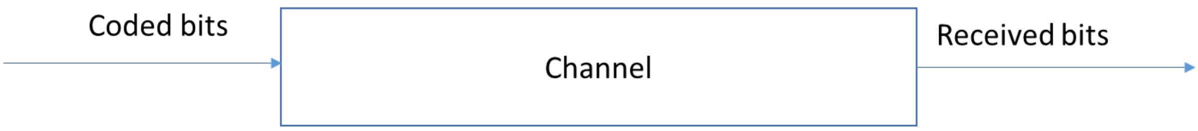
COMP 9121 Week 2

Data-Link Layer

Exercise 1. CRC

(1) Consider the following information bits 1010101 and CRC generator 1101. What are the coded bits to be transmitted? Show that the coded bits are divisible by 1101.

(2) The system can be shown in the following figure.



Received bits – (minus) coded bits is called the error pattern (note that here "–" is equivalent to XOR). Obviously, all "0" error pattern means "no error".

Please show

(a) Error pattern 0000100 can be detected by the receiver.

(b) Error pattern 0001101 cannot be detected by the receiver.

(c) Could you find another error pattern which cannot be detected by the receiver?

Exercise 2. Slotted ALOHA

Consider two nodes, A and B, that use the slotted ALOHA protocol to contend for a channel. Suppose node A has more data to transmit than node B, and node A's retransmission probability p_A is greater than node B's retransmission probability, p_B.

(1) Provide a formula for node A's average throughput. What is the total efficiency of the protocol with these two nodes?

(2) If $p_A = 2p_B$, is node A's average throughput twice as large as that of node B? Why or why not? If not, how can you choose p_A and p_B to make that happen?

(3) In general, suppose there are N nodes, among which node A has retransmission probability $2p$ and all other nodes have retransmission probability p. Provide expressions to compute the average throughputs of node A and of any other node.

Exercise 3. More on Slotted ALOHA

Suppose four active nodes—nodes A, B, C and D—are competing for access to a channel using slotted ALOHA. Assume each node has an infinite number of packets to send. Each node attempts to transmit in each slot with probability *p*. The first slot is numbered slot 1, the second slot is numbered slot 2, and so on.

(1) What is the probability that node A succeeds for the first time in slot 5?

(2) What is the probability that some node (either A, B, C or D) succeeds in slot 4?

(3) What is the probability that the first success occurs in slot 3?

(4) What is the efficiency of this four-node system?

G(x) = 1101

res: 1010101 011

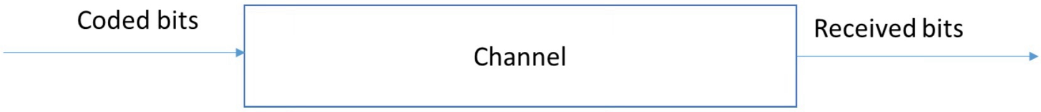
(1) Consider the following information bits 1010101 and CRC generator 1101. What are the coded bits to be transmitted? Show that the coded bits are divisible by 1101.

```
            1101111
1101 ) 1010101 000
       1101              CRC code
       01111
        1101
        00100
        00000
          1001
          1101
          01000
           1101
           01010
            1101
            01110
             1101
             0011
```

Redundancy

```
          1101111
1101 ) 1010101 011
       1101
       01111
        1101
         1001
         1101
         1000
         1101
         1011
         1101
          1101
          1101
             0
```

(2) The system can be shown in the following figure.



Received bits – (minus) coded bits is called the error pattern (note that here "–" is equivalent to XOR). Obviously, all "0" error pattern means "no error".

Please show $G(x) = 1101$

(a) Error pattern 0000100 can be detected by the receiver.

(b) Error pattern 0001101 cannot be detected by the receiver.

(c) Could you find another error pattern which cannot be detected by the receiver?

(a) $E(x) = 100$

(b) $E(x) = 1101$ $\quad 1101 \overline{\big)\ 1101\ }$

(c) 1101 1101

Exercise 2. Slotted ALOHA

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(1) Provide a formula for node A's average throughput. What is the total efficiency of the protocol with these two nodes?

(2) If $p_A = 2p_B$, is node A's average throughput twice as large as that of node B? Why or why not? If not, how can you choose p_A and p_B to make that happen?

