## Neutrino Mass from R-parity Violation in Split Supersymmetry

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## Abstract

We investigate how the observed neutrino data can be accommodated by R-parity violation in Split Supersymmetry. The atmospheric neutrino mass and mixing are explained by the bilinear parameters  $\xi_i$  inducing the neutrino-neutralino mixing as in the usual low-energy supersymmetry. Among various one-loop corrections, only the quark-squark exchanging diagrams involving the order-one trilinear couplings  $\lambda'_{i23,i32}$  can generate the solar neutrino mass and mixing if the scalar mass  $m_S$  is not larger than  $10^9$  GeV. This scheme requires an unpleasant hierarchical structure of the couplings, e.g.,  $\lambda_{i23,i32} \sim 1$ ,  $\lambda'_{i33} \lesssim 10^{-4}$  and  $\xi_i \lesssim 10^{-6}$ . On the other hand, the model has a distinct collider signature of the lightest neutralino which can decay only to the final states,  $l_i W^{(*)}$  and  $\nu Z^{(*)}$ , arising from the bilinear mixing. Thus, the measurement of the ratio;  $\Gamma(eW^{(*)})$ :  $\Gamma(\mu W^{(*)})$ :  $\Gamma(\tau W^{(*)})$  would provide a clean probe of the small reactor and large atmospheric neutrino mixing angles as far as the neutralino mass is larger than 62 GeV.

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The gauge hierarchy problem has been considered as a strong motivation for low-energy supersymmetry, in which all the observable sector soft parameters lie in the TeV scale, ensuring the naturalness of the Higgs mass. However, a rather radical suggestion has been made to abandon the naturalness property and consider a high-scale supersymmetry [1], in which all the scalars are extremely heavy, except for the finely tuned Higgs bosons, and the fermions (including gauginos and Higgsinos) remain light. This approach, dubbed as "Split Supersymmetry" [2], was further advocated by gauge unification and dark matter taken as the guiding principle for new physics. The idea of Split Supersymmetry, being trivially free from the difficulties like flavor-changing neutral current and CP problems and the cosmological gravitino problem, predicts a distinct phenomenology of a long-lived gluino, which could be probed in the future collider or cosmological experiments [3].

In this paper, we wish to investigate the possibility of allowing R-parity violation as the origin of the neutrino masses and mixing [4] in the framework of Split Supersymmetry. Actually the heaviness of the squarks and sleptons allows the large R-parity violation without any difficulty with low energy precision measurements and this interesting feature of Split Supersymmetry is crucially used to produce the proper neutrino masses and mixing. Having abandoned R-parity conservation, the lightest supersymmetric particle is now destabilized, and thus one may have to abandon a nice dark matter candidate, a neutralino. Indeed, the neutralino decays very fast if R-parity violation accounts for the observed neutrino masses not only in the conventional low-energy supersymmetry but also in Split Supersymmetry under discussion [5]. Even though we loose the dark matter as the guiding principle [2], the instability of the lightest supersymmetric particle in the TeV scale neutralino sector could provide another way of probing the idea of Split Supersymmetry combined with the origin of neutrino masses and mixing, as is well known in the conventional framework [6, 7, 8]. As we will see, there are several different features in generating the neutrino mass matrix from R-parity violation in Split Supersymmetry summarized as follows.

- Tree-level neutrino mass matrix pattern, coming from the mixing between neutrinos and neutralinos, is the same as in low-energy supersymmetry. This requires small bilinear parameters;  $\xi_i \lesssim 10^{-6}$ .
- The effect of the slepton–Higgs mixing is negligible as the scalar masses are very high.

  As a result, it is not possible to explain observed neutrino data only by bilinear terms.

- The solar neutrino mass and mixing can only be generated by the usual one-loop diagrams involving down-type quarks and squarks with the trilinear couplings  $\lambda'_{i23}$  and  $\lambda'_{i32}$  of the order one.
- The lightest neutralino can decay only to the gauge bosons,  $W^{\pm(*)}$  and  $Z^{0(*)}$ , through which the tree-level neutrino mass parameters could be probed in the collider experiments for the neutralino mass larger than about 62 GeV.
- The model requires an ad-hoc hierarchical structure of the couplings, namely,  $\lambda'_{i23,i32} \sim 1$ ,  $\lambda'_{i33} \lesssim 10^{-4}$  and  $\lambda_{i33} \lesssim 10^{-3}$ , where the last two constraints come from the limit of the bilinear parameter,  $\xi_i \lesssim 10^{-6}$ .

