Probing the R-parity violating supersymmetric effects in

$$B_c \to J/\psi \ell^- \bar{\nu}_\ell, \eta_c \ell^- \bar{\nu}_\ell$$
 and  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  decays

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Motivated by recent  $R_D$ ,  $R_{D^*}$  and  $R_{J/\psi}$  anomalies in  $B \to D\ell^-\bar{\nu}_\ell$ ,  $B \to D^*\ell^-\bar{\nu}_\ell$  and  $B_c \to J/\psi\ell^-\bar{\nu}_\ell$  decays, respectively, we study possible R-parity violating supersymmetric effects in  $B_c \to J/\psi\ell^-\bar{\nu}_\ell$ ,  $\eta_c\ell^-\bar{\nu}_\ell$  and  $\Lambda_b \to \Lambda_c\ell^-\bar{\nu}_\ell$  decays, which are also induced by  $b \to c\ell^-\bar{\nu}_\ell$  at quark level. We find that (I) the constrained slepton exchange couplings  $\lambda_{i33}\tilde{\lambda}'^*_{i23}$  involving in  $b \to c\tau^-\bar{\nu}_\tau$  transition from relevant latest experimental data still have quite large effects on all (differential) branching ratios and the normalized forward-backward asymmetries of the exclusive semileptonic  $b \to c\tau^-\bar{\nu}_\tau$  decays as well as the ratios of the (differential) branching ratios; (II) after satisfying the data of  $R_D$  and  $R_{D^*}$ , the upper limit of  $R_{J/\psi}$ ,  $R_{\eta_c}$  and  $R_{\Lambda_c}$  could be increased by 10%, 112% and 24%, respectively, from their upper limits of the Standard Model predictions by the  $\lambda_{i33}\tilde{\lambda}'^*_{i23}$  couplings. The results in this work could be used to probe R-parity violating effects and will correlate with searches for direct supersymmetric signals at the running LHCb and the forthcoming Belle-II.

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### 1 Introduction

Lepton flavor universality violation in the exclusive  $b \to c\ell^-\bar{\nu}_\ell$  and  $b \to s\ell^+\ell^-$  decays has attracted a lot of attention in the particle physics community and has significantly constrained many possible New Physics (NP) effects. For ratios  $R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau^-\bar{\nu}_{\tau})}{\mathcal{B}(B \to D^{(*)}\ell'^-\bar{\nu}_{\ell'})}$  with  $\ell' = e$  or  $\mu$ , the world average of the BABAR [1, 2], Belle [3–5] and LHCb [6, 7] measurements are [8]

$$R_D^{Exp.} = 0.407 \pm 0.039 \pm 0.024,$$
  
 $R_{D^*}^{Exp.} = 0.304 \pm 0.013 \pm 0.007,$  (1)

which are exceed the Standard Model (SM) predictions ( $R_D^{SM} = 0.297 \pm 0.017$  [9],  $R_{D^*}^{SM} = 0.252 \pm 0.003$  [10]) by 1.9 $\sigma$  and 3.3 $\sigma$ , respectively. Very recently, LHCb reported a new measurement regarding  $b \to c\ell\nu$  in  $B_c$  decays [11]

$$R_{J/\psi} = \frac{\mathcal{B}(B_c^+ \to J/\psi \tau^+ \nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi \mu^+ \nu_\mu)} = 0.71 \pm 0.17 \pm 0.18, \tag{2}$$

which is about  $2\sigma$  higher than its SM prediction [12, 13].

In the ratios  $R_D$ ,  $R_{D^*}$  and  $R_{J/\psi}$ , the theoretical uncertainties, such as the relevant CKM matrix elements and form factors, are largely canceled, so any deviation from the SM prediction would clearly indicate the presence of NP. A lot of works about  $R_D$  and  $R_{D^*}$  have been done to explain these anomalies in different NP models, for examples, model-independent approaches [14–22], charged Higgs [23–28], lepton flavor violation [29–32], leptoquark [33–36], etc. Otherwise, NP effects in  $B_c \to J/\psi \ell^- \bar{\nu}_\ell$  and  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  decays have also been studied, for instance, in Refs. [13, 37, 38] and Refs. [24, 39–44], respectively.

In the supersymmetry without R-parity, the slepton exchange couplings could give large contributions to  $R_D$  and  $R_{D^*}$ , which we have studied in Ref. [45]. Since  $B_c \to J/\psi \ell^- \bar{\nu}_\ell$ ,  $\eta_c \ell^- \bar{\nu}_\ell$  and  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  decays are also induced at quark level by  $b \to c \ell^- \bar{\nu}_\ell$ , they involve the same set of R-parity violating (RPV) coupling constants as  $B \to D \ell^- \bar{\nu}_\ell$  and  $B \to D^* \ell^- \bar{\nu}_\ell$  decays. In this work, using the latest experimental data of all relevant exclusive  $b \to c \ell^- \bar{\nu}_\ell$  decays, we will analyze the constrained RPV contributions to the branching ratios and their ratios, differential branching ratios as well as normalized forward-backward (FB) asymmetries of the charged leptons in  $B_c \to J/\psi \ell^- \bar{\nu}_\ell$ ,  $\eta_c \ell^- \bar{\nu}_\ell$  and  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  decays.

The paper is organized as follows. In section 2, we briefly review the theoretical expressions of the exclusive  $b \to c\ell^-\bar{\nu}_\ell$  decays. In section 3, using the constrained parameter spaces from

relevant experimental measurements, we make a detailed classification research on the RPV effects on the quantities which have not been measured or not been well measured yet. Our conclusions are given in section 4.

#### 2 Theoretical Framework

The general effective Hamiltonian for  $b \to c \ell_m^- \bar{\nu}_{\ell_n}$  transitions can be written as [46, 47]

$$\mathcal{H}_{eff}(b \to c\ell_m^- \bar{\nu}_{\ell_n}) = \frac{G_F V_{cb}}{\sqrt{2}} \left\{ \left[ G_V \bar{c} \gamma_\mu b - G_A \bar{c} \gamma_\mu \gamma_5 b \right] \bar{\ell}_m^- \gamma^\mu (1 - \gamma_5) \bar{\nu}_{\ell_n} \right. \\ \left. + \left[ G_S \bar{c} b - G_P \bar{c} \gamma_5 b \right] \bar{\ell}_m^- (1 - \gamma_5) \bar{\nu}_{\ell_n} \right. \\ \left. + \left[ \widetilde{G}_V \bar{c} \gamma_\mu b - \widetilde{G}_A \bar{c} \gamma_\mu \gamma_5 b \right] \bar{\ell}_m^- \gamma^\mu (1 + \gamma_5) \bar{\nu}_{\ell_n} \right. \\ \left. + \left[ \widetilde{G}_S \bar{c} b - \widetilde{G}_P \bar{c} \gamma_5 b \right] \bar{\ell}_m^- (1 + \gamma_5) \bar{\nu}_{\ell_n} \right\} + h.c.$$

$$(3)$$

In the SM,  $G_V = G_A = 1$  and all others are zero. If considering both the SM and the RPV contributions, we have [48]

$$G_V = G_A = 1 - \frac{\sqrt{2}}{G_F V_{cb}} \sum_i \frac{\lambda'_{n3i} \tilde{\lambda}'_{m2i}}{8m_{\tilde{d}_{iR}}^2},$$
 (4)

$$G_S = G_P = \frac{\sqrt{2}}{G_F V_{cb}} \sum_i \frac{\lambda_{inm} \tilde{\lambda}'_{i23}}{4m_{\tilde{\ell}_{iL}}^2}, \tag{5}$$

and all others are zero.

