SYLLABUS

Subject Code	:10EE71	IA Marks:25
No. of Lecture Hrs./ Week	: 04	Exam Hours:03
Total No. of Lecture Hrs.	: 52	Exam Marks:100

PART - A

UNIT - 1

NETWORK TOPOLOGY: Introduction, Elementary graph theory – oriented graph, tree, co-tree, basic cut-sets, basic loops; Incidence matrices – Element-node, Bus incidence, Tree-branch path, Basic cut-set, Augmented cut-set, Basic loop and Augmented loop; Primitive network – impedance form and admittance form.

6 Hours

UNIT - 2

NETWORK MATRICES: Introduction, Formation of Y_{BUS} – by method of inspection (including transformer off-nominal tap setting), by method of singular transformation ($Y_{BUS} = A^T yA$); Formation of Bus Impedance Matrix by step by step building algorithm (without mutual coupling elements).

6 Hours

UNIT - 3 & 4

LOAD FLOW STUDIES: Introduction, Power flow equations, Classification of buses, Operating constraints, Data for load flow; Gauss-Seidal Method – Algorithm and flow chart for PQ and PV buses (numerical problem for one iteration only), Acceleration of convergence; Newton Raphson Method – Algorithm and flow chart for NR method in polar coordinates (numerical problem for one iteration only); Algorithm for Fast Decoupled load flow method; Comparison of Load Flow Methods.

14 Hours

PART - B

UNIT - 5 & 6

ECONOMIC OPERATION OF POWER SYSTEM: Introduction, Performance curves, Economic generation scheduling neglecting losses and generator limits, Economic generation scheduling including generator limits and neglecting losses; Iterative techniques; Economic Dispatch including transmission losses – approximate penalty factor, iterative technique for solution of economic dispatch with losses; Derivation of transmission loss formula; Optimal scheduling for Hydrothermal plants – problem formulation, solution procedure and algorithm.

12 Hours

UNIT - 7 & 8

TRANSIENT STABILITY STUDIES: Numerical solution of Swing Equation – Point-by-point method, Modified Euler's method, Runge-Kutta method, Milne's predictor corrector method. Representation of power system for transient stability studies – load representation, network performance equations. Solution techniques with flow charts.

14 Hours

TEXT BOOKS:

1. **Computer Methods in Power System Analysis-** Stag, G. W., and EI-Abiad, A. H.- McGraw Hill International Student Edition. 1968
2. **Computer Techniques in Power System Analysis**- Pai, M. A- TMH, 2nd edition, 2006.

REFERENCE BOOKS:

1. **Modern Power System Analysis-** Nagrath, I. J., and Kothari, D. P., -TMH, 2003.
2. **Advanced Power System Analysis and Dynamics**- Singh, L. P., New Age International (P) Ltd, New Delhi, 2001.
3. **Computer Aided Power System Operations and Analysis**"- Dhar, R. N- TMH, New Delhi, 1984.
4. **Power System Analysis-** Haadi Sadat, -TMH, 2nd , 12th reprint, 2007

CONTENTS

UNIT-1

NETWORK TOPOLOGY

1. INTRODUCTION

The solution of a given linear network problem requires the formation of a set of equations describing the response of the network. The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components. In the bus frame of reference the variables are the node voltages and node currents. The independent variables in any reference frame can be either currents or voltages. Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix. The formulation of the appropriate relationships between the independent and dependent variables is an integral part of a digital computer program for the solution of power system problems. The formulation of the network equations in different frames of reference requires the knowledge of graph theory. Elementary graph theory concepts are presented here, followed by development of network equations in the bus frame of reference.

1.1 ELEMENTARY LINEAR GRAPH THEORY: IMPORTANT TERMS

The geometrical interconnection of the various branches of a network is called the *topology* of the network. The connection of the network topology, shown by replacing all its elements by lines is called a *graph.* A *linear graph* consists of a set of objects called *nodes* and another set called *elements* such that each element is identified with an ordered pair of nodes. An *element* is defined as any line segment of the graph irrespective of the characteristics of the components involved. A graph in which a direction is assigned to each element is called an *oriented graph* or a *directed graph*. It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions. Elements are indicated by numbers and the nodes by encircled numbers. The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

Connected Graph : This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1, with:
e = number of elements = 6
n = number of nodes = 4
b = number of branches = n-1 = 3
l = number of links = e-b = 3
Tree = T(1,2,3) and
Co-tree = T(4,5,6)
Sub-graph : sG is a sub-graph of G if the following conditions are satisfied:
· sG is itself a graph
· Every node of sG is also a node of G
· Every branch of sG is a branch of G

For eg., sG(1,2,3), sG(1,4,6), sG(2), sG(4,5,6), sG(3,4),.. are all valid sub-graphs of the oriented graph of Fig.1c.

Loop : A sub-graph L of a graph G is a loop if
· L is a connected sub-graph of G
· Precisely two and not more/less than two branches are incident on each node in L

In Fig 1c, the set{1,2,4} forms a loop, while the set{1,2,3,4,5} is not a valid, although the set(1,3,4,5) is a valid loop. The KVL (Kirchhoff's Voltage Law) for the loop is stated as follows: *In any lumped network, the algebraic sum of the branch voltages around any of the loops is zero.*

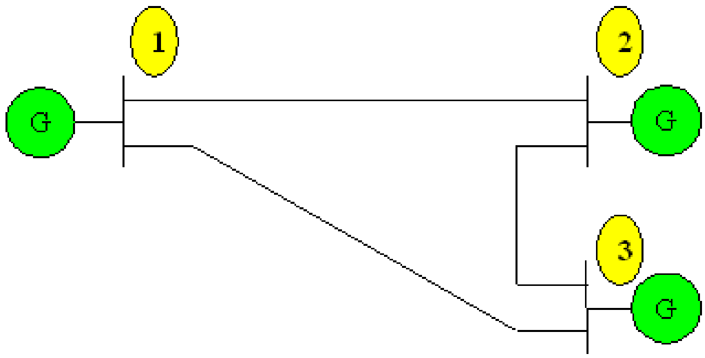


Fig 1a. Single line diagram of a power system

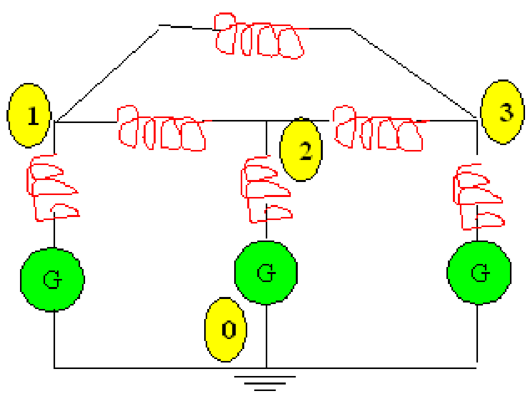


Fig 1b. Reactance diagram

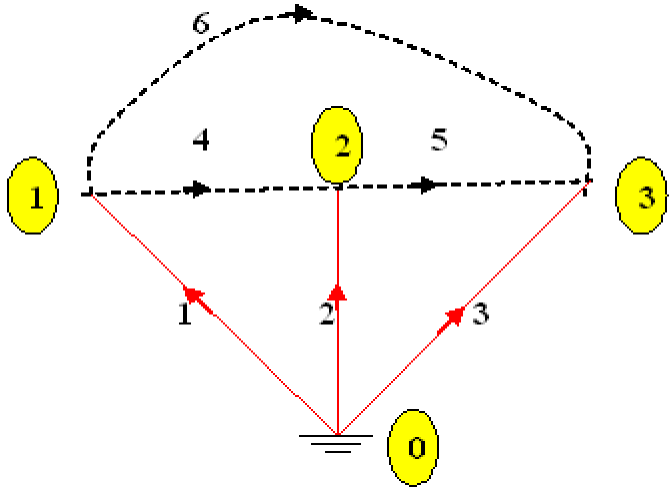


Fig 1c. Oriented Graph

Cutset : It is a set of branches of a connected graph G which satisfies the following conditions :

· The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.

· The removal of all but one of the branches of the set, leaves the remaining graph connected.

Referring to Fig 1c, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected subgraphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.

Tree: It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with *n* nodes,

The number of branches: b = n-1 **(1)**

For the graph of Fig 1c, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

Co-Tree : The set of branches of the original graph G, not included in the tree is called the *co-tree*. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called *links*. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With e as the total number of elements,

The number of links: l = e – b = e – n + 1 **(2)**

For the graph of Fig 1c, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

Tree	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
Co-Tree	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

Basic loops: When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

Basic cut-sets: Cut-sets which contain only one branch and remaining links are called *basic cutsets* or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

1.2 Examples on Basics of LG Theory:

Example-1: Obtain the oriented graph for the system shown in Fig. E1. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.

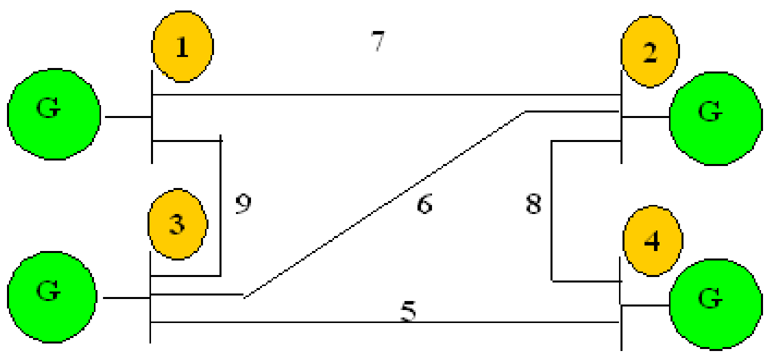


Fig. E1a. Single line diagram of Example System

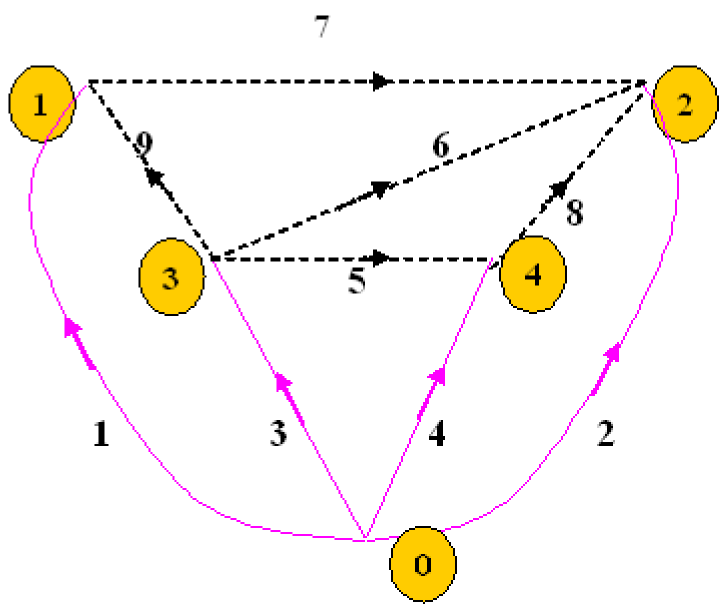


Fig. E1b. Oriented Graph of Fig. E1a.

For the system given, the oriented graph is as shown in figure E1b. some of the valid Tree graphs could be T(1,2,3,4), T(3,4,8,9), T(1,2,5,6), T(4,5,6,7), etc. The basic cutsets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.E1a and tree, T(1,2,3,4) are as shown in Figure E1c and Fig.E1d respectively.

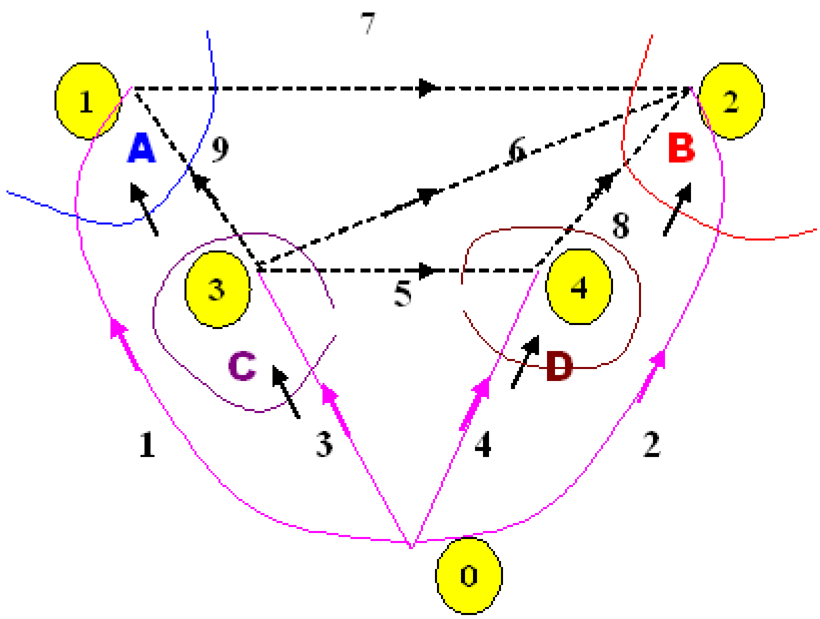


Fig. E1c. Basic Cutsets of Fig. E1a.

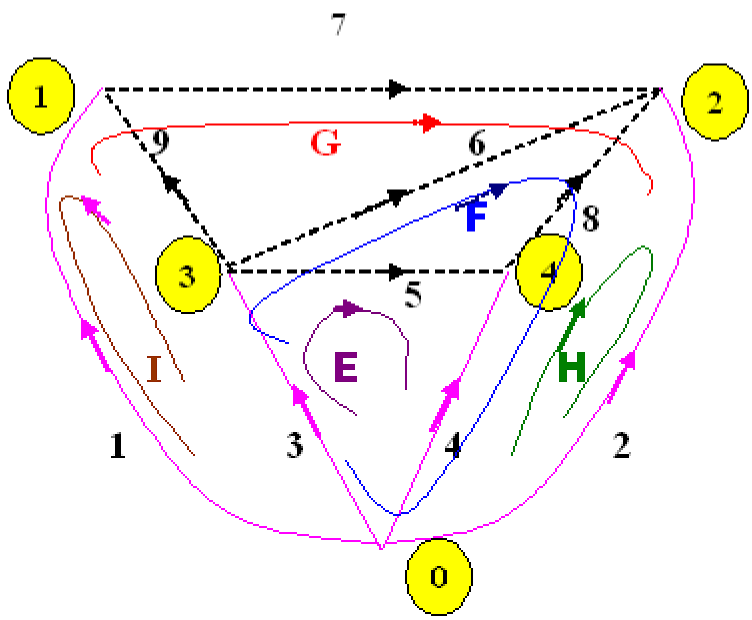


Fig. E1d. Basic Loops of Fig. E1a.

1.3 INCIDENCE MATRICES

Element–node incidence matrix: $\hat{A}$

The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, $\hat{A}$.

An element aij of $\hat{A}$ is defined as under:

aij = 1 if the branch-i is incident to and oriented away from the node-j.
 = -1 if the branch-i is incident to and oriented towards the node-j.
 = 0 if the branch-i is not at all incident on the node-j.

Thus the dimension of $\hat{A}$ is eX*n*, where e is the number of elements and *n* is the number of nodes in the network. For example, consider again the sample system with its oriented graph as in fig. 1c. the corresponding element-node incidence matrix, is obtained as under:

$\hat{A}$ =

Nodes / Elements	0	1	2	3
1	1	-1		
2	1		-1	
3	1			-1
4		1	-1	
5			1	-1
6		1		-1

It is to be noted that the first column and first row are not part of the actual matrix and they only indicate the element number node number respectively as shown. Further, the sum of every row is found to be equal to zero always. Hence, the rank of the matrix is less than *n*. Thus in general, the matrix $\hat{A}$ satisfies the identity:

$$\sum_{j=1}^{n} a_{ij} = 0 \quad \forall \; i = 1,2,\ldots..e. \qquad (3)$$

1.4 Bus incidence matrix: A

By selecting any one of the nodes of the connected graph as the reference node, the corresponding column is deleted from $\hat{A}$ to obtain the bus incidence matrix, A. The dimensions of A are e *(n-1)* and the rank is *n-1*. In the above example, selecting node-0 as reference node, the matrix A is obtained by deleting the column corresponding to node-0, as under:

A =

Buses / Elements	1	2	3
1	-1		
2		-1	
3			-1
4	1	-1	
5		1	-1
6	1		-1

=

A_b	Branches
A_l	Links

It may be observed that for a selected tree, say, T(1,2,3), the bus incidence matrix can be so arranged that the branch elements occupy the top portion of the A-matrix followed by the link elements. Then, the matrix-A can be partitioned into two sub matrices Ab and Al as shown, where,
(i) Ab is of dimension (bxb) corresponding to the branches and
(ii) Al is of dimension (lxb) corresponding to links.

A is a rectangular matrix, hence it is singular. Ab is a non-singular square matrix of dimension-b. Since A gives the incidence of various elements on the nodes with their direction of incidence, the KCL for the nodes can be written as

$AT\ i = 0$ **(4)**

where AT is the transpose of matrix A and *i* is the vector of branch currents. Similarly for the branch voltages we can write,

$v = A\ bus\ E$ **(5)**

Examples on Bus Incidence Matrix:

Example-2: For the sample network-oriented graph shown in Fig. E2, by selecting a tree, T(1,2,3,4), obtain the incidence matrices A and Aˆ . Also show the partitioned form of the matrix-A.

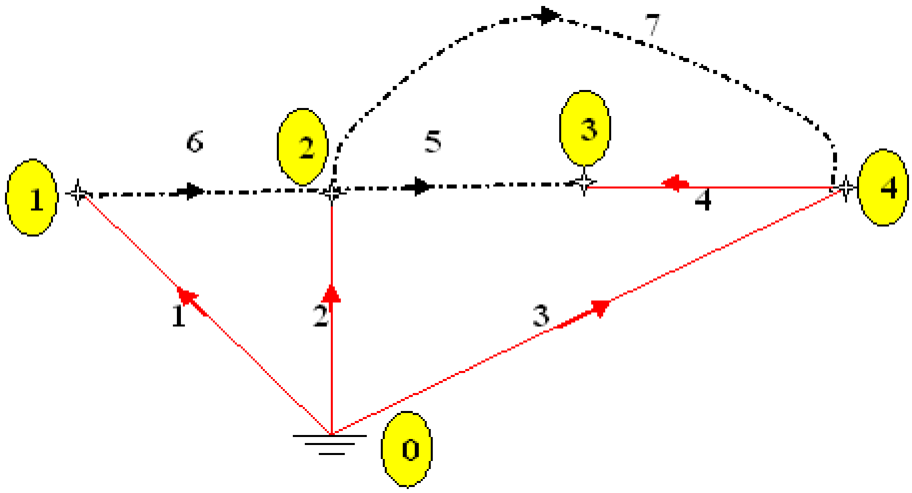


Fig. E2. Sample Network-Oriented Graph

$$\hat{A} = \text{Elements} \begin{array}{c} \text{nodes} \\ \begin{bmatrix} e\backslash n & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & 0 & 1 & -1 & 0 \\ 6 & 0 & 1 & -1 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

$$\mathbf{A} = \text{Elements} \begin{array}{c} \text{buses} \\ \begin{bmatrix} e\backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$$A_b = \text{branches}\;\begin{array}{cc} & \text{buses} \\ b\backslash b & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

$$A_l = \text{links}\;\begin{array}{cc} & \text{buses} \\ l\backslash b & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 5 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

Example-3: For the sample-system shown in Fig. E3, obtain an oriented graph. By selecting a tree, T(1,2,3,4), obtain the incidence matrices A and $\hat{A}$. Also show the partitioned form of the matrix-A.

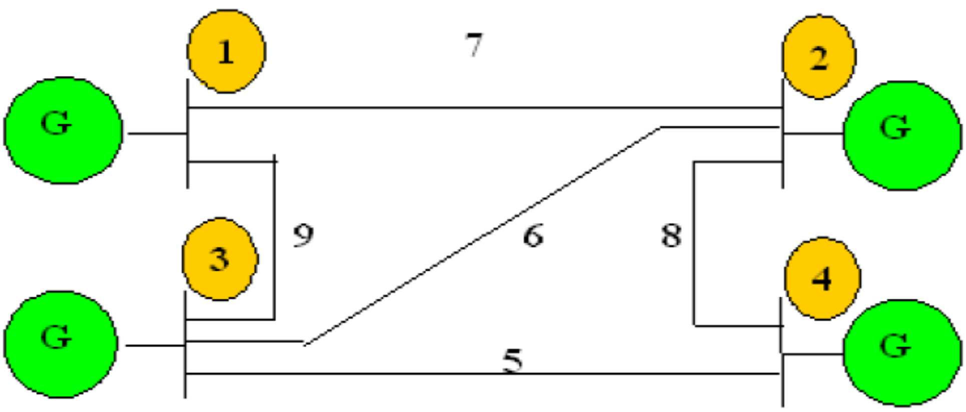


Fig. E3a. Sample Example network

Consider the oriented graph of the given system as shown in figure E3b, below.

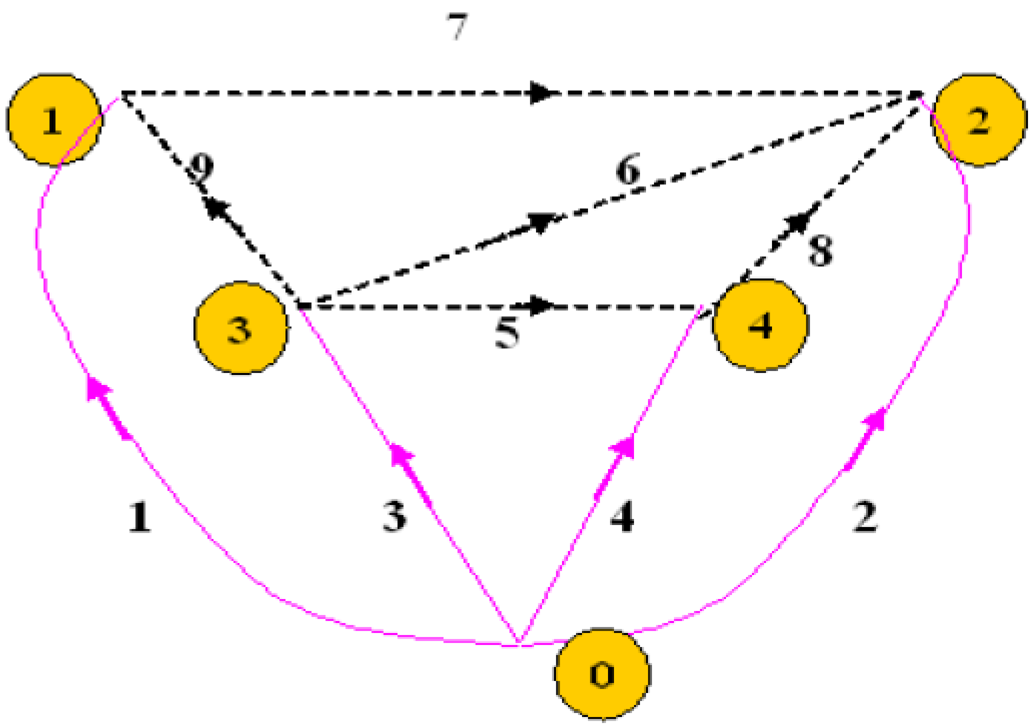


Fig. E3b. Oriented Graph of system of Fig-E3a.

Corresponding to the oriented graph above and a Tree, T(1,2,3,4), the incidence matrices • and A can be obtained as follows:

$\hat{A}$ =

e\n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1			-1	
4	1				-1
5				1	-1
6			-1	1	
7		1	-1		
8			-1		1
9		-1		1	

A =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$A_b =$

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1

$A_l =$

e\b	1	2	3	4
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

1.4 PRIMITIVE NETWORKS

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

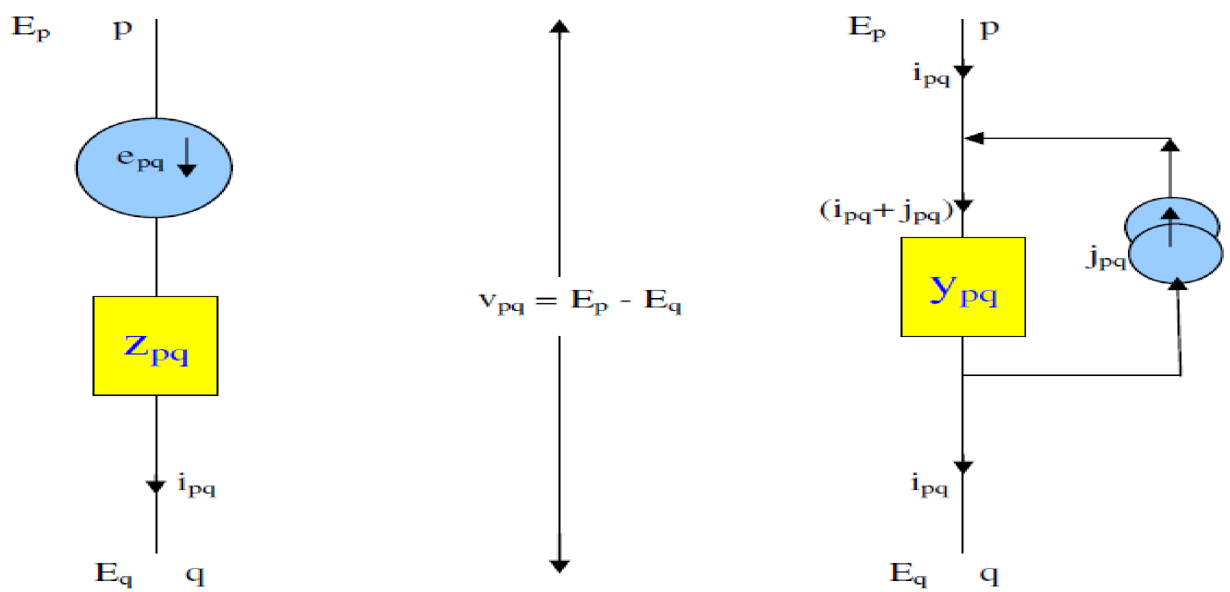


Fig.2 Representation of a primitive network element (a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

vpq = voltage across the element p-q,

epq = source voltage in series with the element p-q,

ipq= current through the element p-q,

jpq= source current in shunt with the element p-q,
zpq= self impedance of the element p-q and
ypq= self admittance of the element p-q.

Performance equation: Each element p-q has two variables, Vpq and ipq. The performance of the given element p-q can be expressed by the performance equations as under:

vpq + epq = zpqipq (in its impedance form)
ipq + jpq = ypqvpq (in its admittance form) **(6)**

Thus the parallel source current jpq in admittance form can be related to the series source voltage, epq in impedance form as per the identity:
jpq = - ypq epq **(7)**
A set of non-connected elements of a given system is defined as a *primitive Network and a*n element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:
v + e = [z] i
i + j = [y] v **(8)**

Primitive network matrices:
A diagonal element in the matrices, [z] or [y] is the self impedance zpq-pq or self admittance, ypq-pq. An off-diagonal element is the mutual impedance, zpq-rs or mutual admittance, ypq-rs, the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix, [y] can be obtained also by inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

Examples on Primitive Networks:

Example-4: Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to: Z1=Z2=0.2, Z3=0.25, Z4=Z5=0.1 and Z6=0.4 units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.

A =

-1	0	0
0	-1	0
0	0	-1
1	-1	0
0	1	-1
1	0	-1

Solution:
The element node incidence matrix, $\hat{A}$ can be obtained from the given A matrix, by pre-augmenting to it an extra column corresponding to the reference node, as under.

