Physics of Atomic Nuclei, Vol. 65, No. 2, 2002, pp. 255–260. Translated from Yadernaya Fizika, Vol. 65, No. 2, 2002, pp. 281–286.

Toward Improving the Accuracy of Testing the Equivalence of the Inertial and the Gravitational Mass under Terrestrial Conditions

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Received March 21, 2001

Abstract—An experiment aimed at testing the equivalence of the inertial and the gravitational mass is considered in which use is made of a facility including a vacuum chamber with two coupled oscillators (a pendulum and dynamical damper that form a vibrational system featuring two degrees of freedom) and falling onto the Sun. The layout of the facility and its basic parameters are presented. The pendulum and the dynamical damper have the same natural frequency, which is equal to the frequency of their rotation about the Sun. This frequency is dependent on the date of the experiment and can be calculated on the basis of the time equation. In the proposed facility, the amplitude of oscillations of the damper is 1.2×10^{-5} rad, which is much greater than the value of 10^{-7} rad previously achieved in the experiment that tested the equivalence principle to the highest precision of about 10^{-12}. This precision can be considerably improved. The result is presented that was obtained from a measurement during the solar eclipse in Moscow on August 11, 1999.

1. INTRODUCTION

The concept of the mass of a body has a double meaning. On one hand, it determines the inertial properties of this body (Newton's second law); on the other hand, it is the source of gravitational forces (Newton's third law). This duality is reflected in the terms "inertial mass" $m^{(i)}$ and "gravitational mass" $m^{(g)}$. An experimental test of the equivalence of these masses is of fundamental importance for confirming Einstein's general theory of relativity.

In the majority of the most precise experiments, use is made of a torsional pendulum, with weights from different materials ("1" and "2") being placed at the ends of its arm. The experiments consist in investigating the dimensionless quantity

$$\Delta = \left(\frac{m^{(g)}}{m^{(i)}}\right)_1 - \left(\frac{m^{(g)}}{m^{(i)}}\right)_2$$

for the situation where the bodies fall onto the Earth or onto the Sun. In the first case (falling onto the Earth), the sought torque T_E is static and is given by

$$T_E = m^{(i)} l \Omega^2 R \Delta \sin\varphi \cos\varphi,$$

where l is the arm length, Ω is the angular velocity of Earth's rotation, R is the radius of the Earth, and φ is the latitude of the place where this experiment is performed. In the second case, the torque is variable, its amplitude T_D being

$$T_D \approx 0.62[\text{cm s}^{-2}] m^{(i)} l \Delta,$$

where $0.62\,\text{cm s}^{-2}$ is the acceleration due to the Sun's gravitational attraction.

An experiment where use is made of bodies falling onto the Earth was first proposed and implemented by Eötvös *et al.* [1], while the analogous experiment to study falling onto the Sun was proposed by the authors of [2].

The literature devoted to the test of the equivalence principle is quite extensive. A considerable number of more recent publications were initiated by the study reported in [3], where the hypothesis of the fifth force was put forth on the basis of an analysis of the data presented in [1].

The most accurate test of the equivalence principle was performed in [4], where the quantity Δ was estimated to a precision of about 10^{-12}.

In the present study, we discuss a way to improve considerably the accuracy of this estimate under terrestrial conditions. A preliminary report is given on the experimental facility that is being developed at the Institute of Theoretical and Experimental Physics (ITEP, Moscow) and which is intended for this purpose. Specialists from the All-Russia Research Institute for Electronic Mechanics (ARRIEM) are taking part in its fabrication.

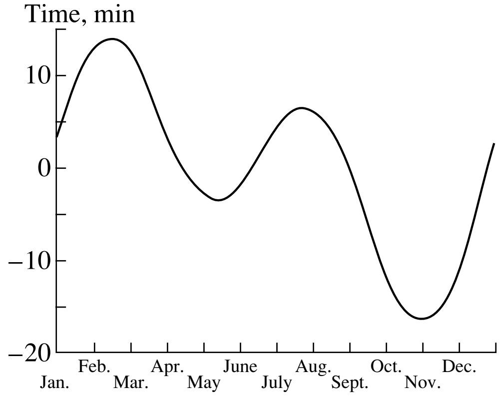

Fig. 1. Graph of the time equation.

