

Thị giác máy tính

Bài 7: 3D Vision



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Nội dung

- Giới thiệu
- Ước lượng 3D
- Hình học lưỡng cực (Epipolar geometry)
- Stereo calibration
- Stereo rectification
- Stereo matching



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Tại sao cần nhiều views?

- 1 view: thông tin về độ sâu và cấu trúc bị mất 1 phần
➔ gây nhập nhằng, ko rõ ràng



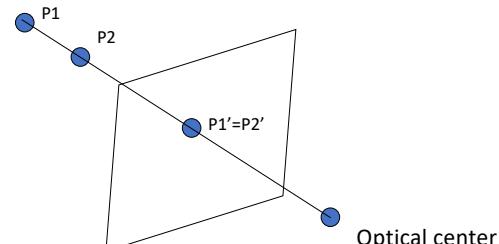
Images from Lana Lazebnik

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Tại sao cần nhiều views?

- 1 view: thông tin về độ sâu và cấu trúc bị mất 1 phần
➔ gây nhập nhằng, ko rõ ràng



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Stereo

- Thị giác người:
 - 2 camera

Pupil/Iris – điều tiết ánh sáng
Retina – chứa bào thị giác, nơi ảnh được tái tạo
Fovea – nơi tập trung cao nhất các tế bào hình nón

FIGURE 7.1

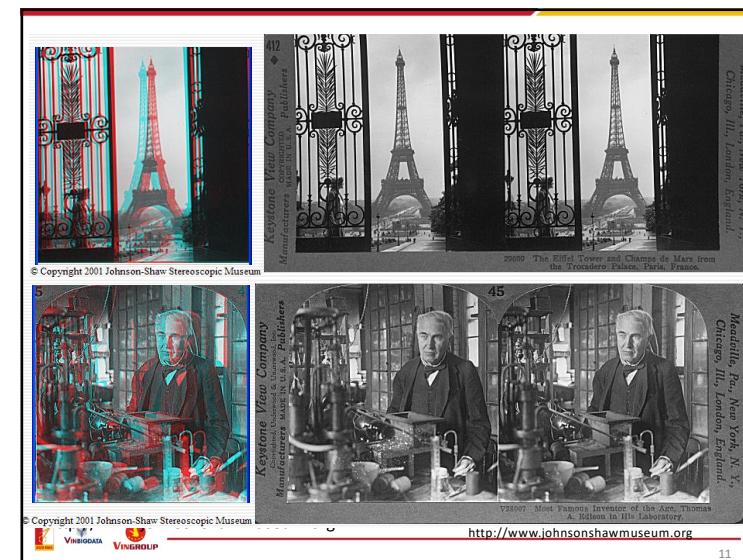
Mắt người tập trung vào 1 điểm trong không gian để các hình ảnh tương ứng tạo ở trung tâm của fovea

From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

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Ước lượng hình dạng 3D

- Từ 1 view:
 - “Shape from X”: shading, texture, focus, motion...
- Sử dụng nhiều hơn 1 view (Stereo):**
 - Tái tạo hình dạng từ 2 camera
 - Xây dựng hình ảnh 3D từ 2 hoặc nhiều ảnh từ các góc nhìn khác nhau

Ý tưởng chính:

scene point

image plane

optical center

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Ước lượng độ sâu với 2 views

Two cameras, simultaneous views

Single moving camera and static scene

Thông tin cần có:

- Thông tin về vị trí camera (“calibration”)
- Các điểm tương đồng giữa 2 ảnh

scene point

image plane

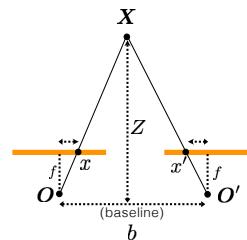
optical center

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Simple stereo system

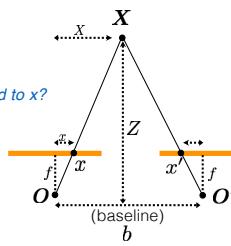


- 2 camera có trục quang học //, tham số camera đã biết (calibrated cameras):



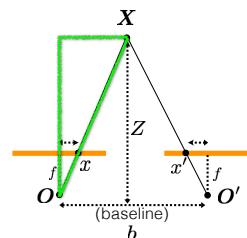
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How is X related to x?



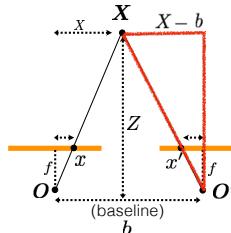
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$$\frac{X}{Z} = \frac{x}{f}$$



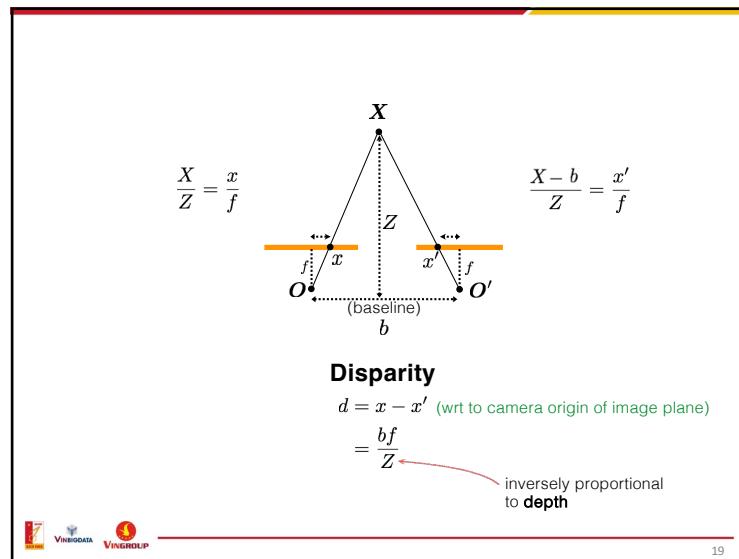
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$$\frac{X}{Z} = \frac{x}{f}$$



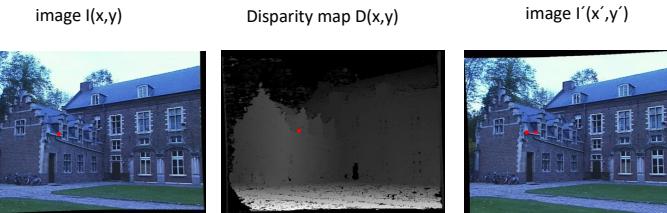
$$\frac{X - b}{Z} = \frac{x'}{f}$$

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Depth from disparity



$$(x', y') = (x + D(x,y), y)$$

Nếu xác định được **các điểm tương ứng** trong 2 ảnh \rightarrow có thể ước lượng độ sâu tương đối của các điểm ảnh...

