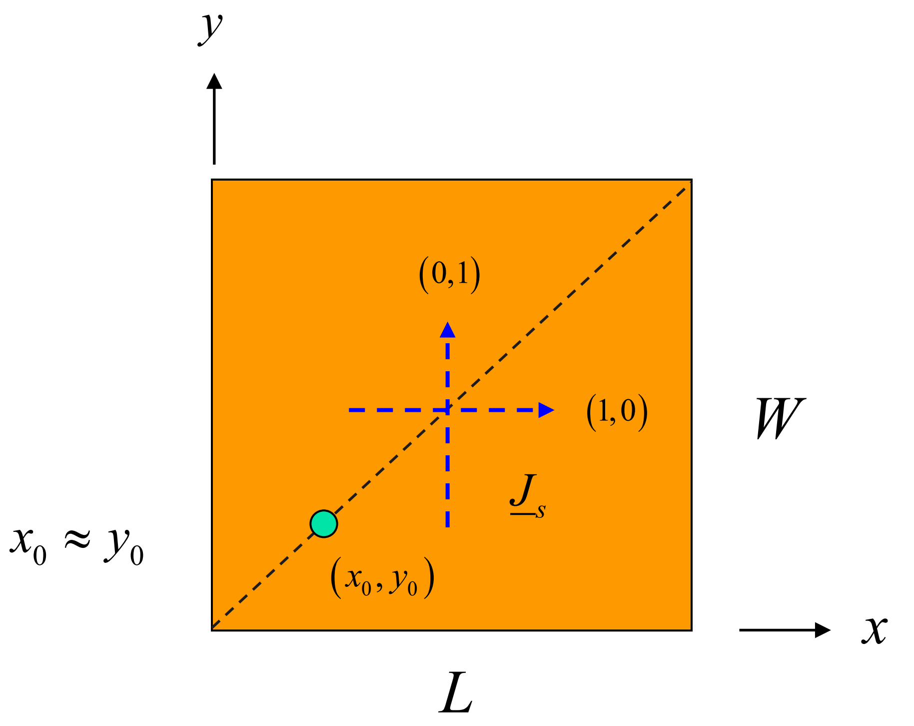
Basic principle: The two dominant modes (1,0) and (0,1) are excited with <u>equal</u> amplitude, but with a $\pm 45^{\text{o}}$ phase.

Circular Polarization

Design equations:

$$f_{\mathrm{CP}} = \frac{f_x + f_y}{2}$$

The optimum CP frequency (the design frequency) is the <u>average</u> of the x and y resonance frequencies.



$$\left.\begin{aligned} f_x &= f_{\mathrm{CP}}\left(1 \mp \frac{1}{2Q}\right) \\ f_y &= f_{\mathrm{CP}}\left(1 \pm \frac{1}{2Q}\right) \end{aligned}\right\}$$

Top sign for LHCP, bottom sign for RHCP.

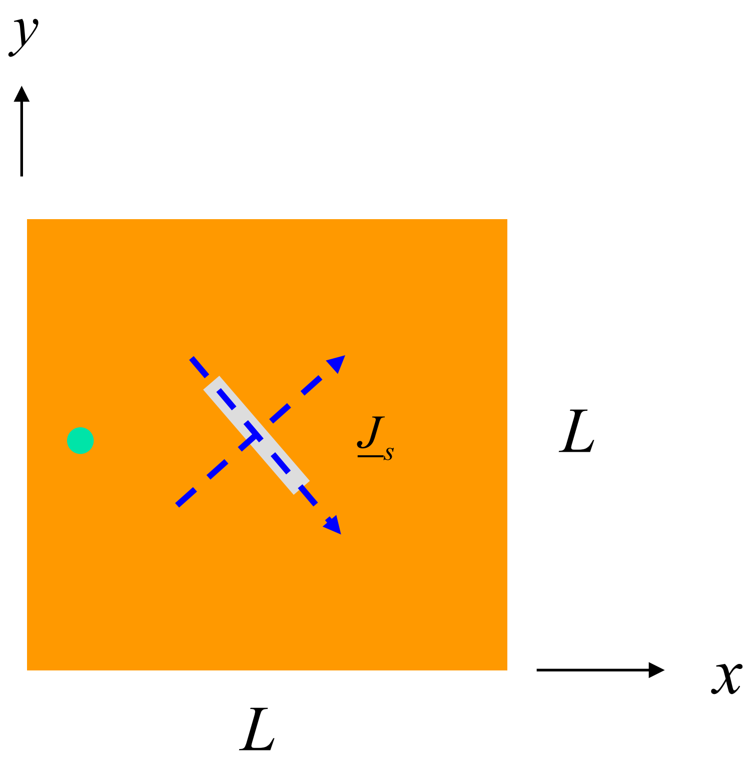
The frequency f_{CP} is also the resonance frequency: $Z_{\mathrm{in}} = R_{\mathrm{in}} = R_x = R_y$

The resonant input resistance of the CP patch at f_{CP} is the same as what a *linearly-polarized square patch* fed at the same value of x_0 or y_0 would have.

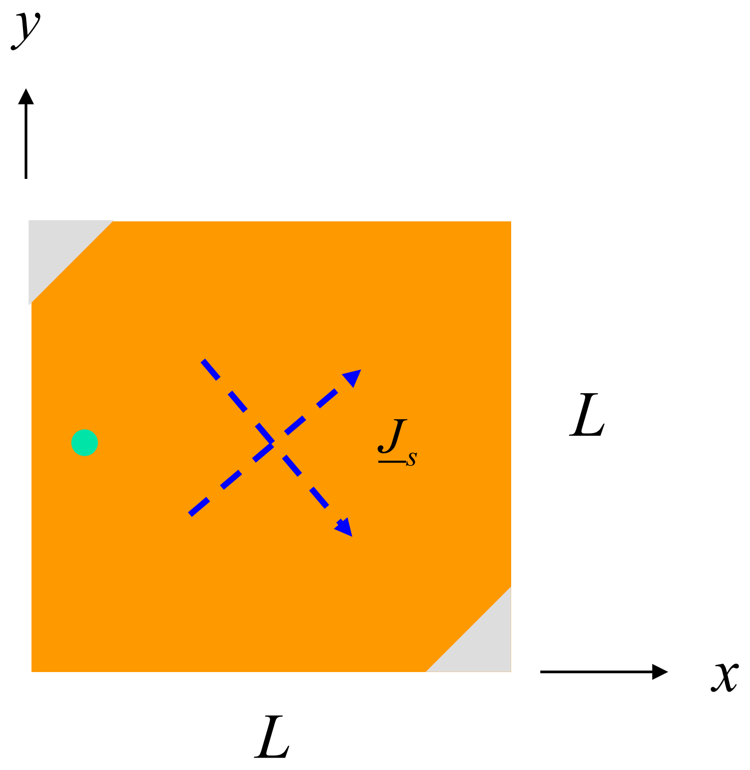
Circular Polarization

Other Variations

Note: <u>Diagonal</u> modes are used as degenerate modes



Patch with slot



Patch with truncated corners

Circular Polarization

Here we compare bandwidths (impedance and axial-ratio BW):

Linearly-polarized (LP) patch: $$\mathrm{BW}_{\mathrm{SWR}}^{\mathrm{LP}} = \frac{1}{\sqrt{2}Q} \qquad (\mathrm{SWR} < 2)$$

Circularly-polarized (CP) single-feed patch:

$$\mathrm{BW}_{\mathrm{SWR}}^{\mathrm{CP}} = \frac{\sqrt{2}}{Q} \qquad (\mathrm{SWR} < 2)$$

$$\mathrm{BW}_{\mathrm{AR}}^{\mathrm{CP}} = \frac{0.348}{Q} \qquad \left(\mathrm{AR} < \sqrt{2} \;\; (3\,\mathrm{dB})\right)$$

The axial-ratio bandwidth is <u>small</u> when using the single-feed method.

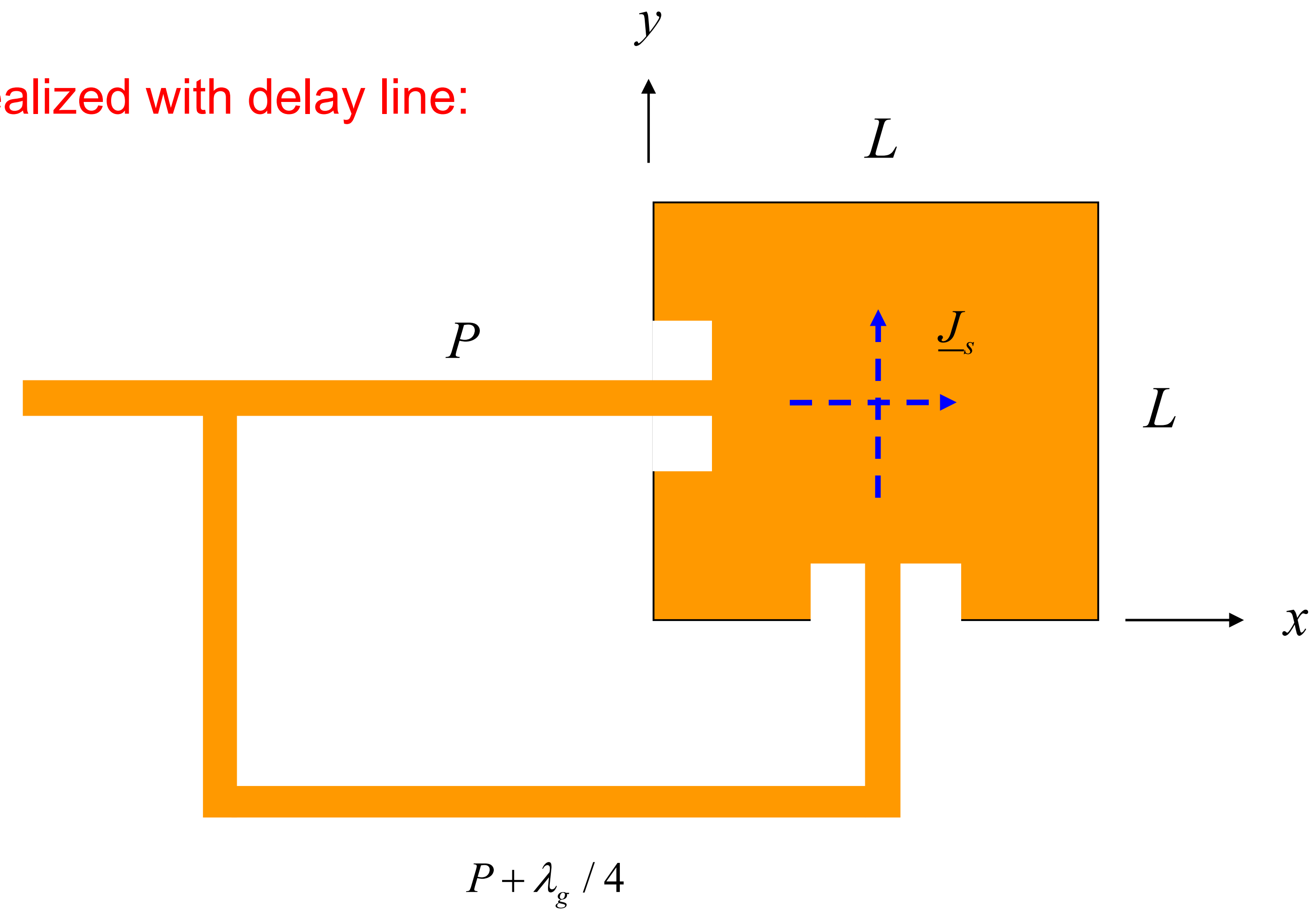
W. L. Langston and D. R. Jackson, "Impedance, Axial-Ratio, and Receive-Power Bandwidths of Microstrip Antennas," *IEEE Trans. Antennas and Propagation*, vol. 52, pp. 2769-2773, Oct. 2004.

Circular Polarization

Dual-Feed Method

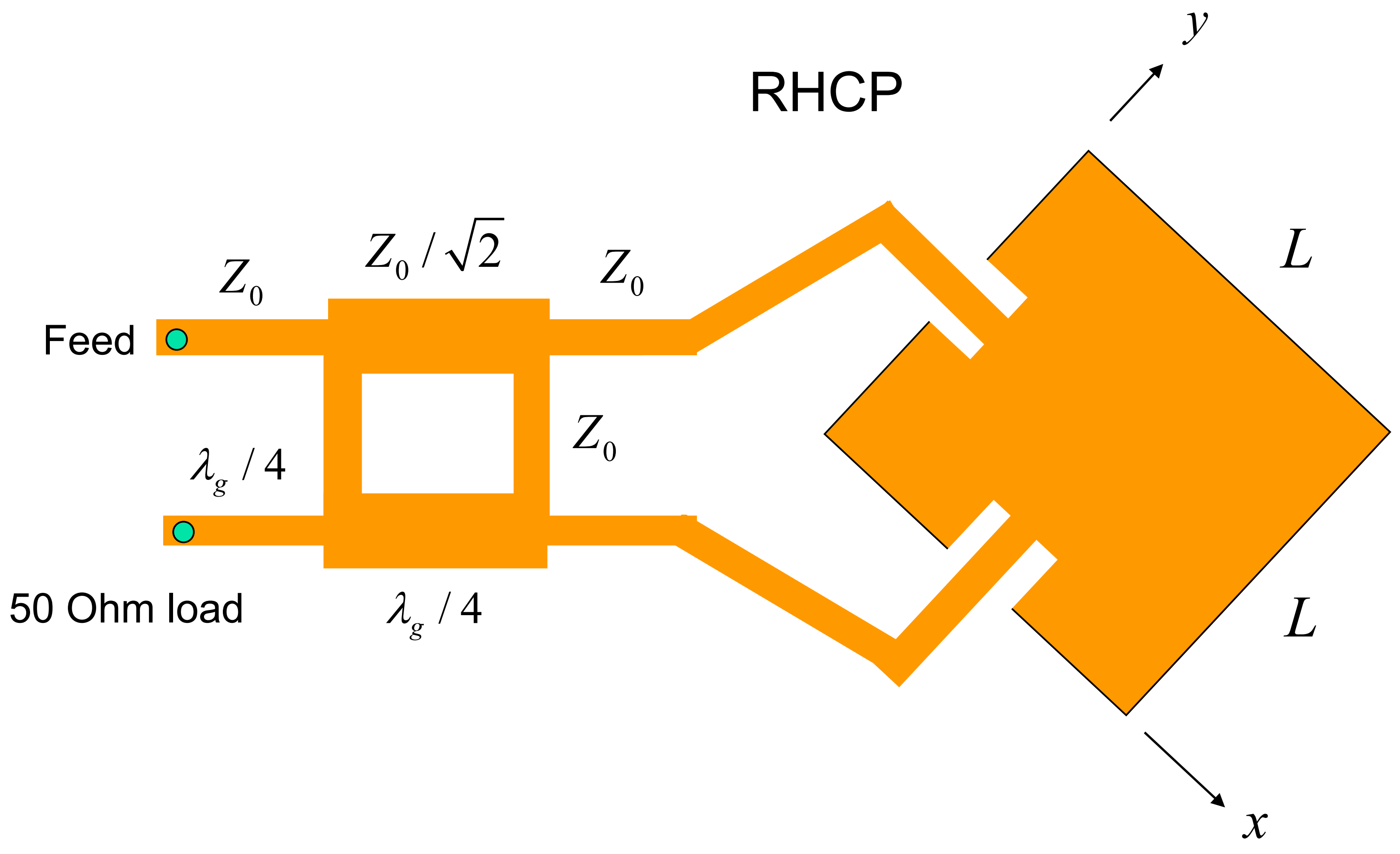
RHCP (in $+z$ direction)

Phase shift realized with delay line:



Circular Polarization

Phase shift realized with 90^o quadrature hybrid (branchline coupler)

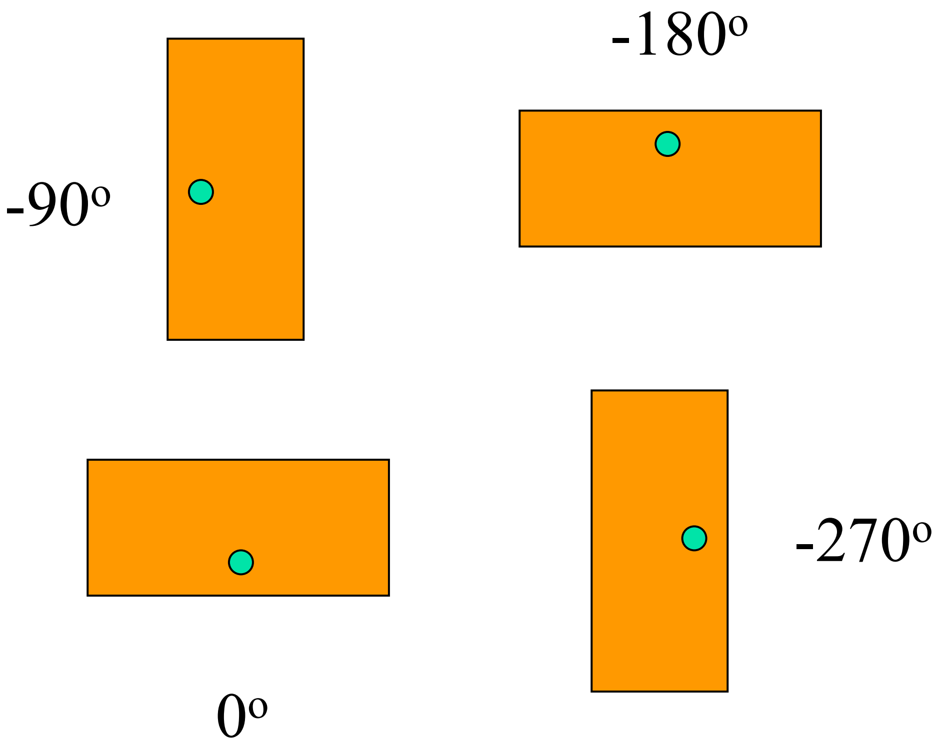


This gives us a higher bandwidth than the simple power divider, but requires a load resistor.