The last point should be contrasted with the case of the low-energy supersymmetry models, where the neutrino data can well be explained by assuming the usual hierarchy of the trilinear couplings  $\lambda_{ijk}$ ,  $\lambda'_{ijk} \leq \lambda'_{i33}$ ,  $\lambda_{i33} \sim 10^{-4} - 10^{-5}$ , which could be the consequence of a family U(1) symmetry [9].

Let us now start our main discussion by considering first the features of the bilinear R-parity violation in Split Supersymmetry. The gauge invariant bilinear terms in the superpotential and the scalar potential are

$$W = \mu H_1 H_2 + \epsilon_i \mu L_i H_2,$$

$$V = B H_1 H_2 + B_i L_i H_2 + m_{L_i H_1}^2 L_i H_1^{\dagger} + h.c.,$$
(1)

where we have used the same notation,  $H_{1,2}$  and  $L_i$ , for the superfields and their scalar components of the Higgs and lepton doublets. In Split Supersymmetry, the dimension-two soft parameters B or  $B_i$  and  $m_{L_iH_1}^2$  could be of the order  $m_S^2$  or  $\epsilon_i m_S^2$  where the high-scale of the scalar masses is likely to be in the range:  $m_S = 10^9 - 10^{13}$  GeV [1, 2].

From the above potential (1), one finds the following R-parity violating parameter;

$$\xi_i \equiv \frac{\langle \tilde{\nu}_i \rangle}{\langle H_1^0 \rangle} - \epsilon_i = \frac{m_{L_i H_1}^2}{m_{L_i}^2} + \frac{B_i}{m_{L_i}^2} t_\beta - \epsilon_i , \qquad (2)$$

where  $t_{\beta} = \tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$  and  $m_{L_i} \sim m_S$  is the soft mass of the *i*-th slepton. The above parameter  $\xi_i$  determines the well-known mixing between the leptons and the gauginos/Higgsinos giving rise to the tree-level neutrino mass matrix (see Fig. 1);

$$M_{ij}^{\nu(0)} = -\frac{M_Z^2}{F_N} \xi_i \xi_j c_\beta^2 \tag{3}$$

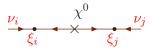


FIG. 1: Tree-level diagrams generating neutrino masses which are induced by the mixing between the neutrinos and gauginos/higgsinos.

where  $F_N \equiv M_1 M_2/(c_W^2 M_1 + s_W^2 M_2) + M_Z^2 s_{2\beta}/\mu$  is the mass parameter deduced from the 4x4 neutralino mass matrix [7]. For the atmospheric neutrino mass scale,  $m_{\nu_3} = \sqrt{\Delta m_{atm}^2} \sim 0.05$  eV, one determines the size of  $\xi \equiv \sqrt{\sum_i |\xi_i|^2}$ :

$$\xi c_{\beta} = 7.4 \times 10^{-7} \left(\frac{F_N}{M_Z}\right)^{\frac{1}{2}} \left(\frac{m_{\nu_3}}{0.05 \text{ eV}}\right)^{\frac{1}{2}}.$$
 (4)

Barring the cancellation among the three terms on the right-hand side of Eq. (2), this implies that each term should not be larger than  $10^{-6}/c_{\beta}$ .

When the mass matrix (3) explains the atmospheric neutrino data, the atmospheric neutrino mixing angle  $\theta_{23}$  and the reactor neutrino mixing angle  $\theta_{13}$  are approximately determined as follows [7]:

$$\sin^2 2\theta_{23} \approx 4 |\hat{\xi}_2|^2 |\hat{\xi}_3|^2$$
  

$$\sin^2 2\theta_{13} \approx 4 |\hat{\xi}_1|^2 (1 - |\hat{\xi}_1|^2)$$
(5)

where  $\hat{\xi}_i$  is the unit vector of  $\xi_i$ :  $\hat{\xi}_i \equiv \xi_i/\xi$ . This feature can be probed in collider experiments as will be discussed later.

While the tree mass matrix (3) can generate only one nonzero mass eigenvalue, presumably  $m_{\nu_3}$ , the second largest mass,  $m_{\nu_2} = \sqrt{\Delta m_{sol}^2} \sim 9$  meV, can be generated from one-loop radiative corrections. Combining these, both the atmospheric and the solar neutrino data can be accommodated in the conventional supersymmetric theories [10, 11]. There is another type of bilinear parameters defined by

$$\eta_i \equiv \xi_i - \frac{B_i}{B},\tag{6}$$

which quantifies the mixing between the sleptons and Higgs bosons and appears in the one-loop diagrams. The parameter  $\eta_i$  can play an important role to produce sizable one-loop

contribution in the usual supersymmetric theories [11]. However, this is not the case in Split Supersymmetry where the one-loop contributions always come with the combination of

$$\eta_i \frac{m_Z^2}{m_{L_i}^2} \tag{7}$$

which becomes very small for  $m_Z \ll m_S$ . Therefore, the bilinear parameter  $\xi_i$  alone cannot explain the observed neutrino data, which excludes the bilinear model as the origin of the neutrino masses and mixing in the context of Split Supersymmetry.