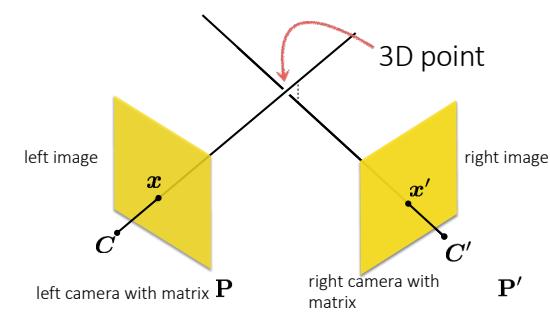


Stereo system – tổng quát

- 2 camera có trục quang học không song song

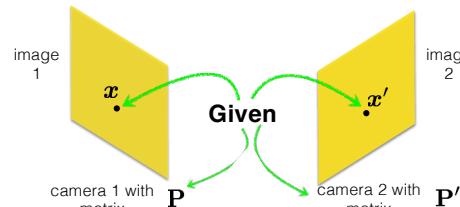


Tái tạo điểm 3D



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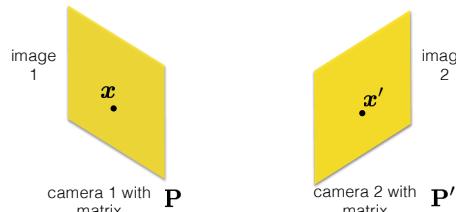
Phương pháp tam giác (triangulation)



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Triangulation

Which 3D points map to x ?

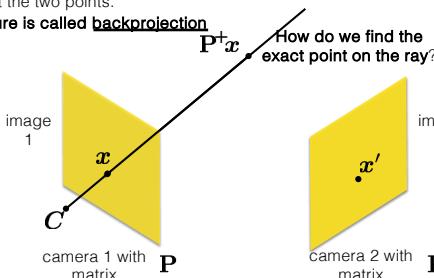


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Triangulation

Create two points on the ray:
1) find the camera center; and
2) apply the pseudo-inverse of P on x .
Then connect the two points.

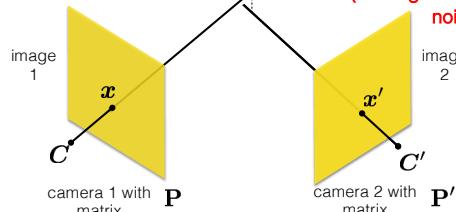
This procedure is called backprojection



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Triangulation

Find 3D object point
*Will the lines intersect?
(no single solution due to noise)*



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Triangulation

Given a set of (noisy) matched points

$$\{\mathbf{x}_i, \mathbf{x}'_i\}$$

and camera matrices

$$\mathbf{P}, \mathbf{P}'$$

Estimate the 3D point

$$\mathbf{X}$$



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$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Do the same after
first expanding out
the camera matrix
and points

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} \mathbf{p}_1^\top \\ \mathbf{p}_2^\top \\ \mathbf{p}_3^\top \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} \mathbf{p}_1^\top \mathbf{X} \\ \mathbf{p}_2^\top \mathbf{X} \\ \mathbf{p}_3^\top \mathbf{X} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{p}_1^\top \mathbf{X} \\ \mathbf{p}_2^\top \mathbf{X} \\ \mathbf{p}_3^\top \mathbf{X} \end{bmatrix} = \begin{bmatrix} y\mathbf{p}_3^\top \mathbf{X} - \mathbf{p}_2^\top \mathbf{X} \\ \mathbf{p}_1^\top \mathbf{X} - x\mathbf{p}_3^\top \mathbf{X} \\ x\mathbf{p}_2^\top \mathbf{X} - y\mathbf{p}_1^\top \mathbf{X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



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Concatenate the 2D points from both images

$$\begin{array}{l} \text{Two rows from camera one} \\ \left[\begin{array}{c} y\mathbf{p}_3^\top - \mathbf{p}_2^\top \\ \mathbf{p}_1^\top - x\mathbf{p}_3^\top \end{array} \right] \mathbf{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \text{Two rows from camera two} \\ \left[\begin{array}{c} y'\mathbf{p}_3'^\top - \mathbf{p}_2'^\top \\ \mathbf{p}_1'^\top - x'\mathbf{p}_3'^\top \end{array} \right] \mathbf{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array}$$

$$\mathbf{A}\mathbf{X} = \mathbf{0}$$

How do we solve homogeneous linear system?

$$\mathbf{S} \quad \mathbf{V} \quad \mathbf{D} \quad !$$



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Ước lượng 3D?

Task: Match point in left image to point in right image



Left image

Right image

How would you do it?

- Dense matching (for each point)
- Avoid search over entire image



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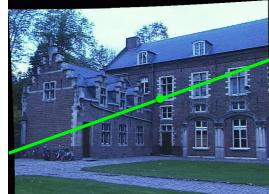
Epipolar
geometry!

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Ước lượng 3D



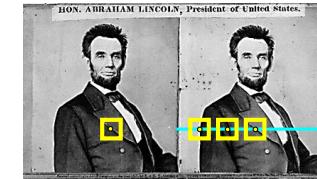
Left image



Right image

1. Select point in one image (how?)
 2. Form epipolar line for that point in second image (how?)
 3. Find matching point along line (how?)
 4. Perform triangulation (how?)
- What are the disadvantages of this procedure?

Ước lượng 3D

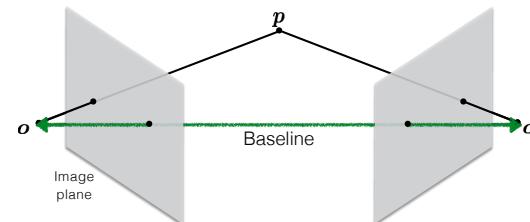


1. Rectify images (make epipolar lines horizontal)
 2. For each pixel
 - a. Find epipolar line
 - b. Scan line for best match
 - c. Compute depth from disparity
- $$Z = \frac{bf}{d}$$

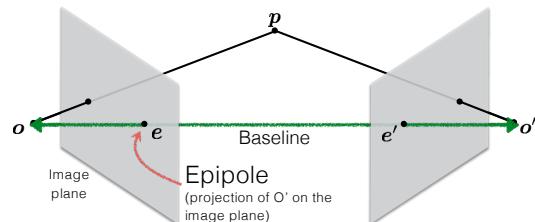
Next ...

