

# Collider signatures of neutrino masses and mixing from $R$ -parity violation

Eung Jin Chun,<sup>1</sup> Dong-Won Jung,<sup>2</sup> Sin Kyu Kang,<sup>3,4</sup> and Jong Dae Park<sup>2</sup>

<sup>1</sup>*Korea Institute for Advanced Study, P.O. Box 201, Cheongryangri, Seoul 130-650, Korea*

<sup>2</sup>*Department of Physics, Seoul National University, Seoul 151-747, Korea*

<sup>3</sup>*Institute for Basic Science, Korea University, Seoul 136-701, Korea*

<sup>4</sup>*Graduate School of Science, Hiroshima University, Higashi-Hiroshima 739-8526, Japan*

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$R$ -parity violation in the supersymmetric standard model can be the source of neutrino masses and mixing. We analyze the neutrino mass matrix coming from either bilinear or trilinear  $R$ -parity violation and its collider signatures, assuming that the atmospheric and solar neutrino data are explained by three active neutrino oscillations. Taking the gauge mediated supersymmetry breaking mechanism, we show that the lightest neutralino decays well inside the detector and the model could be tested by observing its branching ratios in future colliders. In the bilinear model where only the small solar neutrino mixing angle can be accommodated, the relation  $10^3 \text{BR}(\nu e^\pm \tau^\mp) \sim \text{BR}(\nu \mu^\pm \tau^\mp) \sim \text{BR}(\nu \tau^\pm \tau^\mp)$  serves as a robust test of the model. The large mixing angle solution can be realized in the trilinear model which predicts  $\text{BR}(\nu e^\pm \tau^\mp) \sim \text{BR}(\nu \mu^\pm \tau^\mp) \sim \text{BR}(\nu \tau^\pm \tau^\mp)$ . In either case, the relation  $\text{BR}(e jj) \ll \text{BR}(\mu jj) \sim \text{BR}(\tau jj)$  should hold to be consistent with the atmospheric neutrino and CHOOZ experiments.

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## I. INTRODUCTION

In the supersymmetric standard model, gauge invariance and renormalizability allow lepton and baryon number violation and thus may cause too a fast proton decay. Such a problem is usually avoided by introducing a discrete symmetry. Among various possibilities, the  $Z_2$   $R$  parity and  $Z_3$   $B$  parity have been advocated, as they can be remnants of gauge symmetries in string theory [1]. Imposing  $R$  parity has been more popular because of its simplicity and the possibility of having a natural dark matter candidate. The second option of allowing lepton number violation is also of a great interest since it can generate neutrino masses and mixing [2] in an economical way to explain the current neutrino data. There is a huge (but incomplete) body of literature investigating neutrino properties in this framework [3].

$R$ -parity violation may lead to a distinctive collider signature that the usual lightest supersymmetric particle (LSP), which is typically a neutralino, produces clean lepton (or baryon) number violating signals through its decay [4]. In a model of neutrino masses and mixing with  $R$ -parity violation, one can have more specific predictions for various branching ratios of the LSP decay, as the structure of lepton flavor violating couplings is dictated by the pattern of neutrino mixing determined from neutrino experiments [5–7]. This provides a unique opportunity to test the model in the future collider experiments. A necessary condition is of course that the LSP has a short lifetime to produce a bunch of decay signals inside the detector. In the models we will consider, the total LSP decay rate is proportional to the (heaviest) neutrino mass and thus the measurement of the LSP decay length could also be useful to test the model.

It is the purpose of this paper to examine the correspondence between neutrino oscillation parameters and collider signatures characterizing specific models of neutrino masses and mixing from  $R$ -parity violation. For this, we will consider the bilinear and trilinear models to see whether they can

accommodate the atmospheric [8] and solar neutrino oscillations [9] and the constraint coming from the CHOOZ experiment [10], simultaneously. One of our basic assumptions is the universality of soft supersymmetry breaking terms at a high scale, which is usually imposed to avoid flavor problems in the supersymmetric standard model. This implies that the lepton flavor violation occurs only in the superpotential with bilinear and/or trilinear  $R$ -parity violating terms and the supersymmetry breaking mechanism is flavor blind. Then, the tree-level neutrino mass is generated by the renormalization group evolution which breaks universality between the slepton and Higgs soft terms at the weak scale. As a specific scheme, we will consider the mechanism of gauge mediated supersymmetry breaking which solves the supersymmetric flavor problem in a natural way [11]. A comprehensive analysis of neutrino masses and mixing in this context has been performed in Ref. [12].<sup>1</sup>

Under such an assumption, the bilinear model can only realize the small mixing angle of solar neutrino oscillations while the trilinear model can accommodate the large mixing angle as well. In both cases, we will investigate whether the LSP decay length is short enough and what are the predictions for LSP decay signals which could test the model in future collider experiments. Here, another assumption we make is that the LSP is a neutralino. Let us remark that a similar analysis has been made in Ref. [7] considering supergravity models with generic bilinear  $R$ -parity violating terms.

This paper is organized as follows. In Sec. II, we calculate the “effective” trilinear  $R$ -parity violating couplings, rotating away the mixing mass terms between the ordinary particles and superparticles which arise as a consequence of bilinear  $R$ -parity violation. Those couplings are relevant for the LSP decay. In Sec. III, we examine the neutrino mass matrix which is generated through renormalization group evolution

<sup>1</sup>For a recent detailed analysis, see Ref. [13].

and various (finite) one-loop diagrams. From this, we will make a qualitative analysis to examine the sizes of various  $R$ -parity violating couplings which are required to explain the current neutrino oscillation data. In Sec. IV, we will provide a numerical analysis to determine  $R$ -parity conserving and violating input parameters with which the atmospheric and solar neutrino masses and mixing are realized, in the context of gauge mediated supersymmetry breaking models. Calculating the corresponding LSP decay rate and branching ratios of various modes, we will find how the model can be tested in the collider experiments. Finally, we will conclude in Sec. V.

## II. EFFECTIVE $R$ -PARITY VIOLATING VERTICES FROM BILINEAR TERMS

Allowing lepton number violation in the supersymmetric standard model, the superpotential is composed of the  $R$ -parity conserving  $W_0$  and violating  $W_1$  part:

$$W_0 = \mu H_1 H_2 + h_i^e L_i H_1 E_i^c + h_i^d Q_i H_1 D_i^c + h_i^u Q_i H_1 U_i^c, \\ W_1 = \epsilon_i \mu L_i H_2 + \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c. \quad (1)$$

Among soft supersymmetry breaking terms, let us write  $R$ -parity violating bilinear terms:

$$V_{\text{soft}} = B \mu H_1 H_2 + B_i \epsilon_i \mu L_i H_2 + m_{L_i H_1}^2 L_i H_1^\dagger + \text{H.c.} \quad (2)$$

It is clear that the electroweak symmetry breaking gives rise to nonzero vacuum expectation values of sneutrino fields  $\tilde{\nu}_i$  as follows [2]:

$$a_i \equiv \frac{\langle \tilde{\nu}_i \rangle}{\langle H_1 \rangle} = - \frac{\bar{m}_{L_i H_1}^2 + B_i \epsilon_i \mu t_\beta}{m_{\tilde{\nu}_i}^2}, \quad (3)$$

where  $\bar{m}_{L_i H_1}^2 = m_{L_i H_1}^2 + \epsilon_i \mu^2$ ,  $t_\beta = \tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$ , and  $m_{\tilde{\nu}_i}^2 = m_{L_i}^2 + M_{ZC2\beta}^2/2$ . In general, there are three types of independent  $R$ -parity violating bilinear parameters such as  $\epsilon_i$ ,  $a_i$ , and  $B_i/B$ , which give rise to the mixing between the ordinary particles and superparticles. That is, neutrinos and neutralinos, charged leptons and charginos, neutral Higgs bosons and sneutrinos, as well as charged Higgs bosons and charged sleptons have mixing mass terms which are determined by the above  $R$ -parity violating parameters. The mixing between neutrinos and neutralinos particularly serves as the origin of the tree-level neutrino masses which will be discussed later. Note that the above quantities have to be very small to account for tiny neutrino masses. While the effect of such small parameters on the particle and sparticle mass spectra (apart from the neutrino sector) are negligible, they induce small but important  $R$ -parity violating vertices between the particles and sparticles, which make the LSP destabilized and generate one-loop neutrino masses. The derivation of the induced  $R$ -parity violating couplings has been performed in many previous works. The usual approach is to take full diagonalizations of enlarged sparticle-particle

mass matrices with  $R$ -parity violating parts so that the vertices in terms of the mass eigenstates are obtained directly.

