Game Theory: Lecture #1

Outline:

- Sociotechnical systems
- Social models
- Game theory
- Course outline

Sociotechnical Systems

- Engineering goal: Optimize performance subject to constraints

common paradigm
"design specification" $\longrightarrow$ "performance"

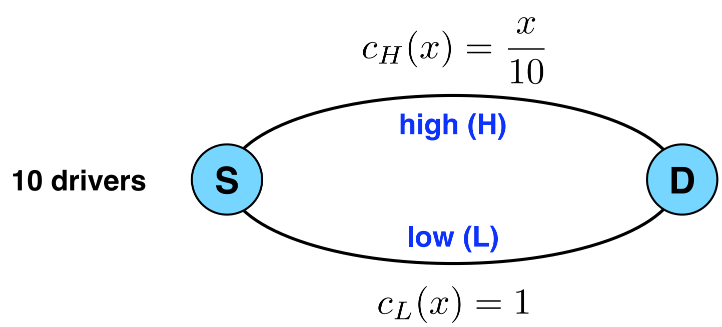
- Considerations:
  - Cost?
  - Weight?
  - Efficiency?
  - Green/environmental?
- Emerging systems: Integration of social and engineering components

emerging paradigm
"design specification" $\longrightarrow$ "social behavior" $\longrightarrow$ "performance"

- Challenges:
  - Societal behavior is a main driving factor of performance.
  - Optimal design **must** account for societal behavior
- Examples:
  - Transportation networks
  - Smart grid
  - College admissions?
- Question: How do you model/influence societal behavior?
- **Game Theory:** Set of tools for modeling/predicting social behavior

Transportation Networks

- Engineering: Develop infrastructure necessary to meet societal demands
- Example: Simplistic transportation network model



- Components:
  - Society: 10 drivers seeking to traverse from S to D over shared network
  - Routes: High (H) or Low (L) path
  - Congestion: $c_H(x)$ denotes congestion on H if x drivers use H
    - Ex: $c_H(5) = 1/2$, $c_H(10) = 1$, $c_L(5) = 1$, $c_L(10) = 1$
- Questions: What is optimal routing decision that minimizes average congestion?
- Curiosity: Will drivers efficiently utilize a transportation network?
- System cost: If x_H user take High road, total congestion is

$$C(x_H, x_L) = x_H \cdot C_H(x_H) + x_L \cdot C_L(x_L) = x_H \cdot \left(\frac{x_H}{10}\right) + (10 - x_H) \cdot (1)$$

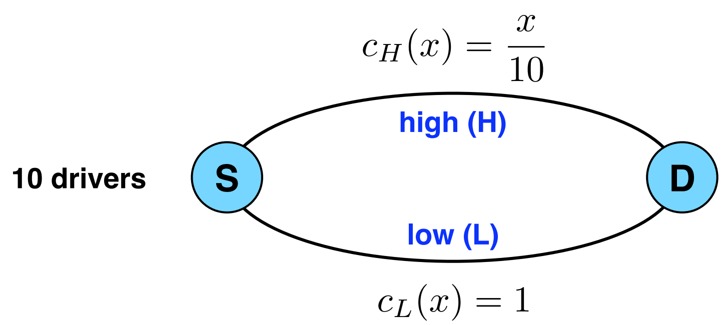
  Optimizing over $x_H \in \{0, 1, \cdots, 10\}$ gives us $x_H^* = 5$ and the total congestion is

$$C(x_H^* = 5, x_L^* = 5) = 5 \cdot \left(\frac{1}{2}\right) + 5 \cdot (1) = 7.5$$

- Summary: A total congestion of 7.5 is the *best-case* scenario.

Transportation Networks (2)

- Question: What is a reasonable prediction of social behavior?



- Fact: Social behavior emerges as a result of individual drivers' decisions
- Reasonable driver model: Each driver minimizes their own experienced congestion
- Recall optimal allocation: 5 drivers H, 5 driver L
  - Congestion of users on H route?
  - Congestion of users on L route?
- Question: Is $n_H^* = 5$ and $n_L^* = 5$ a reasonable prediction of social behavior? **No!**
- Question: What is a reasonable prediction of social behavior?
- Answer: $n_H = 10$, $n_L = 0$

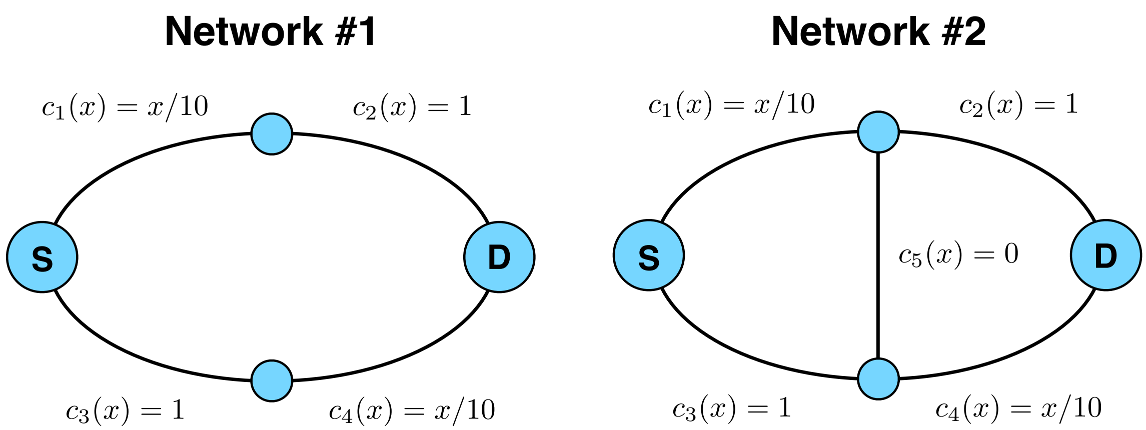
$$C(n_H = 10, n_L = 0) = 10 > C(n_H^* = 5, n_L^* = 5) = 7.5$$

Why?