- Epipolar geometry (hình học lưỡng cực)
- Stereo calibration:
 - tìm các tham số của hệ thống 2 camera
 - Ma trận F (Fundamental matrix), E (Essential matrix)
- Stereo rectification: hiệu chỉnh 2 ảnh sao cho
 - Cặp epipolar line tương ứng giữa 2 ảnh nằm trên cùng 1 hàng
 - Epipolar lines trong mỗi ảnh: song song với nhau

Epipolar geometry

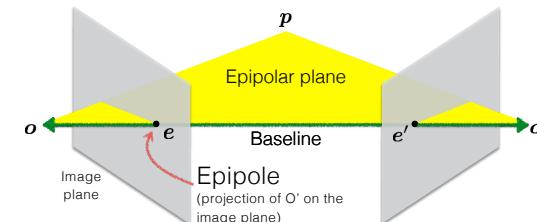


Epipolar geometry



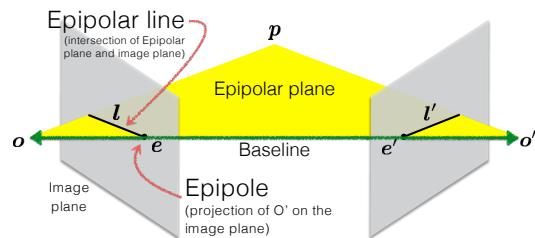
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Epipolar geometry



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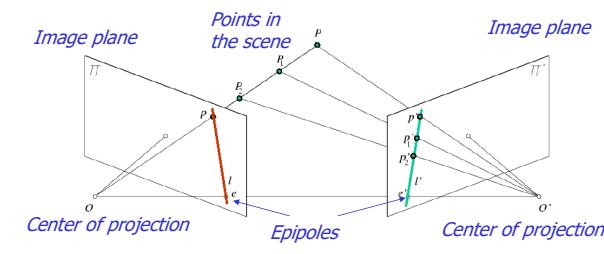
Epipolar geometry



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Epipolar constraint

- Một điểm ảnh trong ảnh bên trái được quan sát thấy trên epipolar line tương ứng ở ảnh bên phải
- Tất cả epipolar lines đều đi qua epipole của trên ảnh đó



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Epipolar constraint

- Một điểm ảnh trong ảnh bên trái được quan sát thấy trên epipolar line tương ứng ở ảnh bên phải
- ➔ Giảm thời gian tìm kiếm điểm tương ứng: chỉ cần tìm các điểm trên epipolar line



Image from Andrew Zisserman

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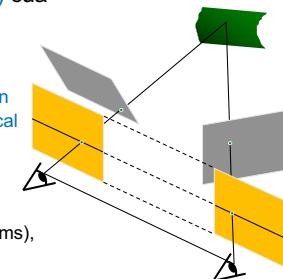
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Stereo image rectification

- Về ứng dụng, rất thuận lợi nếu epipolar lines tương ứng là các hàng (rows) của từng ảnh

➔ rectification:

- reproject image planes onto a common plane parallel to the line between optical centers
- pixel motion is horizontal after this transformation
- Need two homographies (3x3 transforms), one for each input image reprojection



Slide credit: Li Zhang

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What do the epipolar lines look like?

Converging cameras



Parallel cameras

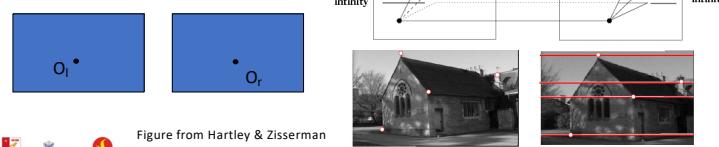
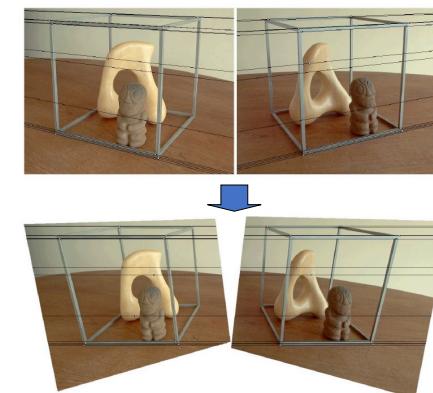


Figure from Hartley & Zisserman

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Stereo image rectification: example



Source: Alyosha Efros

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Stereo Calibration



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Essential and Fundamental matrix : E, F

- Mối tương quan giữa 2 camera:

- Essential matrix E

- thể hiện vị trí tương đối của camera thứ 2 so với camera thứ 1
- Gồm 1 phép dịch T và 1 phép quay R

- Fundamental matrix F

- Chứa thông tin về ma trận E và tham số trong của 2 camera

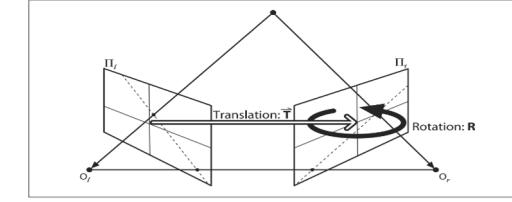
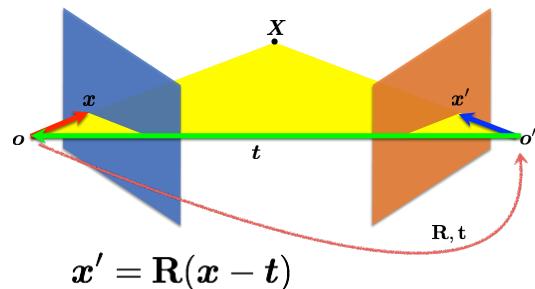


Figure 12-9. The essential geometry of stereo imaging is captured by the essential matrix E, which contains all of the information about the translation T and the rotation R, which describe the location of the second camera relative to the first in global coordinates

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Ma trận E (Essential Matrix)



Giả sử:

- x và x' : được biểu diễn 2D trong hệ tọa độ camera
- Nếu ma trận tham số trong $K = I \Rightarrow$ biểu diễn trong hệ tọa độ camera = biểu diễn trong hệ tọa độ ảnh



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Ma trận E

rigid motion

$$x' = R(x - t)$$

coplanarity

$$(x - t)^\top (t \times x) = 0$$

$$(x'^\top R)(t \times x) = 0$$

$$(x'^\top R)([t_x]x) = 0$$

$$x'^\top (R[t_x])x = 0$$

$$x'^\top E x = 0$$

Essential Matrix
[Longuet-Higgins 1981]

$[t_x]$: skew symmetric matrix for vector t



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Tính chất của ma trận E

Longuet-Higgins equation

$$\mathbf{x}'^\top \mathbf{E} \mathbf{x} = 0$$

Epipolar lines

$$\begin{aligned}\mathbf{x}^\top \mathbf{l} &= 0 & \mathbf{x}'^\top \mathbf{l}' &= 0 \\ \mathbf{l}' &= \mathbf{E} \mathbf{x} & \mathbf{l} &= \mathbf{E}^T \mathbf{x}'\end{aligned}$$

Epipoles

$$\mathbf{e}'^\top \mathbf{E} = 0 \quad \mathbf{E} \mathbf{e} = 0$$

(2D points expressed in camera coordinate system)



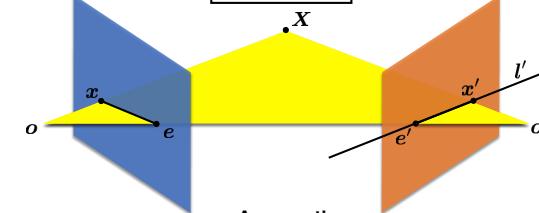
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E và epipolar line