2. WAY TO IMPROVE THE ACCURACY OF THE TEST OF THE EQUIVALENCE PRINCIPLE

The proposed experiment for testing the equivalence principle employs a device having a vacuum chamber equipped with two coupled torsional pendulums and falling onto the Sun. The device is suspended in space with the aid of a magnetic field and rotates at an angular velocity ω that is maintained with a high precision. The coupled torsional pendulums form a vibrational system having two degrees of freedom; each taken separately (that is, in an uncoupled state) has the same natural frequency ω_0, the period τ_0 of the system being given by

$$\tau_0 = 2\pi\sqrt{\frac{32IL}{E\pi d^4}},$$

where L is the length of the pendulum thread, d is the thread diameter, I is the moment of inertia, and E is the shear modulus of the thread material [1.5×10^{12}g/(cm s^2) for tungsten]. One (large) pendulum has a moment of inertia I_1; in order to generate the required variable torque from the Sun, there are two identical weights from different materials at the ends of its arm. The other (small) pendulum from the same material has a moment of inertia $I_2 \ll I_1$, a weight at the end of its thread being much lighter than the weights of the massive pendulum. If the equivalence principle were violated, the effect of the Sun on the weights of the massive pendulum would excite small-amplitude oscillations of frequency ω_0; in the steady-state regime, there would concurrently arise oscillations of the lighter pendulum in antiphase, their amplitude being much larger. The oscillations of the massive pendulum would then be damped by the oscillations of the lighter pendulum and could in principle disappear completely despite the effect of the sought torque on the massive pendulum. In this case, the oscillations of the lighter pendulum in antiphase become very large. In the theory of oscillators, this remarkable phenomenon is commonly referred to as an antiresonance, while the small pendulum is called a dynamical damper. In the experiment being discussed, this property of the oscillations of a dynamical damper is used to improve the sensitivity of searches for sought oscillations. In contrast to conventional applications of an oscillating system with two degrees of freedom in many technical devices, the proposed experiment would employ it, figuratively speaking, in an "inverted" form. Under the antiresonance condition, the angular velocity of the dynamical damper is given by

$$\omega = -\Omega_u \sin\varphi + \sqrt{\Omega_u^2 \sin^2\varphi + \omega_0^2 - \Omega_u^2},$$

where Ω_u is the angular velocity of the Earth with respect to the Sun (it depends on the date of experiment), while the period T can be found from the time equation (Fig. 1)

$$T = (1440 - \eta)\ \text{min},$$

$$\eta = 7.6\sin 0.986^\circ(n-4) - 9.8\sin 1.973^\circ(n-81),$$

where n is the ordinal number of the day in a year from January 1. The numerical value of η can be found in the astronomical calendar.

If the accuracy of the experiment is sufficiently high, the resonance frequency ω_0 can be fixed for a specific date of the experiment in accordance with the time equation; for another date, the torsional pendulum will automatically be off the resonance. If the measuring procedure is invariable, the manifestation of the sought oscillations will become different, which will be a clear signal of their existence. The condition

$$\delta\omega < |\omega_0 - \omega|,$$

where $\delta\omega = \omega_0/Q$ is the half-width of the resonance and Q is its quality factor, is satisfied in this experiment. From this inequality, it follows that the sought oscillations induced by the Sun can be separated in frequency from the background oscillations associated with the rotation about the Earth.

Krylov [5] performed a calculation of steady-state forced oscillations for a stabilizer of ship rolling. The formulas that he obtained are applicable to oscillations of a torsional pendulum and a dynamical damper, the former and the latter playing, respectively, the role of a ship and the role of the stabilizer of rolling. In the antiresonance regime, the amplitude A_Γ of sought forced steady-state oscillations of the dynamical damper will be

$$A_\Gamma = \frac{T_D\tau_1\tau_2}{4I_1}\left[\left(1 + \frac{I_2\omega_0^2\tau_1\tau_2}{4I_1}\right)^2 + \frac{I_2^2\omega_0^2\tau_1^2}{4I_1^2}\right]^{-1/2},$$

where $\tau_1 = \tau_2 = \tau = 2Q/\omega_0$ is the relaxation time for the pendulum and the dynamical damper and I_1 and I_2 are their moments of inertia.