From general effective Hamiltonian given in Eq. (3), we obtain the differential branching ratios. The detail expressions can be found in A.1 and A.2 of Appendix. We only give the final ones in this subsection. For  $B_c \to \eta_c \ell^- \bar{\nu}_\ell$  and  $B \to D \ell^- \bar{\nu}_\ell$  decays,

$$\frac{d\mathcal{B}(B_q \to P\ell^- \bar{\nu}_\ell)}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \tau_{B_q} |\vec{p}_P| q^2}{96\pi^3 m_{B_q}^2} \left( 1 - \frac{m_\ell^2}{q^2} \right)^2 \left\{ H_0^2 (|G_V|^2 + |\widetilde{G}_V|^2) \left( 1 + \frac{m_\ell^2}{2q^2} \right) + \frac{3m_\ell^2}{2q^2} \left[ \left| H_t G_V + \frac{\sqrt{q^2}}{m_\ell} H_S G_S \right|^2 + \left| H_t \widetilde{G}_V + \frac{\sqrt{q^2}}{m_\ell} H_S \widetilde{G}_S \right|^2 \right] \right\}, \tag{6}$$

with

$$H_{0} = \frac{2m_{B_{q}}|\vec{p}_{P}|}{\sqrt{q^{2}}}F_{+}(q^{2}),$$

$$H_{t} = \frac{m_{B_{q}}^{2} - m_{P}^{2}}{\sqrt{q^{2}}}F_{0}(q^{2}),$$

$$H_{S} = \frac{m_{B_{q}}^{2} - m_{P}^{2}}{m_{b} - m_{c}}F_{0}(q^{2}).$$
(7)

For  $B_c \to J/\psi \ell^- \bar{\nu}_\ell$  and  $B \to D^* \ell^- \bar{\nu}_\ell$  decays,

$$\frac{d\mathcal{B}(B_q \to V \ell^- \bar{\nu}_\ell)}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \tau_{B_q} |\vec{p}_V| q^2}{96\pi^3 m_{B_q}^2} \left( 1 - \frac{m_\ell^2}{q^2} \right)^2 \left\{ \left| \mathcal{A}_{AV} \right|^2 + \frac{m_\ell^2}{2q^2} \left( \left| \mathcal{A}_{AV} \right|^2 + 3 \left| \mathcal{A}_{tP} \right|^2 \right) + \left| \widetilde{\mathcal{A}}_{AV} \right|^2 + \frac{m_\ell^2}{2q^2} \left( \left| \widetilde{\mathcal{A}}_{AV} \right|^2 + 3 \left| \widetilde{\mathcal{A}}_{tP} \right|^2 \right) \right\}, \tag{8}$$

where

$$\begin{aligned} \left| \mathcal{A}_{AV} \right|^2 &= \mathcal{A}_0^2 \left| G_A \right|^2 + \mathcal{A}_{\parallel}^2 \left| G_A \right|^2 + \mathcal{A}_{\perp}^2 \left| G_V \right|^2, \\ \left| \widetilde{\mathcal{A}}_{AV} \right|^2 &= \mathcal{A}_0^2 \left| \widetilde{G}_A \right|^2 + \mathcal{A}_{\parallel}^2 \left| \widetilde{G}_A \right|^2 + \mathcal{A}_{\perp}^2 \left| \widetilde{G}_V \right|^2, \\ \left| \mathcal{A}_{tP} \right|^2 &= \left| \mathcal{A}_0 G_A + \frac{\sqrt{q^2}}{m_\ell} \mathcal{A}_P G_P \right|^2, \\ \left| \widetilde{\mathcal{A}}_{tP} \right|^2 &= \left| \mathcal{A}_0 \widetilde{G}_A + \frac{\sqrt{q^2}}{m_\ell} \mathcal{A}_P \widetilde{G}_P \right|^2, \end{aligned}$$
(9)

with

$$\mathcal{A}_{0} = \frac{1}{2m_{V}\sqrt{q^{2}}} \left[ (m_{B_{q}}^{2} - m_{V}^{2} - q^{2})(m_{B_{q}} + m_{V})A_{1}(q^{2}) - \frac{4m_{B_{q}}^{2}|\vec{p}_{V}|^{2}}{m_{B_{q}} + m_{V}} A_{2}(q^{2}) \right],$$

$$\mathcal{A}_{\parallel} = \sqrt{2}(m_{B_{q}} + m_{V})A_{1}(q^{2}),$$

$$\mathcal{A}_{\perp} = -\frac{4m_{B_{q}}V(q^{2})|\vec{p}_{V}|}{\sqrt{2}(m_{B_{q}} + m_{V})},$$

$$\mathcal{A}_{t} = \sqrt{2}m_{B_{q}}|\vec{p}_{V}|A_{0}(q^{2}),$$

$$\mathcal{A}_{P} = -\frac{2m_{B_{q}}|\vec{p}_{V}|A_{0}(q^{2})}{m_{b} + m_{c}}.$$
(10)

For baryonic  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  decays,

$$\frac{d\mathcal{B}(\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell)}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \tau_{\Lambda_b} |\vec{p}_{\Lambda_c}| q^2}{192\pi^3 m_{\Lambda_b}^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[B_1 + \frac{m_\ell^2}{2q^2} B_2 + \frac{3}{2} B_3 + \frac{3m_\ell}{\sqrt{q^2}} B_4\right], \quad (11)$$

where

$$B_{1} = \left| H_{\frac{1}{2}0} \right|^{2} + \left| H_{-\frac{1}{2}0} \right|^{2} + \left| H_{\frac{1}{2}1}^{2} \right|^{2} + \left| H_{-\frac{1}{2}-1} \right|^{2},$$

$$B_{2} = \left| H_{\frac{1}{2}0} \right|^{2} + \left| H_{-\frac{1}{2}0} \right|^{2} + \left| H_{\frac{1}{2}1} \right|^{2} + \left| H_{-\frac{1}{2}-1} \right|^{2} + 3\left( \left| H_{\frac{1}{2}t} \right|^{2} + \left| H_{-\frac{1}{2}t} \right|^{2} \right),$$

$$B_{3} = \left| H_{\frac{1}{2}0}^{SP} \right|^{2} + \left| H_{-\frac{1}{2}0}^{SP} \right|^{2},$$

$$B_{4} = Re \left[ H_{\frac{1}{2}t} H_{\frac{1}{2}0}^{SP*} + H_{-\frac{1}{2}t} H_{-\frac{1}{2}0}^{SP*} \right],$$