In this work, we take an alternative but equivalent approach which is useful when  $R$ -parity violating parameters are small. It is to rotate away only the small  $R$ -parity violating (off-diagonal) blocks of the particle-sparticle mass matrices, leaving untouched  $R$ -parity conserving particle or sparticle masses at the diagonal blocks. In this way, we can draw the induced (or “effective”)  $R$ -parity violating vertices in terms of the electroweak/flavor eigenstate basis. A merit of this method is that one can clearly see the vertex structure of the induced  $R$ -parity violating couplings along with the usual trilinear vertices in  $W_1$  of Eq. (1) added to the usual  $R$ -parity conserving Lagrangian. This is nothing but the usual seesaw diagonalization, which we summarize as follows. Let us take sparticle-particle mass matrix given by

$$\begin{pmatrix} M & \Delta \\ \Delta^\dagger & M' \end{pmatrix}$$

with  $\Delta \ll M, M'$ . Then, the approximate diagonalization (valid up to the second order of  $R$ -parity violating parameters  $\sim \Delta/M$  or  $\sim \Delta/M'$ ) can be done with the help of the rotation matrix given by

$$\begin{pmatrix} 1 - \frac{1}{2} \Theta \Theta^\dagger & -\Theta \\ \Theta^\dagger & 1 - \frac{1}{2} \Theta^\dagger \Theta \end{pmatrix},$$

where  $\Theta$  can be found by solving the relation  $\Delta = M \Theta - \Theta M'$  in the leading order of  $\Delta$ . The upper and lower diagonal blocks are then shifted as  $M \rightarrow M + (\Theta \Delta^\dagger + \Delta \Theta^\dagger)/2$  and  $M' \rightarrow M' - (\Theta^\dagger \Delta + \Delta^\dagger \Theta)/2$ . Note that the neutrino-neutralino mass matrix has vanishing submatrix for the neutrinos  $M=0$  and the above change in  $M$  is just the seesaw generation of small neutrino masses. For the other particles/sparticles, such changes can be safely neglected. After performing such a rotation, we get the “effective”  $R$ -parity violating vertices in the electroweak/flavor basis. Then, it is quite straightforward to find the corresponding couplings in the mass basis following the usual diagonalization of the familiar ( $R$ -parity conserving) particle/sparticle mass matrices.

In this paper, we do not repeat to write the mixing mass terms between sparticles and particles. Instead, we will present the rotation matrices  $\Theta$  in terms of the following three bilinear  $R$ -parity violating variables:

$$\epsilon_i \quad (\text{or} \quad a_i), \quad \xi_i \equiv a_i - \epsilon_i, \quad \eta_i \equiv a_i - B_i/B.$$

In generic supersymmetry breaking models with nonuniversality, the above three types of parameters are independent. But, in the restrictive models imposing the universality condition at the mediation scale of supersymmetry breaking, nonzero values of  $\xi_i$  and  $\eta_i$  arise as a consequence of renormalization group evolution and thus only two types of parameters are independent. In this paper, we usually take  $\epsilon_i$  and  $\xi_i$  as independent ones.

*Neutrino-neutralino diagonalization.* Rotating away the neutrino-neutralino mixing mass terms (by  $\theta^N$ ) can be made by the following redefinition of neutrinos and neutralinos:

$$\begin{pmatrix} \nu_i \\ \chi_j^0 \end{pmatrix} \rightarrow \begin{pmatrix} \nu_i - \theta_{ik}^N \chi_k^0 \\ \chi_j^0 + \theta_{lj}^N \nu_l \end{pmatrix}, \quad (4)$$

where  $(\nu_i)$  and  $(\chi_j^0)$  represent three neutrinos  $(\nu_e, \nu_\mu, \nu_\tau)$  and four neutralinos  $(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$  in the flavor basis, respectively. (For the mass eigenstates, we will use the usual notation  $\tilde{\chi}_i^0$ , which are related to the flavor eigenstates as follows:  $\tilde{\chi}_i^0 = N_{ij} \chi_j^0$ .) The rotation elements  $\theta_{ij}^N$  are given by

$$\theta_{ij}^N = \xi_i c_j^N c_\beta - \epsilon_i \delta_{j3} \quad (5)$$

and

$$(c_j^N) = \frac{M_Z}{F_N} \left( \frac{s_W M_2}{c_W^2 M_1 + s_W^2 M_2}, -\frac{c_W M_1}{c_W^2 M_1 + s_W^2 M_2}, -s_\beta \frac{M_Z}{\mu}, c_\beta \frac{M_Z}{\mu} \right),$$

where  $F_N = M_1 M_2 / (c_W^2 M_1 + s_W^2 M_2) + M_Z^2 s_{2\beta} / \mu$ . Here  $s_W = \sin \theta_W$  and  $c_W = \cos \theta_W$  with the weak mixing angle  $\theta_W$ .

*Charged lepton or chargino diagonalization.* Defining  $\theta^L$  and  $\theta^R$  as the two rotation matrices corresponding to the left-handed negatively and positively charged fermions, we have

$$\begin{pmatrix} e_i \\ \chi_j^- \end{pmatrix} \rightarrow \begin{pmatrix} e_i - \theta_{ik}^L \chi_k^- \\ \chi_j^- + \theta_{lj}^L e_l \end{pmatrix}, \quad \begin{pmatrix} e_i^c \\ \chi_j^+ \end{pmatrix} \rightarrow \begin{pmatrix} e_i^c - \theta_{ik}^R \chi_k^+ \\ \chi_j^+ + \theta_{lj}^R e_l^c \end{pmatrix}, \quad (6)$$

where  $e_i$  and  $e_i^c$  denote the left-handed charged leptons and antileptons  $(\chi_j^-) = (\tilde{W}^-, \tilde{H}_1^-)$  and  $(\chi_j^+) = (\tilde{W}^+, \tilde{H}_2^+)$ . The rotation elements  $\theta_{ij}^{L,R}$  are given by

$$\theta_{ij}^L = \xi_i c_j^L c_\beta - \epsilon_i \delta_{j2}, \quad \theta_{ij}^R = \frac{m_i^e}{F_C} \xi_i c_j^R c_\beta \quad (7)$$

and

$$(c_j^L) = -\frac{M_W}{F_C} \left( \sqrt{2}, 2s_\beta \frac{M_W}{\mu} \right),$$

$$(c_j^R) = -\frac{M_W}{F_C} \left[ \sqrt{2} \left( 1 - \frac{M_2}{\mu} t_\beta \right), \frac{M_2^2 c_\beta^{-1}}{\mu M_W} + 2 \frac{M_W}{\mu} c_\beta \right],$$

and  $F_C = M_2 + M_W^2 s_{2\beta} / \mu$ .

*Neutrino or neutral Higgs boson diagonalization.* Denoting the rotation matrix by  $\theta^S$ , we get

$$\begin{pmatrix} \tilde{\nu}_i \\ H_1^0 \\ H_2^0 \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{\nu}_i - \theta_{i1}^S H_1^0 - \theta_{i2}^S H_2^{0*} - \theta_{i3}^S H_1^{0*} - \theta_{i4}^S H_2^0 \\ H_1^0 + \theta_{i1}^S \tilde{\nu}_i + \theta_{i3}^S \tilde{\nu}_i^* \\ H_2^0 + \theta_{i2}^S \tilde{\nu}_i^* + \theta_{i4}^S \tilde{\nu}_i \end{pmatrix}, \quad (8)$$

where

$$\theta_{i1}^S = -a_i - \eta_i s_\beta^2 m_A^2 [m_{\tilde{\nu}_i}^4 - m_{\tilde{\nu}_i}^2 (m_A^2 + M_Z^2 s_\beta^2) - m_A^2 M_Z^2 s_\beta^2 c_{2\beta}] / F_S, \quad (9)$$

$$\theta_{i2}^S = +\eta_i s_\beta c_\beta m_A^2 [m_{\tilde{\nu}_i}^4 - m_{\tilde{\nu}_i}^2 (m_A^2 + M_Z^2 c_\beta^2) + m_A^2 M_Z^2 c_\beta^2 c_{2\beta}] / F_S,$$

$$\theta_{i3}^S = -\eta_i s_\beta^2 c_\beta^2 m_A^2 M_Z^2 [m_{\tilde{\nu}_i}^2 - m_A^2 c_{2\beta}] / F_S,$$

$$\theta_{i4}^S = +\eta_i s_\beta^3 c_\beta m_A^2 M_Z^2 [m_{\tilde{\nu}_i}^2 + m_A^2 c_{2\beta}] / F_S$$

with  $F_S = (m_{\tilde{\nu}_i}^2 - m_h^2)(m_{\tilde{\nu}_i}^2 - m_H^2)(m_{\tilde{\nu}_i}^2 - m_A^2)$  and  $m_A, m_h$ , and  $m_H$  are the masses of pseudo-scalar, light, and heavy neutral scalar Higgs bosons, respectively. Note that  $m_A^2 = -B\mu / c_\beta s_\beta$  in our convention. For our calculation, we assume that all the  $R$ -parity violating parameters are real and so are all  $\theta$ 's. We also note that the presence of the scalar fields as well as their complex conjugates in Eq. (8) is due to the electroweak symmetry breaking, which is expected to be suppressed by the factor  $M_Z^2 / m_A^2$ .