Circular Polarization

Synchronous Rotation

Multiple elements are rotated in space and fed with phase shifts.



Because of symmetry, radiation from higher-order modes (or probes) tends to be reduced, resulting in good cross-pol.

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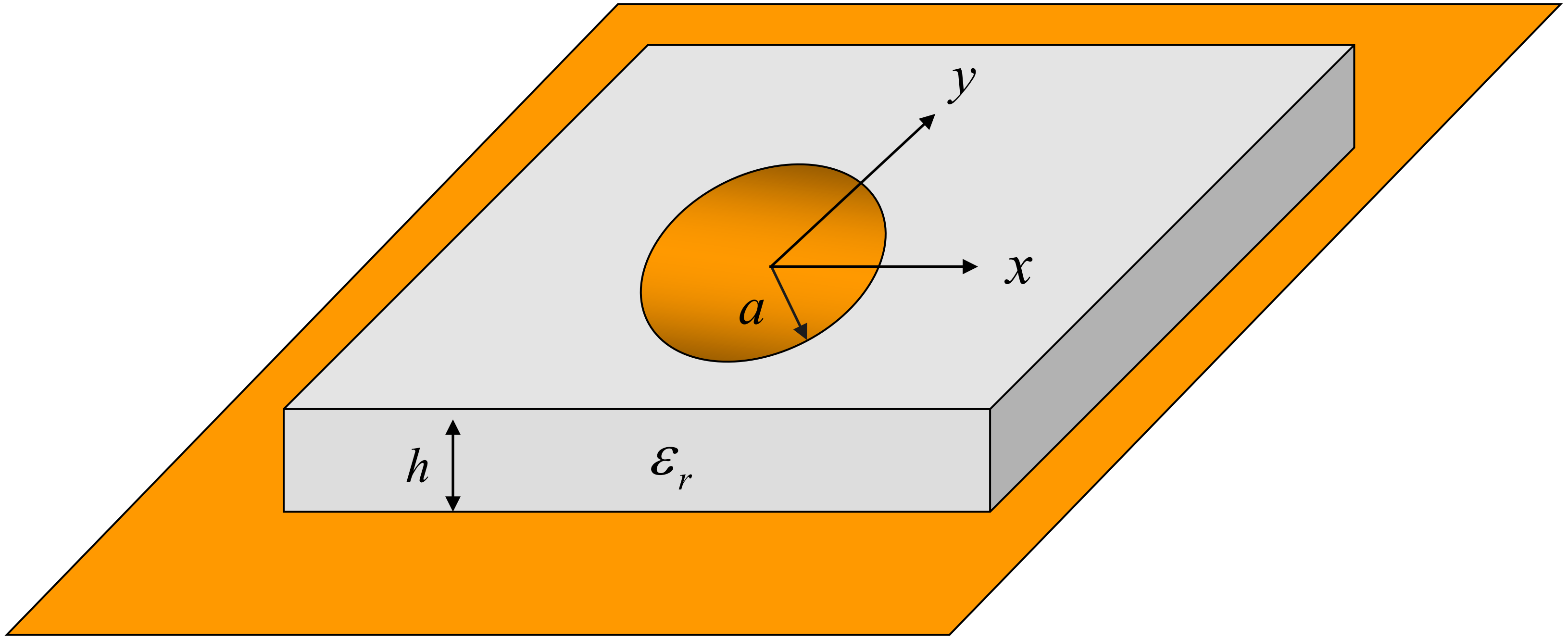
Circular Patch



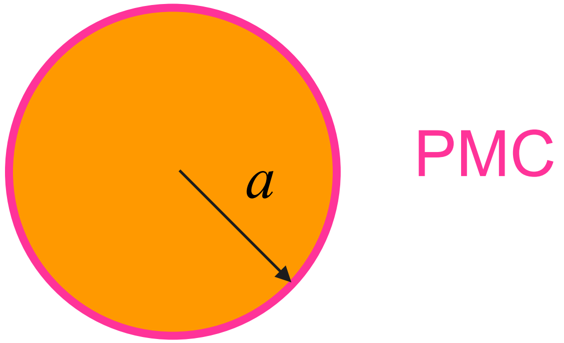
Circular Patch

Resonance Frequency

From separation of variables:

$$E_z = \cos(m\phi) J_m(k_1 \rho)$$

$$k_1 = k_0 \sqrt{\varepsilon_r}$$



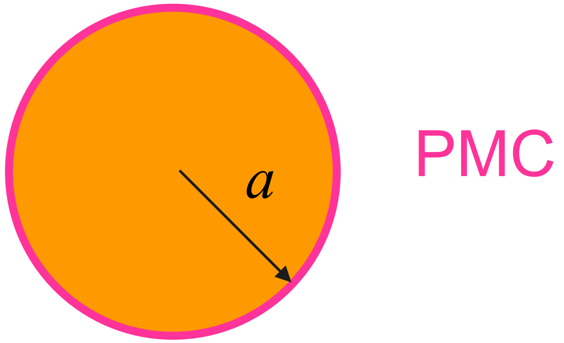
J_m = Bessel function of first kind, order m.

$$\left.\frac{\partial E_z}{\partial \rho}\right|_{\rho=a} = 0 \quad \Rightarrow \quad J_m'(k_1 a) = 0$$

Circular Patch

Resonance Frequency

$$J'_m\left(k_1 a\right) = 0$$



This gives us

$$k_1 a = x'_{mn}$$

(n^{th} root of $J_m{}'$ Bessel function)

$$\Rightarrow \quad f_{mn} = \frac{c}{2\pi\sqrt{\varepsilon_r}}\, x'_{mn}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Circular Patch

Resonance Frequency

Table of values for x'_{mn}

n / m	0	1	2	3	4	5
1	3.832	1.841	3.054	4.201	5.317	5.416
2	7.016	5.331	6.706	8.015	9.282	10.520
3	10.173	8.536	9.969	11.346	12.682	13.987

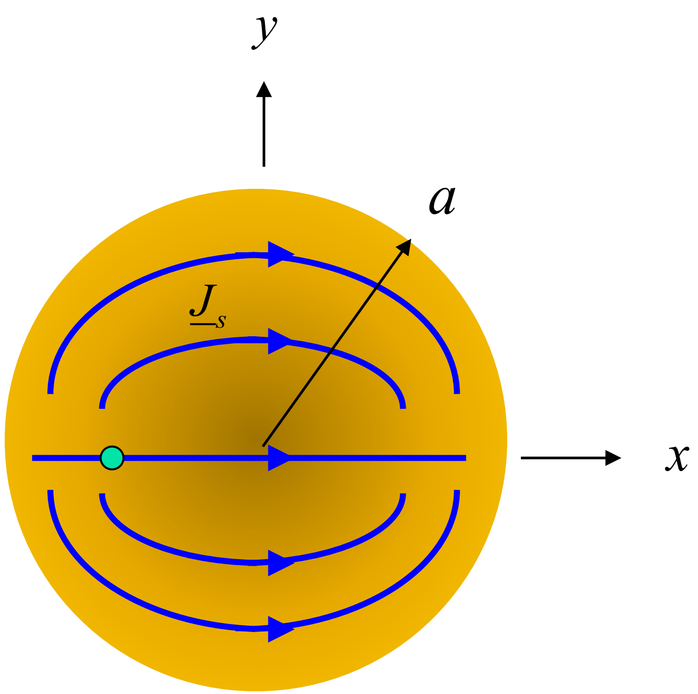
Dominant mode: TM_{11}

$$f_{11} = \frac{c}{2\pi a\sqrt{\varepsilon_r}}\, x'_{11}$$

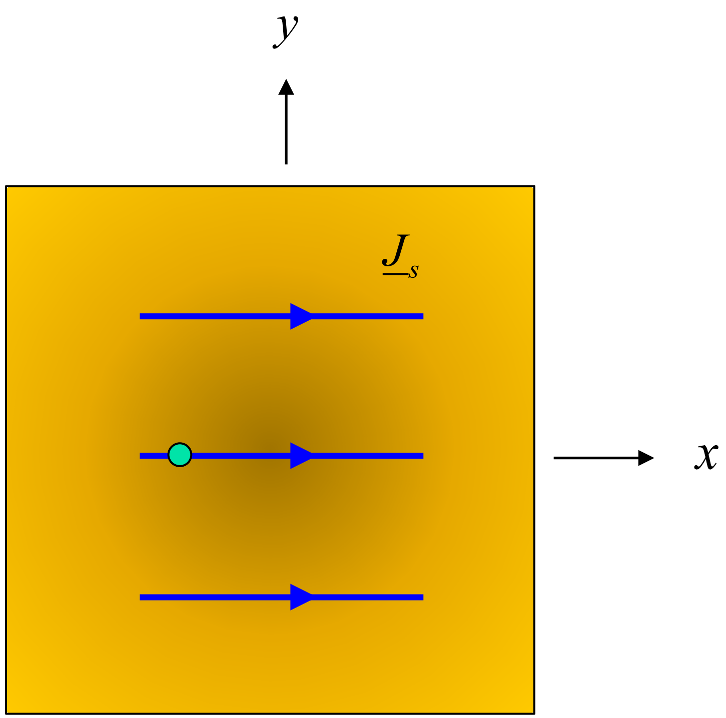
$$x'_{11} \approx 1.841$$

Circular Patch

Dominant mode: TM_{11}



Circular patch (TM_{11})



Square patch (TM_{10})

$W = L$

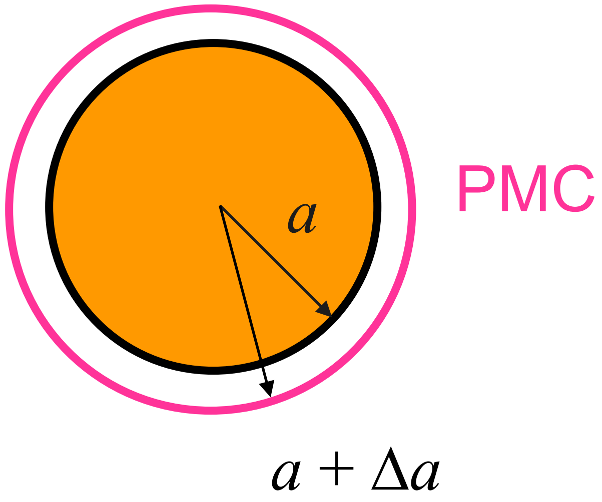
The TM_{11} mode of the circular patch is somewhat similar to the TM_{10} mode of a square patch.

Circular Patch

Fringing extension

$$a_e = a + \Delta a$$

$$f_{11} = \frac{c}{2\pi a_e \sqrt{\varepsilon_r}} x'_{11}$$



"Long/Shen Formula":

$$a_e = a\sqrt{1 + \frac{2h}{\pi a \varepsilon_r}\left[\ln\left(\frac{\pi a}{2h}\right) + 1.7726\right]}$$

or

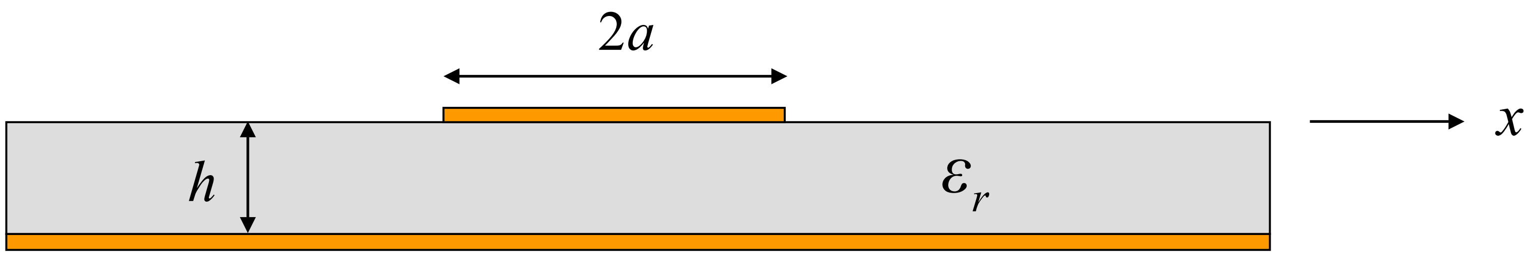
$$\Delta a \approx \frac{h}{\pi \varepsilon_r}\left[\ln\left(\frac{\pi a}{2h}\right) + 1.7726\right]$$

L. C. Shen, S. A. Long, M. Allerding, and M. Walton, "Resonant Frequency of a Circular Disk Printed-Circuit Antenna," IEEE Trans. Antennas and Propagation, vol. 25, pp. 595-596, July 1977.

Circular Patch

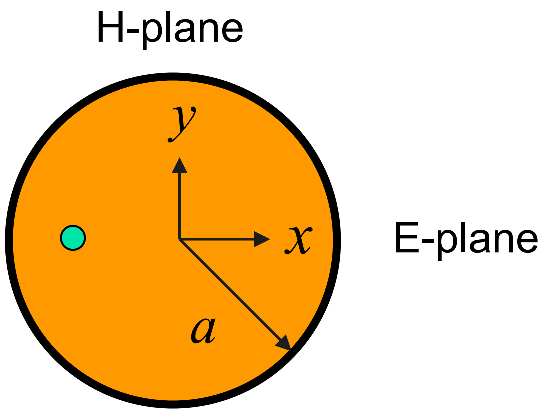
Patterns

(The patterns are based on the magnetic current model.)



Infinite GP and substrate

The origin is at the center of the patch.



The probe is on the x axis.

Inside the patch cavity:

$$E_z(\rho,\phi) = \cos\phi \left(\frac{J_1(k_1\rho)}{J_1(k_1 a)}\right)\left(\frac{1}{h}\right)$$

$$k_1 = k_0\sqrt{\varepsilon_r}$$

(The edge voltage has a maximum value of one volt (at $\phi = \pi$).

