In [1]:
```python
from cmdstanpy import CmdStanModel
import arviz as az
import numpy as np
import scipy.stats as stats
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib as mpl
import stan
import seaborn as sns
import plotly.express as px

plt.style.use("ggplot")  #using style ggplot
```

```
/usr/local/anaconda3/envs/DataAnalytics/lib/python3.8/site-packages/tqdm/auto.py:22: TqdmWarning: IProgress not found. Please update jupyter and ipywidgets. See https://ipywidgets.readthedocs.io/en/stable/user_install.html
  from .autonotebook import tqdm as notebook_tqdm
```

In [2]:
```python
data = pd.read_csv('./cars.csv')
data.head(10)
```

Out[2]:

	Unnamed: 0	mark	model	generation_name	year	mileage	vol_engine	fuel	city	province	price
0	0	opel	combo	gen-d-2011	2015	139568	1248	Diesel	Janki	Mazowieckie	35900
1	1	opel	combo	gen-d-2011	2018	31991	1499	Diesel	Katowice	Śląskie	78501
2	2	opel	combo	gen-d-2011	2015	278437	1598	Diesel	Brzeg	Opolskie	27000
3	3	opel	combo	gen-d-2011	2016	47600	1248	Diesel	Korfantów	Opolskie	30800
4	4	opel	combo	gen-d-2011	2014	103000	1400	CNG	Tarnowskie Góry	Śląskie	35900
5	5	opel	combo	gen-d-2011	2017	121203	1598	Diesel	Warszawa	Mazowieckie	51900
6	6	opel	combo	gen-d-2011	2017	119965	1248	Diesel	Wrocław	Dolnośląskie	44700
7	7	opel	combo	gen-d-2011	2016	201658	1248	Diesel	Lublin	Lubelskie	29000
8	8	opel	combo	gen-d-2011	2014	178666	1598	Diesel	Złotów	Wielkopolskie	28900
9	9	opel	combo	gen-d-2011	2015	113000	1248	Diesel	Strzyżew	Mazowieckie	34900

In [3]:
```python
print(data.mark.unique())
```

```
['opel' 'audi' 'bmw' 'volkswagen' 'ford' 'mercedes-benz' 'renault'
 'toyota' 'skoda' 'alfa-romeo' 'chevrolet' 'citroen' 'fiat' 'honda'
 'hyundai' 'kia' 'mazda' 'mini' 'mitsubishi' 'nissan' 'peugeot' 'seat'
 'volvo']
```

In [4]:
```python
print(data.loc[(data['mark'] == 'bmw') | (data['mark'] == 'mercedes-benz') |
               (data['mark'] == 'audi')].model.unique())
```

```
['a3' '80' 'a1' 'a2' 'a4' 'a4-allroad' 'a5' 'a6' 'a6-allroad' 'a7' 'a8'
 'e-tron' 'q2' 'q3' 'q4-sportback' 'q5' 'q7' 'q8' 'rs3' 'rs5' 'rs6'
 'rs-q3' 's3' 's5' 's8' 'sq5' 'tt' '3gt' '5gt' 'i3' 'm2' 'm3' 'm4' 'm5'
 'm8' 'seria-1' 'seria-2' 'seria-3' 'seria-4' 'seria-5' 'seria-6'
 'seria-7' 'seria-8' 'x1' 'x2' 'x3' 'x4' 'x5' 'x5-m' 'x6' 'x6-m' 'x7'
 'amg-gt' 'citan' 'cl-klasa' 'cla-klasa' 'clk-klasa' 'cls-klasa'
 'gl-klasa' 'gla-klasa' 'glb-klasa' 'glc-klasa' 'gle-klasa' 'glk-klasa'
 'gls-klasa' 'a-klasa' 'b-klasa' 'c-klasa' 'e-klasa' 'g-klasa' 'r-klasa'
 's-klasa' 'v-klasa' 'm-klasa' 'sl' 'slk-klasa' 'sprinter' 'viano' 'vito']
```

In [5]:
```python
segments = {
    "A": ['i3', 'cla-klasa'],
    "B": ['a1', 'a2', 'q2', 'a-klasa'],
    "C": ['seria-1', 'seria-2', 'm2', 'a3', 'a4', 'rs-q3', 's3', 'citan', 'gla-klasa', 'b-klasa'],
    "D": ['seria-3', 'seria-4', 'm4', 'm3', '3gt', '80', 'a4-allroad', 'a5', 'q4-sportback', 'q5', 's5',
          'glb-klasa', 'glc-klasa', 'glk-klasa', 'sl'],
    "E": ['seria-5', 'seria-8', 'm8', 'm5', '5gt', 'x6', 'x6-m', 'x5-m', 'a6', 'a6-allroad', 'e-tron', 'q7',
          'q8', 's8', 'gle-klasa', 'e-klasa', 'g-klasa', 'r-klasa'],
    "F": ['seria-7', 'x7', 'a7', 'a8', 'cl-klasa', 'gl-klasa', 'gls-klasa', 's-klasa'],
    "G": ['seria-6', 'rs6', 'clk-klasa', 'cls-klasa', 'slk-klasa'],
    "I": ['x1', 'x2', 'x3', 'x4', 'x5', 'q3', 'm-klasa'],
    "S": ['amg-gt'],
    "Unknown": ['rs3', 'rs5', 'sq5', 'tt', 'c-klasa', 'v-klasa', 'sprinter', 'viano', 'vito']
}
segments_invert = {}
for key, value in segments.items():
    for val in value:
        segments_invert[str(val)] = key
segments_invert
```

Out[5]:
```
{'i3': 'A',
 'cla-klasa': 'A',
 'a1': 'B',
 'a2': 'B',
 'q2': 'B',
 'a-klasa': 'B',
 'seria-1': 'C',
 'seria-2': 'C',
 'm2': 'C',
 'a3': 'C',
 'a4': 'C',
 'rs-q3': 'C',
 's3': 'C',
 'citan': 'C',
 'gla-klasa': 'C',
 'b-klasa': 'C',
 'seria-3': 'D',
 'seria-4': 'D',
 'm4': 'D',
 'm3': 'D',
 '3gt': 'D',
 '80': 'D',
 'a4-allroad': 'D',
 'a5': 'D',
 'q4-sportback': 'D',
 'q5': 'D',
 's5': 'D',
 'glb-klasa': 'D',
 'glc-klasa': 'D',
 'glk-klasa': 'D',
 'sl': 'D',
 'seria-5': 'E',
 'seria-8': 'E',
 'm8': 'E',
 'm5': 'E',
 '5gt': 'E',
 'x6': 'E',
 'x6-m': 'E',
 'x5-m': 'E',
 'a6': 'E',
 'a6-allroad': 'E',
 'e-tron': 'E',
 'q7': 'E',
 'q8': 'E',
 's8': 'E',
 'gle-klasa': 'E',
 'e-klasa': 'E',
 'g-klasa': 'E',
 'r-klasa': 'E',
 'seria-7': 'F',
 'x7': 'F',
 'a7': 'F',
 'a8': 'F',
 'cl-klasa': 'F',
 'gl-klasa': 'F',
 'gls-klasa': 'F',
 's-klasa': 'F',
 'seria-6': 'G',
 'rs6': 'G',
 'clk-klasa': 'G',
 'cls-klasa': 'G',
 'slk-klasa': 'G',
 'x1': 'I',
 'x2': 'I',
 'x3': 'I',
 'x4': 'I',
 'x5': 'I',
 'q3': 'I',
 'm-klasa': 'I',
 'amg-gt': 'S',
 'rs3': 'Unknown',
 'rs5': 'Unknown',
 'sq5': 'Unknown',
 'tt': 'Unknown',
 'c-klasa': 'Unknown',
 'v-klasa': 'Unknown',
 'sprinter': 'Unknown',
 'viano': 'Unknown',
 'vito': 'Unknown'}
```

In [6]:
```python
data = data.loc[(data['mark'] == 'bmw') | (data['mark'] == 'mercedes-benz') |
               (data['mark'] == 'audi')]
data
```

Out[6]:

	Unnamed: 0	mark	model	generation_name	year	mileage	vol_engine	fuel	city	province	pric
8061	8061	audi	a3	gen-8p-2003-2012	2010	169500	1390	Gasoline	Cieszyn	Śląskie	2990
8062	8062	audi	a3	gen-8p-2003-2012	2018	67990	1500	Gasoline	Kielce	Świętokrzyskie	12985
8063	8063	audi	a3	gen-8p-2003-2012	2008	222248	1896	Diesel	Częstochowa	Śląskie	2290
8064	8064	audi	a3	gen-8p-2003-2012	2010	243000	1968	Diesel	Zawiercie	Śląskie	2790
8065	8065	audi	a3	gen-8p-2003-2012	2009	259658	1598	Diesel	Nowy Dwór Gdański	Pomorskie	2390
...	...	...	...	...	...	...	...	...	...	...	...
62658	62658	mercedes-benz	vito	gen-w639-2003	2007	266860	2148	Diesel	Stalowa Wola	Podkarpackie	2799
62659	62659	mercedes-benz	vito	gen-w639-2003	2013	270000	2987	Diesel	Olsztyn	Warmińsko-mazurskie	6490
62660	62660	mercedes-benz	vito	gen-w639-2003	2005	342600	2200	Diesel	Piotrkowice	Świętokrzyskie	1450
62661	62661	mercedes-benz	vito	gen-w639-2003	2006	282000	2148	Diesel	Świnoujście	Zachodniopomorskie	1250
62662	62662	mercedes-benz	vito	gen-w639-2003	2009	113000	2148	Diesel	Piasek	Śląskie	3880

30237 rows × 11 columns

In [7]:
```python
#checking null value
data.isna().sum()
```

Out[7]:
```
Unnamed: 0            0
mark                  0
model                 0
generation_name    4960
year                  0
mileage               0
vol_engine            0
fuel                  0
city                  0
province              0
price                 0
dtype: int64
```

In [8]:
```python
data.drop(columns = ["generation_name", 'Unnamed: 0', 'city', 'province'],inplace=True) #dropping

#checking null value
data.isna().sum()
```

Out[8]:
```
mark          0
model         0
year          0
mileage       0
vol_engine    0
fuel          0
price         0
dtype: int64
```

In [9]:
```python
data['model_types'] = data['model'].replace(segments_invert)
data
```

Out[9]:

	mark	model	year	mileage	vol_engine	fuel	price	model_types
8061	audi	a3	2010	169500	1390	Gasoline	29900	C
8062	audi	a3	2018	67990	1500	Gasoline	129850	C
8063	audi	a3	2008	222248	1896	Diesel	22900	C
8064	audi	a3	2010	243000	1968	Diesel	27907	C
8065	audi	a3	2009	259658	1598	Diesel	23900	C
...	...	...	...	...	...	...	...	...
62658	mercedes-benz	vito	2007	266860	2148	Diesel	27999	Unknown
62659	mercedes-benz	vito	2013	270000	2987	Diesel	64900	Unknown
62660	mercedes-benz	vito	2005	342600	2200	Diesel	14500	Unknown
62661	mercedes-benz	vito	2006	282000	2148	Diesel	12500	Unknown
62662	mercedes-benz	vito	2009	113000	2148	Diesel	38800	Unknown

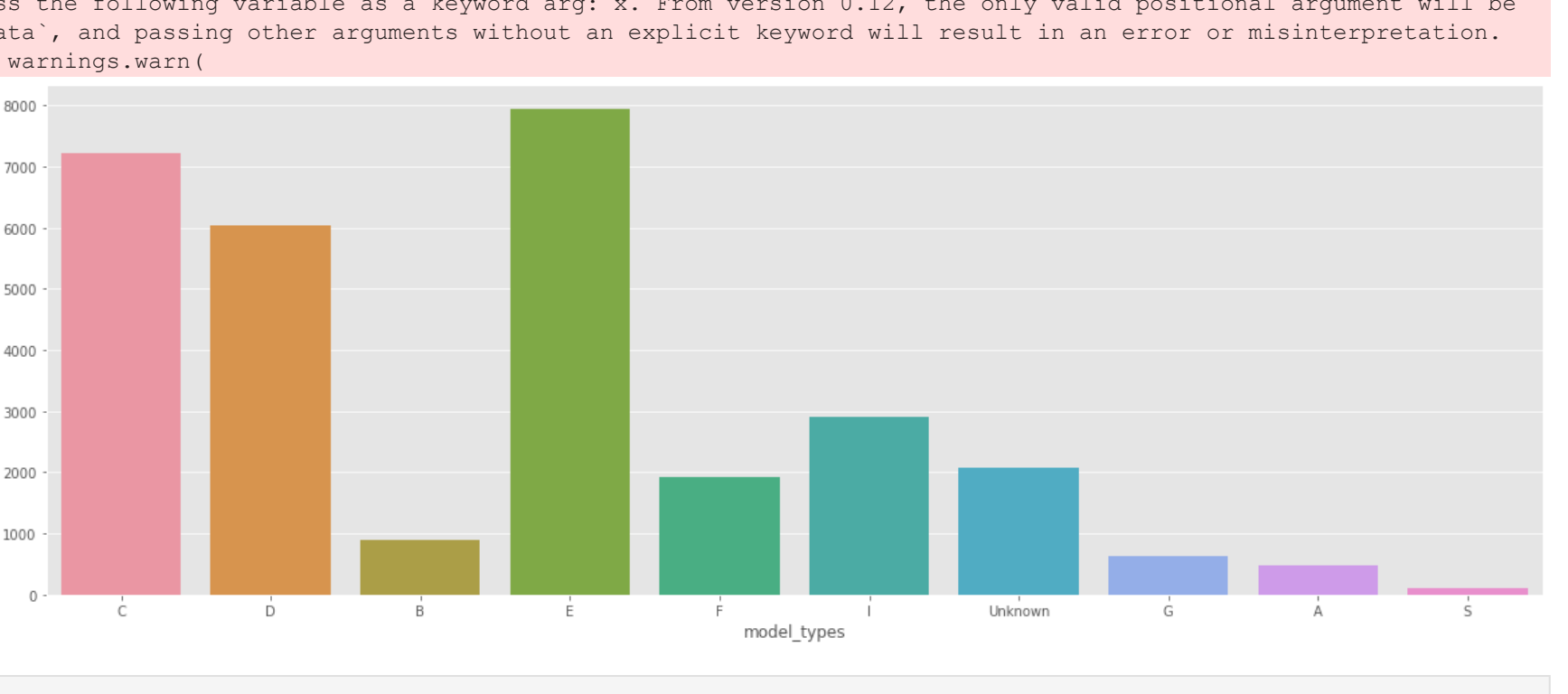
30237 rows × 8 columns

In [10]:
```python
print(data.model_types.unique())
```

```
['C' 'D' 'B' 'E' 'F' 'I' 'Unknown' 'G' 'A' 'S']
```