*Charged slepton/charged Higgs boson diagonalization.* Defining  $\theta^C$  as the rotation matrix, we have

$$\begin{pmatrix} \tilde{e}_i \\ \tilde{e}_i^{c*} \\ H_1^- \\ H_2^- \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{e}_i - \theta_{i1}^C H_1^- - \theta_{i2}^C H_2^- \\ \tilde{e}_i^{c*} - \theta_{i3}^C H_1^- - \theta_{i4}^C H_2^- \\ H_1^- + \theta_{i1}^C \tilde{e}_i + \theta_{i3}^C \tilde{e}_i^{c*} \\ H_2^- + \theta_{i2}^C \tilde{e}_i + \theta_{i4}^C \tilde{e}_i^{c*} \end{pmatrix}, \quad (10)$$

where

$$\theta_{i1}^C = -a_i - \eta_i \frac{s_\beta^2 m_A^2 (m_{Ri}^2 - m_{H^-}^2)}{(m_{H^-}^2 - m_{e_{i1}}^2)(m_{H^-}^2 - m_{e_{i2}}^2)} - \xi_i \frac{m_i^e \mu m_{Di}^2 t_\beta}{(m_{H^-}^2 - m_{e_{i1}}^2)(m_{H^-}^2 - m_{e_{i2}}^2)}, \quad (11)$$

$$\theta_{i2}^C = -\eta_i \frac{s_\beta c_\beta m_A^2 (m_{Ri}^2 - m_{H^-}^2)}{(m_{H^-}^2 - m_{e_{i1}}^2)(m_{H^-}^2 - m_{e_{i2}}^2)} - \xi_i \frac{m_i^e \mu m_{Di}^2}{(m_{H^-}^2 - m_{e_{i1}}^2)(m_{H^-}^2 - m_{e_{i2}}^2)},$$

$$\theta_{i3}^C = +\eta_i \frac{s_\beta^2 m_A^2 m_{Di}^2}{(m_{H^-}^2 - m_{e_{i1}}^2)(m_{H^-}^2 - m_{e_{i2}}^2)} + \xi_i \frac{m_i^e \mu (m_{Li}^2 - m_{H^-}^2) t_\beta}{(m_{H^-}^2 - m_{e_{i1}}^2)(m_{H^-}^2 - m_{e_{i2}}^2)},$$

$$\theta_{i4}^C = +\eta_i \frac{s_{\beta} c_{\beta} m_A^2 m_{Di}^2}{(m_{H^-}^2 - m_{e_{i1}}^2)(m_{H^-}^2 - m_{e_{i2}}^2)} + \xi_i \frac{m_i^e \mu (m_{Li}^2 - m_{H^-}^2)}{(m_{H^-}^2 - m_{e_{i1}}^2)(m_{H^-}^2 - m_{e_{i2}}^2)}.$$

Here,  $m_{H^-}$  stands for the charged-Higgs boson mass, and  $m_{Li}^2$ ,  $m_{Ri}^2$ , and  $m_{Di}^2$  correspond to the LL, RR, and LR components of the  $i$ th charged-slepton mass-squared matrix, respectively, and  $m_{e_{i1,2}}^2$  are its eigenvalues. We remark that the appearance of  $a_i$  in  $\theta_{i1}^S$  and  $\theta_{i1}^C$  is due to the rotations which remove the Goldstone modes from the redefined neutral and charged slepton fields.

With the expressions for the rotation matrices in Eqs. (4)–(11), we can obtain the effective  $R$ -parity violating vertices from the usual  $R$ -parity conserving interaction vertices, which are relevant to the LSP decays. Let us mention again that our effective vertices are written not in the mass basis but in the flavor basis. We list them below by taking only the linear terms in  $\theta$ 's which are enough for our purpose.

$\chi^0 - \nu - Z$  vertices:

$$\mathcal{L}_{\chi^0 \nu Z} = \bar{\chi}_i^0 \gamma^\mu P_L L_{ij}^{\chi^0 \nu Z} \nu_j Z_\mu^0 + \text{H.c.} \quad (12)$$

$$\text{with } L_{ij}^{\chi^0 \nu Z} = \frac{g}{2c_W} [c_1^N, c_2^N, 0, 2c_4^N] \xi_j c_\beta.$$

$\chi^0 - l - W$  vertices:

$$\mathcal{L}_{\chi^0 l W} = \bar{\chi}_i^0 \gamma^\mu [P_L L_{ij}^{\chi^0 l W} + P_R R_{ij}^{\chi^0 l W}] e_j W_\mu^+ + \text{H.c.} \quad (13)$$

$$\text{with } L_{ij}^{\chi^0 l W} = \frac{g}{\sqrt{2}} [c_1^N, c_2^N - \sqrt{2}c_1^L, c_3^N - c_2^L, c_4^N] \xi_j c_\beta,$$

$$R_{ij}^{\chi^0 l W} = \frac{g}{\sqrt{2}} [0, -\sqrt{2}c_1^R, 0, -c_2^R] \xi_j c_\beta.$$

$\chi^0 - \nu - H_{1,2}^0$  vertices:

$$\mathcal{L}_{\chi^0 \nu H_{1,2}^0} = \bar{\chi}_i^0 [P_L L_{ij}^{\chi^0 \nu H_{1,2}^0} + P_R R_{ij}^{\chi^0 \nu H_{1,2}^0}] \nu_j H_{1,2}^{0*} + \text{H.c.} \quad (14)$$

$$\text{with } L_{ij}^{\chi^0 \nu H_{1,2}^0} = \frac{g}{\sqrt{2}} [-t_W(\theta_{j1}^S - \theta_{j3}^N), (\theta_{j1}^S - \theta_{j3}^N), (t_W \theta_{j1}^N - \theta_{j2}^N), 0],$$

$$L_{ij}^{\chi^0 \nu H_2^0} = \frac{g}{\sqrt{2}} [-t_W(\theta_{j4}^S + \theta_{j4}^N), (\theta_{j4}^S + \theta_{j4}^N), 0, (-t_W \theta_{j1}^N + \theta_{j2}^N)],$$

$$R_{ij}^{\chi^0 \nu H_1^0} = \frac{g}{\sqrt{2}} [-t_W \theta_{j3}^S, \theta_{j3}^S, 0, 0],$$

$$R_{ij}^{\chi^0 \nu H_2^0} = \frac{g}{\sqrt{2}} [-t_W \theta_{j2}^S, \theta_{j2}^S, 0, 0].$$

$\chi^0 - l - H_{1,2}^+$  vertices:

$$\mathcal{L}_{\chi^0 l H_{1,2}^+} = \bar{\chi}_i^0 [P_L L_{ij}^{\chi^0 l H_{1,2}^+} + P_R R_{ij}^{\chi^0 l H_{1,2}^+}] e_j H_{1,2}^+ + \text{H.c.} \quad (15)$$

$$\text{with } L_{ij}^{\chi^0 l H_1^+} = \frac{-1}{\sqrt{2}} [g'(\theta_{j1}^C - \theta_{j2}^L), g(\theta_{j1}^C - \theta_{j2}^L), \sqrt{2}(g \theta_{j1}^L + h_j^e \theta_{j3}^C), 0],$$

$$L_{ij}^{\chi^0 l H_2^+} = \frac{-1}{\sqrt{2}} [g' \theta_{j2}^C, g \theta_{j2}^C, h_j^e \theta_{j4}^C, 0],$$

$$R_{ij}^{\chi^0 l H_1^+} = [\sqrt{2}g' \theta_{j3}^C + h_j^e \theta_{j1}^N, h_j^e \theta_{j2}^N, -h_j^e(\theta_{j1}^C - \theta_{j3}^N), h_j^e \theta_{j4}^N],$$

$$R_{ij}^{\chi^0 l H_2^+} = \left[ \sqrt{2}g' \theta_{j4}^C - \frac{g'}{\sqrt{2}} \theta_{j2}^R, -\frac{g}{\sqrt{2}} \theta_{j2}^R, -h_j^e \theta_{j2}^C, -g \theta_{j1}^R \right],$$

In Eqs. (12)–(15), the four components inside brackets correspond to the indices  $i=1, \dots, 4$  indicating the neutralino states ( $\tilde{B}$ ,  $\tilde{W}_3$ ,  $\tilde{H}_1^0$ ,  $\tilde{H}_2^0$ ), respectively, as before. Here, let us remark that all of the above vertices depend only on the variables  $\xi_i$  or  $\eta_i$  which are generated by renormalization group evolution under the universality condition, even though the individual elements  $\theta_{i3}^N$ ,  $\theta_{i2}^L$ ,  $\theta_{i1}^S$ , and  $\theta_{i1}^C$  depend on either  $\epsilon_i$  or  $a_i$ . This fact will be important when we study the LSP decay processes.

*Effective  $LQ\bar{d}$ ,  $L\bar{Q}u$ ,  $LL\bar{e}$ , and  $\nu f\bar{f}^*$  vertices:* Below we list the  $\lambda$ -like or  $\lambda'$ -like couplings which are neither supersymmetric nor  $SU(2)_L$  symmetric:

$$\begin{aligned} \mathcal{L}_{LQ\bar{d}} = & \varepsilon_{ab} [\Lambda_{aij}^{d1} \tilde{L}_{ai} \bar{d}_j P_L Q_{bj} + \Lambda_{aij}^{d2} \tilde{L}_{ai}^{c*} \bar{d}_j P_L Q_{bj} \\ & + \Lambda_{aij}^{d3} (\bar{d}_j P_L L_{ai} \tilde{Q}_{bj} + \bar{L}_{ai}^c P_L Q_{bj} \tilde{d}_j^c) \\ & + \Lambda_{aij}^{d4} \bar{L}_{ai}^c P_L Q_{bj} \tilde{d}_j^*] + \text{H.c.}, \end{aligned} \quad (16)$$

$$\text{where } \Lambda_{aij}^{d1} = [\theta_{i1}^S, \theta_{i1}^C] h_j^d, \quad \Lambda_{aij}^{d2} = [\theta_{i3}^S, \theta_{i3}^C] h_j^d,$$

$$\Lambda_{aij}^{d3} = [\theta_{i3}^N, \theta_{i2}^L] h_j^d, \quad \Lambda_{aij}^{d4} = \frac{g}{\sqrt{2}} [-t_W \theta_{i1}^N + \theta_{i2}^N, \sqrt{2} \theta_{i1}^L],$$

$$\begin{aligned} \mathcal{L}_{L\bar{Q}u} = & \delta_{ab} [\Lambda_{aij}^{u1} \tilde{L}_{ai} \bar{Q}_{bj} P_R u_j + \Lambda_{aij}^{u2} \tilde{L}_{ai}^{c*} \bar{Q}_{bj} P_R u_j \\ & + \Lambda_{aij}^{u3} (\tilde{u}_j^c \bar{Q}_{bj} P_R L_{ai} + \bar{L}_{ai}^c \tilde{Q}_{bj}^* P_R u_j) \\ & + \Lambda_{aij}^{u4} \tilde{u}_j \bar{Q}_{aj} P_R L_{ai}] + \text{H.c.}, \end{aligned} \quad (17)$$