Circular Patch

Patterns

$$E_\theta(r,\theta,\phi) = 2\pi a \frac{E_0}{\eta_0} \operatorname{tanc}(k_{z1}h) \cos\phi \, J_1'(k_0 a \sin\theta) Q(\theta)$$

$$E_\phi(r,\theta,\phi) = -2\pi a \frac{E_0}{\eta_0} \operatorname{tanc}(k_{z1}h) \sin\phi \left(\frac{J_1(k_0 a \sin\theta)}{k_0 a \sin\theta} \right) P(\theta)$$

where

$$\operatorname{tanc}(x) \equiv \tan(x)/x$$

$$P(\theta) = \cos\theta\left(1-\Gamma^{\mathrm{TE}}(\theta)\right) = \cos\theta\left[\frac{-2jN(\theta)}{\tan(k_0 h N(\theta)) - jN(\theta)\sec\theta}\right]$$

$$Q(\theta) = 1-\Gamma^{\mathrm{TM}}(\theta) = \frac{-2j\left(\dfrac{\varepsilon_r}{N(\theta)}\right)\cos\theta}{\tan(k_0 h\, N(\theta)) - j\dfrac{\varepsilon_r}{N(\theta)}\cos\theta}$$

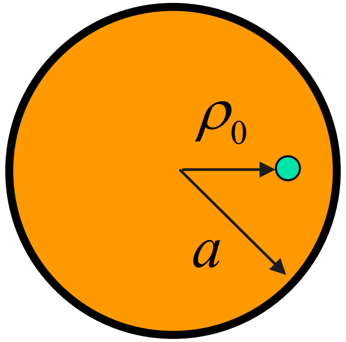
$$E_0 = \left(\frac{-j\omega\mu_0}{4\pi r}\right) e^{-jk_0 r}$$

$$N(\theta) = \sqrt{\varepsilon_r - \sin^2(\theta)}$$

Note: To account for lossy substrate, use $\varepsilon_r \to \varepsilon_{rc} = \varepsilon_r(1 - j\tan\delta)$

Circular Patch

Input Resistance



$$R_{\text{in}} \approx R_{\text{edge}} \left[\frac{J_1^2(k_1 \rho_0)}{J_1^2(k_1 a)} \right]$$

$$k_1 = k_0 \sqrt{\varepsilon_r}$$

Circular Patch

Input Resistance (cont.)

$$R_{\text{edge}} = \left[\frac{1}{2P_{\text{sp}}}\right] e_r$$

e_r = radiation efficiency

Note:

$$P_{\text{tot}} = \frac{1}{2}\frac{\left|V_{\text{edge}}\right|^2}{R_{\text{edge}}}$$

$$P_{\text{tot}} = P_{\text{sp}} / e_r$$

where

$$P_{\text{sp}} = \frac{\pi}{8\eta_0}(k_0 a)^2 \int_0^{\pi/2} \text{tanc}^2\left(k_0 h N(\theta)\right) \cdot \left[\left|Q(\theta)\right|^2 J_1'^{\,2}\left(k_0 a \sin\theta\right) + \left|P(\theta)\right|^2 J_{\text{inc}}^2\left(k_0 a \sin\theta\right)\right] \sin\theta \, d\theta$$

$$J_{\text{inc}}(x) = J_1(x) / x$$

P_{sp} = power radiated into space by circular patch with maximum edge voltage of one volt.

Circular Patch

Input Resistance (cont.)

CAD Formula:

$$P_{\text{sp}} = \frac{\pi}{8\eta_0}(k_0 a)^2 I_c$$

$$I_c \approx \frac{4}{3} p_c$$

$$p_c \approx \sum_{k=0}^{6} \left(k_0 a\right)^{2k} e_{2k}$$

$$e_0 = 1$$
$$e_2 = -0.400000$$
$$e_4 = 0.0785710$$
$$e_6 = -7.27509 \times 10^{-3}$$
$$e_8 = 3.81786 \times 10^{-4}$$
$$e_{10} = -1.09839 \times 10^{-5}$$
$$e_{12} = 1.47731 \times 10^{-7}$$

Circular Patch

Bandwidth and Radiation Efficiency

$$\text{BW} = \frac{1}{\sqrt{2}Q} \qquad e_r = \frac{Q}{Q_{\text{sp}}}$$

$$\frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{\text{sp}}} + \frac{1}{Q_{\text{sw}}}$$

$$Q_{\text{sp}} = \frac{3}{2}\left(\frac{1}{x_{11}'^2}\right)\left(x_{11}'^2 - 1\right)\left(\frac{1}{p_c}\right)\left(\frac{1}{k_0 h}\right)\varepsilon_r$$

$$Q_{\text{sw}} = Q_{\text{sp}}\left(\frac{e_r^{\text{hed}}}{1 - e_r^{\text{hed}}}\right)$$

$$e_r^{\text{hed}} = \frac{1}{1 + \frac{3}{4}\pi(k_0 h)\left(\frac{1}{c_1}\right)\left(1 - \frac{1}{\varepsilon_r}\right)^3}$$

$$c_1 = 1 - \frac{1}{\varepsilon_r} + \frac{0.4}{\varepsilon_r^2}$$

Same relationship between Q_{sw} and Q_{sp} as for the rectangular patch.

Note:

$$e_r^{\text{hed}} = \frac{P_{\text{sp}}}{P_{\text{tot}}} = \frac{P_{\text{sp}}}{P_{\text{sp}} + P_{\text{sw}}}$$

$$\Rightarrow \frac{1}{e_r^{\text{hed}}} = \frac{P_{\text{sp}} + P_{\text{sw}}}{P_{\text{sp}}} = 1 + \frac{P_{\text{sw}}}{P_{\text{sp}}}$$

$$\Rightarrow \frac{P_{\text{sw}}}{P_{\text{sp}}} = \frac{1}{e_r^{\text{hed}}} - 1$$

$$\Rightarrow \frac{Q_{\text{sp}}}{Q_{\text{sw}}} = \left(\frac{1}{e_r^{\text{hed}}} - 1\right) = \frac{1 - e_r^{\text{hed}}}{e_r^{\text{hed}}}$$

$$\Rightarrow \frac{Q_{\text{sw}}}{Q_{\text{sp}}} = \frac{e_r^{\text{hed}}}{1 - e_r^{\text{hed}}}$$

$$Q_c = \left(\frac{\eta_0}{2}\right)\left[\frac{(k_0 h)}{R_s^{\text{ave}}}\right] \qquad R_s^{\text{ave}} = \left(R_s^{\text{patch}} + R_s^{\text{ground}}\right)/2$$

$$R_s = \frac{1}{\sigma\delta} \qquad \delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} \qquad Q_d = 1/\tan\delta$$

These formulas are the same for <u>any</u> shape patch.

Circular Patch

Directivity

$$D \approx \frac{3}{p_c} \frac{\tanc^2(k_1 h)}{\left[1 + \left(\frac{\mu_r}{\varepsilon_r}\right)\tan^2(k_1 h)\right]} \approx \frac{3}{p_c}$$

$$\operatorname{tanc}(x) \equiv \tan(x)/x$$

The term p_c for the circular patch was defined previously.

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Improving Bandwidth

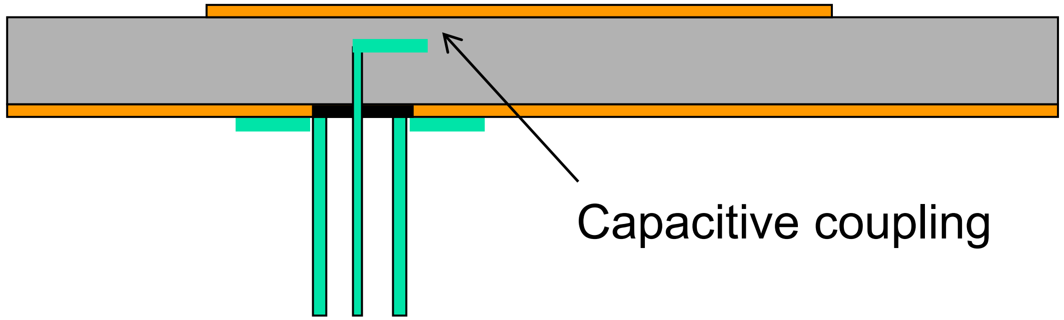
Some of the techniques that have been successfully developed are illustrated here.

The literature may be consulted for additional designs and variations.

Improving Bandwidth

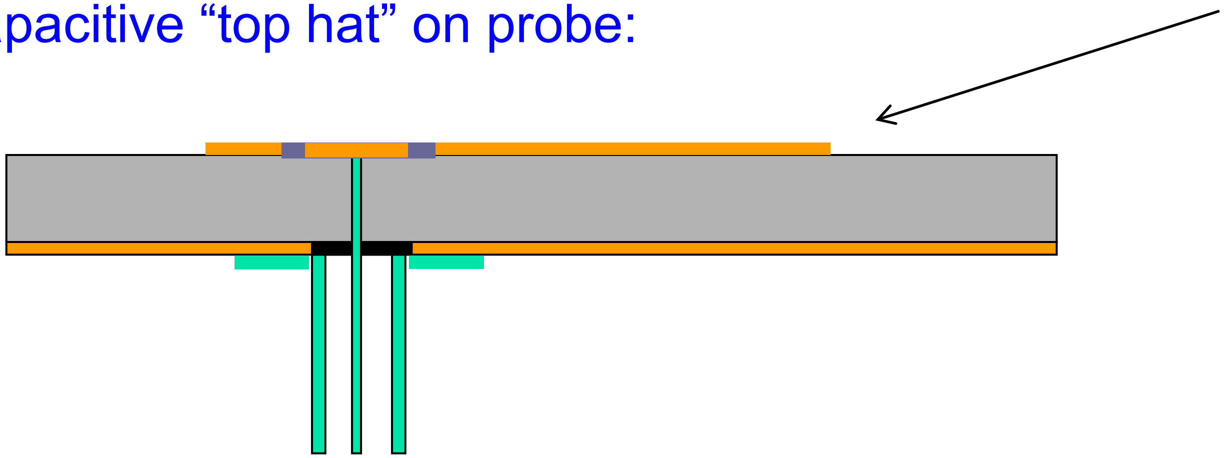
Probe Compensation

L-shaped probe:

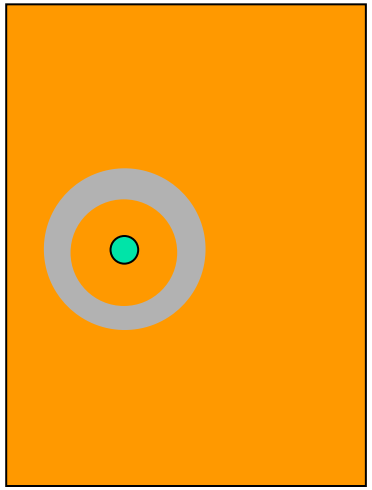


Basic Idea:
As the substrate thickness increases, the probe inductance limits the bandwidth – so we compensate for it with a capacitance.

Capacitive “top hat” on probe:



Top view



Improving Bandwidth

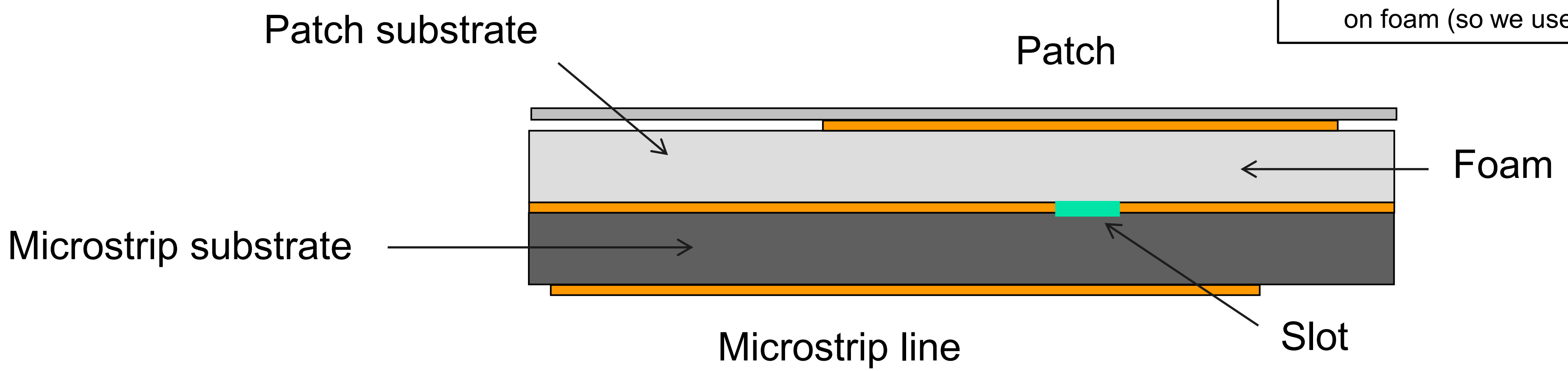
SSFIP: Strip Slot Foam Inverted Patch (a version of the ACP).

ACP = Aperture Coupled Patch

- Bandwidths greater than 25% have been achieved.
- Increased bandwidth is due to the thick foam substrate and also a dual-tuned resonance (patch+slot).

Note: There is no probe inductance to worry about with the ACP feed.

Note:
It would be difficult to fabricate a patch directly on foam (so we use an inverted patch).



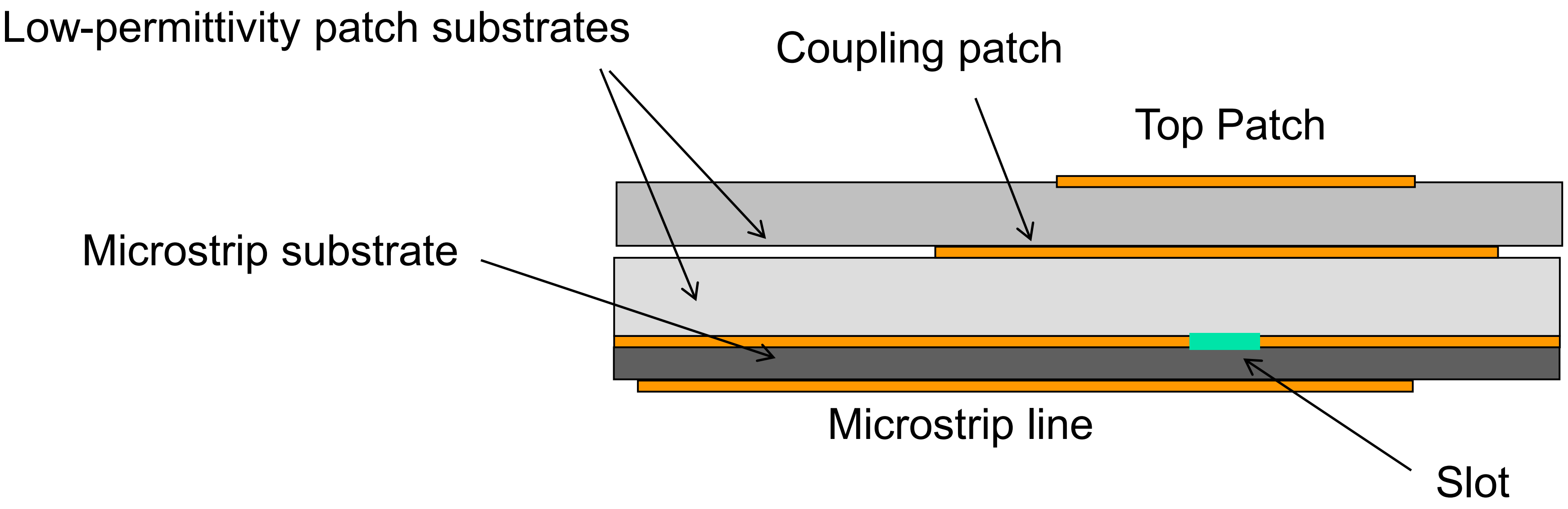
J.-F. Zürcher and F. E. Gardiol, *Broadband Patch Antennas,* Artech House, Norwood, MA, 1995.

