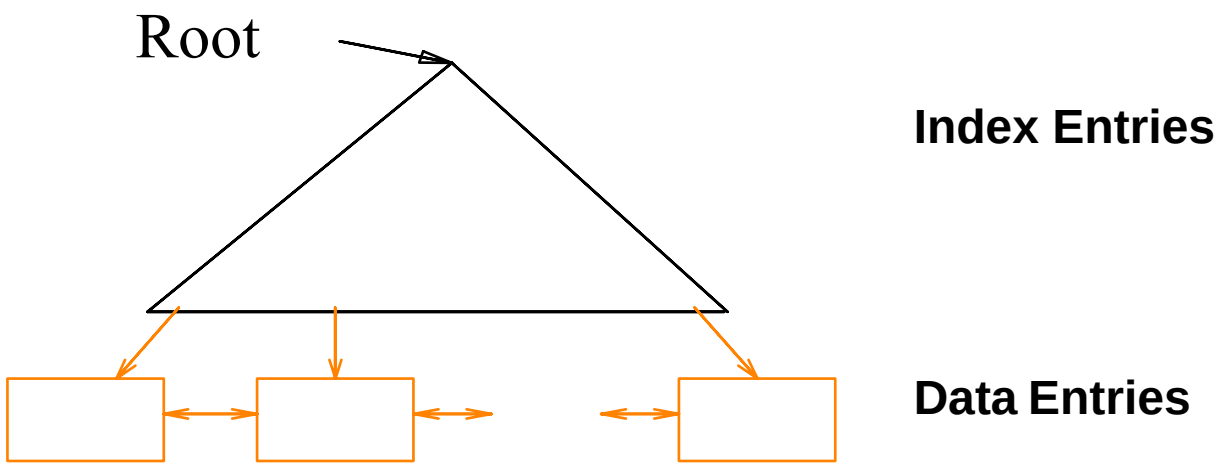
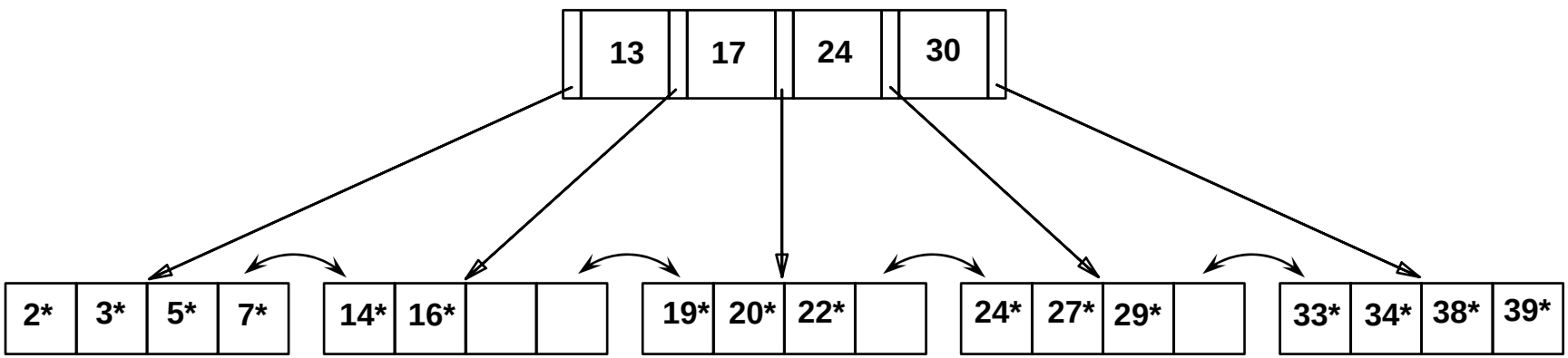
Lecture 6.2: B+ Tree



Example B+ Tree

- Search begins at root, and key comparisons direct it to a leaf.
- Search for
  - 5*
  - 19*
  - , all data entries >= 24* ...



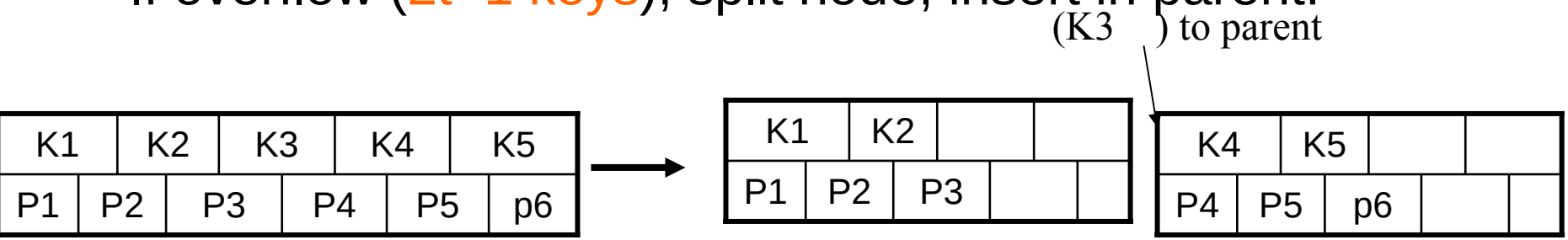
Inserting a Data Entry into a B+ Tree

- Find correct leaf *X.*
- Put data entry onto *X*.
  - If *X* has enough space, *done*!
  - Else, must *split* *X (into X and a new node X2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *X2* into parent of *X*.
- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key.

Insertion in a B+ Tree

Insert (K, P)

- Find leaf where K belongs, insert
- If no overflow (2t keys or less), halt.
- If overflow (2t+1 keys), split node, insert in parent:

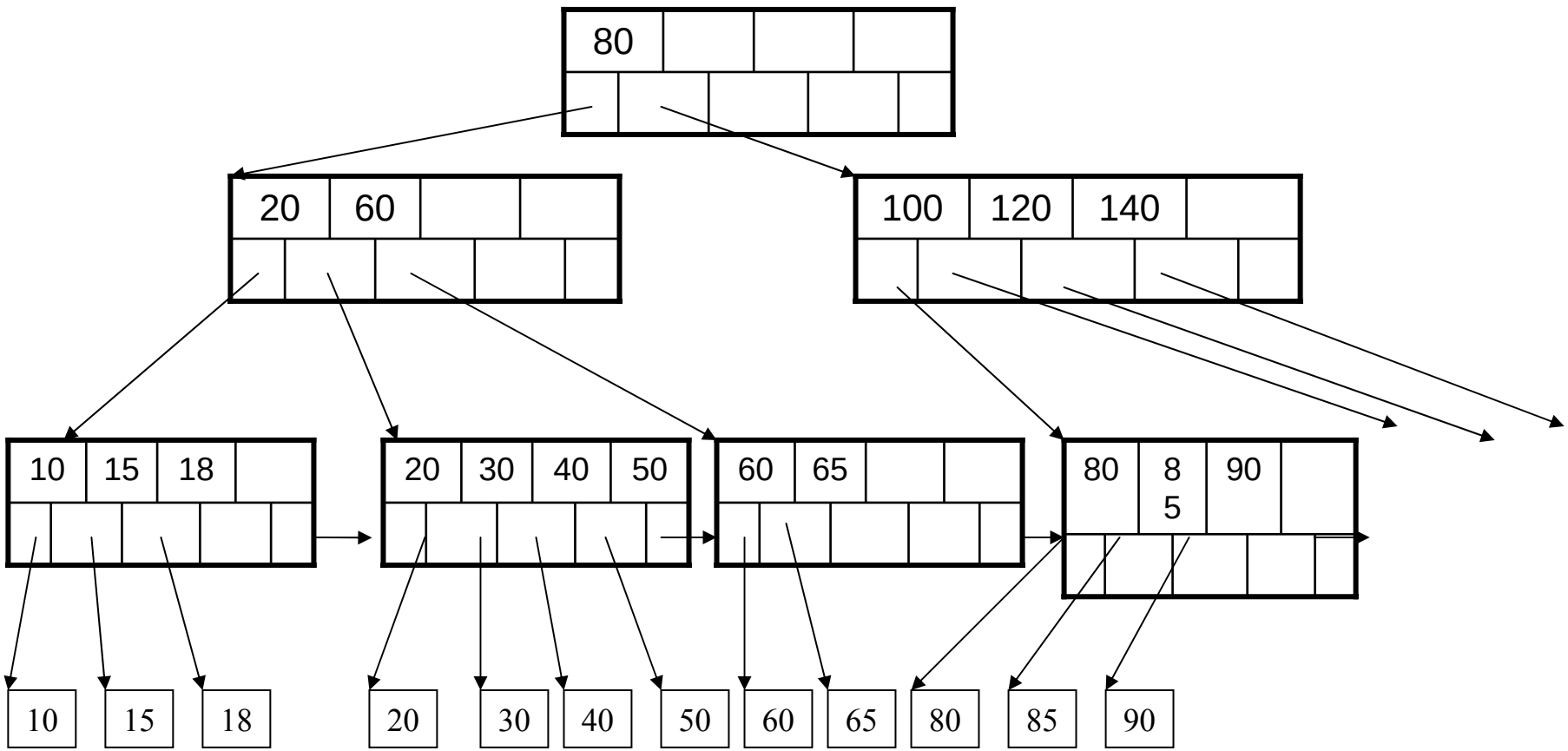


- If leaf, keep K3 too in right node
- When root splits, new root has 1 key only

Insertion in a B+ Tree

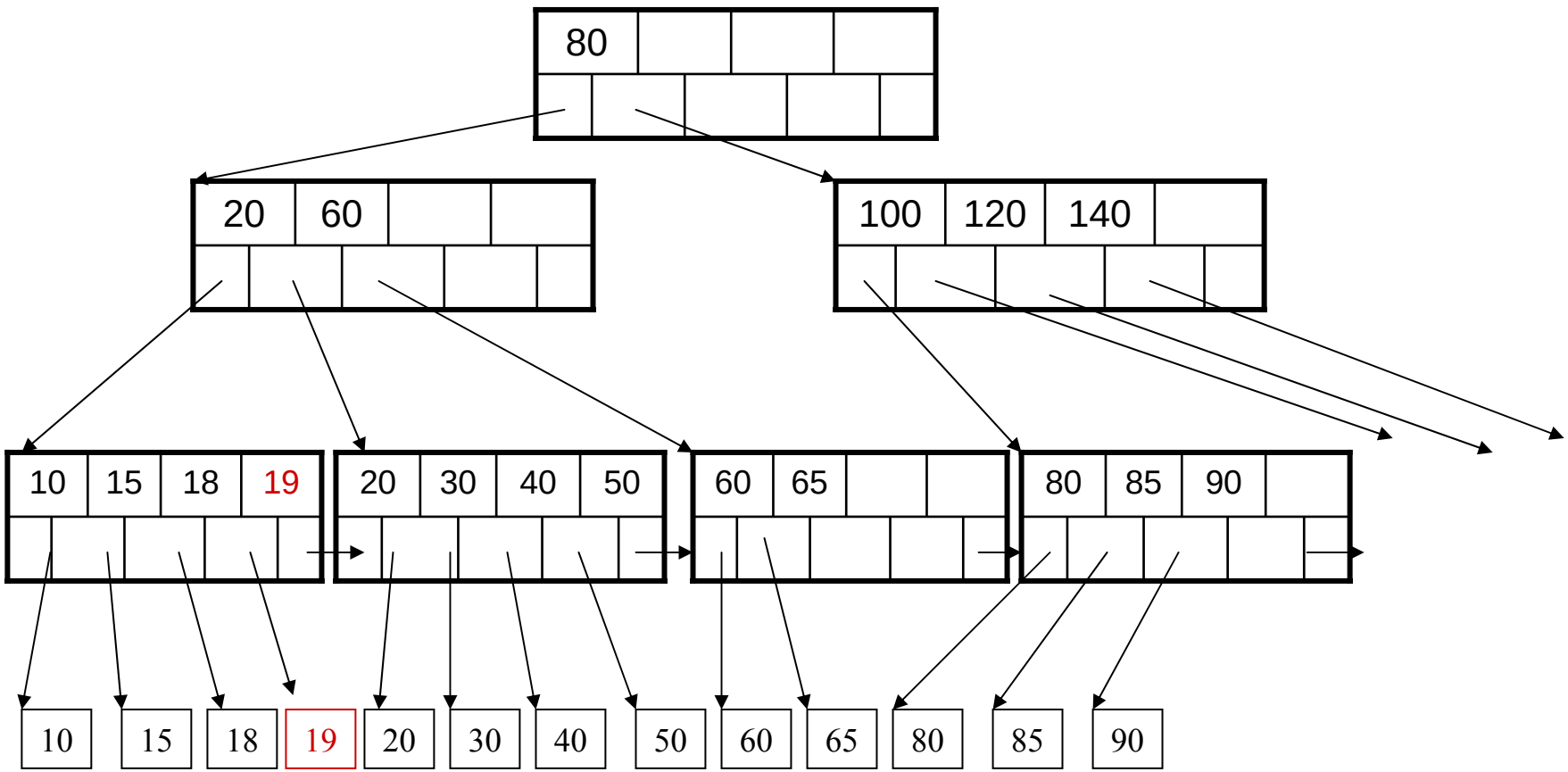
Bậc của cây t=2

Insert K=19



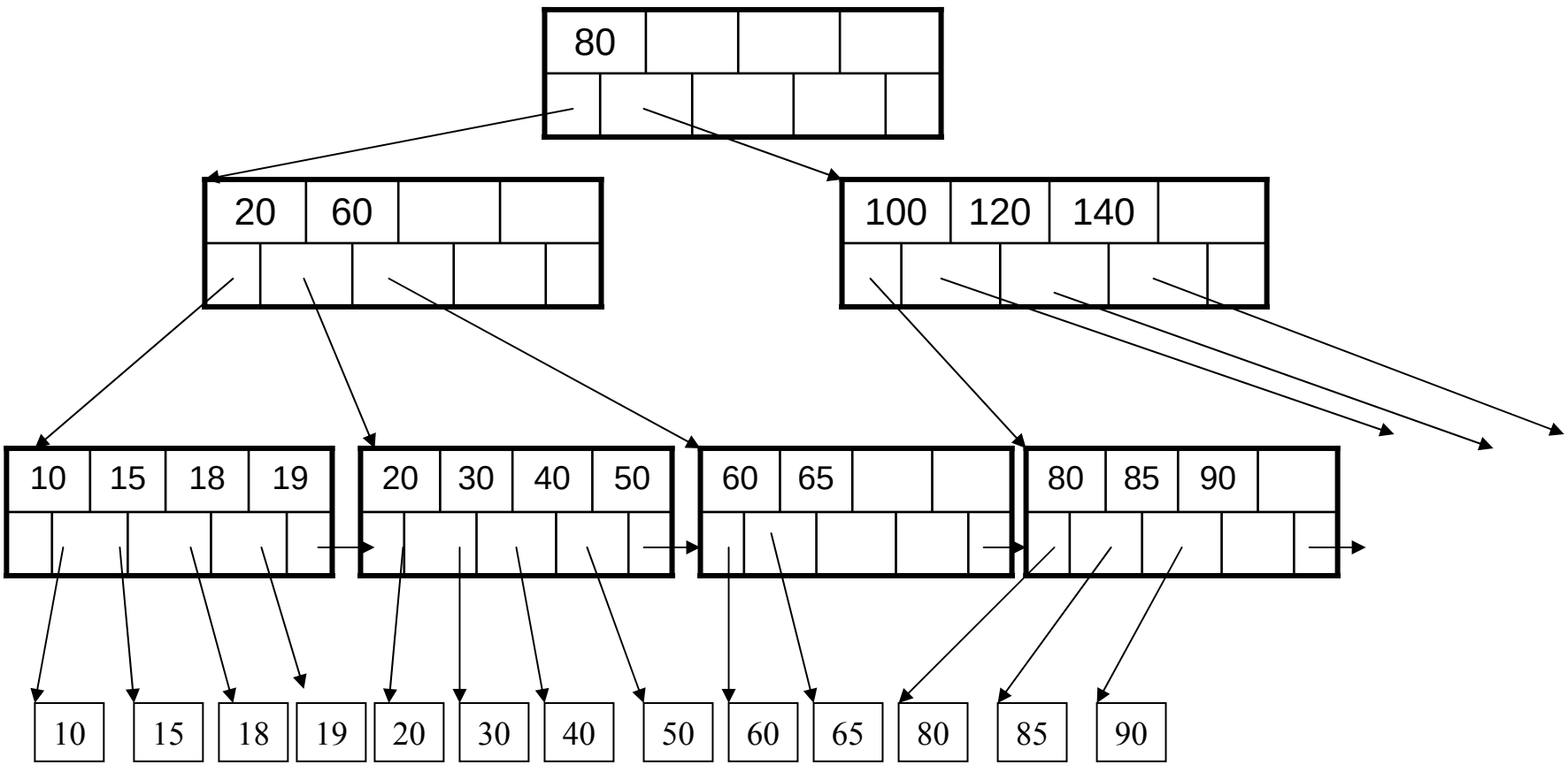
Insertion in a B+ Tree