$$(12)$$

with

$$H_{\lambda_{2}\lambda_{1}} \equiv H_{\lambda_{2}\lambda_{1}}^{V} - H_{\lambda_{2}\lambda_{1}}^{A},$$

$$H_{\frac{1}{2}0}^{V} = G_{V} \frac{\sqrt{Q_{-}}}{\sqrt{q^{2}}} [(m_{\Lambda_{b}} + m_{\Lambda_{c}}) f_{1}q^{2} - q^{2} f_{2}(q^{2})],$$

$$H_{\frac{1}{2}0}^{A} = G_{A} \frac{\sqrt{Q_{+}}}{\sqrt{q^{2}}} [(m_{\Lambda_{b}} - m_{\Lambda_{c}}) g_{1}q^{2} + q^{2} g_{2}(q^{2})],$$

$$H_{\frac{1}{2}1}^{V} = G_{V} \sqrt{2Q_{-}} [-f_{1}q^{2} + (m_{\Lambda_{b}} + m_{\Lambda_{c}}) f_{2}(q^{2})],$$

$$H_{\frac{1}{2}1}^{A} = G_{A} \sqrt{2Q_{+}} [-g_{1}q^{2} - (m_{\Lambda_{b}} - m_{\Lambda_{c}}) g_{2}(q^{2})],$$

$$H_{\frac{1}{2}t}^{V} = G_{V} \frac{\sqrt{Q_{+}}}{\sqrt{q^{2}}} [(m_{\Lambda_{b}} - m_{\Lambda_{c}}) f_{1}q^{2} + q^{2} f_{3}(q^{2})],$$

$$H_{\frac{1}{2}t}^{A} = G_{A} \frac{\sqrt{Q_{-}}}{\sqrt{q^{2}}} [(m_{\Lambda_{b}} + m_{\Lambda_{c}}) g_{1}q^{2} - q^{2} g_{3}(q^{2})],$$

$$H_{\frac{1}{2}0}^{SP} \equiv H_{\frac{1}{2}0}^{S} - H_{\frac{1}{2}0}^{P},$$

$$H_{\frac{1}{2}0}^{S} = G_{S} \frac{\sqrt{Q_{+}}}{m_{b} - m_{c}} [(m_{\Lambda_{b}} - m_{\Lambda_{c}}) f_{1}(q^{2}) + q^{2} f_{3}(q^{2})],$$

$$H_{\frac{1}{2}0}^{P} = G_{P} \frac{\sqrt{Q_{-}}}{m_{b} + m_{c}} [(m_{\Lambda_{b}} + m_{\Lambda_{c}}) g_{1}(q^{2}) - q^{2} g_{3}(q^{2})],$$

$$(14)$$

where  $Q_{\pm} = (m_{\Lambda_b} \pm m_{\Lambda_c})^2 - q^2$ . Either from parity or from explicit calculation, we have the relations  $H^V_{-\lambda_2-\lambda_1} = H^V_{\lambda_2\lambda_1}$ ,  $H^A_{-\lambda_2-\lambda_1} = -H^A_{\lambda_2\lambda_1}$ ,  $H^S_{\lambda_2\lambda_1} = H^S_{-\lambda_2-\lambda_1}$  and  $H^P_{\lambda_2\lambda_1} = -H^P_{-\lambda_2-\lambda_1}$ .

In order to further study the RPV effects, we need calculate other two important physical quantities in  $M_1 \to M_2 \ell^- \bar{\nu}_\ell$  decays to reduce the error. The ratio of differential branching ratio may be written as

$$\frac{dR_{M_2}}{dq^2} = \frac{d\Gamma(M_1 \to M_2 \tau^- \bar{\nu}_\tau)/ds}{d\Gamma(M_1 \to M_2 \ell'^- \bar{\nu}_{\ell'})/ds}.$$
(15)

Noted that  $R_{M_2}$  is obtained by separately integrating the numerators and denominators of

above  $dR_{M_2}/dq^2$ . The normalized forward-backward asymmetry is defined as

$$A_{FB}^{M_1 \to M_2 \ell^- \bar{\nu}_{\ell}}(q^2) = \frac{\int_{-1}^{0} d\cos\theta_{\ell} \left[ \frac{d^2 \Gamma(M_1 \to M_2 \ell^- \bar{\nu}_{\ell})}{dq^2 d\cos\theta_{\ell}} \right] - \int_{0}^{1} d\cos\theta_{\ell} \left[ \frac{d^2 \Gamma(M_1 \to M_2 \ell^- \bar{\nu}_{\ell})}{dq^2 d\cos\theta_{\ell}} \right]}{\frac{d\Gamma(M_1 \to M_2 \ell^- \bar{\nu}_{\ell})}{dq^2}}.$$
 (16)

### 3 Numerical Results and Discussions

The main theoretical input parameters are the transition form factors, the CKM matrix element  $V_{cb}$ , the masses, the mean lives, etc. Relevant transition form factors are taken from Refs. [12,42,49,50], the CKM matrix element is taken from the UTfit Collaboration [51], and others are gotten from PDG [52]. The 95% confidence level (CL) theoretical uncertainties of the input parameters are considered in our results.

In our calculation, we consider only one NP coupling at one time and keep its interference with the SM amplitude to study the RPV effects. Due to the strong helicity suppression, the squark exchange couplings have no very obvious effects on the differential branching ratios and the normalized FB asymmetries of the semileptonic exclusive  $b \to c\ell^-\bar{\nu}_\ell$  decays. So we will only focus on the slepton exchange couplings in our following discussions. We assume the masses of the corresponding slepton are 500 GeV, for other values of the slepton masses, the bounds on the couplings in this paper can be easily obtained by scaling them by factor of  $\tilde{f}^2 \equiv \left(\frac{m_{\tilde{\ell}}}{500 \text{GeV}}\right)^2$ .

A part of latest relevant experimental ranges at 95% CL are listed in the second column of Tab. 1. The following experimental constraints at 95% CL of  $B_u$  decays will be also considered in our analysis.

$$\mathcal{B}(B_u \to D^{*0} \ell'^- \bar{\nu_{\ell'}}) = (5.69 \pm 0.19) \times 10^{-2},$$

$$\mathcal{B}(B_u \to D^{*0} \tau^- \bar{\nu_{\tau}}) = (1.88 \pm 0.20) \times 10^{-2},$$

$$\mathcal{B}(B_u \to D \ell'^- \bar{\nu_{\ell'}}) = (2.27 \pm 0.11) \times 10^{-2},$$

$$\mathcal{B}(B_u \to D \tau^- \bar{\nu_{\tau}}) = (7.7 \pm 2.5) \times 10^{-3}.$$
(17)

Noted that the slepton exchange contributions to  $B_u^- \to D_u^{(*)0} \ell^- \nu_\ell$  and  $B_d^0 \to D_d^{(*)+} \ell^- \nu_\ell$  are very similar to each other, since the SU(2) flavor symmetry implies  $M(B_u^- \to D_u^{(*)0} \ell^- \nu_\ell) \approx M(B_d^0 \to D_d^{(*)+} \ell^- \nu_\ell)$ . So we would take  $B_d^0 \to D_d^{(*)+} \ell^- \nu_\ell$  decays as examples in the following. Since the experimental measurements of  $\mathcal{B}(B \to D^* \tau^- \bar{\nu}_\tau)$  and  $\mathcal{R}(D^*)$  obviously deviate from their SM predictions, we do not impose some obvious deviated measurements and just leave

them as predictions of the restricted parameter spaces of the RPV couplings, and then compare them with the experimental results. Two schemes of 95% CL experimental bounds will be used in this work.

