Contents

```
clear; clc; close all;
sd = 1; rng(sd);

% add paths
addpath(genpath('../src'))

% define parameters
d = 2;      % dimension of unknown parameters
M = 1000;   % number of samples
```

-- STANDARDIZE SAMPLES --

```
% generate samples
samples = sample_banana(M);
x_samples = samples(:,1);
y_samples = samples(:,2);

% standardize samples with a Gaussian linear diagonal map
G = GaussianPullbackDensity(d, true);
G = G.optimize([y_samples, x_samples]);
norm_samples = G.evaluate([y_samples, x_samples]);

% split into x and y samples
ynorm_samples = norm_samples(:,1);
xnorm_samples = norm_samples(:,2);
```

-- LEARN TRANSPORT MAP --

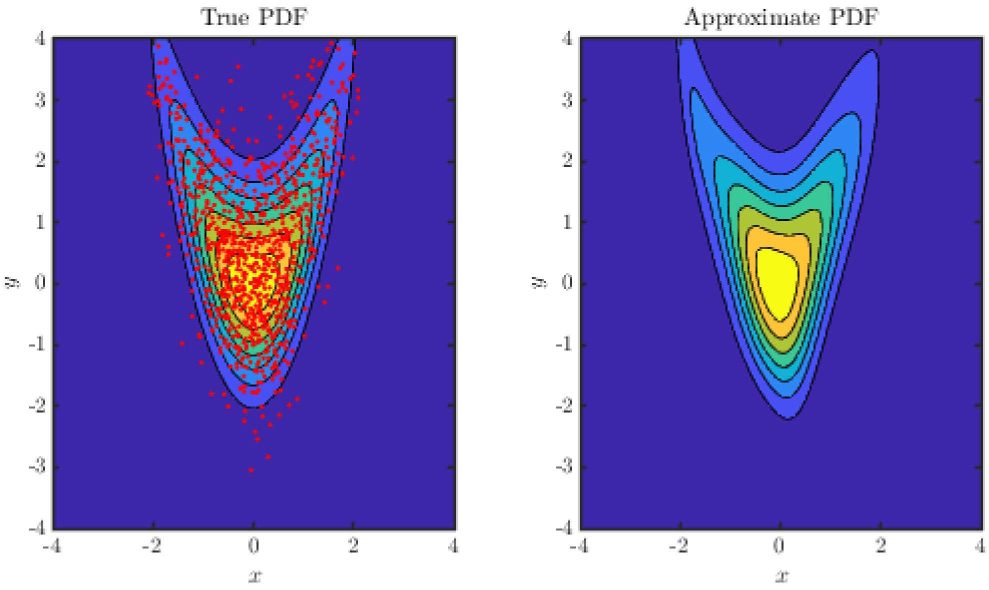
```
% define reference distribution
ref = IndependentProductDitribution({Normal(), Normal()});

% setup map with greedy basis selection (start from S(x) = Id(x))
basis = HermiteProbabilistPoly();
TM = identity_map(1:d, basis);
PB = PullbackDensity(TM, ref);

% specify a maximum number of terms (5 for S^1, 40 for S^2)
[PB, ~] = PB.greedy_optimize([ynorm_samples, xnorm_samples], [], [10,30], 'max_terms');

% compose map with linear transformation for pre-conditioning
CM = ComposedPullbackDensity({G, PB}, ref);
```

```
Term   0 - Training error: 1.418439, Validation error: NaN
Term   2 - Training error: 1.418438, Validation error: NaN
Term   3 - Training error: 1.273878, Validation error: NaN
Term   4 - Training error: 1.273215, Validation error: NaN
Term   5 - Training error: 1.265608, Validation error: NaN
Term   6 - Training error: 1.265072, Validation error: NaN
Term   7 - Training error: 1.265071, Validation error: NaN
Term   8 - Training error: 1.263520, Validation error: NaN
Term   9 - Training error: 1.261428, Validation error: NaN
Term  10 - Training error: 1.261427, Validation error: NaN
Term   0 - Training error: 1.418439, Validation error: NaN
Term   2 - Training error: 1.418429, Validation error: NaN
Term   3 - Training error: 1.415890, Validation error: NaN
Term   4 - Training error: 1.409103, Validation error: NaN
Term   5 - Training error: 1.404070, Validation error: NaN
Term   6 - Training error: 1.403844, Validation error: NaN
Term   7 - Training error: 1.196712, Validation error: NaN
Term   8 - Training error: 1.175529, Validation error: NaN
Term   9 - Training error: 1.175528, Validation error: NaN
Term  10 - Training error: 1.000332, Validation error: NaN
Term  11 - Training error: 0.996817, Validation error: NaN
Term  12 - Training error: 0.995597, Validation error: NaN
Term  13 - Training error: 0.986088, Validation error: NaN
Term  14 - Training error: 0.985842, Validation error: NaN
Term  15 - Training error: 0.985221, Validation error: NaN
Term  16 - Training error: 0.985173, Validation error: NaN
Term  17 - Training error: 0.984171, Validation error: NaN
Term  18 - Training error: 0.984069, Validation error: NaN
Term  19 - Training error: 0.983910, Validation error: NaN
Term  20 - Training error: 0.981600, Validation error: NaN
Term  21 - Training error: 0.981595, Validation error: NaN
Term  22 - Training error: 0.968504, Validation error: NaN
Term  23 - Training error: 0.968037, Validation error: NaN
Term  24 - Training error: 0.967870, Validation error: NaN
Term  25 - Training error: 0.967058, Validation error: NaN
Term  26 - Training error: 0.966220, Validation error: NaN
Term  27 - Training error: 0.965892, Validation error: NaN
Term  28 - Training error: 0.965886, Validation error: NaN
Term  29 - Training error: 0.965161, Validation error: NaN
Term  30 - Training error: 0.964836, Validation error: NaN
```