Parameters of coupled oscillators

	Pendulum	Damper
Thread length, L	250 cm	53 cm
Thread diameter, d	5×10^{-3} cm	5×10^{-4} cm
Arm length, l	15 cm	4 cm
Mass of weights, m	600 g	4 g
Moment of inertia, I	1.35×10^5 g cm^2	64 g cm^2
Resonance frequency, ω_0	1.7×10^{-3} rad/s	1.7×10^{-3} rad/s
Relaxation time, τ	10^5 s	10^5 s
Quality factor, Q	85	85
Resonance half-width, $\delta\omega$	2×10^{-5} rad/s	2×10^{-5} rad/s

The damper-induced enhancement of the amplitude $A_{\rm M}$ of pendulum oscillations is

$$\frac{A_\Gamma}{A_{\rm M}} = Q.$$

We note that, in an experiment without a dynamical damper, the amplitude of the pendulum, $A_{\rm M}$, due to the sought torque T_D is equal to the resonance value A_r:

$$A_r = \frac{T_D \tau_1}{2 I_1 \omega_0}.$$

If the experiment features a damper and if the condition $I_2 \omega_0^2 \tau_1 \tau_2 (4 I_1) \gg 1$ is satisfied, the increase in this amplitude is

$$\frac{A_\Gamma}{A_r} = \frac{I_1}{I_2 Q}.$$

The precision that can be achieved in the experiment is restricted by thermal noise causing the deflection angle $\delta\Theta$ [6],

$$\delta\Theta = \left(\frac{1}{\omega_0}\right)^2 \sqrt{\frac{6kT^0}{I_2 \tau_2 t}},$$

where k is the Boltzmann constant, T^0 is the absolute temperature, and t is the measurement time.

The estimated parameters of the apparatus are presented in the table. For the experiment performed in February, the time equation yields $\Omega_u = 7.345 \times 10^{-5}$ rad/s (the period is equal to 24 h $-$ 14.3 min $=$ 8.5542×10^4 s). At the measurement time of $t = 6 \times 10^5$ s, the condition $t \gg \tau$ is satisfied, and the forced pendulum oscillations in question can be considered to be steady-state.

For the value of $\Delta = 10^{-12}$ and the parameters of the pendulum and the dynamical damper from the table, we obtain

$$T_D = 2.8 \times 10^{-9} \text{g cm}^2/\text{s}^2,$$

$$\omega = 1.636 \times 10^{-3} \text{ rad/s},$$

$$\omega_0 - \omega = 6.4 \times 10^{-5} \text{ rad/s},$$

$$\mathrm{A}_\Gamma = 1.2 \times 10^{-5} \text{ rad},$$

$$\frac{\mathrm{A}_\Gamma}{A_{\rm M}} = 85,$$

$$\frac{A_\Gamma}{A_r} = 24.8,$$

$$\delta\Theta \approx 8.8 \times 10^{-8} \text{ rad}.$$

From a comparison of the above value of A_Γ with the value of 10^{-7} rad reported in [4], we conclude that the accuracy of testing the equivalence principle can be considerably improved. In order to isolate the effect, use is made of the Fourier analysis of oscillations of the main pendulum and dynamical damper. At $\Delta = 10^{-12}$, the amplitude of the sought oscillations of the dynamical damper will be greater than the amplitude of the main pendulum by the factor $A_\Gamma / A_{\rm M}$.