(3) In general, suppose there are N nodes, among which node A has retransmission probability $2p$ and all other nodes have retransmission probability p. Provide expressions to compute the average throughputs of node A and of any other node.

X: no. of packets attempted for transmission in one timeslot.

Normalized throughput $= P(X=1)$ # N is all nodes which want to use this chanel

$= Np(1-p)^{N-1}$ # we need do binomial for p

(1). throughput：吞吐量

A's efficiency: $\boxed{P_A(1-P_B)}$

$$Np(1-p)^{N-1} = 2p(1-p) = p(1-p) + p(1-p)$$

$\Downarrow$ why? 我们需要所有节点的组合可能

Total Efficiency : $\boxed{P_A(1-P_B) + P_B(1-P_A)}$

(2) Assume $P_A = 2P_B$

A's : $P_A(1-P_B) = 2P_B - 2P_B^2$

B's : $P_B(1-P_A) = P_B - 2P_B^2$

$\Rightarrow$ A's $\neq$ 2B's

In order to make it happen, we have

$$A's = 2B's$$

$$P_A - P_A P_B = 2P_B - 2P_A P_B \Rightarrow P_A + P_A P_B = 2P_B \Rightarrow P_A = \frac{2P_B}{1+P_B}$$

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(1) notes

If $N=3$, there are A, B, C Nodes

$$NP(1-P)^{N-1} = 3P(1-P)^2$$

$$\Downarrow$$

$$P_A(1-P_B)(1-P_C) + P_B(1-P_A)(1-P_C) + P_C(1-P_B)(1-P_A)$$

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(3)

A's average throughput:

$P_A = 2P$ $\quad P_{other} = P$ (i.e. $P_B = P_C = \cdots P_N = P$)

Hence it's average throughput $= P_A(1-P_{other})^{N-1}$

$$= \boxed{2P(1-P)^{N-1}}$$

Other's average throughput $= P_{other}(1-P_A)^{N-2}(1-P_A)$

$$= \boxed{P(1-P)^{N-2}(1-2P)}$$

X

Exercise 3. More on Slotted ALOHA

Suppose four active nodes—nodes A, B, C and D—are competing for access to a channel using slotted ALOHA. Assume each node has an infinite number of packets to send. Each node attempts to transmit in each slot with probability p. The first slot is numbered slot 1, the second slot is numbered slot 2, and so on.

(1) What is the probability that node A succeeds for the first time in slot 5?

(2) What is the probability that some node (either A, B, C or D) succeeds in slot 4?

(3) What is the probability that the first success occurs in slot 3?

(4) What is the efficiency of this four-node system?

(1) let $P(A)$ represent the probability node A succeeds in a slot

$$P(\text{A firstly succeed in slot5}) = \underbrace{[1-P(A)]^4}_{\text{前4次A都失败}} \underbrace{P(A)}_{\text{第5次成功}}$$

(2) $P(\text{some node succeed in slot 4})$ # in any slot is the same answer

$$= P(\text{A succeed in slot4}) + \cdots + P(\text{D succeed in slot 4})$$

$$= (1-p)^3 p + \cdots + (1-p)^3 p$$ # $(1-p)^3$: 其它3个node都失败

p: 指定node成功

$$= 4(1-p)^3 p = P(X=1)$$

(3) $P(\text{first success in slot 3})$

$$= P(\text{No node succeed in first 2 slots}) \cdot P(\text{some node success in slot3})$$

$$= [1 - P(\text{some nodes succeed in a slot})]^2 \, P(\text{Some nodes succeed in slot3})$$

$$= \underbrace{[1-(4(1-p)^3 p)]^2}_{\text{前2个node}} \cdot \underbrace{(4(1-p)^3 p)}_{\text{3rd}}$$

(4)

Efficiency = Normalized Throughput

$$= P(X=1)$$

$$= \cancel{\binom{N}{k} p^{k} (1-p)^{N-k}} \qquad Np(1-p)^{N-1}$$

$$= \cancel{\binom{4}{1} p^{1} (1-p)^{3}}$$

$$= \frac{4!}{3!\,1!} p(1-p)^{3}$$

$$= \frac{4\times3\times2\times1}{3\times2\times1\times1} \times \cdots$$

$$= 4p(1-p)^{3}$$

#会发现其实就是(2)所求的