Homework-Week 2

Goals

- Understand and implement the bracketed methods for estimating roots
- Understand and implement the open methods for estimating roots
- Understand and implement simple iterative methods for solving coupled nonlinear equations
- Understand the limitations of all algorithms used in this homework

Mandatory Functions

- *incremental*-Finds regions of roots using the incremental area. Requires the function(anonymous), upper and lower limit, and the number of regions between the limits as input. The output should be an array of regions where there are zero crossings.
- *bisection*-Finds the root of a function using the bisection approach. Inputs are the function(anonymous), upper and lower limits, maximum number of iterations, and maximum absolute relative error. The outputs should be the root and a table of iteration results.
- *false_position*-Finds the roots of a function using the false position approach. Inputs are the function(anonymous), upper and lower limits, maximum number of iterations, and maximum absolute relative error. The outputs should be the root and a table of iteration results.
- *fixed_point*-Finds the root of a function using the fixed point approach. Inputs are the function(anonymous),initial guess, maximum number of iterations, and maximum absolute relative error. The outputs should be the root and a table of iteration results.
- *newton_rhapson*-Finds the root of a function using the Newton-Rhapson approach. Inputs are the function(anonymous),the derivative of the function(anonymous),initial guess, maximum number of iterations, and maximum absolute relative error. The outputs should be the root and a table of iteration results.
- *modified_secant*-Finds the root of a function using the modified secant approach. Inputs are the function(anonymous),initial guess,perturbation fraction, maximum number of iterations, and maximum absolute relative error. The outputs should be the root and a table of iteration results.
- *nonlin_fixed_point* Solves a systems of nonlinear equations using the fixed point approach. Inputs are the function (anonymous), initial guess, maximum iterations, and maximum absolute relative error. The outputs should be the root and a table of iteration results.
- *nonlin_newton_rhapson* Solves a systems of nonlinear equations using the Newton-Rhapson approach. Inputs are the function (anonymous), matrix of derivatives(anonynous functions),initial guess, maximum iterations, and maximum absolute relative error. The outputs should be the root and a table of iteration results.

Problems

Problem 1

Determine the roots of $f(x) = -12 - 21x - 18x^2 - 2.75x^3$ between $x_l = -10$ and $x_u = 10$, and a stopping criterion of 0.1%. You should use your *icremental* function first to find all roots, and use those ranges to get better results using the *bisection* and *false_position* functions.

Problem 2

You are designing a spherical tank to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as

$$V = \pi h^2 \frac{3R - h}{3}$$

Where V=volume (m^3), h=depth of water in tank(m), and R=the tank radius(m). If R=3 m, to what depth must the tank be filled so that it holds 30 m^3? Calculate the answer using both the false position and midpoint approaches. Employ a lower limit of $x_l = 0$ and $x_u = R$.

Problem 3

An oscillating current in an electric circuit is described by $i(t) = 9e^{-t}sin(2\pi t)$, where t is in seconds. Determine the lowest value of t such that $i = 3\,\mathrm{A}$. Use and compare the results using the Newton-Rhapson, modified secant, and fixed-point approaches.

Problem 4

The resistivity ρ of doped silicon is based on the charge q on an electron, the electron density n, and the electron mobility μ. The electron density is given in terms of the doping density N and the intrinsic carrier density n_i. The electron mobility is described by the temperature T, the reference temperature T_0, and the reference mobility μ_0. The equations requried to compute the resistivity are

$$\rho = \frac{1}{qn\mu}$$

where

$$n = \frac{1}{2}(N + \sqrt{N^2 + 4n_i^2})$$

and

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{-2.42}$$

Determine N, given $T_0 = 300\ K$, $T = 1000\ K$, $\mu_0 = 1300\ cm^2(V\ s)^{-1}$, $q = 1.6 \times 10^{-19}\ C$, $n_i = 6.21 \times 10^9\ cm^{-3}$, and a desired $\rho = 6 \times 10^6\ Vs\ cm/C$. Use (a)bisection, and (b) the modified secant approach.

Problem 5

A total charge Q is uniformly distributed around a ring-shaped conductor with radius a. A charge q is located at a distance x from the center of the ring. The force exerted on the charge by the ring is given by

$$F = \frac{1}{4\pi\ e_0} \frac{qQx}{(x^2 + a^2)^{3/2}}$$

where $e_0 = 8.85 \times 10^{-12}\ C^2/(N\ m^2)$. Find the distance x where the force is 1N if q and Q are $2 \times 10^{-5}\ C$ for a ring with a radius of 0.9 m. Use the Newton-Rhapson, fixed point, and modified secant approaches to find the solution.

Problem 6

The effective impedance of a parallel RLC circuit is

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

where Z=impedance (Ohms), and ω= the angular frequency (rad/s). Find the ω that results in an impedance of $75\,\Omega$ using both bisection and false position with initial guesses of 1 and 1000 for the following parmeters: $R = 225\,\Omega$, $C = 0.6 \times 10^{-6}\ F$, and $L = 0.5\ H$. Ensure the absolute relative error is no greater than 0.1%.

Problem 7

The resonant frequency of a series RLC circuit is

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

With $L = 5\ H$ and $C = 10^{-4}\ F$, determine the value of R that causes the resonant frequency to be 1 kHz. Use the Newton-Rhapson, bisection, and false position methods.

Problem 8

Determine the roots of the following simultaneous nonlinear equations using (a) fixed point iteration, and (b) the Newton-Rhapson method:

$$y = -x^2 + x + 0.75$$

$$y + 5xy = x^2$$

Employ inital guesses of $x = y = 1.2$ and discuss the results.

Problem 9

Determine the roots of the following simultaneous nonlinear equations:

$$(x-4)^2 + (y-4)^2 = 5$$

$$x^2 + y^2 = 16$$

Use a graphical approach to obtain your initial guesses. Determine refined estimates with the two-equation Newton-Rhapson method