Nếu biết **tọa độ 1 điểm** trong hệ quy chiếu camera 1, và **biết E**

⇒ xác định được **epipolar line** tương ứng ở camera 2

$$\mathbf{E} \mathbf{x} = \mathbf{l}'$$



Assumption:

2D points expressed in camera coordinate system
(i.e., intrinsic matrices are identities)



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Trường hợp tổng quát: E

- Giả sử 2 camera đều xác định được tham số trong (calibrated camera) K, K'

$$\hat{\mathbf{x}}'^\top \mathbf{E} \hat{\mathbf{x}} = 0$$

The essential matrix operates on image points expressed in **2D coordinates expressed in the camera coordinate system**

$$\hat{\mathbf{x}}' = \mathbf{K}'^{-1} \mathbf{x}'$$

$$\hat{\mathbf{x}} = \mathbf{K}^{-1} \mathbf{x}$$

camera point image point



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Trường hợp tổng quát: E

$$\hat{\mathbf{x}}$$

tọa độ trong hệ
quy chiếu camera

$$\hat{\mathbf{x}}'^\top \mathbf{E} \hat{\mathbf{x}} = 0$$

$$\mathbf{x}' = \mathbf{K}'^{-1} \mathbf{x}'$$

camera point

$$\hat{\mathbf{x}} = \mathbf{K}^{-1} \mathbf{x}$$

image point

$$\mathbf{x}$$

tọa độ trong hệ
quy chiếu ảnh

$$\begin{aligned}\mathbf{K}'^{-\top} \mathbf{E} \mathbf{K}^{-1} \mathbf{x} &= 0 \\ \mathbf{x}'^\top (\mathbf{K}'^{-\top} \mathbf{E} \mathbf{K}^{-1}) \mathbf{x} &= 0 \\ \mathbf{x}'^\top \mathbf{F} \mathbf{x} &= 0\end{aligned}$$

→ F: Fundamental matrix



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Tính chất của ma trận F

Longuet-Higgins equation $\mathbf{x}'^\top \mathbf{F} \mathbf{x} = 0$

Epipolar lines

$$\begin{aligned}\mathbf{x}^\top \mathbf{l} &= 0 & \mathbf{x}'^\top \mathbf{l}' &= 0 \\ \mathbf{l}' &= \mathbf{F} \mathbf{x} & \mathbf{l} &= \mathbf{F}^T \mathbf{x}'\end{aligned}$$

Epipoles

$$\mathbf{e}'^\top \mathbf{F} = 0 \quad \mathbf{F} \mathbf{e} = 0$$

(2D points expressed in image coordinates)



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Essential and Fundamental matrix : E, F

- Ma trận **E** :

- E: chỉ phụ thuộc vào tham số ngoài
- Nếu đã biết tham số trong K, K' \rightarrow E cho phép xác định tham số ngoài

$$\mathbf{E} = \mathbf{R}[\mathbf{t}_x]$$

- Ma trận **F**:

- F phụ thuộc vào tham số trong và tham số ngoài
- Nếu K, K', R, t không biết \rightarrow ước lượng F? (*weak calibration*)

$$\mathbf{F} = \mathbf{K}'^{-\top} \mathbf{E} \mathbf{K}^{-1}$$



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Weak calibration – Ước lượng F

- Weak calibration
 - Intrinsics and extrinsics parameters: unknown
 - i.e. estimate F without calibration matrix K ou P of 2 camera
- F : 3x3 matrix,
 - rank(F)= 2
 - 8 unknown values
- Need solve many (>8) pair of corresponding pixels



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Thuật toán 8 điểm

$$\mathbf{x}'_m^\top \mathbf{F} \mathbf{x}_m = 0$$

$$\left[\begin{array}{ccc} x'_m & y'_m & 1 \end{array} \right] \left[\begin{array}{ccc} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{array} \right] \left[\begin{array}{c} x_m \\ y_m \\ 1 \end{array} \right] = 0$$

1 cặp điểm \rightarrow 1 phương trình

$$\begin{aligned}x_m x'_m f_1 + x_m y'_m f_2 + x_m f_3 + \\ y_m x'_m f_4 + y_m y'_m f_5 + y_m f_6 + \\ x'_m f_7 + y'_m f_8 + f_9 = 0\end{aligned}$$



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Thuật toán 8 điểm

$$\mathbf{x}_m'^\top \mathbf{F} \mathbf{x}_m = 0$$

$$\begin{bmatrix} x_1x'_1 & x_1y'_1 & x_1 & y_1x'_1 & y_1y'_1 & y_1 & x'_1 & y'_1 & 1 \\ \vdots & \vdots \\ x_Mx'_M & x_My'_M & x_M & y_Mx'_M & y_My'_M & y_M & x'_M & y'_M & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix} = 0$$

$$\mathbf{A}\mathbf{x} = 0$$

Total Least Squares

$$\begin{aligned} \text{minimize} \quad & \|\mathbf{Ax}\|^2 \\ \text{subject to} \quad & \|\mathbf{x}\|^2 = 1 \end{aligned} \quad \text{SVD!}$$



Ước lượng F (opencv)

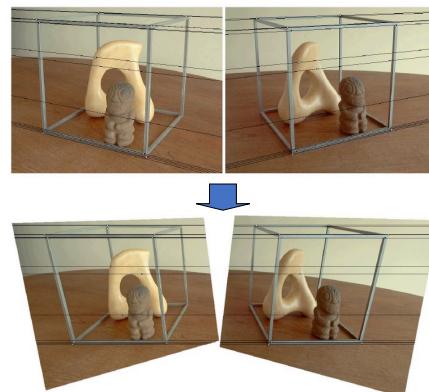
- Phát hiện các điểm quan trọng trên 2 ảnh
 - Vd: sử dụng DoG, SIFT
- So khớp để tìm ra các cặp điểm tương đồng
 - VD: nearest neighbor + ratio of distance
- Giải bài toán tối ưu $\rightarrow \mathbf{F}$
 - (thư viện có hàm hỗ trợ: findFundamentalMatrix)



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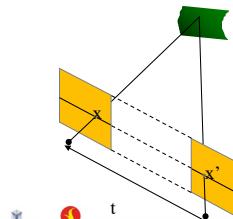
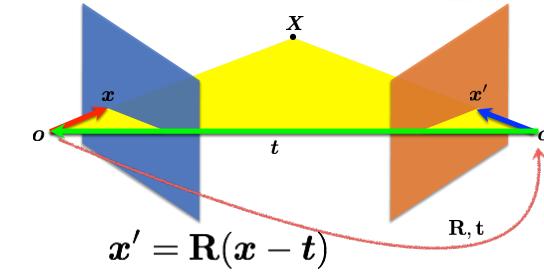
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Stereo rectification