3. EXPERIMENTAL FACILITY

The layout of the experimental facility for testing the equivalence principle is shown in Fig. 2. The device is suspended in space with the aid of an electromagnet (*5*) (see [7]) and, through a flexible link (*3*), is rotated with a frequency $\omega \approx 1.636 \times 10^{-3}$ rad/s (the period is about 1 h) with respect to the Earth by an electric motor (*1*), which is controlled by an electronic device (*2*). The rotation is monitored by a laser and recording electronics (*11*), as well as by 36 mirrors (*12*). Upon the switching of the electromagnet on and a smooth increase in the current through it, the experimental facility of weight about 150 kg is gently lifted from the support (not shown in the figure) and is

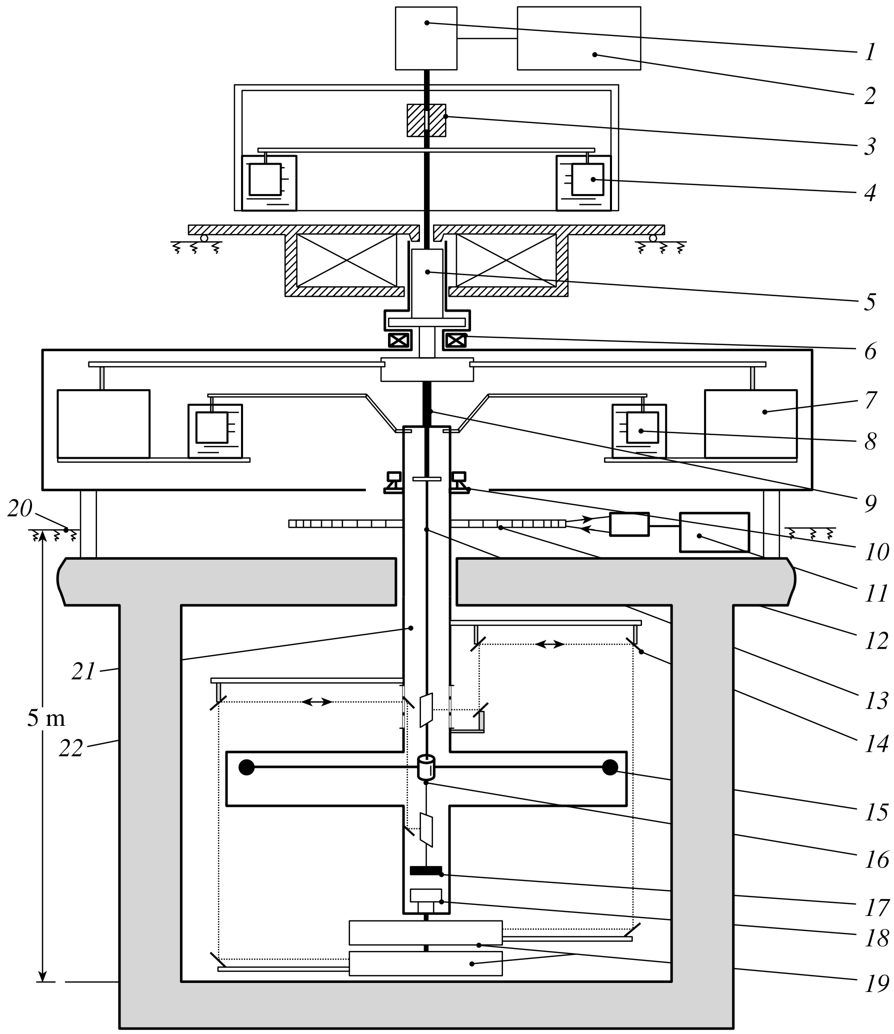

Fig. 2. Layout of the experimental facility: (*1*) electric motor, (*2*) electronic control for the electric motor, (*3*) flexible link between the electric motor and the experimental facility, (*4*) damper of the experimental facility, (*5*) magnetic suspension device, (*6*) coils for generating a rotating magnetic field, (*7*) weight of the experimental facility, (*8*) damper of the vacuum chamber, (*9*) thread for suspending the vacuum chamber, (*10*) magnet for damping pendulum oscillations, (*11*) laser and electronics for measuring the rotation, (*12*) 36 mirrors mounted on the device for measuring the rotation, (*13*) tungsten thread of the torsional pendulum, (*14*) mirrors for measuring the oscillations of the pendulum and dynamical damper, (*15*) weight of the torsional pendulum, (*16*) tungsten thread of the dynamical damper, (*17*) weight of the dynamical damper, (*18*) adsorption vacuum pump, (*19*) autocollimators, (*20*) Earth's surface, (*21*) vacuum chamber, and (*22*) concrete walls of the basement.

suspended at a current of about 120 mA in the space at a given altitude.