In [11]:
```python
# how much repeat type models  in the dataset

plt.figure(figsize=(20,7))
sns.countplot(data["model_types"])
plt.show()
```

```
/usr/local/anaconda3/envs/DataAnalytics/lib/python3.8/site-packages/seaborn/_decorators.py:36: FutureWarning: Pass the following variable as a keyword arg: x. From version 0.12, the only valid positional argument will be `data`, and passing other arguments without an explicit keyword will result in an error or misinterpretation.
  warnings.warn(
```



In [12]:
```python
data['car_age'] = 2022 - data['year']
data=data.loc[(data['car_age']>2)]
```

In [13]:
```python
data = data.loc[(data['model_types'] == 'F')]
```

In [14]:
```python
# Standarizing mileage, car_age and price
# mileage
mean_mileage = data.mileage.mean()
std_mileage = data.mileage.std()
data['c_mileage'] = data.mileage - mean_mileage
data['s_mileage'] = (data.mileage - mean_mileage)/std_mileage

# car_age
mean_car_age = data.car_age.mean()
std_car_age = data.car_age.std()
data['c_car_age'] = data.car_age - mean_car_age
data['s_car_age'] = (data.car_age - mean_car_age)/std_car_age

# price
mean_price = data.price.mean()
std_price = data.price.std()
data['s_price'] = (data.price - mean_price)/std_price
data.head(10)
```

Out[14]:

	mark	model	year	mileage	vol_engine	fuel	price	model_types	car_age	c_mileage	s_mileage	c_car_age	s_ca
20650	audi	a7	2011	190000	3000	Gasoline	107700	F	11	14580.49846	0.148886	-0.279729	-0.0
20651	audi	a7	2012	189400	2967	Diesel	67000	F	10	13980.49846	0.142760	-1.279729	-0.2
20652	audi	a7	2010	250000	2967	Diesel	60000	F	12	74580.49846	0.761567	0.720271	0.1
20653	audi	a7	2016	84000	2995	Gasoline	116000	F	6	-91419.50154	-0.933516	-5.279729	-0.8
20654	audi	a7	2017	90000	2995	Gasoline	149999	F	5	-85419.50154	-0.872248	-6.279729	-1.0
20655	audi	a7	2013	109967	2967	Diesel	125000	F	9	-65452.50154	-0.668358	-2.279729	-0.3
20656	audi	a7	2013	82575	2773	Gasoline	100000	F	9	-92844.50154	-0.948067	-2.279729	-0.3
20657	audi	a7	2017	75000	2967	Diesel	199000	F	5	-100419.50154	-1.025418	-6.279729	-1.0
20658	audi	a7	2012	214000	2967	Diesel	88400	F	10	38580.49846	0.393959	-1.279729	-0.2
20659	audi	a7	2016	98000	1984	Gasoline	165900	F	6	-77419.50154	-0.790557	-5.279729	-0.8

In [15]:
```python
def is_sorted(a):
    '''Check if numpy 1d-array is sorted '''
    if type(a) != np.ndarray:
        raise TypeError('Argument must be a numpy array but is {}'.format(type(a)))
    if len(a.shape) > 1:
        raise ValueError('Array must be 1 dimensional but has shape {}'.format(a.shape))
    return np.all(a[:-1] <= a[1:])
```

In [16]:
```python
LIGHT="#FFFCDC"
LIGHT_HIGHLIGHT="#FEF590"
MID="#FDED2A"
MID_HIGHLIGHT="#F0DC05"
DARK="#EECA02"
DARK_HIGHLIGHT="#BB9700"
GREEN="#00FF00"
LIGHT_GREY="#DDDDDD"
```

In [17]:
```python
def sort_1d_array_and_2d_array_by_1d_array(x, fx):
    if (type(x) != np.ndarray) or (type(fx) != np.ndarray):
        raise TypeError('At least one of the arguments is not a numpy array type(x)={}, type(fx)={}',format(typ
    if len(x)!=fx.shape[1]:
        raise ValueError('2d array number of columns is not matching the 1d array. Expected {} got {}'.format(l
    arr2D = np.concatenate([np.expand_dims(x,axis=0),fx],axis=0)
    sortedArr = arr2D [ :, arr2D[0].argsort()]
    return sortedArr[0,:], sortedArr[1:,:]
```

In [18]:
```python
def get_quantiles(fx, probs=None):
    if probs is None:
        probs = [10, 20, 30, 40, 50, 60, 70, 80, 90]
    if len(probs) % 2 == 0:
        raise ValueError('Number of quantiles must be even')
    if len(probs) > 11:
        raise ValueError('Too many quantiles (max is 11)')
    if probs[int(len(probs)/2)] != 50:
        raise ValueError('Middle quantile should be 50 but is {}'.format(probs[int(len(probs)/2)]))
    return np.percentile(fx, probs, axis=0)
```

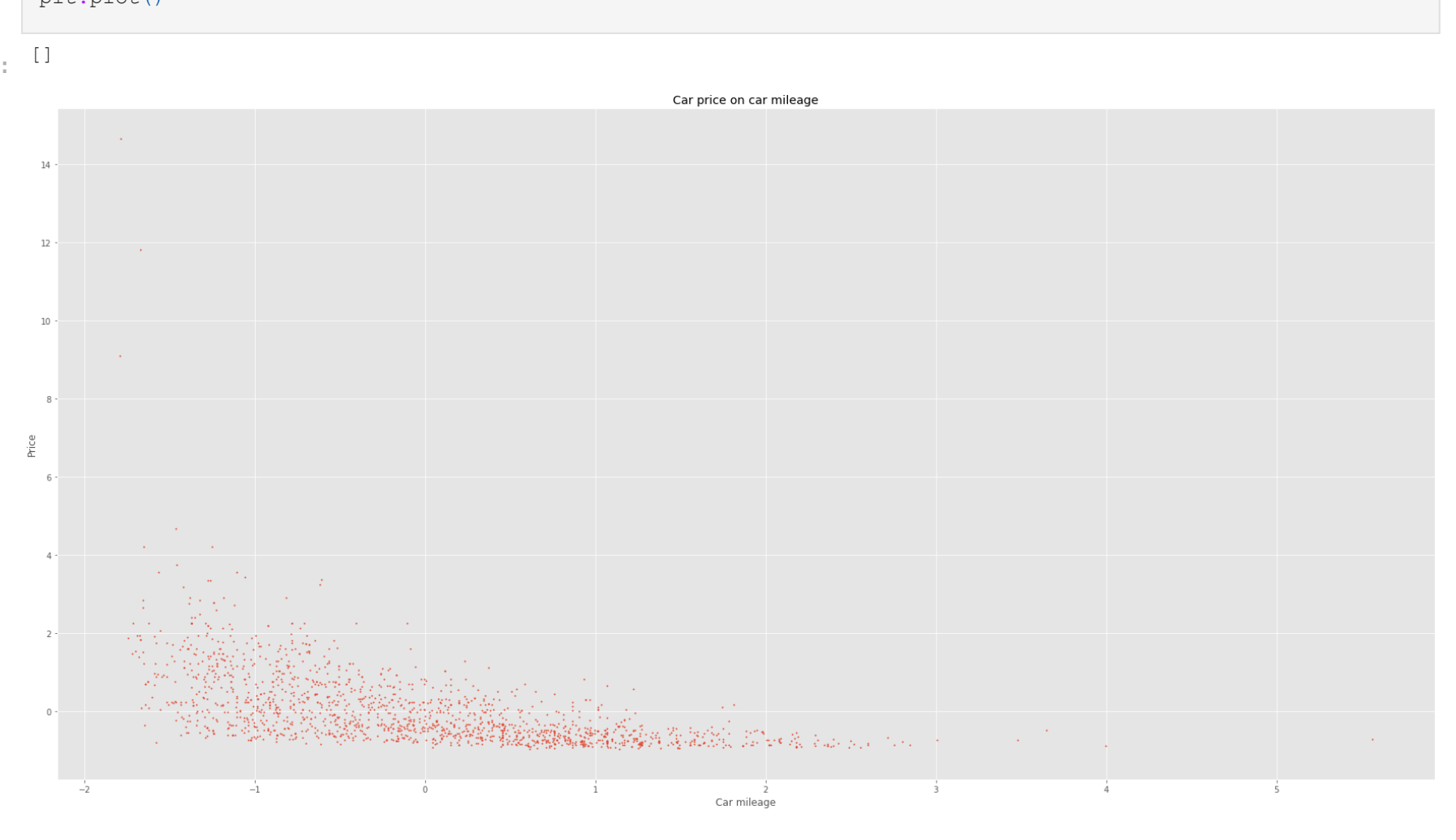
In [19]:
```python
def ribbon_plot(x, fx, ax=None,zorder=0,probs=None,supress_warning=False):
    '''Plot a ribbon plot for regression and similar.
    Plot consists of quantiles (by 10%) of a variate (fx) as a function of covariate (x). x has shape (n, )
    fx has shape (N,n)
    '''
    if ax is None:
        ax = plt.gca()
    if not is_sorted(x):
        x, fx = sort_1d_array_and_2d_array_by_1d_array(x, fx)
    if (len(set(x)) != len(x)) and (not supress_warning):
        warnings.warn("x variable has repeated values, which can influence the plot")
    perc_interv = get_quantiles(fx, probs)
    nq=perc_interv.shape[0]
    colortab = [LIGHT,LIGHT_HIGHLIGHT,MID,MID_HIGHLIGHT,DARK,DARK_HIGHLIGHT]

    for i in range(int(nq/2)): ax.fill_between(x,
                            perc_interv[i,:],
                            perc_interv[-(i+1),:],
                            color=colortab[i],
                            zorder=zorder)
    ax.plot(x,perc_interv[int(nq/2),:],color=colortab[int(nq/2)],zorder=zorder)
    return ax
```

In [20]:
```python
print("Standard diviation of mileage:",data.s_mileage.std())
print("Mean of mileage:",data.s_mileage.mean())
print("Standard diviation of price:",data.s_price.std())
print("Mean of price:",data.s_price.mean())
print("Max value of price:",data.s_price.max())
print("Maximal value of mileage:",data.s_mileage.max())
print("Maximal value of year:",data.s_car_age.max())
```

```
Standard diviation of mileage: 1.0
Mean of mileage: 3.502367151004807e-17
Standard diviation of price: 1.0
Mean of price: -7.004734302009614e-17
Max value of price: 14.65114792583687
Maximal value of mileage: 5.560897178995661
Maximal value of year: 3.241587436917497
```

In [27]:
```python
fig, axes = plt.subplots(1, 1, figsize=(30, 15))
plt.plot(data.s_mileage, data.s_price, 'o', markersize=1)
axes.set_xlabel("Car mileage")
axes.set_ylabel("Price")
axes.set_title("Car price on car mileage")
plt.plot()
```

Out[27]: []



First predictor - one param: mileage

First predictor is basing the predictions of the price of car on the basis of the distance traveled. We can check whether parameter values and distributions "make sens"

In [28]:
```python
with open('one_param_ppc_mileage.stan', 'r') as file:
    print(file.read())
```

```
data {
    int N;
    real mileage[N];
}

generated quantities {
            real alpha = normal_rng(0,1);
            real beta = normal_rng(0,1);
            real sigma = exponential_rng(0.01);
            real price[N];
            for (i in 1:N) {
                price[i] = normal_rng(mileage[i]*beta+alpha,sigma);
            }
}
```

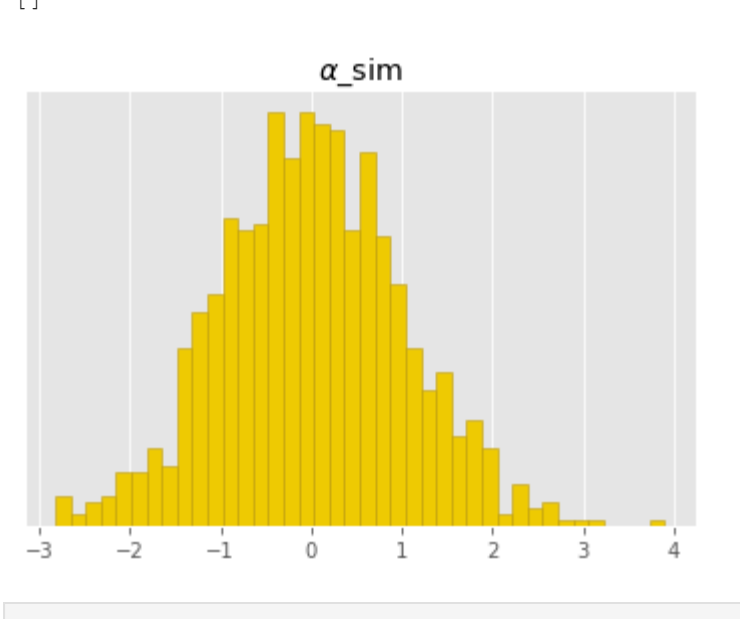
In [29]:
```python
model1_ppc=CmdStanModel(stan_file='one_param_ppc_mileage.stan')
R = 1000
data_sim={'N': 50, 'mileage': np.linspace(data.s_mileage.min(),data.s_mileage.max())}
sim_one_param=model1_ppc.sample(data=data_sim,
                                iter_sampling=R,
                                iter_warmup=0,
                                chains=1,
                                fixed_param=True,
                                seed=29042020)

alpha_sim=sim_one_param.stan_variable('alpha')
beta_sim=sim_one_param.stan_variable('beta')
```

```
INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |██████████| 00:00 Sampling completed
```

