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Onsite Round - Editorial

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Problem A. Are the Nodes Reachable?

Problem Setter: Raihat Zaman Neloy
Tester: Rumman Mahmud, Pritom Kundu, Nafis Sadique
Cetegory: Ad-hoc
Total Solved: 0
First to Solve: N/A

We can split the problem is two subproblem.

When $ans < 32$. How do we solve it? First for each vertex calculate the list of vertex reachable from it. Also calculate the list of vertex from which it is reachable. Both can be done in $O(\frac{max(n) \cdot m}{64})$ using bitsets. Now assume that vertex a is reachable from vertex b. In that case vertices $b-1$ or $b+1$ will be reachable by adding an edge of cost 1. We can find all the vertices from which vertex a is reachable with 1 cost by simply binary shifting the bitset by 1 in both direction and apply the bitwise OR operator on all three (no change, left shift, right shift). If we do this again then we find the vertices 2 distance away and so on. We keep doing this for distances upto 31. The complexity to do this is $O(\frac{n^2*32}{64})$.

Now, lets split the bitsets into 64 sized buckets. Technically we could've used an unsigned long long array to perform the bitset operations in the first place. For each vertex we will have 33 bitset buckets. 32 of them are for the shifted bitsets from upstream and one for the downstream. Then for the query, we check if the first upstream bucket of V with 31 shift and the downstream bucket of U has any match. We can do that by simply performing a bitwise AND operation. If there is a match then we check 30 shift and keep reducing it. If no match is found then we move to the next bucket. At the end in this approach we will need $O(\frac{n}{64} + 32)$ operations per query.

When $ans \geq 32$. We again keep a bucket of vertices reachable from a vertex. Vertices numbered from [1..64] are in one bucket, [65, 128] are in second buckets and so on. For each bucket we only keep the minimum and maximum id of the vertex reachable from it. Similarly we do it for the upstream vertices. For a query, we merge those two list of vertices (min and max of each bucket), sort them and find the smallest gap in consecutive vertices (one must be upstream while the other must be downstream). This takes $O(\frac{n*2}{64})$ per query.

Alternatively, we can do the same using 128 bit integers. The performance is comparable.

Problem B. Yet Another Crossover Episode

Problem Setter: Shahjalal Shohag
Tester: Rumman Mahmud, Pritom Kundu, Jubayer Rahman, Nafis Sadique
Cetegory: Dynamic Programming, Bit Manipulation
Total Solved: 7
First to Solve: MBSTU_Novatos

We know that $x\&y \leq \min(x, y)$ and it looks like $\gcd(x, y) \leq \min(x, y)$.

So $\gcd(x \oplus y, x\&y) \leq \min(x \oplus y, x\&y) \leq \min(x, y)$.

Also notice that i can be equal to j, so it looks like the answer is $\max(a_1, a_2, \ldots, a_n)$ because this is the maximum possible gcd.

But this is wrong due to the case when $x = 0$ or $y = 0$ because then $\gcd(x, y) = x$ when $y = 0$ and $\gcd(x, y) = y$ when $x = 0$.

So when $a_i \oplus a_j = 0$, we have $a_i = a_j$, so $\gcd(a_i \oplus a_j, a_i\&a_j) = \gcd(0, a_i\&a_i) = \gcd(0, a_i) = a_i$. So for this case

we need to take care of the maximum number of the array only.

And when $a_i \& a_j = 0$, we have $a_i \oplus a_j = a_i + a_j$ because there is no common bit in a_i and a_j so xor and addition are the same in this case.

So we need to find the maximum $a_i + a_j$ where $a_i \& a_j = 0$. So let's fix a_i and then find the maximum a_j such that $a_i \& a_j = 0$ which means we need to find the maximum a_j such that a_j is a submask of the flipped bits of a_i.

Finding the maximum submask of a mask that exists in the array is a standard problem that can be solved using Sum Over Subsets (SOS) DP. We can also find the count of such submasks using the same DP.

So overall we need to consider the maximum element of the array and the maximum sum of two elements such that there is no common bit between them.

Time Complexity: $O(m \cdot 2^m)$ where m is the number of bits in the maximum element of the array which is bounded by 23.