The laboratory room formed by four weights (*7*) of about 30 kg each (only two of them are shown in the figure) that rotates at a constant speed, its moment of inertia being about 2×10^8 g cm^2, is the main structural part of the facility. In the rotating laboratory room, background perturbations (including seismic perturbations) are suppressed by the magnetic suspension device and the damper (*4*). In the laboratory room, a vacuum chamber (*21*) is suspended with a thread (*9*) about 3 mm in diameter; in turn, two coupled oscillators (*13*) and (*16*) are suspended within the vacuum chamber. In the laboratory room, the background oscillations of the vacuum chamber are suppressed by the damper (*8*). Prior to the beginning of the experiment, the chamber is evacuated, and an adsorption pump (*18*) is used in the course of the experiment to maintain a vacuum at the required level of 10^{-5} torr. The initial torsional oscillations of the pendulum and the dynamical damper are forced to relax by the rotating magnetic field with the aid of a follow-up system (not shown in the figure), while pendulum oscillations are suppressed by a magnetic damper (*10*).

In the facility, the sought oscillations are measured

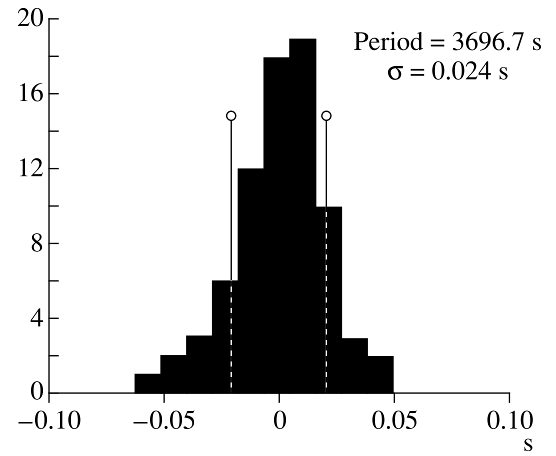

Fig. 3. Experimental histogram of deviations of the periods of facility rotation from its average value.

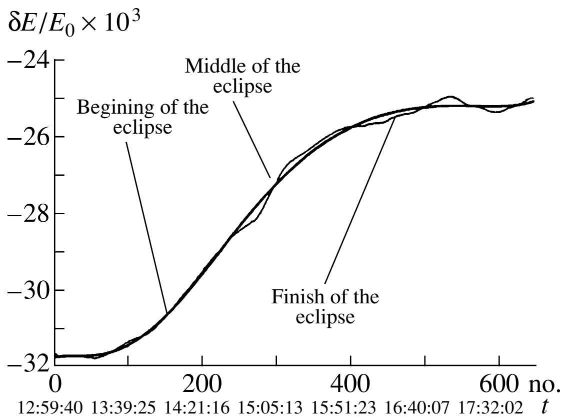

Fig. 4. Relative energy loss per revolution in inertial free rotation of bodies suspended in space with the aid of magnetic field during the solar eclipse in Moscow on August 11 (the symbol "no." stands for the ordinal number of a period, while the symbol t stands for time of its detection.).

by autocollimators (*19*) that are attached to the rotating vacuum chamber. Information is transferred in a contactless way through the infrared channel. The light beams of autocollimators are incident on the mirror of the pendulum and of the dynamical damper from opposite sides; owing to this, the effect of background chamber oscillations decreases in the difference of their readings. Therewith the amplitude of the sought oscillations of the dynamical damper undergoes virtually no changes, since, in the antiresonance regime, it is much greater than the amplitude of oscillations of the torsional pendulum. In the experimental facility, the dynamical damper, which is a sensitive detector of the sought oscillations, is thereby shielded from background oscillations (including seismic oscillations) by the magnetic suspension device, the damper of the laboratory room, the damper of the vacuum chamber, and the damper of pendulum oscillations.