$$\text{where } \Lambda_{aij}^{u1} = [-\theta_{i2}^S, \theta_{i2}^C] h_j^u, \quad \Lambda_{aij}^{u2} = [-\theta_{i4}^S, \theta_{i4}^C] h_j^u,$$

$$\Lambda_{aij}^{u3} = [-\theta_{i4}^N, \theta_{i2}^R] h_j^u,$$

$$\Lambda_{ai}^{u4} = \frac{g}{\sqrt{2}} \left[ -\frac{1}{3} t_w \theta_{i1}^N - \theta_{i2}^N, -\sqrt{2} \theta_{i1}^R \right],$$

$$\begin{aligned} \mathcal{L}_{LL\bar{e}} = & \varepsilon_{ab} [\Lambda_{aij}^{l1} \tilde{L}_{ai} \bar{e}_j P_L L_{bj} + \Lambda_{aij}^{l2} \tilde{L}_{ai}^* \bar{e}_j P_L L_{bj} \\ & + \Lambda_{aij}^{l3} (\bar{e}_j P_L L_{ai} \tilde{L}_{bj} - \bar{L}_{ai} P_L L_{bj} \tilde{e}_j^c) + \Lambda_{ai}^{l4} \tilde{L}_{ai}^c P_L L_{bj} \tilde{e}_j^*] \\ & + \Lambda_{ai}^{l5} \tilde{\nu}_j \tilde{L}_{aj}^c P_R L_{ai} + \text{H.c.}, \end{aligned} \quad (18)$$

$$\text{where } \Lambda_{aij}^{l1} = [\theta_{i1}^S, \theta_{i1}^C] h_j^e, \quad \Lambda_{aij}^{l2} = [\theta_{i3}^S, \theta_{i3}^C] h_j^e,$$

$$\Lambda_{aij}^{l3} = [\theta_{i3}^N, \theta_{i2}^L] h_j^e,$$

$$\Lambda_{ai}^{l4} = \frac{g}{\sqrt{2}} [t_w \theta_{i1}^N + \theta_{i2}^N, -\sqrt{2} \theta_{i1}^L],$$

$$\Lambda_{ai}^{l5} = \frac{g}{\sqrt{2}} [t_w \theta_{i1}^N - \theta_{i2}^N, -\sqrt{2} \theta_{i1}^R].$$

In Eqs. (16)–(18), the two components in the brackets correspond to the two states of the  $SU(2)_L$  doublets with indices  $a, b=1,2$ , and  $L^c \equiv (\nu, e^c)$  is defined as a lepton  $SU(2)_L$  doublet while  $\tilde{L}^c = (\tilde{\nu}, \tilde{e}^c)$  is its scalar counterpart. Finally, we have

$$\mathcal{L}_{\nu f \tilde{f}} = \Lambda_i^\nu \bar{\nu}_i P_R \left[ \frac{2}{3} u_j \tilde{u}_j^c - \frac{1}{3} d_j \tilde{d}_j^c - e_j \tilde{e}_j^c \right] + \text{H.c.}, \quad (19)$$

where  $\Lambda_i^\nu = \sqrt{2} g' \theta_{i1}^N$ .

As one can see, the above vertices are nonsupersymmetric and  $SU(2)_L$  breaking. But, among various terms in Eqs. (16) and (18), one can separate out the supersymmetric couplings  $\epsilon_i h_j^d$  and  $\epsilon_i h_j^e$  leaving all the vertices depending only on  $\xi_i$  or  $\eta_i$  similarly to the vertices in Eqs. (12)–(15). Then, combining those with the couplings in the superpotential (1), we can define the effective supersymmetric couplings as  $\tilde{\lambda}'_{ijk} = \epsilon_i h_j^d \delta_{jk} + \lambda'_{ijk}$  and  $\tilde{\lambda}_{ijk} = \epsilon_i h_j^e \delta_{jk} + \lambda_{ijk}$ . We will see that these couplings determine the quantity  $\xi_i$  or  $\eta_i$  through the renormalization group evolution of the bilinear (soft) terms.

### III. RADIATIVE NEUTRINO MASS MATRIX FROM R-PARITY VIOLATION

After performing the rotations described in the previous section, the three neutrinos in the “weak basis” get important mass corrections arising from the seesaw mechanism associated with the heavy four neutralinos. As is well known, this gives the “tree-level” neutrino matrix of the form

$$M_{ij}^{\text{tree}} = -\frac{M_Z^2}{F_N} \xi_i \xi_j c_\beta^2, \quad (20)$$

which makes massive only one neutrino  $\nu_3$  in the direction of  $\tilde{\xi}$ . The other two get masses from finite one-loop corrections and thus  $\nu_3$  is usually the heaviest component. We fix

the value of  $m_{\nu_3}$  from the atmospheric neutrino data [8] and thus the overall size of  $\xi \equiv |\tilde{\xi}|$  as

$$\xi c_\beta = 0.74 \times 10^{-6} \left( \frac{F_N}{M_Z} \right)^{1/2} \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right)^{1/2}, \quad (21)$$

where  $F_N$  is defined in Eq. (5) and its typical value is given by  $M_2$ . Furthermore, among three neutrino mixing angles defined by the mixing matrix

$$\begin{aligned} U = & \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \\ & \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (22)$$

with  $c_{ij} = \cos \theta_{ij}$  and  $s_{ij} = \sin \theta_{ij}$ , etc., two angles are almost determined by the tree-level mass matrix (20) as follows:

$$\sin^2 2\theta_{\text{atm}} \approx \sin^2 2\theta_{23} \approx 4 \frac{\xi_2^2 \xi_3^2}{\xi^2},$$

$$\sin^2 2\theta_{\text{chooz}} \approx \sin^2 2\theta_{13} \approx 4 \frac{\xi_1^2}{\xi^2} \left( 1 - \frac{\xi_1^2}{\xi^2} \right). \quad (23)$$

The atmospheric neutrino and CHOOZ experiments [8,10] require  $\sin^2 2\theta_{\text{atm}} \approx 1$  and  $\sin^2 2\theta_{\text{chooz}} < 0.2$ . The angle  $\theta_{12}$  can be determined only after including one-loop corrections and is responsible for the solar neutrino mixing,  $\theta_{\text{sol}} \approx \theta_{12}$ , if the neutrino mass matrix is to explain the atmospheric and solar neutrino oscillations as will be discussed in the next section.

An important property of the tree-level neutrino mass matrix (20) is that it depends on the bilinear  $R$ -parity violating quantities  $\xi_i = a_i - \epsilon_i$  which can be reexpressed as

$$\xi_i = \epsilon_i \frac{\Delta m_i^2 + \Delta B_i \mu t_\beta}{m_{\tilde{\nu}_i}^2} - \frac{m_{L_i H_1}^2}{m_{\tilde{\nu}_i}^2}, \quad (24)$$

where  $\Delta m_i^2 \equiv m_{H_1}^2 - m_{L_i}^2$ ,  $\Delta B_i \equiv B - B_i$ , and  $m_{\tilde{\nu}_i}^2 = m_{L_i}^2 + M_Z^2 c_{2\beta}^2/2$ . In other words, the nonzero values of  $\xi_i$  arise from the mismatch of soft mass parameters for the Higgs field  $H_1$  and slepton field  $L_i$  having the same gauge quantum numbers. If one assumes the universality condition, one has  $\Delta m_i^2 = \Delta B_i = m_{L_i H_1}^2 = 0$  at the mediation scale of supersymmetry breaking, and their nonzero values are generated by Yukawa coupling effects through the renormalization group evolution down to the weak scale.

Under the assumption that the  $R$ -parity violating couplings follow the usual hierarchies as the quark and lepton Yukawa couplings, that is,  $\lambda'_i \equiv \lambda'_{i33}$  and  $\lambda_i \equiv \lambda_{i33}$  give the dominant contributions, the renormalization group equations (RGEs) of the bilinear terms are given by



$$16\pi^2 \frac{d}{dt} \Delta m_i^2 = 6h_b^2 X_b + 2(1 - \delta_{i3})h_\tau^2 X_\tau,$$

$$16\pi^2 \epsilon_i \frac{d}{dt} \Delta B_i = \epsilon_i (6h_b^2 A_b + 2(1 - \delta_{i3})h_\tau^2 A_\tau) + (6\lambda_i' h_b A_i' + 2\lambda_i h_\tau A_i) - \Delta B_i (3\lambda_i' h_b + \lambda_i h_\tau), \quad (25)$$

$$16\pi^2 \frac{d}{dt} m_{L_i H_1}^2 = -(6\lambda_i' h_b X_b + 2\lambda_i h_\tau X_\tau) + m_{L_i H_1}^2 [3h_b^2 + (1 + \delta_{i3})h_\tau^2] + (6\lambda_i' h_b + 2\lambda_i h_\tau) \Delta m_i^2 - (6\lambda_i' h_b A_b \Delta A_i' + 2\lambda_i h_\tau A_\tau \Delta A_i),$$

where  $t = \ln Q$  with the renormalization scale  $Q$ ,  $X_b = m_{Q_3}^2 + m_{D_3^c}^2 + m_{H_1}^2 + A_b^2$ ,  $X_\tau = m_{L_3}^2 + m_{E_3^c}^2 + m_{H_1}^2 + A_\tau^2$ . Here,  $A$ 's are the trilinear soft parameters corresponding to the  $h_b, h_\tau, \lambda_i'$ , and  $\lambda_i$  couplings, and finally  $\Delta A_i' \equiv A_i' - A_b$ ,  $\Delta A_i \equiv A_i - A_\tau$ . Under the one-step approximation, the above RGE can be solved as