$\hat{A}$ =

1	-1	0	0
1	0	-1	0
1	0	0	-1
0	1	-1	0
0	0	1	-1
0	1	0	-1

Based on the conventional definitions of the elements of $\hat{A}$, the oriented graph can be formed as under:

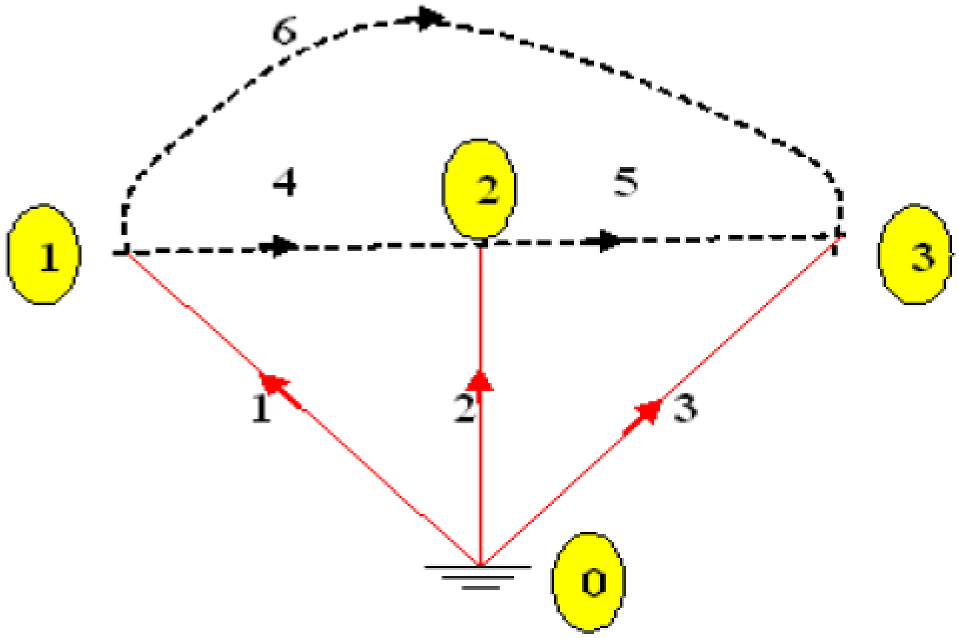


Fig. E4 Oriented Graph

Thus the primitive network matrices are square, symmetric and diagonal matrices of order e=no. of elements = 6. They are obtained as follows.

[z] =

0.2	0	0	0	0	0
0	0.2	0	0	0	0
0	0	0.25	0	0	0
0	0	0	0.1	0	0
0	0	0	0	0.1	0
0	0	0	0	0	0.4

And

[y] =

5.0	0	0	0	0	0
0	5.0	0	0	0	0
0	0	4.0	0	0	0
0	0	0	10	0	0
0	0	0	0	10	0
0	0	0	0	0	2.5

Example-5: Consider three passive elements whose data is given in Table E5 below. Form the primitive network impedance matrix.

Table E5

Element number	Self impedance ($z_{pq\text{-}pq}$)		Mutual impedance, ($z_{pq\text{-}rs}$)	
	Bus-code, (p-q)	Impedance in p.u.	Bus-code, (r-s)	Impedance in p.u.
1	1-2	j 0.452		
2	2-3	j 0.387	1-2	j 0.165
3	1-3	j 0.619	1-2	j 0.234

Solution:

$$[z] = \begin{array}{c} \\ 1\text{-}2 \\ 2\text{-}3 \\ 1\text{-}3 \end{array} \begin{array}{c} \begin{array}{ccc} 1\text{-}2 & 2\text{-}3 & 1\text{-}3 \end{array} \\ \left[\begin{array}{ccc} j\,0.452 & j\,0.165 & j\,0.234 \\ j\,0.165 & j\,0.387 & 0 \\ j\,0.234 & 0 & j\,0.619 \end{array}\right] \end{array}$$

Note:
· The size of [z] is e´ e, where e= number of elements,
· The diagonal elements are the self impedances of the elements
· The off-diagonal elements are mutual impedances between the corresponding elements.
· Matrices [z] and [y] are inter-invertible.

UNIT-2

NETWORK MATRICES

2. FORMATION OF Y_{BUS} AND Z_{BUS}

The bus admittance matrix, YBUS plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:
1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. ZBUS Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:
Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

EBUS = ZBUS IBUS
IBUS = YBUS EBUS **(9)**

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

EBR = ZBR IBR
IBR = YBR EBR **(10)**

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

ELOOP = ZLOOP ILOOP
ILOOP = YLOOP ELOOP **(11)**

Of the various network matrices refered above, the bus admittance matrix (YBUS) and the bus impedance matrix (ZBUS) are determined for a given power system by the rule of inspection as explained next.

2.1 Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: I1 =Y1V1 + Y3 (V1-V3) + Y6 (V1 – V2)
At node 2: I2 =Y2V2 + Y5 (V2-V3) + Y6 (V2 – V1)

At node 3: 0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 – V2) **(12)**

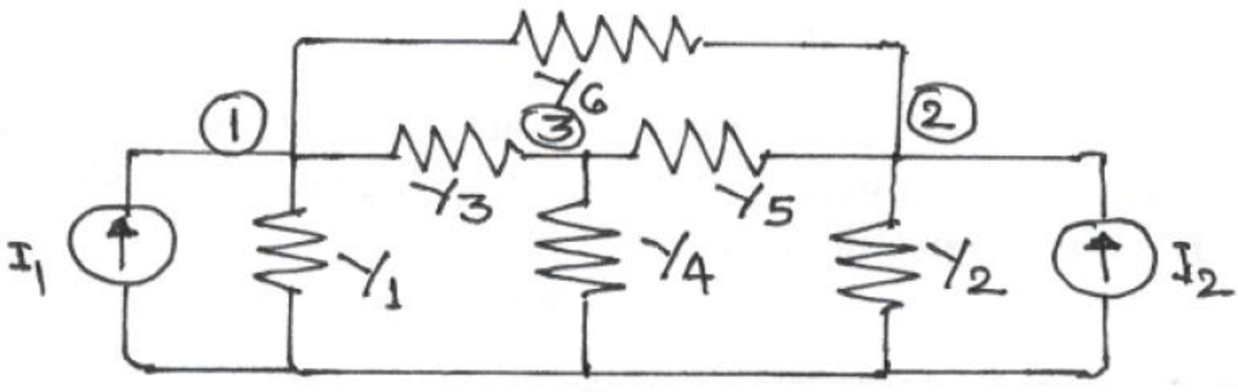


Fig. 3 Example System for finding YBUS

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad \textbf{(13)}$$

In other words, the relation of equation (9) can be represented in the form
IBUS = YBUS EBUS **(14)**
Where, YBUS is the bus admittance matrix, IBUS & EBUS are the bus current and bus voltage vectors respectively. By observing the elements of the bus admittance matrix, YBUS of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:
Diagonal elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,
Off Diagonal elements: An off-diagonal element (Yij) of the bus admittance matrix, YBUS, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any. This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

Yii = S yij (j = 1,2,…….n)
Yij = - yij (j = 1,2,…….n) **(15)**

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

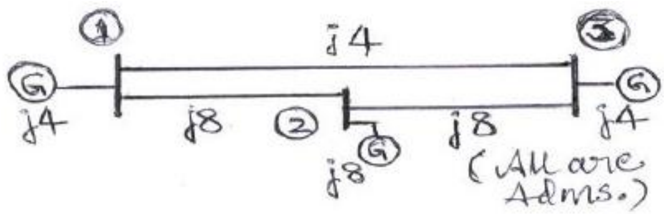
2.2 Bus impedance matrix

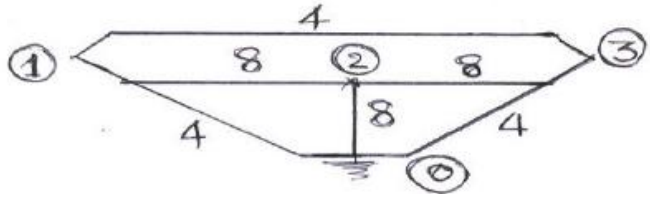
In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are interinvertible.
Note: It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

Examples on Rule of Inspection:

Example 6: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

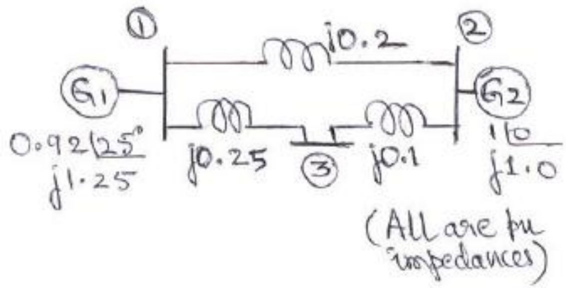
$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



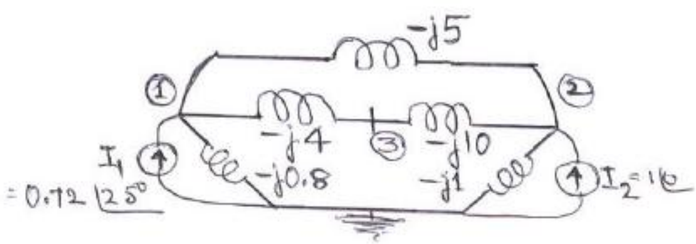


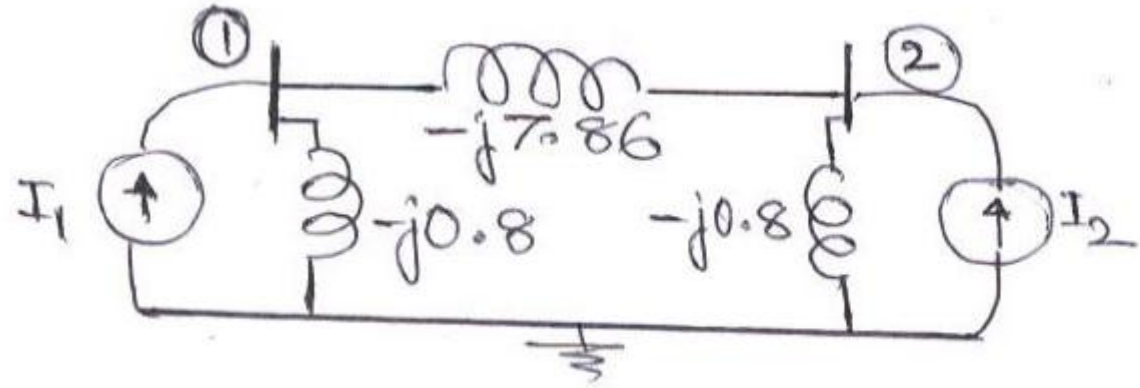
Example 7: Obtain YBUS for the impedance network shown aside by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

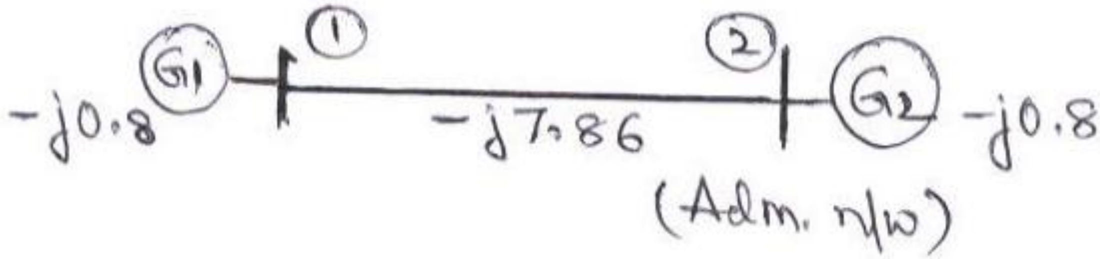
$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$



$$Z_{BUS} = Y_{BUS}^{-1}$$







$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS} = j\begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}$$

2.3 SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1nodes are present, out of which one of them is designated as the reference node).
For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

IBUS = YBUS EBUS **(17)**
Where EBUS = vector of bus voltages measured with respect to reference bus
IBUS = Vector of currents injected into the bus
YBUS = bus admittance matrix
The performance equation of the primitive network in admittance form is given by
i + j = [y] v
Pre-multiplying by At (transpose of A), we obtain

At i +At j = At [y] v **(18)**
However, as per equation (4),

At i =0,
since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff's law is zero. Similarly, At j gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

At j = IBUS **(19)**
Thus from (18) we have, IBUS = At [y] v **(20)**
However, from (5), we have
v =A EBUS
And hence substituting in (20) we get,

IBUS = At [y] A EBUS **(21)**
Comparing (21) with (17) we obtain,

YBUS = At [y] A **(22)**
The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

ZBUS = YBUS^{-1} **(23)**
Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

Examples on Singular Transformation:

Example 8: For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.

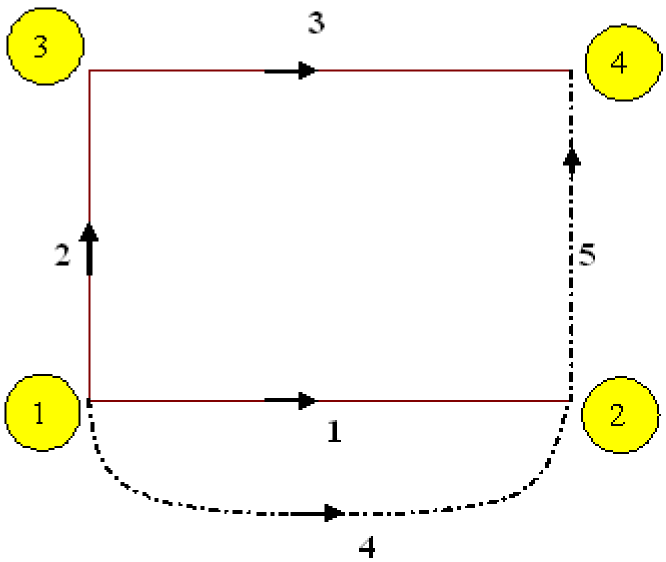


Fig E8 System for Example-8

Table E8: Data for Example-8

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:
The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix [y] = [z]-1 and given by,

$$[y] = \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t\,[y]\,A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances or admittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by *(n-1)* nodal equations, where *n* is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

FORMATION OF BUS IMPEDANCE MATRIX

2.4 NODE ELIMINATION BY MATRIX ALGEBRA

Nodes can be eliminated by the matrix manipulation of the standard node equations. However, *only those nodes at which current does not enter or leave the network can be considered for such elimination*. Such nodes can be eliminated either in one group or by taking the eligible nodes one after the other for elimination, as discussed next.

CASE-A: Simultaneous Elimination of Nodes:

Consider the performance equation of the given network in bus frame of reference in admittance form for a n-bus system, given by:

$$\mathbf{IBUS = YBUS\ EBUS} \qquad (1)$$

Where IBUS and EBUS are n-vectors of injected bus current and bus voltages and YBUS is the square, symmetric, coefficient bus admittance matrix of order n. Now, of the n buses present in the system, let p buses be considered for node elimination so that the reduced system after elimination of p nodes would be retained with m (= n-p) nodes only. Hence the corresponding performance equation would be similar to (1) except that the coefficient matrix would be of order m now, i.e.,

$$\mathbf{IBUS = YBUS^{new}\ EBUS} \qquad (2)$$

Where $YBUS^{new}$ is the bus admittance matrix of the reduced network and the vectors

IBUS and EBUS are of order m. It is assumed in (1) that IBUS and EBUS are obtained with their elements arranged such that the elements associated with p nodes to be eliminated are in the lower portion of the vectors. Then the elements of YBUS also get located accordingly so that (1) after matrix partitioning yields,

$$\begin{vmatrix} I_{BUS-m} \\ I_{BUS-p} \end{vmatrix} = \begin{array}{c c} & \begin{array}{cc} m & p \end{array} \\ \begin{array}{c} m \\ p \end{array} & \begin{vmatrix} Y_A & Y_B \\ Y_C & Y_D \end{vmatrix} \end{array} \begin{vmatrix} E_{BUS-m} \\ E_{BUS-p} \end{vmatrix} \quad (3)$$

Where the self and mutual values of YA and YD are those identified only with the nodes to be retained and removed respectively and YC=YBt is composed of only the corresponding mutual admittance values, that are common to the nodes m and p.
Now, for the p nodes to be eliminated, it is necessary that, each element of the vector IBUS-p should be zero. Thus we have from (3):

$$I_{BUS-m} = Y_A E_{BUS-m} + Y_B E_{BUS-p}$$
$$I_{BUS-p} = Y_C E_{BUS-m} + Y_D E_{BUS-p} = 0 \quad \textbf{(4)}$$

Solving,

$$\mathbf{E_{BUS-p} = - YD^{-1}Y_C E_{BUS-m}} \quad \textbf{(5)}$$

Thus, by simplification, we obtain an expression similar to (2) as,

$$\mathbf{I_{BUS-m}} = \{Y_A - Y_B Y_D\text{-}1 Y_C\} \mathbf{E_{BUS-m}} \quad \textbf{(6)}$$

Thus by comparing (2) and (6), we get an expression for the new bus admittance matrix in terms of the sub-matrices of the original bus admittance matrix as:

$$\mathbf{Y_{BUSnew} = \{Y_A - Y_B Y_D \text{ -}1 Y_C\}} \quad \textbf{(7)}$$

This expression enables us to construct the given network with only the necessary nodes retained and all the unwanted nodes/buses eliminated. However, it can be observed from (7) that the expression involves finding the inverse of the sub-matrix YD (of order p). This would be computationally very tedious if p, the nodes to be eliminated is very large, especially for real practical systems. In such cases, it is more advantageous to eliminate the unwanted nodes from the given network by considering one node only at a time for elimination, as discussed next.

CASE-B: Separate Elimination of Nodes:

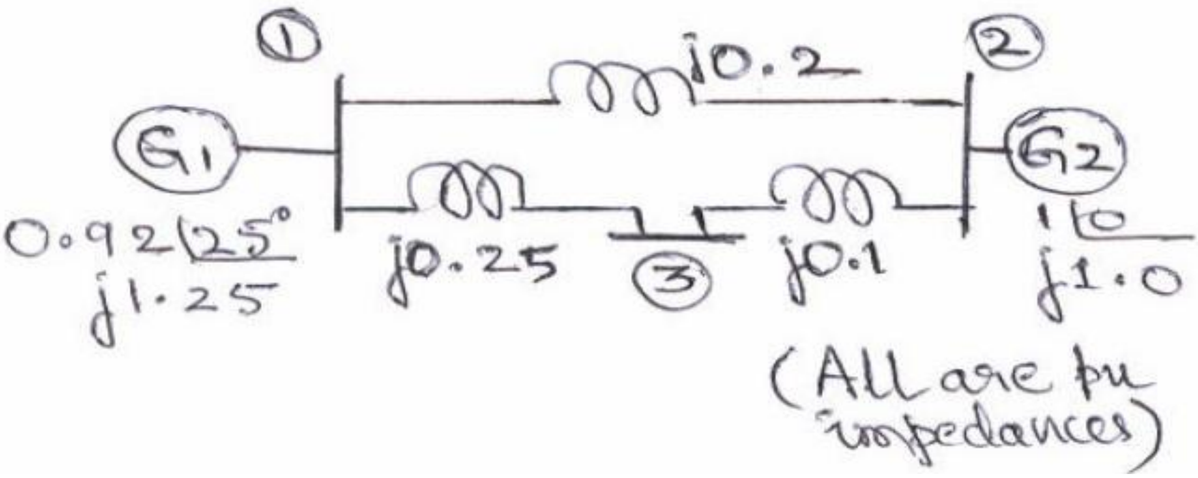
Here again, the system buses are to be renumbered, if necessary, such that the node to be removed always happens to be the last numbered one. The sub-matrix YD then would be a single element matrix and hence it inverse would be just equal to its own reciprocal value. Thus the generalized algorithmic equation for finding the elements of the new bus admittance matrix can be obtained from (6) as,

$$\mathbf{Yij^{new} = Yij^{old} - Yin\ Ynj / Ynn} \text{ " i,j = 1,2,…… n.} \quad \textbf{(8)}$$

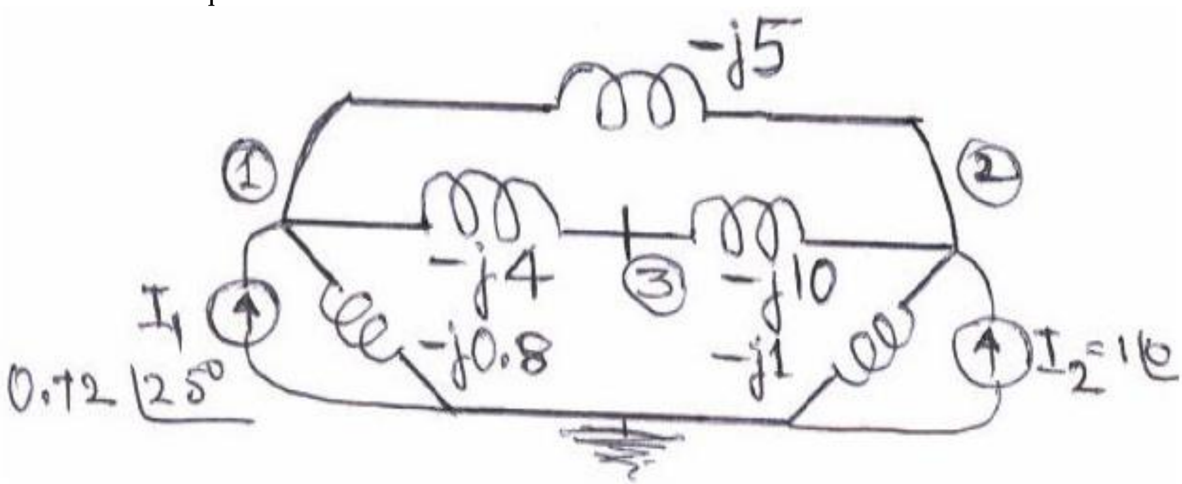
Each element of the original matrix must therefore be modified as per (7). Further, this procedure of eliminating the last numbered node from the given system of n nodes is to be iteratively repeated p times, so as to eliminate all the unnecessary p nodes from the original system.

Examples on Node elimination:

Example-1: Obtain YBUS for the impedance network shown below by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



The admittance equivalent network is as follows:



The bus admittance matrix is obtained by RoI as:

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

The reduced matrix after elimination of node 3 from the given system is determined as per the equation:

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS}^{new} = \begin{array}{c|cc} n/n & 1 & 2 \\ \hline 1 & -j8.66 & j7.86 \\ 2 & j7.86 & -j8.66 \end{array}$$

Alternatively,

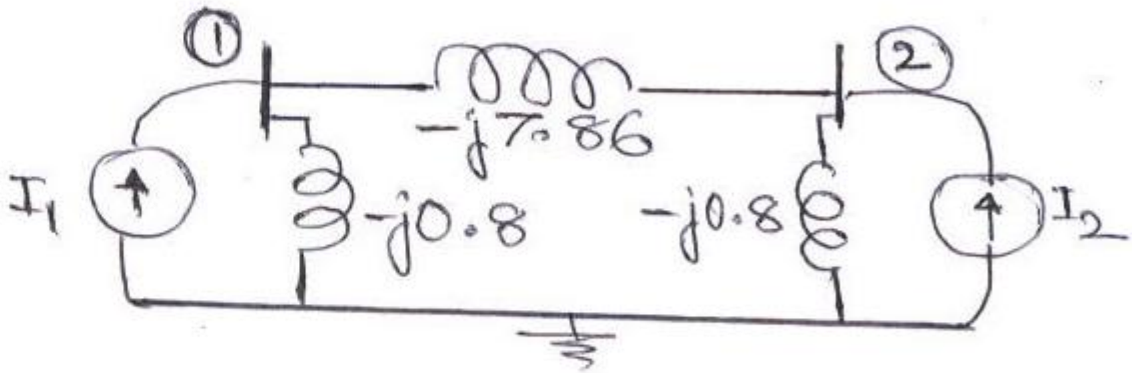
$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{i3}\, Y_{3j} \,/\, Y_{33} \qquad \forall\ i,j = 1,2.$$

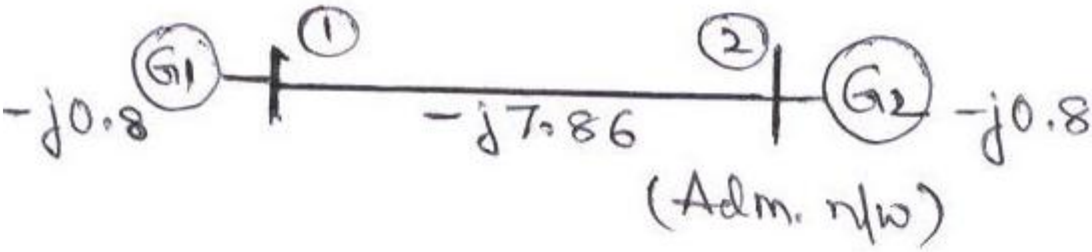
$$Y_{11} = Y_{11} - Y_{13}Y_{31} / Y_{33} = -j8.66$$

$$Y_{22} = Y_{22} - Y_{23}Y_{32} / Y_{33} = -j8.66$$

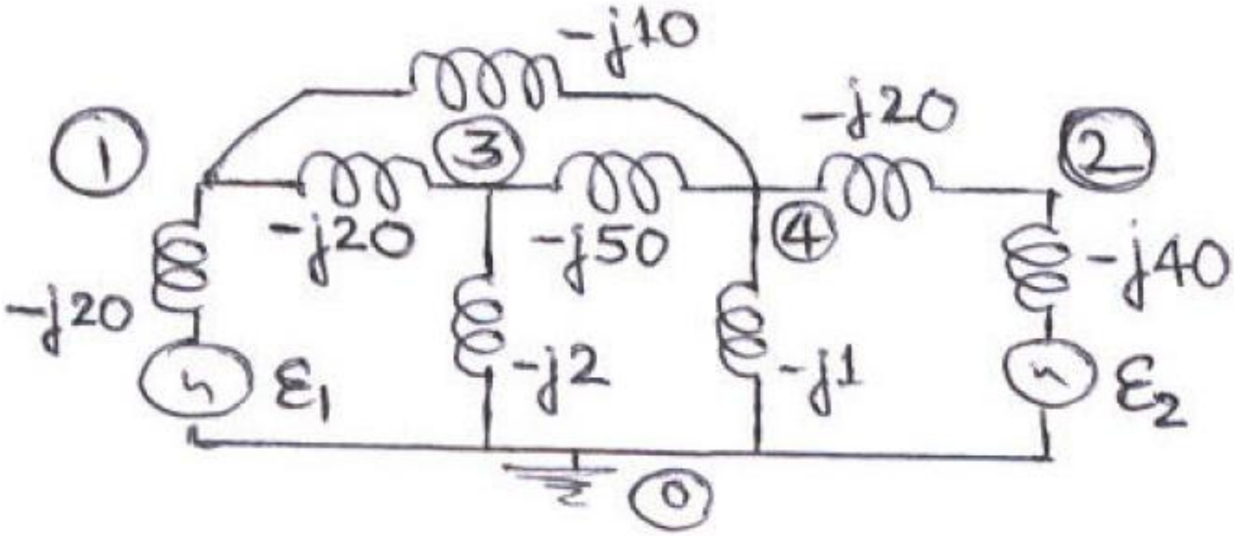
$$Y_{12} = Y_{21} = Y_{12} - Y_{13}Y_{32} / Y_{33} = j7.86$$

Thus the reduced network can be obtained again by the rule of inspection as shown below.