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Epipolar lines nằm // theo trục hoành thi:

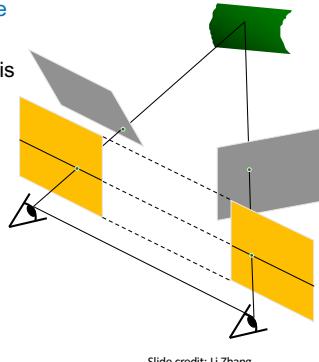
$$R = I \quad t = (T, 0, 0)$$

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Stereo image rectification

- Reproject image planes onto a common plane parallel to the line between optical centers
- pixel motion is horizontal after this transformation
- Need two homographies (3x3 transforms), one for each input image reprojection



Slide credit: Li Zhang

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Stereo Rectification

1. **Rotate** the right camera by \mathbf{R}
(aligns camera coordinate system orientation only)
2. Rotate (**rectify**) the left camera so that the epipole is at infinity
3. Rotate (**rectify**) the right camera so that the epipole is at infinity
4. Adjust the **scale**

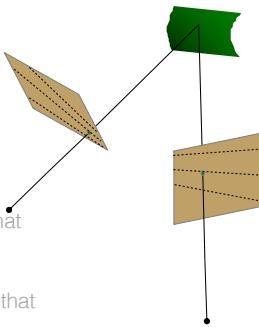


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Stereo Rectification

1. Compute \mathbf{E} to get \mathbf{R}
2. **Rotate** the right camera by \mathbf{R}
(aligns camera coordinate system orientation only)
3. Rotate (**rectify**) the left camera so that the epipole is at infinity
4. Rotate (**rectify**) the right camera so that the epipole is at infinity
5. Adjust the **scale**



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Stereo Rectification

1. Compute \mathbf{E} to get \mathbf{R}
2. **Rotate** the right camera by \mathbf{R}
(aligns camera coordinate system orientation only)
3. Rotate (**rectify**) the left camera so that the epipole is at infinity
4. Rotate (**rectify**) the right camera so that the epipole is at infinity
5. Adjust the **scale**

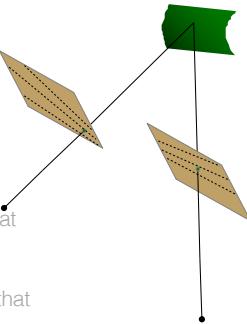


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Stereo Rectification

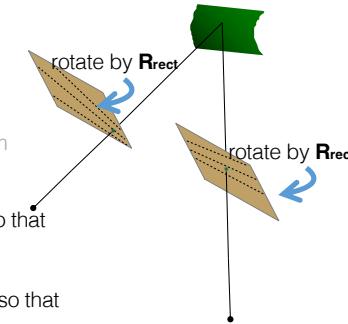
1. Compute \mathbf{E} to get \mathbf{R}
- 2. Rotate** the right camera by \mathbf{R}
(aligns camera coordinate system orientation only)
3. Rotate (**rectify**) the left camera so that the epipole is at infinity
4. Rotate (**rectify**) the right camera so that the epipole is at infinity
5. Adjust the **scale**



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Stereo Rectification

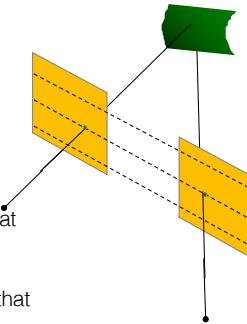
1. Compute \mathbf{E} to get \mathbf{R}
- 2. Rotate** the right camera by \mathbf{R}
(aligns camera coordinate system orientation only)
- 3. Rotate (rectify)** the left camera so that the epipole is at infinity - \mathbf{R}_{rect}
- 4. Rotate (rectify)** the right camera so that the epipole is at infinity - \mathbf{R}_{rect}
5. Adjust the **scale**



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Stereo Rectification

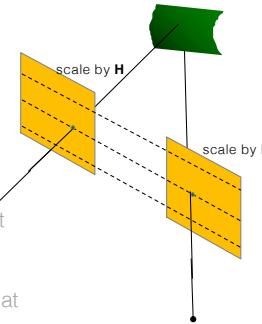
1. Compute \mathbf{E} to get \mathbf{R}
- 2. Rotate** the right camera by \mathbf{R}
(aligns camera coordinate system orientation only)
- 3. Rotate (rectify)** the left camera so that the epipole is at infinity - \mathbf{R}_{rect}
- 4. Rotate (rectify)** the right camera so that the epipole is at infinity - \mathbf{R}_{rect}
5. Adjust the **scale**



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Stereo Rectification

1. Compute \mathbf{E} to get \mathbf{R}
- 2. Rotate** the right camera by \mathbf{R}
(aligns camera coordinate system orientation only)
- 3. Rotate (rectify)** the left camera so that the epipole is at infinity
- 4. Rotate (rectify)** the right camera so that the epipole is at infinity
5. Adjust the **scale**



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Stereo Rectification

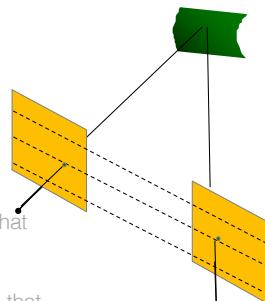
1. Compute \mathbf{E} to get \mathbf{R}

2. Rotate the right camera by \mathbf{R}
(aligns camera coordinate system orientation only)

3. Rotate (rectify) the left camera so that the epipole is at infinity

4. Rotate (rectify) the right camera so that the epipole is at infinity

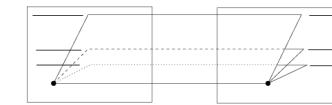
5. Adjust the scale



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Epipolar lines // khi Epipoles ở vô cực

Parallel cameras



epipole at infinity



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Stereo Rectification Algorithm

1. Estimate \mathbf{E} using the 8 point algorithm (SVD)
2. Estimate the epipole \mathbf{e} (SVD of \mathbf{E})
3. Build \mathbf{R}_{rect} from \mathbf{e}
4. Decompose \mathbf{E} into \mathbf{R} and \mathbf{T}
5. Set $\mathbf{R}_1 = \mathbf{R}_{\text{rect}}$ and $\mathbf{R}_2 = \mathbf{R}\mathbf{R}_{\text{rect}}$
6. Rotate each left camera point (warp image)

$$[x' \ y' \ z'] = \mathbf{R}_1 [x \ y \ z]$$
7. Rectified points as $\mathbf{p} = f/z' [x' \ y' \ z']$
8. Repeat 6 and 7 for right camera points using \mathbf{R}_2



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Stereo matching



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1. Rectify images
(make epipolar lines horizontal)
2. For each pixel
a. Find epipolar line
b. Scan line for best match ← How would you do this?
c. Compute depth from disparity

$$Z = \frac{bf}{d}$$

VINHDATA VINGROUP

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Template matching

How do we detect the template in the following image?

output

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

image

Increases for higher local intensities.

