

CSC 2262: LU Factorization Problem

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & -1 \\ -8 & 2 & 2 \end{bmatrix}, \quad A = LU$$

A to U

- Replace R_i by $R_i - kR_j$

$$\begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & -1 \\ -8 & 2 & 2 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 1 \\ -8 & 2 & 2 \end{bmatrix} \xrightarrow{R_3 + 4R_1} \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 6 & -2 \end{bmatrix}$$

$$\xrightarrow{R_3 + 3R_2} \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} = U$$

Constructing L from K multipliers

$$K = 2, -4, -3$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix}$$

Conclusion

$$A = LU \Rightarrow \begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & -1 \\ -8 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$A \qquad \qquad L \qquad \qquad U$