Improving Bandwidth

Stacked Patches

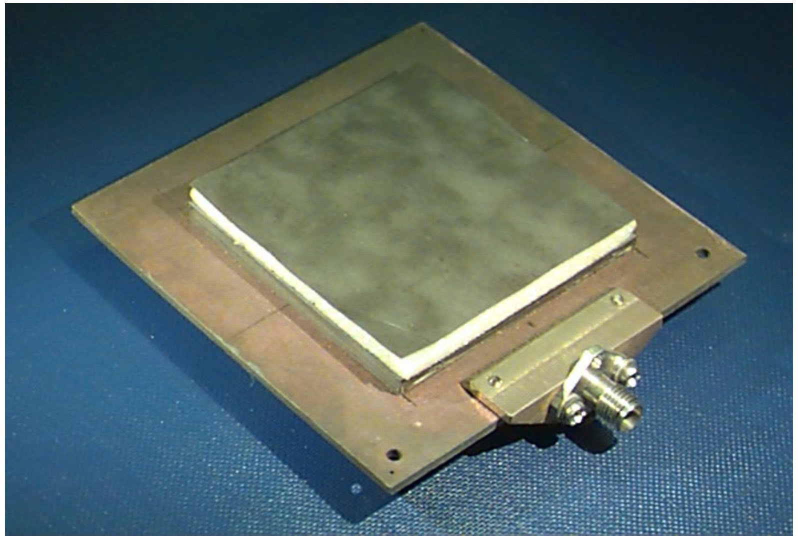
- Bandwidth increase is due to thick low-permittivity antenna substrates and a dual or triple-tuned resonance.
- Bandwidths of 25% have been achieved using a probe feed.
- Bandwidths of 100% have been achieved using an ACP feed.

Note: There is no probe inductance to worry about with the ACP feed.

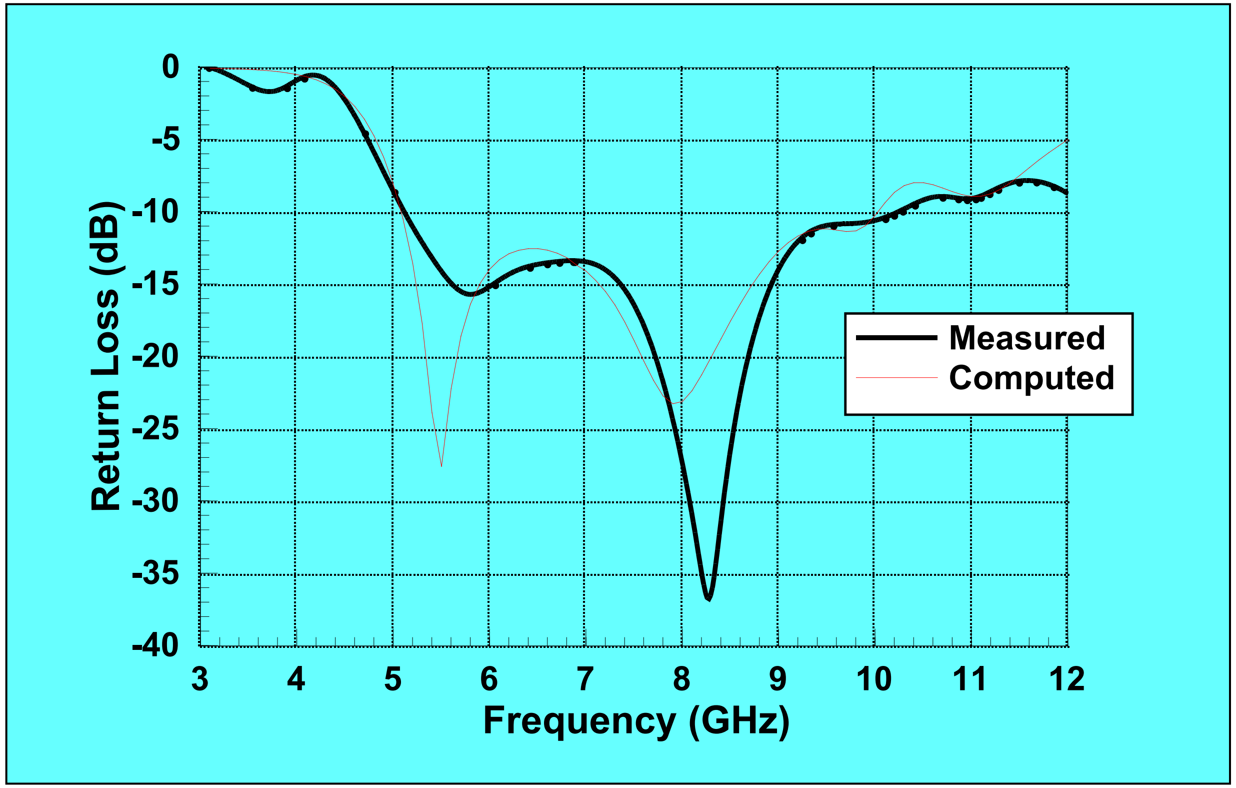


Improving Bandwidth

Stacked Patches



Stacked patch with ACP feed

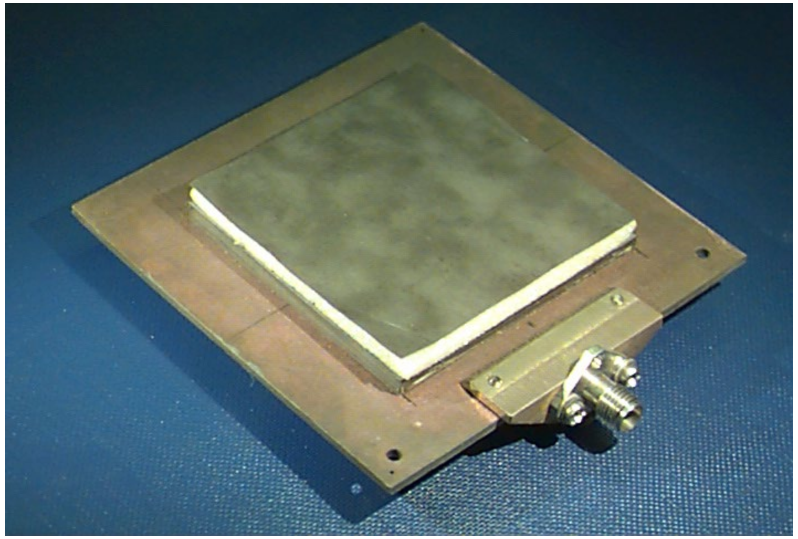


Bandwidth (S_{11} = -10 dB) is about 100%

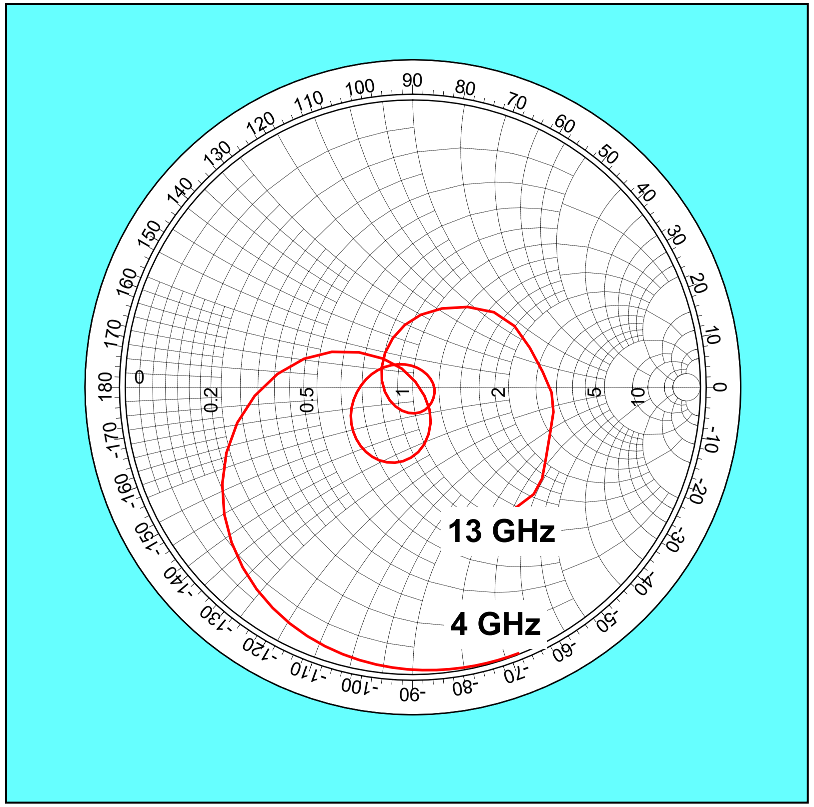
(Photo courtesy of Dr. Rodney B. Waterhouse)

Improving Bandwidth

Stacked Patches



Stacked patch with ACP feed

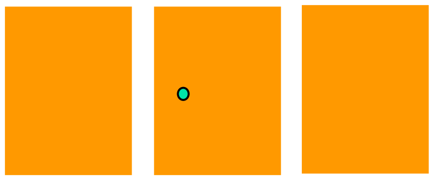


Two extra loops are observed on the Smith chart.

(Photo courtesy of Dr. Rodney B. Waterhouse)

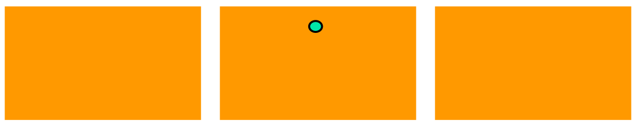
Improving Bandwidth

Parasitic Patches

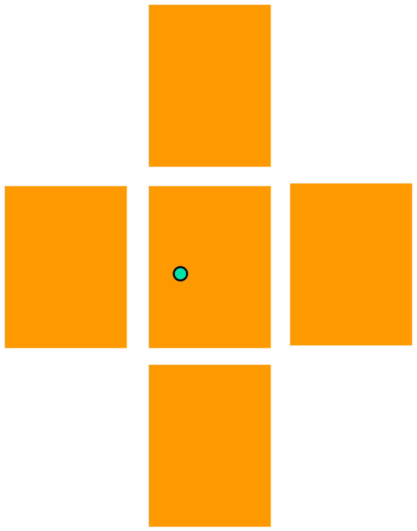


Radiating Edges Gap Coupled Microstrip Antennas (REGCOMA).

Much of this work was pioneered by K. C. Gupta.



Non-Radiating Edges Gap Coupled Microstrip Antennas (NEGCOMA)

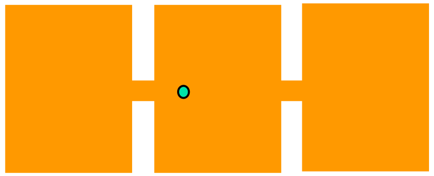


Four-Edges Gap Coupled Microstrip Antennas (FEGCOMA)

Bandwidth improvement factor:
REGCOMA: 3.0, NEGCOMA: 3.0, FEGCOMA: 5.0?

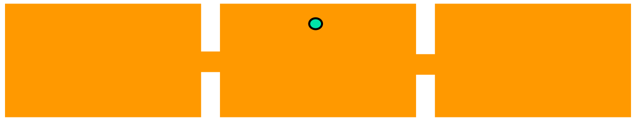
Improving Bandwidth

Direct-Coupled Patches

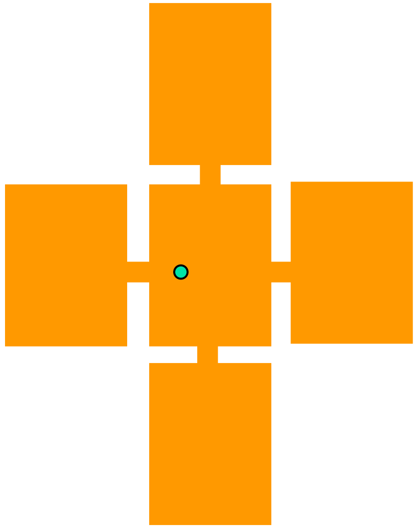


Radiating Edges Direct Coupled Microstrip Antennas (REDCOMA).

Much of this work was pioneered by K. C. Gupta.



Non-Radiating Edges Direct Coupled Microstrip Antennas (NEDCOMA)

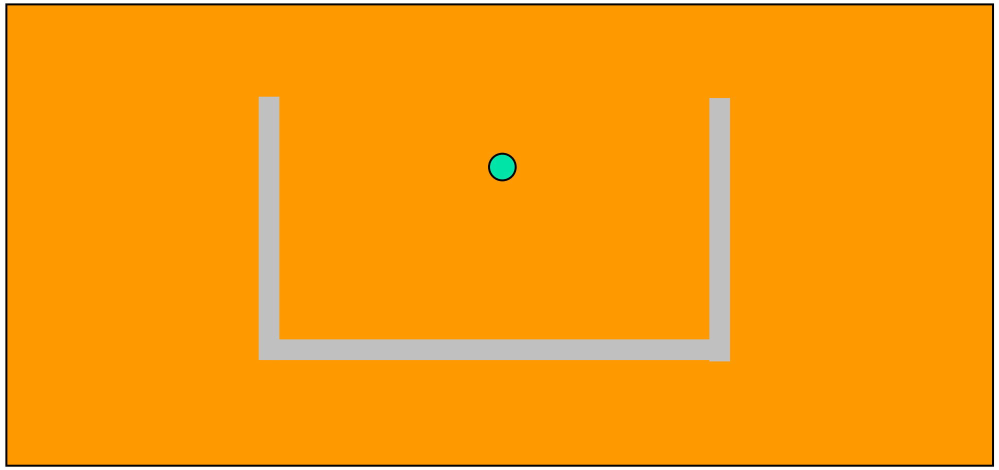


Four-Edges Direct Coupled Microstrip Antennas (FEDCOMA)

Bandwidth improvement factor:
REDCOMA: 5.0, NEDCOMA: 5.0, FEDCOMA: 7.0

Improving Bandwidth

U-Shaped Slot



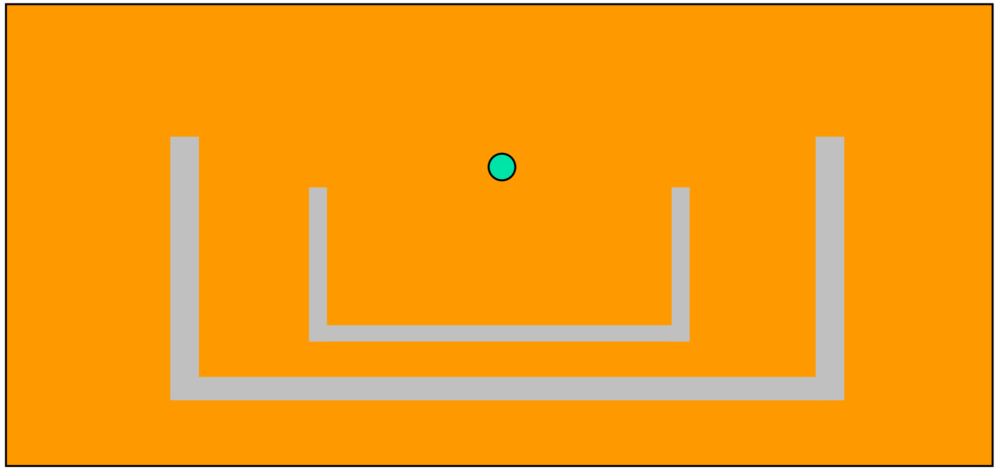
The introduction of a U-shaped slot can give a significant bandwidth (10%-40%).

(This is due to a double resonance effect, with two different modes.)

"Single Layer Single Patch Wideband Microstrip Antenna," T. Huynh and K. F. Lee, *Electronics Letters*, vol. 31, no. 16, pp. 1310-1312, 1986.

Improving Bandwidth

Double U-Slot

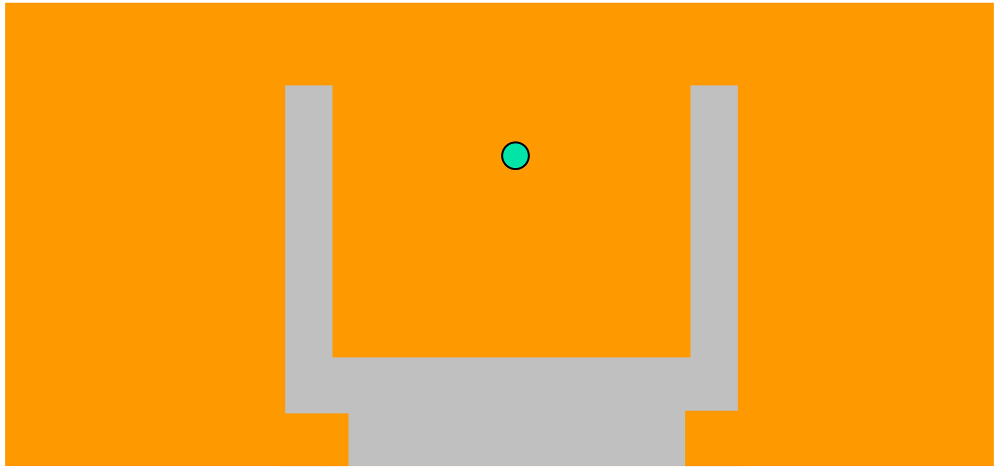


A 44% bandwidth was achieved.