$$\epsilon_i \Delta m_i^2 - m_{L_i H_1}^2 = \frac{1}{8\pi^2} (3\tilde{\lambda}_i' h_b X_b + \tilde{\lambda}_i h_\tau X_\tau) \ln \frac{M_m}{m_\tau},$$

$$\epsilon_i \Delta B_i = \frac{1}{8\pi^2} (3\tilde{\lambda}_i' h_b \tilde{A}_i' + \tilde{\lambda}_i h_\tau \tilde{A}_i) \ln \frac{M_m}{m_\tau}, \quad (26)$$

where  $M_m$  is the mediation scale of supersymmetry breaking and  $m_\tau$  is a typical stop mass scale where we calculate the sneutrino vacuum expectation values. Here, we have defined  $\tilde{\lambda}_i' = \epsilon_i h_b + \lambda_i'$ ,  $\tilde{\lambda}_i = \epsilon_i (1 - \delta_{i3}) h_\tau + \lambda_i$ ,  $\tilde{\lambda}_i' \tilde{A}_i' = \epsilon_i h_b A_b + \lambda_i' A_i'$ , and  $\tilde{\lambda}_i \tilde{A}_i = \epsilon_i (1 - \delta_{i3}) h_\tau A_\tau + \lambda_i A_i$ . Note that  $\lambda_3$  as well as  $\tilde{\lambda}_3$  vanish. In gauge mediated supersymmetry breaking models where the mediation scale  $M_m$  is low, the above approximate solution is quite reliable.

We are ready to discuss the typical sizes of the supersymmetric bilinear and trilinear parameters  $\epsilon_i$  and  $\lambda_i'$ ,  $\lambda_i$  (or  $\tilde{\lambda}_i'$ ,  $\tilde{\lambda}_i$ ) which will be relevant for the study of the LSP decay. Assuming that there is no fine cancellation among various terms in Eq. (24) and the term with  $X_b$  gives the largest contribution in Eq. (26), we obtain

$$\frac{\xi_i c_\beta}{\tilde{\lambda}_i'} \sim \frac{3}{8\pi^2} \frac{m_b}{v} \frac{X_b}{m_{\tilde{\nu}_i}^2} \ln \frac{M_m}{m_\tau}. \quad (27)$$

In gauge mediated supersymmetry breaking models [11], the sfermion soft masses are determined by gauge-boson/gaugino loop corrections which implies  $X_b/m_{\tilde{\nu}_i}^2 \approx 2\alpha_3^2/\alpha_2^2$ . Further assuming the supersymmetry breaking scale  $\Lambda_S$  close to  $M_m$ , we take  $M_m/m_\tau \approx 4\pi/\alpha_3$ . This gives

$$\epsilon_i h_b \text{ or } \lambda_i' \sim 20 \xi_i c_\beta. \quad (28)$$

When the tree mass matrix gives the atmospheric neutrino mass scale as discussed, we get  $\xi_i c_\beta \sim 10^{-6}$  and thus

$$\epsilon_i \sim 20 \xi_i c_\beta / h_b \sim 2 \times 10^{-3} / t_\beta \quad (29)$$

for  $F_N = M_Z$ . This shows that the parameters  $\epsilon_i$  and  $a_i$  can be very large while maintaining  $\xi_i = a_i - \epsilon_i$  very small for low  $\tan \beta$ .

Let us consider another possibility that  $\tilde{\lambda}_i$  gives dominant contribution in Eq. (26). Following the similar steps as above, we obtain

$$\epsilon_i h_\tau \text{ or } \lambda_i \sim 560 \xi_i c_\beta, \quad (30)$$

where we took  $X_\tau = 3m_{\tilde{\nu}_i}^2$ . This implies that the contribution of  $\lambda_i$  to  $\xi_i$  is comparable to that of  $\lambda_i'$  if  $\lambda_i \sim 30\lambda_i'$ . Later, we will see that the large mixing angle explaining the solar neutrino data can be obtained for  $\lambda_{1,2} \sim 5\lambda_{2,3}'$ .

So far, we neglected the radiative corrections in the determination of vacuum expectation values of the sneutrino as well as Higgs fields. To obtain reliable minimization conditions for the electroweak symmetry breaking, one has to consider the effective scalar potential

$$V_{\text{eff}} = V_0 + V_1,$$

where  $V_0$  is the tree-level potential and  $V_1 = (1/64\pi^2) \text{Str } \mathcal{M}^4 [\ln(\mathcal{M}^2/Q^2) - \frac{3}{2}]$  includes one-loop corrections. With  $R$ -parity violation,  $V_1$  is a function of not only the Higgs fields but also the sneutrinos [12,14,15]. In deriving Eq. (24), we neglected  $V_1$  and used the tree-level minimization conditions for the neutral Higgs and sneutrino fields. Since the nonzero  $\xi_i$ 's are also generated by the renormalization effect, the inclusion of  $V_1$  in the determination of sneutrino vacuum expectation values is crucial, in particular, in the case of a low-scale supersymmetry breaking mediation. In gauge mediation models, such one-loop corrections can give rise to an order of magnitude change in neutrino mass-squared values which are well measured in the atmospheric and solar neutrino experiments [12]. After including such effects, Eq. (24) is modified to

$$\xi_i = + \epsilon_i \frac{\Delta m_i^2 + \Delta B_i \mu t_\beta}{m_{\tilde{\nu}_i}^2 + \Sigma_{L_i}^{(2)}} - \frac{m_{L_i H_1}^2}{m_{\tilde{\nu}_i}^2 + \Sigma_{L_i}^{(2)}} + \epsilon_i \frac{\Sigma_{H_1} - \Sigma_{L_i}^{(2)} - \epsilon_i^{-1} \Sigma_{L_i}^{(1)}}{m_{\tilde{\nu}_i}^2 + \Sigma_{L_i}^{(2)}}, \quad (31)$$

where  $\Sigma_{H_1} = \partial V_1 / H_1^* \partial H_1$ ,  $\Sigma_{L_i}^{(1)} = \partial V_1 / H_1^* \partial L_i$ , and  $\Sigma_{L_i}^{(2)} = \partial V_1 / L_i^* \partial L_i$  [12]. In our numerical calculation in the following section, we include such improvements.

In order to get the full neutrino mass matrix, one-loop radiative corrections to the neutrino mass matrix should be included:

$$M_{ij}^\nu = M_{ij}^{\text{tree}} + M_{ij}^{\text{loop}}.$$

Below, we will discuss whether the above mass matrix  $M^\nu$  can explain the atmospheric and solar neutrino data, simultaneously. A numerical calculation in this direction was first

performed in Ref. [16] without including the effect of the one-loop effective potential  $V_1$  in the context of minimal supergravity models. Inclusion of full one-loop corrections has been made in Refs. [12] and [15], in gauge mediation and supergravity models, respectively. As mentioned above, the one-loop mass matrix  $M^{\text{loop}}$  lifts twofold degeneracy of the tree-level mass matrix and thus all the three neutrinos get masses which are generically hierarchical. It is instructive to compare the tree and loop mass components, in order to get an idea about the mass of the neutrino  $\nu_2$  which determine the solar neutrino mass scale. The largest contribution to  $M^{\text{loop}}$  usually comes from the one-loop diagrams with  $\lambda'_i$  and  $\lambda_i$  (more generally with the induced ones  $\tilde{\lambda}'_i$  and  $\tilde{\lambda}_i$ ) which takes the form

$$M_{ij}^{\text{loop}} = 3 \frac{\tilde{\lambda}'_i \tilde{\lambda}'_j}{8\pi^2} \frac{m_b^2 (A_b + \mu t_\beta)}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \ln \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2} + \frac{\tilde{\lambda}_i \tilde{\lambda}_j}{8\pi^2} \frac{m_\tau^2 (A_\tau + \mu t_\beta)}{m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2} \ln \frac{m_{\tilde{\tau}_1}^2}{m_{\tilde{\tau}_2}^2}, \quad (32)$$

where  $m_{\tilde{b}_i}$  and  $m_{\tilde{\tau}_i}$  are the sbottom and stau mass eigenvalues, respectively.

When  $\tilde{\lambda}'_i > \tilde{\lambda}_i$  so that  $\tilde{\lambda}'_i$  give dominant contributions to both  $\xi_i$  and  $M^{\text{loop}}$ , one has

$$\frac{M_{ij}^{\text{loop}}}{M_{ij}^{\text{tree}}} \sim \frac{3}{8\pi^2} \frac{\tilde{\lambda}'_i \tilde{\lambda}'_j}{\xi_i \xi_j c_\beta^2} \frac{m_b^2 \mu t_\beta}{M_Z m_{\tilde{b}}^2} \sim 10^{-3} t_\beta \quad (33)$$

assuming the relation (28) and  $2.5\mu = m_{\tilde{b}} = 500$  GeV. In this case, the second neutrino mass eigenvalue is determined by the subleading contribution of  $\tilde{\lambda}_i$  to either  $\xi_i$  or  $M^{\text{loop}}$ . Thus, we expect  $m_{\nu_2}/m_{\nu_3} < 10^{-3} t_\beta$ , or, equivalently,

$$\frac{\Delta m_{21}^2}{\Delta m_{32}^2} < 10^{-6} t_\beta^2. \quad (34)$$

Note that the solar neutrino experiments require  $\Delta m_{21}^2 = 10^{-5} - 10^{-10}$  eV<sup>2</sup> depending on the type of solar neutrino oscillation solutions. As the atmospheric neutrino oscillation requires  $\Delta m_{32}^2 \approx 3 \times 10^{-3}$  eV<sup>2</sup>, it would be much easier to get the so-called vacuum oscillation or the low  $\Delta m^2$  MSW solution. To realize the large mixing (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution which is now strongly favored by the recent SNO data [9], a large  $\tan \beta$  is needed. Such a tendency has also been observed by numerical calculations in the context of minimal supergravity models [16,15]. However, under the assumption that  $\tilde{\lambda}'_i > \tilde{\lambda}_i$ , it is impossible to get a large mixing angle for the solar neutrino oscillation due to the CHOOZ constraint. This will become clear when we discuss the bilinear model in the next section.