Problem C. Cut the Stick, Share You Must

Problem Setter: Rumman Mahmud
Tester: Shahjalal Shohag, Pritom Kundu, Nafis Sadique
Cetegory: Math, Number Theory
Total Solved: 15
First to Solve: IUT_CocolaChampionBiscuit

Since n & k are large ($1 \le k < n \le 10^6$), we need to find an optimal solution. Here are some hints before we dive into the actual solution. Let's define a function $P(n, k)$ that returns the number of ways to split n into k partitions where the GCD of the partitions is a prime number.

- **Hint 1: How would you solve it if the GCD was 1 instead of a prime number?**

- **Hint 2: How does changing the GCD to a prime affect n?**

- **Hint 3: Define $f(n, k)$ to return the number of ways to split n into k partitions where the GCD of the partitions is 1. Define another function $g(n, k, p)$ to return the number of ways to split n into k partitions where the GCD of the partitions is exactly p (with p being prime). Can you draw a relation between $g(n, k, p)$ and $f(\frac{n}{p}, k)$?**

Solution
A valid split will have a prime GCD. Let's say the GCD of a valid sequence is p. This means each partition will be a multiple of p. If we divide each partition by p, the new sequence will have GCD 1. The GCD cannot be greater than 1, as otherwise, the GCD of the original sequence cannot equal p. Using this observation, $g(n, k, p) = f(\frac{n}{p}, k)$.
To compute $P(n, k)$, sum over all prime factors of n:

$$P(n, k) = \sum_{p \in U(n)} f\left(\frac{n}{p}, k\right)$$

Here, $U(n)$ is the set of unique prime factors of n. The remaining challenge is finding the number of ways to split n into k partitions with a GCD of 1. The Möbius function and binomial coefficients can be used for this computation. Details are left as an exercise.

Complexity

- **Precomputing unique prime factors of all numbers:** $O(N \log N)$

- **Precomputing factorial and inverse factorials modulo** M**:** $O(N + \log M)$

- **Complexity of finding valid partitions per test case: For each prime factor** p_i**, finding splits with GCD** 1 **has a complexity of** $O(2^{\omega(\frac{n}{p_i})})$**. Repeating this for all unique prime factors of** n **gives** $O(\sum_{p \in U(n)} 2^{\omega(\frac{n}{p})})$**, where** $\omega(n)$ **is the number of unique prime factors of** n**.**

- **Overall complexity:** $O(N \log N + T \cdot \sum_{p \in U(n)} 2^{\omega(\frac{n}{p})})$

The maximum number of unique primes is 7. For example, the product of the first 7 primes is $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 = 510510$. Adding another prime would exceed N. Hence, the maximum value of $\sum_{p \in U(n)} 2^{\omega(\frac{n}{p})}$ is $7 \cdot 2^6 = 448$. Which means, per query you will need to perform 448 operations in the worst case. Which is good enough for the TL.

Bonus

We can further reduce the complexity by only considering the divisors of n that have at most one square prime factor. We need to modify the Mobius function a bit to achieve that. To not spoil the fun, I will keep it as an exercise for you as well.

Problem D. CatGPT

Problem Setter: Anik Sarker, Rafid Bin Mostofa
Tester: Rumman Mahmud, Pritom Kundu, Nafis Sadique
Cetegory: Graph, Offline Processing
Total Solved: 40
First to Solve: FaimDeSang

There can be only 26 clowders. We can use this information. First, lets process all the event in the ascending order of R_i. We then process the cats from left to right. After processing each cat we keep in memory the most recent position we saw a cat of clowder C_i. Then we process the events ended at the current position. Because we know the last position for the clowder, we know that event will merge clowders that have their *last position* $> L_i$. We keep the list of merged clowders for the event in memory as well.

For the queries, we also process them in the ascending order of y_i. But this time we iterate over the events from left to right. Since we know for an event, the clowders that will be merged, we can generate a list of unordered (i, j) pairs that will be merged for an event. There can't be more than $\frac{25 \cdot 26}{2}$ pairs. We keep track of the last event they were merged, similar to the previous section. Finally for an event, we process the queries that ended in that event. From the list of the last position of the pairs, we can have a graph of 26 vertices. We merge vertices using union-find. Then we check the largest merged clowder and output their size.

Problem E: Quasi-binary Representations

Problem Setter: Pritom Kundu
Tester: Shahjalal Shohag, Jubayer Rahman, Nafis Sadique
Cetegory: Dynamic Programming, Math
Total Solved: 5
First to Solve: DU_Primordius

Suppose we have placed the first k digits. The number formed the digits by we have placed must be less than or equal to the value formed by first k bits in the binary representation. Suppose, the difference is d. We need to make up this difference from the lower bits. The key observation is if $d > k$, then whatever digits we place in the lower position, we cannot make the difference up.