Let us ask whether the inclusion of trilinear R-parity violating can be a viable option for the generation of the desired neutrino mass matrix. The general lepton number violating trilinear terms in the superpotential are

$$W = \lambda'_{ijk} L_i Q_j D_k^c + \lambda_{ijk} L_i L_j E_k^c \tag{8}$$

where we take i < j for  $\lambda_{ijk}$  which is antisymmetric under the exchange,  $i \leftrightarrow j$ . For our discussion, it is important to realize that certain trilinear couplings can radiatively generate the bilinear parameters (2), and thus can be strongly constrained. The most strongly constrained couplings are  $\lambda'_{i33}$  or  $\lambda_{i33}$  which contribute to the renormalization group evolution of the bilinear parameter, e.g.,  $\epsilon_i$  as follows [12]:

$$16\pi^2 \frac{d\epsilon_i}{dt} \sim -3\lambda'_{ijj} h_{d_j} - \lambda_{ijj} h_{e_j} \,. \tag{9}$$

Solving the above equation by one-step approximation, we find

$$\epsilon_i c_\beta = \frac{1}{16\pi^2} \left(3\lambda'_{ijj} \frac{m_{d_j}}{v} + \lambda_{ijj} \frac{m_{e_j}}{v}\right) \ln \frac{M_X}{m_S}.$$
 (10)

For the values of  $M_X = 10^{16}$  GeV and  $m_S = 10^9$  GeV, and the condition for the bilinear parameter  $|\xi_i|c_\beta \approx |\epsilon_i|c_\beta$  (4), we obtain

$$\lambda'_{i11} \frac{m_d}{m_b}, \ \lambda'_{i22} \frac{m_s}{m_b}, \ \lambda'_{i33} \lesssim 10^{-4} \,.$$
 (11)

for  $F_N = M_Z$ . Applying the similar argument, the bounds on the couplings  $\lambda_{ijj}$  are found to be

$$\lambda_{i11} \frac{m_e}{m_\tau}, \ \lambda_{i22} \frac{m_\mu}{m_\tau}, \ \lambda_{i33} \lesssim 7 \times 10^{-4} \,.$$
 (12)

Such small trilinear couplings (11,12) cannot generate a sizable one-loop correction to the neutrino masses explaining the solar neutrino data. As an example, consider the typical

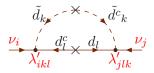


FIG. 2: One-loop diagrams generating sizable neutrino masses from the quark-squark exchange.

one-loop contribution to neutrino masses coming from the quark-squark exchange (see Fig. 2):

$$M_{ij}^{\nu(1)} \approx \frac{3}{8\pi^2} \lambda'_{ikl} \lambda'_{jlk} \frac{m_{d_k} m_{d_l} A_d}{m_S^2}$$
 (13)

where we have taken the squark mass to be  $m_S$ .

Taking  $\lambda'_{i33} = 10^{-4}$  and  $A = m_S = 10^9$  GeV, one finds that the one-loop mass (13) become far below the solar neutrino mass scale  $\sim 9$  meV. The only way to obtain such a sizable neutrino mass scale is to have order-one trilinear couplings avoiding the above constraint (11). It turns out that the unique possibility for this is to allow the couplings

$$\lambda'_{i23}$$
 and  $\lambda'_{i32}$ .

The above couplings can induce the bilinear parameter at two-loop level, which can be viewed as the generation of the induced coupling of  $\lambda'_{i33}$  [12]:

$$\lambda'_{i33}|_{induced} \sim \frac{1}{16\pi^2} \lambda'_{i32} h_t^2 V_{ts} \frac{m_s}{m_b} \quad \text{or} \quad \frac{1}{16\pi^2} \lambda'_{i23} h_t^2 V_{ts}$$
 (14)

where  $V_{ts}$  is the CKM mixing element of quarks. Thus, the couplings  $\lambda'_{i23}$  and  $\lambda'_{i32}$  can be of the order one and can generate the following neutrino mass elements:

$$M_{ij}^{\nu(1)} \approx \frac{3}{8\pi^2} \lambda'_{i23} \lambda'_{j32} \frac{m_b m_s}{m_S} \sim 10^{-2} \text{ eV}$$
 (15)

for  $m_S \sim 10^9$  GeV. For much larger  $m_S$ , the above contribution becomes much too small and cannot be used for our purpose. The elements  $M_{ij}^{\nu(1)}$  with (i, j = 1, 2) in Eq. (15) essentially determine the large solar neutrino mixing angle  $\theta_{12}$  which can be arranged appropriate by the choice of the trilinear couplings;  $\lambda'_{i23}\lambda'_{j32} \sim 1$  for all i and j. We do not bother to present the detailed values for this as it will not be necessary in the following discussions.

Concerning the  $\lambda$  couplings, the leading contributions come from the diagrams in Fig. 3 which do not have the  $1/m_S$  suppression [13]. Given the constraints (12), only the couplings

 $\lambda_{ikl}$  (with  $i \neq k \neq l$ ) can be large enough to generate sizable neutrino mass components. Calculation of the diagrams in Fig. 3 gives us

$$M_{ik}^{\nu(1)} \approx \frac{g}{8\pi^2} \lambda_{ikl} \xi_l c_\beta^2 \frac{m_{e_l} M_W}{\sqrt{2}\mu} \ln \frac{m_{L_i}}{m_{L_k}}$$
 (16)

which is a fairly good approximation in the typical region of parameter space:  $M_2$ ,  $\mu > M_W$  and  $t_{\beta} > 3$ . Comparing this with the tree contributions in Eq. (3), we get the loop-to-tree ratio:

$$\frac{m_{\nu_2}}{m_{\nu_3}} \approx \frac{g}{8\pi^2} \frac{m_{e_l} M_W}{M_Z^2} \frac{\lambda_{ikl} \xi_l}{|\xi|^2} \frac{F_N}{\mu},$$
(17)

which shows that the neutrino mass elements of the order of the solar neutrino mass scale  $m_{\nu_2} \sim 9$  meV can be obtained if the couplings are as large as

$$\lambda_{ikl} \frac{m_{e_l}}{m_{\tau}} \sim 10^3 \xi \sim 10^{-3} t_{\beta} \,.$$
 (18)

From this, one finds that  $\lambda_{231}$  has no effect due to the strong suppression by  $m_e/m_{\tau}$  and thus only sizable  $M_{12}$  and  $M_{13}$  components can be generated from  $\lambda_{123}$  and  $\lambda_{132}$ , respectively. However, this pattern cannot accommodate the solar neutrino mass-squared difference and the mixing angle properly.

Therefore, we conclude that the atmospheric and solar neutrino data can be explained by the combination of the tree-level and the loop contribution given by in Eqs (3) and (15), respectively, in the context of Split Supersymmetry. For this, we need the small bilinear couplings and order-one trilinear couplings:

$$\xi_1 \ll \xi_2 \approx \xi_3 \sim 10^{-6} \quad \text{and} \quad \lambda'_{i32}, \lambda'_{i23} \sim 1,$$
 (19)

and the scalar masses of the order  $m_S \sim 10^9$  GeV. Note that the bilinear parameter  $\xi$  can come from the tree-level input  $\epsilon_i$  of the same order, or from the radiative generation by the trilinear coupling  $\lambda'_{i33}$  of the order  $10^{-4}$  as in Eq. (11).

Split Supersymmetry predicts distinct signatures from the R-parity violating decays of the lightest neutralino,  $\chi_1^0$ , which are different from the conventional supersymmetric theories. Similarly to the gluino case, the neutralino decay processes through squark exchange are highly suppressed. But, the lightest neutralino decay can occur through the bilinear mixing of leptons and neutralinos governed by the parameter  $\xi_i$ . This has to be contrasted with the low-energy supersymmetry case where the effect of trilinear couplings can be sizable to



FIG. 3: One-loop diagrams generating sizable neutrino masses with no ultra heavy mass suppression but with lepton mass suppression.

