# COMP5318/COMP4318 Machine Learning and Data Mining

Week 11 Tutorial exercises
Hidden Markov Models

Exercise 1. *Hidden Markov Models*

You were locked in a room for several days, and you were asked about the weather outside. The only piece of evidence you had is whether Alicia, who comes into the room to bring your meals, is carrying an umbrella or not.

The following Hidden Markov Model models the situation. The initial state probabilities are: $A_0(Sunny)=0.4$, $A_0(Rainy)=0.3$ and $A_0(Foggy)=0.3$.
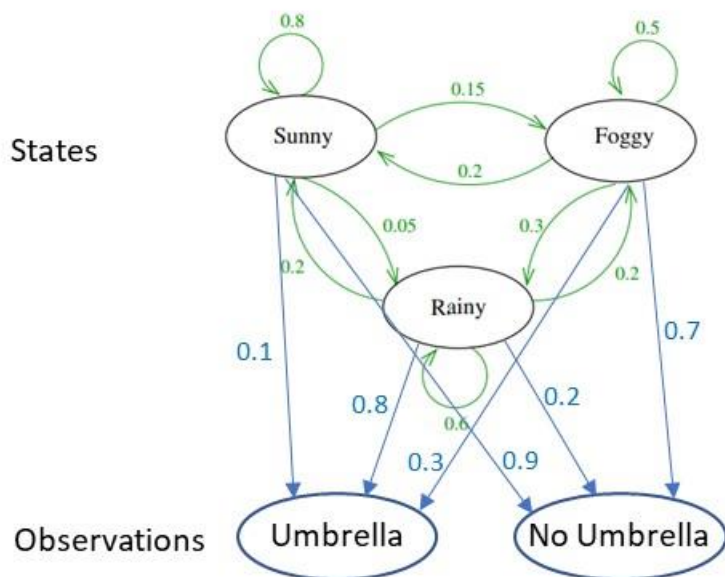
Suppose that on the first day you observed that Alicia had no umbrella, then on the second day Alicia carried an umbrella. What is the most likely sequence of hidden states?

Solution:

Which algorithm should we use – the Forward algorithm or the Viterbi algorithm? This is Hidden Markov Model Problem 2, the task is to find the most likely sequence of states. Hence, we should use the Viterbi algorithm.

States: Sunny (S), Rainy (R), Foggy (F)
Observations: Umbrella (U), No Umbrella (NA)
Observation sequence: No Umbrella (NA), Umbrella (U)

Step 1: Initialization

Day 1: NA observed

$V_S(1) = A_0(S)e_S(NA)$= 0.4*0.9=0.36
$V_R(1) = A_0(R)e_R(NA)$= 0.3*0.2=0.06
$V_F(1) = A_0(F)e_F(NA)$= 0.3*0.7=0.21

Step 2: Iteration

Day 2: U observed

$V_S(2) = e_S(U)$*max$(V_S(1)a_{SS}, V_R(1)a_{RS}, V_F(1)a_{FS})=$
        = 0.1*max(**0.36*0.8,** 0.06*0.2, 0.21*0.2)=0.1*0.36*0.8=0.0288
$Ptr_S(2) = $ argmax$(\mathbf{0.36 * 0.8},\ 0.06 * 0.2\ , 0.21 * 0.2) = 1$, i.e. S

$V_R(2) = e_R(U)$*max$(V_S(1)a_{SR},\ V_R(1)a_{RR},\ V_F(1)a_{FR})=$
        = 0.8*max(0.36*0.05, 0.06*0.6, **0.21*0.3**)=0.8*0.21*0.3=0.0504
$Ptr_R(2) = $ argmax$(0.36 * 0.05, 0.06 * 0.6,\ \mathbf{0.21 * 0.3}) = 3$, i.e. F

$V_F(2) = e_F(U)$*max$(V_S(1)a_{SF},\ V_R(1)a_{RF},\ V_F(1)a_{FF})=$
        = 0.3*max(0.36*0.15, 0.06*0.2, **0.21*0.5**)=0.3*0.21*0.5=0.0315
$Ptr_F(2) = $ argmax$(0.36 * 0.15, 0.28 * 0.2, \mathbf{0.21 * 0.5}) = 3$, i.e. F

Step 3: Termination and trace-back

Final state = argmax$(V_S(2), V_R(2), V_F(2))$=argmax(0.0288, **0.0504**, 0.0315)=2, i.e. R

Trace-back through pointers:
$Ptr_R(2) = $F

Hence, the most likely sequence of hidden states is F, R (Foggy, Rainy)