After insertion



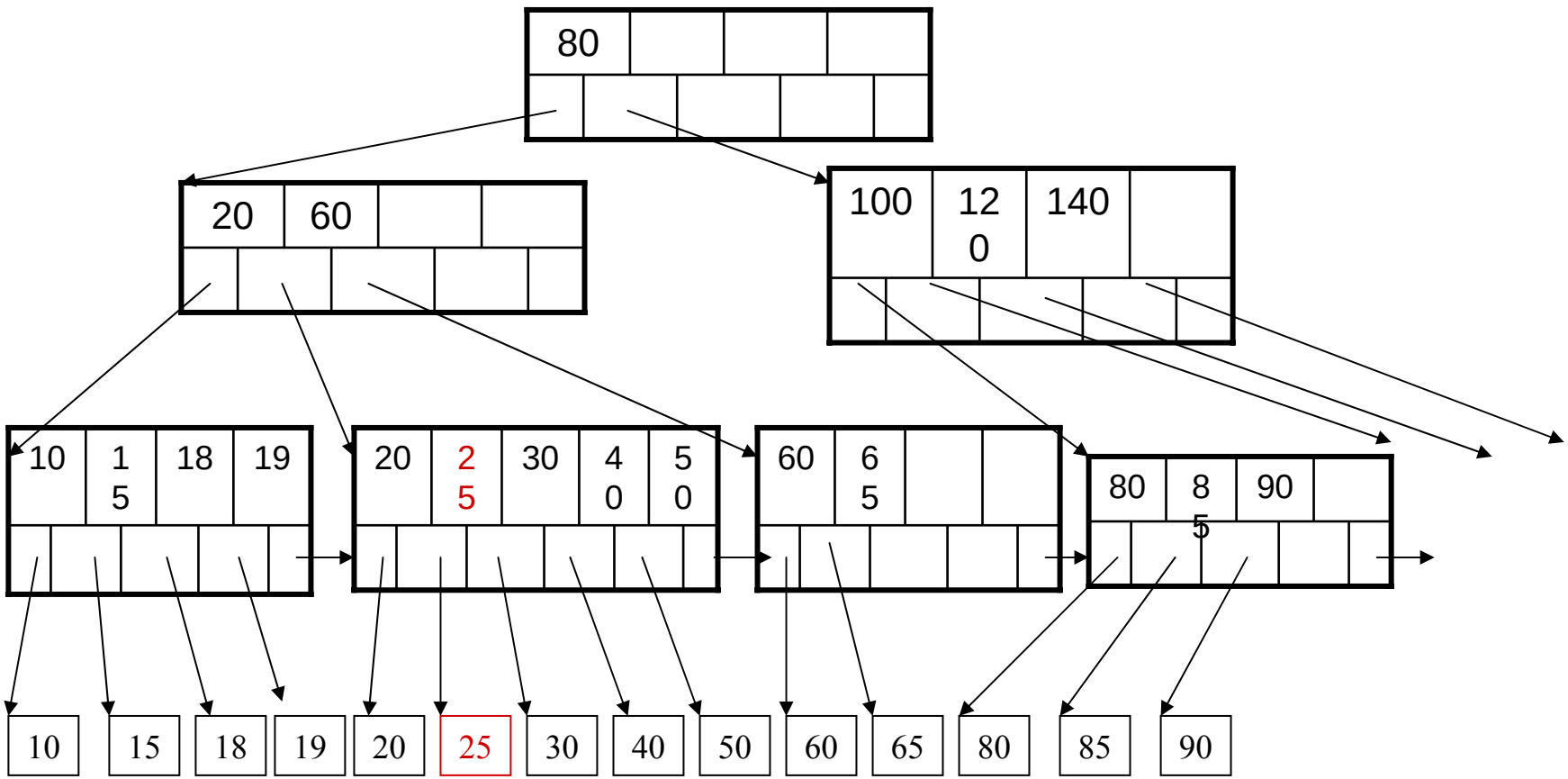
Insertion in a B+ Tree

Now insert 25



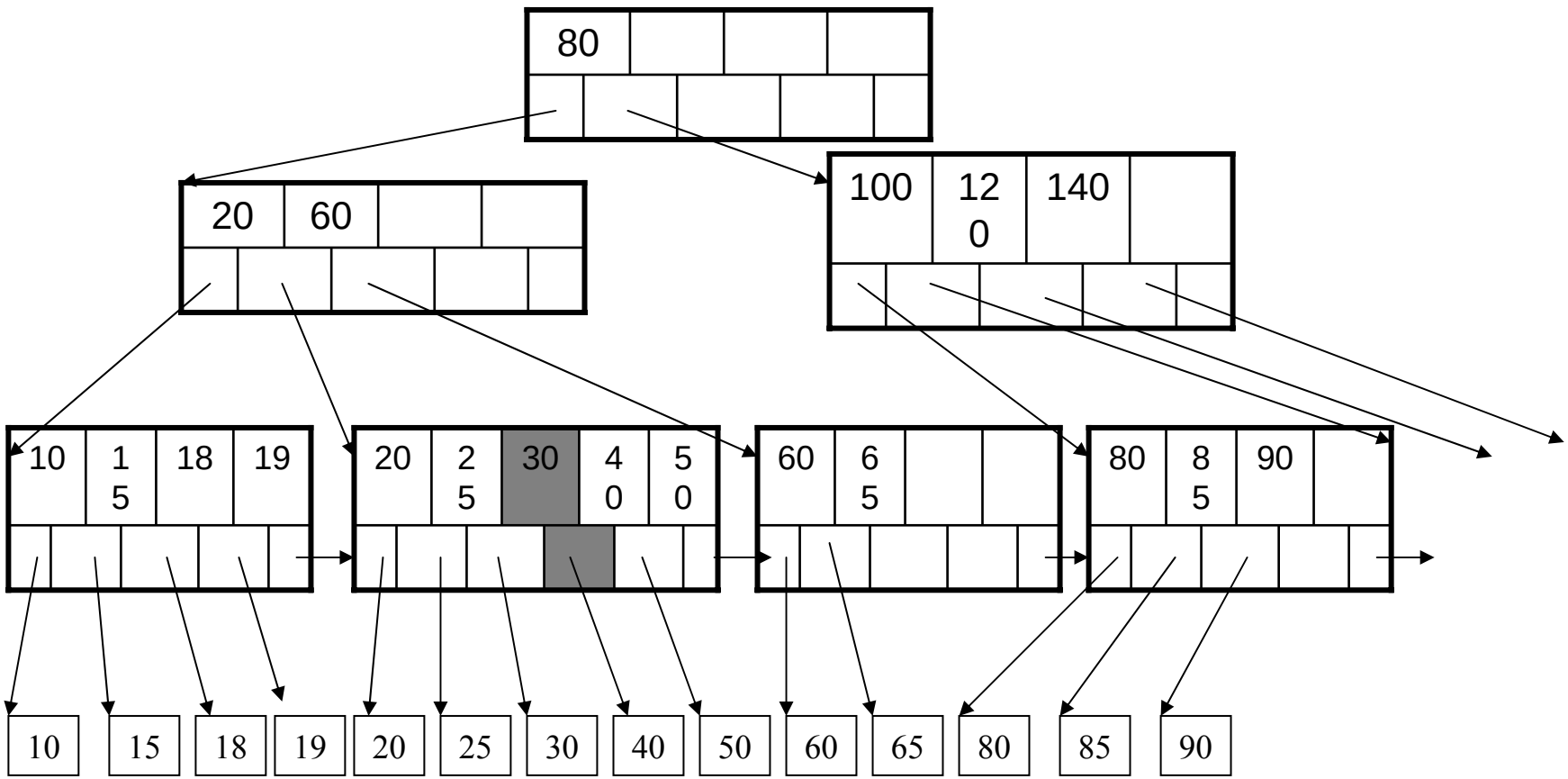
Insertion in a B+ Tree

After insertion



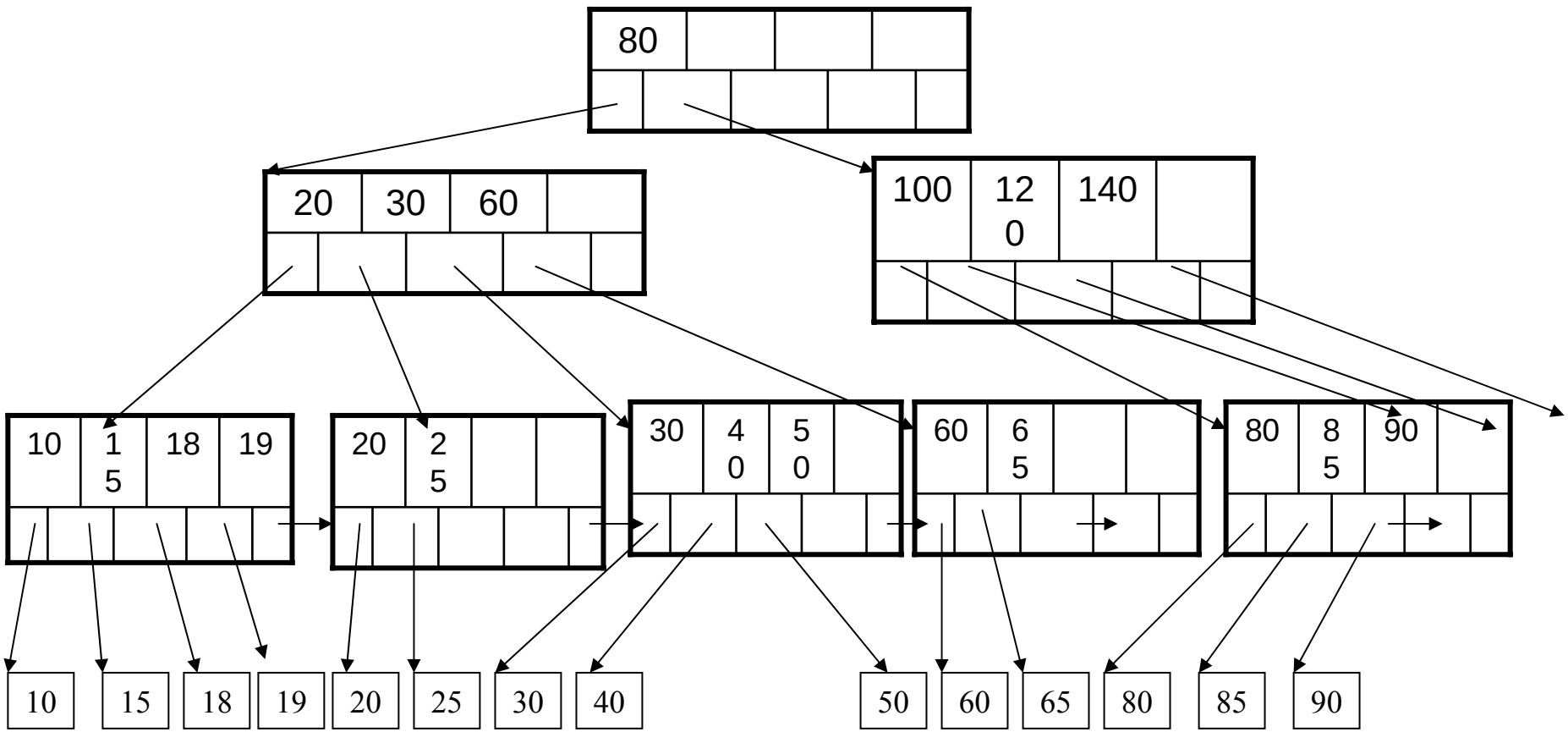
Insertion in a B+ Tree

But now have to split !