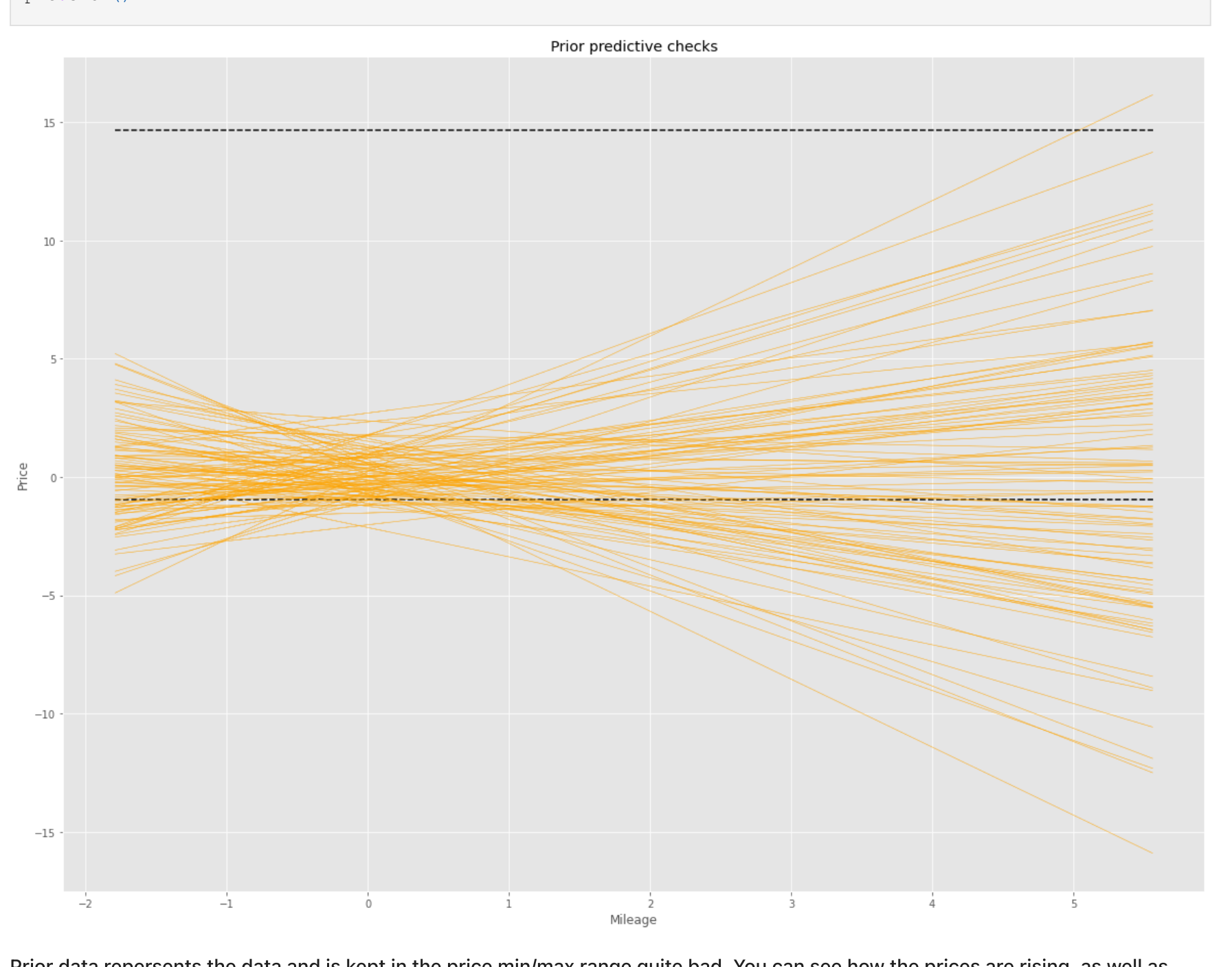
```
INFO:cmdstanpy:CmdStan done processing.
```

In [30]:
```python
fig, axes = plt.subplots(1, 1, figsize=(6, 4))
ax = axes
ax.hist(alpha_sim, bins=40, color=DARK, edgecolor=DARK_HIGHLIGHT, density=True)
ax.set_title(r'$\alpha$_sim')
ax.set_yticks(())
```

Out[30]: []



In [31]:
```python
fig, axes = plt.subplots(1, 1, figsize=(20, 15))
axes.hlines([data.s_price.min(), data.s_price.max()], xmin=data.s_mileage.min(),xmax=data.s_mileage.max(),lines

for i in range(100):
    axes.plot(data.s_mileage, alpha_sim[i]+beta_sim[i]*data.s_mileage,color='orange',alpha=0.5,linewidth=0.5)
axes.set_xlabel('Mileage')
axes.set_ylabel('Price')
axes.set_title(r'Prior predictive checks')
plt.show()
```



Prior data repersents the data and is kept in the price min/max range quite bad. You can see how the prices are rising, as well as decreasing in relation to the distance traveled. It's bad information, so we should improve first predictor.

Time to improve first predictor - one param: mileage

Now it's time to change the distribution parameters. The change in the alpha mean is conditioned by instinctive knowledge about the prices of cars in analysed segment after standardization.

In [48]:
```python
with open('one_param_ppc_mileage_improved.stan', 'r') as file:
    print(file.read())
```

```
data {
    int N;
    real mileage[N];
}

generated quantities {
            real alpha = normal_rng(0.7,0.5);
            real beta = normal_rng(-0.5,0.5);
            real <lower = 0> sigma = exponential_rng(0.3);
            real price[N];
            for (i in 1:N) {
                price[i] = normal_rng(mileage[i]*beta+alpha,sigma);
            }
}
```

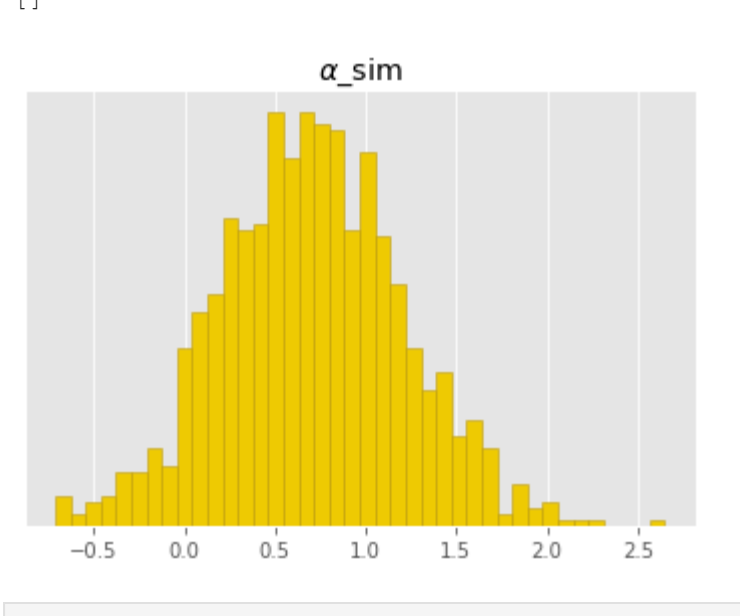
In [49]:
```python
model1_improved_ppc=CmdStanModel(stan_file='one_param_ppc_mileage_improved.stan')
R = 1000
data_sim={'N': 50, 'mileage': np.linspace(data.s_mileage.min(),data.s_mileage.max())}
sim2=model1_improved_ppc.sample(data=data_sim,
                                iter_sampling=R,
                                iter_warmup=0,
                                chains=1,
                                fixed_param=True,
                                seed=29042020)

alpha_sim_improved=sim2.stan_variable('alpha')
beta_sim_improved=sim2.stan_variable('beta')
```

```
INFO:cmdstanpy:compiling stan file /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.stan to exe file /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved
INFO:cmdstanpy:compiled model executable: /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved
WARNING:cmdstanpy:Stan compiler has produced 2 warnings:
WARNING:cmdstanpy:
--- Translating Stan model to C++ code ---
bin/stanc  --o=/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.hpp /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.stan
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.stan', line 3, column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.stan', line 10, column 12: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc

--- Compiling, linking C++ code ---
clang++ -std=c++1y -Wno-unknown-warning-option -Wno-tautological-compare -Wno-sign-compare -D_REENTRANT -Wno-ignored-attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I stan/lib/stan_math/lib/boost_1.75.0 -I stan/lib/stan_math/lib/sundials_6.0.0/include -I stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -DBOOST_DISABLE_ASSERTS          -c -include-pch stan/src/stan/model/model_header.hpp.gch -x c++ -o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.hpp
clang++ -std=c++1y -Wno-unknown-warning-option -Wno-tautological-compare -Wno-sign-compare -D_REENTRANT -Wno-ignored-attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I stan/lib/stan_math/lib/boost_1.75.0 -I stan/lib/stan_math/lib/sundials_6.0.0/include -I stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -DBOOST_DISABLE_ASSERTS               -Wl,-L,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb" -Wl,-rpath,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb"
/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.o src/cmdstan/main.o
      -Wl,-L,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb" -Wl,-rpath,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb"   stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_nvecserial.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_cvodes.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_idas.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_kinsol.a  stan/lib/stan_math/lib/tbb/libtbb.dylib stan/lib/stan_math/lib/tbb/libtbbmalloc.dylib stan/lib/stan_math/lib/tbb/libtbbmalloc_proxy.dylib -o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved
rm -f /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_mileage_improved.o

INFO:cmdstanpy:CmdStan start processing
chain 1 |██████████| 00:00 Sampling completed
```

```
INFO:cmdstanpy:CmdStan done processing.
```

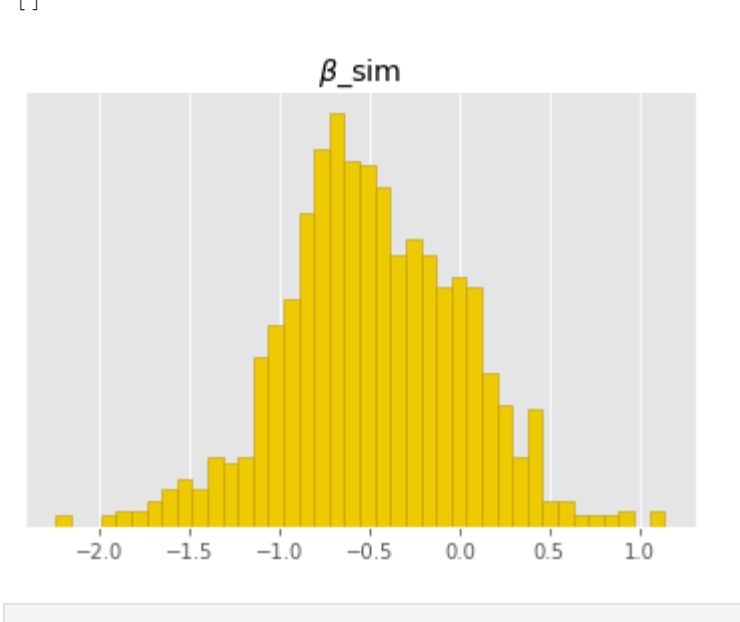
In [50]:
```python
fig, axes = plt.subplots(1, 1, figsize=(6, 4))
ax = axes
ax.hist(alpha_sim_improved, bins=40, color=DARK, edgecolor=DARK_HIGHLIGHT, density=True)
ax.set_title(r'$\alpha$_sim')
ax.set_yticks(())
```

Out[50]: []

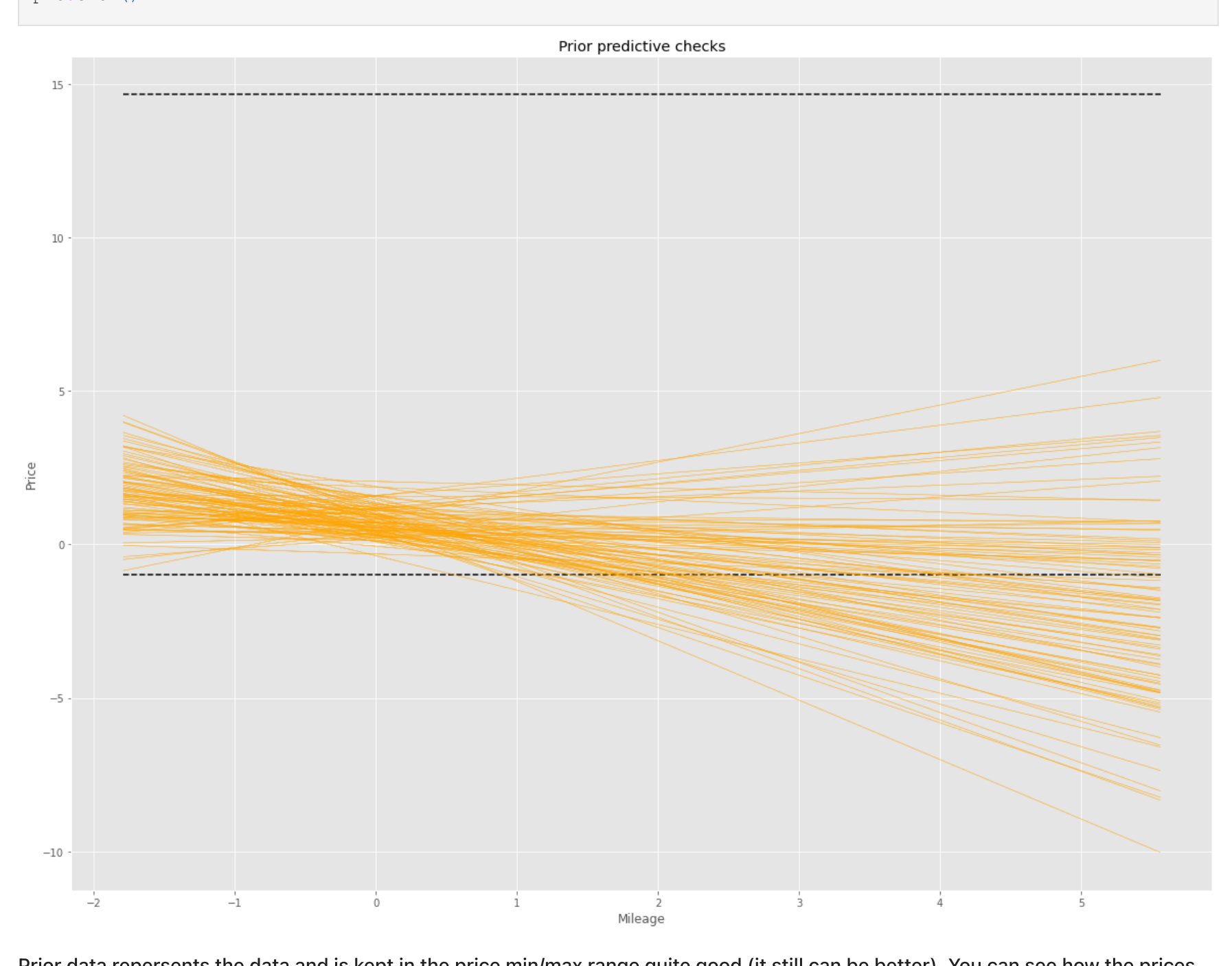


In [51]:
```python
fig, axes = plt.subplots(1, 1, figsize=(6, 4))
ax = axes
ax.hist(beta_sim_improved, bins=40, color=DARK, edgecolor=DARK_HIGHLIGHT, density=True)
ax.set_title(r'$\beta$_sim')
ax.set_yticks(())
```

Out[51]: []



In [52]:
```python
fig, axes = plt.subplots(1, 1, figsize=(20, 15))
axes.hlines([data.s_price.min(), data.s_price.max()], xmin=data.s_mileage.min(),xmax=data.s_mileage.max(),lines

for i in range(100):
    axes.plot(data.s_mileage, alpha_sim_improved[i]+beta_sim_improved[i]*data.s_mileage,color='orange',alpha=0.
axes.set_xlabel('Mileage')
axes.set_ylabel('Price')
axes.set_title(r'Prior predictive checks')
plt.show()
```



Prior data repersents the data and is kept in the price min/max range quite good (it still can be better). You can see how the prices are mainly decreasing in relation to the distance traveled. It's good information, so we know that prior predictive is succesfull.