Let us now consider the opposite case that  $\tilde{\lambda}_i \gg \tilde{\lambda}'_i$  so that  $\tilde{\lambda}_i$  give dominant contributions to  $\xi_i$  and  $M^{\text{loop}}$  [which is the case when  $\tilde{\lambda}_i > 30\tilde{\lambda}'_i$  as in Eq. (30)], one finds

$$\frac{M_{ij}^{\text{loop}}}{M_{ij}^{\text{tree}}} \sim \frac{1}{8\pi^2} \frac{\tilde{\lambda}_i \tilde{\lambda}_j}{\xi_i \xi_j c_\beta^2} \frac{m_\tau^2 \mu t_\beta}{M_Z m_{\tilde{\tau}}^2} \sim t_\beta \quad (35)$$

for  $\mu = 1.5m_{\tilde{\tau}} = 200$  GeV. Note that  $m_{\nu_2}/m_{\nu_3}$  can be even larger than one and the resultant neutrino mass components satisfy  $M_{11,12,22}^\nu > M_{i3}^\nu$  as  $\tilde{\lambda}_3 \equiv 0$ . Such a case is not favorable as it cannot give a large mixing angle for the the atmospheric neutrino oscillation. From the above discussion, we can infer that the atmospheric and LMA solar neutrino oscillation can be realized if the couplings  $\tilde{\lambda}_i$  and  $\tilde{\lambda}'_i$  satisfy a relation in-between Eqs. (33) and (35), which will be the case where  $\lambda_i$  is moderately larger than  $\lambda'_i$ .<sup>2</sup> We will analyze the neutrino masses and mixing in such a scheme and its collider signature in the following section.

#### IV. ATMOSPHERIC AND SOLAR NEUTRINO OSCILLATIONS AND LSP DECAYS IN GMSB MODELS

Let us make a numerical analysis to find how the neutrino mass matrix from  $R$ -parity violation explain both the atmospheric and solar neutrino oscillations and what are the corresponding collider signatures coming from the LSP decay. Our discussions are specialized in the models with gauge mediated supersymmetry breaking (GMSB) in which the universality condition is automatic and thus supersymmetric flavor problems are naturally avoided. For our discussion, we will take the minimal number of the messenger multiplets  $(5 + \bar{5})$  and the messenger supersymmetry breaking scale  $\Lambda_S$  not too far from the mediation scale  $M_m$  [11]. We will also concentrate on the cases where the LSP (being a neutralino) is lighter than the  $W$  boson so that only three-body decays are allowed.

Due to the (effective)  $R$ -parity violating couplings introduced in the previous sections, the LSP, denoted by  $\tilde{\chi}_1^0$ , decays through the mediation of on or off-shell  $W, Z$  gauge bosons, Higgs bosons, sleptons, and squarks, producing the following three-fermion final states:

$$\nu\nu\nu, \quad \nu l_i^\pm l_j^\mp, \quad \nu q \bar{q}', \quad t_i^\pm q q'.$$

Here we do not distinguish the neutrino flavors, and the final quark states will be identified with jets. The modes  $\nu l_i^\pm l_j^\mp$  and  $t_i^\pm jj$  are of a particular interest since the flavor dependence of  $R$ -parity violating couplings, which are relevant to the neutrino mixing angles, will be encoded in their branching ratios.

Before performing the numerical analysis, let us make some qualitative discussions on the LSP decay. When the LSP is heavier than the  $W$  boson, the decay modes  $\tilde{\chi}_1^0 \rightarrow l_i^\pm W^\mp$  will have sizable branching fractions [5,6] and measuring them will give a direct information on the ratios  $\xi_1^2 : \xi_2^2 : \xi_3^2$  from which we can probe the neutrino mixing

<sup>2</sup>The LMA solution may also be obtained in the bilinear model if one relaxes the universality condition [15,17].

angles  $\theta_{23}$  and  $\theta_{13}$  through Eq. (23) [6]. The decay rate of the mode  $\tilde{\chi}_1^0 \rightarrow l_i W$  is given by [6]

$$\Gamma(l_i W) = \frac{G_F m_{\tilde{\chi}_1^0}^3}{4\sqrt{2}\pi} \left[ |C_1^L|^2 + \left| C_1^R \frac{m_i^e}{F_C} \right|^2 \right] |\xi_i|^2 c_\beta^2 I_2(M_W^2/m_{\tilde{\chi}_1^0}^2) \quad (36)$$

$$\text{with } C_1^L = \frac{1}{\sqrt{2}} [N_{11} c_1^N + N_{12} (c_2^N - \sqrt{2} c_1^L) + N_{13} (c_3^N - c_2^L) + N_{14} c_4^N]$$

$$C_1^R = N_{12} c_1^R + \frac{1}{\sqrt{2}} N_{14} c_2^R.$$

Here,  $N_{1j}$  are the components of the neutralino diagonalization matrix for the LSP and  $I_2(x) = (1-x)^2(1+2x)$ . Taking  $\xi_i c_\beta = 10^{-6}$ ,  $C_1^L = 1$ , and  $m_{\tilde{\chi}_1^0} = M_Z$ , we get  $\Gamma(IW) \approx 10^{-14}$  GeV corresponding to the decay length  $\tau \approx 2$  cm. Thus, the measurement of  $\text{BR}(eW)$ :  $\text{BR}(\mu W)$ :  $\text{BR}(\tau W)$ , or  $\xi_1^2$ :  $\xi_2^2$ :  $\xi_3^2$ , will be certainly feasible in the future colliders. Note that the contribution  $C_1^R m_\tau / F_C \sim m_\tau t_\beta / \mu$  can be neglected unless  $\tan \beta$  is very large.

If the LSP is lighter than the  $W$  boson, only three-body decay modes are allowed and thus the desired decay modes may be too suppressed to be observed. As a comparison with the above two body decay, let us consider the process  $\tilde{\chi}_1^0 \rightarrow l_i W^* \rightarrow l_i f f'$  whose decay rate is

$$\Gamma(l_i W^*) = \frac{3G_F^2 m_{\tilde{\chi}_1^0}^5}{64\pi^3} \left[ |C_1^L|^2 + \left| C_1^R \frac{m_i^e}{F_C} \right|^2 \right] |\xi_i|^2 c_\beta^2 I_3(m_{\tilde{\chi}_1^0}^2/M_W^2), \quad (37)$$

where  $I_3(x) = [12x - 6x^2 - 2x^3 + 12(1-x)\ln(1-x)]/x^4$ . This gives  $\Gamma(l_i W^*) \approx 8 \times 10^{-17}$  GeV for  $C_1^L = 1$ ,  $\xi_i c_\beta = 10^{-6}$ , and  $m_{\tilde{\chi}_1^0} = 50$  GeV. If this is the dominant decay channel, the total decay length will be  $\tau \sim 2.5$  m making it hard to observe sufficient LSP decay signals. However, it will turn out that the dominant LSP decay diagrams involve the effective  $\tilde{\lambda}_i'$  or  $\tilde{\lambda}_i$  couplings which make the LSP decay well inside the detector. This can be understood from the previous discussions showing that  $\tilde{\lambda}_i', \tilde{\lambda}_i \gg \xi_i c_\beta$ . Furthermore, the corresponding decay modes  $\tilde{\chi}_1^0 \rightarrow \nu j j$  or  $\nu l_i^\pm l_j^\mp$  are dominated by the diagrams with the exchange of the sneutrino or charged slepton (in particular, the right-handed stau) which are relatively light. To get an order of magnitude estimation, let us consider the decay rate for  $\tilde{\chi}_1^0 \rightarrow \nu l_i^\pm l_j^\mp$

$$\Gamma(\nu l_i^\pm l_j^\mp) = \frac{\alpha' \tilde{\lambda}_{kij}^2 m_{\tilde{\chi}_1^0}^5}{768\pi^2 m_{e_j^c}^4} |N_{11}|^2 J(m_{\tilde{\chi}_1^0}^2, m_{\nu_i}^2, m_{e_j^c}^2, m_{e_k^c}^2), \quad (38)$$

where  $\alpha' = g'^2/4\pi$  and  $J$  is a order-one function of the particle masses which is normalized to be one in the limit of

$m_{\tilde{\chi}_1^0}^2 = m_{e_k^c}^2 = 0$ . Taking  $\lambda_{i33} = 2 \times 10^{-5}$ ,  $m_{\tilde{\chi}_1^0} = 50$  GeV,  $m_{e_j^c} = 70$  GeV, and  $J = 1$ , we get  $\Gamma(\nu_i \tau \tau) \approx \Gamma(\nu_i l_i^\pm \tau^\mp) \approx 10^{-14}$  GeV which corresponds to  $\tau \sim 2$  cm. As we will see, this is a typical order of magnitude for the total decay rate of the LSP when  $R$ -parity violation accounts for the atmospheric and solar neutrino masses and mixing.

Let us now present our numerical results for the two possible schemes of  $R$ -parity violation: (i) the bilinear model which has only three input parameters  $\epsilon_i$  and (ii) the trilinear model where we introduce five input parameters  $\lambda_i'$  and  $\lambda_i$ .