Example-2: Obtain YBUS for the admittance network shown below by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



Y_{BUS}=

n/n	1	2	3	4
1	-j50	0	j20	j10
2	0	-j60	0	j72
3	j20	0	-j72	j50
4	j10	j72	j50	-j81

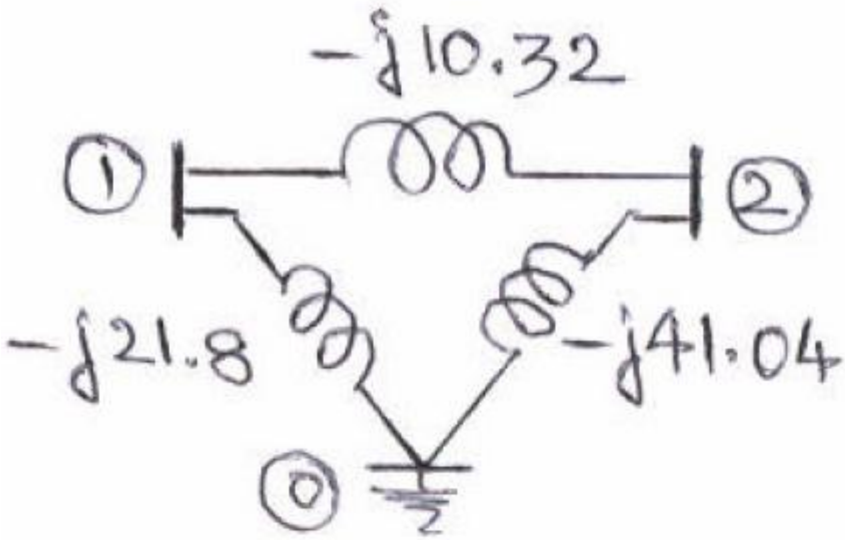
$$= \begin{vmatrix} Y_A & Y_B \\ Y_C & Y_D \end{vmatrix}$$

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

Y_{BUS}^{new}=

n/n	1	2
1	-j32.12	j10.32
2	j10.32	-j51.36

Thus the reduced system of two nodes can be drawn by the rule of inspection as under:



2.5 ZBUS building

FORMATION OF BUS IMPEDANCE MATRIX

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\overline{E}_{bus} = [Z_{bus}]\overline{I}_{bus} \qquad (9)$$

When expanded so as to refer to a n bus system, (9) will be of the form

$$E_1 = Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots + .Z_{1k}I_k \ldots + Z_{1n}I_n$$
$$\vdots$$
$$\vdots$$
$$E_k = Z_{k1}I_1 + Z_{k2}I_2 + \ldots\ldots + Z_{kk}I_k + \ldots. + Z_{kn}I_n$$
$$\vdots$$
$$\vdots$$
$$E_n = Z_{n1}I_1 + Z_{n2}I_2 + \ldots\ldots\ldots + Z_{nk}I_k + \ldots.. + Z_{nn}I_n \qquad (10)$$

Now assume that the bus impedance matrix Zbus is known for a partial network of m buses and a known reference bus. Thus, Zbus of the partial network is of dimension mxm. If now a new element is added between buses p and q we have the following two possibilities:

(i) p is an existing bus in the partial network and q is a new bus; in this case p-q is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both *p* and *q* are buses existing in the partial network; in this case *p*-*q* is a **link** added to the p-network as shown in Fig 1b.

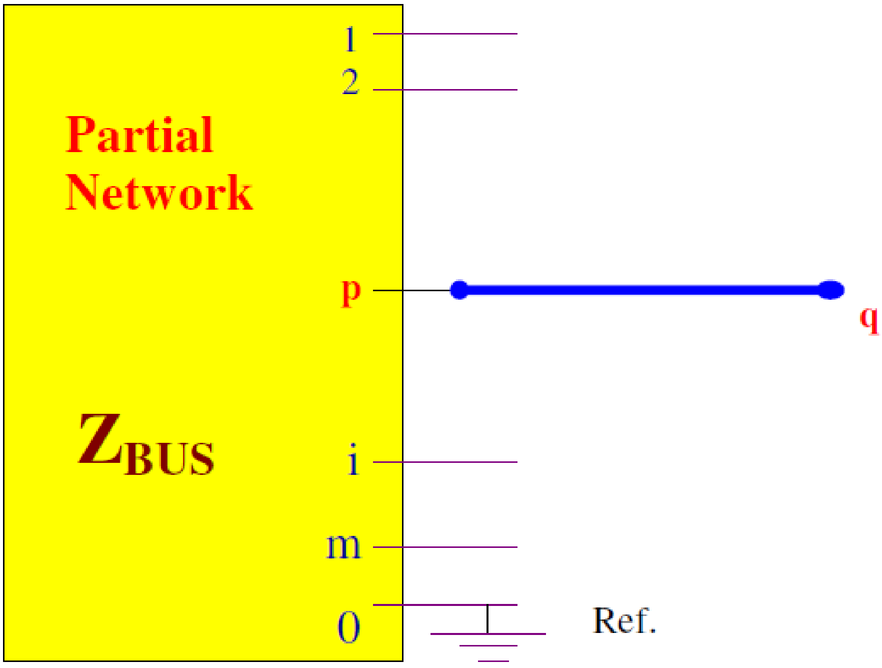


Fig 1a. Addition of branch p-q

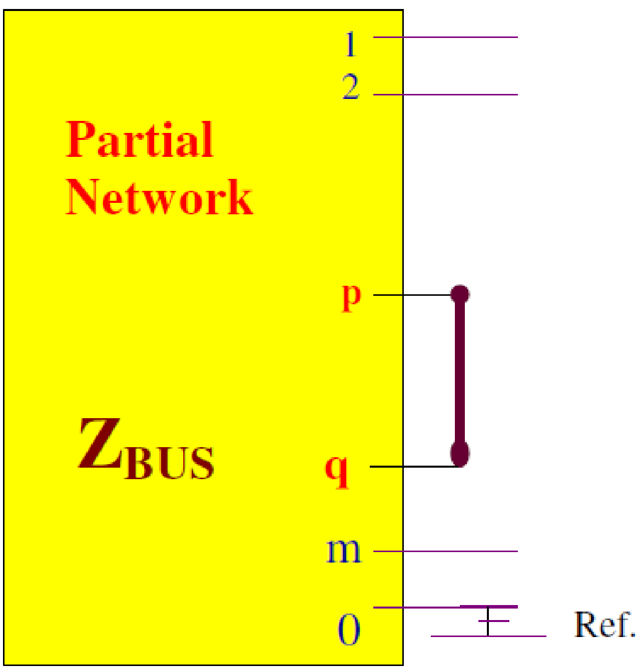


Fig 1b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix. If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

ADDITION OF A BRANCH

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \qquad (11)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector ypq-rs is not equal to zero and **Zij= Zji** " i,j=1,2,…m,q
(12)

To find Zqi:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

Ek = Zki Ii = Zki " k = 1, 2,…i…....p,….m, q
(13)
Hence, Eq = Zqi ; Ep = Zpi ………

Also, Eq=Ep -vpq ; so that Zqi = Zpi - vpq " i =1, 2,…i…....p,….m, _q
(14)

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \tag{15}$$

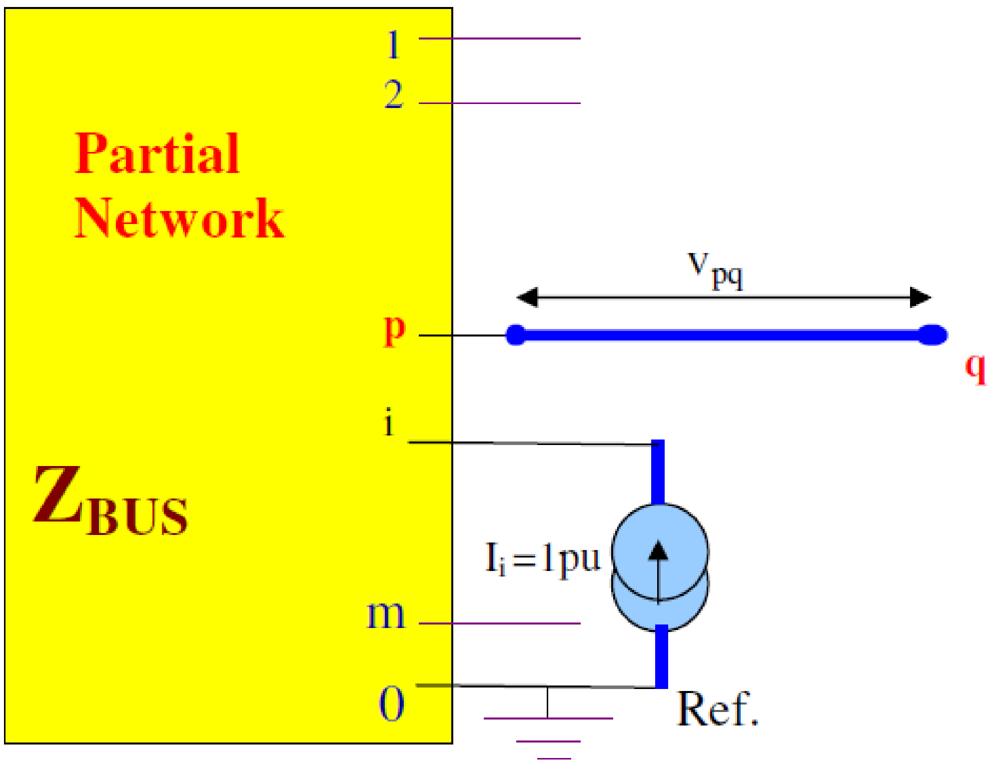


Fig.2 Calculation for Zqi

where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \qquad (16)$$

Solving, $v_{pq} = -\frac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{17}$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \quad i = 1,2.....m; i \neq q \tag{18}$$

To find zqq:

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, $I_q = 1.0$ pu. As before, we have the relations as under:

$E_k = Z_{kq}\, I_q = Z_{kq}$ $\quad \forall$ k = 1, 2,…i…...p,….m, q (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq}v_{pq} + \bar{y}_{pq,rs}\bar{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{21}$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rq} - \bar{Z}_{sq}\right)}{y_{pq,pq}} \tag{22}$$

Special Cases

The following special cases of analysis concerning ZBUS building can be considered with respect to the addition of branch to a p-network.

Case (a): If there is no mutual coupling then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then $E_p=0$. thus,

$$Z_{pi} = 0 \qquad i = 1,2.....m : i \neq q$$

And $Z_{pq} = 0.$

Hence, from (18) (22) $Z_{qi} = 0 \qquad i = 1,2......m; i \neq q$

And $Z_{qq} = z_{pq,pq}$ \ (23)

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1,2....m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \qquad (24)$$

ADDITION OF A LINK

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \qquad (25)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector $y_{pq\text{-}rs}$ is not equal to zero and $\mathbf{Z_{ij} = Z_{ji}}$ $\forall$ i,j=1,2,…m,l. (26)

To find Zli:

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, el in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki} I_i = Z_{ki} \qquad \forall \; k = 1, 2, \ldots i \ldots\ldots p, \ldots.m, l \tag{27}$$

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_p = Z_{pi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$;

$$\text{So that } Z_{li} = Z_{pi} - Z_{qi} - v_{pq} \quad \forall \; i=1,2,\ldots i \ldots.p,\ldots q,\ldots.m, \neq l \tag{28}$$

To find vpq:
In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \tag{29}$$

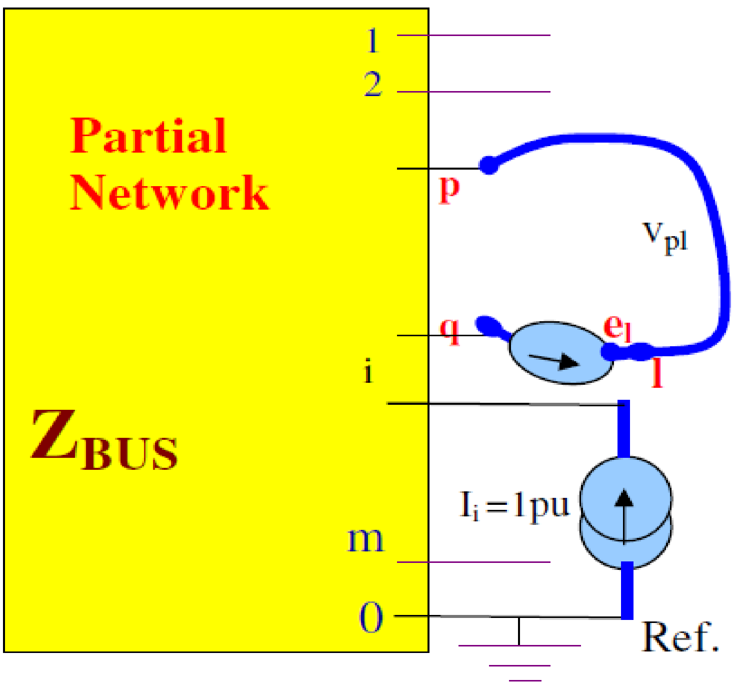


Fig.3 Calculation for Z_{li}

where i_{pl} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pl} is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl}v_{pl} + \bar{y}_{pl,rs}\bar{v}_{rs} = 0 \tag{30}$$

Solving, $v_{pl} = -\dfrac{\bar{y}_{pl,rs}\bar{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \tag{31}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And

$$y_{pl,pl} = y_{pq,pq} \tag{32}$$

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2.....m; i \neq l \tag{33}$$

To find Z_{ll}:

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, I_l = 1.0 pu. As before, we have the relations as under:

$E_k = Z_{kl}\, I_l = Z_{kl} \quad \forall\ k = 1, 2, \ldots i \ldots p, \ldots q, \ldots m, l$ (34)

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

So that $Z_{ll} = Z_{pl} - Z_{ql} - v_{pl} \quad \forall\ i=1,2,\ldots i \ldots p, \ldots q, \ldots m, \neq l$ (35)

Since now the current in the added element is $i_{pl} = -I_l = -1.0$, we have from (29)

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs} \bar{v}_{rs} = -1$$

Solving, $v_{pl} = -1 + \dfrac{\bar{y}_{pl,rs} \bar{v}_{rs}}{y_{pl,pl}}$

$$v_{pl} = -1 + \frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \tag{36}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $y_{pl,pl} = y_{pq,pq}$ (37)

Using (34), (36) and (37) in (35), we get

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \tag{38}$$

Special Cases Contd….

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1,2 \ldots m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \tag{39}$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

Case (d): If there is no mutual coupling, then elements of *pq rs y* , are zero. Further, if *p* is not the reference node, then

$$Z_{li} = Z_{pi} - Z_{qi}$$

$$\begin{aligned} Z_{ll} &= Z_{pl} - Z_{ql} - z_{pq,pq} \\ &= Z_{pp} + Z_{qq} - 2\,Z_{pq} + z_{pq,pq} \end{aligned} \tag{40}$$

2.6 MODIFICATION OF ZBUS FOR NETWORK CHANGES

An element which is not coupled to any other element can be removed easily. The Zbus is modified as explained in sections above, by adding in parallel with the element (to be removed), a link whose impedance is equal to the negative of the impedance of the element to be removed. Similarly, the impedance value of an element which is not coupled to any other element can be changed easily. The Zbus is modified again as explained in sections above, by adding in parallel with the element (whose impedance is to be changed), a link element of impedance value chosen such that the parallel equivalent impedance is equal to the desired value of impedance. When mutually coupled elements are removed, the Zbus is modified by introducing appropriate changes in the bus currents of the original network to reflect the changes introduced due to the removal of the elements.

Examples on ZBUS building

Example 1: For the positive sequence network data shown in table below, obtain ZBUS by building procedure.

Sl. No.	p-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

Solution:
The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.

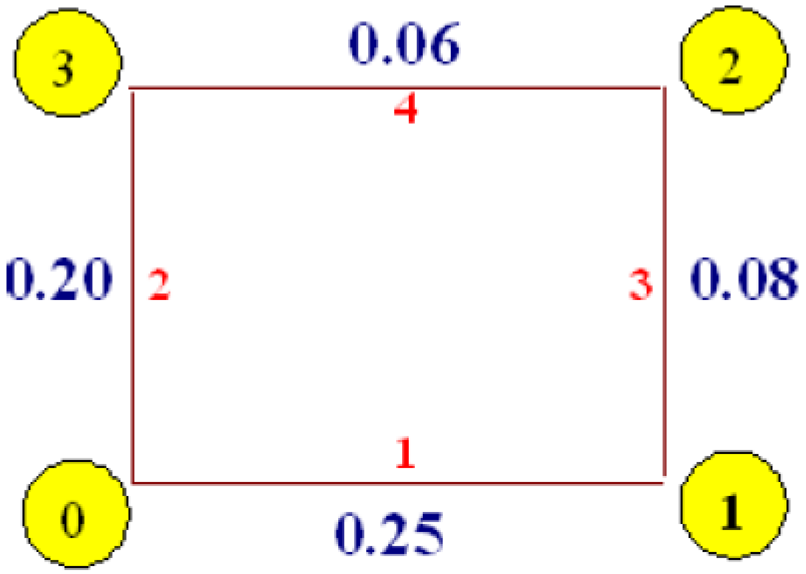
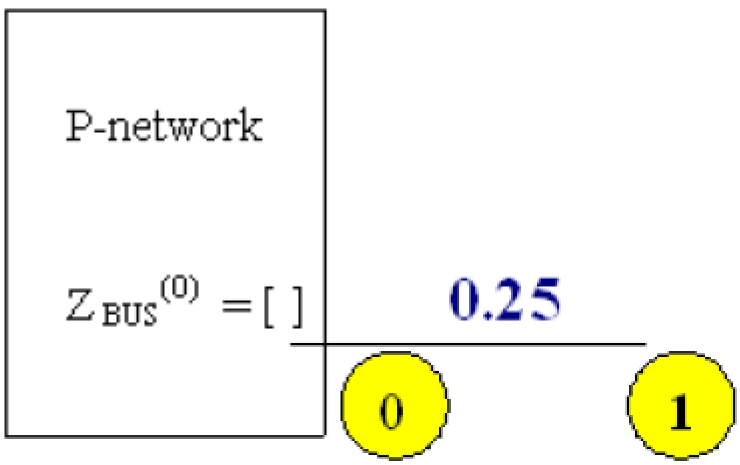


Fig. E1: Example System

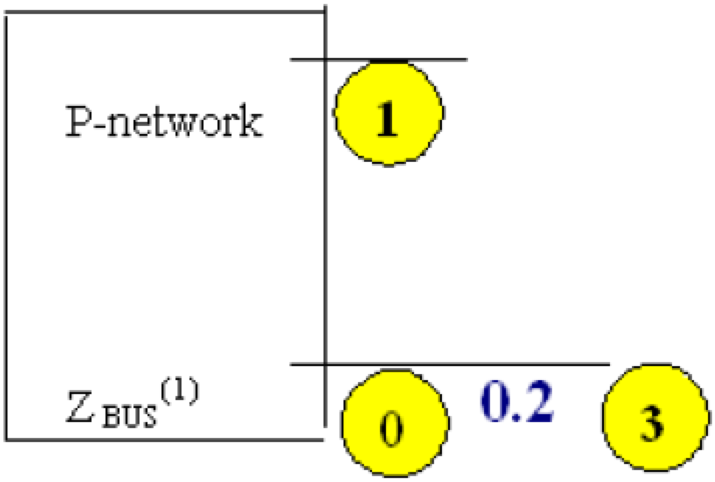
Consider building ZBUS as per the various stages of building through the consideration of the corresponding partial networks as under:

Step-1: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



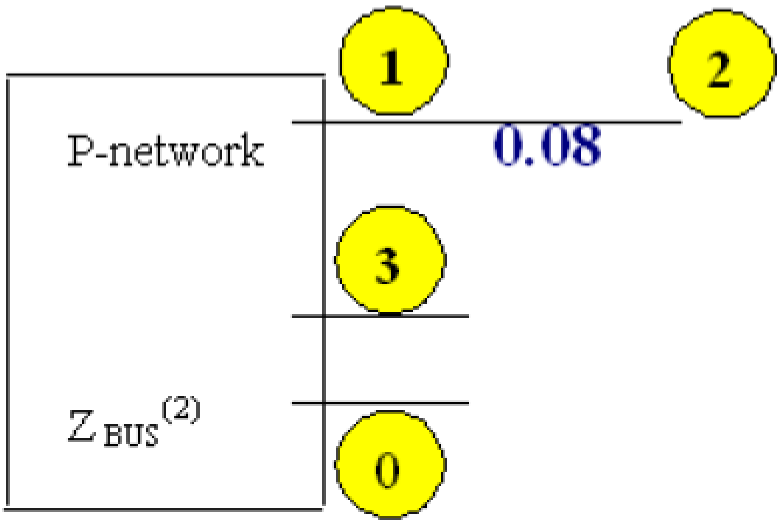
$$Z_{BUS}^{(1)} = \begin{array}{cc} & 1 \\ 1 & \boxed{0.25} \end{array}$$

Step-2: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



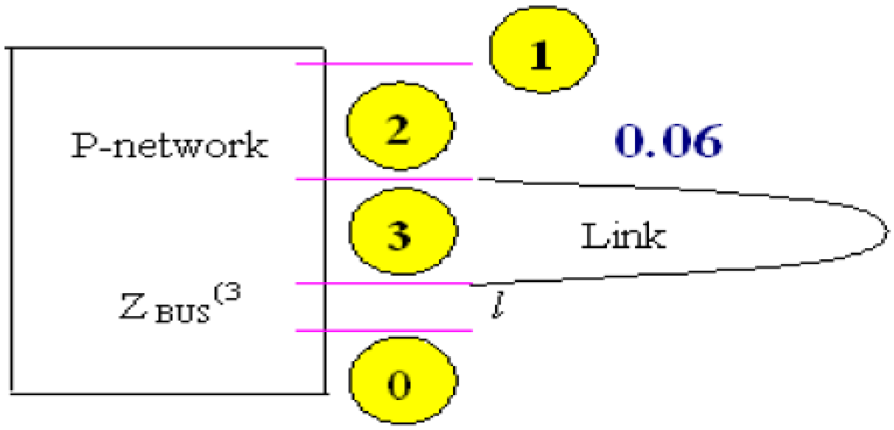
$$Z_{BUS}^{(2)} = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{cc} 1 & 3 \\ \hline 0.25 & 0 \\ 0 & 0.2 \end{array}$$

Step-3: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;



$$Z_{BUS}^{(3)} = \begin{array}{c} \\ 1 \\ 3 \\ 2 \end{array} \begin{array}{ccc} 1 & 3 & 2 \\ \hline 0.25 & 0 & 0.25 \\ 0 & 0.2 & 0 \\ 0.25 & 0 & 0.33 \end{array}$$

Step-4: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;



$Z_{BUS}^{(4)} =$

	1	3	2	l
1	0.25	0	0.25	0.25
3	0	0.2	0	-0.2
2	0.25	0	0.33	0.33
l	0.25	-0.2	0.33	0.59

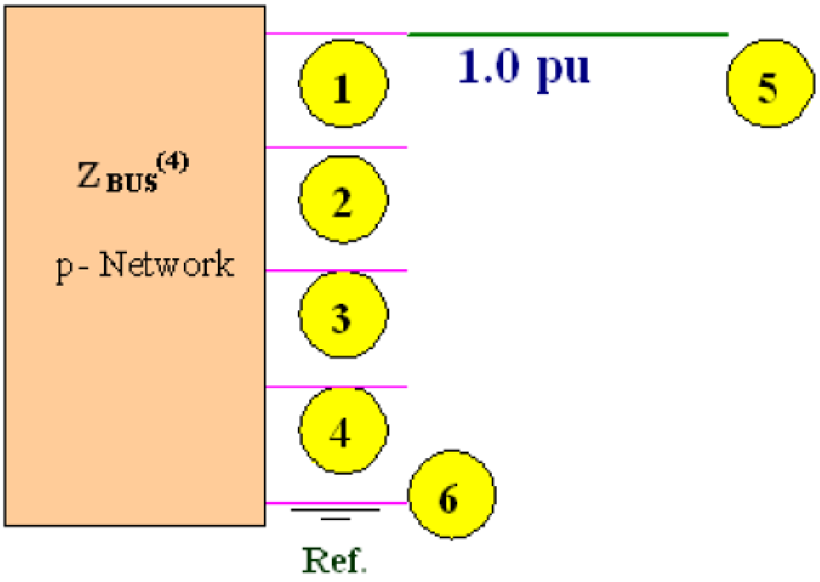
The fictitious node *l* is eliminated further to arrive at the final impedance matrix as under:

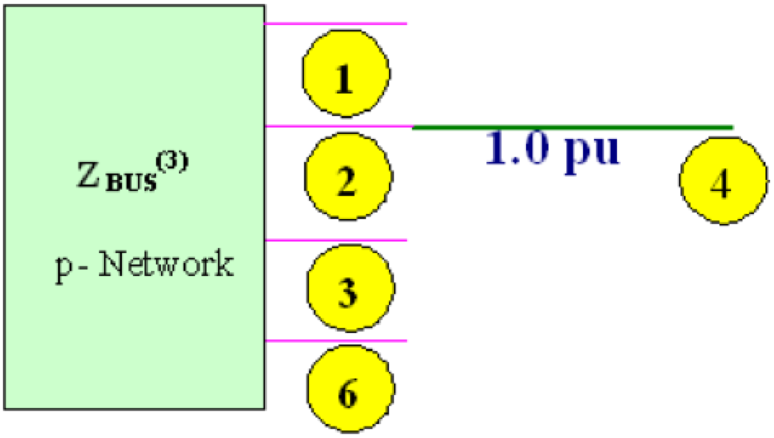
$Z_{BUS}^{(final)} =$

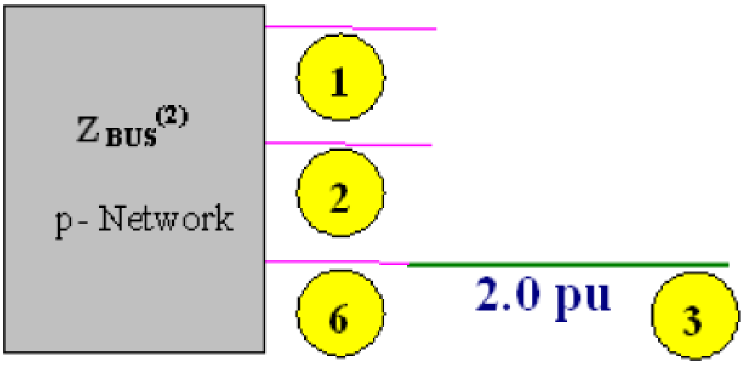
	1	3	2
1	0.1441	0.0847	0.1100
3	0.0847	0.1322	0.1120
2	0.1100	0.1120	0.1454

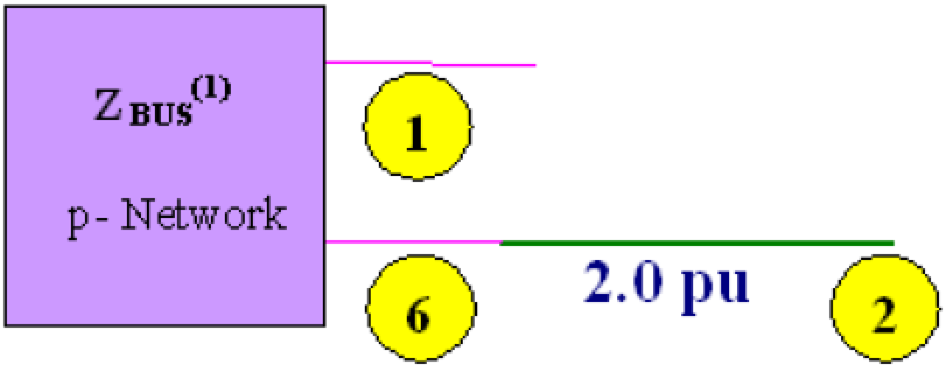
$Z_{BUS} =$

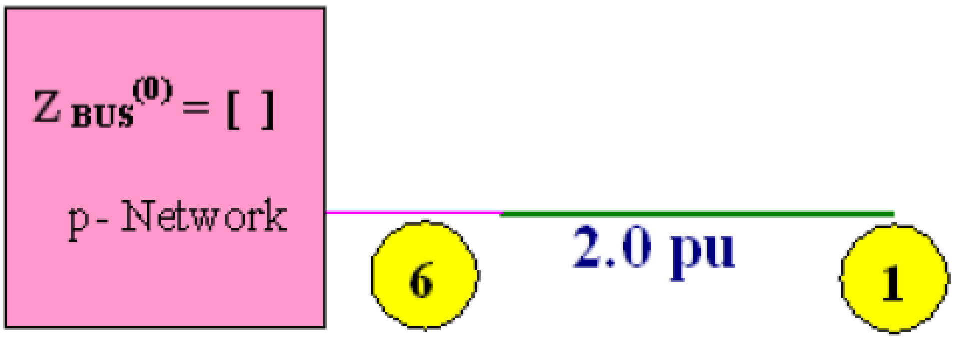
	1	2	3	4	5
1	2	0	0	0	2
2	0	2	0	2	0
3	0	0	2	0	0
4	0	2	0	3	0
5	2	0	0	0	3

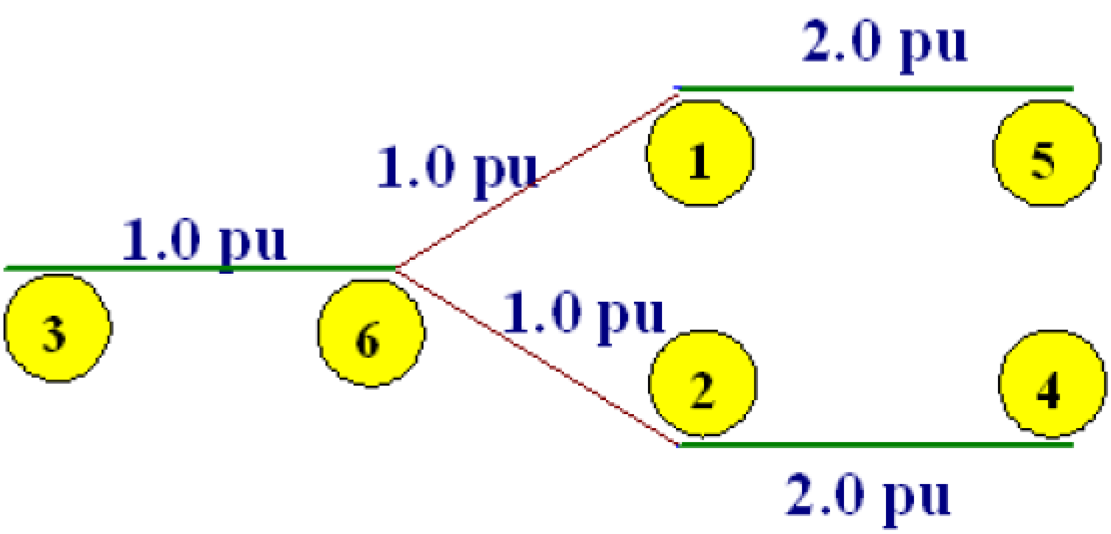

Z BUS(4)
p- Network
1
2
3
4
5
6
1.0 pu
Ref.