 $S_1$ : All relevant experimental bounds except for  $\mathcal{B}(B \to D^* \tau^- \bar{\nu}_{\tau})$  and  $\mathcal{R}(D^*)$  at 95% CL.

 $S_2$ : All relevant experimental bounds except for  $\mathcal{B}(B \to D^*\tau^-\bar{\nu}_{\tau})$  at 95% CL.

Slepton exchange couplings  $\lambda_{i11}\tilde{\lambda}'^*_{i23}$ ,  $\lambda_{i22}\tilde{\lambda}'^*_{i23}$  and  $\lambda_{i33}\tilde{\lambda}'^*_{i23}$  contribute to the exclusive  $b\to ce^-\bar{\nu}_e$ ,  $b\to c\mu^-\bar{\nu}_\mu$  and  $b\to c\tau^-\bar{\nu}_\tau$  decays, respectively. For  $\lambda_{i11}\tilde{\lambda}'^*_{i23}$  and  $\lambda_{i22}\tilde{\lambda}'^*_{i23}$ , which contribute to both  $b\to c\ell'^-\bar{\nu}_{\ell'}$  and  $b\to s\ell'^+\ell'^-$  transitions, the stronger constraints come from the exclusive  $b\to s\ell'^+\ell'^-$  decays ( $|\lambda_{i11}\tilde{\lambda}'^*_{i23}|<5.75\times10^{-4}$ ,  $|\lambda_{i22}\tilde{\lambda}'^*_{i23}|<1.63\times10^{-5}$ ) [53,54], which will be used in our numerical results. Fig.1 shows the allowed coupling spaces of  $\lambda_{i33}\tilde{\lambda}'^*_{i23}$  from the latest 95% CL experimental measurements of the exclusive  $b\to c\tau^-\bar{\nu}_\tau$  in the cases of  $S_1$  and  $S_2$ . Fig.1 shows us that both moduli and RPV weak phases of  $\lambda_{i33}\tilde{\lambda}'^*_{i23}$  are constrained in the cases of both  $S_1$  and  $S_2$ . In the  $S_1$ , we obtain  $|\lambda_{i33}\tilde{\lambda}'^*_{i23}| \le 0.67$ , and the slight difference between this constrained space within  $S_1$  case and one in Fig. 3 of Ref. [45] comes from the updated input parameters and experimental measurements. If considering the  $R_{D^*}$  bound, i.e., in the cases of  $S_2$ , very strong bounds on  $\lambda_{i33}\tilde{\lambda}'^*_{i23}$  are obtained,  $0.11 \le |\lambda_{i33}\tilde{\lambda}'^*_{i23}| \le 0.30$  and  $|\phi_{RPV}| \le 82^\circ$ .

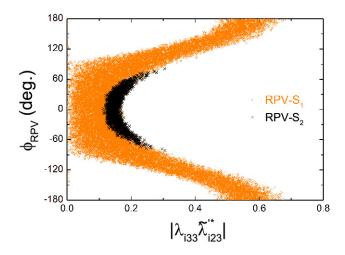


Figure 1: The allowed spaces due to slepton exchange couplings from the latest 95% CL experimental bounds of the exclusive  $b \to c \ell^- \bar{\nu}_\ell$  decays.

Now we discuss the constrained slepton exchange effects in  $B_c \to J/\psi \ell^- \bar{\nu}_\ell$ ,  $\eta_c \ell^- \bar{\nu}_\ell$  and  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  decays. Our numerical predictions for the branching ratios and their ratios are summarized in last columns of Tab. 1. We also show their sensitivities to the moduli and weak phases of the slepton exchange couplings  $\lambda_{i33}\tilde{\lambda}'^*_{i23}$  in Figs. 2-3. For convenient analysis and comparison, we also give all SM predictions of  $B_c \to J/\psi \ell^- \bar{\nu}_\ell$ ,  $\eta_c \ell^- \bar{\nu}_\ell$  and  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  decays as well as the similar updated predictions of  $B \to D^{(*)} \ell^- \bar{\nu}_\ell$  decays, which have been given in Ref. [45]. We have the following remarks for the branching ratios and their ratios:

• Experimental constaints: In the case of  $S_1$ , the 95% CL experimental upper limits of  $\mathcal{B}(B_d \to D^*\ell'^-\nu_{\ell'})$  and  $R_D$  as well as lower limits of  $\mathcal{B}(B_d \to D\ell'^-\nu_{\ell'})$  and  $R_D$  give effective constraints on the RPV couplings. In the case of  $S_2$ , the lower limit of  $R_{D^*}$  can give further effective constraint.

Table 1: Experimental ranges and our numerical predictions for the branching ratios (in units of  $10^{-2}$ ) and their ratios.

Exp. ranges	SM predictions	$RPV-S_1$	$RPV-S_2$
• • •	$[\ 0.86\ ,\ 1.67\ ]$	$[\ 0.77\ ,\ 1.64\ ]$	$[\ 0.77\ ,\ 1.61\ ]$
• • •	$[\ 0.26\ ,\ 0.46\ ]$	$[\ 0.22\ ,\ 0.47\ ]$	$[\ 0.26\ ,\ 0.47\ ]$
• • •	$[\ 0.27\ ,\ 0.88\ ]$	$[\ 0.25\ ,\ 0.87\ ]$	$[\ 0.25\ ,\ 0.83\ ]$
• • •	$[0.084, \ 0.263]$	$[0.087, \ 0.478]$	[0.138, 0.464]
	$[\ 4.75\ ,\ 6.33\ ]$	$[\ 4.38\ ,\ 6.22\ ]$	$[\ 4.38\ ,\ 6.22\ ]$
	$[\ 1.60\ ,\ 2.08\ ]$	$[\ 1.57\ ,\ 2.97\ ]$	$[\ 1.77\ ,\ 2.52\ ]$
[ 4.71 , 5.15 ]	$[\ 4.45\ ,\ 5.32\ ]$	[ 4.88 , 5.15 ]	[ 4.88 , 5.15 ]
[ 1.42 , 1.92 ]	$[\ 1.12\ ,\ 1.35\ ]$	[ 1.12 , 1.43 ]	[ 1.35 , 1.43 ]
[ 1.95 , 2.43 ]	$[\ 1.82\ ,\ 2.46\ ]$	[ 1.95 , 2.32 ]	[ 1.95 , 2.32 ]
$[\ 0.60\ ,\ 1.46\ ]$	$[\ 0.50\ ,\ 0.76\ ]$	$[\ 0.62\ ,\ 1.14\ ]$	$[\ 0.85\ ,\ 1.14\ ]$
[0.225, 1.195]	$[0.271, \ 0.314]$	$[0.252, \ 0.346]$	[0.285, 0.346]
	$[0.192, \ 0.613]$	[0.204, 1.300]	[0.274, 1.300]
	$[0.322, \ 0.356]$	$[0.328, \ 0.527]$	$[0.373, \ 0.443]$
[0.317, 0.497]	$[0.255, \ 0.328]$	$[0.317, \ 0.497]$	$[0.414, \ 0.497]$
[ <mark>0.275</mark> , 0.333]	$[0.242, \ 0.262]$	$[0.226, \ 0.283]$	$[0.275, \ 0.283]$
	[ 4.71 , 5.15 ] [ 1.42 , 1.92 ] [ 1.95 , 2.43 ] [ 0.60 , 1.46 ] [ 0.225, 1.195] [ 0.317, 0.497]	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