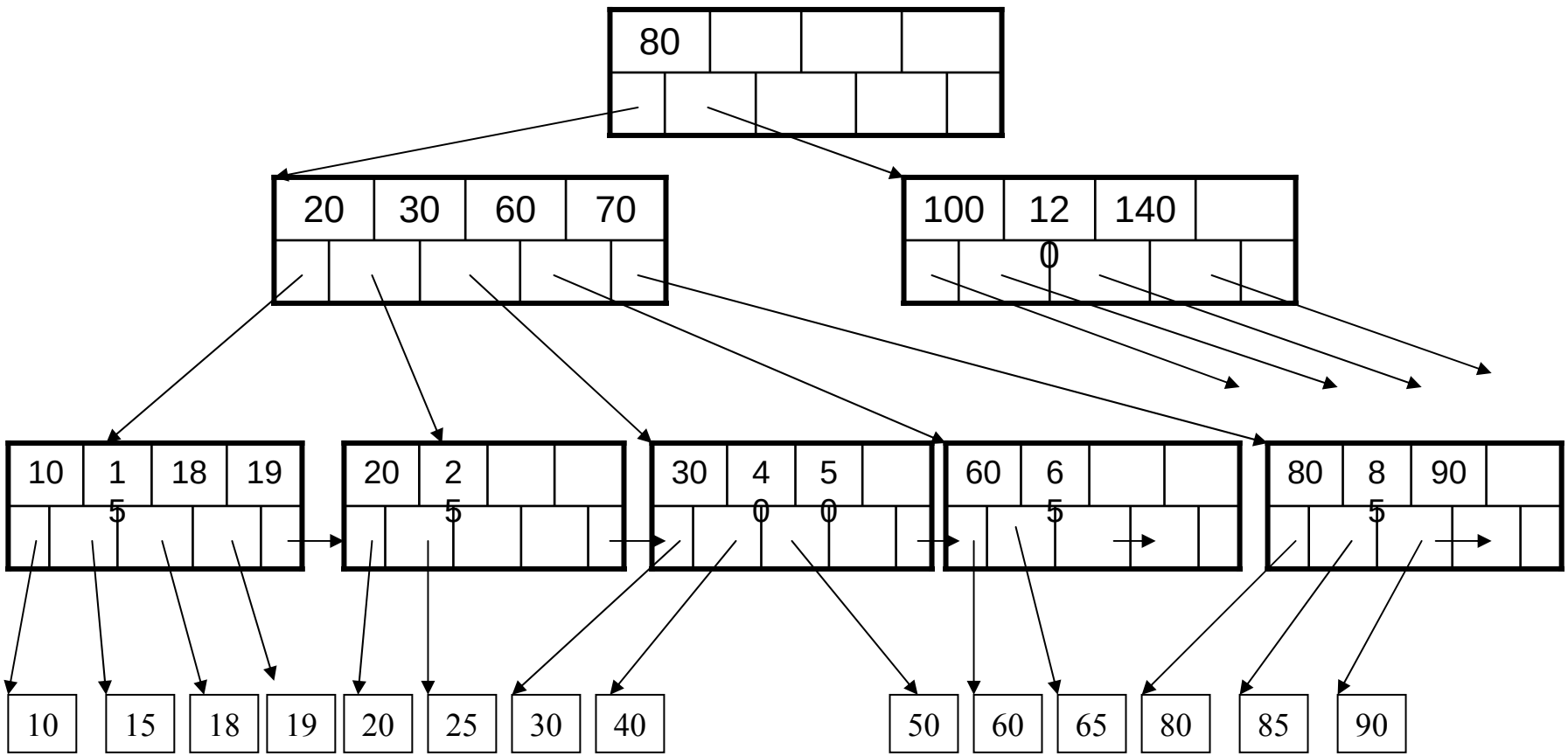
Insertion in a B+ Tree

After the split



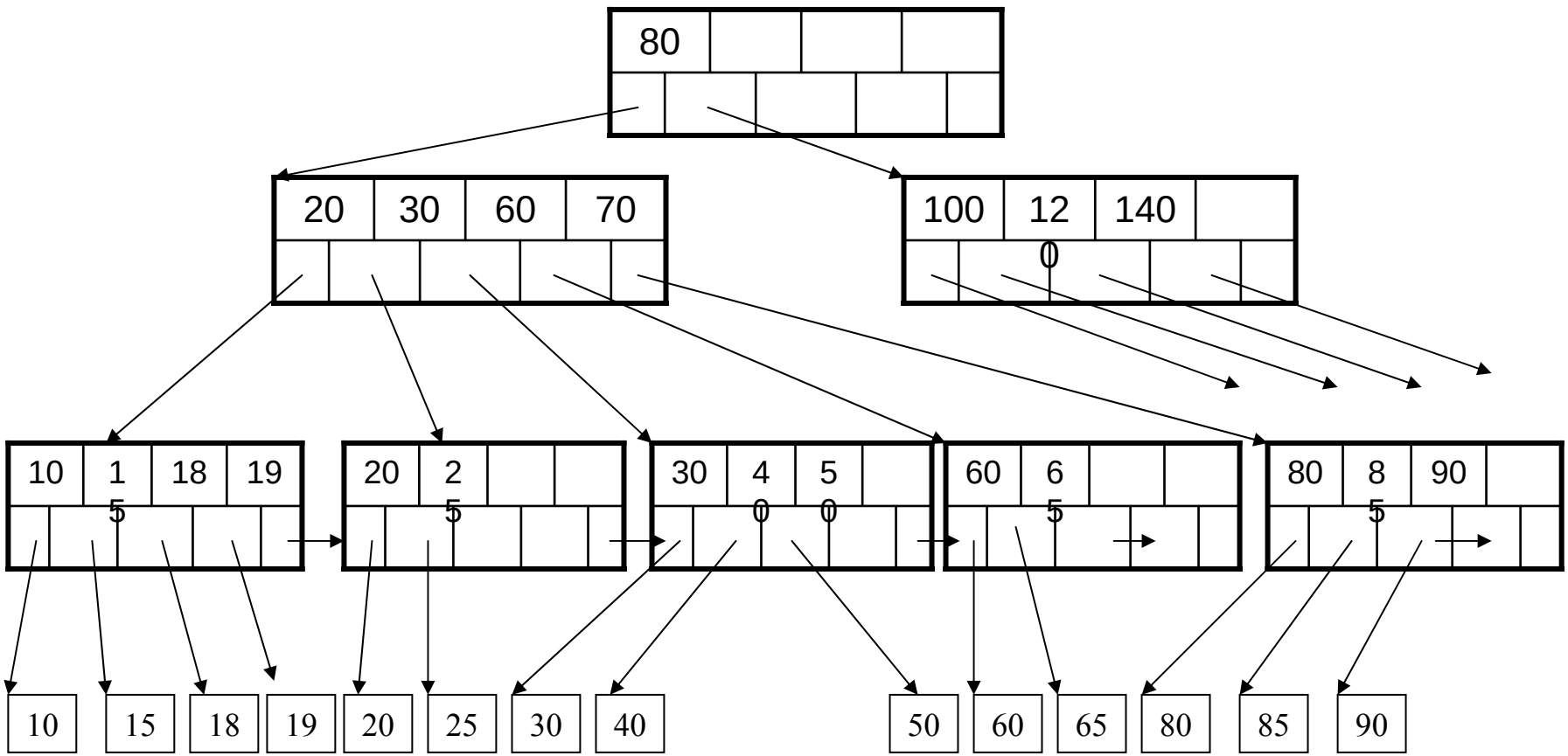
Insertion in a B+ Tree

Another B+ Tree



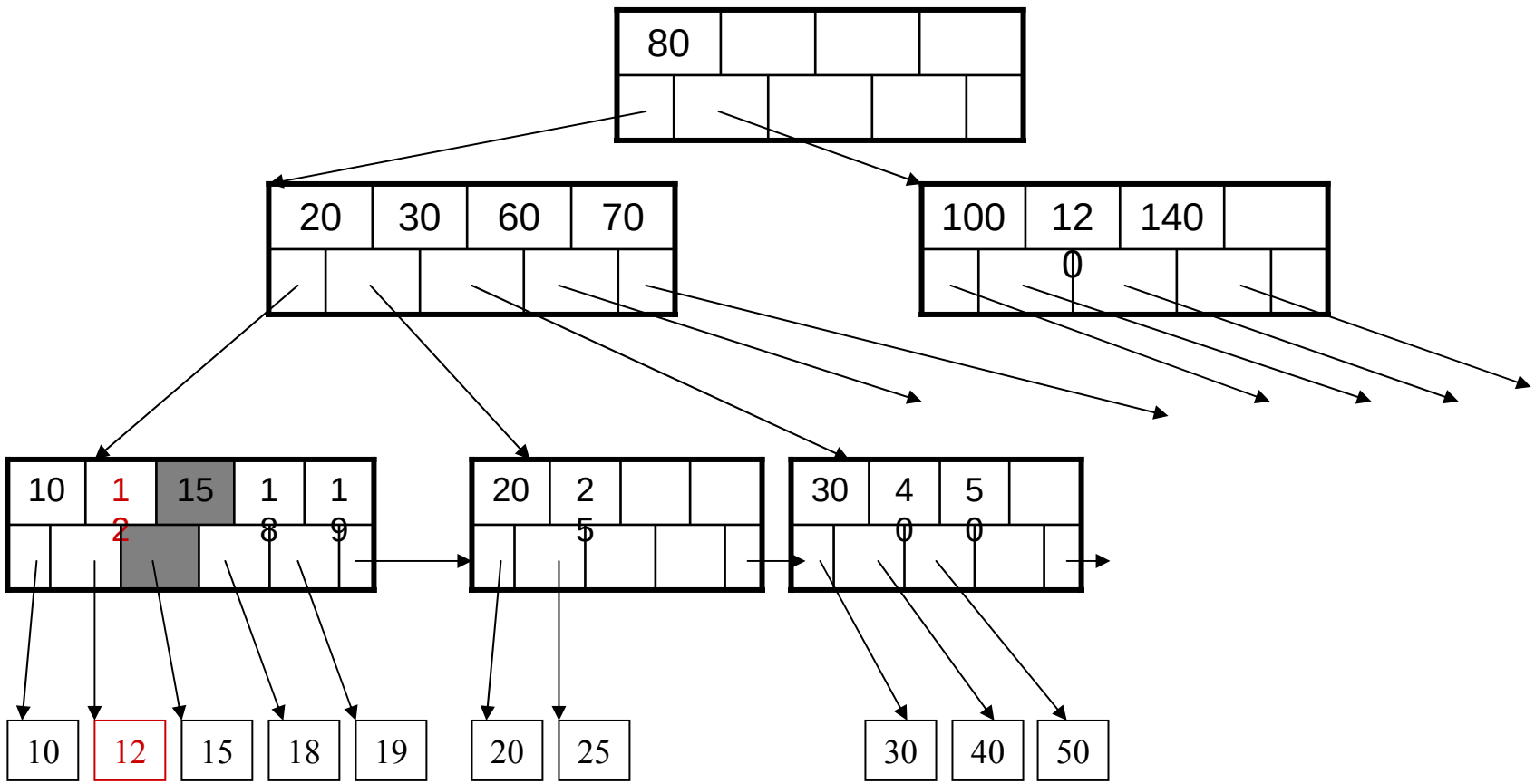
Insertion in a B+ Tree

Now Insert 12



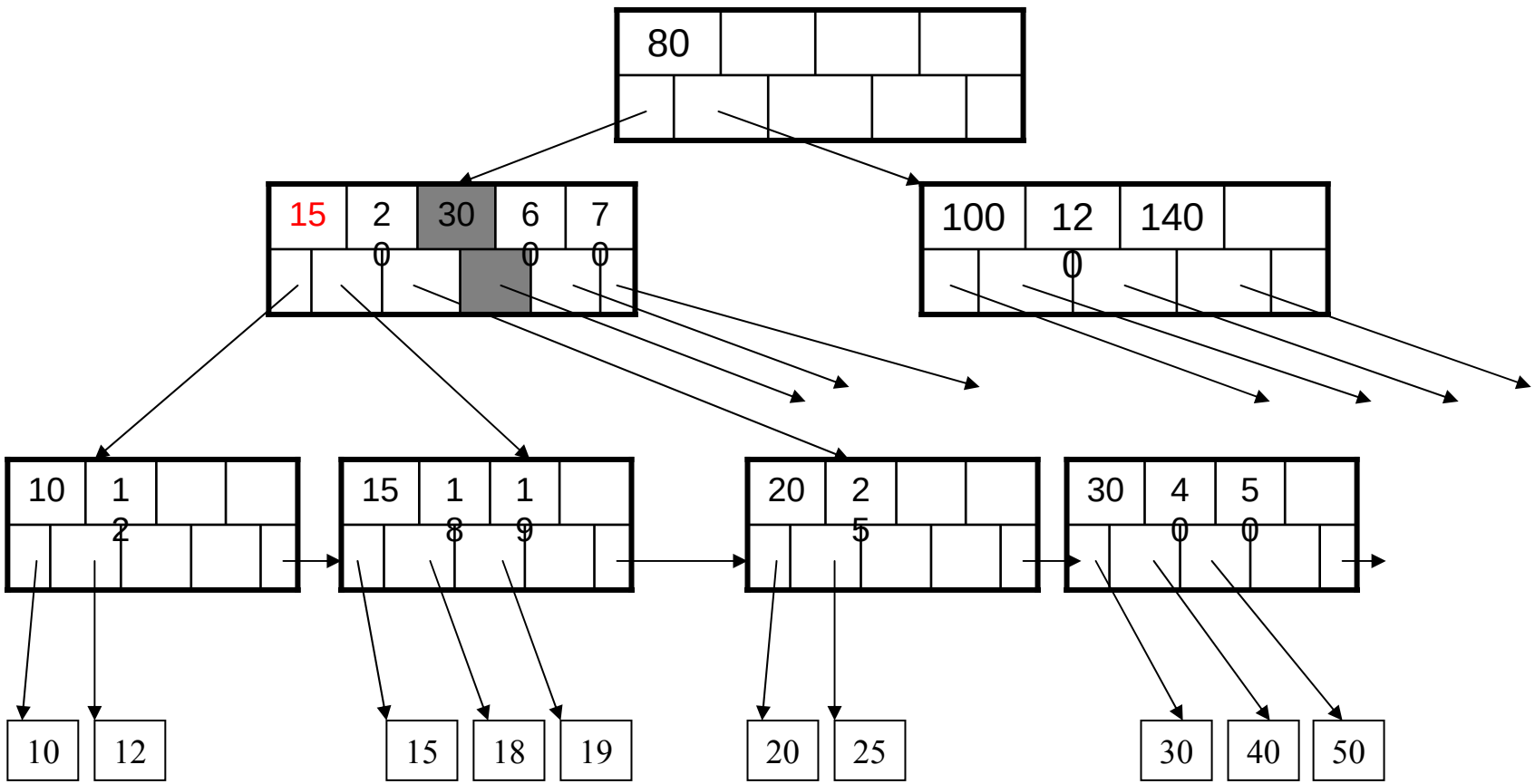
Insertion in a B+ Tree

Need to split leaf