*Bilinear model: a scheme for the SMA solution.* The bilinear model with the universality condition is known to accommodate only the small mixing angle solution (SMA) of solar neutrino oscillations [12,15], which is now strongly disfavored by the recent SNO data [9]. This model is an attractive option as it is the minimal  $R$ -parity violating model and provides fairly neat correlations between the neutrino oscillation parameters and the collider signatures. In this scheme, the effective trilinear couplings are given by  $\tilde{\lambda}_i' = \epsilon_i h_b$  and  $\tilde{\lambda}_i = \epsilon_i h_\tau$  in Eq. (26) and thus both the tree and the loop mass matrix takes the form  $M_{ij}^\nu \propto \epsilon_i \epsilon_j$ . The other flavor dependence comes from the  $h_b, h_\tau$  Yukawa coupling effects, which is weak unless  $\tan \beta$  is very large. Now that the relation Eq. (33) is applied here, the determination of the overall size of  $\xi$  in Eq. (21) and two mixing angles  $\theta_{23}$  and  $\theta_{13}$  in Eq. (23) holds almost precisely. Thus, the atmospheric and CHOOZ neutrino experiments require  $|\xi_1| \ll |\xi_2| \approx |\xi_3|$  which can be directly translated to the condition  $|\epsilon_1| \ll |\epsilon_2| \approx |\epsilon_3|$ . This leads to the neutrino mass matrix structure  $M_{11}^\nu < M_{12}^\nu < M_{22,23,33}^\nu$ . As a consequence, only a small mixing angle for the solar neutrino oscillation can be accounted for in the bilinear model. Indeed, the solar mixing angle  $\theta_{12}$  is almost fixed by the relation  $\tan \theta_{12} \approx \tan \theta_{13}$ , and thus we get the relation  $\sin^2 2\theta_{\text{sol}} \approx \sin^2 2\theta_{\text{chooz}}$ .

Given  $\Delta m_{32}^2 \approx 3 \times 10^{-3}$  eV<sup>2</sup> for the atmospheric neutrino oscillation, Eq. (34) tells us that  $\Delta m_{21}^2 < 3 \times 10^{-9} t_\beta^2$  eV<sup>2</sup>. This estimation is by no means exact but can show some qualitative features. For instance, it implies that the right value of  $\Delta m_{21}^2 \sim 5 \times 10^{-6}$  eV<sup>2</sup> is hardly achieved with small  $\tan \beta$  in the GMSB models under consideration. In our numerical calculation, we looked for the SMA solutions varying the parameters  $\tan \beta$ ,  $\Lambda_S$ , and  $M_m$  as well as two  $R$ -parity violating parameters  $\epsilon_1^0$  and  $\epsilon_2^0$  defined at the scale  $M_m$  while keeping  $\epsilon_2^0 = \epsilon_3^0$ . We could find a reasonable parameter space only for  $\tan \beta \approx 10-25$ , limiting ourselves to  $\tan \beta < 25$  because the (right-handed) stau becomes the LSP for larger  $\tan \beta$ .

In Tables I and II, we present two typical sets of parameters accommodating the SMA solution and the other neutrino data. In the tables,  $\epsilon_i^0$  denotes input values set at the scale  $M_m$  where supersymmetry breaking is mediated. As can be seen, the effective couplings  $\tilde{\lambda}_i'$  and  $\tilde{\lambda}_i$  are much larger than  $\xi_i c_\beta$  and the dominant decay modes are  $\nu j j$  and  $\nu l l$  where the diagrams with the exchanges of sneutrinos and charged sleptons give main contributions. One of its consequences is that the total decay rate is larger than  $10^{-14}$  GeV



TABLE I. A bilinear model with the input parameters  $\tan\beta$ ,  $\Lambda_S$ ,  $M_m$ , and  $\epsilon_i^0$ , allowing for the SMA solution. The values of  $\epsilon_i^0$  are set at the mediation scale  $M_m$ . The effective trilinear/bilinear  $R$ -parity violating parameters  $\tilde{\lambda}'_i$ ,  $\tilde{\lambda}_i/\xi_i^{(0)}$  defined in the text, are shown in the upper part. Here,  $\xi_i^0$  and  $\xi_i$  are the tree-level and one-loop improved values, respectively. In the lower part are shown the important branching ratios of the LSP with mass  $m_{\tilde{\chi}_1^0}$  and its total decay rate  $\Gamma$ . The three columns correspond to the lepton flavors  $i=e, \mu$ , and  $\tau$ , respectively. For the mode  $\nu jj$ , we do not distinguish the neutrino flavors. The resulting neutrino oscillation parameters are presented in the last two lines.  $[(\Delta m_{31}^2, \Delta m_{21}^2) = (2.5 \times 10^{-3}, 6.1 \times 10^{-6}) \text{ eV}^2 (\sin^2 2\theta_{\text{atm}}, \sin^2 2\theta_{\text{sol}}, \sin^2 2\theta_{\text{chooz}}) = (0.99, 0.0018, 0.0017).]$

Set 1	$\tan\beta=10$	$\Lambda_S=40 \text{ TeV}$	$M_m=150 \text{ TeV}$
$\epsilon_i^0$	$7.53 \times 10^{-6}$	$2.51 \times 10^{-4}$	$2.51 \times 10^{-4}$
$\tilde{\lambda}'_i$	$1.40 \times 10^{-6}$	$4.68 \times 10^{-5}$	$4.68 \times 10^{-5}$
$\tilde{\lambda}_i$	$7.60 \times 10^{-7}$	$2.53 \times 10^{-5}$	0
$\xi_i^0$	$2.40 \times 10^{-7}$	$8.01 \times 10^{-6}$	$7.86 \times 10^{-6}$
$\xi_i$	$1.81 \times 10^{-7}$	$6.06 \times 10^{-6}$	$6.63 \times 10^{-6}$
BR	$e$	$\mu$	$\tau$
$\nu jj$		$3.58 \times 10^{-1}$	
$l_i^\pm jj$	$1.10 \times 10^{-6}$	$1.22 \times 10^{-3}$	$1.18 \times 10^{-3}$
$\nu l_i^\pm \tau^\mp$	$4.56 \times 10^{-4}$	$3.12 \times 10^{-1}$	$3.27 \times 10^{-1}$
$m_{\tilde{\chi}_1^0}=49 \text{ GeV}$		$\Gamma=7.18 \times 10^{-14} \text{ GeV}$	

making the decay length smaller than a few cm. We have checked that the total decay rate is in the region of  $\Gamma \sim 5 \times 10^{-14} \text{ GeV}$  for the LSP mass  $m_{\tilde{\chi}_1^0}=25-80 \text{ GeV}$ . That is, the decay length is around  $\tau \sim 0.4 \text{ cm}$ . This has to be contrasted with the supergravity case [7] where one typically gets  $\tau > 1 \text{ cm}$  for a light LSP. It is also worthwhile to note that the SMA solution requires  $\xi_1/\xi_2 \approx 0.03 \approx \epsilon_1/\epsilon_2$  and thus the bilinear model typically predicts the following relation:

$$10^3 \text{ BR}(\nu e^\pm \tau^\mp) \sim \text{BR}(\nu \mu^\pm \tau^\mp) \approx \text{BR}(\nu \tau^\pm \tau^\mp). \quad (39)$$

TABLE II. Same as in Table I but with  $\tan\beta=25$ .  $[(\Delta m_{31}^2, \Delta m_{21}^2) = (3.0 \times 10^{-3}, 4.1 \times 10^{-6}) \text{ eV}^2, (\sin^2 2\theta_{\text{atm}}, \sin^2 2\theta_{\text{sol}}, \sin^2 2\theta_{\text{chooz}}) = (0.99, 0.0019, 0.0017).]$

Set 2	$\tan\beta=25$	$\Lambda_S=45 \text{ TeV}$	$M_m=90 \text{ TeV}$
$\epsilon_i^0$	$8.94 \times 10^{-7}$	$2.98 \times 10^{-5}$	$2.98 \times 10^{-5}$
$\tilde{\lambda}'_i$	$4.25 \times 10^{-7}$	$1.42 \times 10^{-5}$	$1.42 \times 10^{-5}$
$\tilde{\lambda}_i$	$2.30 \times 10^{-7}$	$7.67 \times 10^{-6}$	0
$\xi_i^0$	$4.36 \times 10^{-7}$	$1.45 \times 10^{-5}$	$1.45 \times 10^{-5}$
$\xi_i$	$4.04 \times 10^{-7}$	$1.35 \times 10^{-5}$	$1.28 \times 10^{-5}$
BR	$e$	$\mu$	$\tau$
$\nu jj$		$1.11 \times 10^{-1}$	
$l_i^\pm jj$	$1.55 \times 10^{-6}$	$1.72 \times 10^{-3}$	$1.72 \times 10^{-3}$
$\nu l_i^\pm \tau^\mp$	$6.22 \times 10^{-4}$	$4.07 \times 10^{-1}$	$4.07 \times 10^{-1}$
$m_{\tilde{\chi}_1^0}=58 \text{ GeV}$		$\Gamma=4.30 \times 10^{-14} \text{ GeV}$	

TABLE III. A trilinear model realizing the LMA solution. Here the couplings  $\tilde{\lambda}'_i$  and  $\tilde{\lambda}_i$  can be considered as input parameters defined at the weak scale. The rests are the same as in the previous tables.  $[(\Delta m_{31}^2, \Delta m_{21}^2) = (3.1 \times 10^{-3}, 5.0 \times 10^{-5}) \text{ eV}^2, (\sin^2 2\theta_{\text{atm}}, \sin^2 2\theta_{\text{sol}}, \sin^2 2\theta_{\text{chooz}}) = (0.99, 0.80, 0.0001).]$