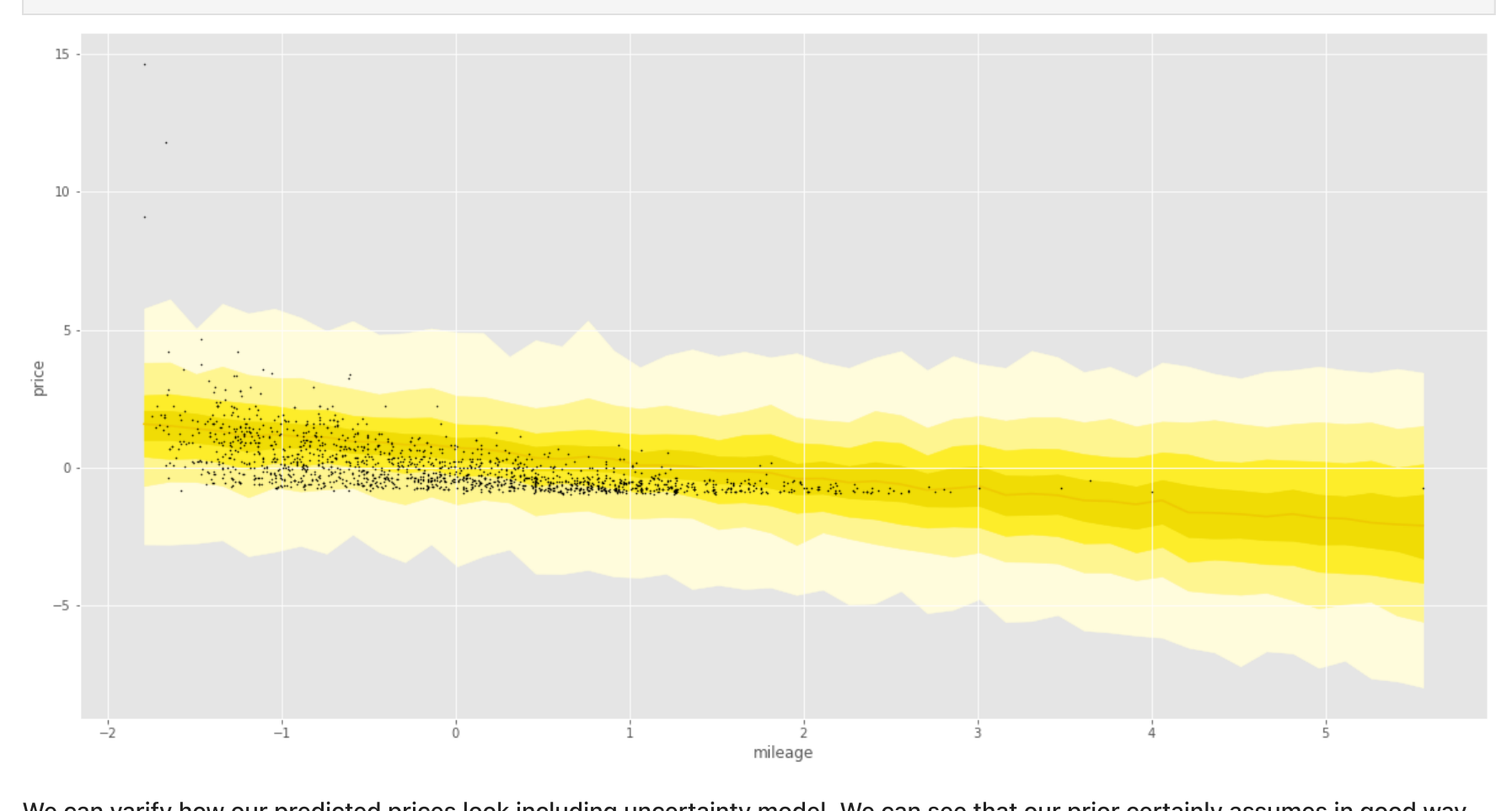
In [53]:
```python
az.summary(sim2,var_names=['alpha','beta','sigma'],round_to=2,kind='stats')
```

Out[53]:

	mean	sd	hdi_3%	hdi_97%
alpha	0.70	0.50	-0.17	1.69
beta	-0.49	0.50	-1.39	0.48
sigma	3.29	3.24	0.00	9.39

Here we can see that the majority of tha parameter alpha is concentrated around 0.7 (it's about 250k PLN). The beta is slightly diffused with most of the parameter concentrated around -0.5.

In [54]:
```python
price_sim2=sim2.stan_variable('price')
fig, axes = plt.subplots(1, 1, figsize=(20,10))
axes=ribbon_plot(data_sim['mileage'], price_sim2, axes)
axes.scatter(data.s_mileage, data.s_price, color='black', alpha=0.9, s=1)
axes.set_xlabel('mileage')
axes.set_ylabel('price')
plt.show()
```



We can varify how our predicted prices look including uncertainty model. We can see that our prior certainly assumes in good way standarized prices.

SECOND PREDICTOR - one param: car age

The distribution used is the lognormal distribution multiplied by -1 due to declining data.

In [55]:
```python
with open('one_param_ppc_age.stan', 'r') as file:
    print(file.read())
```

```
data {
        int N;
        real car_age[N];
}

generated quantities {
            real alpha = normal_rng(0.7,0.5);
            real beta_car_age = -lognormal_rng(-1.5,1);
            real <lower = 0> sigma = exponential_rng(0.3);
            real price[N];
            for (i in 1:N) {
                price[i] = normal_rng(car_age[i]*beta_car_age+alpha,sigma);
            }
}
```

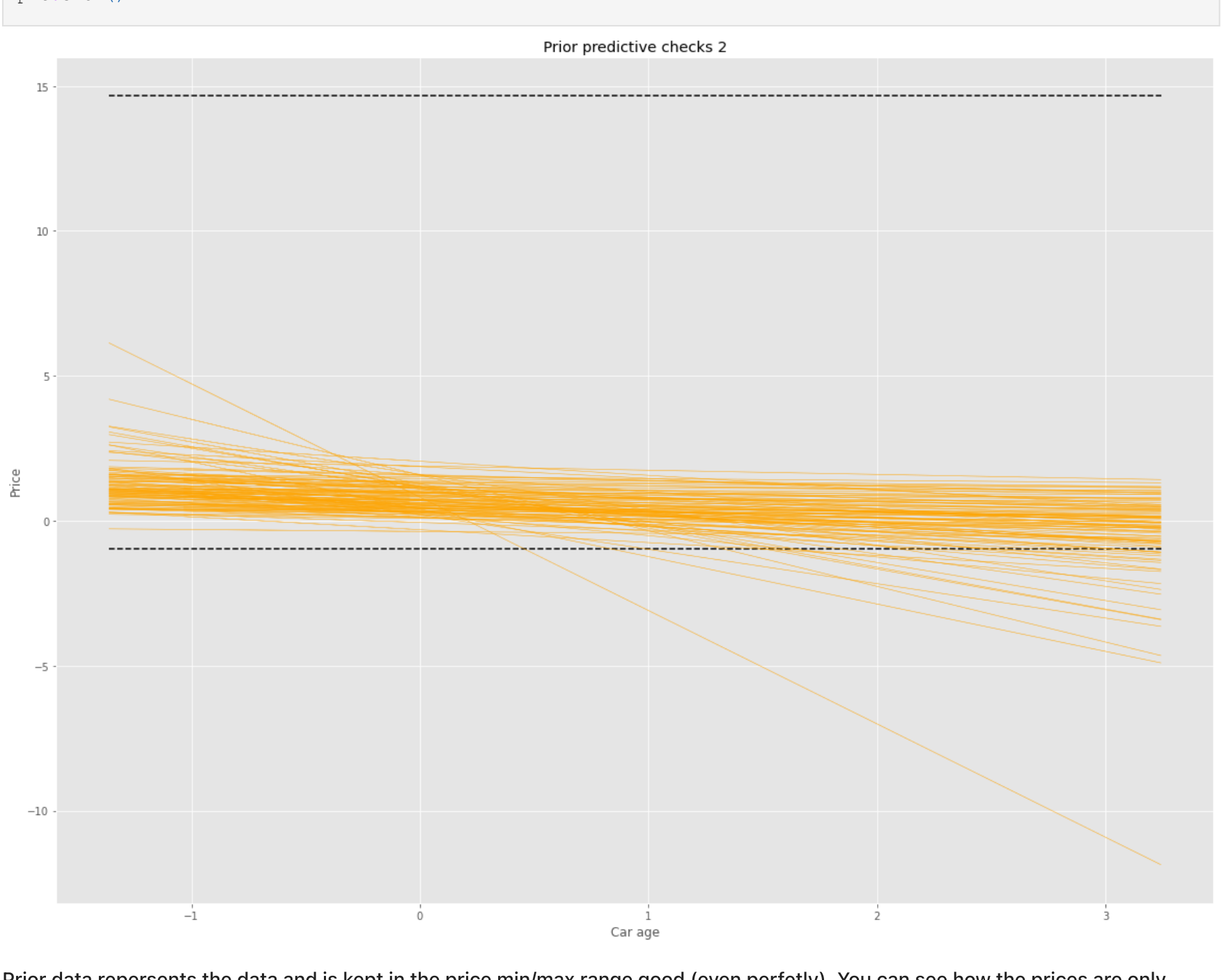
In [56]:
```python
model2_ppc=CmdStanModel(stan_file='one_param_ppc_age.stan')
R = 1000
data_sim2={'N': 50, 'car_age': np.linspace(data.s_car_age.min(),data.s_car_age.max())}
sim3=model2_ppc.sample(data=data_sim2,
                       iter_sampling=R,
                       iter_warmup=0,
                       chains=1,
                       fixed_param=True,
                       seed=29042020)

alpha_sim3=sim3.stan_variable('alpha')
beta_sim3=sim3.stan_variable('beta_car_age')
```

```
INFO:cmdstanpy:compiling stan file /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.stan to exe file /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age
INFO:cmdstanpy:compiled model executable: /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age
WARNING:cmdstanpy:Stan compiler has produced 2 warnings:
WARNING:cmdstanpy:
--- Translating Stan model to C++ code ---
bin/stanc  --o=/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.hpp /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.stan
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.stan', line 3, column 7: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.stan', line 10, column 12: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc

--- Compiling, linking C++ code ---
clang++ -std=c++1y -Wno-unknown-warning-option -Wno-tautological-compare -Wno-sign-compare -D_REENTRANT -Wno-ignored-attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I stan/lib/stan_math/lib/boost_1.75.0 -I stan/lib/stan_math/lib/sundials_6.0.0/include -I stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -DBOOST_DISABLE_ASSERTS          -c -include-pch stan/src/stan/model/model_header.hpp.gch -x c++ -o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.hpp
clang++ -std=c++1y -Wno-unknown-warning-option -Wno-tautological-compare -Wno-sign-compare -D_REENTRANT -Wno-ignored-attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I stan/lib/stan_math/lib/boost_1.75.0 -I stan/lib/stan_math/lib/sundials_6.0.0/include -I stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -DBOOST_DISABLE_ASSERTS               -Wl,-L,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb" -Wl,-rpath,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb"  /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.o src/cmdstan/main.o        -Wl,-L,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb" -Wl,-rpath,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb"   stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_nvecserial.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_cvodes.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_idas.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_kinsol.a  stan/lib/stan_math/lib/tbb/libtbb.dylib stan/lib/stan_math/lib/tbb/libtbbmalloc.dylib stan/lib/stan_math/lib/tbb/libtbbmalloc_proxy.dylib -o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age
rm -f /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/one_param_ppc_age.o

INFO:cmdstanpy:CmdStan start processing
chain 1 |                                  | 00:00 Sampling completed
```

```
INFO:cmdstanpy:CmdStan done processing.
```

In [57]:

```python
fig, axes = plt.subplots(1, 1, figsize=(20, 15))
axes.hlines([data.s_price.min(), data.s_price.max()], xmin=data.s_car_age.min(),xmax=data.s_car_age.max(),lines
for i in range(100):
    axes.plot(data.s_car_age, alpha_sim3[i]+beta_sim3[i]*data.s_car_age,color='orange',alpha=0.5,linewidth=0.5)
axes.set_xlabel('Car age')
axes.set_ylabel('Price')
axes.set_title(r'Prior predictive checks 2')
plt.show()
```



Prior data repersents the data and is kept in the price min/max range good (even perfetly). You can see how the prices are only decreasing in relation to the distance traveled. It's good information, so we know that prior predictive is succesfull.