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Template matching

How do we detect the template in the following image?

output

$$h[m, n] = \sum_{k, l} (g[k, l] - \bar{g}) f[m + k, n + l]$$

image

template mean

thresholding

True detection

False detections

Solution 2: Filter the image using a **zero-mean template**. What went wrong?
Not robust to high-contrast areas

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Template matching

How do we detect the template in the following image?

output

$$h[m, n] = \sum_{k, l} (g[k, l] - f[m + k, n + l])^2$$

image

1-output

filter

thresholding

True detection

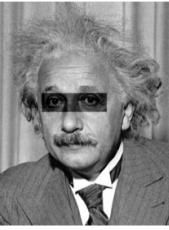
Solution 3: Use sum of squared differences (SSD). What could go wrong?

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Template matching

How do we detect the template  in the following image?



filter 

output

$$h[m, n] = \sum_{k,l} (g[k, l] - f[m + k, n + l])^2$$

image

1-output



Not robust to local intensity changes

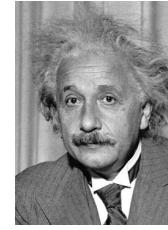
Solution 3: Use sum of squared differences (SSD).

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Template matching

How do we detect the template  in the following image?



Observations so far:

- subtracting mean deals with brightness bias
- dividing by standard deviation removes contrast bias

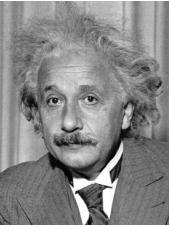
Can we combine the two effects?

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Template matching

How do we detect the template  in the following image?



filter  template mean

output

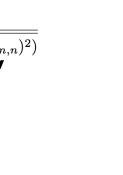
$$h[m, n] = \frac{\sum_{k,l} (g[k, l] - \bar{g})(f[m + k, n + l] - \bar{f}_{m,n})}{\sqrt{(\sum_{k,l} (g[k, l] - \bar{g})^2) (\sum_{k,l} (f[m + k, n + l] - \bar{f}_{m,n})^2)}}$$

local patch mean

What will the output look like?

image

1-output



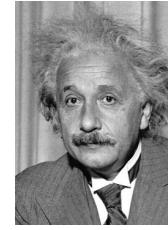
Solution 4: Normalized cross-correlation (NCC).

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Template matching

How do we detect the template  in the following image?



1-output



thresholding



True detections

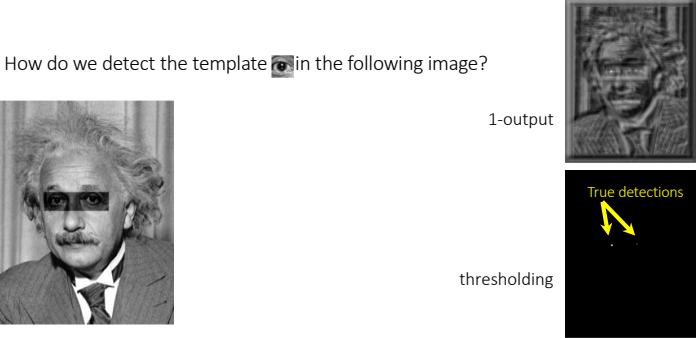
Solution 4: Normalized cross-correlation (NCC).

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Template matching

How do we detect the template  in the following image?



Solution 4: **Normalized cross-correlation (NCC)**.

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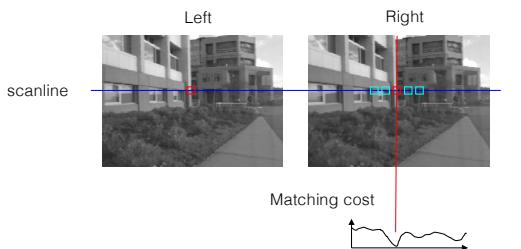
What is the best method?

It depends on whether you care about speed or invariance.

- Zero-mean: Fastest, very sensitive to local intensity.
- Sum of squared differences: Medium speed, sensitive to intensity offsets.
- Normalized cross-correlation: Slowest, invariant to contrast and brightness.

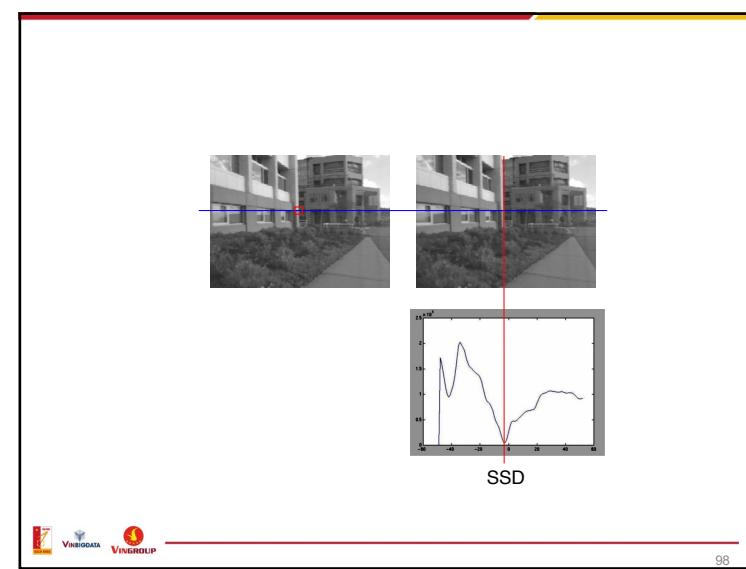
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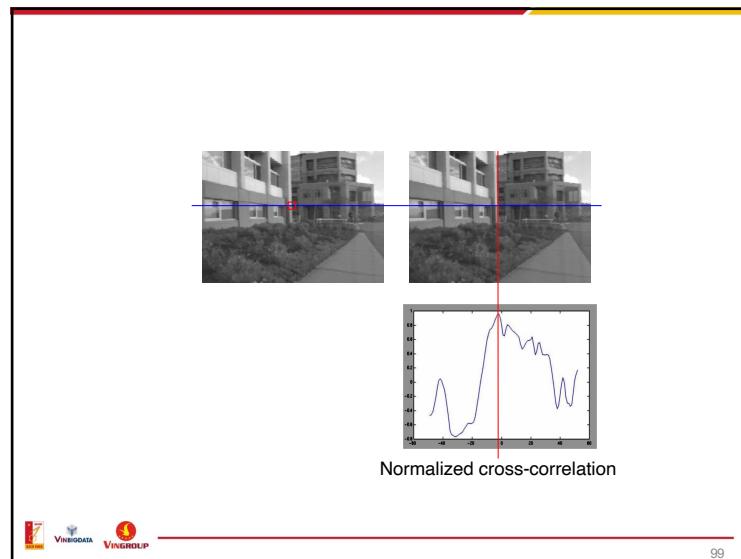
Stereo Block Matching