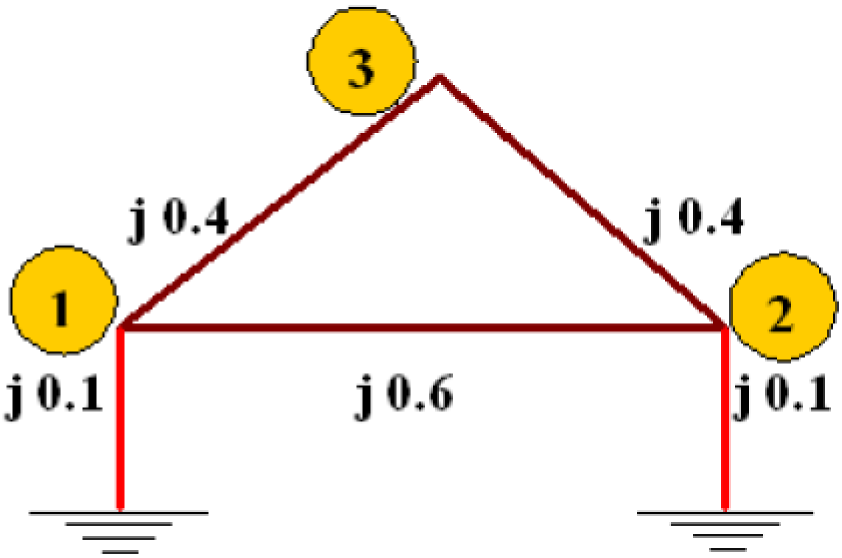

Z BUS(3)
p- Network
1
2
3
6
4
1.0 pu


Z BUS(2)
p- Network
1
2
6
3
2.0 pu


Z BUS(1)
p- Network
1
6
2.0 pu
2

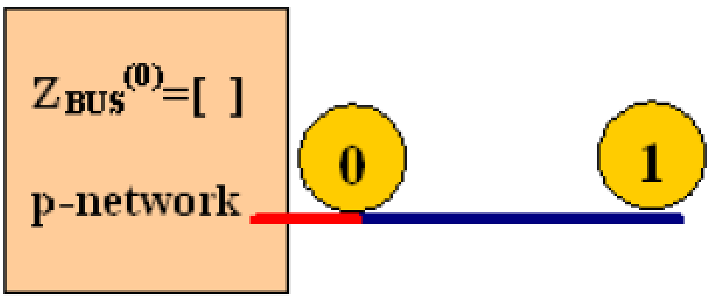

Z BUS(0) = [ ]
p- Network
6
2.0 pu
1


2.0 pu
1.0 pu
1
5
1.0 pu
3
6
1.0 pu
2
4
2.0 pu



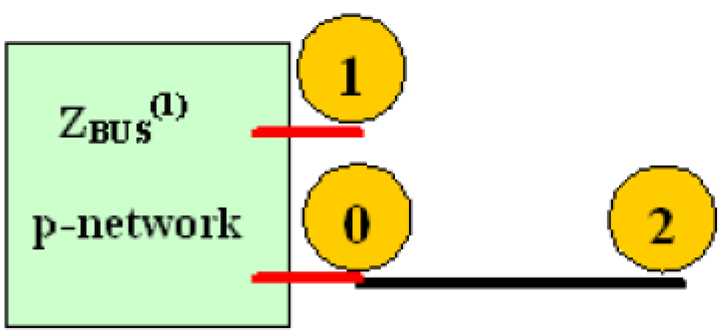
Solution: The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

Step1: Add branch 1 between node 1 and reference node. (q =1, p = 0)



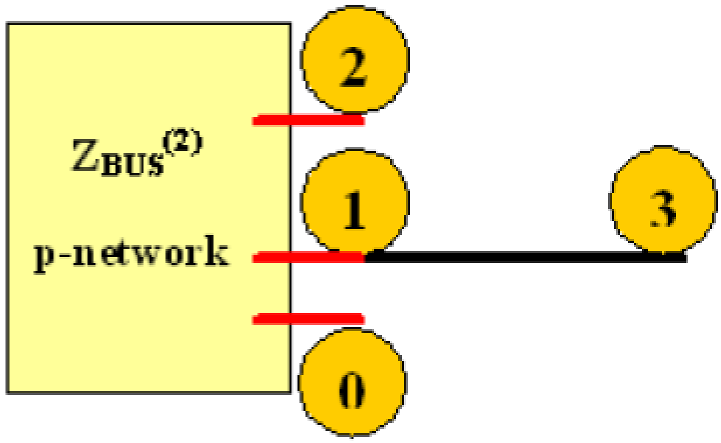
$$Z_{bus}^{(1)} = \begin{array}{cc} & 1 \\ 1 & \left[j0.1 \right] \end{array}$$

Step2: Add branch 2, between node 2 and reference node. (q = 2, p = 0).



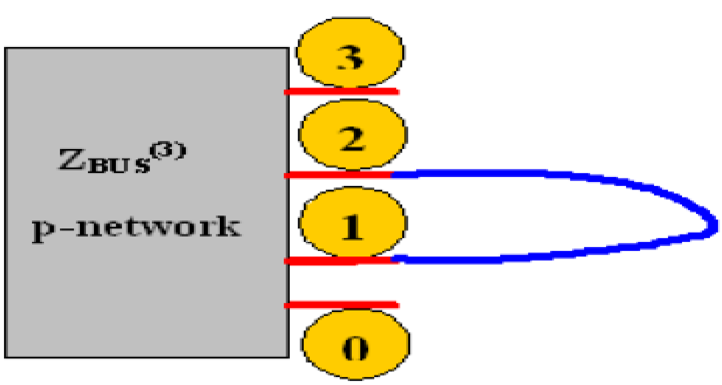
$$Z_{Bus} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & j0.1 & 0 \\ 2 & 0 & j0.15 \end{array}$$

Step3: Add branch 3, between node 1 and node 3 (p = 1, q = 3)



$$Z_{Bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j0.1 & 0 & j0.1 \\ 2 & 0 & j0.15 & 0 \\ 3 & j0.1 & 0 & j0.5 \end{array}$$

Step 4: Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



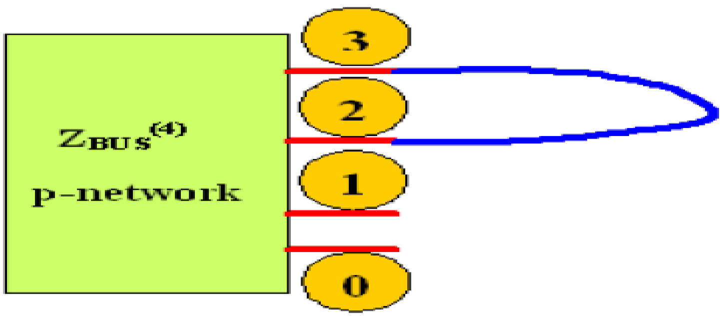
$$Z_{bus} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ l \end{array} & \begin{bmatrix} j0.1 & 0 & j0.1 & j0.1 \\ 0 & j0.15 & 0 & -j0.15 \\ j0.1 & 0 & j0.5 & j0.1 \\ j0.1 & -j0.15 & j0.1 & j0.85 \end{bmatrix} \end{array}$$

Now the extra node-*l* has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\, Y_{nj} / Y_{nn}} \qquad \forall\ i,j = 1,2,3.$$

$$Z_{bus} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix} \end{array}$$

Step 5: Add link between node 2 and node 3 (p = 2, q=3)



$$Z_{l1} = Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058$$
$$Z_{l2} = Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588$$

$$Z_{l3} = Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058$$

$$Z_{ll} = Z_{2l} - Z_{3l} + z_{23,23}$$

$$= j0.10588 - (-j0.47058) + j0.4 = j0.97646$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{array}$$

Node l is eliminated as shown in the previous step:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{array}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under:

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{array}$$

As a check, it can be observed that the bus admittance matrix, Y_{BUS} can also be obtained by the rule of inspection to arrive at the same answer.

UNIT-3 &4

LOAD FLOW STUDIES

3.1 REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. Solution Linear equations:
* **Direct methods**:
- Cramer's (Determinant) Method,
- Gauss Elimination Method (only for smaller systems),
- LU Factorization (more preferred method), etc.

*** Iterative methods**:
- Gauss Method
- Gauss-Siedel Method (for diagonally dominant systems)

3. Solution of Nonlinear equations:
Iterative methods only:
- Gauss-Siedel Method (for smaller systems)
- Newton-Raphson Method (if corrections for variables are small)

4. Solution of differential equations:
Iterative methods only:
- Euler and Modified Euler method,
- RK IV-order method,
- Milne's predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:
_ Selection of initial solution/ estimates
_ Determination of fresh/ new estimates during each iteration
_ Selection of number of iterations as per tolerance limit
_ Time per iteration and total time of solution as per the solution method selected
_ Convergence and divergence criteria of the iterative solution
_ Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:
In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate. The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the off diagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.The Newton-Raphson method is very much useful for solution of non –linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

3.2 LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load flow studies play a vital role in power system studies. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:
_ The Kirchhoff's relations holding good,
_ Capability limits of reactive power sources,
_ Tap-setting range of tap-changing transformers,
_ Specified power interchange between interconnected systems,
_ Selection of initial values, acceleration factor, convergence limit, etc.

3.3 Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G,Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 00 , as per flat-start procedure of iterative
solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of YBUS based LFA:

The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and hence only n(n+1)/2 elements of YBUS need to be stored for a n-bus system. Further, in the YBUS matrix, Yij = 0, if an incident element is not present in the system connecting the buses 'i' and 'j'. since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, YBUS of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\text{Percentage sparsity of a given matrix of } n^{th} \text{ order:} = \frac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$$

$$S = (Z / n^2) \times 100 \ \% \qquad (1)$$

The percentage sparsity of Y_{BUS}, in practice, could be as high as 80-90%, especially
for very large, practical power systems. This sparsity feature of YBUS is extensively used in reducing the load flow calculations and in minimizing the memory required to store the

coefficient matrices. This is due to the fact that only the non-zero elements YBUS can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While YBUS is thus highly sparse, it's inverse, ZBUS, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

3.4 THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \qquad \textbf{(2)}$$

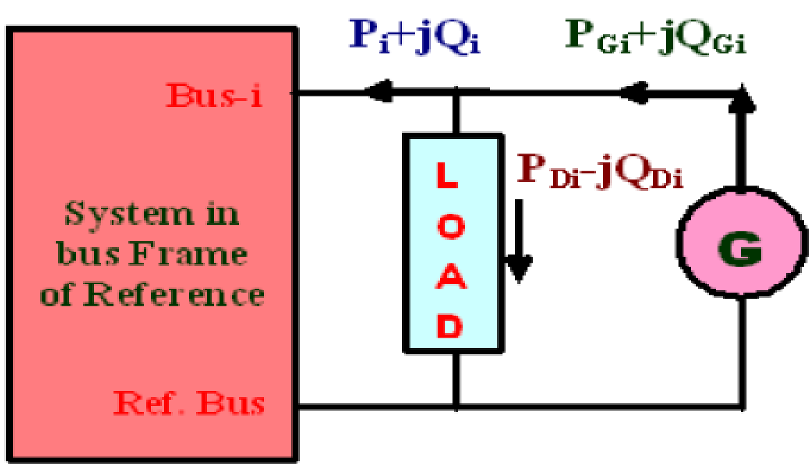


Fig.1 power flows at a bus-i

where Si = net complex power injected into bus *i*, SGi = complex power injected by the generator at bus *i*, and SDi = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

Si = _Sij " i = 1, 2, ………..n
(3)

where Sij is the sum over all lines connected to the bus and *n* is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

Ii = IGi – IDi " i = 1, 2, ………..n **(4)** where IGi is the current injected by the generator at the bus and IDi is the current drawn by the load (demand) at that bus. In the bus frame of reference

IBUS = YBUS VBUS
(5)

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix} \quad \text{is the vector of currents injected at the buses,}$$

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix} \quad \text{is the vector of complex bus voltages.}$$

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(6)}$$

The complex power S_i is given by

$$S_i = V_i\ I_i^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\ V_j \right)^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \qquad \textbf{(7)}$$

Let $\quad V_i \triangleq |V_i| \angle \delta_i = |V_i| (\cos\ \delta_i + j \sin\ \delta_i)$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (\cos\delta_{ij} + j\sin\delta_{ij})\,(G_{ij} - j\,B_{ij}) \tag{8}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) \tag{9}$$

$$Q_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, (G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) \tag{10}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $\quad Y_{ij} = |Y_{ij}| \angle\theta_{ij}$ **(11)**

Again, we get from (7),

$$S_i = |V_i| \angle\delta_i \sum_{j=1}^{n} |Y_{ij}| \angle -\theta_{ij} \, |V_j| \angle -\delta_j \tag{12}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \cos(-\theta_{ij} + \delta_i - \delta_j)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \cos -(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n, \tag{13}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \sin -(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i| \, |V_j| \, |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \tag{14}$$

Equations (9)-(10) and (13)-(14) are the 'power flow equations' or the 'load flow equations' in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, Pi, Qi, |Vi|, and di. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

3.4 DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses-*n*, number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0o), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus *i*, the data required includes the bus number, active power generation PGi, the specified voltage magnitude *i sp V* , , minimum reactive power limit Qi,min, and maximum reactive power limit Qi,max.

Load data: For all loads the data required includes the the bus number, active power demand PDi, and the reactive power demand QDi.

Transmission line data: For every transmission line connected between buses *i* and *k* the data includes the starting bus number *i*, ending bus number *k*,.*r*esistance of the line, reactance of the line and the half line charging admittance.

Transformer data:
For every transformer connected between buses *i* and *k* the data to be given includes: the starting bus number *i*, ending bus number *k*, resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio *a*.

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance (Gsh + j Bsh).

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:
Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \quad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \quad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \quad \forall\ i = 2, 3, \ldots\ldots\ldots n \quad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

3.5 Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0\angle 0^0$. This is normally refered as the ***flat start*** solution.
4. Update the voltages. In any $(k+1)st$ iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} \mathrm{V}_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} \mathrm{V}_j^{(k)}\right] \quad \forall \text{ i=2,3,...n} \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current *(k+1)st* iteration. Hence these values are used. For buses *(i+1)…..n,* values from previous, *kth* iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall \text{ i = 2,3,...n} \qquad \textbf{(19)}$$

Where,e is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = \mathrm{P}_1 - \mathrm{jQ}_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

7. Compute all line flows.
8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:
At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\,\mathrm{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij}\, V_j\right\}$$

Where *Im* stands for the imaginary part. At any $(k+1)^{st}$ iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\,\mathrm{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij}\, V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij}\, V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:
In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e (*k* +1) *i Q* computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the *(k+1)st* iteration and the voltage is calculated with the value of Qi set as follows:

If $Q_i < Q_{i,min}$ If $Q_i > Q_{i,max}$

Then $Q_i = Q_{i,min}$. Then $Q_i = Q_{i,max}$.

(23)

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \qquad \textbf{(24)}$$

where α is a real number. When α =1, the value of (*k* +1) is the computed value. If 1<α <2 then the value computed is extrapolated. Generally _ is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

3.6 Examples on GS load flow analysis:

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if $V_1 = 1\angle 0^0$ pu.*

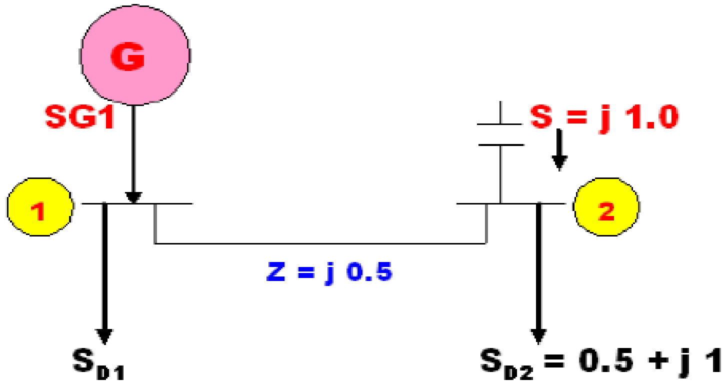


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\,1.0) = -\,0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}\,V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776 \ \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\ 0.23529 \ = 0.970145 \ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\ 0.249999 \ = 0.970261 \ \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\ 0.248963 = 0.966237 \ \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\ 0.25 \ = 0.966237 \ \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10-6 pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472 \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472 \angle 14.931^0$$

$$= 0.5 + \mathrm{j}\ 0.133329 \text{ pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

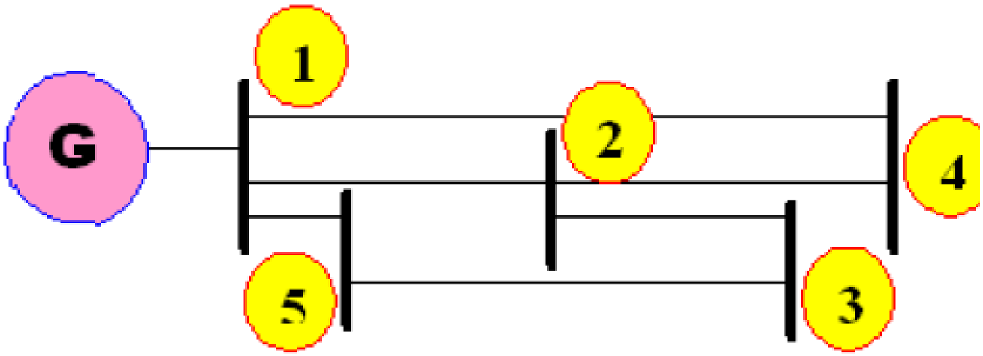
$$= 0.517472 \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -\ 0.5 + \mathrm{j}\ 0.0 \text{ pu}$$

The total loss in the line is given by S12 + S21 = j 0.133329 pu Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{SP} \rvert$ (pu)	δ
1	-	-	-	-	1.02	0°
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

$P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

$Y_{bus} =$

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be $V_1^0 = 1.02+j0.0$ in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294)\times 1.0\angle 0^0\}\right]$$

$$= 0.98140\angle -3.0665^{\circ} = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3||V_4|(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3||V_5|(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^o$; $\delta_2 = -3.0665^o$; $\delta_3 = 0^o$; $\delta_4 = 0^o$ and $\delta_5 = 0^o$

$\therefore$ $\delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^o$ ($\delta_{ik} = \delta_i - \delta_k$); $\delta_{32} = 3.0665^o$

$Q_3 = 1.04\,[1.02\,(0.0+j0.0) + 0.9814\,\{-1.17647 \times \sin(3.0665^o) - 4.70588$ $\times\cos(3.0665^o)\} + 1.04\{-9.41176\times\cos(0^o)\} + 1.0\,\{0.0 + j0.0\} + 1.0\{-4.70588\times\cos(0^0)\}]$

$= 1.04\,[-4.6735 + 9.78823 - 4.70588] = 0.425204$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588) \times (0.98140\angle -3.0665^o)\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times (1\angle 0^o)\}\right]$$

$$= 1.05569 \angle 3.077^o = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^0 \text{pu}$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-0.58823 + j2.35294) \times (0.98140\angle -3.0665^o)\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle -7.303^o \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^o - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times 1.04\angle 3.077^o\}\right]$$

$$= 0.994618\angle -1.56^o = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^0$ pu $\qquad V_2 = 0.98140\ \angle -3.066^0$ pu

$V_3 = 1.04\angle 3.077^0$ pu $\qquad V_4 = 0.955715\angle -7.303^0$ pu

and $\qquad V_5 = 0.994618\angle -1.56^0$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) All buses except bus 1 are PQ Buses

(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and 0.25_Q2_1.0 pu.

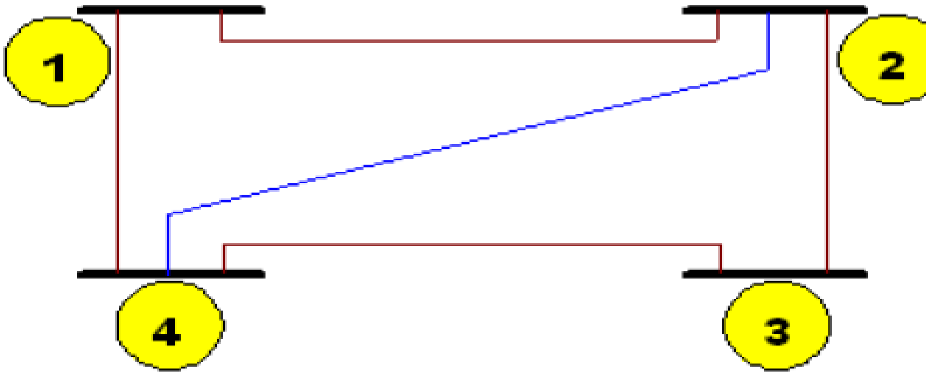


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04\angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$Y_{BUS} =$

3.0 – j9.0	–2.0 + j6.0	– 1.0 + j3.0	0
–2.0 + j6.0	3.666 – j11.0	– 0.666 + j2.0	– 1.0 + j3.0
–1.0 + j3.0	–0.666 + j2.0	3.666 – j11.0	–2.0 + j6.0
0	–1.0 + j3.0	– 2.0 + j6.0	3.0 – j9.0

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0\angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}\,V_1^o - Y_{23}\,V_3^0 - Y_{24}\,V_4^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.02014\angle 2.605^o$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.02014\angle 2.605^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.03108\angle -4.831^o$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.02014\angle 2.605^o)\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.03108\angle -4.831^o)\}\right]$$

$$= 1.02467\angle -0.51^o$$

Hence

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.02014\ \angle 2.605^0$ pu

$V_3^1 = 1.03108\ \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467\ \angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\left[|V_1|\left(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}\right) + |V_2|\left(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22}\right) + |V_3|\left(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}\right) + |V_4|\left(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24}\right)\right]$$

$$= 1.04\,[1.04\,\{-6.0\} + 1.04\,\{11.0\} + 1.0\{-2.0\} + 1.0\,\{-3.0\} = 0.208 \text{ pu.}$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\} - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^o)\}\right]$$

$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04\angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\angle 0^0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\} - \{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.035587\angle -4.951^{\circ} \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^o)\}\right]$$

$$= 0.9985\angle -0.178^{\circ}$$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $\qquad V_4^1 = 0.9985\angle -0.178^0$ pu

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \le Q_2 \le 1$ pu. If $0.25 \le Q_2 \le 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad V_2^1 = 1.05645 \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^0 \text{ pu}$$

Limitations of GS load flow analysis

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of *n* non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots\ldots x_n) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(25)}$$

Let $x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots x_n^{\,0}$, be the initial guess of unknown variables and $\Delta x_1^{\,0}, \Delta x_2^{\,0} \ldots\ldots \Delta x_n^{\,0}$ be the respective corrections. Therefore,

$$f_i(x_1^{\,0} + \Delta x_1^{\,0}, x_2^{\,0} + \Delta x_2^{\,0} \ldots\ldots\ldots x_n^{\,0} + \Delta x_n^{\,0}) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(26)}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^{\,0}, x_2^{\,0} \ldots\ldots x_n^{\,0}) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^{\,0} + \left(\frac{\partial f_i}{\partial x_2}\right)\Delta x_2^{\,0} + \ldots.. + \left(\frac{\partial f_i}{\partial x_n}\right)\Delta x_n^{\,0}\right]$$

$$+ \text{Higher order terms} = 0 \qquad \forall\ i = 1,2\ldots\ldots n \qquad \textbf{(27)}$$

Where, $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at ($x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots x_0^{\,n}$). If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^{\,0} \\ f_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ f_n^{\,0} \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \left(\frac{\partial f_n}{\partial x_1}\right) & \left(\frac{\partial f_n}{\partial x_2}\right) & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_n}{\partial x_n}\right) \end{bmatrix} \begin{bmatrix} \Delta x_1^{\,0} \\ \Delta x_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_n^{\,0} \end{bmatrix} = 0 \qquad \textbf{(28)}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $\qquad F^0 = -J^0 \Delta X^0$

Or $\qquad \Delta X^0 = -[J^0]^{-1} F^0 \qquad$ **(29)**

And $\qquad X^1 = X^0 + \Delta X^0 \qquad$ **(30)**

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ... x_n) = 0$, where $x_1, x_2, ... x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \quad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \quad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \qquad \textbf{(33)}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \qquad \textbf{(34)}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \qquad \textbf{(35)}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J₃

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J₂

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i|\sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J₄

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

DECOUPLED LOAD FLOW

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are,

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation the implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero. , L

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \qquad \textbf{(37)}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1} \Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} = \left[L^{(r)}\right]^{-1} \Delta Q^{(r)}$

(v)Update, $|V^{(r+1)}| = |V^{(r)}| + |\Delta V^{(r)}|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$$

The matrix (37) reduces to

$$[\Delta P] = \left[|V_i||V_j|B'_{ij}\right][\Delta\delta]$$

$$[\Delta Q] = \left[|V_i||V_j|B''_{ij}\right]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(38)}$$

Where B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = \left[B'_{ij}\right][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = \left[B''_{ij}\right]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(39)}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)

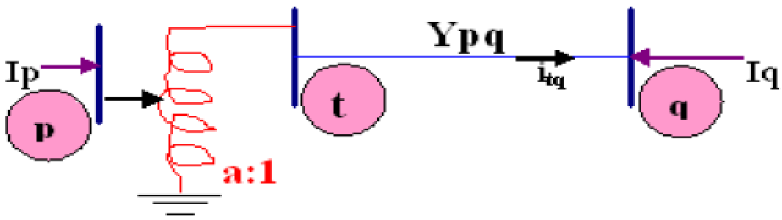


Fig. 2. Equivalent circuit of a tap setting transformer

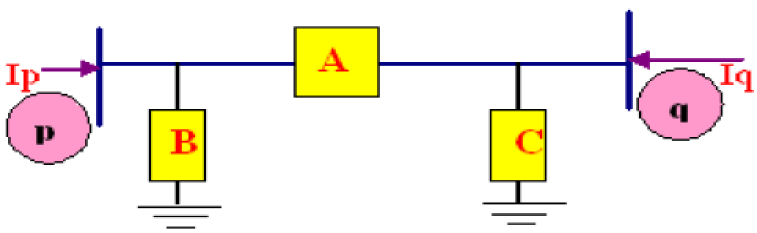


Fig. 3. π-Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the π-equivalent circuit parameters are given by the expressions as under:

(i) Fixed tap setting transformers (on no load)

A = Ypq/ a

B = 1/a (1/a -1) Ypq

C = (1-1/a) Ypq

(i) Tap changing under load (TCUL) transformers (on load)

A = Ypq

B = (1/a -1) (1/a + 1 – Eq/Ep) Ypq

C = (1-1/a) (Ep/Eq) Ypq

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.