Y. X. Guo, K. M. Luk, and Y. L. Chow, "Double U-Slot Rectangular Patch Antenna," *Electronics Letters*, vol. 34, no. 19, pp. 1805-1806, 1998.

Improving Bandwidth

E Patch



A modification of the U-slot patch.

A bandwidth of 34% was achieved (40% when using a capacitive “top hat” or “washer” to compensate for the probe inductance).

B. L. Ooi and Q. Shen, “A Novel E-shaped Broadband Microstrip Patch Antenna,” *Microwave and Optical Technology Letters*, vol. 27, no. 5, pp. 348-352, 2000.

Multi-Band Antennas

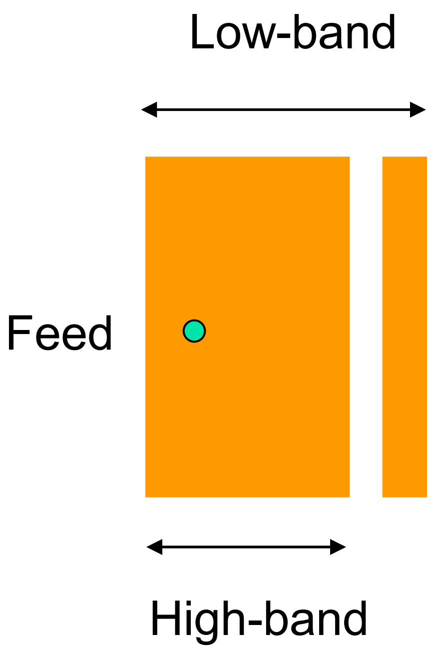
A multi-band antenna is sometimes more desirable than a broadband antenna, if multiple narrow-band channels are to be covered.

General Principle:

Introduce multiple resonance paths into the antenna.

Multi-Band Antennas

Dual-band E patch



Dual-band patch with parasitic strip

Outline

- ❖ Overview of microstrip antennas
- ❖ History of microstrip antennas
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- ❖ Circular polarization
- ❖ Circular patch
- ❖ Improving bandwidth
- ❖ Miniaturization
- ❖ Reducing surface waves and lateral radiation

Miniaturization

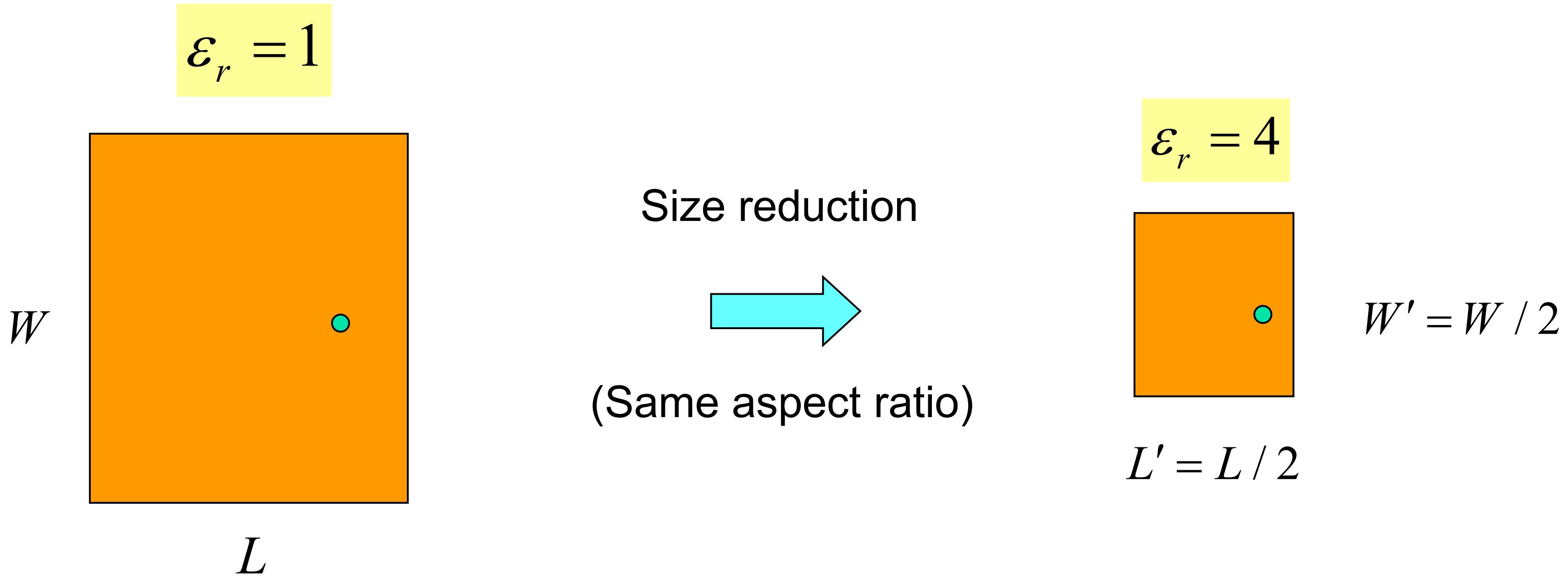
- High Permittivity
- Quarter-Wave Patch
- PIFA
- Capacitive Loading
- Slots
- Meandering

Note: Miniaturization usually comes at a price of reduced bandwidth!

Usually, bandwidth is proportional to the volume of the patch cavity, as we will see in the examples.

Miniaturization

High Permittivity



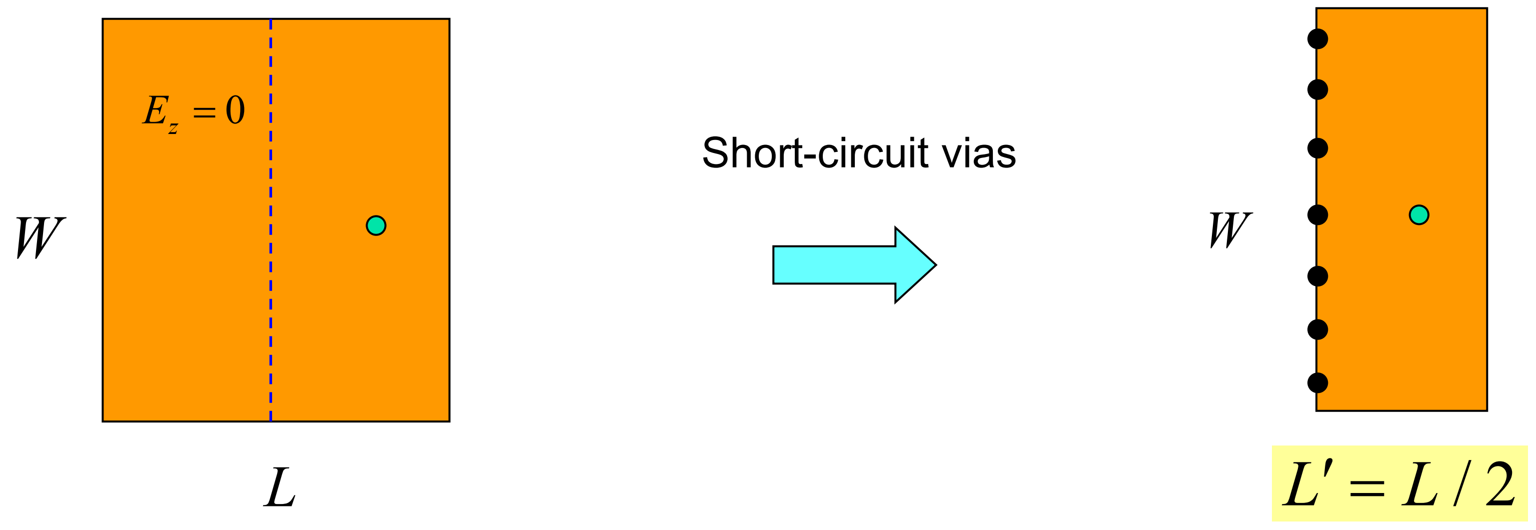
The volume decreases by a factor of 4.

The smaller patch has about one-fourth the bandwidth of the original patch.

(Bandwidth is inversely proportional to the permittivity.)

Miniaturization

Quarter-Wave patch



The volume decreases by a factor of 2.

The new patch has about one-half the bandwidth of the original patch.

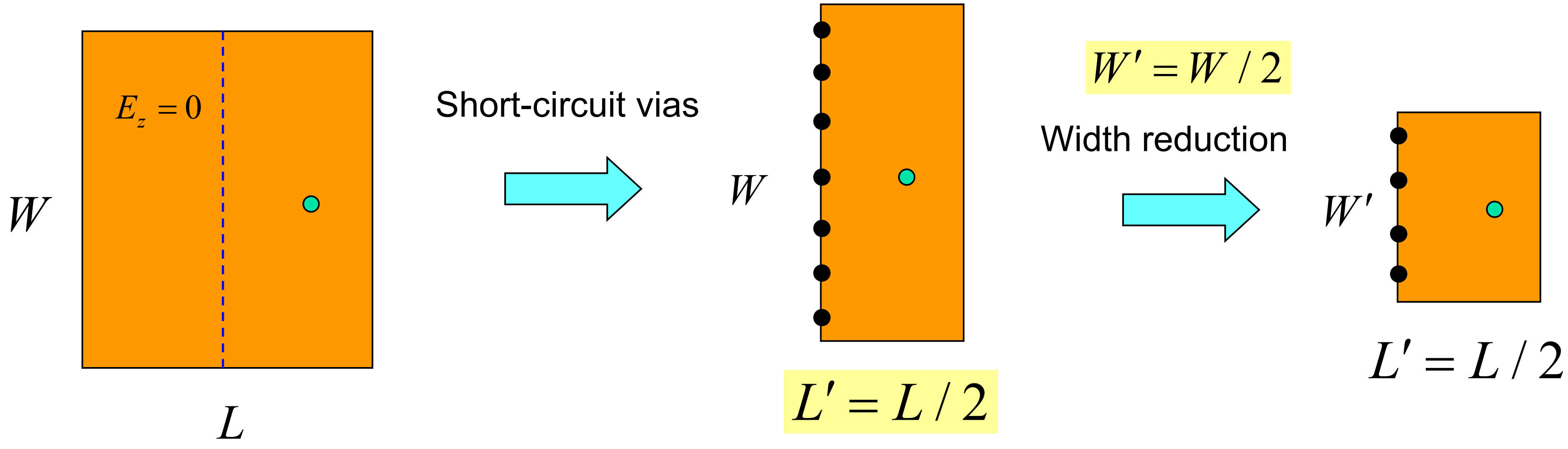
Neglecting losses: $$Q = Q_{\text{sp}} = \omega_0 \frac{U_s}{P_{\text{sp}}} \qquad \left.\begin{aligned} U_s' &= U_s / 2 \\ P_{\text{sp}}' &= P_{\text{sp}} / 4 \end{aligned}\right\} \Rightarrow Q' = 2Q$$

Note: We now have 1/2 of the previous radiating magnetic current (only one radiating edge).

Miniaturization

Smaller Quarter-Wave patch

A quarter-wave patch with the *same aspect ratio* W/L as the original patch

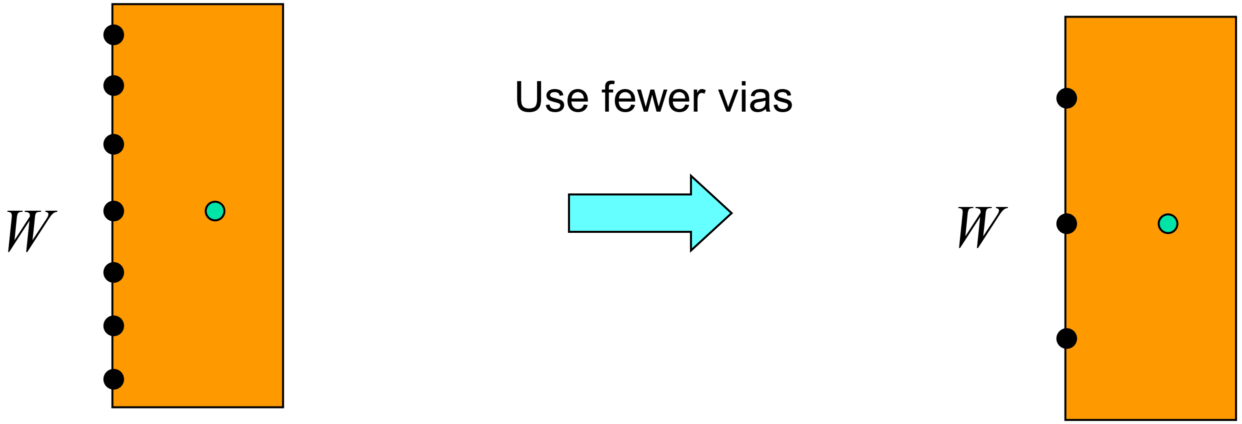


The volume decreases by a factor of 4 from the original patch.

The new patch has about one-half the bandwidth of the original quarter-wave patch (bandwidth is proportional to the patch width), and hence one-fourth the bandwidth of the regular patch.

Miniaturization

Quarter-Wave Patch with Fewer Vias



$L' = L/2$

L''

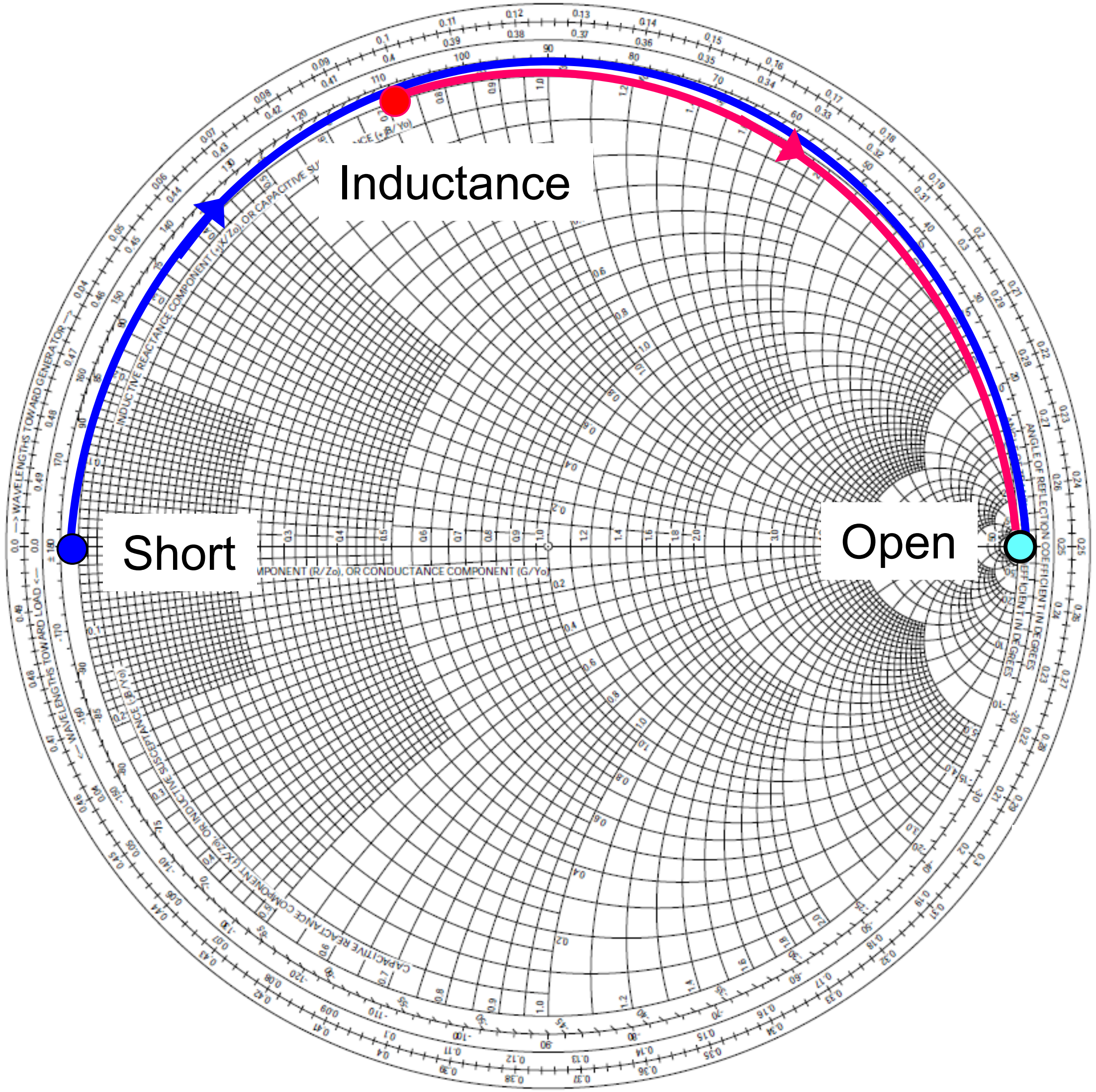
$$L'' < L'$$

Fewer vias gives more miniaturization!