be detected in the future collider experiments [14]. Let us discuss if the neutralino decay length is short enough to allow the observation of its decay modes in the colliders. When  $\chi_1^0$  is heavier than Z, it decays to lW and  $\nu Z$  having the following rates:

$$\Gamma(l_{i}W) \approx \frac{G_{F}m_{\chi_{1}^{0}}^{3}}{4\sqrt{2}\pi}|\xi_{i}|^{2}c_{\beta}^{2}\mathcal{I}(\frac{m_{W}^{2}}{m_{\chi_{1}^{0}}^{2}}),$$

$$\Gamma(\nu_{i}Z) \approx \frac{G_{F}m_{\chi_{1}^{0}}^{3}}{16\sqrt{2}\pi}|\xi_{i}|^{2}c_{\beta}^{2}\mathcal{I}(\frac{m_{Z}^{2}}{m_{\chi_{1}^{0}}^{2}}),$$

$$\Gamma_{2-body} = \Gamma(lW + \nu Z) \approx \frac{G_{F}m_{\chi_{1}^{0}}^{3}}{16\sqrt{2}\pi}|\xi|^{2}c_{\beta}^{2}\left(4\mathcal{I}(\frac{m_{W}^{2}}{m_{\chi_{1}^{0}}^{2}}) + \mathcal{I}(\frac{m_{Z}^{2}}{m_{\chi_{1}^{0}}^{2}})\right).$$
(20)

where we have ignored the fermion masses and the kinematical factors are given as follows.

$$\mathcal{I}(x) \equiv (1-x)^2 (1+2x) \theta(x-1).$$

When  $\chi_1^0$  is lighter than W, it has only 3-body decay modes through the virtual W and Z and its decay rates are then

$$\Gamma(l_i W^*) \approx \frac{3G_F^2 m_{\chi_1^0}^5}{64\pi^3} |\xi_i|^2 c_\beta^2$$

$$\Gamma_{3-body} = \Gamma(lW^* + \nu Z^*) \approx \frac{5.4G_F^2 m_{\chi_1^0}^5}{64\pi^3} |\xi|^2 c_\beta^2.$$
(21)

From the above equations (20) and (21), one finds the decay length;

$$\frac{1}{\Gamma} = 1.8 \, mm \quad \text{or} \quad 1 \, m \,.$$

for  $|\xi|c_{\beta} = 7.4 \times 10^{-7}$  (4) and  $m_{\chi_1^0} = 100$  GeV or 62 GeV, respectively. Thus, the neutralino mass is required to be larger than 62 GeV. Independently of the 2-body or 3-body decay case, the measurement of the branching ratios for the mode  $l_iW$  will determine the relative sizes of  $|\xi_i|$ 

$$|\xi_1|^2 : |\xi_2|^2 : |\xi_3|^2 = B(eW^{(*)}) : B(\mu W^{(*)}) : B(\tau W^{(*)})$$

which could provide the test of the model predicting the relation (5). In the usual low-energy supersymmetry, the above conclusion may not be secured if the neutralino allows only the 3-body decay modes for which the effect of trilinear couplings can be even larger than the bilinear effect [14].

In conclusion, we have shown how the observed neutrino data can be accommodated in the context of Split Supersymmetry with R-parity violation. As most one-loop diagrams generating the neutrino mass matrix are suppressed by the high scale of the scalar masses, there appears a rather unique way to accommodate the desired neutrino mass matrix. The tree-level neutrino mass matrix coming from the bilinear parameters can nicely explain the atmospheric neutrino mass and mixing by the same way as in the usual low-energy supersymmetry. This requires the bilinear parameters to be of the order  $10^{-6}$  and put some bounds on the trilinear couplings like  $\lambda'_{i33} \lesssim 10^{-4}$ . Among various one-loop corrections, the quark-squark exchange diagrams can produce the solar neutrino mass and mixing taking the order-one couplings  $\lambda'_{i23,i32}$  if the scalar mass is at its lower end:  $m_S \sim 10^9$  GeV.

In this scheme, the lightest neutralino can decay only through the bilinear mixing into the final states of  $l_iW^{(*)}$  and  $\nu Z^{(*)}$ . The corresponding decay length can be less than 1 m if the neutralino mass is larger than 62 GeV. Thus, the observation of such features and the measurement of the branching ratios following the relation:  $B(eW^{(*)}) \ll B(\mu W^{(*)}) \approx B(\tau W^{(*)})$  would provide a clean probe of the model prediction coming from the small reactor and large atmospheric neutrino mixing angles.

We would add some words on fine tuning in the parameter space to explain neutrino masses. The seemingly unnatural parameter space with the small bilinear couplings ( $\lesssim 10^{-6}$  and the hierarchy in the trilinear couplings (e.g.,  $\lambda'_{i33} \ll \lambda'_{i23,i32} \sim 1$ ) is rather uniquely chosen to fit the experimental data of neutrino masses and mixing. This makes more complicate the Yukawa hierarchy problem, which appears to be the generic feature of the R-parity violating Split Supersymmetry.

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