Set 3	$\tan\beta=5$	$\Lambda_S=40 \text{ TeV}$	$M_m=80 \text{ TeV}$
$\tilde{\lambda}'_i$	$1.07 \times 10^{-7}$	$1.07 \times 10^{-5}$	$0.96 \times 10^{-5}$
$\tilde{\lambda}_i$	$4.07 \times 10^{-5}$	$4.07 \times 10^{-5}$	0
$\xi_i^0$	$2.66 \times 10^{-7}$	$2.99 \times 10^{-6}$	$2.48 \times 10^{-6}$
$\xi_i$	$2.57 \times 10^{-7}$	$2.67 \times 10^{-6}$	$2.66 \times 10^{-6}$
BR	$e$	$\mu$	$\tau$
$\nu jj$		$7.92 \times 10^{-3}$	
$l_i^\pm jj$	$5.17 \times 10^{-7}$	$6.55 \times 10^{-5}$	$4.49 \times 10^{-5}$
$\nu l_i^\pm \tau^\mp$	$2.35 \times 10^{-1}$	$2.35 \times 10^{-1}$	$5.22 \times 10^{-1}$
$m_{\tilde{\chi}_1^0}=46 \text{ GeV}$		$\Gamma=1.20 \times 10^{-13} \text{ GeV}$	

As pointed out in Ref. [7], the modes  $l_i jj$  are of a great interest. Their decay rates are dominated by the  $W$  exchange diagrams as the contribution of the largest coupling  $\tilde{\lambda}'_{i33}$  giving  $\tilde{\chi}_1^0 \rightarrow l_i t \bar{b}$  is kinematically forbidden and the coupling  $\tilde{\lambda}'_{i22} = \epsilon_i h_s$  gives the subleading effect compared to  $\xi_i c_\beta$  which enters the  $\chi$ - $l$ - $W$  vertices. Therefore, the ratio

$$\text{BR}(ejj):\text{BR}(\mu jj):\text{BR}(\tau jj) \quad (40)$$

is almost the same as the ratio  $\xi_1^2:\xi_2^2:\xi_3^2$  to determine  $\theta_{\text{atm}}$  and  $\theta_{\text{chooz}}$  through Eq. (23) as in the case of  $m_{\tilde{\chi}_1^0} > M_W$ . Here, we remark that the branching fraction  $\text{BR}(ejj)$  is too small to be observed in the future linear colliders. Assuming the integrated luminosity  $1000 \text{ fb}^{-1}$  per year, the branching ratios below  $10^{-5}$  would not be feasible [7]. However, the measurement of  $\text{BR}(ejj) \ll \text{BR}(\mu jj) \approx \text{BR}(\tau jj)$  will provide a robust test for the bilinear model.

*Trilinear model: a scheme for the LMA solution.* Let us now consider a more general situation that both bilinear and trilinear  $R$ -parity violating terms are present. In this case, it is convenient to rotate away the supersymmetric bilinear terms  $\epsilon_i$  to the trilinear couplings as we defined the effective ones in the previous sections. In this way, we are allowed to introduce only five couplings,  $\tilde{\lambda}'_i$  and  $\tilde{\lambda}_i$  which are related to the third generation quarks and leptons. This would be the simplest trilinear model.

The trilinear model provides the possibility to realize the LMA solution which is the most favored at present. As discussed before, in order to get the LMA solution, sizable contributions to  $M_{11,12,22}^\nu$  are needed to enlarge the solar neutrino mixing while keeping the hierarchy of  $M_{ij} < M_{i3,33}$  to realize the large atmospheric neutrino mixing. From the numerical calculation scanning the five trilinear parameters, we find that the LMA solution is realized if  $\tilde{\lambda}_{1,2} \sim 5\tilde{\lambda}'_{2,3}$ . The conditions  $\tilde{\lambda}_1 \sim \tilde{\lambda}_2$  and  $\tilde{\lambda}'_2 \sim \tilde{\lambda}'_3$  are needed to get two large mixing angles, while the small CHOOZ angle requires  $\tilde{\lambda}'_1$

TABLE IV. Same as in Table III with  $\tan\beta=25$ . [ $(\Delta m_{31}^2, \Delta m_{21}^2)=(3.0\times 10^{-3}, 5.0\times 10^{-5})$  eV<sup>2</sup>,  $(\sin^2 2\theta_{\text{atm}}, \sin^2 2\theta_{\text{sol}}, \sin^2 2\theta_{\text{chooz}})=(0.91, 0.84, 0.16)$ .]

Set 4	$\tan\beta=25$	$\Lambda_S=40$ TeV	$M_m=80$ TeV
$\lambda'_i$	$7.45\times 10^{-8}$	$4.48\times 10^{-6}$	$7.43\times 10^{-6}$
$\lambda_i$	$1.61\times 10^{-5}$	$2.82\times 10^{-5}$	0
$\xi_i^0$	$4.32\times 10^{-7}$	$7.09\times 10^{-6}$	$1.09\times 10^{-5}$
$\xi_i$	$1.62\times 10^{-6}$	$1.02\times 10^{-5}$	$1.28\times 10^{-5}$
BR	$e$	$\mu$	$\tau$
$\nu jj$		$1.54\times 10^{-3}$	
$l_i^\pm jj$	$7.08\times 10^{-8}$	$1.91\times 10^{-5}$	$4.53\times 10^{-5}$
$\nu l_i \bar{l}_3$	$1.15\times 10^{-1}$	$3.53\times 10^{-1}$	$5.31\times 10^{-1}$
$m_{\tilde{\chi}_1^0}=50$ GeV		$\Gamma=5.62\times 10^{-13}$ GeV	

$<0.2\tilde{\lambda}'_{2,3}$ . Under such conditions, we could not find any restrictions on the GMSB input parameters  $\tan\beta$ ,  $\Lambda_S$ , and  $M_m$ .

In Tables III and IV, we present two examples allowing the LMA solution for  $\tan\beta=5$  and 25, respectively. We fixed  $M_m=2\Lambda_S$ . The total decay rate is found to be in the vicinity of  $\Gamma\sim 5\times 10^{-13}$  GeV for all the LSP mass below 80 GeV. Therefore, the decay length is of the order 0.4 mm. This enhancement compared to the bilinear case is due to the largeness of the  $\tilde{\lambda}_i$  couplings which also make the modes  $\nu ll$  dominating. As a consequence, we infer the following distinct feature of the LMA solution:

$$\text{BR}(\nu e \bar{\tau}) \sim \text{BR}(\nu \mu \bar{\tau}) \sim \text{BR}(\nu \tau \bar{\tau}) \quad (41)$$

where the individual branching ratio is larger than 10%. The relation for the atmospheric neutrino mixing angle in Eq. (23) is not as exact as in the bilinear case, but it still holds to a good approximation as can be seen from the tables. On the other hand, the expression for the CHOOZ angle determined from  $\xi_i$ 's is not applicable any more and we cannot draw any conclusive prediction for the value of it. Still, the relation

$\text{BR}(e jj) \ll \text{BR}(\mu jj) \sim \text{BR}(\tau jj)$  holds to a certain degree, but these branching fractions become as small as  $10^{-5}$  making it difficult to be measured in the planned colliders.

## V. CONCLUSION

The supersymmetric standard model without  $R$ -parity is an attractive framework for the neutrino masses and mixing as certain neutrino oscillation parameters can be probed by measuring the decay length and various branching fractions of the neutralino LSP in the future colliders experiments. Taking two simple models of  $R$ -parity violation, the bilinear model with three input parameters and the trilinear model with five parameters, we analyzed the neutrino mass matrix which explains both the atmospheric and solar neutrino data and its consequences on collider searches. One of our basic assumptions is the universality of soft terms for which we considered gauge mediation models of supersymmetry breaking. A notable consequence of such an assumption is that the LSP (lighter than the  $W$  boson) decays mainly through the (effective) trilinear couplings  $\lambda'_i$  and  $\lambda_i$  and its decay length is found to be around  $\tau \sim 0.1$  cm.

The observation of the decay modes  $\nu l_i^\pm l_j^\mp$  and  $l_i^\pm jj$  will be important as they reflect the lepton number violating structure of a certain model. The bilinear model which can accommodate only the SMA solution of the solar neutrino oscillation predicts the relation  $10^3 \text{BR}(\nu e^\pm \tau^\mp) \sim \text{BR}(\nu \mu^\pm \tau^\mp) \approx \text{BR}(\nu \tau^\pm \tau^\mp)$ . The dominant decay modes are found to be  $\tilde{\chi}_1^0 \rightarrow \nu \mu^\pm \tau^\mp$ ,  $\nu \tau^\pm \tau^\mp$ , and  $\nu jj$ , which are all of the order 10%. The trilinear model can realize the strongly favored LMA solution which can be tested by observing the dominant decay modes  $\tilde{\chi}_1^0 \rightarrow \nu e^\pm \tau^\mp$ ,  $\nu \mu^\pm \tau^\mp$ , and  $\nu \tau^\pm \tau^\mp$  satisfying the relation  $\text{BR}(\nu e^\pm \tau^\mp) \sim \text{BR}(\nu \mu^\pm \tau^\mp) \sim \text{BR}(\nu \tau^\pm \tau^\mp)$ . In both cases, the relation,  $\text{BR}(e jj) \ll \text{BR}(\mu jj) \sim \text{BR}(\tau jj)$  should be consistent with the atmospheric and CHOOZ neutrino data.

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