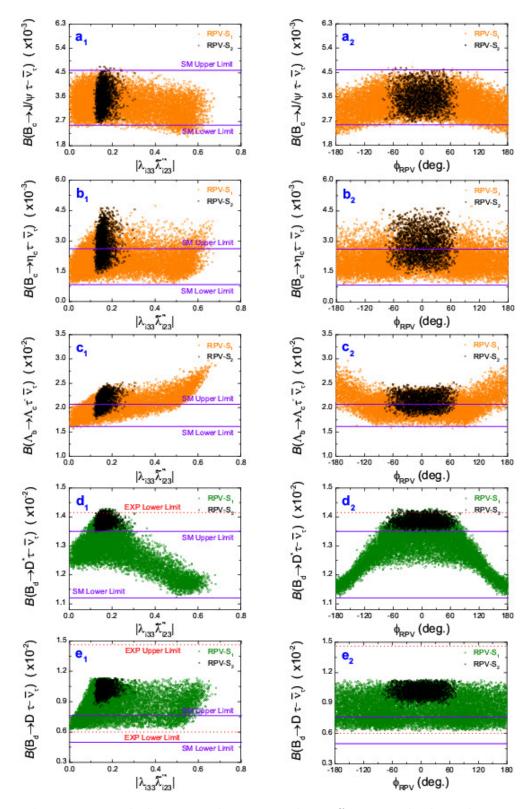


Figure 2: The constrained slepton exchange coupling effects on the branching ratios of the exclusive semileptonic  $b \to c\tau^-\bar{\nu}_{\tau}$  decays.

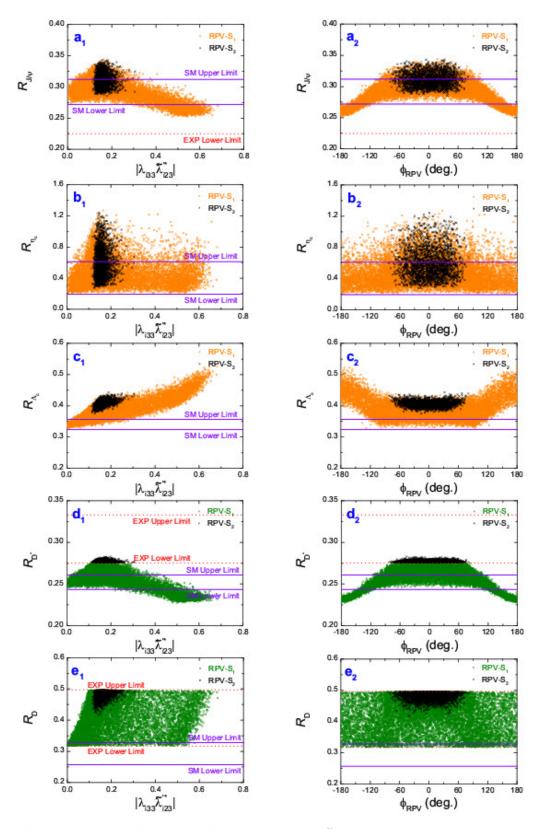


Figure 3: The constrained slepton exchange coupling effects on  $R_{J/\psi},~R_{\eta_c},~R_{\Lambda_c},~R_{D^*}$  and  $R_D$ .

- Branching ratios with  $\ell = \ell'$ : The constrained slepton couplings have but not large effects on  $\mathcal{B}(B_c \to J/\psi \ell'^- \bar{\nu}_{\ell'}, \eta_c \ell'^- \bar{\nu}_{\ell'})$  and  $\mathcal{B}(\Lambda_b \to \Lambda_c \ell'^- \bar{\nu}_{\ell'})$ . And these branching ratios are not very sensitive to relevant slepton exchange couplings, so we will not display their sensitivities to RPV couplings as similar as Fig. 2.
- Branching ratios with  $\ell = \tau$ : As displayed in Fig. 2, the constrained slepton couplings have very obvious effects on all five branching ratios with  $\ell = \tau$ , and they are very sensitive to both moduli and weak phases of  $\lambda_{i33}\tilde{\lambda}_{i23}^{\prime *}$ . If also considering the experimental bounds of  $R_{D^*}$ , the lower limits of  $\mathcal{B}(B_c \to J/\psi\tau^-\bar{\nu}_{\tau}, \eta_c\tau^-\bar{\nu}_{\tau})$  and  $\mathcal{B}(B \to D^*\tau^-\bar{\nu}_{\tau}, D\tau^-\bar{\nu}_{\tau})$  as well as both upper and lower limits of  $\mathcal{B}(\Lambda_b \to \Lambda_c\tau^-\bar{\nu}_{\tau})$  are further constrained. As for  $\mathcal{B}(B_d \to D_d^*\tau^-\bar{\nu}_{\tau})$ , which experimental constraints are not used, within both  $S_1$  and  $S_2$  cases, only very narrow ranges,  $[1.42, 1.43] \times 10^{-2}$ , satisfy its present measurement.
- Ratios of the branching ratios: As shown in Fig. 3, the ratios of the branching ratios are also very sensitive to  $\lambda_{i33}\tilde{\lambda}'^*_{i23}$  couplings, the further experimental constraints of  $R_{D^*}$  give obvious lower limits of  $R_{J/\psi}$ ,  $R_{\Lambda_c}$  and  $R_D$ . Present experimental measurement of  $R_{J/\psi}$  with the large uncertainty could not give any further constraint on the slepton exchange couplings. The upper limit of  $R_{J/\psi}$ ,  $R_{\eta_c}$  and  $R_{\Lambda_c}$  could be increased by 10%, 112% and 24%, respectively, from their upper limits of the SM predictions. The upper limit of RPV prediction of  $R_{J/\psi}$  is about half of the central value of the experimental measurement, but is within 1.5 $\sigma$ .