(The edge now has an inductive impedance instead of being a short circuit: the miniaturization is explained on the next slide.)

Miniaturization

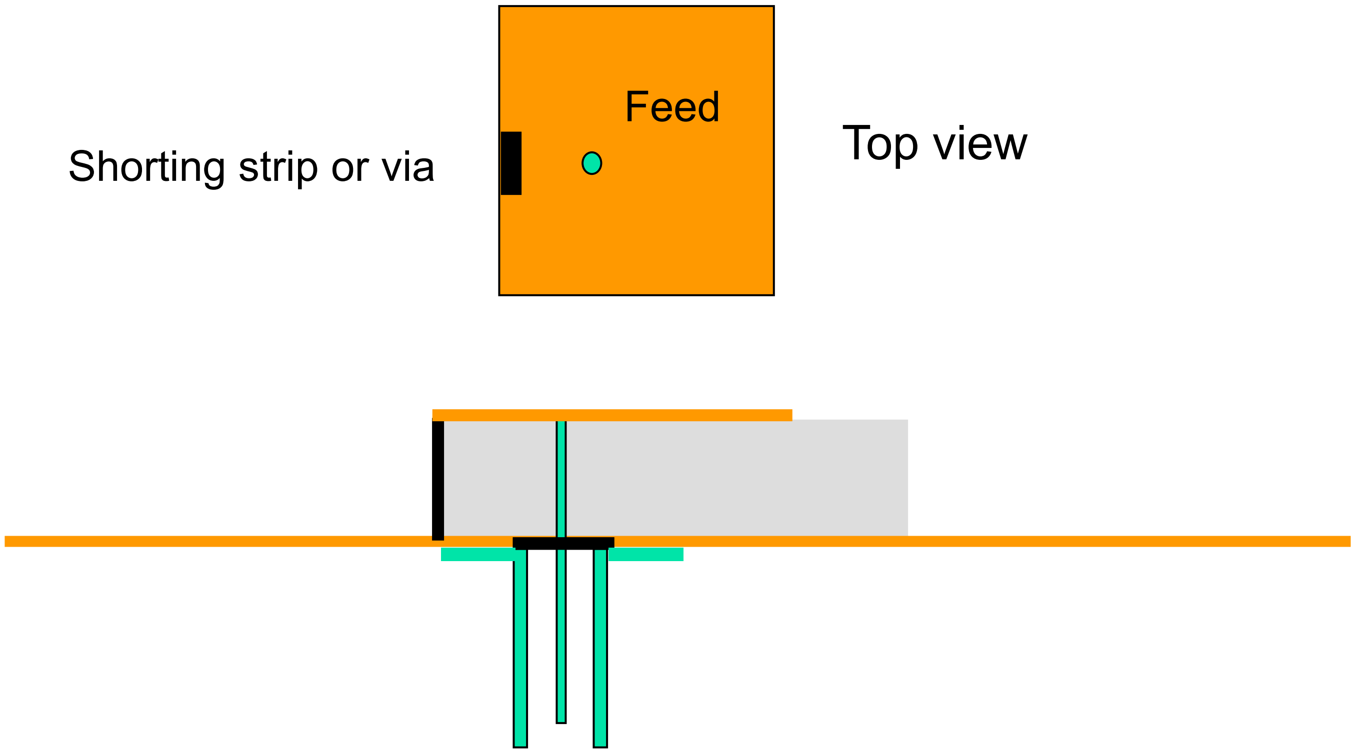
Quarter-Wave Patch with Fewer Vias



The Smith chart provides a simple explanation for the extra length reduction vs. using a short-circuit wall.

Miniaturization

Planar Inverted F (PIFA)

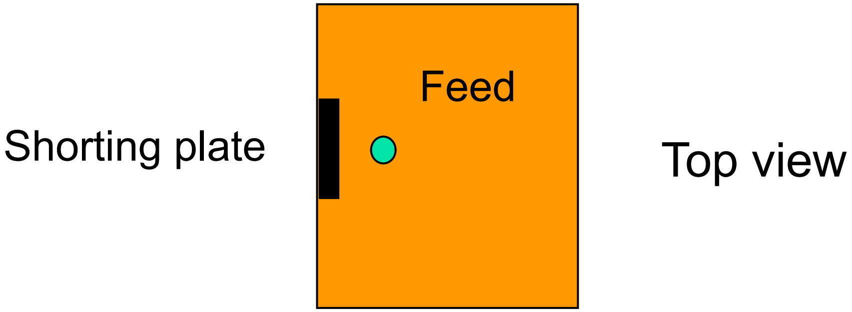


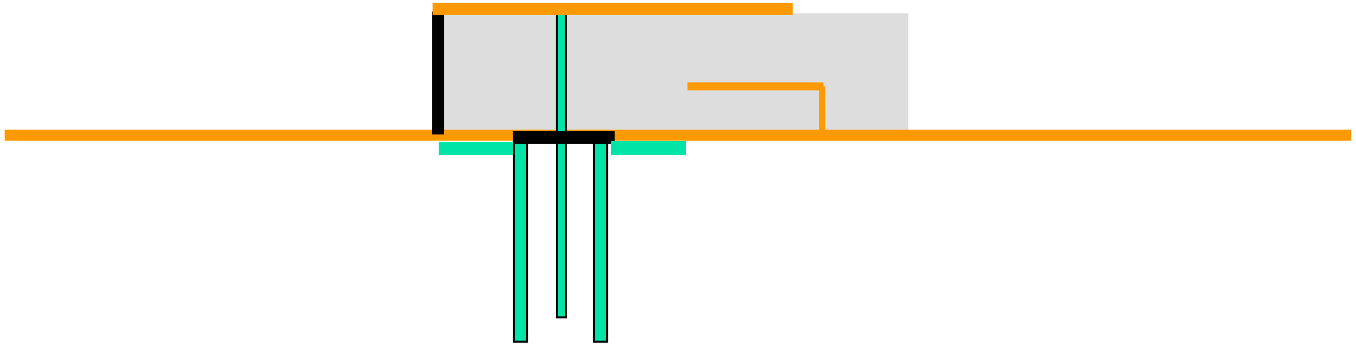
A single shorting strip or via is used.

This antenna can be viewed as a limiting case of the via-loaded patch, or as an LC resonator.

Miniaturization

PIFA with Capacitive Loading



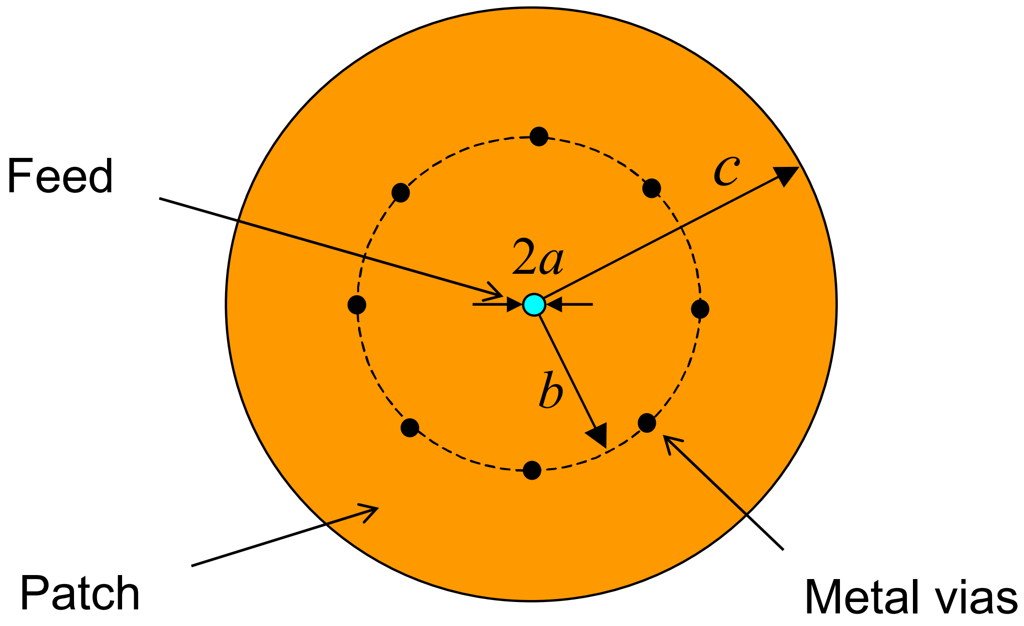


The capacitive loading allows for the length of the PIFA to be reduced.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Miniaturization

Circular Patch Loaded with Vias



The patch has a monopole-like pattern

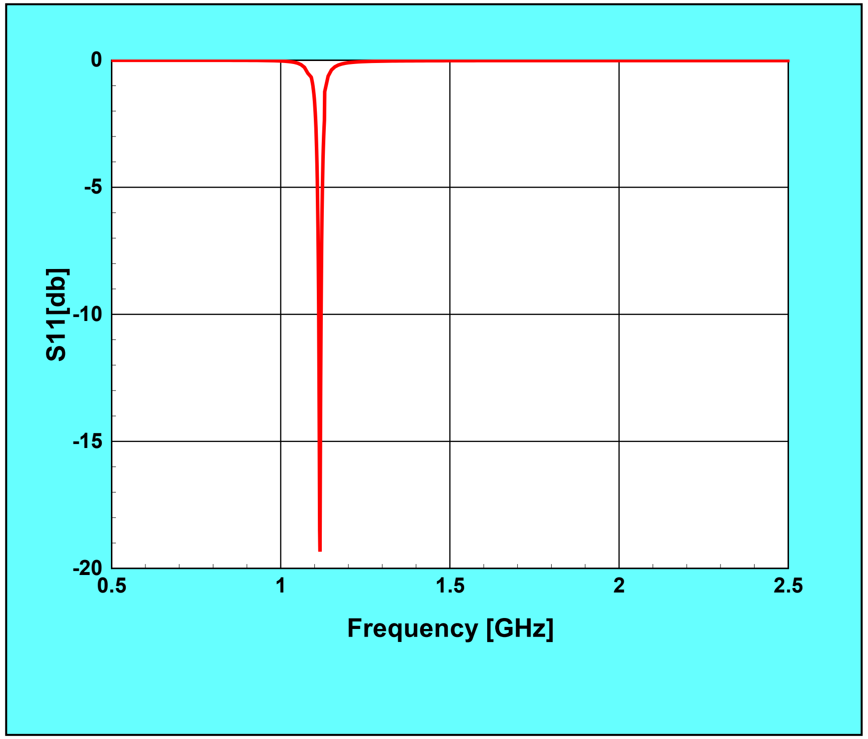
The patch operates in the $(0,0)$ mode, as an LC resonator.

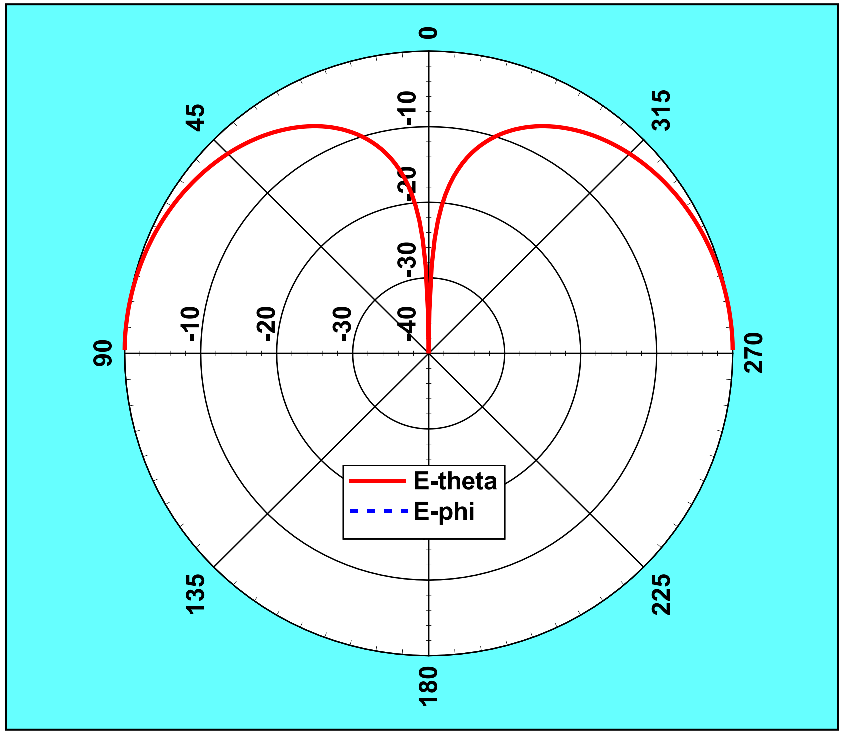
(Hao Xu Ph.D. dissertation, University of Houston, 2006)

Miniaturization

Circular Patch Loaded with Vias

Example: Circular Patch Loaded with 2 Vias





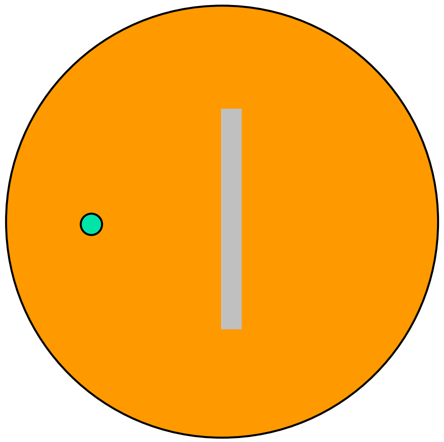
Unloaded: resonance frequency = 5.32 GHz. Loaded: resonance frequency = 1.11 GHz.

(Miniaturization factor = 4.8)

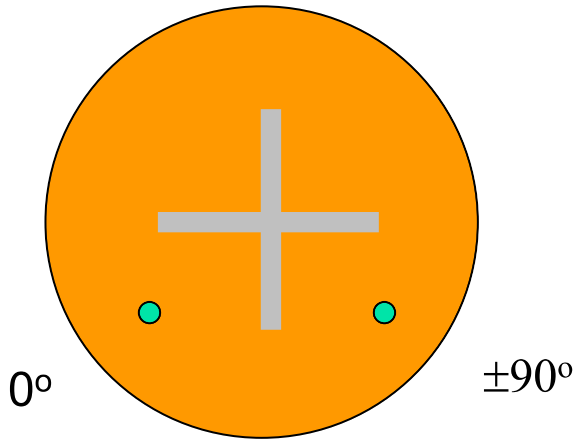
Miniaturization

Slotted Patch

Top view



Linear

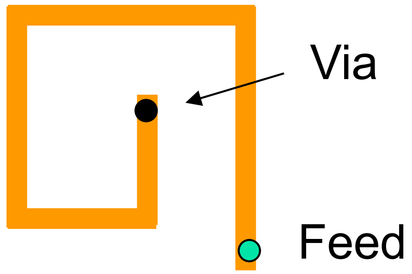


CP

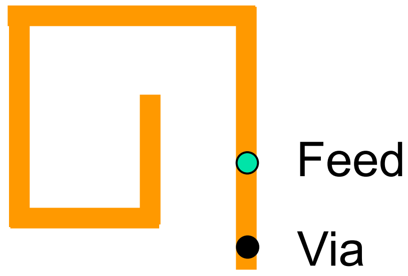
The slot forces the current to flow through a longer path, increasing the effective dimensions of the patch.