-- PLOT FULL DENSITY --

```
% check approximation
xx = linspace(-4,4,100);
[X, Y] = meshgrid(xx, xx);

% evaluate approximate and true density
true_pi   = exp(log_pdf_banana([X(:), Y(:)]));
approx_pi = exp(CM.log_pdf([Y(:), X(:)]));

true_pi   = reshape(true_pi, size(X,1), size(X,2));
approx_pi = reshape(approx_pi, size(X,1), size(X,2));

% plot densities and samples
figure('position',[0,0,600,300])

subplot(1,2,1)
contourf(X, Y, true_pi)
```

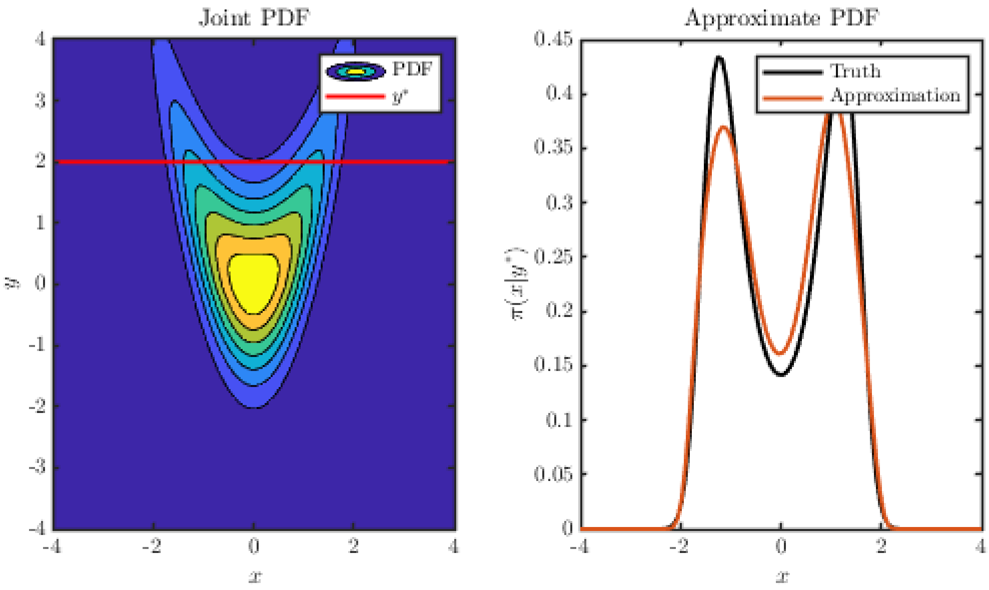
```
hold on
plot(samples(:,1), samples(:,2), '.r','MarkerSize',6)
axis([-4,4,-4,4])
lim = caxis;
xlabel('$x$')
ylabel('$y$')
title('True PDF')

subplot(1,2,2)
contourf(X, Y, approx_pi)
axis([-4,4,-4,4])
caxis(lim)
hold on
xlabel('$x$')
ylabel('$y$')
title('Approximate PDF')
```



-- PLOT CONDITIONAL DENSITY --

```
% check approximation
yst = 2;
xx = linspace(-4,4,100);

% evaluate approximate and true density
true_cond_pi_tilde = exp(log_pdf_banana([xx.', repmat(yst,length(xx),1)]));
true_cond_pi_norm_const = trapz(xx, true_cond_pi_tilde);
true_cond_pi = true_cond_pi_tilde/true_cond_pi_norm_const;
approx_pi = exp(CM.log_pdf([repmat(yst,length(xx),1), xx.'],2));

% plot densities and samples
figure('position',[0,0,600,300])

subplot(1,2,1)
contourf(X, Y, true_pi)
hold on
plot(xx, yst*ones(length(xx),1), '-r')
axis([-4,4,-4,4])
legend('PDF','$y^*$')
xlabel('$x$')
ylabel('$y$')
```

```
title('Joint PDF')
hold off

subplot(1,2,2)
hold on
plot(xx, true_cond_pi, '-k')
plot(xx, approx_pi)
xlim([-4,4])
legend('Truth','Approximation')
xlabel('$x$')
ylabel('$\pi(x|y^*)$')
title('Approximate PDF')
hold off
```