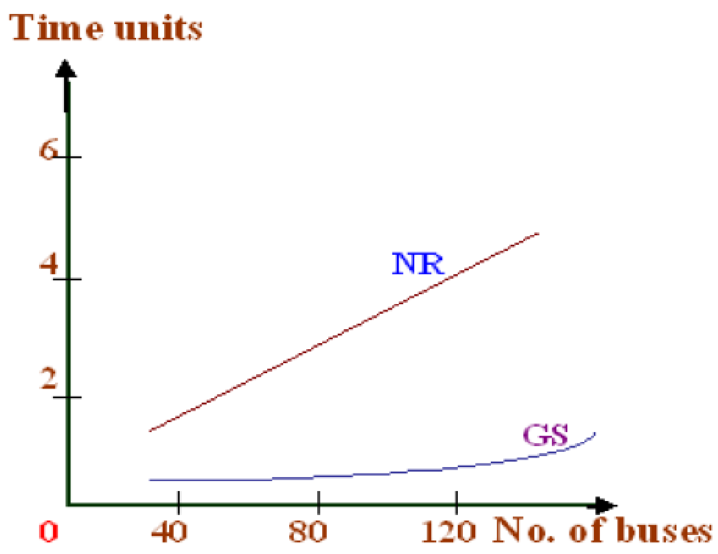


Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

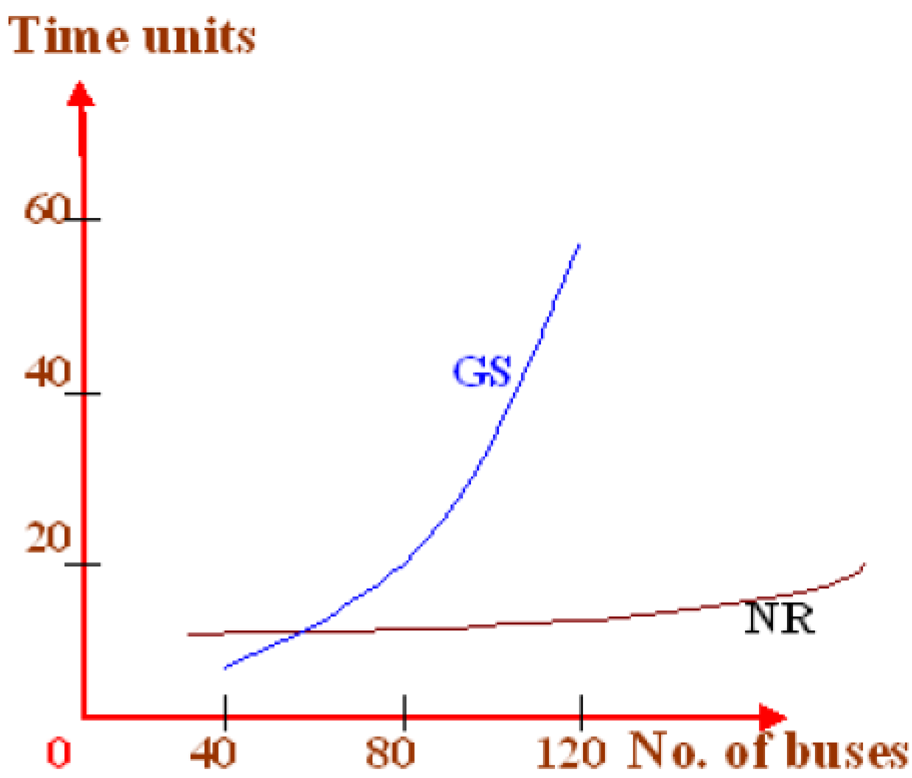


Figure 5. Total time of Iteration in GS and NR methods

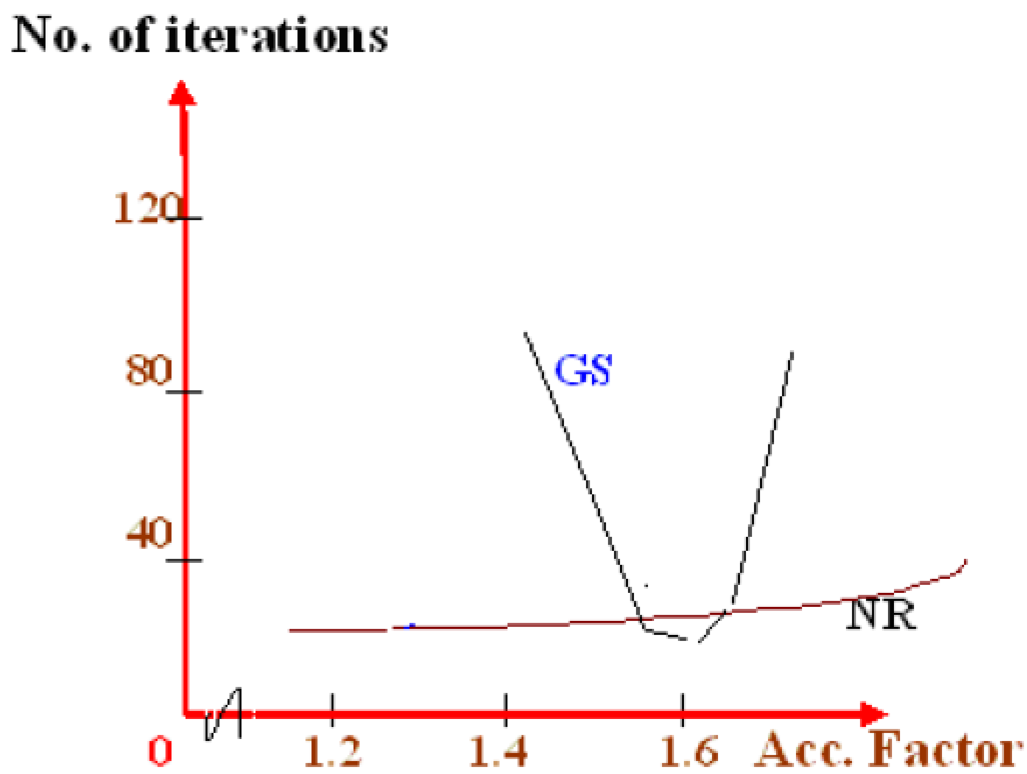


Figure 6. Influence of acceleration factor on load flow methods

FINAL WORD

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular. The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy.

PART-B

UNIT-5 & 6

ECONOMIC OPEARATION OF POWER SYSTEM

5.1 INTRODUCTION

One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system. Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants. The operation economics can again be subdivided into two parts.
i) Problem of *economic dispatch*, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.
ii) Problem of *optimal power flow,* which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.
During operation of the plant, a generator may be in one of the following states:
i) Base supply without regulation: the output is a constant.
ii) Base supply with regulation: output power is regulated based on system load.
iii) Automatic non-economic regulation: output level changes around a base setting as area control error changes.
iv) Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting.

Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons. The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.

Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

5.2 PERFORMANCE CURVES

INPUT-OUTPUT CURVE

This is the fundamental curve for a thermal plant and is a plot of the input in British thermal units (Btu) per hour versus the power output of the plant in MW as shown in Fig1.

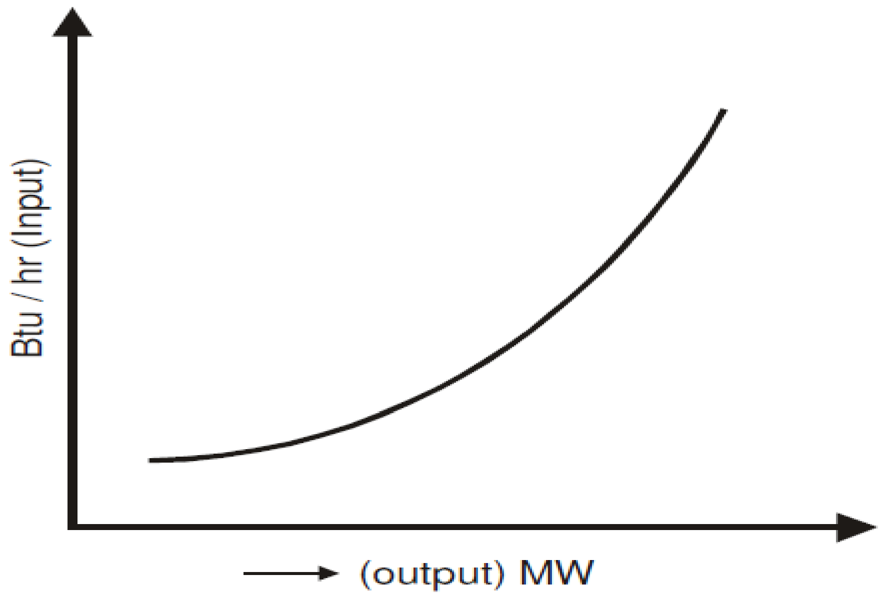


Fig 1: Input – output curve

HEAT RATE CURVE

The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input – output curve at any point. The reciprocal of heat – rate is called fuel –efficiency. The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in Fig .2

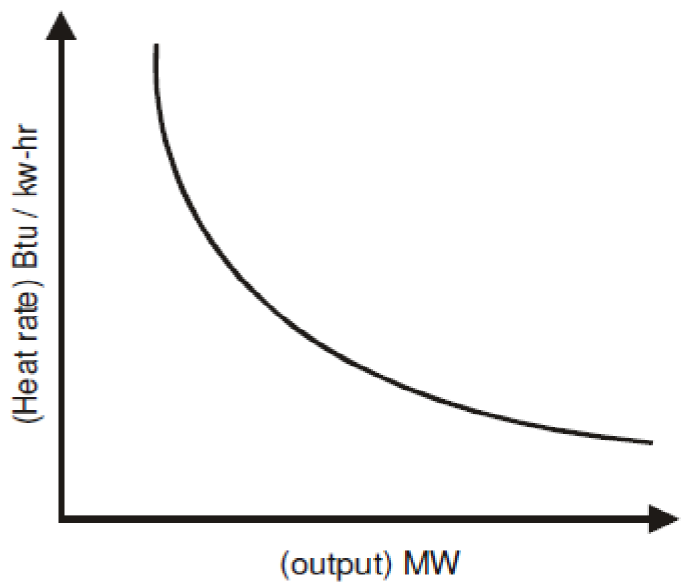


Fig .2 Heat rate curve.

INCREMENTAL FUEL RATE CURVE

The incremental fuel rate is equal to a small change in input divided by the corresponding change in output.

$$\text{Incremental fuel rate} = \frac{\Delta Input}{\Delta Output}$$

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig 3

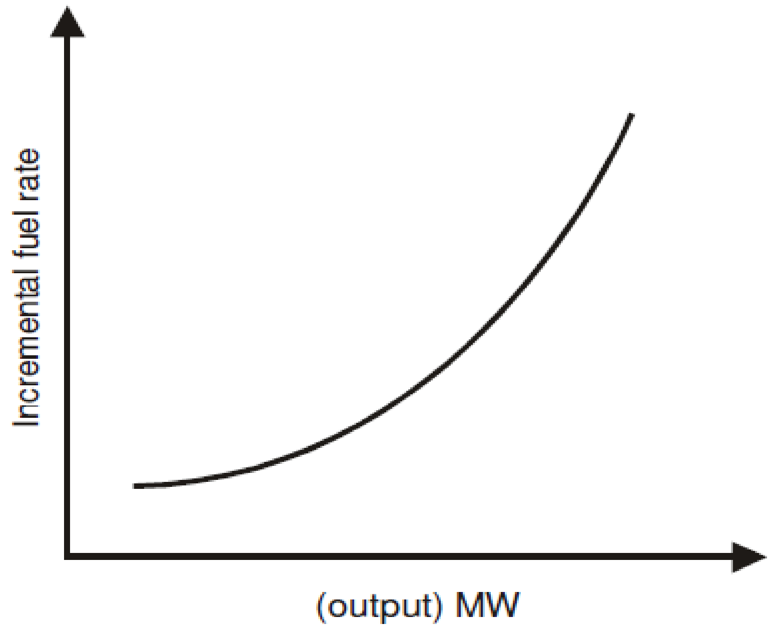


Fig 3: Incremental fuel rate curve

<u>Incremental cost curve</u>

The incremental cost is the product of incremental fuel rate and fuel cost (Rs / Btu or $ / Btu). The curve in shown in Fig. 4. The unit of the incremental fuel cost is Rs / MWh or $ /MWh.

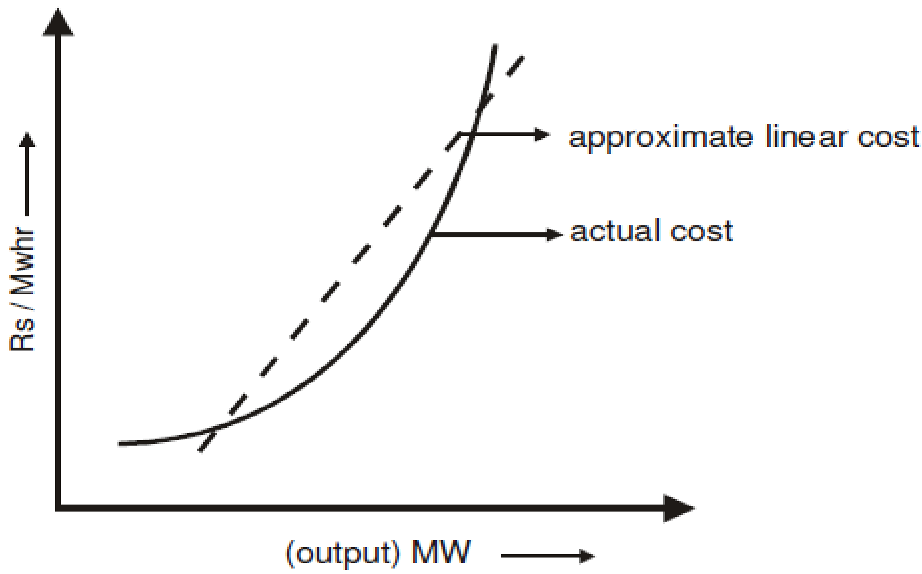


Fig 4: Incremental cost curve

In general, the fuel cost Fi for a plant, is approximated as a quadratic function of the generated output PGi.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad Rs / h$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \qquad Rs / MWh$$

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labour, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve. While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between PGmin, the minimum loading limit, below which it is technically infeasible to operate a unit and PGmax, which is the maximum output limit.

5.3 ECONOMIC GENERATION SCHEDULING NEGLECTING LOSSES AND GENERATOR LIMITS

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand PD. Consider a system with **n**g number of generating plants supplying the total demand PD. If Fi is the cost of plant ***i*** in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

where F_T = total cost.
P_{Gi} = generation of plant i.
P_D = total demand.

This is a constrained optimization problem, which can be solved by Lagrange's method.

LAGRANGE METHOD FOR SOLUTION OF ECONOMIC SCHEDULE

The problem is restated below:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$P_D = \sum_{i=1}^{n_g} P_{Gi} = 0$$

The augmented cost function is given by

$$£ = F_T + \lambda\left(P_D - \sum_{i=1}^{n_g} P_{Gi} \right)$$

The minimum is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = 0 \text{ and } \frac{\partial £}{\partial \lambda} = 0$$

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

$$\frac{\partial £}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The second equation is simply the original constraint of the problem. The cost of a plant

Fi depends only on its own output PGi, hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda \; ; \qquad i = 1 \ldots\ldots n_g$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1 \ldots\ldots n_g$$

The above equation is called the co-ordination equation. Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand. From we have

$$P_{Gi} = \frac{\lambda - b_i}{2c_i}$$

We know in a loss less system

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Substituting (8.16) we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of λ is obtained from (8.17) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}}$$

It can be seen that l is dependent on the demand and the coefficients of the cost function.

Example 1.

The fuel costs of two units are given by

$F_1 = 1.5 + 20\ P_{G1} + 0.1\ P_{G1}^2$ Rs/h

$F_2 = 1.9 + 30\ P_{G2} + 0.1\ P_{G2}^2$ Rs/h

P_{G1}, P_{G2} are in MW. Find the optimal schedule neglecting losses, when the demand is 200 MW.

Solution:

$$\frac{dF_1}{dP_{G1}} = 20 + 0.2P_{G1} \quad \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 30 + 0.2P_{G2} \quad \text{Rs / MWh}$$

$$P_D = P_{G1} + P_{G2} = 200 \text{ MW}$$

For economic schedule

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \lambda$$

$$20 + 0.2\ P_{G1} = 30 + 0.2\ (200 - P_{G1})$$

Solving we get, $P_{G1} = 125$ MW

$P_{G2} = 75$ MW

$$\lambda = 20 + 0.2\ (125) = 45 \text{ Rs / MWh}$$

Example 2

The fuel cost in \$ / h for two 800 MW plants is given by

$F_1 = 400 + 6.0\ P_{G1} + 0.004\ P_{G1}^2$

$F_2 = 500 + b_2\ P_{G2} + c_2\ P_{G2}^2$

where P_{G1}, P_{G2} are in MW

(a) The incremental cost of power, λ is \$8 / MWh when total demand is 550MW. Determine optimal generation schedule neglecting losses.

(b) The incremental cost of power is \$10/MWh when total demand is 1300 MW. Determine optimal schedule neglecting losses.

(c) From (a) and (b) find the coefficients b_2 and c_2.

Solution:

a) $$P_{G1} = \frac{\lambda - b_1}{2c_1} = \frac{8.0 - 6.0}{2 \times 0.004} = 250 \text{ MW}$$

$$P_{G2} = P_D - P_{G1} = 550 - 250 = 300 \text{ MW}$$

b) $$P_{G1} = \frac{\lambda - b_1}{2C_1} = \frac{10-6}{2\times 0.004} = 500 \ \text{MW}$$

$$P_{G2} = P_D - P_{G1} = 1300 - 500 = 800 \ \text{MW}$$

c) $$P_{G2} = \frac{\lambda - b_2}{2c_2}$$

From (a) $$300 = \frac{8.0 - b_2}{2c_2}$$

From (b) $$800 = \frac{10.0 - b_2}{2c_2}$$

Solving we get $b_2 = 6.8$

$c_2 = 0.002$

5.4 ECONOMIC SCHEDULE INCLUDING LIMITS ON GENERATOR (NEGLECTING LOSSES)

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi\,(min)} \le P_{Gi} \le P_{Gi\,(max)}\ ; i = 1, \ldots\ldots\ldots n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

Example 3

Incremental fuel costs in $ / MWh for two units are given below:

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 2.0 \ \$ / \text{MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.012P_{G2} + 1.6 \ \$ / \text{MWh}$$

The limits on the plants are P_{min} = 20 MW, P_{max} = 125 MW. Obtain the optimal schedule if the load varies from 50 – 250 MW.

Solution:

The incremental fuel costs of the two plants are evaluated at their lower limits and upper limits of generation.

At $P_{G\,(min)}$ = 20 MW.

$$\lambda_{1(min)} = \frac{dF_1}{dP_{G1}} = 0.01 \text{x } 20 + 2.0 = 2.2\$ / \text{MWh}$$

$$\lambda_{2(min)} = \frac{dF_2}{dP_{G2}} = 0.012 \text{ x } 20 + 1.6 = 1.84 \ \$ / \text{MWh}$$

At $P_{G\,(Max)}$ =125 Mw

$\lambda_{1(max)}$ = 0.01 x 125 + 2.0 = 3.25 $ / MWh

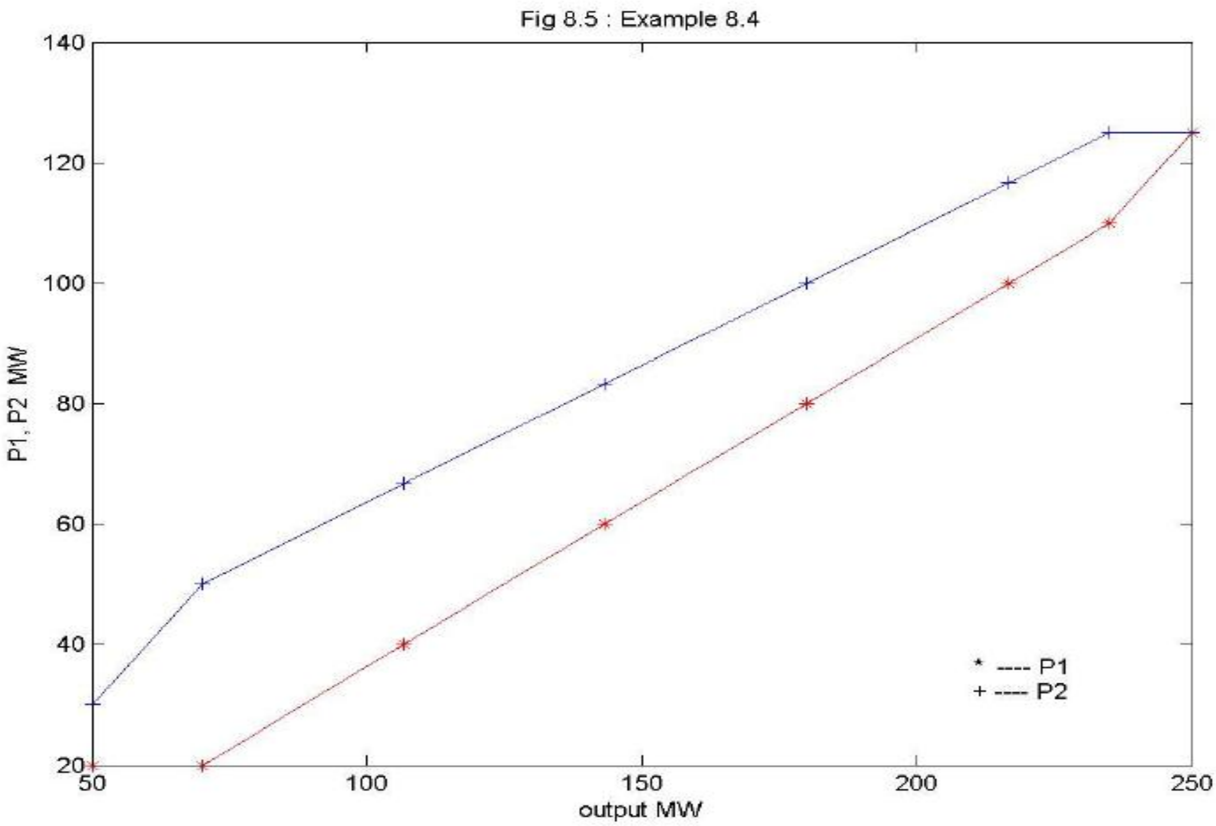
$\lambda_{2(max)}$ = 0.012 x 125 + 1.6 = 3.1 $ / MWh

Now at light loads unit 1 has a higher incremental cost and hence will operate at its lower limit of 20 MW. Initially, additional load is taken up by unit 2, till such time its incremental fuel cost becomes equal to 2.2$ / MWh at PG2 = 50 MW. Beyond this, the two units are operated with equal incremental fuel costs. The contribution of each unit to meet the demand is obtained by assuming different values of l; When l = 3.1 $ / MWh, unit 2 operates at its upper limit. Further loads are taken up by unit 1. The computations are show in Table

Table Plant output and output of the two units

$\frac{dF_1}{dP_{G1}}$ \$/MWh	$\frac{dF_2}{dP_{G2}}$ \$/MWh	Plant λ \$/MWh	P_{G1} MW	P_{G2} MW	Plant Output MW
2.2	1.96	1.96	20⁺	30	50
2.2	2.2	2.2	20⁺	50	70
2.4	2.4	2.4	40	66.7	106.7
2.6	2.6	2.6	60	83.3	143.3
2.8	2.8	2.8	80	100	180
3.0	3.0	3.0	100	116.7	216.7
3.1	3.1	3.1	110	125*	235

For a particular value of l, PG1 and PG2 are calculated using (8.16). Fig 8.5 Shows plot of each unit output versus the total plant output.



For any particular load, the schedule for each unit for economic dispatch can be obtained.

Example 4.