In [58]:

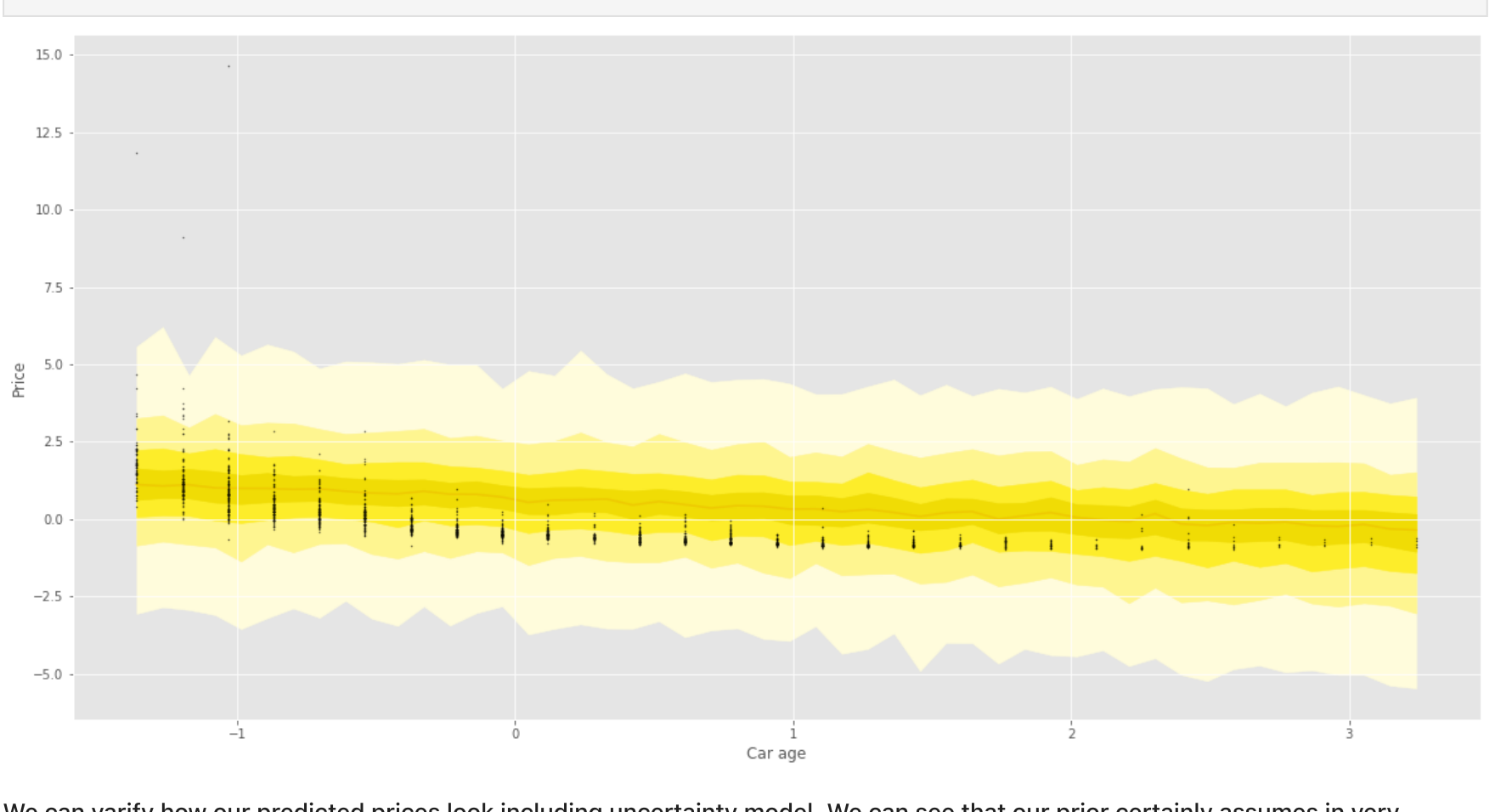
```python
az.summary(sim3,var_names=['alpha','beta_car_age','sigma'],round_to=2,kind='stats')
```

Out[58]:

	mean	sd	hdi_3%	hdi_97%
alpha	0.70	0.50	-0.17	1.69
beta_car_age	-0.38	0.51	-1.05	-0.01
sigma	3.29	3.24	0.00	9.39

In [59]:

```python
price_sim=sim3.stan_variable('price')
fig, axes = plt.subplots(1, 1, figsize=(20,10))
axes=ribbon_plot(data_sim2['car_age'], price_sim, axes)
axes.scatter(data.s_car_age, data.s_price, color='black', alpha=0.5, s=1)
axes.set_xlabel('Car age')
axes.set_ylabel('Price')
plt.show()
```



We can varify how our predicted prices look including uncertainty model. We can see that our prior certainly assumes in very good way standarized prices.

FIRST MODEL - ONLY WITH MILEAGE PARAMETER (1' DEGREE)

In [81]:

```python
with open('price_fit.stan', 'r') as file:
    print(file.read())
```

```
data {
    int N;
    vector[N] mileage;
    real price[N];
}

parameters {
    real alpha;
    real beta;
    real<lower=0> sigma;
}

transformed parameters {
    vector[N] mu = mileage*beta+alpha;
}

model {
    alpha ~ normal(0.2,0.5);
    beta ~ normal(-0.5,0.5);
    sigma ~ exponential(0.3);
    price ~ normal(mu,sigma);
}

generated quantities {
    real prices[N];
    real log_lik[N];
    for (i in 1:N) {
        prices[i] = normal_rng(mu[i],sigma);
        log_lik[i] = normal_lpdf(price[i] | mu[i], sigma);
    }
}
```

In [82]:

```python
model1_fit=CmdStanModel(stan_file='price_fit.stan')
model1_fit.compile(force=False)
d_short=data.head(1000)
data_fit1=dict(N=len(d_short),
               mileage=d_short.s_mileage.values,
               price=d_short.s_price.values)
fit1=model1_fit.sample(data=data_fit1,seed=28052020)
alpha_fit=fit1.stan_variable('alpha')
beta_fit=fit1.stan_variable('beta')
mu_fit=fit1.stan_variable('mu')
price_pred=fit1.stan_variable('prices')
log_lik=fit1.stan_variable('log_lik')
az.summary(fit1,var_names=['alpha','beta','sigma'],round_to=2,kind='stats')
```

```
INFO:cmdstanpy:compiling stan file /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.stan to exe file /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit
INFO:cmdstanpy:compiled model executable: /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit
WARNING:cmdstanpy:Stan compiler has produced 3 warnings:
WARNING:cmdstanpy:
--- Translating Stan model to C++ code ---
bin/stanc  --o=/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.hpp /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.stan
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.stan', line 4, column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.stan', line 25, column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.stan', line 26, column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc

--- Compiling, linking C++ code ---
clang++ -std=c++1y -Wno-unknown-warning-option -Wno-tautological-compare -Wno-sign-compare -D_REENTRANT -Wno-ignored-attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I stan/lib/stan_math/lib/boost_1.75.0 -I stan/lib/stan_math/lib/sundials_6.0.0/include -I stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -DBOOST_DISABLE_ASSERTS          -c -include-pch stan/src/stan/model/model_header.hpp.gch -x c++ -o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.hpp
clang++ -std=c++1y -Wno-unknown-warning-option -Wno-tautological-compare -Wno-sign-compare -D_REENTRANT -Wno-ignored-attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I stan/lib/stan_math/lib/boost_1.75.0 -I stan/lib/stan_math/lib/sundials_6.0.0/include -I stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -DBOOST_DISABLE_ASSERTS               -Wl,-L,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb" -Wl,-rpath,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb"  /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.o src/cmdstan/main.o        -Wl,-L,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb" -Wl,-rpath,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb"   stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_nvecserial.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_cvodes.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_idas.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_kinsol.a  stan/lib/stan_math/lib/tbb/libtbb.dylib stan/lib/stan_math/lib/tbb/libtbbmalloc.dylib stan/lib/stan_math/lib/tbb/libtbbmalloc_proxy.dylib -o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit
rm -f /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit.o

INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |                                                  | 00:00 Status
chain 2 |                                                  | 00:00 Status

chain 3 |                                                  | 00:00 Status


chain 1 |                    | 00:00 Iteration:  200 / 2000 [ 10%]  (Warmup)

chain 3 |                    | 00:00 Iteration:  100 / 2000 [  5%]  (Warmup)
chain 2 |                    | 00:00 Iteration:  200 / 2000 [ 10%]  (Warmup)


chain 1 |                    | 00:00 Iteration:  900 / 2000 [ 45%]  (Warmup)

chain 3 |                    | 00:00 Iteration:  800 / 2000 [ 40%]  (Warmup)
chain 2 |                    | 00:00 Iteration:  900 / 2000 [ 45%]  (Warmup)


chain 4 |                    | 00:00 Iteration:  900 / 2000 [ 45%]  (Warmup)

chain 1 |                  | 00:01 Iteration: 1400 / 2000 [ 70%]  (Sampling)
chain 2 |                  | 00:01 Iteration: 1400 / 2000 [ 70%]  (Sampling)


chain 4 |                  | 00:01 Iteration: 1400 / 2000 [ 70%]  (Sampling)

chain 1 |                  | 00:01 Iteration: 1800 / 2000 [ 90%]  (Sampling)

chain 3 |                  | 00:01 Iteration: 1800 / 2000 [ 90%]  (Sampling)
chain 2 |                  | 00:01 Iteration: 1800 / 2000 [ 90%]  (Sampling)


chain 1 |                                  | 00:02 Sampling completed
chain 2 |                                  | 00:02 Sampling completed
chain 3 |                                  | 00:02 Sampling completed
chain 4 |                                  | 00:02 Sampling completed
```

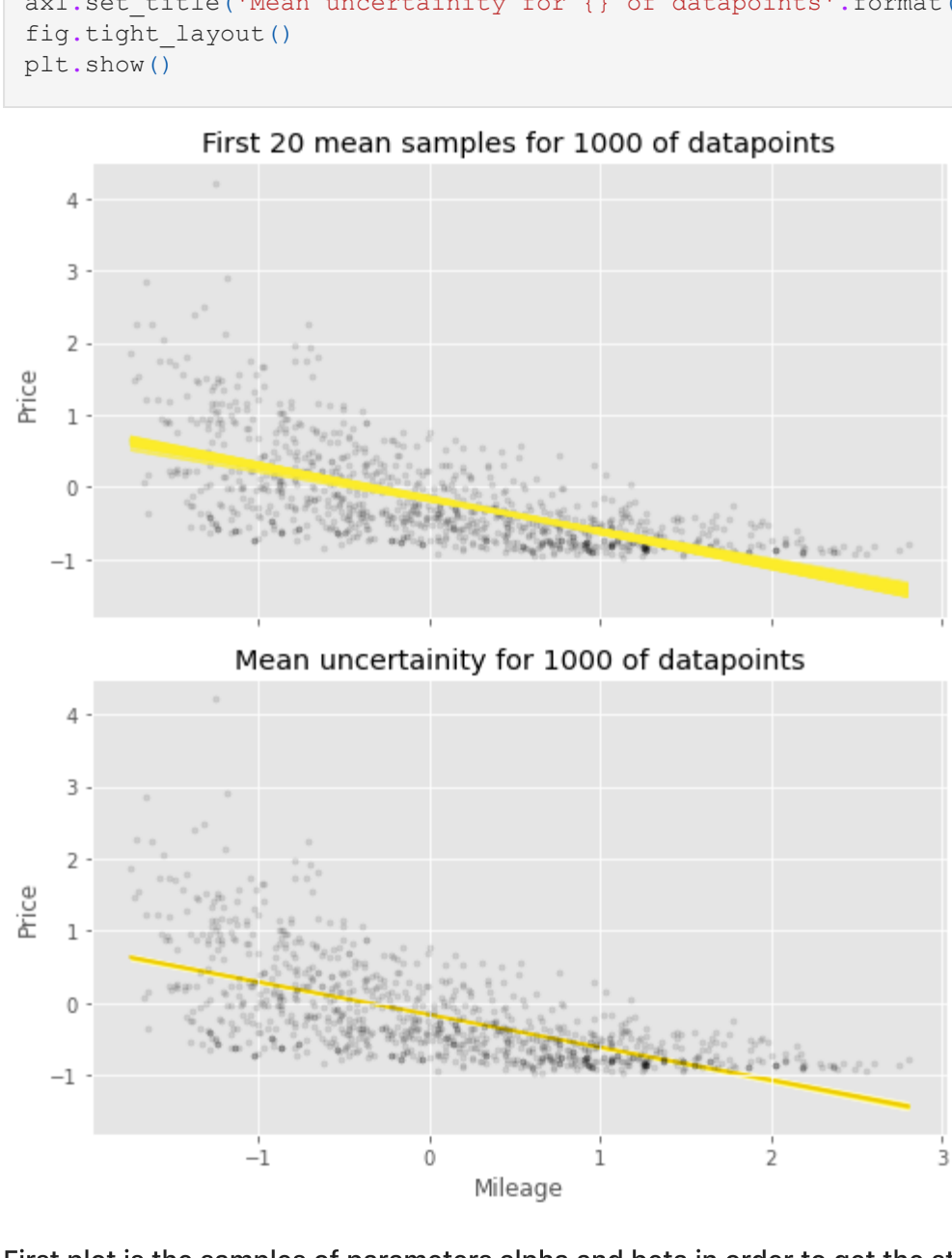
```
INFO:cmdstanpy:CmdStan done processing.
```

Out[82]:

	mean	sd	hdi_3%	hdi_97%
alpha	-0.16	0.02	-0.19	-0.13
beta	-0.45	0.02	-0.49	-0.42
sigma	0.52	0.01	0.49	0.54

In [83]:

```python
fig, axes = plt.subplots(2,1, figsize=(7, 8),sharey=True,sharex=True)
ax0=axes[0]
for i in range(100):
    ax0.plot(d_short.s_mileage,
             alpha_fit[i]+beta_fit[i]*d_short.s_mileage,
             color=MID,
             alpha=0.5,
             linewidth=0.5)
ax0.scatter(d_short.s_mileage, d_short.s_price, color='black', alpha=0.1, s=10)
ax0.set_title('First 20 mean samples for {} of datapoints'.format(len(d_short)))
ax0.set_ylabel('Price')
ax1=axes[1]
ax1=ribbon_plot(d_short.s_mileage.values, mu_fit, ax1, supress_warning=True)
ax1.scatter(d_short.s_mileage, d_short.s_price, color='black', alpha=0.1, s=10)
ax1.set_xlabel('Mileage')
ax1.set_ylabel('Price')
ax1.set_title('Mean uncertainity for {} of datapoints'.format(len(d_short)))
fig.tight_layout()
plt.show()
```

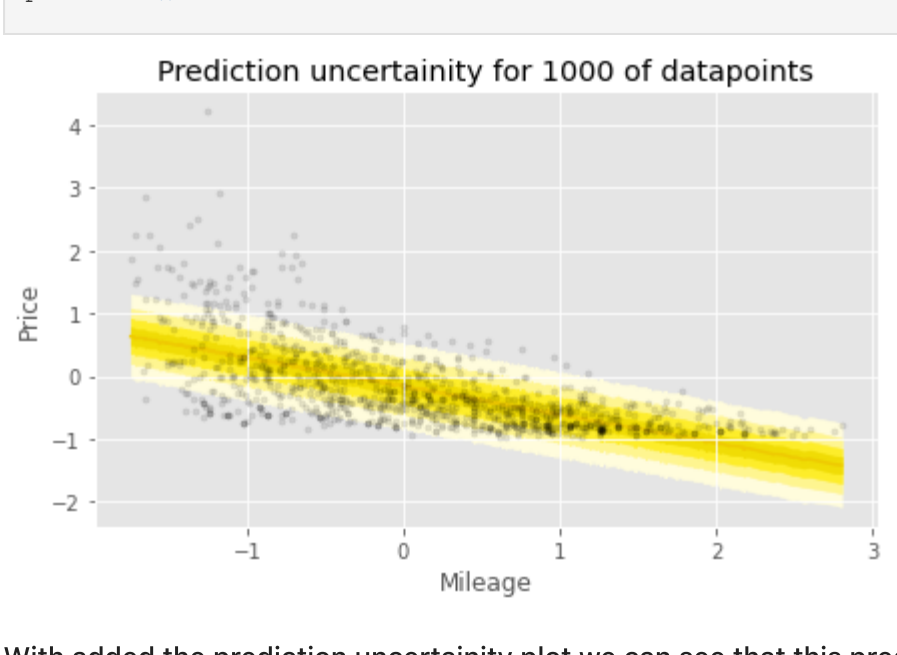


First plot is the samples of parameters alpha and beta in order to get the standarized mean and we can see how this mean relationship gives our model thinks this is the underlying phenomena adn we can see that these samples are generally really consistent.