This allows us to do a dp solution. Define $dp[i][x]$ be the number of ways to place the remaining digits assuming

that we have placed the first i digits and have a difference of x so far. Then,

$$dp[i][x] = \sum_{\substack{0 \leq d \leq k \\ x+d \equiv n_i \pmod 2}}^{k} dp[i-1][(x + d - n_i)/2]$$

This solution is $O(k^2 \log n)$ solution which is not enough. In order to pass optimize transitions to $O(1)$ using prefix sums. Overall time will be $O(k \log n)$ per testcase.

Problem F: Flowers

Problem Setter: Hasinur Rahman
Tester: Nafis Sadique
Cetegory: Hashing, Divide & Conquer
Total Solved: 0
First to Solve: N/A

To solve this problem, we need to express the shapes of the trees as a hash value. We need to design the hash function so that it will remain consistent about the tree structure whether we calculate it from the whole flower or while merging it.

About the hash function: We will calculate the hash of the structure depending on the LRU path-string of the preorder traversal of a flower. Here, 'L' = Left child traversal from parent, 'R' = Right child traversal from the parent, 'U' = Getting back to the parent after visiting all its children. It will ensure a uniquely shaped tree traversal. Note that, if you don't include the 'U', it will not express a unique shape.

About the flower merging: Let's say two flowers A, and B are getting merged. B will be the child of the leftmost leftless node of A. First, we will divide the hash of A into two parts, LeftHash and RightHash and we know the hash of the structure of A. Now, think about how to merge them to get the hash of the merged structure.

About tracking Unique Shapes: For each tree, we need to maintain a set of hash values. While merging, If there are two sets S_1 and S_2, we need to merge them following the rule: Always merge the light set and the heavy set. This will lead to better-amortized complexity.

Time complexity: $\mathcal{O}(N \times \lg N)$
Memory complexity: $\mathcal{O}(N)$

Problem G: Library Function vs Keyword

Problem Setter: Shahriar Manzur
Tester: Jubayer Rahman, Nafis Sadique, Rumman Mahmud
Cetegory: Ad-hoc
Total Solved: 228
First to Solve: BRACU_Crows

For $strlen(line)$, we simply need to find the position of the first $null$('\0') character. The number of characters before that is the answer. If no $null$ characters exist then the answer would be the string length.

For $sizeof(line)$, count the number of $null$ in the string. Then the answer is:

$$string\ length - number\ of\ null\ characters + 1.$$

Problem H: Hand Cricket

Problem Setter: Kazi Md Irshad
Tester: Pritom Kundu, Jubayer Rahman, Nafis Sadique

Cetegory: Data Structure, Math, Probability
Total Solved: 0
First to Solve: N/A

The strategies are in Nash equilibrium. Therefore, Alice cannot improve her strategy. She will have the same expected points for any index. Let this expected value be X. Let $p_1, p_2, \ldots, p_N$ be Bob's strategy. Then for all i:

$$A_i \cdot (1 - p_i) = X$$

$$p_i = 1 - \frac{X}{A_i}$$

$$1 = \sum_{i=1}^{N} (1 - \frac{X}{A_i})$$

As sum of p_i is 1. Adding this equation over all i gives:

$$X = \frac{N - 1}{\sum_{i=1}^{N} \frac{1}{A_i}}$$

Which means for range $[L, R]$ the answer is $X = \frac{R - L}{\sum_{i=L}^{R} \frac{1}{A_i}}$ Alice needs to increase X. Reducing $\sum_{i=1}^{N} \frac{1}{A_i}$ will increase X. It can be proved that Alice always increments the smallest A_i. To implement this efficiently, we will build a persistent segment tree on the array values. The jth root of the segment tree will account for array elements in the range $[1, j]$. The tree maintains the following:

- The sum of all A_i values in the range.

- The sum of inverses of A_i in the range.

- The count of elements available in the range.

We will walk the segment tree from the root for the $(L - 1)$-th element and the root of the R-th element. During this traversal, we will maintain:

- How much of K is left.

- The value and count of the small numbers that have been incremented.

On a segment tree node we can check if the whole subtree can be incremented to its maximum in $O(1)$ and also get the sum of inverses in $O(1)$ if K depletes.

The complexity is $O(Q log N)$. However we have allowed slower solutions like $O(Q log^2 N)$ (using binary search and persistent segment tree), and $O(Q log^3 N)$ (using binary search and merge sort tree).