In example 3, what is the saving in fuel cost for the economic schedule compared to the case where the load is shared equally. The load is 180 MW.
Solution:
From Table it is seen that for a load of 180 MW, the economic schedule is PG1 = 80 MW and PG2 = 100 MW. When load is shared equally PG1 = PG2 = 90 MW. Hence, the generation of unit 1 increases from 80 MW to 90 MW and that of unit 2 decreases from 100 MW to 90 MW, when the load is shared equally. There is an increase in cost of unit 1 since PG1 increases and decrease in cost of unit 2 since PG2 decreases.

$$\text{Increase in cost of unit 1} = \int_{80}^{90} \left(\frac{dF_1}{dP_{G1}} \right) dP_{G1}$$

$$= \int_{80}^{90} (0.01P_{G1} + 2.0)dP_{G1} = 28.5 \text{ \$ / h}$$

$$\text{Decrease in cost of unit 2} = \int_{100}^{90} \left(\frac{dF_2}{dP_{G2}} \right) dP_{G2}$$

$$= \int_{100}^{90} (0.012P_{G2} + 1.6)dP_{G2} = -27.4 \text{ \$ / h}$$

Total increase in cost if load is shared equally = 28.5 – 27.4 = 1.1 \$ / h

Hence the saving in fuel cost is 1.1 \$ / h if coordinated economic schedule is used.

5.5 ECONOMIC DISPATCH INCLUDING TRANSMISSION LOSSES

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation. The mathematical formulation is now stated as

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such That $$\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$$

where P_L is the total loss.

The Lagrange function is now written as

$$£ = F_T - \lambda\left(\sum_{i=1}^{n_g} P_{Gi} - P_D - P_L\right) = 0$$

The minimum point is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right) = 0\,;\ \ i = 1\ldots\ldots n_g$$

$$\frac{\partial £}{\partial \lambda} = \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L = 0 \qquad \text{(Same as the constraint)}$$

Since $\dfrac{\partial F_T}{\partial P_{Gi}} = \dfrac{dF_i}{dP_{Gi}}$, (8.27) can be written as

$$\frac{dF_i}{dP_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

$$\lambda = \frac{dF_i}{dP_{Gi}}\left(\frac{1}{1 - \dfrac{\partial P_L}{\partial P_{Gi}}}\right)$$

The term $\dfrac{1}{1-\dfrac{\partial P_L}{\partial P_{Gi}}}$ is called the penalty factor of plant ***i***, $\mathbf{L_i}$. The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i\,;\ i = 1\ldots\ldots n_g$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered. A rigorous general expression for the loss PL is given by

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn} + \Sigma_n P_{Gn} B_{no} + B_{oo}$$

where Bmn, Bno , Boo called loss – coefficients , depend on the load composition. The assumption here is that the load varies linearly between maximum and minimum values. A simpler expression is

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn}$$

The expression assumes that all load currents vary together as a constant complex fraction of the total load current. Experiences with large systems has shown that the loss of accuracy is not significant if this approximation is used. An average set of loss coefficients may be used over the complete daily cycle in the coordination of incremental production costs and incremental transmission losses. In general, Bmn = Bnm and can be expanded for a two plant system as

$$P_L = B_{11} P_{G1} + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$$

Example 5

A generator is supplying a load. An incremental change in load of 4 MW requires generation to be increased by 6 MW. The incremental cost at the plant bus is Rs 30 /MWh. What is the incremental cost at the receiving end?

Solution:

$$\frac{dF_1}{dP_{G1}} = 30$$

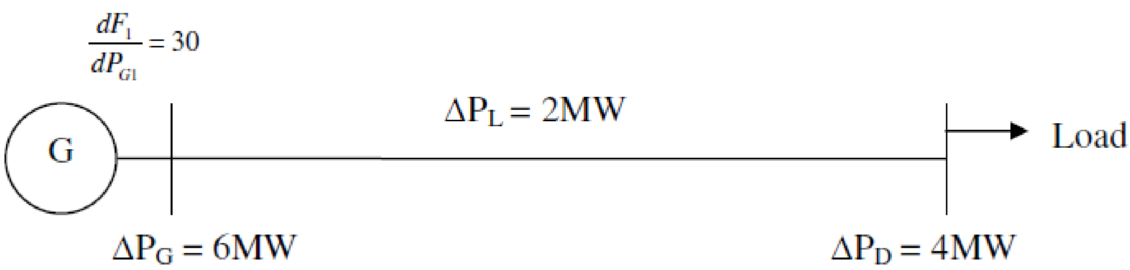


Fig ; One line diagram of example 5

$$\Delta P_L = \Delta P_G - \Delta P_D = 2 \text{ MW}$$

λ at receiving end is given by

$$\lambda = \frac{dF_1}{dP_{G1}} \times \frac{\Delta P_G}{\Delta P_D} = 30 \times \frac{6}{4} = 45 \text{ Rs / MWh}$$

$$\text{or } \lambda = \frac{dF_1}{dP_{G1}} \times \frac{1}{1 - \frac{\Delta P_L}{\Delta P_G}} = 30 \times \frac{1}{1 - \frac{2}{6}} = 45 \text{ Rs / MWh}$$

Example 6

In a system with two plants, the incremental fuel costs are given by

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 20 \ \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.015P_{G2} + 22.5 \ \text{Rs / MWh}$$

The system is running under optimal schedule with $P_{G1} = P_{G2} = 100$ MW.

If $\frac{\partial P_L}{\partial P_{G2}} = 0.2$, find the plant penalty factors and $\frac{\partial P_L}{\partial P_{G1}}$.

Solution:

For economic schedule,

$$\frac{dF_i}{dP_{Gi}} L_i = \lambda ; \qquad L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} ;$$

For plant 2, $P_{G2} = 100$ MW

$$\therefore (0.015 \times 100 + 22.5)\frac{1}{1 - 0.2} = \lambda.$$

Solving, $\lambda = 30 \ \text{Rs / MWh}$

$$L_2 = \frac{1}{1 - 0.2} = 1.25$$

$$\frac{dF_1}{dP_{G1}} L_1 = \lambda \Rightarrow (0.01 \text{x} 100 + 20) \ L_1 = 30$$

$$L_1 = 1.428$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}}$$

$$1.428 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} \ ; \text{Solving} \quad \frac{\partial P_L}{\partial P_{G1}} = 0.3$$

Example 7

A two bus system is shown in Fig. 8.8 If 100 MW is transmitted from plant 1 to the load, a loss of 10 MW is incurred. System incremental cost is Rs 30 / MWh. Find P_{G1}, P_{G2} and power received by load if

$$\frac{dF_1}{dP_{G1}} = 0.02P_{G1} + 16.0 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.04P_{G2} + 20.0 \text{ Rs / MWh}$$

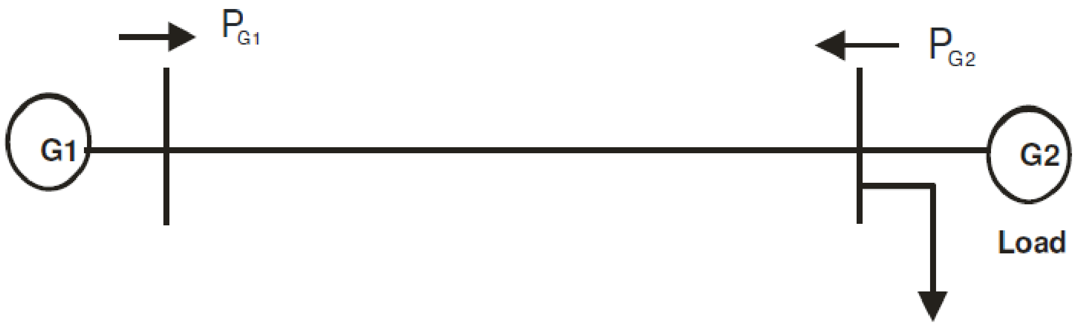


Fig One line diagram of example 7

Solution;

Since the load is connected at bus 2 , no loss is incurred when plant two supplies the load.

Therefore in (8.36) $B_{12} = 0$ and $B_{22} = 0$

$$P_L = B_{11}P_{G1}{}^2; \qquad \frac{\partial P_L}{\partial P_{G1}} = 2B_{11}P_{G1}; \qquad \frac{\partial P_L}{\partial P_{G2}} = 0.0$$

From data we have $P_L = 10$ MW, if $P_{G1} = 100$ MW

$10 = B_{11}\ (100)^2$

$B_{11} = 0.001 \quad MW^{-1}$

Coordination equation with loss is

$$\frac{dF_i}{dP_{Gi}} + \lambda\frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

For plant 1 $\quad \dfrac{dF_1}{dP_{G1}} + \lambda\dfrac{\partial P_L}{\partial P_{G1}} = \lambda$

$(0.02\ P_{G1} + 16.0) + 30\ (2 \times 0.001 \times P_{G1}) = 30$

$0.08\ P_{G1} = 30 - 16.0$. From which, $P_{G1} = 175$ MW

For Plant 2 $\quad \dfrac{dF_2}{dP_{G1}} + \lambda\dfrac{\partial P_L}{\partial P_{G2}} = \lambda$

$0.04\ P_{G2} + 20.0 = 30$ or $P_{G2} = 250$ MW

Loss $= B_{11}\ P_{G1}{}^2 = 0.001 \times (175)^2 = 30.625$ MW

$P_D = (P_{G1} + P_{G2}) - P_L = 394.\ 375$ MW

5.6 DERIVATION OF TRANSMISSION LOSS FORMULA

An accurate method of obtaining general loss coefficients has been presented by Kron. The method is elaborate and a simpler approach is possible by making the following assumptions:

(i) All load currents have same phase angle with respect to a common reference

(ii) The ratio X / R is the same for all the network branches.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig a

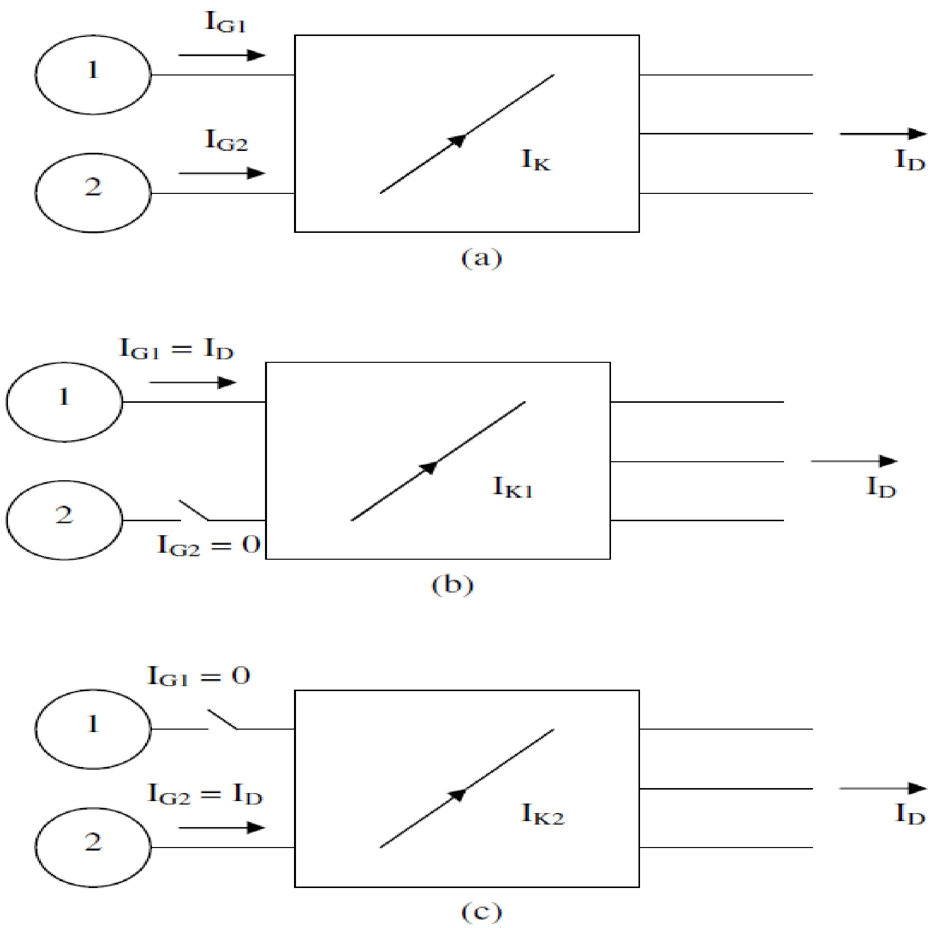


Fig Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{K1}. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

N_{K1} and N_{K2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1} I_{G1} + N_{K2} I_{G2}$$

where I_{G1}, I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D. Because of the assumptions made, I_{K1} and I_D have same phase angle, as do I_{K2} and I_D. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}|\angle\sigma_1 \text{ and } I_{G2} = |I_{G2}|\angle\sigma_2 .$$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} with respect to a common reference. We can write

$$|I_K|^2 = (N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2)^2 + (N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2)^2$$

$$= \begin{array}{l} N_{K1}^2|I_{G1}|^2[\cos^2\sigma_1 + \sin^2\sigma_1] + N_{K2}^2|I_{G2}|^2[\cos^2\sigma_2 + \sin^2\sigma_2] \\ + 2[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2] \end{array}$$

$$= N_{K1}^2|I_{G1}|^2 + N_{K2}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where P_{G1}, P_{G2} are three phase real power outputs of plant1 and plant 2; V_1, V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K + \frac{P_{G2}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^2 R_K$$

$$P_L = P_{G1}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}{}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}{}^2 + \ldots\ldots + \frac{P_{Gn}{}^2}{|V_n|^2 (\cos\phi_n)^2} \sum_K N_{Kn}{}^2 R_K$$
$$+ 2 \sum_{\substack{p,q=1 \\ p \neq q}}^{n} \frac{P_{GP} P_{Gq} \cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_k N_{KP} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{Pq} P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

Example 8

Calculate the loss coefficients in pu and MW^{-1} on a base of 50MVA for the network of Fig below. Corresponding data is given below.

$I_a = 1.2 - j\,0.4$ pu	$Z_a = 0.02 + j\,0.08$ pu
$I_b = 0.4 - j\,0.2$ pu	$Z_b = 0.08 + j\,0.32$ pu
$I_c = 0.8 - j\,0.1$ pu	$Z_c = 0.02 + j\,0.08$ pu
$I_d = 0.8 - j\,0.2$ pu	$Z_d = 0.03 + j\,0.12$ pu
$I_e = 1.2 - j\,0.3$ pu	$Z_e = 0.03 + j\,0.12$ pu

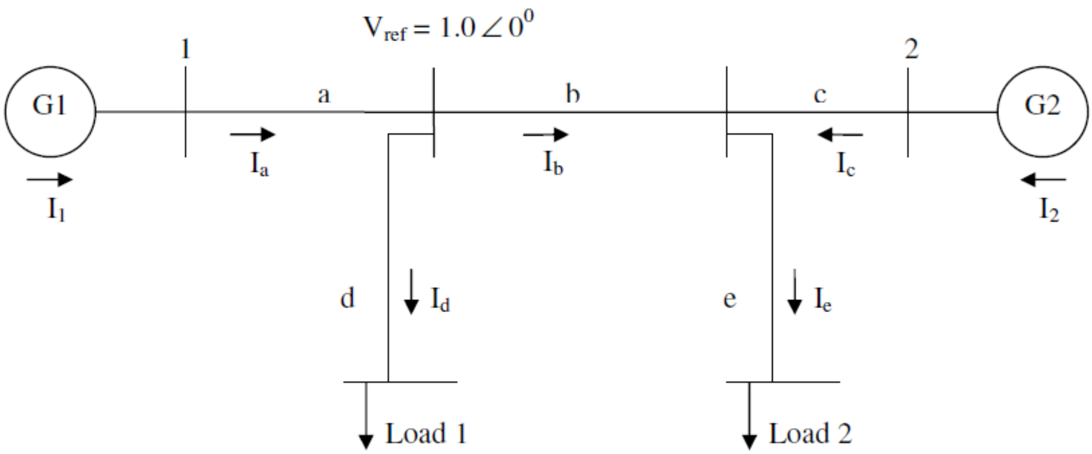


Fig : Example 8

Solution:

Total load current

$I_L = I_d + I_e = 2.0 - j\,0.5 = 2.061\ \angle -14.03^0 A$

$I_{L1} = I_d = 0.8 - j\,0.2 = 0.8246\ \angle -14.03^0\ A$

$$\frac{I_{L1}}{I_L} = 0.4; \qquad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$$

If generator 1, supplies the load then $I_1 = I_L$. The current distribution is shown in Fig a.

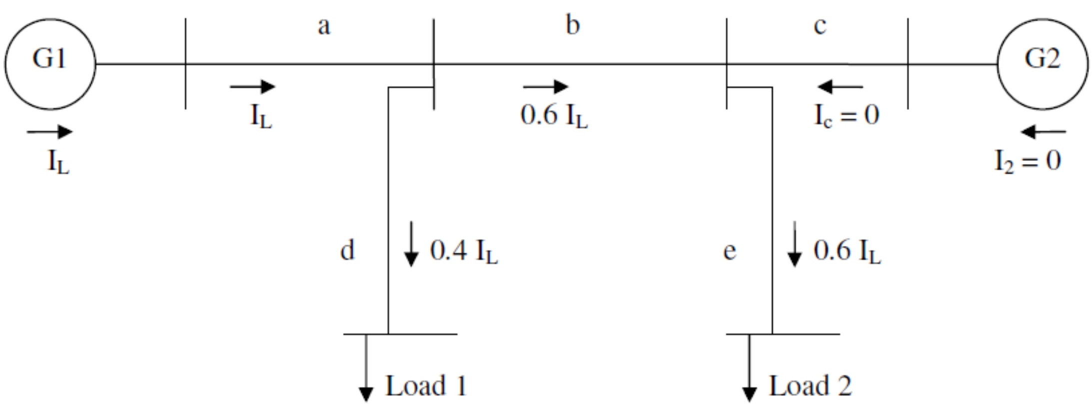


Fig a : Generator 1 supplying the total load

$$N_{a1} = \frac{I_a}{I_L} = 1.0;\ N_{b1} = \frac{I_b}{I_L} = 0.6;\ N_{C1} = 0;\ N_{d1} = 0.4;\ N_{e1} = 0.6.$$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

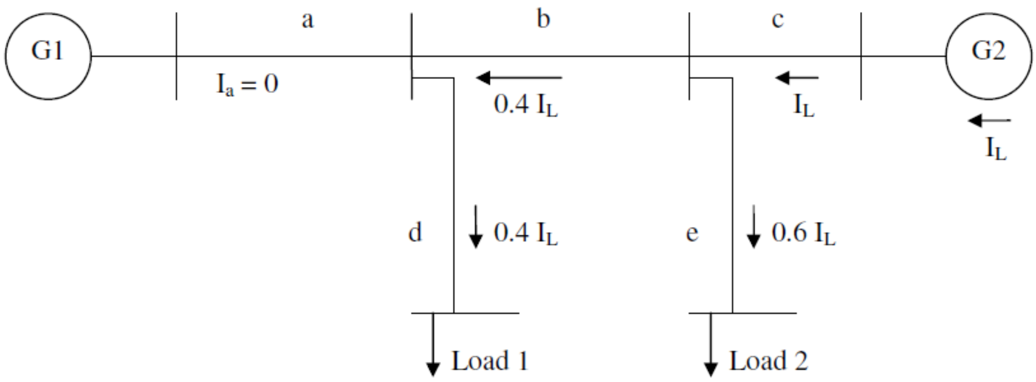


Fig b: Generator 2 supplying the total load

$N_{a2} = 0$; $N_{b2} = -0.4$; $N_{c2} = 1.0$; $N_{d2} = 0.4$; $N_{e2} = 0.6$

From Fig 8.10, $V_1 = V_{ref} + Z_a I_a$

$$= 1\angle 0^0 + (1.2 - j\,0.4)\,(0.02 + j0.08)$$

$$= 1.06\angle 4.78^0 = 1.056 + j\,0.088 \text{ pu.}$$

$$V_2 = V_{ref} - I_b Z_b + I_c Z_c$$

$$= 1.0\ \angle 0^0 - (0.4 - j\,0.2)\,(0.08 + j\,0.32) + (0.8 - j\,0.1)\,(0.02 + j\,0.08)$$

$$= 0.928 - j\,0.05 = 0.93\ \angle\text{-}3.10^0 \text{ pu.}$$

Current Phase angles

$$\sigma_1 = \text{angle of } I_1 (=I_a) = \tan^{-1}\left(\frac{-0.4}{1.2}\right) = -18.43^0$$

$$\sigma_2 = \text{angle of } I_2 (=I_c) = \tan^{-1}\left(\frac{-0.1}{0.8}\right) = -7.13^0$$

$$\cos(\sigma_1 - \sigma_2) = 0.98$$

Power factor angles

$$\phi_1 = 4.78^0 + 18.43 = 23.21^0; \cos\phi_1 = 0.92$$

$$\phi_2 = 7.13^0 - 3.10^0 = 4.03^0; \cos\phi_2 = 0.998$$

$$B_{11} = \frac{\sum_K N_{K1}{}^2 R_K}{|V_1|^2 (\cos\phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2 (0.920)^2}$$

$= 0.0677$ pu

$= 0.0677 \text{ x } \dfrac{1}{50} = 0.1354 \times 10^{-2}\,\text{MW}^{-1}$

$$B_{12} = \frac{Cos(\sigma_1 - \sigma_2)}{|V_1||V_2|(\cos\phi_1)(\cos\phi_2)} \sum_K N_{K1} N_{K2} R_K$$

$$= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)}[-0.4\times 0.6\times 0.08 + 0.4\times 0.4\times 0.03 + 0.6\times 0.6\times 0.03]$$

= - 0.00389 pu

= - 0.0078 x 10^{-2} MW^{-1}

$$B_{22} = \frac{\sum_K N_{K2}{}^2 R_K}{|V_2|^2 \left(\cos\phi_2{}^2\right)}$$

$$= \frac{(-0.4)^2 0.08 + 1.0^2 \times 0.02 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(0.93)^2 (0.998)^2}$$

$$= 0.056\text{pu} = 0.112\times 10^{-2}\text{MW}^{-1}$$

UNIT-7 & 8

TRANSIENT STABILITY STUDIES

7.1 INTRODUCTION

Power system stability of modern large inter-connected systems is a major problem for secure operation of the system. Recent major black-outs across the globe caused by system instability, even in very sophisticated and secure systems, illustrate the problems facing secure operation of power systems. Earlier, stability was defined as the ability of a system to return to normal or stable operation after having been subjected to some form of disturbance. This fundamentally refers to the ability of the system to remain in synchronism. However, modern power systems operate under complex interconnections, controls and extremely stressed conditions. Further, with increased automation and use of electronic equipment, the quality of power has gained utmost importance, shifting focus on to concepts of voltage stability, frequency stability,
inter-area oscillations etc.

The IEEE/CIGRE Joint Task Force on stability terms and conditions have proposed the following definition in 2004: "*Power System stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded, so that practically the entire system remains intact".* The Power System is an extremely non-linear and dynamic system, with operating parameters continuously varying. Stability is hence, a function of the initial operating condition and the nature of the disturbance. Power systems are continually subjected to small disturbances in the form of load changes. The system must be in a position to be able to adjust to the changing conditions and operate satisfactorily. The system must also withstand large disturbances, which may even cause structural changes due to isolation of some faulted elements. A power system may be stable for a particular (large) disturbance and unstable for another disturbance. It is impossible to design a system which is stable under all disturbances. The power system is generally designed to be stable under those disturbances which have a high degree of occurrence. The response to a disturbance is extremely complex and involves practically all the equipment of the power system. For example, a short circuit leading to a line isolation by circuit breakers will cause variations in the power flows, network bus voltages and generators rotor speeds. The voltage variations will actuate the voltage regulators in the system and generator speed variations will actuate the prime mover governors; voltage and frequency variations will affect the system loads. In stable systems, practically all generators and loads remain connected, even though parts of the system may be isolated to preserve bulk operations. On the other hand, an unstable system condition could lead to cascading outages and a shutdown of a major portion of the power system.

ROTOR ANGLE STABILITY

Rotor angle stability refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. Instability results in some generators accelerating (decelerating) and losing synchronism with other generators. Rotor angle stability depends on the ability of each synchronous machine to maintain equilibrium between electromagnetic torque and mechanical torque. Under steady state, there is equilibrium between the input mechanical torque and output electromagnetic torque of each generator, and its speed remains a constant. Under a disturbance, this equilibrium is upset and the generators accelerate/decelerate according to the mechanics of a rotating body. Rotor angle stability is further categorized as follows:

Small single (or small disturbance) rotor angle stability

It is the ability of the power system to maintain synchronism under small disturbances. In this case, the system equation can be linearized around the initial operating point and the stability depends only on the operating point and not on the disturbance. Instability may result in

(i) A non oscillatory or a periodic increase of rotor angle

(ii) (ii) Increasing amplitude of rotor oscillations due to insufficient damping.

The first form of instability is largely eliminated by modern fast acting voltage regulators and the second form of instability is more common. The time frame of small signal stability is of the order of 10-20 seconds after a disturbance.

Large-signal rotor angle stability or transient stability

This refers to the ability of the power system to maintain synchronism under large disturbances, such as short circuit, line outages etc. The system response involves large excursions of the generator rotor angles. Transient stability depends on both the initial operating point and the disturbance parameters like location, type, magnitude etc. Instability is normally in the form of a periodic angular separation. The time frame of interest is 3-5 seconds after disturbance. The term dynamic stability was earlier used to denote the steady-state stability in the presence of automatic controls (especially excitation controls) as opposed to manual controls. Since all generators are equipped with automatic controllers today, dynamic stability has lost relevance and the Task Force has recommended against its usage.

7.2 MECHANICS OF ROTATORY MOTION

Since a synchronous machine is a rotating body, the laws of mechanics of rotating bodies are applicable to it. In rotation we first define the fundamental quantities. The angle _m is defined, with respect to a circular arc with its center at the vertex of the angle, as the ratio of the arc length *s* to radius *r*.

$$\theta_m = \frac{s}{r} \tag{1}$$

The unit is radian. Angular velocity ω_m is defined as

$$\omega_m = \frac{d\theta_m}{dt} \tag{2}$$

and angular acceleration as

$$\alpha = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \tag{3}$$

The torque on a body due to a tangential force F at a distance r from axis of rotation is given by

$$T = r\,F \tag{4}$$

The total torque is the summation of infinitesimal forces, given by

$$T = \int r\, dF \tag{5}$$

The unit of torque is N-m. When torque is applied to a body, the body experiences angular acceleration. Each particle experiences a tangential acceleration $a = r\alpha$, where r is the distance of the particle from axis of rotation. The tangential force required to accelerate a particle of mass *dm* is

$$dF = a\, dm = r\, \alpha\, dm \tag{6}$$

The torque required for the particle is

$$dT = r\, dF = r^2\, \alpha\, dm \tag{7}$$

and that required for the whole body is given by

$$T = \alpha \int r^2 dm \; = I\, \alpha \tag{8}$$

Here

$$I = \int r^2 dm \tag{9}$$

is called the moment of inertia of the body. The unit is Kg – m^2. If the torque is assumed to be the result of a number of tangential forces F, which act at different points of the body

$$T = \sum r\, F$$

Now each force acts through a distance

$$ds = r\, d\theta_m$$

The work done is $\sum F \,.\, ds$

$$dW = \sum F\, r\, d\theta_m = d\theta_m\; T$$

$$W = \int T\, d\theta_m \tag{10}$$

and $$T = \frac{dW}{d\theta_m} \tag{11}$$

Thus the unit of torque may also be Joule per radian.

The power is defined as rate of doing work. Using (11)

$$P = \frac{dW}{dt} = \frac{T\,d\theta_m}{dt} = T\,\omega_m \qquad (12)$$

The angular momentum M is defined as

$$M = I\,\omega_m \qquad (13)$$

and the kinetic energy is given by

$$KE = \frac{1}{2} I\,\omega_m^{\,2} = \frac{1}{2} M\,\omega_m \qquad (14)$$

From (14) we can see that the unit of M is seen to be J-sec/rad.