Now we discuss the constrained slepton exchange coupling effects on the differential branching ratios and their ratios of the exclusive semileptonic  $b \to c\ell^-\bar{\nu}_\ell$  decays, which are shown in Fig. 4. From the first and last columns of Fig.4, one can see that the constrained slepton exchange couplings have very large effects on all five differential branching ratios with  $\ell = \tau$  and all five ratios of the differential branching ratios. In  $S_2$  case, the 95% CL experimental measurement of  $R_{D^*}$  given obviously further constraints on the lower limits of  $d\mathcal{B}(B_c \to J/\psi \tau \nu_\tau)/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \tau \nu_\tau)/dq^2$ ,  $d\mathcal{B}(B \to D^* \tau \nu_\tau)/dq^2$ ,  $dR_{J/\psi}/dq^2$ ,  $dR_{\eta_c}/dq^2$ ,  $dR_{\Lambda_c}/dq^2$ ,  $dR_{D^*}/dq^2$  as well as both upper and lower limits of  $d\mathcal{B}(\Lambda_b \to \Lambda_c \tau \nu_\tau)/dq^2$ ,  $d\mathcal{B}(B \to D \tau \nu_\tau)/dq^2$  and  $dR_D/dq^2$ . From the second column of Fig.4, one can see that the constrained slepton exchange couplings still have some effects on  $d\mathcal{B}(B_c \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ ,  $d\mathcal{B}(B_c \to \eta_c \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(\Lambda_b \to J/\psi \ell' \nu_{\ell'})/dq^2$ .

 $\Lambda_c \ell' \nu_{\ell'})/dq^2$ , nevertheless, they have no obvious effects on  $d\mathcal{B}(B \to D^* \ell' \nu_{\ell'})/dq^2$  and  $d\mathcal{B}(B \to D\ell' \nu_{\ell'})/dq^2$ , since the present accurate experimental measurements of  $\mathcal{B}(B \to D^{(*)} \ell' \nu_{\ell'})$  give very strongly constraints on the slepton exchange coupling contributions. In addition,  $R_{D^*}$  could not give obviously further constraints on all five differential branching ratios with  $\ell = \ell'$ .

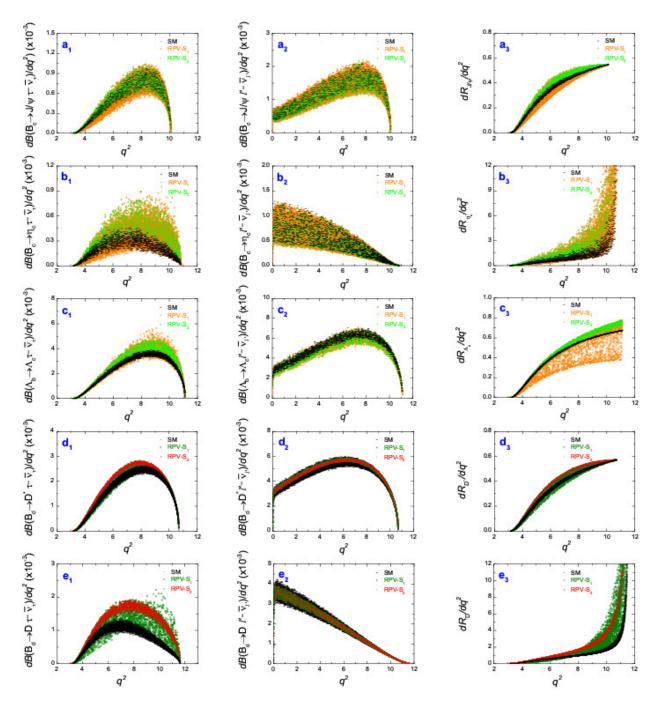


Figure 4: The constrained slepton exchange coupling effects on the differential branching ratios and their ratios of the exclusive semileptonic  $b \to c \ell^- \bar{\nu}_\ell$  decays.

Fig. 5 displays the constrained slepton exchange coupling effects on the normalized forward-backward asymmetries of the exclusive semileptonic  $b \to c \ell^- \bar{\nu}_\ell$  decays. Since the SM and RPV predictions of the normalized forward-backward asymmetries in cases of  $\ell = \mu$  and  $\ell = e$  are quite different, we show them all in Fig. 5. For  $\ell = \tau$  case, as shown in the first column of Fig. 5, the significant effects on all five normalized forward-backward asymmetries are allowed in case of  $S_1$ , the experimental measurement of  $R_D^*$  could give very strongly further bounds on

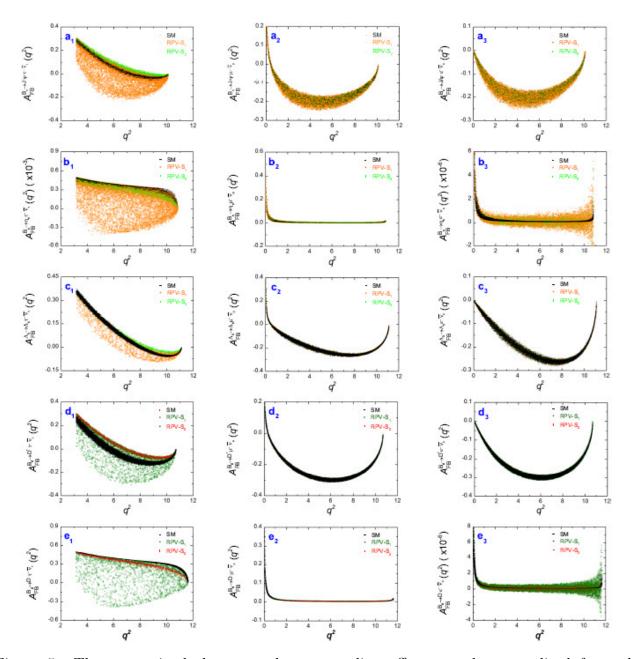


Figure 5: The constrained slepton exchange coupling effects on the normalized forward-backward asymmetries of the exclusive semileptonic  $b \to c \ell^- \bar{\nu}_\ell$  decays.

these normalized forward-backward asymmetries. So the measurement of these five normalized forward-backward asymmetries could test our RPV predictions and further shrink or reveal the parameter spaces of the slepton exchange couplings. In addition, as shown in Fig. 5  $(b_3,e_3)$ , the constrained slepton exchange couplings still provide quite obvious effects on  $A_{FB}^{B_c \to \eta_c e^{-\bar{\nu}_e}}(q^2)$  and  $A_{FB}^{B_d \to De^{-\bar{\nu}_e}}(q^2)$ , their sign could be changed, nevertheless, both quantities are tiny.

#### 4 Conclusion

Motivated by  $R_D$ ,  $R_{D^*}$  and  $R_{J/\psi}$  anomalies reported by LHCb, BABAR and Belle Collaborations, we have studied RPV supersymmetric effects in  $B_c \to J/\psi \ell^- \bar{\nu}_\ell$ ,  $\eta_c \ell^- \bar{\nu}_\ell$  and  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  decays, which are also induced by  $b \to c \ell^- \bar{\nu}_\ell$  at quark level. Since the squark exchange couplings have tiny effects in these decays, we have only focused on the slepton exchange couplings in this work.