- Slide a window along the epipolar line and compare contents of that window with the reference window in the left image
- Matching cost: **SSD** or **normalized correlation**

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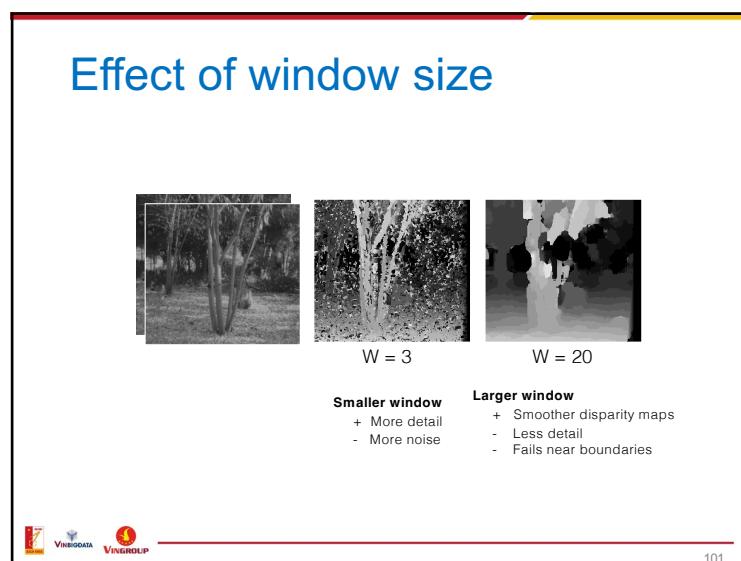


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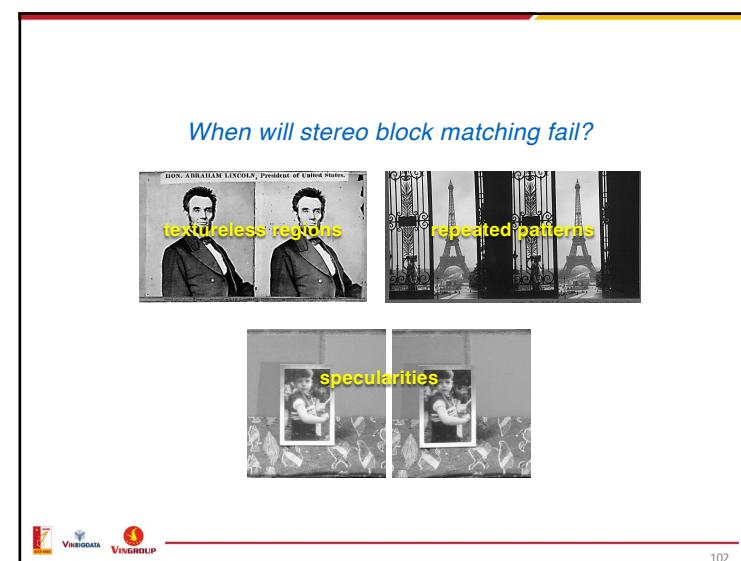
Similarity Measure	Formula
Sum of Absolute Differences (SAD)	$\sum_{(i,j) \in W} I_1(i,j) - I_2(x+i, y+j) $
Sum of Squared Differences (SSD)	$\sum_{(i,j) \in W} (I_1(i,j) - I_2(x+i, y+j))^2$
Zero-mean SAD	$\sum_{(i,j) \in W} I_1(i,j) - \bar{I}_1(i,j) - I_2(x+i, y+j) + \bar{I}_2(x+i, y+j) $
Locally scaled SAD	$\sum_{(i,j) \in W} I_1(i,j) - \frac{\bar{I}_1(i,j)}{\bar{I}_2(x+i, y+j)} I_2(x+i, y+j) $
Normalized Cross Correlation (NCC)	$\frac{\sum_{(i,j) \in W} I_1(i,j) \cdot I_2(x+i, y+j)}{\sqrt{\sum_{(i,j) \in W} I_1^2(i,j) \cdot \sum_{(i,j) \in W} I_2^2(x+i, y+j)}}$

SAD SSD NCC Ground truth

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Improving stereo matching

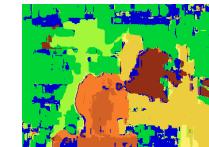


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Block matching

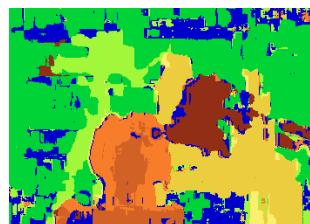
Ground truth



What are some problems with the result?



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How can we improve depth estimation?

Too many discontinuities.

We expect disparity values to change slowly.

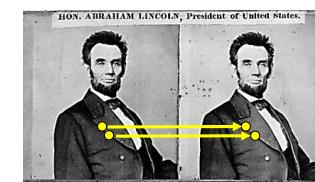
→ Let's make an assumption:
depth should change smoothly



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Stereo matching as ... Energy Minimization



What defines a good stereo correspondence?

1. **Match quality**
 - Want each pixel to find a good match in the other image
2. **Smoothness**
 - If two pixels are adjacent, they should (usually) move about the same amount



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energy function
(for one pixel)

$$E(d) = \underbrace{E_d(d)}_{\text{data term}} + \lambda \underbrace{E_s(d)}_{\text{smoothness term}}$$

Want each pixel to find a good match in the other image
(block matching result)

Adjacent pixels should (usually) move about the same amount
(smoothness function)

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$$E(d) = E_d(d) + \lambda E_s(d)$$

$$E_d(d) = \sum_{(x,y) \in I} C(x, y, d(x, y))$$

data term (x, y) ∈ I SSD distance between windows centered at (x, y) and J(x + d(x, y), y)

$$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p, d_q)$$

smoothness term (p, q) ∈ E

\mathcal{E} : set of neighboring pixels

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$$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p, d_q)$$

smoothness term (p, q) ∈ E

$$V(d_p, d_q) = |d_p - d_q|$$

L₁ distance

$$V(d_p, d_q) = \begin{cases} 0 & \text{if } d_p = d_q \\ 1 & \text{if } d_p \neq d_q \end{cases}$$

"Potts model"