SWING EQUATION:

From (8)

$$I\alpha = T$$

or

$$\frac{I\,d^2\theta_m}{dt^2} = T \qquad (15)$$

Here T is the net torque of all torques acting on the machine, which includes the shaft torque (due to prime mover of a generator or load on a motor), torque due to rotational losses (friction, windage and core loss) and electromagnetic torque.

Let T_m = shaft torque or mechanical torque corrected for rotational losses

T_e = Electromagnetic or electrical torque

For a generator T_m tends to accelerate the rotor in positive direction of rotation and for a motor retards the rotor.

The accelerating torque for a generator

$$T_a = T_m - T_e \qquad (16)$$

Under steady-state operation of the generator, T_m is equal to T_e and the accelerating torque is zero. There is no acceleration or deceleration of the rotor masses and the machines run at a constant synchronous speed. In the stability analysis in the following sections, T_m is assumed to be a constant since the prime movers (steam turbines or hydro turbines) do no act during the short time period in which rotor dynamics are of interest in the stability studies.

Now (15) has to be solved to determine θ_m as a function of time. Since θ_m i measured with respect to a stationary reference axis on the stator, it is the measure of th absolute rotor angle and increases continuously with time even at constant synchronou speed. Since machine acceleration /deceleration is always measured relative t synchronous speed, the rotor angle is measured with respect to a synchronously rotatin reference axis. Let

$$\delta_m = \theta_m - \omega_{sm} t \tag{17}$$

where ω_{sm} is the synchronous speed in mechanical rad/s and δ_m is the angul displacement in mechanical radians.

Taking the derivative of (17) we get

$$\frac{d\delta_m}{dt} = \frac{d\theta_m}{dt} - \omega_{sm}$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d^2\theta_m}{dt^2} \tag{18}$$

Substituting (18) in (15) we get

$$I\frac{d^2\delta_m}{dt^2} = \mathrm{T_a} = \mathrm{T_m} - \mathrm{T_e} \quad \text{N-m} \tag{19}$$

Multiplying by ω_m on both sides we get

$$\omega_m I\frac{d^2\delta_m}{dt^2} = \omega_m(\mathrm{T_m} - \mathrm{T_e}) \quad \text{N-m} \tag{20}$$

From (12) and (13), we can write

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_a \qquad W \tag{21}$$

where M is the angular momentum, also called inertia constant

P_m = shaft power input less rotational losses

P_e = Electrical power output corrected for losses

P_a = acceleration power

M depends on the angular velocity ω_m, and hence is strictly not a constant, because ω_m deviates from the synchronous speed during and after a disturbance. However, under stable conditions ω_m does not vary considerably and M can be treated as a constant. (21) is called the *"Swing equation"*. The constant M depends on the rating of the machine and varies widely with the size and type of the machine. Another constant called H constant (also referred to as inertia constant) is defined as

$$H = \frac{\begin{array}{c}\text{stored kinetic energy in mega joules}\\ \text{at sychronous speed}\end{array}}{\text{Machine rating in MVA}} \ MJ/MVA \tag{22}$$

H falls within a narrow range and typical values are given in Table 9.1.

If the rating of the machine is G MVA, from (22) the stored kinetic energy is GH Mega Joules. From (14)

$$GH = \frac{1}{2} M \omega_{sm} \quad \text{MJ} \tag{23}$$

or

$$M = \frac{2GH}{\omega_{sm}} \quad \text{MJ-s/mech rad} \tag{24}$$

The swing equation (21) is written as

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = \frac{P_a}{G} = \frac{P_m - P_e}{G} \tag{25}$$

In (.25) δ_m is expressed in mechanical radians and ω_{sm} in mechanical radians per second (the subscript m indicates mechanical units). If δ and ω have consistent units then mec

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{26}$$

Here ω_s is the synchronous speed in electrical rad/s ($\omega_s = \left(\frac{p}{2}\right)\omega_{sm}$) and P_a is acceleration power in per unit on same base as H. For a system with an electrical frequency f Hz, (26) becomes

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{27}$$

when δ is in electrical radians and

$$\frac{H}{180f}\frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{28}$$

when δ is in electrical degrees.

(27) and (28) also represent the swing equation. It can be seen that the swing equation is a second order differential equation which can be written as two first order differential equations:

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = P_m - P_e \quad \text{pu} \tag{29}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{30}$$

in which ω, ω_s and δ are in electrical units. In deriving the swing equation, damping has been neglected.

<u>Table 1 : H constants of synchronous machines</u>

Type of machine		H (MJ/MVA)
Turbine generator condensing	1800 rpm	9 – 6
	3600 rpm	7 – 4
Non condensing	3600 rpm	4 – 3
Water wheel generator		
	Slow speed < 200 rpm	2 – 3
	High speed > 200 rpm	2 – 4
Synchronous condenser		
	Large	1.25 } 25% less for hydrogen cooled
	Small	1.0 } 25% less for hydrogen cooled
Synchronous motor with load varying from 1.0 to 5.0		2.0

In defining the inertia constant H, the MVA base used is the rating of the machine. In a multi machine system, swing equation has to be solved for each machine, in which case, a common MVA base for the system has to chosen. The constant H of each machine must be consistent with the system base.

Let

G_{mach} = Machine MVA rating (base)

G_{system} = System MVA base

In (9.25), H is computed on the machine rating $G = G_{mach}$

Multiplying (9.25) by $\frac{G_{mach}}{G_{system}}$ on both sides we get

$$\left(\frac{G_{mach}}{G_{system}}\right)\frac{2H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = \frac{P_m - P_e}{G_{mach}}\left(\frac{G_{mach}}{G_{system}}\right) \tag{31}$$

$$\frac{2H_{system}}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = P_m - P_e \quad \text{pu (on system base)}$$

$$\text{where } H_{system} = H\frac{G_{mach}}{G_{system}} \tag{32}$$

In the stability analysis of a multi machine system, computation is considerably reduced if the number of swing equations to be solved is reduced. Machines within a plant normally swing together after a disturbance. Such machines are called coherent machines and can be replaced by a single equivalent machine, whose dynamics reflects the dynamics of the plant.

Example 1:

A 50Hz, 4 pole turbo alternator rated 150 MVA, 11 kV has an inertia constant of 9 MJ / MVA. Find the (a) stored energy at synchronous speed (b) the rotor acceleration if the input mechanical power is raised to 100 MW when the electrical load is 75 MW, (c) the speed at the end of 10 cycles if acceleration is assumed constant at the initial value.

Solution:

(a) Stored energy = GH = 150 × 9 = 1350 MJ

(b) $P_a = P_m - P_e = 100 - 75 = 25$ MW

$$M = \frac{GH}{180f} = \frac{1350}{180 \times 50} = 0.15 \text{ MJ} - \text{s}/^{\circ}\text{e}$$

$$0.15\frac{d^2\delta}{dt^2} = 25$$

$$\text{Acceleration } \alpha = \frac{d^2\delta}{dt^2} = \frac{25}{0.15} = 166.6\ ^\circ\text{e/s}^2$$

$$= 166.6 \times \frac{2}{P}\ ^\circ\text{m/s}^2$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360}\ rps/s$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \times 60 \text{ rpm/s}$$

$$= 13.88 \text{ rpm/s}$$

* Note $^\circ$e = electrical degree; $^\circ$m = mechanical degree; P=number of poles.

(c) $10 \text{ cycles} = \frac{10}{50} = 0.2\,\text{s}$

$N_S = \text{Synchronous speed} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$

Rotor speed at end of 10 cycles = $N_S + \alpha \times 0.2$

$= 1500 + 13.88 \times 0.2 = 1502.776$ rpm

Example 2:

Two 50 Hz generating units operate in parallel within the same plant, with the following ratings:

Unit 1: 500 MVA, 0.8 pf, 13.2 kV, 3600 rpm: H = 4 MJ/MVA

Unit 2: 1000 MVA, 0.9 pf, 13.8 kV, 1800 rpm: H = 5 MJ/MVA

Calculate the equivalent H constant on a base of 100 MVA.

Solution:

$$H_{1system} = H_{1mach} \times \frac{G_{1mach}}{G_{system}}$$

$$= 4 \times \frac{500}{100} = 20 \text{ MJ/MVA}$$

$$H_{2system} = H_{2mach} \times \frac{G_{2mach}}{G_{system}}$$

$$= 5 \times \frac{1000}{100} = 50 \text{ MJ/MVA}$$

$$H_{eq} = H_1 + H_2 = 20 + 50 = 70 \text{ MJ/MVA}$$

This is the equivalent inertia constant on a base of 100 MVA and can be used when the two machines swing coherently.

7.3 POWER–ANGLE EQUATION:

In solving the swing equation, certain assumptions are normally made (i) Mechanical power nput Pm is a constant during the period of interest, immediately after the disturbance (ii) Rotor speed changes are insignificant. (iii) Effect of voltage regulating loop during the transient is neglected i.e the excitation is assumed to be a constant. As discussed in section 9.4, the power–angle relationship plays a vital role in the solution of the swing equation.

POWER–ANGLE EQUATION FOR A NON–SALIENT POLE MACHINE:

The simplest model for the synchronous generator is that of a constant voltage behind an impedance. This model is called the classical model and can be used for cylindrical rotor (non–salient pole) machines. Practically all high–speed turbo alternators are of cylindrical rotor construction, where the physical air gap around the periphery of the rotor is uniform. This type of generator has approximately equal magnetic reluctance, regardless of the angular position of the rotor, with respect to the armature mmf.

The power output of the generator is given by the real part of $E_g I_a^*$.

$$I_a = \frac{E_g \angle\delta - V_t \angle 0°}{R_a + jx_d} \tag{38}$$

Neglecting R_a, $$I_a = \frac{E_g \angle\delta - V_t \angle 0°}{j x_d}$$

$$P = \Re \left\{ (E_g \angle\delta) \left(\frac{E_g \angle 90° - \delta}{x_d} - \frac{V_t \angle 90°}{x_d} \right)^* \right\}$$

$$= \frac{E_g^2 \cos 90°}{x_d} - \frac{E_g V_t \cos(90° + \delta)}{x_d}$$

$$= \frac{E_g V_t \sin\delta}{x_d} \tag{39}$$

(Note- $\Re$ stands for real part of)

The maximum power that can be transferred for a particular excitation is given by $\frac{E_g V_t}{x_d}$ at $\delta = 90°$.

7.4 POWER ANGLE EQUATION FOR A SALIENT POLE MACHINE:

Here because of the salient poles, the reluctance of the magnetic circuit in which flows the flux produced by an armature mmf in line with the quadrature axis is higher than that of the magnetic circuit in which flows the flux produced by the armature mmf in line with the direct axis. These two components of armature mmf are proportional to the corresponding components of armature current. The component of armature current producing an mmf acting in line with direct axis is called the direct component, Id. The component of armature current producing an mmf acting in line with the quadrature axis is called the quadrature axis component, Iq.

$$\text{Power output } P = V_t I_a \cos\theta$$

$$= E_d I_d + E_q I_q \qquad (40)$$

$$E_d = V_t \sin\delta \qquad (41a)$$

$$E_q = V_t \cos\delta \qquad (41b)$$

$$I_d = \frac{E_g - E_q}{x_d} = I_a \sin(\delta+\theta) \qquad (41c)$$

$$I_q = \frac{E_d}{x_q} = I_a \cos(\delta+\theta) \qquad (41d)$$

Substituting (9.41c) and (9.41d) in (9.40), we obtain

$$P = \frac{E_g V_t \sin\delta}{x_d} + \frac{V_t^2 (x_d - x_q)\sin 2\delta}{2 x_d x_q} \qquad (42)$$

(9.42) gives the steady state power angle relationship for a salient pole machine. The second term does not depend on the excitation and is called the reluctance power component. This component makes the maximum power greater than in the classical model. However, the angle at which the maximum power occurs is less than 90^o.

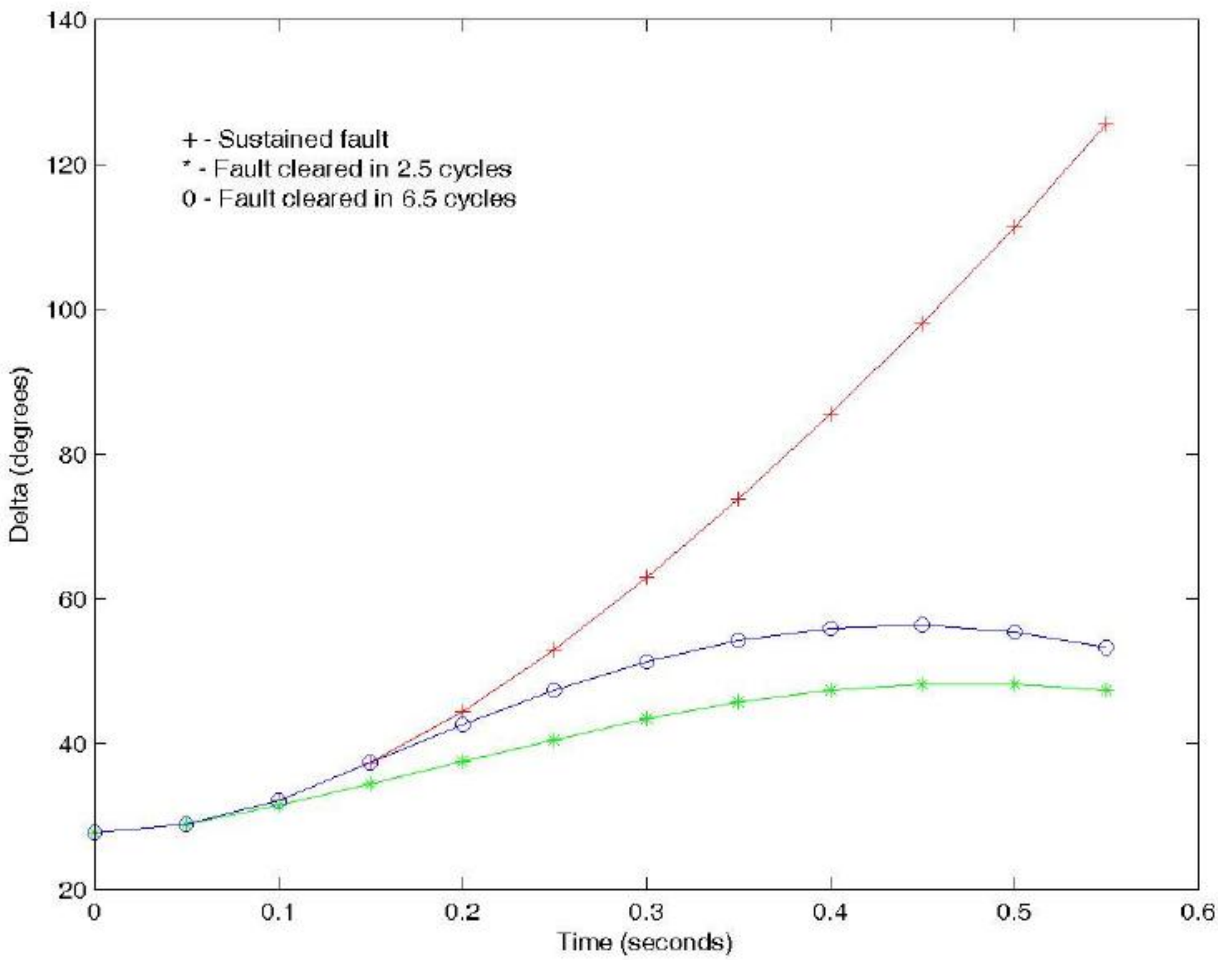
7.5 TRANSIENT STABILITY:

As defined earlier, transient stability is the ability of the system to remain stable under large disturbances like short circuits, line outages, generation or load loss etc. The evaluation of the transient stability is required offline for planning, design etc. and online for load management, emergency control and security assessment. Transient stability analysis deals with actual solution of the nonlinear differential equations describing the dynamics of the machines and their controls and interfacing it with the algebraic equations describing the interconnections through the transmission network. Since the disturbance is large, linearized analysis of the swing equation (which describes the rotor dynamics) is not possible. Further, the fault may cause structural changes in the network, because of which the power angle curve prior to fault, during the fault and post fault may be different. Due to these reasons, a general stability criteria for transient stability cannot be established, as was done in the case of steady state stability (namely PS > 0). Stability can be established, for a given fault, by actual solution of the swing equation. The time taken for the fault to be cleared (by the circuit breakers) is called the *clearing time*. If the fault is cleared fast enough, the probability of the system remaining stable after the clearance is more. If the fault persists for a longer time, likelihood of instability is increased. *Critical clearing time* is the maximum time available for clearing the fault, before the system loses stability. Modern circuit breakers are equipped with auto reclosure facility, wherein the breaker automatically recloses after two sequential openings. If the fault still persists, the breakers open permanently. Since most faults are transient, the first reclosure is in general successful. Hence, transient stability has been greatly enhanced by auto closure breakers.

Some common assumptions made during transient stability studies are as follows:

1. Transmission line and synchronous machine resistances are neglected. Since resistance introduces a damping term in the swing equation, this gives pessimistic results.
2. Effect of damper windings is neglected which again gives pessimistic results.
3. Variations in rotor speed are neglected.
4. Mechanical input to the generator is assumed constant. The governor control loop is neglected. This also leads to pessimistic results.
5. The generator is modeled as a constant voltage source behind a transient reactance, neglecting the voltage regulator action.
6. Loads are modeled as constant admittances and absorbed into the bus admittance matrix.

The above assumptions, vastly simplify the equations. A digital computer program for transient stability analysis can easily include more detailed generator models and effect of controls, the discussion of which is beyond the scope of present treatment. Studies on the transient stability of an SMIB system, can shed light on some important aspects of stability of larger systems. The figure below shows an example of how the clearing time has an effect on the swing curve of the machine.



7.6 Modified Euler's method:

Euler's method is one of the easiest methods to program for solution of differential equations using a digital computer . It uses the Taylor's series expansion, discarding all second–order and higher–order terms. Modified Euler's algorithm uses the derivatives at the beginning of a time step, to predict the values of the dependent variables at the end of the step (t1 = t0 +Δt). Using the predicted values, the derivatives at the end of the interval are computed. The average of the two derivatives is used in updating the variables.

Consider two simultaneous differential equations:

$$\frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values x0, y0, t0 at the beginning of a time step and a step size *h* we solve as follows:

Let

$$D_x = f_x(x_0, y_0, t_0) = \left.\frac{dx}{dt}\right|_0$$

$$D_y = f_y(x_0, y_0, t_0) = \left.\frac{dy}{dt}\right|_0$$

$$\left.\begin{aligned} x^P &= x_0 + D_x\, h \\ y^P &= y_0 + D_y\, h \end{aligned}\right\} \quad \text{Predicted values}$$

$$D_{xP} = \left.\frac{dx}{dt}\right|_P = f_x(x^P, y^P, t_1)$$

$$D_{yP} = \left.\frac{dy}{dt}\right|_P = f_y(x^P, y^P, t_1)$$

$$x_1 = x_o + \left(\frac{D_x + D_{xP}}{2}\right) h$$

$$y_1 = y_o + \left(\frac{D_y + D_{yP}}{2}\right) h$$

x_1 and y_1 are used in the next iteration. To solve the swing equation by Modified Euler's method, it is written as two first order differential equations:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{\max}\sin\delta}{M}$$

Starting from an initial value _o, _o at the beginning of any time step, and choosing a step size _t s, the equations to be solved in modified Euler's are as follows:

$$\left.\frac{d\delta}{dt}\right|_0 = D_1 = \omega_0$$

$$\left.\frac{d\omega}{dt}\right|_0 = D_2 = \frac{P_m - P_{\max}\sin\delta_0}{M}$$

$$\delta^P = \delta_0 + D_1\,\Delta t$$

$$\omega^P = \omega_0 + D_2\,\Delta t$$

$$\left.\frac{d\delta}{dt}\right|_P = D_{1P} = \omega^P$$

$$\left.\frac{d\omega}{dt}\right|_P = D_{2P} = \frac{P_m - P_{\max}\sin\delta^P}{M}$$

$$\delta_1 = \delta_0 + \left(\frac{D_1 + D_{1P}}{2}\right)\Delta t$$

$$\omega_1 = \omega_0 + \left(\frac{D_2 + D_{2P}}{2}\right)\Delta t$$

δ_1 and ω_1 are used as initial values for the successive time step. Numerical errors are introduced because of discarding higher–order terms in Taylor's expansion. Errors can be decreased by choosing smaller values of step size. Too small a step size, will increase computation, which can lead to large errors due to rounding off. The Runge- Kutta method which uses higher–order terms is more popular.

Example :A 50 Hz, synchronous generator having inertia constant H = 5.2 MJ/MVA and $x_d^{'}$ = 0.3 pu is connected to an infinite bus through a double circuit line as shown in Fig. 9.21. The reactance of the connecting HT transformer is 0.2 pu and reactance of each line is 0.4 pu. $|E_g|$ = 1.2 pu and $|V|$ = 1.0 pu and P_e = 0.8 pu. Obtain the swing curve using modified Eulers method for a three phase fault occurs at the middle of one of the transmission lines and is cleared by isolating the faulted line.

Solution:

Before fault transfer reactance between generator and infinite bus

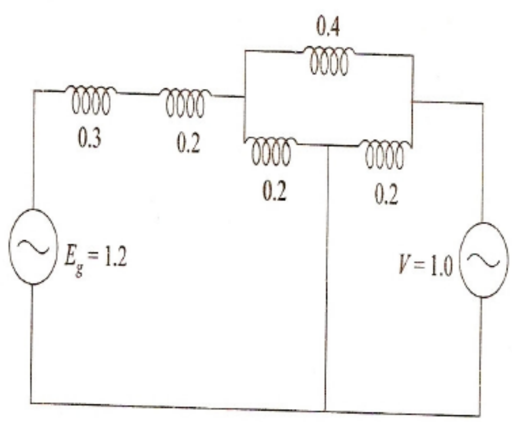
$$X_I = 0.3 + 0.2 + \frac{0.4}{2} = 0.7 \text{ pu}$$

$$P_{max\ I} = \frac{1.2 \times 1.0}{0.7} = 1.714 \text{ pu.}$$

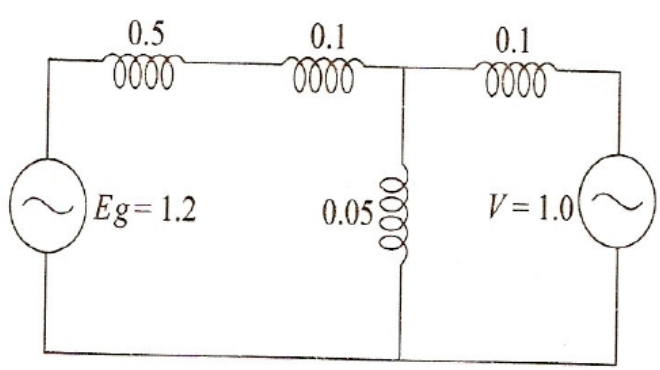
Initial $P_e = 0.8$ pu $= P_m$

Initial operating angle $\delta_o = \sin^{-1} \dfrac{0.8}{1.714} = 27.82^o = 0.485$ rad.

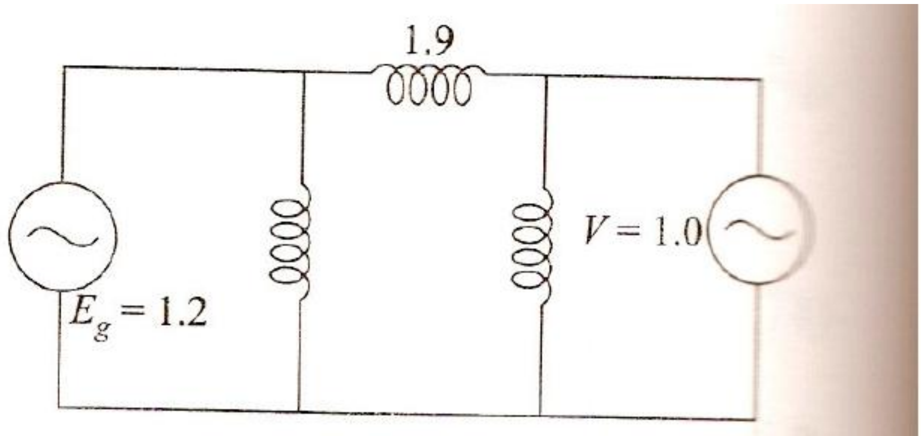
When fault occurs at middle of one of the transmission lines, the network and its reduction is as shown in Fig a to Fig c.



(a)



(b)



(c)

The transfer reactance is 1.9 pu.

$$P_{\max II} = \frac{1.2 \times 1.0}{1.9} = 0.63 \text{ pu}$$

Since there is no outage, $P_{\max III} = P_{\max I} = 1.714$

$$\delta_{\max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{\max III}}\right) = \pi - \sin^{-1}\left(\frac{0.8}{1.714}\right) = 2.656 \text{ rad}$$

$$\cos \delta_{cr} = \frac{P_m\,(\delta_{\max} - \delta_o) - P_{\max II} \cos\delta_o + P_{\max III}\,\cos\delta_{\max}}{P_{\max III} - P_{\max II}}$$

$$= \frac{0.8(2.656 - 0.485) - 0.63 \cos(0.485) + 1.714 \cos(2.656)}{1.714 - 0.63}$$

$$= \frac{1.7368 - 0.5573 - 1.5158}{1.084} = -0.3102$$

$$\delta_{cr} = \cos^{-1}(-0.3102) = 1.886 \text{ rad} = 108.07^\circ$$

with line outage

$$X_{III} = 0.3 + 0.2 + 0.4 = 0.9 \text{ pu}$$

$$P_{\max III} = \frac{1.2 \times 1.0}{0.9} = 1.333 \text{ pu}$$

$$\delta_{\max} = \pi - \sin^{-1}\frac{0.8}{1.333} = 2.498 \text{ rad}$$

7.7

Modified Eulers method

$\delta_o = 27.82^o = 0.485$ rad

$$\omega_o = 0.0 \text{ rad / sec (at } t = 0^0)$$

Choosing a step size of 0.05 s, the swing is computed. Table a gives the values of the derivatives and predicted values. Table b gives the initial values δ_o, ω_o and the values at the end of the interval δ_1, ω_1. Calculations are illustrated for the time step t = 0.2 s.

$$\delta_o = 0.761$$

$$\omega_o = 2.072$$

$$P_m = 0.8$$

$$M = \left[\frac{5.2}{\Pi \times 50}\right] = 0.0331 \text{ s}^2 / \text{rad}$$

$$P_{max} \text{ (after fault clearance)} = 1.333 \text{ pu}$$

$$D_1 = 2.072$$

$$D_2 = \frac{0.8 - 1.333 \sin(0.761)}{0.0331} = -3.604$$

$$\delta^P = 0.761 + (2.072 \times 0.05) = 0.865$$

$$\omega^P = 2.072 + (-3.604 \times 0.05) = 1.892$$

$$D_{1P} = 1.892$$

$$D_{2P} = \frac{0.8 - 1.333 \sin(0.865)}{0.0331} = -6.482$$

$$\delta_1 = 0.761 + \left(\frac{2.072 + 1.892}{2}\right) 0.05 = 0.860$$

$$\omega_1 = 2.072 + \left(\frac{-3.604 - 6.482}{2}\right) 0.05 = 1.82$$

δ_1, ω_1 are used as initial values in next time step.