The second one shows that compute quantiles are very well concentrated.

In [84]:

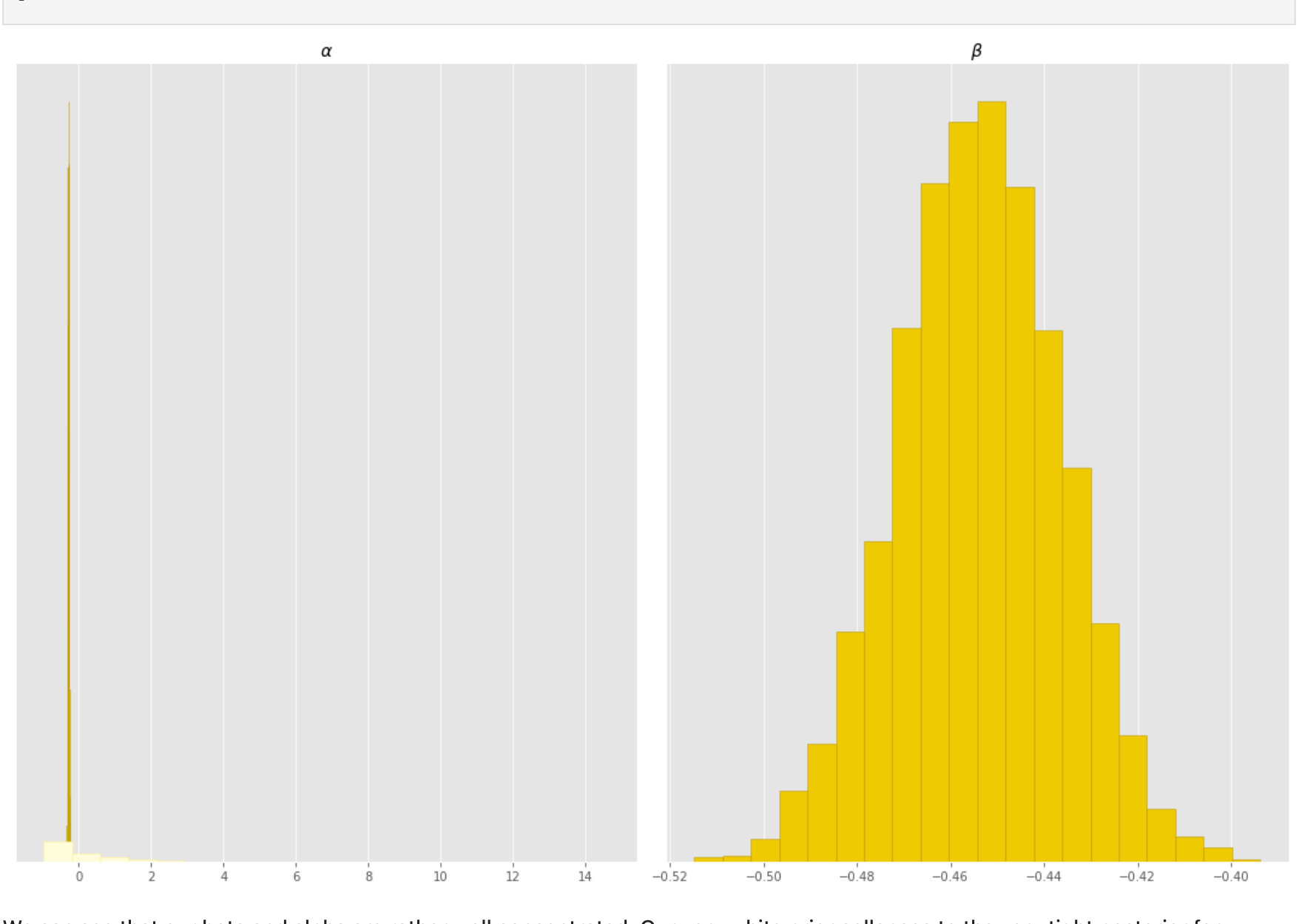
```python
fig, axes = plt.subplots(1, 1, figsize=(7, 4))
axes = ribbon_plot(d_short.s_mileage.values, price_pred, axes, supress_warning=True)
axes.scatter(d_short.s_mileage, d_short.s_price, color='black', alpha=0.1, s=10)
axes.set_xlabel('Mileage')
axes.set_ylabel('Price')
axes.set_title('Prediction uncertainity for {} of datapoints'.format(len(d_short)))
plt.show()
```



With added the prediction uncertainity plot we can see that this prediciton unceratainity is covering more and more available samples (can be better). That means all the relationship is significantly non-linear.

In [132…

```python
fig, axes = plt.subplots(1, 2, figsize=(15, 10))
ax = axes[0]
ax.hist(alpha_fit, bins=20, color=DARK, edgecolor=DARK_HIGHLIGHT, density=True)
ax.hist(data.s_price, bins=20, color=LIGHT, edgecolor=LIGHT_HIGHLIGHT, density=True)
ax.set_title(r'$\alpha$')
ax.set_yticks(())
ax2 = axes[1]
ax2.hist(beta_fit, bins=20, color=DARK, edgecolor=DARK_HIGHLIGHT, density=True)
ax2.set_title(r'$\beta$')
ax2.set_yticks(())
fig.tight_layout()
plt.show()
```



We can see that our beta and alpha are rather well concentrated. Our very white prior collapses to the very tight posterior for alpha.

SECOND MODEL - 2 PREDICTORS (ONE WITH 2' DEGREE POLYNOMIAL FUNCTION)

Here we use higher order predictors for mileage and additional predictor for car age

In [66]:

```python
data['s_mileage2']=data.s_mileage*data.s_mileage
data['s_mileage3']=data.s_mileage*data.s_mileage*data.s_mileage
data
```

Out[66]:

	mark	model	year	mileage	vol_engine	fuel	price	model_types	car_age	c_mileage	s_mileage	c_car_age
20650	audi	a7	2011	190000	3000	Gasoline	107700	F	11	14580.49846	0.148886	-0.279729
20651	audi	a7	2012	189400	2967	Diesel	67000	F	10	13980.49846	0.142760	-1.279729
20652	audi	a7	2010	250000	2967	Diesel	60000	F	12	74580.49846	0.761567	0.720271
20653	audi	a7	2016	84000	2995	Gasoline	116000	F	6	-91419.50154	-0.933516	-5.279729
20654	audi	a7	2017	90000	2995	Gasoline	149999	F	5	-85419.50154	-0.872248	-6.279729
...	...	...	...	...	...	...	...	...	...	...	...	...
62018	mercedes-benz	s-klasa	2018	16100	3982	Gasoline	268900	F	4	-159319.50154	-1.626866	-7.279729
62019	mercedes-benz	s-klasa	2018	37111	3982	Gasoline	325000	F	4	-138308.50154	-1.412315	-7.279729
62020	mercedes-benz	s-klasa	2017	99000	5461	Gasoline	499000	F	5	-76419.50154	-0.780346	-6.279729
62021	mercedes-benz	s-klasa	2015	14600	4663	Gasoline	258298	F	7	-160819.50154	-1.642183	-4.279729
62022	mercedes-benz	s-klasa	2017	59950	2987	Diesel	295000	F	5	-115469.50154	-1.179098	-6.279729

1623 rows × 16 columns

In [67]:

```python
X=data[['s_mileage', 's_mileage2']]
X
```

Out[67]:

	s_mileage	s_mileage2
20650	0.148886	0.022167
20651	0.142760	0.020380
20652	0.761567	0.579984
20653	-0.933516	0.871452
20654	-0.872248	0.760816
...	...	...
62018	-1.626866	2.646692
62019	-1.412315	1.994634
62020	-0.780346	0.608939
62021	-1.642183	2.696764
62022	-1.179098	1.390273

1623 rows × 2 columns

In [115…

```python
with open('price_fit_2.stan', 'r') as file:
    print(file.read())
```

```
data {
    int N;
    int K; //number of predictions
    matrix[N,K] X; //design matrix
    vector[N] car_age;
    real price[N];
}

parameters {
    real alpha;
    vector[K] beta_mileage;
    real beta_car_age;
    real sigma;
}

transformed parameters {
    vector[N] mu = X*beta_mileage+car_age*(-beta_car_age)+alpha;
}

model {
    alpha ~ normal(0.2,0.5);
    beta_car_age ~ lognormal(-1.5, 1);
    beta_mileage ~ normal(-0.2,0.5);
    sigma ~ exponential(0.2);
    price ~ normal(mu,sigma);
}

generated quantities {
    real prices[N];
    real log_lik[N];
    for (i in 1:N) {
        prices[i] = normal_rng(mu[i],sigma);
        log_lik[i] = normal_lpdf(price[i] | mu[i], sigma);
    }
}
```

In [116…

```python
model2_fit=CmdStanModel(stan_file='price_fit_2.stan')
model2_fit.compile(force=False)
d_short=data.head(1000)
X=X.head(1000)
data_fit2=dict(N=len(d_short),
               K=2,
               X=X,
               car_age=d_short.s_car_age.values,
               price=d_short.s_price.values)

fit2 = model2_fit.sample(data=data_fit2,seed=28052020)
alpha_fit = fit2.stan_variable('alpha')
beta_mileage_fit = fit2.stan_variable('beta_mileage')
beta_car_age_fit = fit2.stan_variable('beta_car_age')


mu_fit_2 = fit2.stan_variable('mu')
price_pred_2 = fit2.stan_variable('prices')
log_lik = fit2.stan_variable('log_lik')
az.summary(fit2,var_names=['alpha','beta_mileage','beta_car_age','sigma'],round_to=2,kind='stats')
```

```
INFO:cmdstanpy:compiling stan file /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.stan to exe file /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2
INFO:cmdstanpy:compiled model executable: /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2
WARNING:cmdstanpy:Stan compiler has produced 3 warnings:
WARNING:cmdstanpy:
--- Translating Stan model to C++ code ---
bin/stanc  --o=/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.hpp /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.stan
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.stan', line 6, column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.stan', line 29, column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc
Warning in '/Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.stan', line 30, column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated and
    will be removed in Stan 2.32.0. Instead use the array keyword before the
    type. This can be changed automatically using the auto-format flag to
    stanc

--- Compiling, linking C++ code ---
clang++ -std=c++1y -Wno-unknown-warning-option -Wno-tautological-compare -Wno-sign-compare -D_REENTRANT -Wno-ignored-attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I stan/lib/stan_math/lib/boost_1.75.0 -I stan/lib/stan_math/lib/sundials_6.0.0/include -I stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -DBOOST_DISABLE_ASSERTS          -c -include-pch stan/src/stan/model/model_header.hpp.gch -x c++ -o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.hpp
clang++ -std=c++1y -Wno-unknown-warning-option -Wno-tautological-compare -Wno-sign-compare -D_REENTRANT -Wno-ignored-attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I stan/lib/stan_math/lib/boost_1.75.0 -I stan/lib/stan_math/lib/sundials_6.0.0/include -I stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -DBOOST_DISABLE_ASSERTS               -Wl,-L,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb" -Wl,-rpath,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb"  /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.o src/cmdstan/main.o        -Wl,-L,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb" -Wl,-rpath,"/Users/kamilbaradziej/.cmdstan/cmdstan-2.29.1/stan/lib/stan_math/lib/tbb"   stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_nvecserial.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_cvodes.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_idas.a stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_kinsol.a  stan/lib/stan_math/lib/tbb/libtbb.dylib stan/lib/stan_math/lib/tbb/libtbbmalloc.dylib stan/lib/stan_math/lib/tbb/libtbbmalloc_proxy.dylib -o /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2
rm -f /Users/kamilbaradziej/PythonProjects/ModellingofCarPrices/price_fit_2.o

INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |                                                  | 00:00 Status
chain 2 |                                                  | 00:00 Status

chain 3 |                                                  | 00:00 Status


chain 4 |                                                  | 00:00 Status

chain 3 |                    | 00:00 Iteration:    1 / 2000 [  0%]  (Warmup)


chain 1 |                    | 00:00 Iteration:  100 / 2000 [  5%]  (Warmup)
chain 2 |                    | 00:00 Iteration:  100 / 2000 [  5%]  (Warmup)

chain 1 |                    | 00:00 Iteration:  300 / 2000 [ 15%]  (Warmup)
chain 2 |                    | 00:00 Iteration:  300 / 2000 [ 15%]  (Warmup)


chain 4 |                    | 00:00 Iteration:  300 / 2000 [ 15%]  (Warmup)

chain 3 |                    | 00:00 Iteration:  400 / 2000 [ 20%]  (Warmup)
chain 2 |                    | 00:00 Iteration:  500 / 2000 [ 25%]  (Warmup)


chain 1 |                    | 00:00 Iteration:  500 / 2000 [ 25%]  (Warmup)

chain 3 |                    | 00:00 Iteration:  600 / 2000 [ 30%]  (Warmup)
chain 2 |                    | 00:00 Iteration:  700 / 2000 [ 35%]  (Warmup)


chain 1 |                    | 00:00 Iteration:  800 / 2000 [ 40%]  (Warmup)


chain 4 |                    | 00:00 Iteration:  900 / 2000 [ 45%]  (Warmup)

chain 3 |                    | 00:00 Iteration:  900 / 2000 [ 45%]  (Warmup)
chain 1 |                  | 00:00 Iteration: 1001 / 2000 [ 50%]  (Sampling)


chain 4 |                  | 00:00 Iteration: 1001 / 2000 [ 50%]  (Sampling)

chain 3 |                  | 00:00 Iteration: 1001 / 2000 [ 50%]  (Sampling)
chain 1 |                  | 00:01 Iteration: 1200 / 2000 [ 60%]  (Sampling)

chain 3 |                  | 00:01 Iteration: 1200 / 2000 [ 60%]  (Sampling)


chain 4 |                  | 00:01 Iteration: 1200 / 2000 [ 60%]  (Sampling)
chain 1 |                  | 00:01 Iteration: 1400 / 2000 [ 70%]  (Sampling)

chain 3 |                  | 00:01 Iteration: 1400 / 2000 [ 70%]  (Sampling)


chain 4 |                  | 00:01 Iteration: 1400 / 2000 [ 70%]  (Sampling)
chain 1 |                  | 00:01 Iteration: 1600 / 2000 [ 80%]  (Sampling)

chain 3 |                  | 00:01 Iteration: 1600 / 2000 [ 80%]  (Sampling)


chain 4 |                  | 00:01 Iteration: 1600 / 2000 [ 80%]  (Sampling)
chain 1 |                  | 00:02 Iteration: 1700 / 2000 [ 85%]  (Sampling)

chain 3 |                  | 00:02 Iteration: 1700 / 2000 [ 85%]  (Sampling)


chain 4 |                  | 00:02 Iteration: 1700 / 2000 [ 85%]  (Sampling)
chain 1 |                  | 00:02 Iteration: 1800 / 2000 [ 90%]  (Sampling)

chain 3 |                  | 00:02 Iteration: 1800 / 2000 [ 90%]  (Sampling)


chain 4 |                  | 00:02 Iteration: 1800 / 2000 [ 90%]  (Sampling)
chain 1 |                  | 00:02 Iteration: 1900 / 2000 [ 95%]  (Sampling)

chain 3 |                  | 00:02 Iteration: 1900 / 2000 [ 95%]  (Sampling)


chain 1 |                                  | 00:02 Sampling completed
chain 2 |                                  | 00:02 Sampling completed
chain 3 |                                  | 00:02 Sampling completed
chain 4 |                                  | 00:02 Sampling completed
```

```
INFO:cmdstanpy:CmdStan done processing.
```