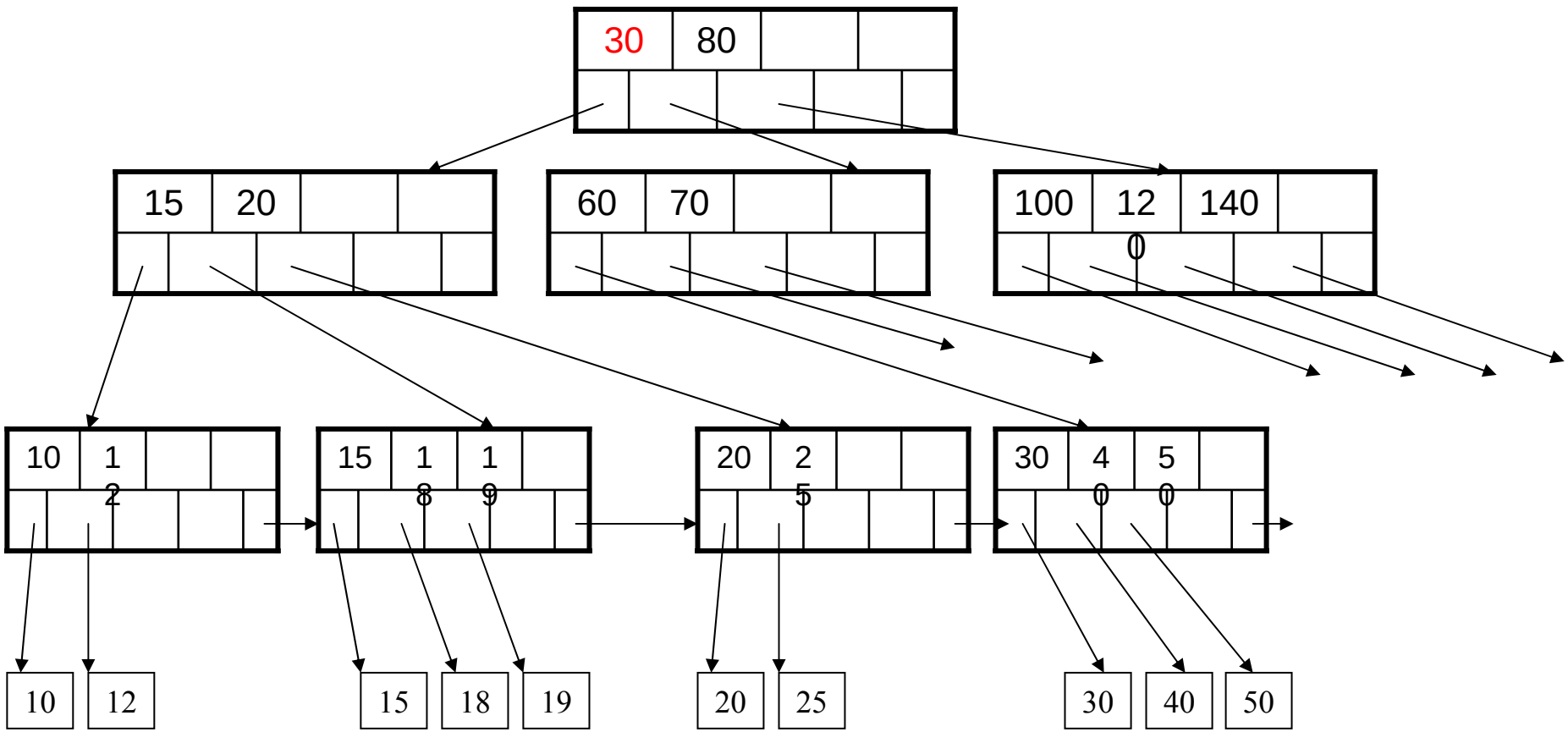
Insertion in a B+ Tree

Need to split branch



Insertion in a B+ Tree

After split



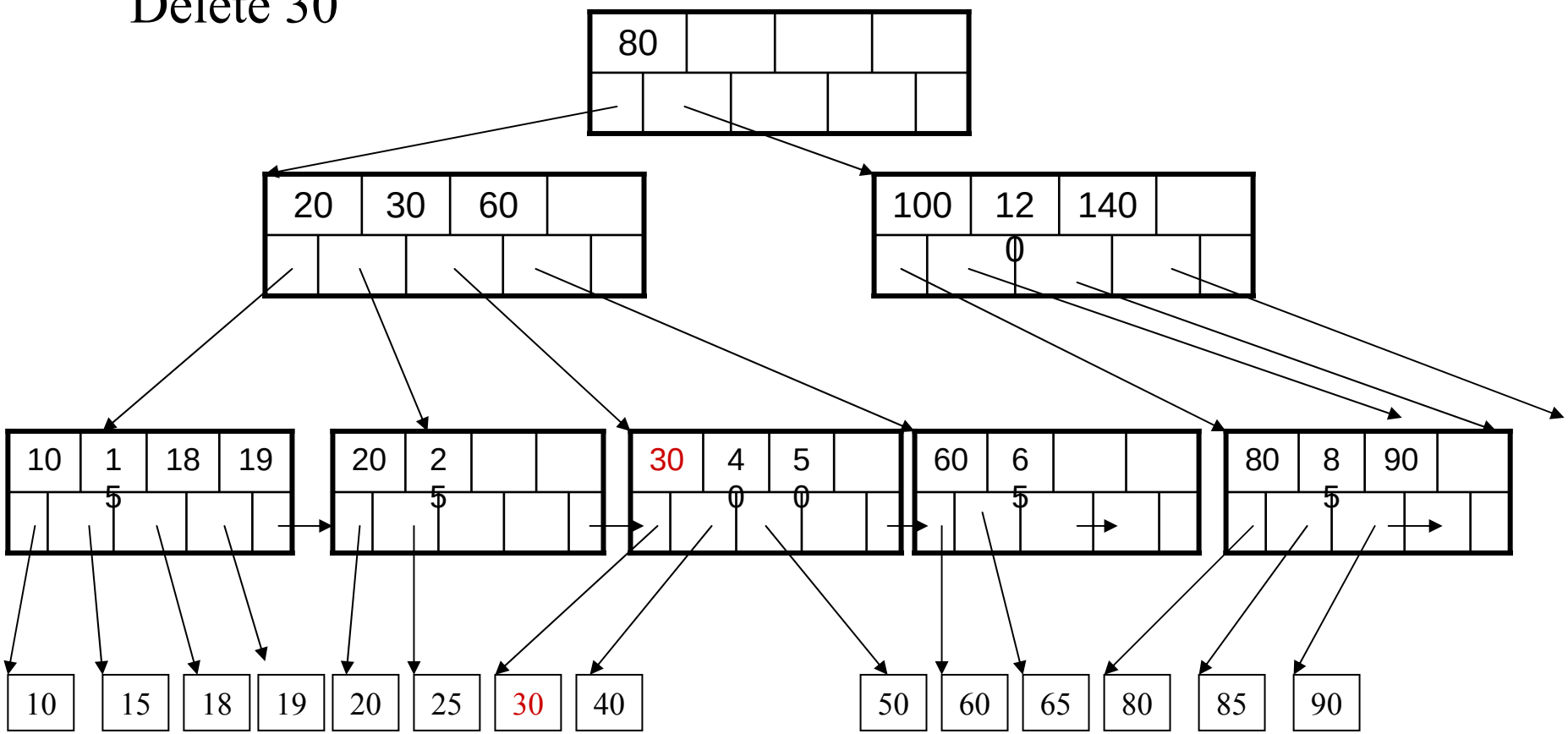
Deleting a Data Entry from a B+ Tree

- Start at root, find leaf *X* where entry belongs.
- Remove the entry.
  - If X is at least half-full, *done!*
  - If X has only **t-1** entries,
    - Try to re-distribute, borrowing from *sibling (adjacent node with same parent as X).*
    - If re-distribution fails, *merge* *X* and sibling.
- If merge occurred, must delete entry (pointing to *X* or sibling) from parent of *X*.
- Merge could propagate to root, decreasing height.