Problem I: In Search of a Kind Person

Problem Setter: Md. Imran Bin Azad
Tester: Shahjalal Shohag, Rumman Mahmud, Raihat Zaman Neloy, Muhiminul Islam Osim
Cetegory: Ad-hoc
Total Solved:
First to Solve: IUT_cgpa_matters

According to the problem statement we need to find the smallest i such that $max(a[1], a[2], ..., a[i-1]) < a[i] < min(a[i+1], a[i+2], ...a[n])$. To find these, we can keep two array where one will track the maximum from 1 to $i - 1$ and the other array will track the minimum from $i + 1$ to n. The special cases are if such i is the first or last element in the given array.

Problem J: The Taxman

Problem Setter: Aminul Haq
Tester: Rumman Mahmud, Muhiminul Islam Osim
Cetegory: Binary Search, Math
Total Solved: 159
First to Solve: DU_Singularity

This problem can be solved either purely through math or by using binary search combined with a bit of math.

For the binary search approach, we can search for the closest income that generates the given tax. The search boundary is adjusted based on the difference between the tax calculated for our current value and the input tax. The lower bound of the search is 12,500, and the upper bound is approximately $\sim 3 \times 10^9$. With the given constraints, we found that 60 iterations are sufficient to produce the accepted output.

Special case: For 0, any value between 0 and 12500 is accepted.

```
double binarySearch(double target) {
    double low = 12500;
    double high = 3e9;
    int iterations = 60;
    while(low <= high && iterations-- > 0) {
        double mid = (low + high) / 2;
        double tax = calculateTax(mid);
        if(tax > target) {
            high = mid;
        } else {
            low = mid;
        }
    }
    return low;
}
```

For the math-based solution, we can calculate the income range where the salary falls by precomputing the tax for each range. For example, for an income of 50,000, the tax is 7,500.00, and for 125,000, the tax is 42,500.00. By comparing the input tax with these precomputed values, the target salary can be directly determined. One helpful observation here is that the income range from over 100,000 to 125,000 is effectively taxed at 60

Problem K: Packet Transmission

Problem Setter: Ashraful Islam
Tester: Pritom Kundu, Nafis Sadique
Cetegory: Lowest Common Ancestor
Total Solved: 0
First to Solve: N/A

Every query we receive can be classified in the following way.

- The query packets (source, destination pair) share a common path in the tree.

- The packets are going in the same direction. In that case we need to assume one of the packets will never wait. So, the other packet will arrive at the first vertex of the common path, wait for the first packet to arrive and then go after it. Since an edge can't be used by multiple packets at the same time, the other packet will have to wait until the first packet crossed that edge. Keep doing that until they reach the end of common path and then they can go their own ways. If we think carefully, we can see that the other packet will have to wait an additional time totalling the maximum edge cost on the common path. However if the first packet comes early then the wait time reduces. Do the same the other way around and take the minimum of the two.

- The packets are going in the different directions. So, we need both packets to start their journey. Eventually they might meet on the two sides of an edge where if one starts crossing, the other must wait. Make one of them wait and the other one go and calculate the time it would take for both them to reach their destinations. Do it both ways and take the minimum time. However if they don't meet at the edge then they don't have to stop and their actual time to reach both their destinations is the answer.

- The query packets use completely different paths. In that case the time it takes for both to reach their destination is the answer.

We can calculate all of these using Sparse Table to calculate Least Common Ancestors in a tree. Alternatively we can use heavy-light decomposition, but that may be very slow. There are lot of corner cases, so careful implementation is necessary.

Problem L: Unhappy Team

Problem Setter: Nafis Sadique
Tester: Shahjalal Shohag, Rumman Mahmud, Jubayer Rahman
Cetegory: Dynamic Programming
Total Solved: 0
First to Solve: N/A

The trick is to select the unhappiness score of someone and count how many times that score appear in the top K values. We can keep a DP state $(bit\_mask, bigger\_scores\_count, selected\_score\_appeared)$. Basically we will put people one after another, everytime calculating their unhappiness score. If the score is smaller than the selected score then we ignore it. If the score is bigger then $bigger\_scores\_count$ is incremented by one. If the score is the same is the selected score then we can consider it as bigger or mark $selected\_score\_appeared$ as $true$. This DP will have the complexity of $O(2^n \cdot n^3)$. With some other minor optimizations, this runs reasonably fast.