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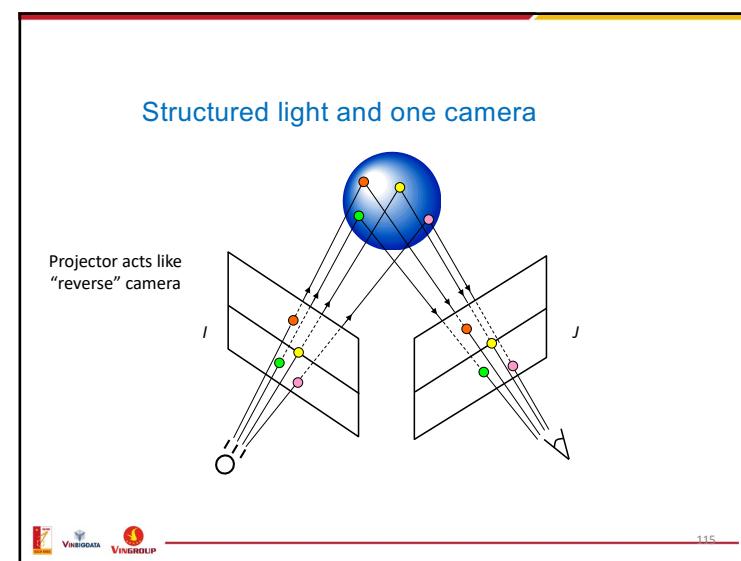
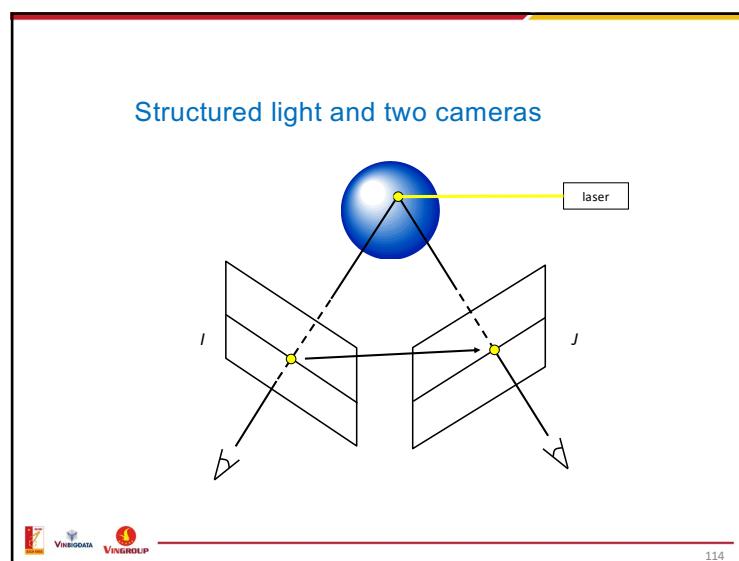
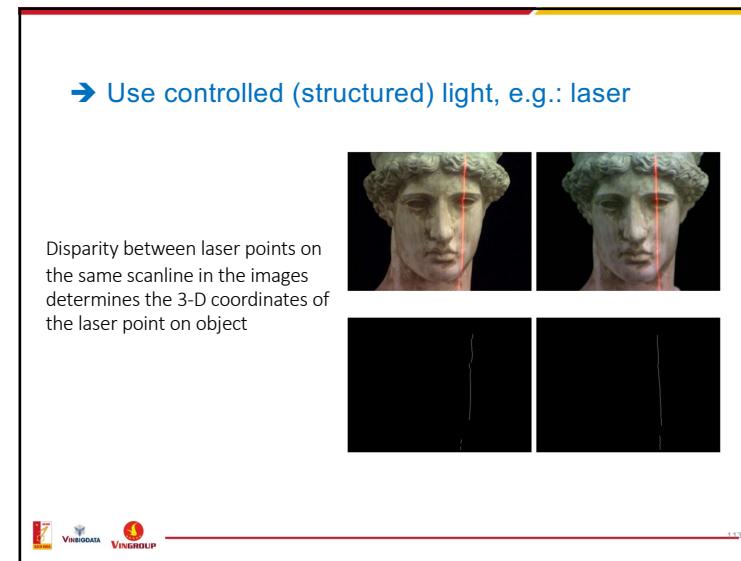
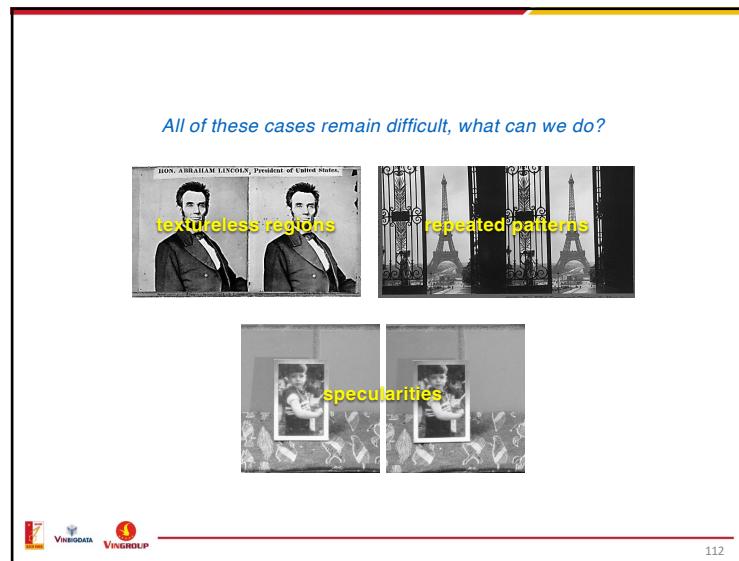
Match only

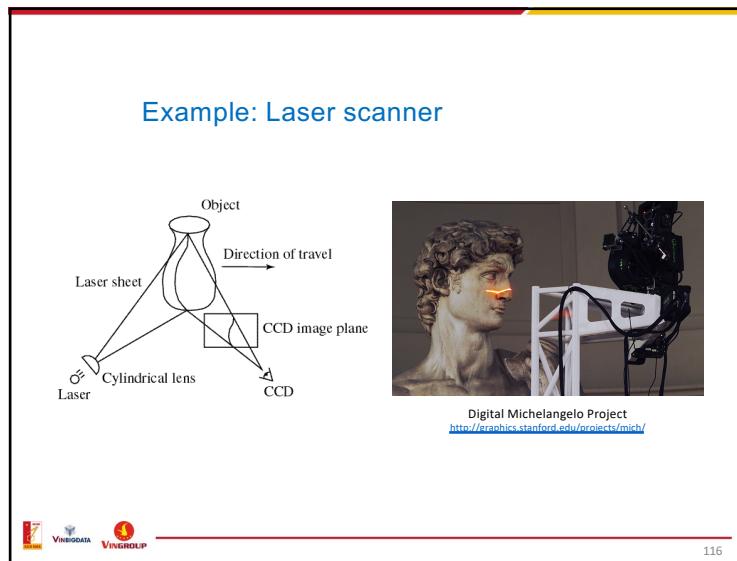
Ground Truth

Match & smoothness (via graph cut)

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](#), PAMI 2001

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References

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- Lectures: 16-385, Computer Vision - Spring 2017, 2020
- Z. Zhang, "A flexible new technique for camera calibration," in *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 22, no. 11, pp. 1330-1334, Nov. 2000, doi: 10.1109/34.888718.
- Computer Vision II: Multiple View Geometry
<https://vision.in.tum.de/teaching/online/mvg>

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