The slepton exchange couplings  $\lambda_{i33}\tilde{\lambda}_{i23}^{\prime*}$  involve in the exclusive  $b\to c\tau^-\bar{\nu}_{\tau}$  decays. The latest relevant experimental measurements at 95% CL give obvious bounds on the both moduli and weak phases of  $\lambda_{i33}\tilde{\lambda}_{i23}^{\prime*}$ , and these couplings could explain the recent  $R_D$ ,  $R_{D^*}$  and  $R_{J/\psi}$  anomalies at the same time. We have found that, if considering all relevant experimental bounds except for  $\mathcal{B}(B_d\to D_d^*\tau^-\bar{\nu}_{\tau})$  and  $\mathcal{R}(D^*)$  at 95% CL, the constrained slepton couplings have great effects on all five (differential) branching ratios with  $\ell=\tau$ , five ratios of the (differential) branching ratios and the normalized forward-backward asymmetries of  $\tau^-$ . And the most of branching ratios with  $\ell=\tau$  and ratios of the branching ratios are very sensitive to the both moduli and weak phases of  $\lambda_{i33}\tilde{\lambda}_{i23}^{\prime*}$ . The experimental lower limit of  $R_{D^*}$  could give further very obvious constraints on slepton exchange couplings. As for  $\mathcal{B}(B_d\to D_d^*\tau^-\bar{\nu}_{\tau})$ , which experimental constraints are not used, a narrow range of RPV prediction could still satisfy its present measurement. The upper limit of  $R_{J/\psi}$ ,  $R_{\eta_c}$  and  $R_{\Lambda_c}$  could be increased by 10%, 112% and 24%, respectively, from their upper limits of the SM predictions. The upper limit of RPV prediction of  $R_{J/\psi}$  is 1.5 $\sigma$  away from its experimental measurement.

The slepton exchange couplings  $\lambda_{i11}\tilde{\lambda}'^*_{i23}$  and  $\lambda_{i22}\tilde{\lambda}'^*_{i23}$  involve in the exclusive  $b\to ce^-\bar{\nu}_e, se^+e^-$  and  $b\to c\mu^-\bar{\nu}_\mu, s\mu^+\mu^-$  decays, respectively. The constrained couplings of  $\lambda_{i11}\tilde{\lambda}'^*_{i23}$  and  $\lambda_{i22}\tilde{\lambda}'^*_{i23}$  from the exclusive  $b\to se^+e^-, s\mu^+\mu^-$  decays have quite small effects on the branching ratios and their ratios of the exclusive semileptonic  $b\to ce^-\bar{\nu}_e$  and  $b\to c\mu^-\bar{\nu}_\mu$  decays, nevertheless, the

constrained  $\lambda_{i11}\tilde{\lambda}'^*_{i23}$  couplings still have obviously effects on the normalized forward-backward asymmetries of  $B_c \to \eta_c e^-\bar{\nu}_e$  and  $B_d \to D e^-\bar{\nu}_e$ , but both  $A_{FB}^{B_c \to \eta_c e^-\bar{\nu}_e}(q^2)$  and  $A_{FB}^{B_d \to D e^-\bar{\nu}_e}(q^2)$  are tiny.

The large amount of data is expected in the near future from LHCb and BELLE II, and the precise measurements of the ratios of the branching ratios and the normalized forward-backward asymmetries of the exclusive semileptonic  $b \to c\tau\nu_{\tau}$  decays would enable us to test our RPV predictions and further shrink or reveal the parameter spaces of the slepton exchange couplings.

## 5 Acknowledgments

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### A Appendix

# A.1 Formulae of the $B_q \to M \ell^- \bar{\nu}_{\ell}$ decays

The hadronic matrix elements for  $B_q \to P/V$  transition can be parameterized by the form factors as

$$\langle P(p')|\bar{q'}\gamma_{\mu}b|B_{q}(p)\rangle = F_{+}(q^{2})\left[(p+p')_{\mu} - \frac{m_{B_{q}}^{2} - m_{P}^{2}}{q^{2}}q_{\mu}\right] + F_{0}(q^{2})\frac{m_{B_{q}}^{2} - m_{P}^{2}}{q^{2}}q_{\mu},$$

$$\langle V(p',\epsilon^{*})|\bar{q'}\gamma_{\mu}b|B_{q}(p)\rangle = \frac{2iV(q^{2})}{m_{B_{q}} + m_{V}}\varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p'^{\rho}p^{\sigma},$$

$$\langle V(p',\epsilon^{*})|\bar{q'}\gamma_{\mu}\gamma_{5}b|B_{q}(p)\rangle = 2m_{V}A_{0}(q^{2})\frac{\epsilon^{*} \cdot q}{q^{2}}q_{\mu} + (m_{B_{q}} + m_{V})A_{1}(q^{2})\left[\epsilon_{\mu}^{*} - \frac{\epsilon^{*} \cdot q}{q^{2}}q_{\mu}\right]$$

$$-A_{2}(q^{2})\frac{\epsilon^{*} \cdot q}{(m_{B_{q}} + m_{V})}\left[(p+p')_{\mu} - \frac{m_{B_{q}}^{2} - m_{V}^{2}}{q^{2}}q_{\mu}\right],$$

$$(18)$$

where q=p-p' is the momentum transfer,  $F_+, F_0$  and  $V, A_0, A_1, A_2$  are the form factors of  $B_q \to P$  and  $B_q \to V$  transitions, respectively. Noted that, in our numerical results, we take the  $B \to D/D^*$  form factors from Refs. [49, 50] and the  $B_c \to \eta_c, J/\psi$  form factors from Refs. [12].

The double differential branching ratios of  $B_q \to P \ell^- \bar{\nu}_\ell$  decays can be represented as

$$\frac{d\mathcal{B}(B_q \to P\ell^-\bar{\nu}_\ell)}{dq^2 d\cos\theta_\ell} = \frac{G_F^2 |V_{cb}|^2 \tau_{B_q} |\vec{p}_P| q^2}{128\pi^3 m_{B_q}^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left\{ H_0^2 \sin^2\theta_\ell \left( |G_V|^2 + |\widetilde{G}_V|^2 \right) + \frac{m_\ell^2}{q^2} \left| H_0 G_V \cos\theta_l - \left( H_t G_V + \frac{\sqrt{q^2}}{m_\ell} H_S G_S \right) \right|^2 + \frac{m_\ell^2}{q^2} \left| H_0 \widetilde{G}_V \cos\theta_l - \left( H_t \widetilde{G}_V + \frac{\sqrt{q^2}}{m_\ell} H_S \widetilde{G}_S \right) \right|^2 \right\}, \tag{19}$$

where  $|\vec{p}_M| \equiv \sqrt{\lambda(m_{B_q}^2, m_M^2, q^2)}/2m_{B_q}$  with  $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2(ab + bc + ca)$ .