- Take away: Social behavior far worse than optimal system behavior.

Transportation Networks (3)

- Engineering goal: Augment network to improve resulting behavior
- Challenge: Must account for social dynamics in design process (often overlooked)
- Example:



- Components:
  - 10 drivers seeking to traverse from S to D over each shared network
  - Network #1: Two paths, $P_1 = \{1, 2\}$, $P_2 = \{3, 4\}$
  - Network #2: Four paths, $P_1 = \{1, 2\}$, $P_2 = \{3, 4\}$, $P_3 = \{1, 5, 4\}$, $P_4 = \{3, 5, 2\}$
  - Congestion additive for drivers, i.e., $c_{P_1}(\cdot) = c_1(\cdot) + c_2(\cdot)$
- Intuition: Quality of emergent behavior Network #2 should be better than Network #1
- What is reasonable prediction of social behavior for each network?
  - **Network #1:** $n_{P_1} = 5$, $n_{P_2} = 5$

$$C(n_{P_1} = 5, n_{P_2} = 5) = 5 \cdot (1 + 1/2) + 5 \cdot (1 + 1/2) = 15.$$

  - **Network #2:** $n_{P_3} = 10$, $n_{P_1} = 0$, $n_{P_2} = 0$, $n_{P_4} = 0$

$$C(n_{P_1} = 0, n_{P_2} = 0, n_{P_3} = 10, n_{P_4} = 0) = 10 \cdot (1 + 1) = 20.$$

- Conclusion: "Better" network resulted in worse performance???

Sociotechnical Systems

emerging paradigm

"design specification" $\longrightarrow$ "social behavior" $\longrightarrow$ "performance"

- Challenges:
  - Optimal design **must** account for social behavior
  - Predicting social behavior non-trivial (and often not optimal)
  - Emergent social behavior often non-intuitive
- Further challenges: "Information" exchange often part of underlying system
- Example: Electrical vehicle charging stations
  - Facility: 5 independent charging stations
  - Customers: 10 users visit facility and seek to utilize charging stations
  - Private information: Different energy requests and time constraints

  Which customers should be able to utilize charging stations? schedule?
- Central problem: Decision-making entity does not have access to users' private information
- Possible resolution:
  - Ask users to report their private information
  - Implement mechanism that derives schedule given information received
- If users give correct information, then the resulting schedule should be desireable
- Concerns:
  - Will users provide truthful information?
  - Can users manipulate mechanism by conveying false information?
  - How does the underlying mechanism impact performance?

Information Based Systems

- **Example:** College admissions
  - Colleges ask for information regarding applicants
  - Every applicant states that UCSB is their top choice
  - Only 25% actually accept their admissions
  - UCSB ECE: Adding 50-75 graduate students per year (admit around 200-250)
- **Example:** Medical residency programs (limited open spots, must be filled)
- **Example:** Lottery-based system for school assignment
  (Mechanism used by Boulder Valley School District)
- Setup:
  - Schools: $\{1, \ldots, m\}$.
  - Number open spots: $\{n_1, \ldots, n_m\}$, $n_i \geq 0$
  - Students: $\{s_1, \ldots, s_n\}$
  - School ranking for each applicant s: $\{q_1^s, \ldots, q_m^s\}$
  - Interpretation: $q_i^s > q_j^s$ means school i is preferred to school j by student s
- Goal: Assign students to schools to maximize social benefit (happiness)
- **Implemented mechanism:** Students report their top three school choices
  - *Round #1:* Randomly pick each student. Assign to top choice if available.
  - *Round #2:* Randomly pick each student not assigned in Round #1. Assign to second choice if available.
  - *Round #3:* Randomly pick each student not assigned in Round #1 or #2. Assign to third choice if available.
  - If not assigned, then student assigned to home school.
- Q: Will students report truthfully?
- Q: How do you ensure desirable behavior if users will not provide accurate information?

Course Outline

emerging paradigm

"design specification" $\longrightarrow$ "social behavior" $\longrightarrow$ "performance"

- Fact: Engineers must successfully understand social behavior to meet the design objectives
- Game theory = Study of *strategic* social behavior, applicable to many various systems
- Detailed (tentative) course outline available on the main syllabus; main topics will be
  - Social Choice (voting)
  - Matching
  - Games, equilibrium, and special types of games
  - Designing games for efficiency
  - Cost sharing
  - Mechanism design
  - Learning in games
  - Game theory for purely engineered systems
- Note: To fully understand social behavior, we will often remove the engineering component. (due to time limitations)