Deletion from a B+ Tree

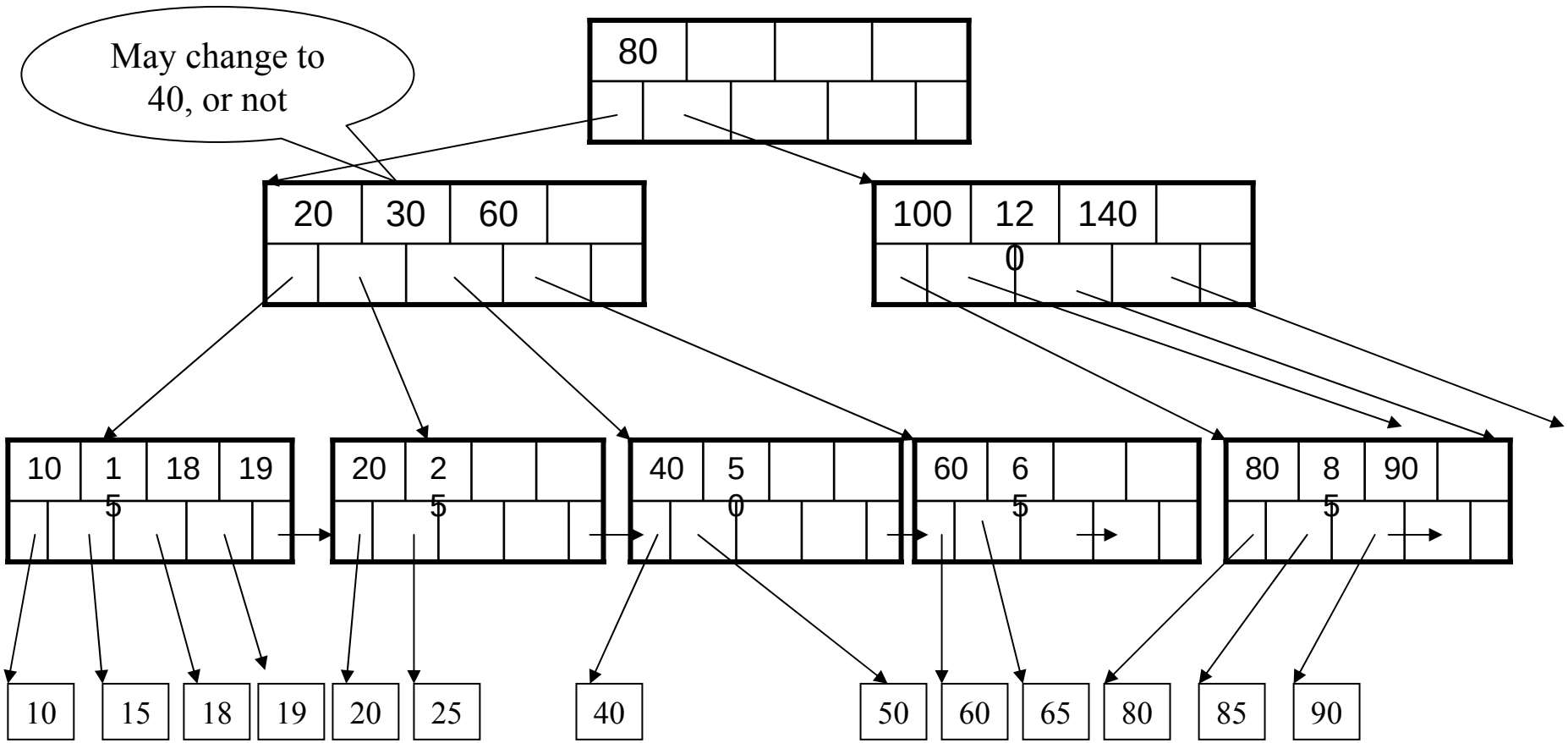
Bậc của cây t=2

Delete 30



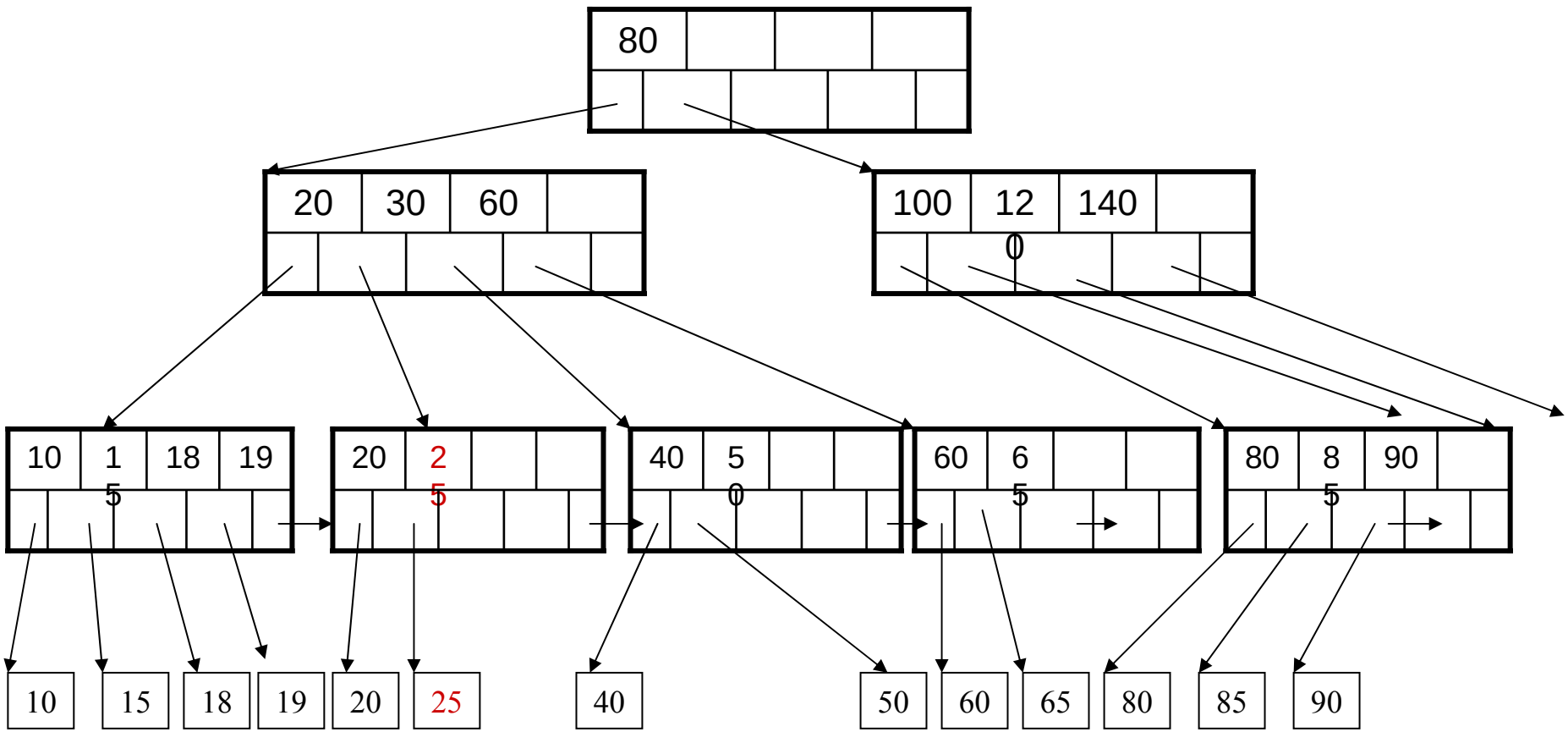
Deletion from a B+ Tree

After deleting 30



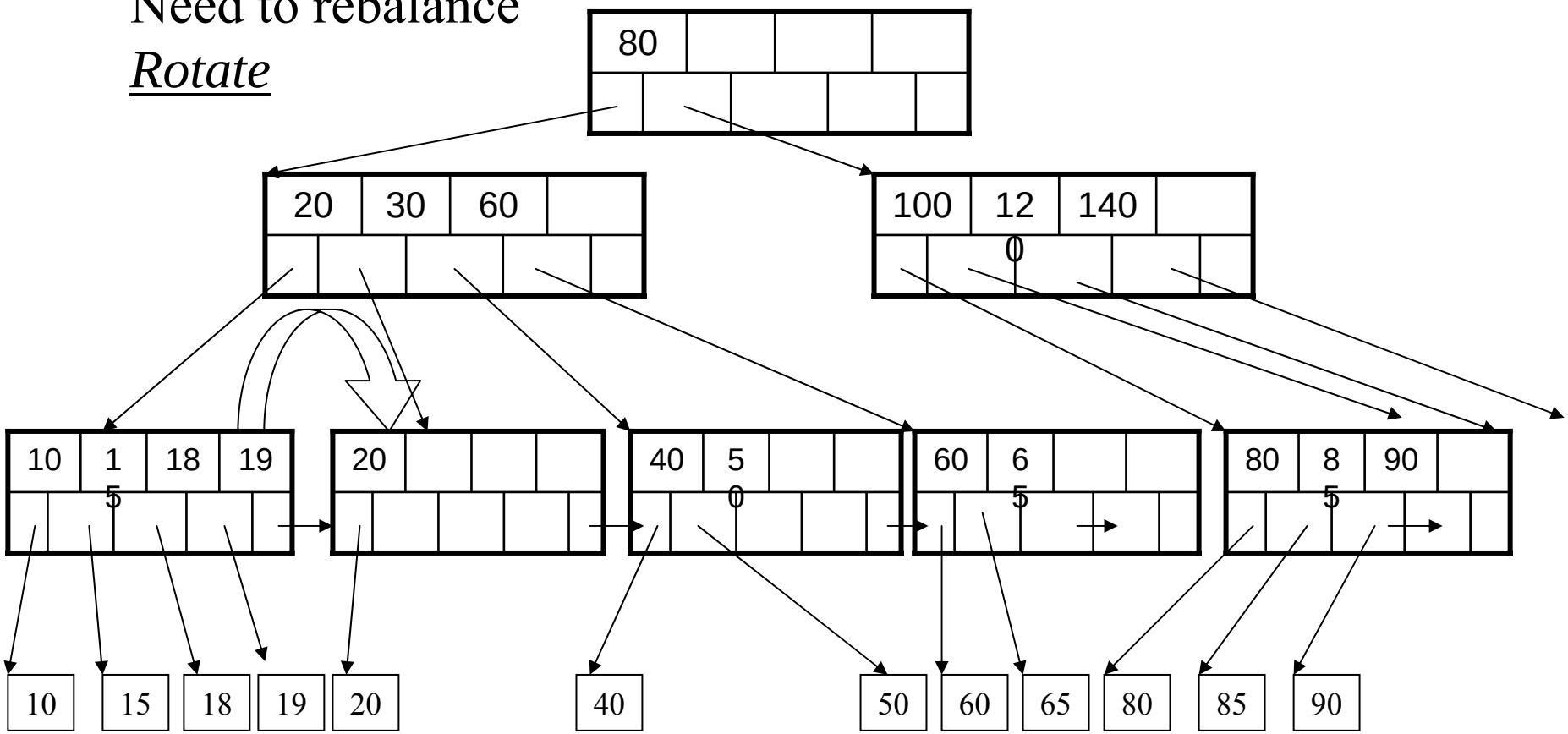
Deletion from a B+ Tree

Now delete 25



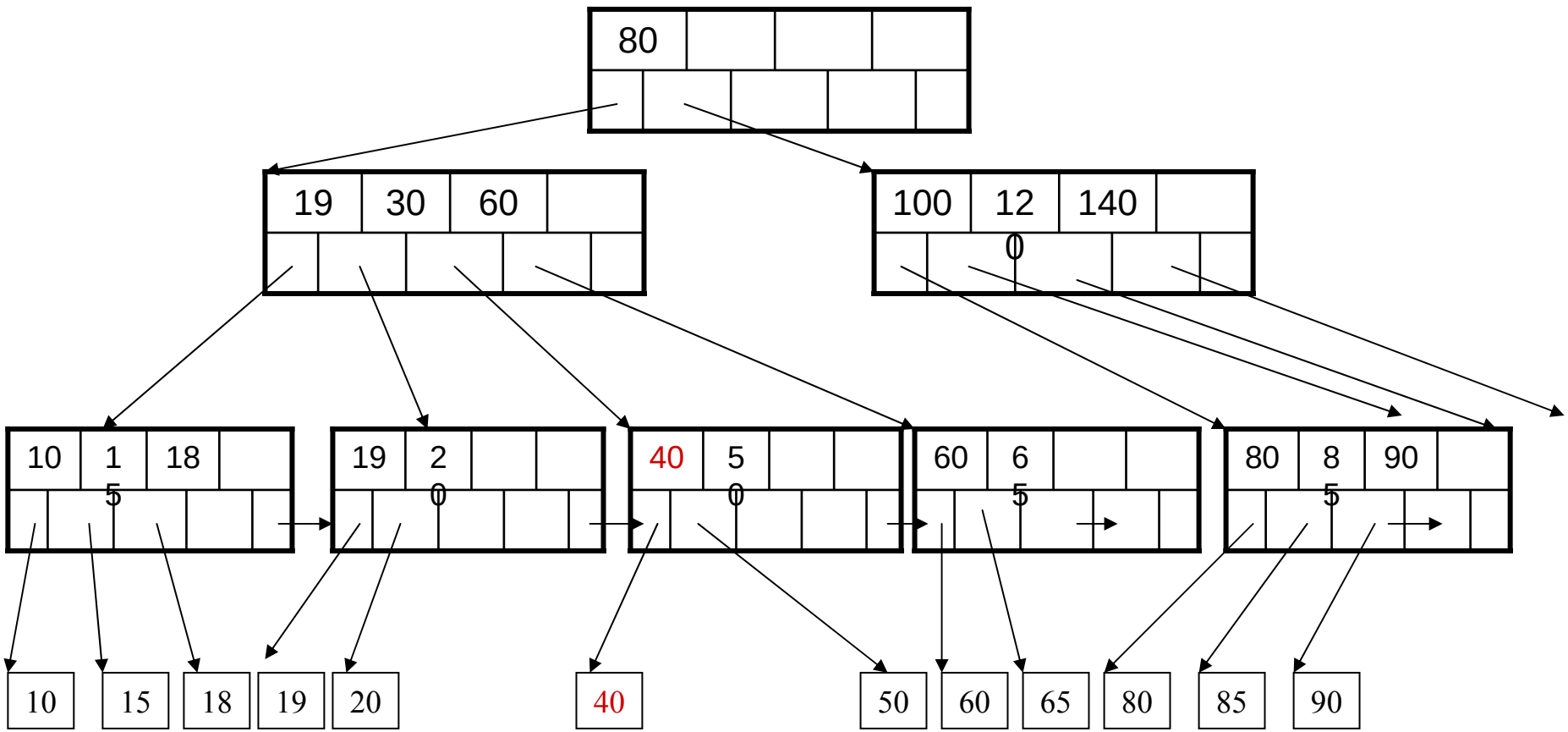