The double differential branching ratios of  $B_q \to V \ell^- \bar{\nu}_\ell$  decays are

$$\frac{d\mathcal{B}(B_{q} \to V\ell^{-}\bar{\nu}_{\ell})}{dq^{2}d\cos\theta_{l}} = \frac{G_{F}^{2}|V_{cb}|^{2}\tau_{B_{q}}|\vec{p}_{V}|q^{2}}{256\pi^{3}m_{B_{q}}^{2}} \left(1 - \frac{m_{\ell}^{2}}{q^{2}}\right)^{2} \left\{2\mathcal{A}_{0}^{2}\left(|G_{A}|^{2} + |\tilde{G}_{A}|^{2}\right)\sin^{2}\theta_{\ell}\right. \\
+ \left(1 + \cos^{2}\theta_{\ell}\right)\left[\mathcal{A}_{\parallel}^{2}\left(|G_{A}|^{2} + |\tilde{G}_{A}|^{2}\right) + \mathcal{A}_{\perp}^{2}\left(|G_{V}|^{2} + |\tilde{G}_{V}|^{2}\right)\right] \\
- 4\cos\theta_{\ell}Re\left[\mathcal{A}_{\parallel}\mathcal{A}_{\perp}\left(G_{A}G_{V}^{*} - \tilde{G}_{A}\tilde{G}_{V}^{*}\right)\right] \\
+ \frac{m_{\ell}^{2}}{q^{2}}\sin^{2}\theta_{\ell}\left[\mathcal{A}_{\parallel}^{2}\left(|G_{A}|^{2} + |\tilde{G}_{A}|^{2}\right) + \mathcal{A}_{\perp}^{2}\left(|G_{V}|^{2} + |\tilde{G}_{V}|^{2}\right)\right] \\
+ \frac{2m_{\ell}^{2}}{q^{2}}\left|\mathcal{A}_{0}G_{A}\cos\theta_{l} - \left(\mathcal{A}_{t}G_{A} + \frac{\sqrt{q^{2}}}{m_{l}}\mathcal{A}_{P}G_{P}\right)\right|^{2} \\
+ \frac{2m_{\ell}^{2}}{q^{2}}\left|\mathcal{A}_{0}\tilde{G}_{A}\cos\theta_{l} - \left(\mathcal{A}_{t}\tilde{G}_{A} + \frac{\sqrt{q^{2}}}{m_{l}}\mathcal{A}_{P}\tilde{G}_{P}\right)\right|^{2}\right\}. \tag{20}$$

# A.2 Formulae of the $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$ decays

The hadronic matrix elements for  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  transition can be parameterized as [44]

$$<\Lambda_{c}(p_{2},\lambda_{2})|\bar{c}\gamma_{\mu}b|\Lambda_{b}(p_{1},\lambda_{1})> = \bar{u}_{2}(p_{2},\lambda_{2})[f_{1}(q^{2})\gamma_{\mu}+if_{2}(q^{2})\sigma_{\mu\nu}q^{\nu}+f_{3}(q^{2})q_{\mu}]u_{1}(p_{1},\lambda_{1}),$$

$$<\Lambda_{c}(p_{2},\lambda_{2})|\bar{c}\gamma_{\mu}\gamma^{5}b|\Lambda_{b}(p_{1},\lambda_{1})> = \bar{u}_{2}(p_{2},\lambda_{2})[g_{1}(q^{2})\gamma_{\mu}+ig_{2}(q^{2})\sigma_{\mu\nu}q^{\nu}+g_{3}(q^{2})q_{\mu}]\gamma_{5}u_{1}(p_{1},\lambda_{1}),$$

$$<\Lambda_{c}(p_{2},\lambda_{2})|\bar{c}b|\Lambda_{b}(p_{1},\lambda_{1})> = \bar{u}_{2}(p_{2},\lambda_{2})\left[f_{1}(q^{2})\frac{\not{q}}{m_{b}-m_{c}}+f_{3}(q^{2})\frac{q^{2}}{m_{b}-m_{c}}\right]u_{1}(p_{1},\lambda_{1}),$$

$$<\Lambda_{c}(p_{2},\lambda_{2})|\bar{c}\gamma^{5}b|\Lambda_{b}(p_{1},\lambda_{1})> = \bar{u}_{2}(p_{2},\lambda_{2})\left[-g_{1}(q^{2})\frac{\not{q}}{m_{b}+m_{c}}-g_{3}(q^{2})\frac{q^{2}}{m_{b}+m_{c}}\right]\gamma_{5}u_{1}(p_{1},\lambda_{1}),$$

where  $q = (p_1 - p_2)$ ,  $\sigma_{\mu\nu} = i[\gamma_{\mu}, \gamma_{\nu}]/2$ ,  $\lambda_i$  is the helicity of baryons, and  $f_i(q^2)$ ,  $g_i(q^2)$  are  $\Lambda_b \to \Lambda_c$  form factors. And the form factors are taken from Ref. [42] in our results.

The double differential branching ratios of  $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$  can be written as [44]

$$\frac{d\mathcal{B}(\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell)}{dq^2 d \cos \theta_\ell} = \frac{G_F^2 |V_{cb}|^2 \tau_{\Lambda_b} |\vec{p}_{\Lambda_c}| q^2}{512 \pi^3 m_{\Lambda_b}^2} \left( 1 - \frac{m_\ell^2}{q^2} \right)^2 \left[ A_1 + \frac{m_\ell^2}{q^2} A_2 + 2A_3 + \frac{4m_\ell}{\sqrt{q^2}} A_4 \right], \quad (21)$$
where  $|\vec{p}_{\Lambda_c}| = \sqrt{\lambda (m_{\Lambda_b}^2, m_{\Lambda_c}^2, q^2)} / 2m_{\Lambda_b}$ , and
$$A_1 = 2 \sin^2 \theta_\ell \left( |H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2 \right) + \left( 1 - \cos \theta_\ell \right)^2 |H_{\frac{1}{2}1}|^2 + \left( 1 + \cos \theta_\ell \right)^2 |H_{-\frac{1}{2}-1}|^2,$$

$$A_2 = 2 \cos^2 \theta_\ell \left( |H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2 \right) + \sin^2 \theta_\ell \left( |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 \right)$$

$$+ 2 \left( |H_{\frac{1}{2}t}|^2 + |H_{-\frac{1}{2}t}|^2 \right) - 4 \cos \theta_\ell Re \left[ H_{\frac{1}{2}t} H_{\frac{1}{2}0}^* + H_{-\frac{1}{2}t} H_{-\frac{1}{2}0}^* \right],$$

$$A_3 = |H_{\frac{1}{2}0}^{SP}|^2 + |H_{-\frac{1}{2}0}^{SP}|^2,$$

$$A_4 = -\cos \theta_\ell Re \left[ H_{\frac{1}{2}0} H_{\frac{1}{2}0}^{SP*} + H_{-\frac{1}{2}0} H_{-\frac{1}{2}0}^{SP*} \right] + Re \left[ H_{\frac{1}{2}t} H_{\frac{1}{2}0}^{SP*} + H_{-\frac{1}{2}t} H_{-\frac{1}{2}0}^{SP*} \right].$$
(22)

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