Bradley Quinn   Question 2   Stability of a System

1) Find A^{-1}, using Gauss-Jordan Elim and $[A|I_n] \Rightarrow ERO \Rightarrow [I_n|A^{-1}]$

$$\left[\begin{array}{cc|cc} 19 & 20 & 1 & 0 \\ 20 & 21 & 0 & 1 \end{array}\right] \xrightarrow{-1(R_1)+R_2} \left[\begin{array}{cc|cc} 19 & 20 & 1 & 0 \\ 1 & 1 & -1 & 1 \end{array}\right] \xrightarrow{P(R_1,R_2)} \left[\begin{array}{cc|cc} 1 & 1 & -1 & 1 \\ 19 & 20 & 1 & 0 \end{array}\right] \cdots$$

$$\xrightarrow{-19(R_1)+R_2} \left[\begin{array}{cc|cc} 1 & 1 & -1 & 1 \\ 0 & 1 & 20 & -19 \end{array}\right] \xrightarrow{-1(R_2)+R_1} \left[\begin{array}{cc|cc} 1 & 0 & -21 & 20 \\ 0 & 1 & 20 & -19 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -21 & 20 \\ 20 & -19 \end{bmatrix}, \quad A = \begin{bmatrix} 19 & 20 \\ 20 & 21 \end{bmatrix}$$

2) Find Cond (A)

For the matrix row norms,

$$\text{Cond}(A) = \underbrace{\max_{1 \le i \le n} \sum_{j=1}^{n} |a_{ij}|}_{\|A\|} \cdot \underbrace{\max_{1 \le i \le n} \sum_{j=1}^{n} |a^{-1}_{ij}|}_{\|A^{-1}\|}$$

$$\text{Cond}(A) = (41) \cdot (41) = 1681$$

- Because we aim to minimize $\frac{\|x - \hat{x}\|_v}{\|x\|_v}$ the difference between our expected vector norm and our approximated vector norm we would like to have our Cond(A) value as near to 1 as possible, a well-conditined system. Cond(A) is way larger than 1 and elements of A and A^{-1} therefore this system is ill-conditioned, to respect, b_1 and b_2.