Out[116…

	mean	sd	hdi_3%	hdi_97%
alpha	-0.28	0.02	-0.31	-0.25
beta_mileage[0]	-0.26	0.02	-0.29	-0.23
beta_mileage[1]	0.13	0.01	0.11	0.15
beta_car_age	0.43	0.01	0.40	0.46
sigma	0.37	0.01	0.36	0.39

Values are standarized so we can see that mileage variables and car age is similar important by looking at mean for these betas in summary above.

In [117…

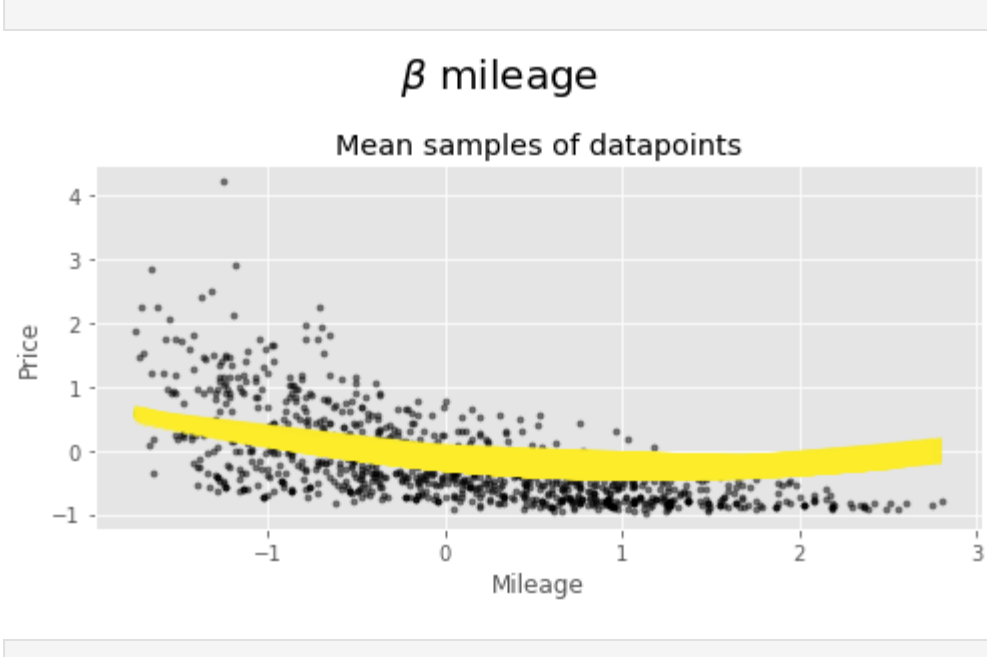
```python
log_lik
```

Out[117…

```
array([[ 6.28153e-02, -3.53514e-01, -3.81695e-02, ...,  5.50041e-03,
        -1.37115e-01,  5.73437e-02],
       [ 7.21114e-02, -3.66336e-01, -3.02584e-02, ...,  9.04882e-03,
        -8.70202e-02,  6.43394e-02],
       [ 7.17877e-02, -3.85074e-01, -3.17052e-02, ...,  4.67290e-07,
        -8.67940e-02,  6.56272e-02],
       ...,
       [ 6.38371e-02, -3.88691e-01, -4.91255e-02, ..., -4.39679e-03,
        -1.49565e-01,  5.74659e-02],
       [ 3.12068e-02, -3.39759e-01, -4.89232e-02, ..., -1.42160e-02,
        -1.51897e-01,  1.59053e-02],
       [ 1.15026e-01, -3.31925e-01,  2.59448e-02, ...,  5.48564e-02,
        -3.69794e-05,  9.96370e-02]])
```

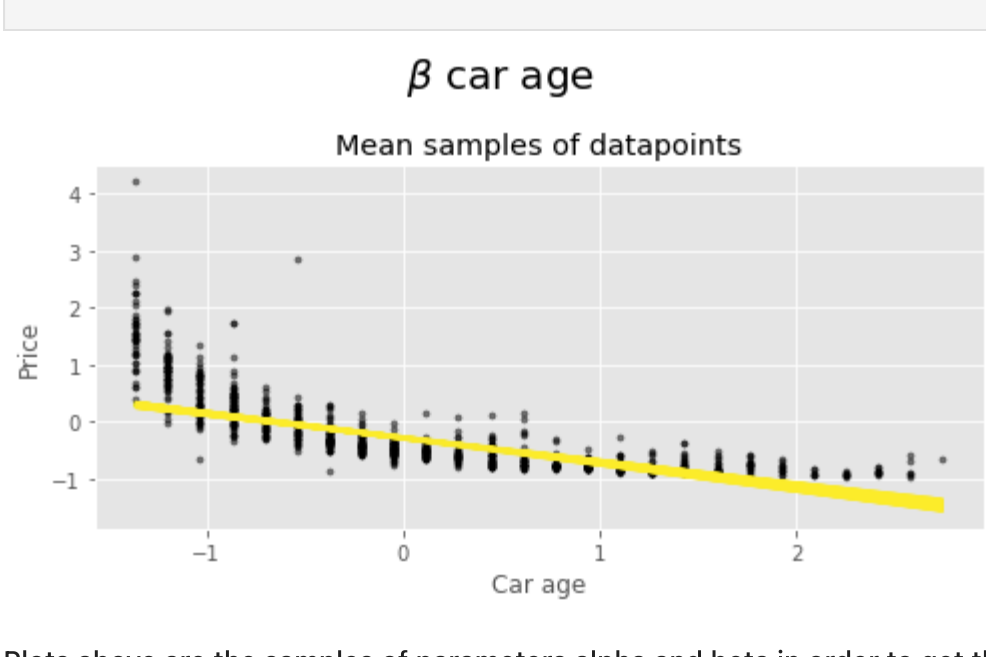
In [118…

```python
fig, axes = plt.subplots(1,1, figsize=(7, 4),sharey=True,sharex=True)
for i in range(459):
    axes.plot(d_short.s_mileage,
              alpha_fit[i]+beta_mileage_fit[i][0]*d_short.s_mileage+beta_mileage_fit[i][1]*d_short.s_mileage2,
              color=MID,
              alpha=0.5,
              linewidth=0.5)
axes.scatter(d_short.s_mileage,d_short.s_price,color='black',alpha=0.5,s=10)
axes.set_title('Mean samples of datapoints'.format(len(d_short)))
fig.suptitle(r'$\beta$ mileage', fontsize=20)
axes.set_ylabel('Price')
axes.set_xlabel('Mileage')
fig.tight_layout()
plt.show()
```



In [119…

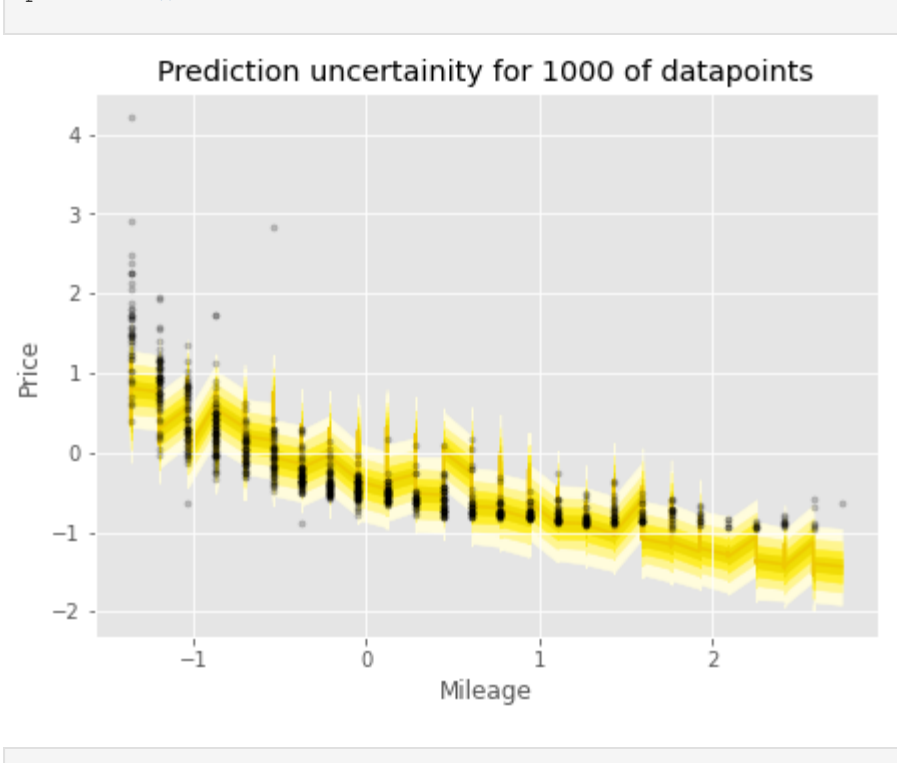
```python
fig, axes = plt.subplots(1,1, figsize=(7, 4),sharey=True,sharex=True)
for i in range(459):
    axes.plot(d_short.s_car_age,
              alpha_fit[i]-beta_car_age_fit[i]*d_short.s_car_age,
              color=MID,
              alpha=0.5,
              linewidth=0.5)
axes.scatter(d_short.s_car_age,d_short.s_price,color='black',alpha=0.5,s=10)
axes.set_title('Mean samples of datapoints'.format(len(d_short)))
fig.suptitle(r'$\beta$ car age', fontsize=20)
axes.set_ylabel('Price')
axes.set_xlabel('Car age')
fig.tight_layout()
plt.show()
```



Plots above are the samples of parameters alpha and beta in order to get the standarized mean and we can see how this mean relationship gives our model thinks this is the underlying phenomena adn we can see that these samples are generally really consistent in that two cases.