Table a : Calculation of derivatives in modified Euler's method

t	D_1	D_2	δ^P	ω^P	D_{1P}	D_{2P}
0^+	0.0	15.296	0.485	0.765	0.765	15.296
0.05	0.765	14.977	0.542	1.514	1.514	14.350
0.10	1.498	14.043	0.636	2.200	2.200	12.860
0.15	2.17	- 0.299	0.761	2.155	2.155	- 3.600
0.20	2.072	- 3.604	0.865	1.892	1.892	- 6.482
0.25	1.820	- 6.350	0.951	1.502	1.502	- 8.612
0.30	1.446	- 8.424	1.015	1.025	1.025	- 10.041
0.35	0.984	- 9.827	1.054	0.493	0.493	- 10.843
0.40	0.467	- 10.602	1.065	- 0.063	- 0.063	- 11.060
0.45	- 0.074	- 10.803	1.048	- 0.614	- 0.614	- 10.720
0.50	- 0.612	- 10.46	1.004	- 1.135	- 1.135	- 9.800

Table b : calculations of δ_o, ω_o and δ_1, ω_1 in modified Euler's method

T	P_{max} (pu)	δ_o rad	ω_o rad / sec	δ_1 rad	ω_1 rad / sec	δ_1 deg
0^-	1.714	0.485	0.0	–	–	-
0^+	0.630	0.485	0.0	0.504	0.765	28.87
0.05	0.630	0.504	0.765	0.561	1.498	32.14
0.10	0.630	0.561	1.498	0.653	2.170	37.41
0.15	1.333	0.653	2.170	0.761	2.072	43.60
0.20	1.333	0.761	2.072	0.860	1.820	49.27
0.25	1.333	0.860	1.820	0.943	1.446	54.03
0.30	1.333	0.943	1.446	1.005	0.984	57.58
0.35	1.333	1.005	0.984	1.042	0.467	59.70
0.40	1.333	1.042	0.467	1.052	- 0.074	60.27
0.45	1.333	1.052	- 0.074	1.035	- 0.612	59.30
0.50	1.333	1.035	- 0.612	0.991	- 1.118	56.78

7.8

Runge - Kutta method

In Range - Kutta method, the changes in dependent variables are calculated from a given set of formulae, derived by using an approximation, to replace a truncated Taylor's series expansion. The formulae for the Runge - Kutta fourth order approximation, for solution of two simultaneous differential equations are given below;

$$\text{Given } \frac{dx}{dt} = f_x\,(x, y, t)$$

$$\frac{dy}{dt} = f_y\,(x, y, t)$$

Starting from initial values x_0, y_0, t_0 and step size h, the updated values are

$$x_1 = x_0 + \frac{1}{6}\,(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}\,(l_1 + 2l_2 + 2l_3 + l_4)$$

where $k_1 = f_x\,(x_0, y_0, t_0)\,h$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_4 = f_x\,(x_0 + k_3, y_0 + l_3, t_0 + h)\,h$$

$$l_1 = f_y\,(x_0, y_0, t_0)\,h$$

$$l_2 = f_y\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_4 = f_y\,(x_0 + k_3, y_0 + l_3, t_0 + h)\,h$$

The two first order differential equations to be solved to obtain solution for the swing equation are:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{\max}\sin\delta}{M}$$

Starting from initial value δ_0, ω_0, t_0 and a step size of Δt the formulae are as follows

$$k_1 = \omega_0 \ \Delta t$$

$$l_1 = \left[\frac{P_m - P_{\max}\sin\delta_0}{M}\right] \Delta t$$

$$k_2 = \left(\omega_0 + \frac{l_1}{2}\right) \Delta t$$

$$l_2 = \left[\frac{P_m - P_{\max}\sin\left(\delta_0 + \frac{k_1}{2}\right)}{M}\right] \Delta t$$

$$k_3 = \left(\omega_0 + \frac{l_2}{2}\right) \Delta t$$

$$l_3 = \left[\frac{P_m - P_{\max}\sin\left(\delta_0 + \frac{k_2}{2}\right)}{M}\right] \Delta t$$

$$k_4 = (\omega_0 + l_3)\ \Delta t$$

$$l_4 = \left[\frac{P_m - P_{\max}\sin(\delta_0 + k_3)}{M}\right] \Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6}\ [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\omega_1 = \omega_0 + \frac{1}{6}\ [l_1 + 2l_2 + 2l_3 + l_4]$$

Example

Obtain the swing curve for previous example using Runge - Kutta method.

Solution:

$$\delta_0 = 27.82^0 = 0.485 \text{ rad.}$$

$$\omega_0 = 0.0 \text{ rad / sec. (at } t = 0^-)$$

Choosing a step size of 0.05 s, the coefficient k_1, k_2, k_3, k_4 and l_1, l_2, l_3, and l_4 are calculated for each time step. The values of δ and ω are then updated. Table a gives the coefficient for different time steps. Table b gives the starting values δ_0, ω_0 for a time step and the updated values δ_1, ω_1 obtained by Runge - Kutta method. The updated values are used as initial values for the next time step and process continued. Calculations are illustrated for the time step t = 0.2 s.

$$\delta_0 = 0.756$$

$$M = 0.0331 \text{ s}^2 / \text{rad}$$

$$\omega_0 = 2.067$$

$$P_m = 0.8$$

$$P_{max} = 1.333 \text{ (after fault is cleared)}$$

$$k_1 = 2.067 \times 0.05 = 0.103$$

$$l_1 = \left[\frac{0.8 - 1.333 \sin(0.756)}{0.0331}\right] \times 0.05 = -0.173$$

$$k_2 = \left[2.067 - \frac{0.173}{2}\right] 0.05 = 0.099$$

$$l_2 = \left[\frac{0.8 - 1.333 \sin\left(0.756 + \dfrac{0.103}{2}\right)}{0.0331}\right] \times 0.05 = -0.246$$

$$k_3 = \left[2.067 - \frac{0.246}{2}\right] 0.05 = 0.097$$

$$l_3 = \left[\frac{0.8 - 1.333 \sin\left(0.756 + \dfrac{0.099}{2}\right)}{0.0331}\right] \times 0.05 = -0.244$$

$$k_4 = (2.067 - 0.244)\, 0.05 = 0.091$$

$$l_4 = \left[\frac{0.8 - 1.333 \sin(0.756 + 0.097)}{0.0331}\right] \times 0.05 = -0.308$$

$$\delta_1 = 0.756 + \frac{1}{6}\,[0.103 + 2 \times 0.099 + 2 \times 0.097 + 0.091] = 0.854$$

$$\omega_1 = 2.067 + \frac{1}{6}\,[-0.173 + 2 \times -0.246 + 2 \times -0.244 - 0.308] = 1.823$$

Now $\delta = 0.854$ and $\omega = 1.823$ are used as initial values for the next time step. The computations have been rounded off to three digits. Greater accuracy is obtained by reducing the step size.

Table a : Coefficients in Runge - Kutta method

T	k_1	l_1	k_2	l_2	k_3	l_3	K_4	l_4
0.0	0.0	0.764	0.019	0.764	0.019	0.757	0.038	0.749
0.05	0.031	0.749	0.056	0.736	0.056	0.736	0.075	0.703
0.10	0.075	0.704	0.092	0.674	0.091	0.667	0.108	0.632
0.15	0.108	– 0.010	0.108	– 0.094	0.106	– 0.095	0.103	– 0.173
0.20	0.103	– 0.173	0.099	– 0.246	0.097	– 0.244	0.091	– 0.308
8.25	0.091	– 0.309	0.083	– 0.368	0.082	– 0.363	0.073	– 0.413
0.30	0.073	– 0.413	0.063	– 0.455	0.061	– 0.450	0.050	– 0.480
0.35	0.050	– 0.483	0.038	– 0.510	0.037	– 0.504	0.025	– 0.523
0.40	0.025	– 0.523	0.012	– 0.536	0.011	– 0.529	– 0.001	– 0.534
0.45	– 0. 001	– 0.534	– 0.015	– 0.533	– 0.015	– 0.526	– 0.027	– 0.519
0.50	– 0.028	– 0.519	– 0.040	– 0.504	– 0.040	– 0.498	– 0.053	– 0.476

Table b: δ, ω computations by Runge - Kutta method

t (sec)	P_{max} (pu)	δ_0 (rad)	ω_0 rad/sec	δ_1 rad	ω_1 rad/sec	δ_1 deg
0^-	1.714	0.485	0.0			
0^+	0.630	0.485	0.0	0.504	0.759	28.87
0.05	0.630	0.504	0.756	0.559	1.492	32.03
0.10	0.630	0.559	1.492	0.650	2.161	37.24
0.15	1.333	0.650	2.161	0.756	2.067	43.32
0.20	1.333	0.756	2.067	0.854	1.823	48.93

0.25	1.333	0.854	1.823	0.936	1.459	53.63
0.30	1.333	0.936	1.459	0.998	1.008	57.18
0.35	1.333	0.998	1.008	1.035	0.502	59.30
0.40	1.333	1.035	0.502	1.046	– 0.029	59.93
0.45	1.333	1.046	– 0.029	1.031	– 0.557	59.07
0.50	1.333	1.031	– 0.557	0.990	– 1.057	56.72

Note: δ_0, ω_0 indicate values at beginning of interval and δ_1, ω_1 at end of interval. The fault is cleared at 0.125 seconds. $\therefore$ $P_{max} = 0.63$ at t = 0.1 sec and $P_{max} = 1.333$ at t = 0.15 sec, since fault is already cleared at that time. The swing curves obtained from modified Euler's method and Runge - Kutta method are shown in Fig. It can be seen that the two methods yield very close results.

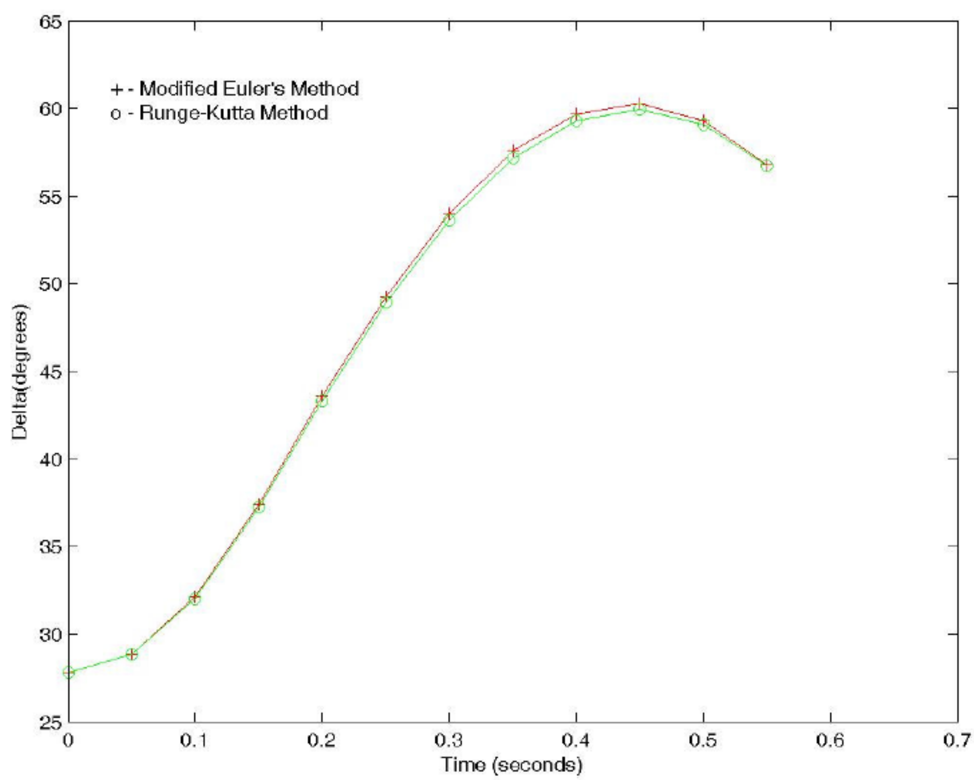


Fig: : Swing curves with Modified Euler' and Runge-Kutta methods

Milne's Predictor Corrector method:

The Milne's formulae for solving two simultaneous differential equations are given below.

Consider $\frac{dx}{dt} = f_x(x, y, t)$

$$\frac{dy}{dt} = f_y(x, y, t)$$

With values of x and y known for four consecutive previous times, the predicted value for n + 1[th] time step is given by

$$x_{n+1}^P = x_{n-3} + \frac{4h}{3}\left[2x'_{n-2} - x'_{n-1} + 2x'_n\right]$$

$$y_{n+1}^P = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Where x' and y' are derivatives at the corresponding time step. The corrected values are

$$x_{n+1} = x_{n-1} + \frac{h}{3}\left[x'_{n-1} + 4x'_n + x'_{n+1}\right]$$

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

where $x'_{n+1} = f_x\left(x_{n+1}^P, y_{n+1}^P, t_{n+1}\right)$

$$y'_{n+1} = f_y\left(x_{n+1}^P, y_{n+1}^P, t_{n+1}\right)$$

To start the computations we need four initial values which may be obtained by modified Euler's method, Runge - Kutta method or any other numerical method which is self starting, before applying Milne's method. The method is applied to the solution of swing equation as follows:

Define $\delta'_n = \left.\frac{d\delta}{dt}\right|_n = \omega_n$

$$\omega'_n = \left.\frac{d\omega}{dt}\right|_n = \frac{P_m - P_{max}\sin\delta_n}{M}$$

$$\delta_{n+1}^P = \delta_{n-3} + \frac{4\Delta t}{3}\left[2\delta'_{n-2} - \delta'_{n-1} + 2\delta'_n\right]$$

$$\omega_{n+1}^{P} = \omega_{n-3} + \frac{4\Delta t}{3}\left[2\omega'_{n-2} - \omega'_{n-1} + 2\omega'_{n}\right]$$

$$\delta_{n+1} = \delta_{n-1} + \frac{\Delta t}{3}\left[\delta'_{n-1} + 4\delta'_{n} + \delta'_{n+1}\right]$$

$$\omega_{n+1} = \omega_{n-1} + \frac{\Delta t}{3}\left[\omega'_{n-1} + 4\omega'_{n} + \omega'_{n+1}\right]$$

where $\delta'_{n+1} = \omega_{n+1}^{P}$

$$\omega'_{n+1} = \frac{P_m - P_{\max}\sin\delta_{n+1}^{P}}{M}$$

Example

Solve example using Milne's method.

Solution:

To start the process, we take the first four computations from Range Kutta method

t = 0.0 s	$\delta_1 = 0.504$	$\omega_1 = 0.759$
t = 0.05 s	$\delta_2 = 0.559$	$\omega_2 = 1.492$
t = 0.10 s	$\delta_3 = 0.650$	$\omega_3 = 2.161$
t = 0.15 s	$\delta_4 = 0.756$	$\omega_4 = 2.067$

The corresponding derivatives are calculated using the formulae for δ'_n and ω'_n. We get

$\delta'_1 = 0.759$	$\omega'_1 = 14.97$
$\delta'_2 = 1.492$	$\omega'_2 = 14.075$
$\delta'_3 = 2.161$	$\omega'_3 = 12.65$
$\delta'_4 = 2.067$	$\omega'_4 = -3.46$

We now compute δ_5 and ω_5, at the next time step i.e t = 0.2 s.

$$\delta_5^{P} = \delta_1 + \frac{4\,\Delta t}{3}\left[2\delta'_2 - \delta'_3 + 2\delta'_4\right]$$

$$= 0.504 + \frac{4\times 0.05}{3}\left[2\times 1.492 - 2.161 + 2\times 2.067\right] = 0.834$$

$$\omega_5^{P} = \omega_1 + \frac{4\,\Delta t}{3}\left[2\omega'_2 - \omega'_3 + 2\omega'_4\right]$$

$$= 0.759 + \frac{4 \times 0.05}{3}\left[2 \times 14.075 - 12.65 + 2 \times (-3.46)\right] = 1.331$$

$$\delta_5' = 1.331$$

$$\omega_5' = \frac{0.8 - 1.333\sin(0.834)}{0.0331} = -5.657$$

$$\delta_5 = \delta_3 + \frac{\Delta t}{3}\left[\delta_3' + 4\delta_4' + \delta_5'\right]$$

$$= 0.65 + \frac{0.05}{3}\left[2.161 + 4 \times 2.067 + 1.331\right] = 0.846$$

$$\omega_5 = \omega_3 + \frac{\Delta t}{3}\left[\omega_3' + 4\omega_4' + \omega_5'\right]$$

$$= 2.161 + \frac{0.05}{3}\left[12.65 - 4 \times 3.46 - 5.657\right] = 2.047$$

$$\delta_5' = \omega_5 = 2.047$$

$$\omega_5' = \frac{0.8 - 1.333\sin(0.846)}{0.0331} = -5.98$$

The computations are continued for the next time step in a similar manner.

MULTI MACHINE TRANSIENT STABILITY ANALYSIS

A typical modern power system consists of a few thousands of nodes with heavy interconnections. Computation simplification and memory reduction have been two major issues in the development of mathematical models and algorithms for digital computation of transient stability. In its simplest form, the problem of a multi machine power system under going a disturbance can be mathematically stated as follows:

$$\dot{x}(t) = f_I(x(t)) \qquad -\infty \le t \le 0$$

$$\dot{x}(t) = f_{II}(x(t)) \qquad 0 < t \le t_{ce}$$

$$\dot{x}(t) = f_{III}(x(t)) \qquad t_{ce} < t < \infty$$

$x(t)$ is the vector of state variables to describe the differential equations governing the generator rotor dynamics, dynamics of flux decay and associated generator controller dynamics (like excitation control, PSS, governor control etc). The function f_I describes the dynamics prior to the fault. Since the system is assumed to be in steady state, all the state variable are constant. If the fault occurs at t = 0, f_{II} describes the dynamics during fault, till the fault is cleared at time t_{cl}. The post–fault dynamics is governed by f_{III}. The state of the system x_{cl} at the end of the fault-on period (at $t = t_{cl}$) provides the initial condition for the post–fault network described which determines whether a system is stable or not after the fault is cleared. Some methods are presented in the following sections to evaluate multi machine transient stability. However, a detailed exposition is beyond the scope of the present book.

REDUCED ORDER MODEL

This is the simplest model used in stability analysis and requires minimum data. The following assumptions are made:

- Mechanical power input to each synchronous machine is assumed to be constant.
- Damping is neglected.
- Synchronous machines are modeled as constant voltage sources behind transient reactance.
- Loads are represented as constant impedances.

With these assumptions, the multi machine system is represented as in Fig. 9.26.

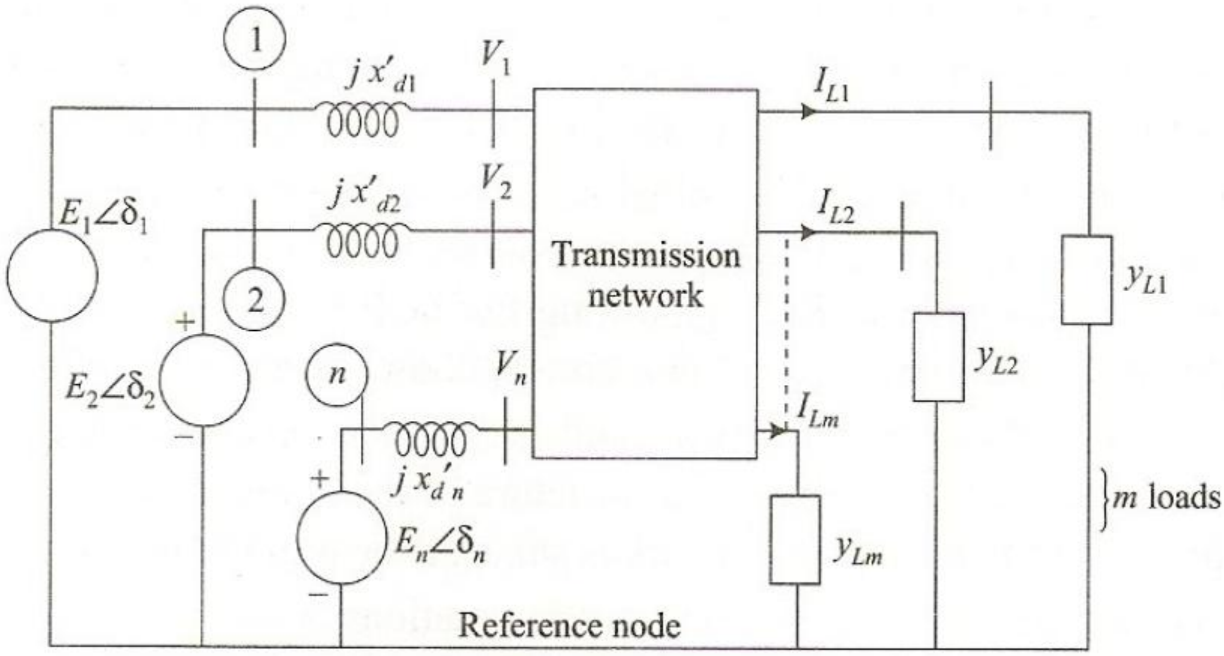


Fig 9.26 Multi machine system

Nodes 1, 2 …… n are introduced in the model and are called internal nodes (the terminal node is the external node connected to the transmission network). The swing equations are formed for the various generators using the following steps:

Step 1: All system data is converted to a common base.

Step 2: A prefault load flow is performed, to determine the prefault steady state voltages, at all the external buses. Using the prefault voltages, the loads are converted into equivalent shunt admittance, connected between the respective bus and the reference node. If the complex load at bus i is given by

$$S_i = P_{Li} + Q_{Li}$$

the equivalent admittance is given by

$$\mathrm{Y}_{\mathrm{Li}} = \frac{S^*_{Li}}{|V_{Li}|^2} = \frac{P_{Li} - jQ_{Li}}{|V_{Li}|^2}$$

Step 3: The internal voltages are calculated from the terminal voltages, using

$$|E_i|\angle\delta_i' = |V_i| + j\,x_{di}'\,I_i$$

$$= |V_i| + jx_{di}'\frac{S_{Gi}^*}{|V_i|}$$

$$= |V_i| + j\,x_{di}'\frac{(P_{Gi} - j\,Q_{Gi})}{|V_i|}$$

δ_i' is the angle of E_i with respect to V_i. If the angle of V_i is β_i, then the angle of E_i, with respect to common reference is given by $\delta_i = \delta_i' + \beta_i$. P_{Gi} and Q_{Gi} are obtained from load flow solution.

Step4: The bus admittance matrix Y_{bus} formed to run the load flow is modified to include the following.

(i) The equivalent shunt load admittance given by, connected between the respective load bus and the reference node.

(ii) Additional nodes are introduced to represent the generator internal nodes. Appropriate values of admittances corresponding to x_d', connected between the internal nodes and terminal nodes are used to update the Y_{bus}.

(iii) Y_{bus} corresponding to the faulted network is formed. Generally transient stability analysis is performed, considering three phase faults, since they are the most severe. The Y_{bus} during the fault is obtained by setting the elements of the row and column corresponding to the faulted bus to zero.

(iv) Y_{bus} corresponding to the post–fault network is obtained, taking into account line outages if any. If the structure of the network does not change, the Y_{bus} of the post-fault network is same as the prefault network.

Step 5: The admittance form of the network equations is

$$I = Y_{bus}\,V$$

Since loads are all converted into passive admittances, current injections are present only at the *n* generator internal nodes. The injections at all other nodes are zero. Therefore, the current vector I can be partitioned as

$$I = \left[\frac{I_n}{0}\right]$$

where I_n is the vector of current injections corresponding to the *n* generator internal nodes. Y_{bus} and V are also partitioned appropriately, so that

$$\left[\frac{I_n}{0}\right] = \left[\begin{array}{c|c} \frac{Y_1}{Y_3} & \frac{Y_2}{Y_4} \end{array}\right]\left[\frac{E_n}{V_t}\right]$$

where E_n is the vector of internal emfs of the generators and V_t is the vector of external bus voltages. From (9.91) we can write

$$I_n = Y_1 E_n + Y_2 V_t$$

$$0 = Y_3 E_n + Y_4 V_t$$

we get

$$V_t = -Y_4^{-1} Y_3 E_n$$

$$I_n = \left(Y_1 - Y_2\, Y_4^{-1}\, Y_3\right) E_n = \hat{Y}\, E_n$$

where $\hat{Y} = Y_1 - Y_2\, Y_4^{-1}\, Y_3$ is called the reduced admittance matrix and has dimension $n \times n$. $\hat{Y}$ gives the relationship between the injected currents and the internal generator voltages. It is to be noted we have eliminated all nodes except the *n* internal nodes.

<u>Step 6</u>: The electric power output of the generators are given by

$$P_{Gi} = \Re\,[E_i\, I_i^*.]$$

Substituting for I_i from (9.94) we get

$$P_{Gi} = |E_i|^2\, \hat{G}_{ii} + \sum_{j=1\neq i}^{n} |E_i||E_j| \left(\hat{B}_{ij} \sin(\delta_i - \delta_j) + \hat{G}_{ij} \cos(\delta_i - \delta_j)\right)$$

(This equation is derived in chapter on load flows)

<u>Step 7</u>: The rotor dynamics representing the swing is now given by

$$M_i \frac{d^2\delta_i}{dt^2} = P_{Mi} - P_{Gi} \qquad i = 1\ldots\ldots\ldots.n$$

The mechanical power P_{Mi} is equal to the pre-fault electrical power output, obtained from pre-fault load flow solution.

<u>Step 8</u>: The *n* second order differential equations can be decomposed into *2n* first order differential equations which can be solved by any numerical method .

Though reduced order models, also called classical models, require less computation and memory, their results are not reliable. Further, the interconnections of the physical network of the system is lost.

FACTORS AFFECTING TRANSIENT STABILITY:

The relative swing of a machine and the critical clearing time are a measure of the stability of a generating unit. From the swing equation, it is obvious that the generating units with smaller H, have larger angular swings at any time interval. The maximum power transfer $P_{max} = \dfrac{E_g V}{x'_d}$, where V is the terminal voltage of the generators. Therefore an increase in x'_d, would reduce P_{max}. Hence, to transfer a given power P_e, the angle δ would increase since $P_e = P_{max} \sin \delta$, for a machine with larger x'_d. This would reduce the critical clearing time, thus, increasing the probability of losing stability.

Generating units of present day have lower values of H, due to advanced cooling techniques, which have made it possible to increase the rating of the machines without significant increase in the size. Modern control schemes like generator excitation control, Turbine valve control, single-pole operation of circuit breakers and fast-acting circuit breakers with auto re-closure facility have helped in enhancing overall system stability.

Factors which can improve transient stability are
(i) Reduction of transfer reactance by using parallel lines.
(ii) Reducing transmission line reactance by reducing conductor spacing and increasing conductor diameter, by using hollow cores.
(iii) Use of bundled conductors.
(iv) Series compensation of the transmission lines with series capacitors. This also increases the steady state stability limit. However it can lead to problem of sub-synchronous resonance.
(v) Since most faults are transient, fast acting circuit breakers with rapid re-closure facility can aid stability.
(vi) The most common type of fault being the single-line-to-ground fault, selective single pole opening and re-closing can improve stability.
(vii) Use of braking resistors at generator buses. During a fault, there is a sudden decrease in electric power output of generator. A large resistor, connected at the generator bus, would partially compensate for the load loss and help in decreasing the acceleration of the generator. The braking resistors are switched during a fault through circuit breakers and remain for a few cycles after fault is cleared till system voltage is restored.
(viii) Short circuit current limiters, which can be used to increase transfer impedance during fault, there by reducing short circuit currents.
(ix) A recent method is fast valving of the turbine where in the mechanical power is lowered quickly during the fault, and restored once fault is cleared.
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