For the experimental facility to be rotated in such a way that the relative deviation of its period was less than the attained value of about 6×10^{-6} (see Fig. 3), two versions of driver are investigated. The first employs a step-type motor with a reduction of 2×10^6, while the second relies on an electronically controlled special motor developed in ARRIEM.

4. CONCLUSIONS

The proposed experiment is advantageous in that the accuracy can be improved by using (1) heavy (a few hundred kilograms) bodies that are suspended in space with the aid of a magnetic field and which can freely rotate with a period of one hour; (2) the resonance enhancement of the sought oscillations; (3) an oscillatory system with two degrees of freedom and special features of oscillations of coupled oscillators; (4) a multistep damping of background oscillations, whose effect can be reduced owing to a special detection of the sought oscillations; and (5) the time equation for test measurements.

As a result, it becomes possible to investigate, with an improved precision, the following fundamental problems of gravitation: the dependence of gravitational interaction on the material of the substance, the effect of screening, the interaction of ordinary matter with dark matter, and so on.

We note that the use of the experimental features indicated above gives impetus to the development of the technique of physical experiments aimed at revealing and analyzing weak physical interactions. In this context, we would like to mention the circumstance that may be of interest.

In 2000, there appeared the study of Qian-shen Wang *et al.* [8], who discussed the experiment that was performed during the solar eclipse in March 1997 and which revealed an unusual variation in the acceleration due to the gravitational attraction of the Earth. In connection with this observation, they put forth the assumption that the gravitational field is screened. This highlights the need for precision measurements that would seek the possible new properties of the gravitational interaction. We therefore deem it advisable to recall the experiment performed at the facility in question with a magnetic suspension of bodies of weight about 120 kg that were executing inertial free rotation in a suspended state during the solar eclipse in Moscow on August 11, 1999 [9]. The results are presented in Fig. 4. The graph depicts the time variation of the ratio $\delta E/E_0$, where δE is the rotational-energy loss per revolution and E_0 is the initial rotational energy. In the case of ordinary damping of rotation, the graph would have the form of a straight line (exponential damping). In all probability, the observed shape of the curve is associated with a magnetic hysteresis and with the rotation of the magnetization vector in the magnetic field of the suspension device as the horizontal inclination of Earth's surface changes owing to tidal forces; however, we cannot rule out the possible existence of a new phenomenon in gravitational interaction.

ACKNOWLEDGMENTS

We are grateful to L.B. Okun, Yu.G. Abov, G.A. Lobov, A.L. Suvorov, V.Ya. Averbukh, V.E. Remezov, V.V. Shichkov, É.M. Khromenko, and P.V. Mokrov for their interest in this study.

This work was supported by the Russian Foundation for Basic Research (project no. 95-02-03534-a).

REFERENCES

1. R. V. Eötvös, D. Pekár, and E. Fekete, Ann. Phys. (Leipzig) **68**, 11 (1922).
2. R. G. Roll, R. Krotkov, and R. H. Dicke, Ann. Phys. (N.Y.) **26**, 442 (1964).
3. E. Fischbach, D. Sudarsky, A. Szafer, *et al.*, Phys. Rev. Lett. **56**, 3 (1986).
4. V. B. Braginskiĭ and V. I. Panov, Zh. Éksp. Teor. Fiz. **61**, 873 (1971) [Sov. Phys. JETP **34**, 463 (1972)].
5. A. N. Krylov, *On Some Differential Equations of Mathematical Physics Having Application in Technical Problems* (Akad. Nauk SSSR, Moscow, 1949).
6. E. G. Adelberger *et al.*, Phys. Rev. D **42**, 3267 (1990).
7. S. M. Kalebin, G. V. Rukolaine, A. N. Polozov, *et al.*, Nucl. Instrum. Methods Phys. Res. A **267**, 35 (1988).
8. Qian-shen Wang, Chuan-zhen Wu, Hong-gang Guo, *et al.*, Phys. Rev. D **62**, 041101(R) (2000).
9. S. M. Kalebin, V. S. Artamonov, D. A. Lyakin, *et al.*, *Scientific Session of Moscow Engineering Physics Institute: Collection of Scientific Works* (Moscow, 2000), Vol. 7, p. 25.

Translated by A. Isaakyan