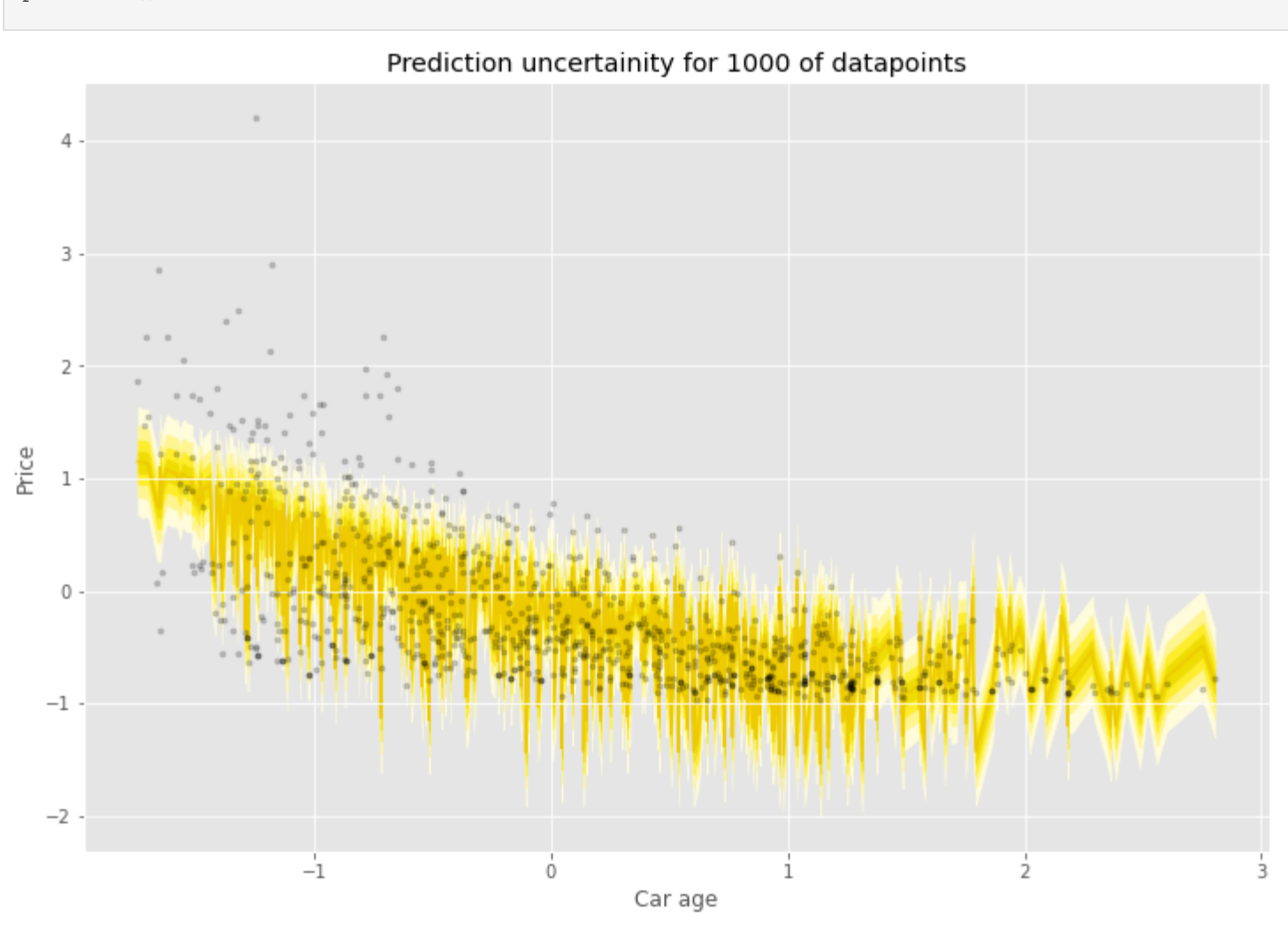
In [120…

```python
fig, axes = plt.subplots(1, 1, figsize=(7, 5))
axes = ribbon_plot(d_short.s_car_age.values, price_pred_2, axes, supress_warning=True)
axes.scatter(d_short.s_car_age, d_short.s_price, color='black', alpha=0.2,s=10)
axes.set_xlabel('Mileage')
axes.set_ylabel('Price')
axes.set_title('Prediction uncertainity for {} of datapoints'.format(len(d_short)))
plt.show()
```



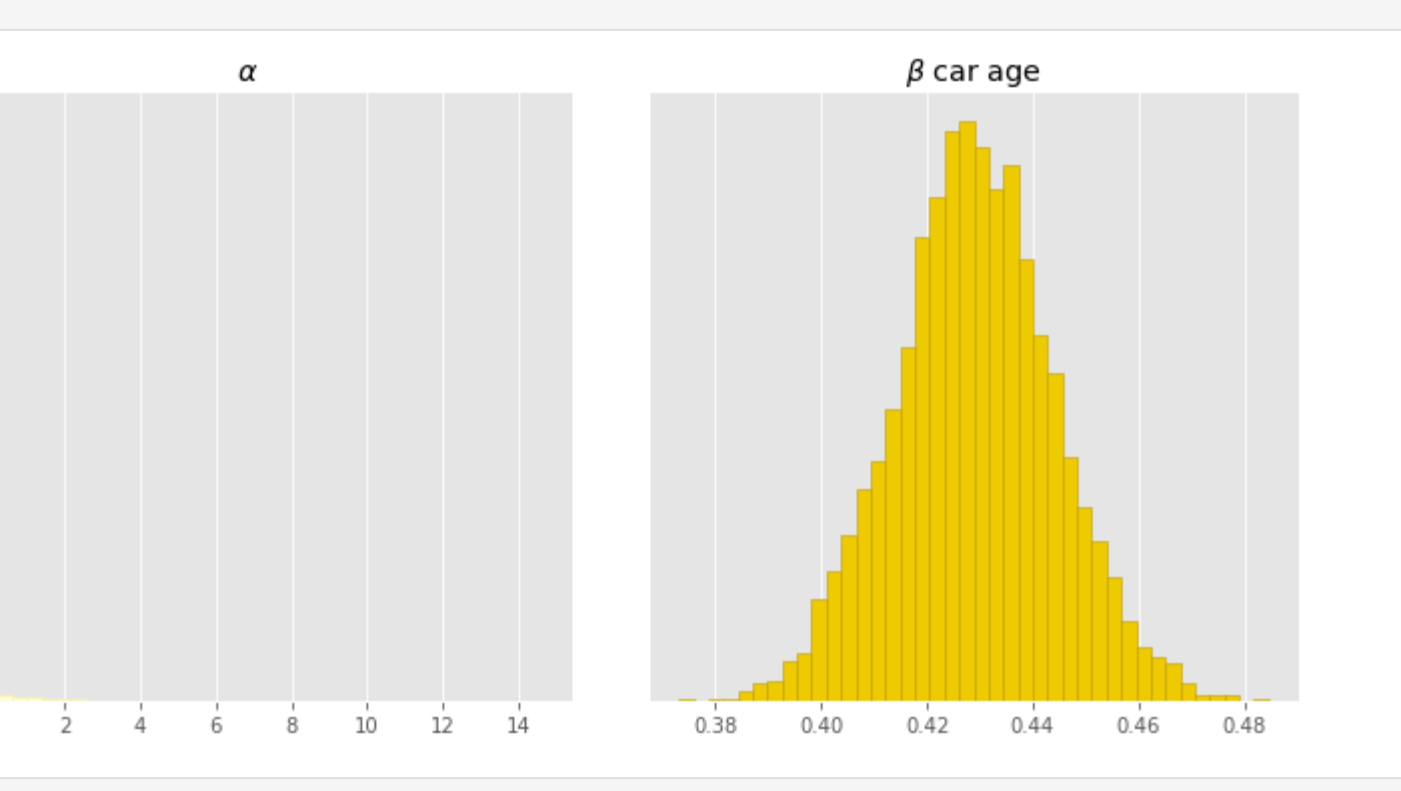
In [121…

```python
fig, axes = plt.subplots(1, 1, figsize=(12, 8))
axes = ribbon_plot(d_short.s_mileage.values, price_pred_2, axes, supress_warning=True)
axes.scatter(d_short.s_mileage, d_short.s_price, color='black', alpha=0.2,s=10)
axes.set_xlabel('Car age')
axes.set_ylabel('Price')
axes.set_title('Prediction uncertainity for {} of datapoints'.format(len(d_short)))
plt.show()
```

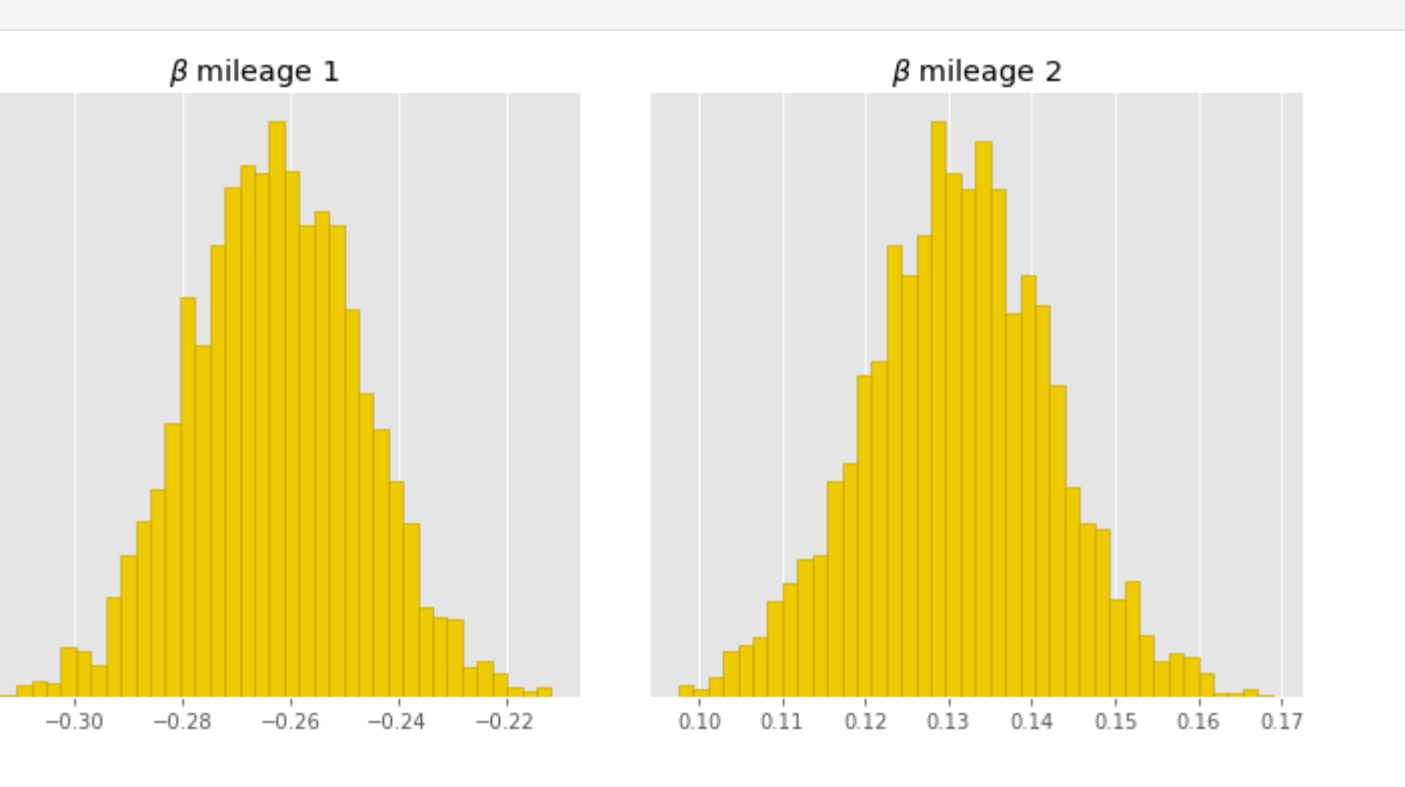


The plots of prediction uncertainity shows that compute quantiles are very well concentrated. The higher order predictors here get even better representation of our data

In [122…
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(10, 5))
axes[0].hist(alpha_fit,bins=40,color=DARK,edgecolor=DARK_HIGHLIGHT,density=True)
axes[0].hist(data.s_price,bins=40,color=LIGHT,edgecolor=LIGHT_HIGHLIGHT,density=True)
axes[0].set_title(r'$\alpha$')
axes[0].set_yticks(())

fig.tight_layout()
axes[1].hist(beta_car_age_fit,bins=40,color=DARK,edgecolor=DARK_HIGHLIGHT,density=True)
axes[1].set_title(r'$\beta$ car age')
axes[1].set_yticks(())
plt.show()
```



In [123…
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(10, 5))
axes[0].hist(beta_mileage_fit[:,0],bins=40,color=DARK,edgecolor=DARK_HIGHLIGHT,density=True)
axes[0].set_title(r'$\beta$ mileage 1')
axes[0].set_yticks(())

fig.tight_layout()
axes[1].hist(beta_mileage_fit[:,1],bins=40,color=DARK,edgecolor=DARK_HIGHLIGHT,density=True)
axes[1].set_title(r'$\beta$ mileage 2')
axes[1].set_yticks(())
plt.show()
```



Comparing models

In [124…
```
# Comparing 2 models: model_1 - with mielage only, model_2 - with mielage and car age
idata1 = az.from_cmdstanpy(posterior = fit1, log_likelihood = "log_lik")
idata2 = az.from_cmdstanpy(posterior = fit2, log_likelihood = "log_lik")
compare_dict = {"model_1": idata1, "model_2": idata2}
az.compare(compare_dict, ic = "loo") # loo stands for leave one out cross validation
```

Out[124…

	rank	loo	p_loo	d_loo	weight	se	dse	warning	loo_scale
model_2	0	-432.320566	10.518179	0.000000	0.999509	56.588405	0.000000	False	log
model_1	1	-767.656092	5.598634	335.335527	0.000491	40.058480	23.539642	False	log

In [125…
```
# Comparing 2 models: model_1 - with mielage only, model_2 - with mielage and car age
idata1 = az.from_cmdstanpy(posterior = fit1, log_likelihood = "log_lik")
idata2 = az.from_cmdstanpy(posterior = fit2, log_likelihood = "log_lik")
compare_dict = {"model_1": idata1, "model_2": idata2}
az.compare(compare_dict, ic = "waic") # loo stands for leave one out cross validation
```

```
/usr/local/anaconda3/envs/DataAnalytics/lib/python3.8/site-packages/arviz/stats/stats.py:1458: UserWarning: For
one or more samples the posterior variance of the log predictive densities exceeds 0.4. This could be indicatio
n of WAIC starting to fail.
See http://arxiv.org/abs/1507.04544 for details
  warnings.warn(
/usr/local/anaconda3/envs/DataAnalytics/lib/python3.8/site-packages/arviz/stats/stats.py:1458: UserWarning: For
one or more samples the posterior variance of the log predictive densities exceeds 0.4. This could be indicatio
n of WAIC starting to fail.
See http://arxiv.org/abs/1507.04544 for details
  warnings.warn(
```

Out[125…

	rank	waic	p_waic	d_waic	weight	se	dse	warning	waic_scale
model_2	0	-432.339402	10.537014	0.00000	0.999509	56.605168	0.000000	True	log
model_1	1	-767.694951	5.637493	335.35555	0.000491	40.082278	23.531161	True	log

In [126…
```
az.plot_compare(az.compare(compare_dict))
```

Out[126…
```
&lt;AxesSubplot:xlabel='Log'&gt;
```