Deletion from a B+ Tree

After deleting 25
Need to rebalance
Rotate



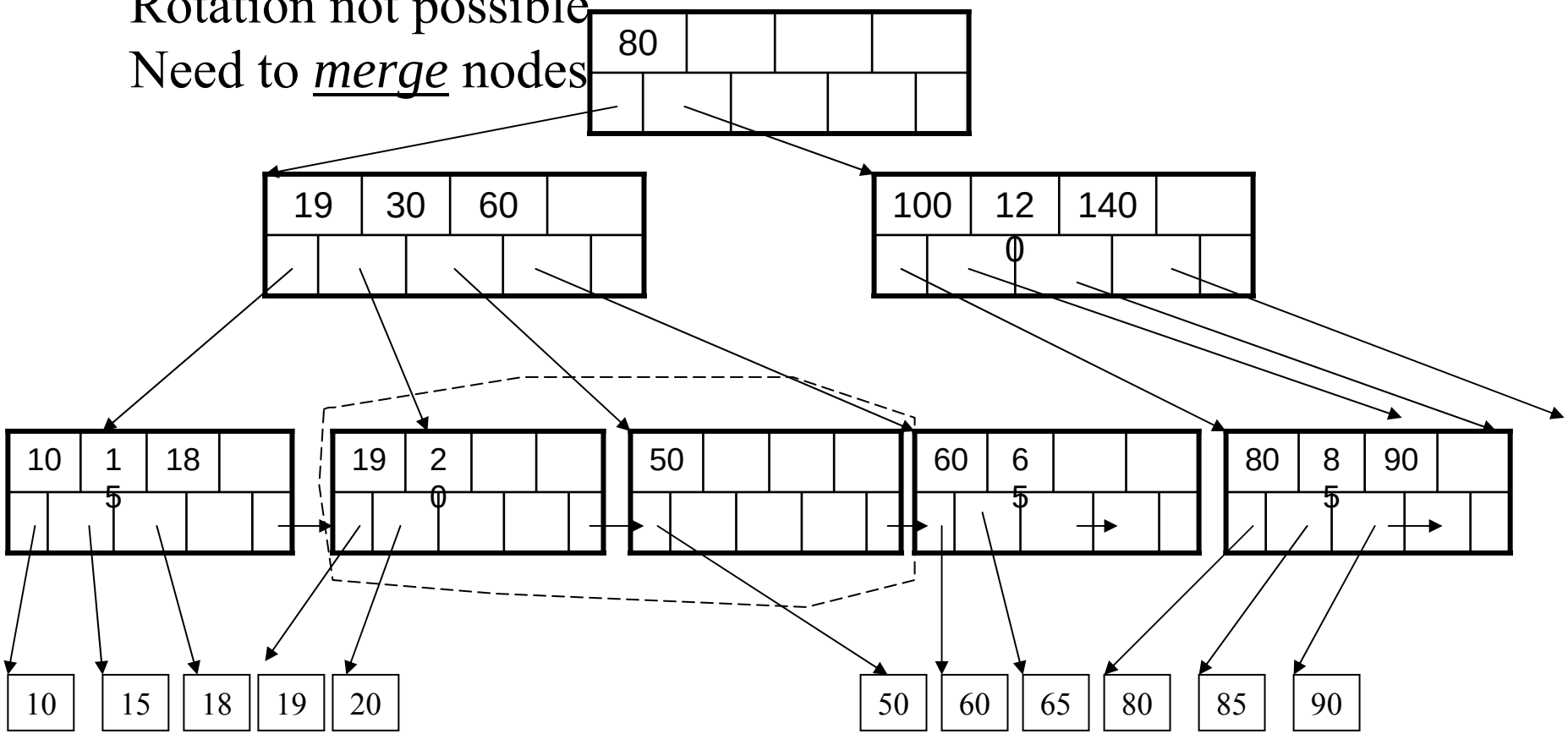
Deletion from a B+ Tree

Now delete 40



Deletion from a B+ Tree

After deleting 40
Rotation not possible
Need to *merge* nodes



Deletion from a B+ Tree

Final tree