Miniaturization

Meandering



Meandered quarter-wave patch



Meandered PIFA

- Meandering forces the current to flow through a longer path, increasing the effective dimensions of the patch.
- Meandering also increases the capacitance of the PIFA line.

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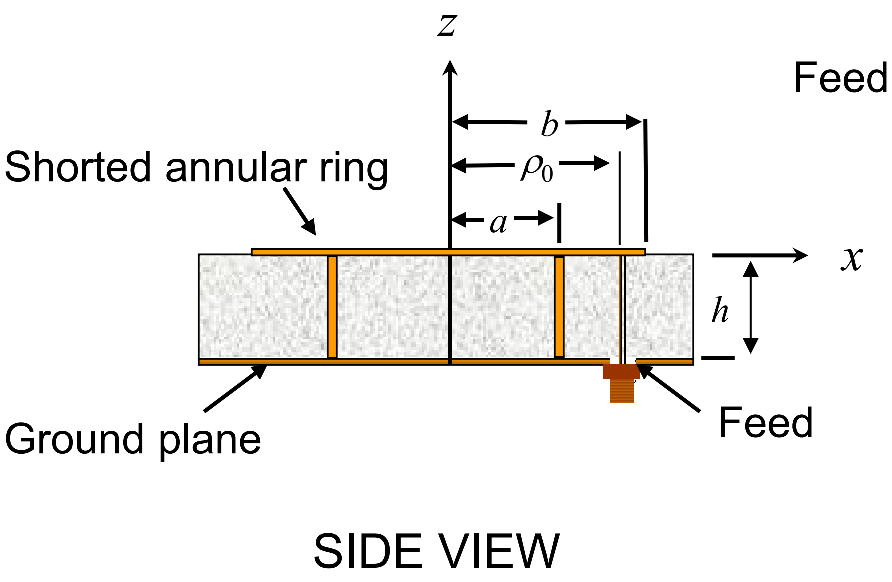
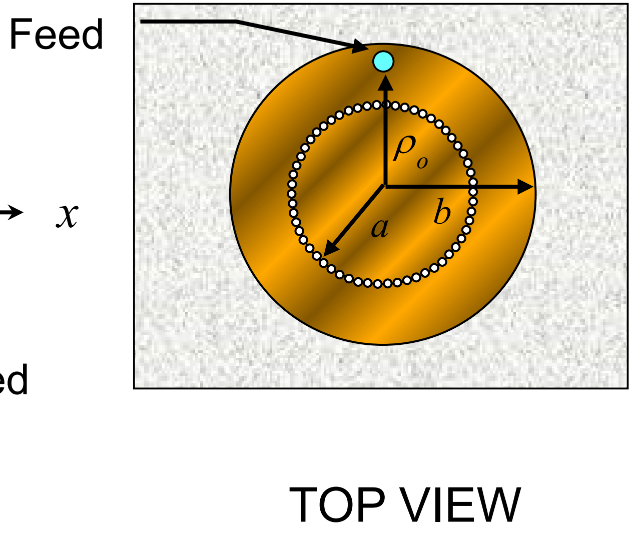
- ❖ Miniaturization
- ❖ Reducing surface wave and lateral radiation

Reducing Surface and Lateral Waves

Reduced Surface Wave (RSW) Antenna





D. R. Jackson, J. T. Williams, A. K. Bhattacharyya, R. Smith, S. J. Buchheit, and S. A. Long, "Microstrip Patch Designs that do Not Excite Surface Waves," IEEE Trans. Antennas Propagat., vol. 41, No 8, pp. 1026-1037, August 1993.

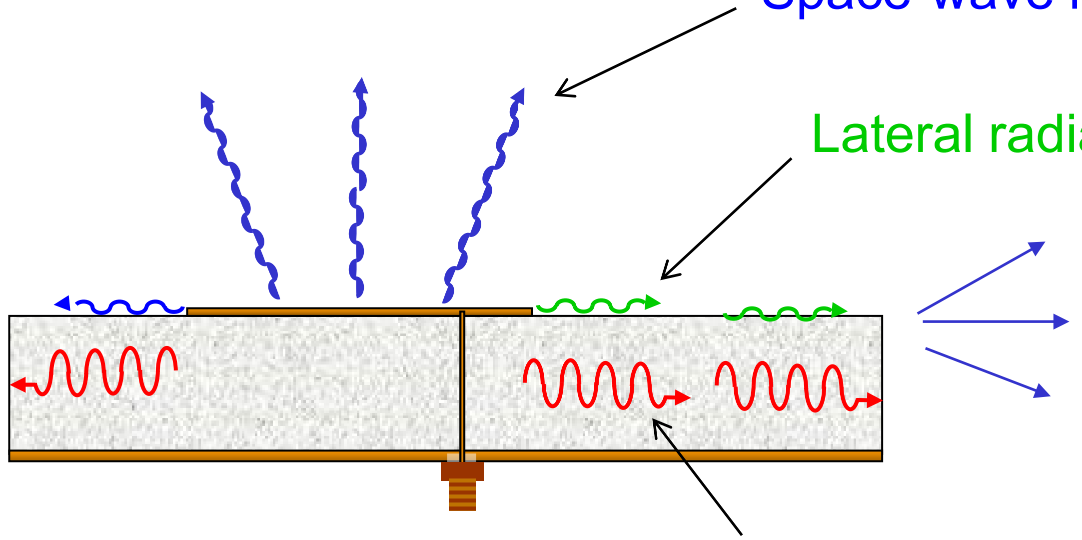
Reducing Surface and Lateral Waves

Reducing surface-wave excitation and lateral radiation reduces edge diffraction and mutual coupling.

- Edge diffraction degrades the radiation pattern on a finite ground plane.
- Mutual coupling causes undesirable coupling between antennas.

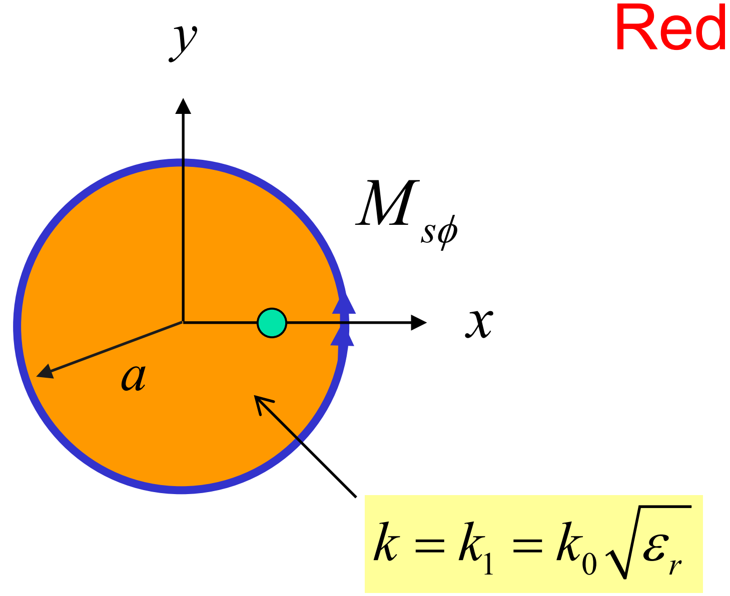
Reducing Surface and Lateral Waves

Reducing surface-wave excitation and lateral radiation reduces edge diffraction.



Reducing Surface and Lateral Waves

Reducing the Surface Wave Excitation



TM$_{11}$ mode inside patch:

$$E_z(\rho,\phi) = V_0\left(\frac{-1}{hJ_1(k_1 a)}\right)\cos\phi\, J_1(k_1\rho)$$

At edge: $E_z = -\dfrac{V_0}{h}\cos\phi$

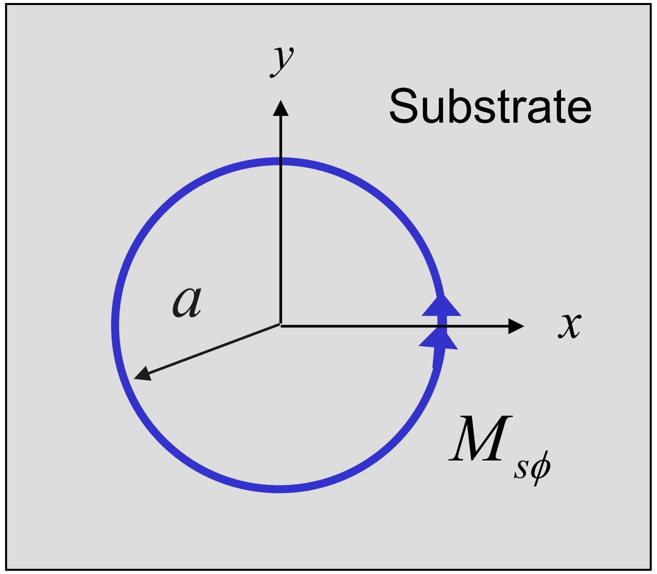
(The voltage at the patch edge is one volt at $\phi = 0$.)

$$\underline{M}_s = -\underline{\hat{n}}\times\underline{E} = -\underline{\hat{\rho}}\times\left(\underline{\hat{z}}E_z\right)$$

$$M_{s\phi}(\phi) = E_z(a,\phi)$$

$$\boxed{M_{s\phi} = -\frac{V_0}{h}\cos\phi}$$

Reducing Surface and Lateral Waves



$$M_{s\phi} = -\frac{V_0}{h}\cos\phi$$

Surface-Wave Excitation:

$$E_z^{\mathrm{TM}_0} = A_{\mathrm{TM}_0}\cos\phi\, H_1^{(2)}\left(\beta_{\mathrm{TM}_0}\rho\right)e^{-jk_{z0}z}$$

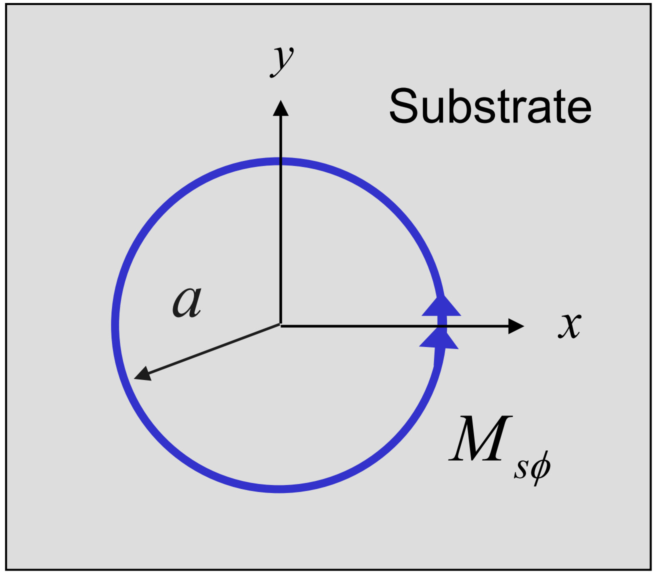
$(z > h)$

$$A_{\mathrm{TM}_0} = AJ_1'\left(\beta_{\mathrm{TM}_0}a\right)$$

$A =$ constant

Set $$J_1'\left(\beta_{\mathrm{TM}_0}a\right) = 0$$

Reducing Surface and Lateral Waves



Choose:

$$\beta_{\mathrm{TM}_0} a = x'_{1n}$$

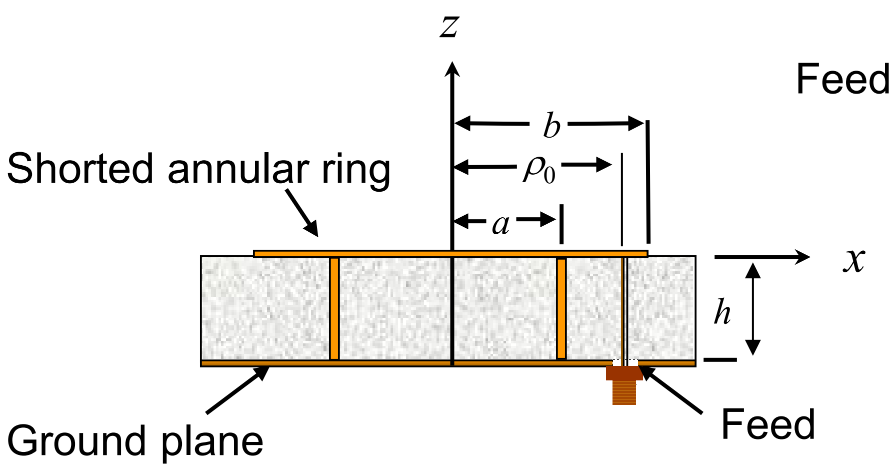
For TM_{11} mode: $x'_{11} \approx 1.841$

Hence $\boxed{\beta_{\mathrm{TM}_0} a = 1.841}$

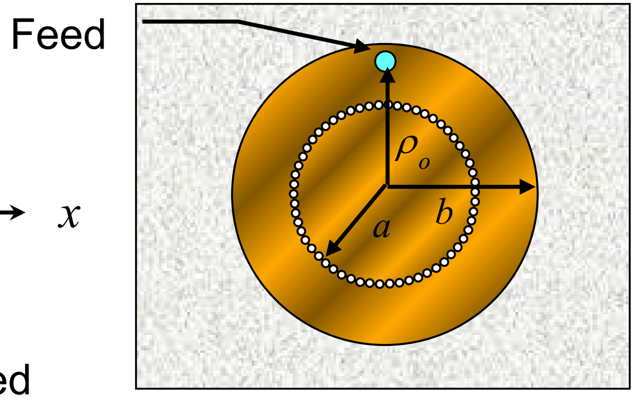
Patch resonance: $k_1 a = x'_{11} = 1.841$

Note: $\beta_{\mathrm{TM}_0} < k_1$ $\Rightarrow$ The RSW patch is too big to be resonant.