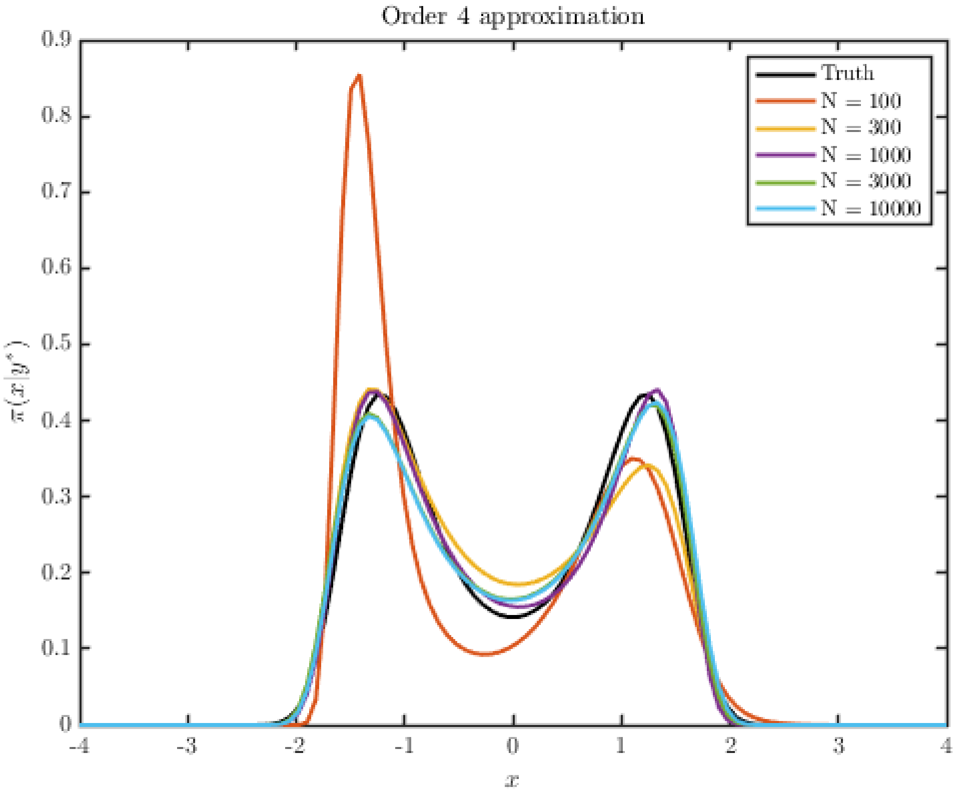


-- PLOT APPROXIMATION VS SAMPLE-SIZE --

```
M_vect = [100,300,1000,3000,10000];  % define sample-sizes
order = 4;                           % define map order

% define cell to store results
approx_pi = cell(length(M_vect),1);

for i=1:length(M_vect)

    % generate samples
    samples = sample_banana(M_vect(i));
    x_samples = samples(:,1);
    y_samples = samples(:,2);

    % standardize samples with a Gaussian linear diagonal map
    G = GaussianPullbackDensity(d, true);
    G = G.optimize([y_samples, x_samples]);
    norm_samples = G.evaluate([y_samples, x_samples]);

    % split into x and y samples
    ynorm_samples = norm_samples(:,1);
    xnorm_samples = norm_samples(:,2);

    % define reference distribution
    ref = IndependentProductDitribution({Normal(), Normal()});
    % learn map with total-order basis
    TM = total_order_map(1:d, HermiteProbabilistPoly(), order);
    PB = PullbackDensity(TM, ref);
    PB = PB.optimize([ynorm_samples, xnorm_samples]);

    % compose map with linear transformation for pre-conditioning
    CM = ComposedPullbackDensity({G, PB}, ref);

    % evaluate and plot approximate density
    approx_pi{i} = exp(CM.log_pdf([repmat(yst,length(xx),1), xx.'],2));

end

figure
hold on
plot(xx, true_cond_pi, '-k','LineWidth',2,'DisplayName','Truth')
for i=1:length(M_vect)
    plot(xx, approx_pi{i}, 'DisplayName', ['N = ' num2str(M_vect(i))])
end
legend('show')
xlabel('$x$')
ylabel('$\pi(x|y^*)$')
title(['Order ' num2str(order) ' approximation'])
hold off
snapnow
```

-- DEFINE MODEL --

```
function X = sample_banana(N)
    x1 = randn(N,1);
    x2 = x1.^2 + randn(N,1);
    X = [x1, x2];
end

function log_pi = log_pdf_banana(X)
    log_pi_x1 = log(normpdf(X(:,1)));
    log_pi_x2 = log(normpdf(X(:,2) - X(:,1).^2));
    log_pi = log_pi_x1 + log_pi_x2;
end

% -- END OF FILE --
```



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