Reducing Surface and Lateral Waves



SIDE VIEW



TOP VIEW

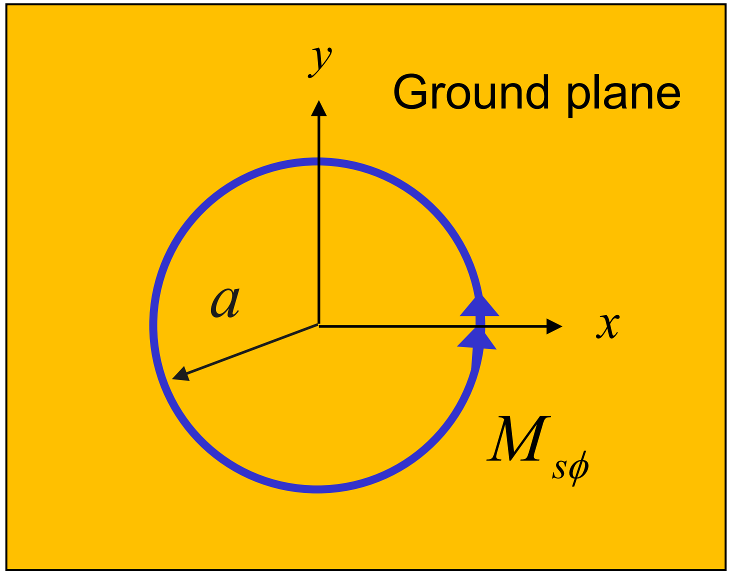
$$\beta_{\mathrm{TM}_0} b = x'_{11} = 1.841$$

The radius a is chosen to make the patch resonant:

$$\frac{J_1(k_1 a)}{Y_1(k_1 a)} = \frac{J'_1\left(\dfrac{k_1 x'_{11}}{\beta_{\mathrm{TM}_0}}\right)}{Y'_1\left(\dfrac{k_1 x'_{11}}{\beta_{\mathrm{TM}_0}}\right)}$$

Reducing Surface and Lateral Waves

Reducing the Lateral Radiation



$$M_{s\phi} = -\frac{V_0}{h}\cos\phi$$

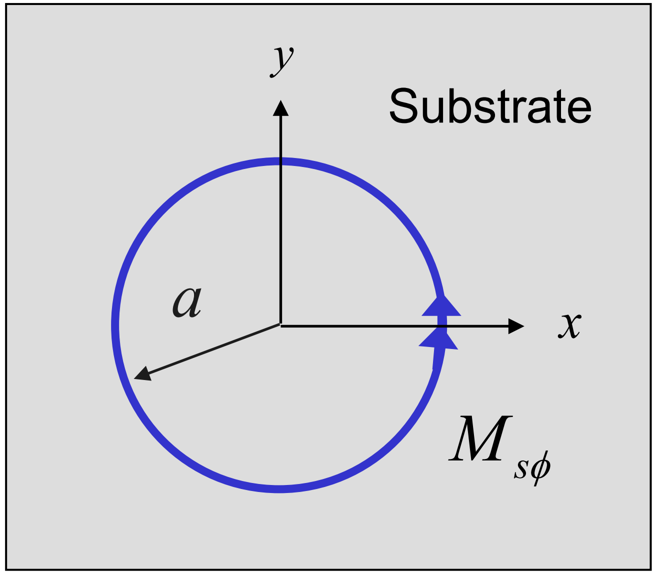
Assume no substrate outside of patch (or very thin substrate):

Space-Wave Field: $$E_z^{\mathrm{SP}} = A_{\mathrm{SP}}\cos\phi\left(\frac{1}{\rho}\right)e^{-jk_0\rho} \qquad z = h \qquad A_{\mathrm{SP}} = CJ_1'(k_0 a)$$

$C =$ constant

Set $J_1'(k_0 a) = 0$ $\Rightarrow$ $k_0 a = 1.841$

Reducing Surface and Lateral Waves



For a thin substrate:

$$\beta_{\mathrm{TM}_0} \approx k_0$$

The same design therefore reduces both surface-wave fields and lateral-radiation fields.

Note: The diameter of the RSW antenna is found from:

$$k_0 a = 1.841$$

$$\Rightarrow \quad \frac{2a}{\lambda_0} = 0.586$$

Note:
The size is approximately independent of the permittivity.
(The patch cannot be miniaturized by choosing a higher permittivity!)

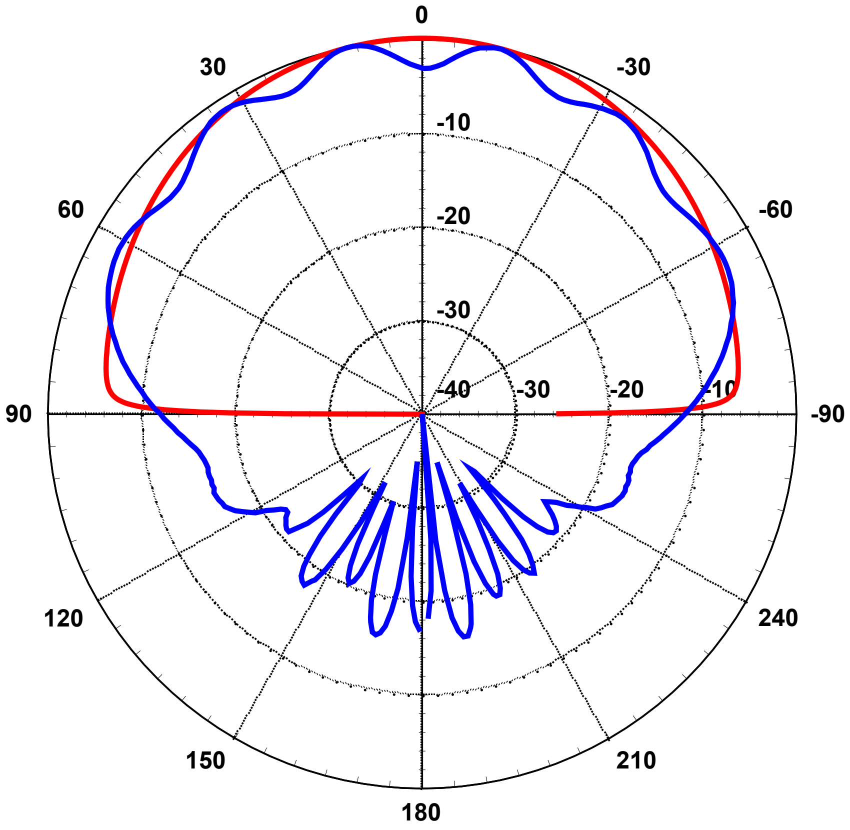
Reducing Surface and Lateral Waves

E-plane Radiation Patterns

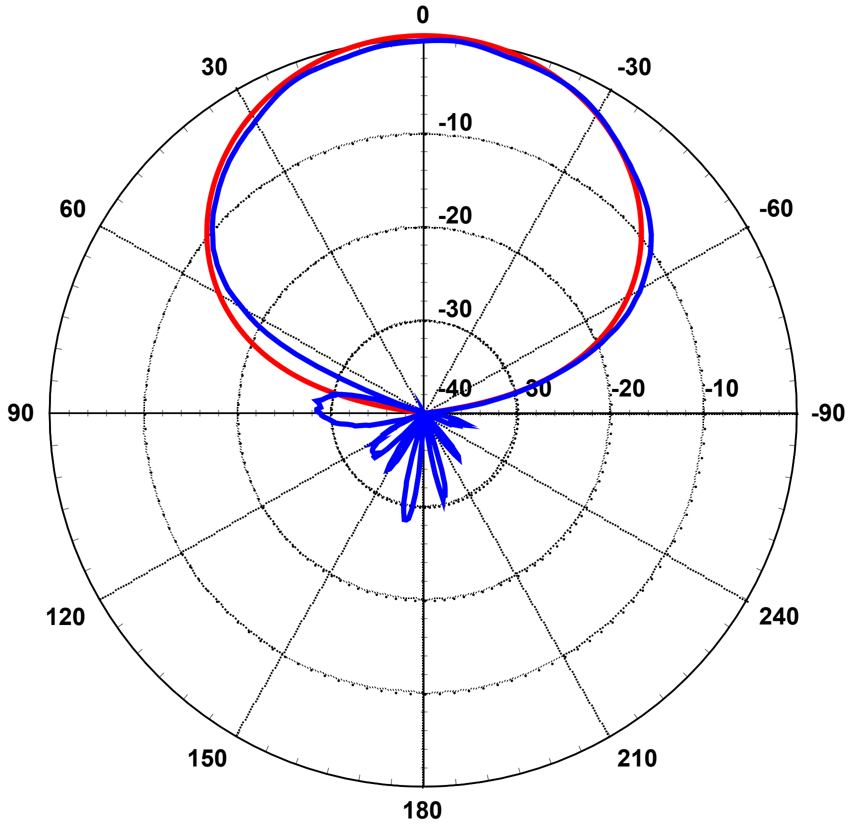
Measurements were taken on a 1 m diameter circular ground plane (GP) at 1.575 GHz.

Measurement

Theory (infinite substrate & GP)



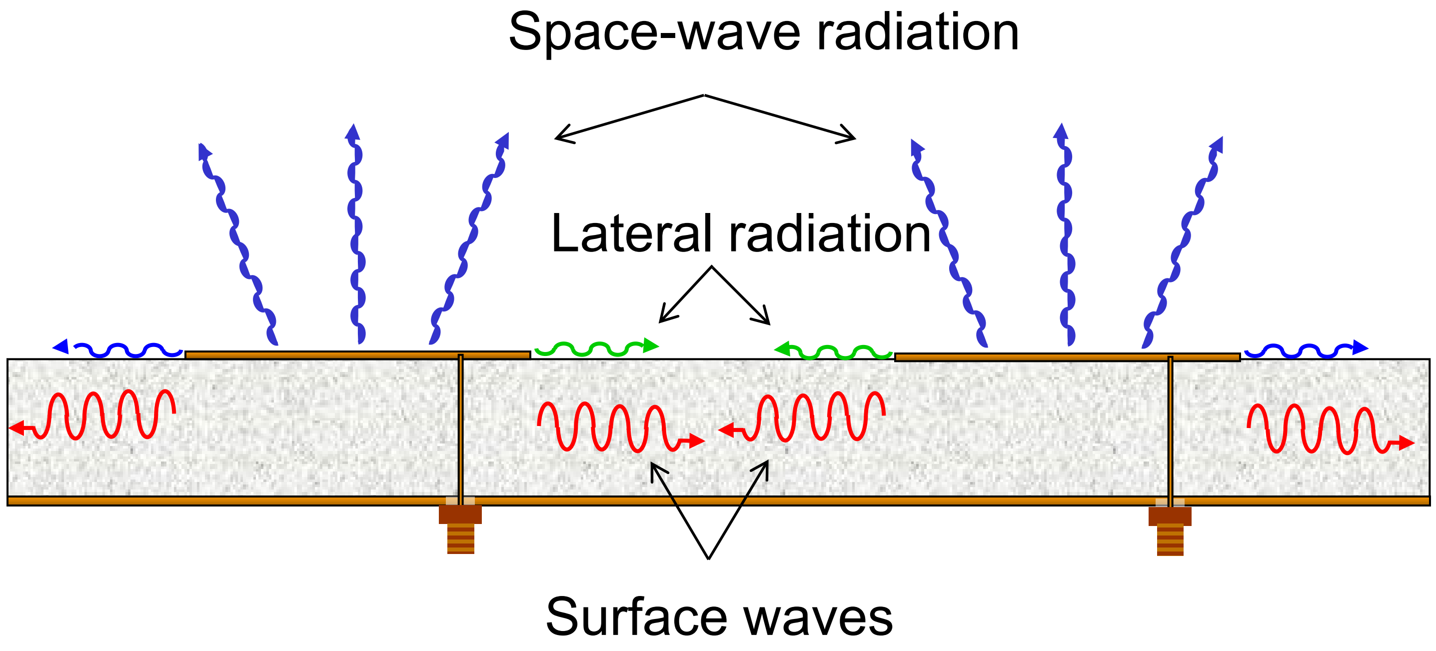
Conventional



RSW

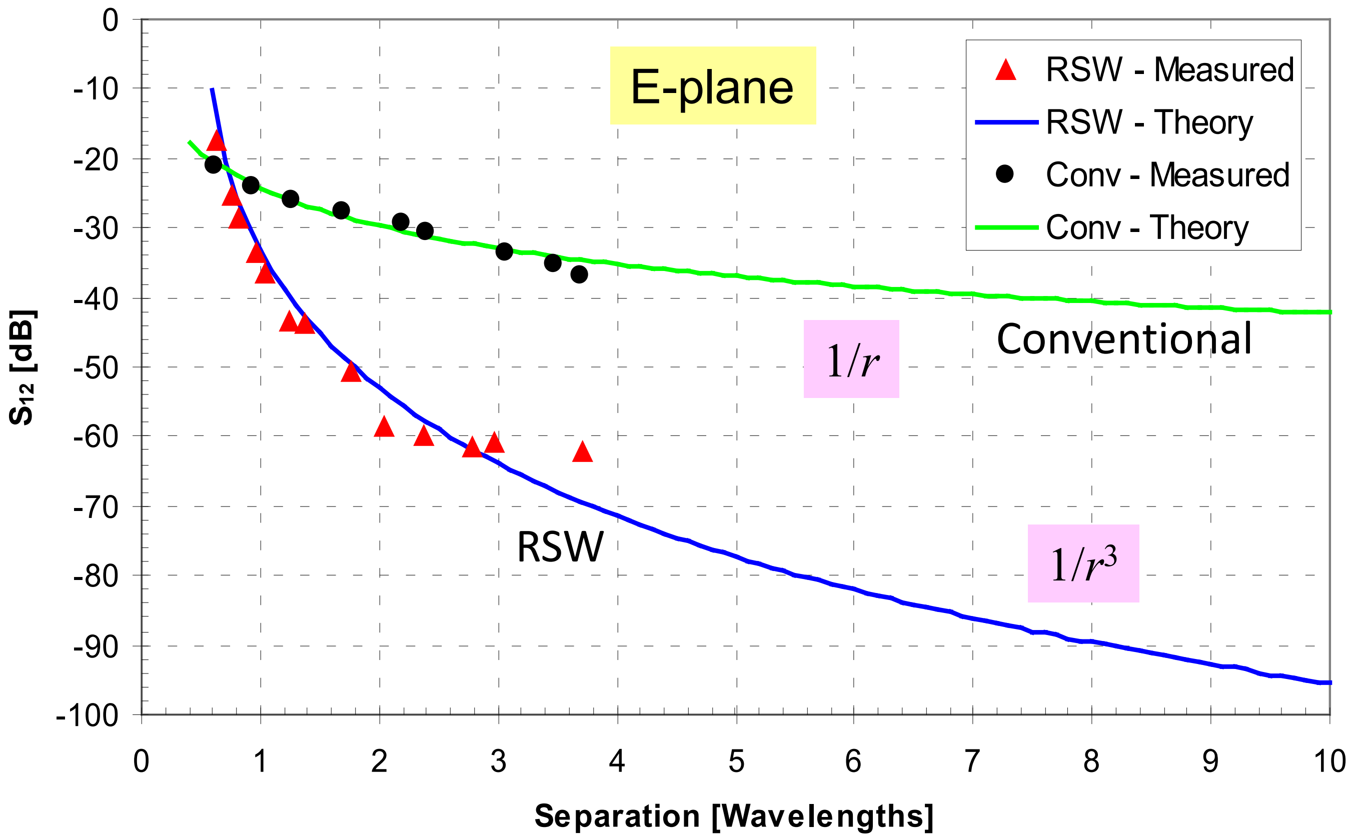
Reducing Surface and Lateral Waves

Reducing surface-wave excitation and lateral radiation reduces mutual coupling.



Reducing Surface and Lateral Waves

Reducing surface-wave excitation and lateral radiation reduces mutual coupling.



"Mutual Coupling Between Reduced Surface-Wave Microstrip Antennas," M. A. Khayat, J. T. Williams, D. R. Jackson, and S. A. Long, IEEE Trans. Antennas and Propagation, vol. 48, pp. 1581-1593, Oct. 2000.

References

General references about microstrip antennas:

Microstrip Patch Antennas, K. F. Fong Lee and K. M. Luk, Imperial College Press, 2011.

Microstrip and Patch Antennas Design, 2nd Ed., R. Bancroft, Scitech Publishing, 2009.

Microstrip Patch Antennas: A Designer's Guide, R. B. Waterhouse, Kluwer Academic Publishers, 2003.

Microstrip Antenna Design Handbook, R. Garg, P. Bhartia, I. J. Bahl, and A. Ittipiboon, Editors, Artech House, 2001.

Advances in Microstrip and Printed Antennas, K. F. Lee, Editor, John Wiley, 1997.

References (cont.)

General references about microstrip antennas (cont.):

CAD of Microstrip Antennas for Wireless Applications, R. A. Sainati, Artech House, 1996.

Microstrip Antennas: The Analysis and Design of Microstrip Antennas and Arrays, D. M. Pozar and D. H. Schaubert, Editors, Wiley/IEEE Press, 1995.

Millimeter-Wave Microstrip and Printed Circuit Antennas, P. Bhartia, Artech House, 1991.

The Handbook of Microstrip Antennas (two volume set), J. R. James and P. S. Hall, INSPEC, 1989.

Microstrip Antenna Theory and Design, J. R. James, P. S. Hall, and C. Wood, INSPEC/IEE, 1981.

References (cont.)

More information about the CAD formulas presented here for the rectangular patch may be found in:

Microstrip Antennas, D. R. Jackson, Ch. 7 of Antenna Engineering Handbook, 5th Ed., J. L. Volakis, Editor, McGraw Hill, 2018.

Computer-Aided Design of Rectangular Microstrip Antennas, D. R. Jackson, S. A. Long, J. T. Williams, and V. B. Davis, Ch. 5 of *Advances in Microstrip and Printed Antennas*, K. F. Lee, Editor, John Wiley, 1997.

References (cont.)

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Compact and Broadband Microstrip Antennas, K.-L. Wong, John Wiley, 2003.

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