CS 4730/5730 Algorithmic Game Theory

Homework #2

Due: Thursday, February 21, 2019

If written: due in class

If typed and submitted on Canvas: 11:59 pm

Submission requirements:

5% extra credit if you submit *digital, typed* writeups in PDF format to the "Homework 2 Writeup" assignment on the Canvas site. However, you may also turn in handwritten assignments in class – but assignments submitted in person (or handwritten, scanned, and submitted on Canvas) will not receive the 5% extra credit.

Assignment:

1. Consider the Roommate Problem discussed in lecture where we have the following two sets of preferences

	A	B	C	D
A	-	3	1	2
B	2	-	1	3
C	2	3	-	1
D	1	3	2	-

Roommates' Preferences

	A	B	C	D
A	-	1	3	2
B	2	-	1	3
C	2	1	-	3
D	2	3	1	-

Roommates' Preferences

Is there a stable division into pairs for either of the preference structures above? If so, provide an example. If not, show that no stable matching system exists by checking all the possibilities.

2. Consider the Marriage Problem discussed in lecture with the following preference structure

	Ann	Beth	Cher	Dot
Al	3	3	2	3
Bob	4	1	3	2
Cal	2	4	4	1
Dan	1	2	1	4

Women's Preferences

	Ann	Beth	Cher	Dot
Al	1	2	3	4
Bob	1	4	3	2
Cal	2	1	3	4
Dan	4	2	3	1

Men's Preferences

(a) What is the resulting proposal associated with the Gale-Shapley algorithm with the men proposing? Is this proposal stable? Verify.

(b) What is the resulting proposal associated with the Gale-Shapley algorithm with the women proposing? Is this proposal stable? Verify.

3. Is there a unique stable matching for the following preference structure? For convention, let the men $= \{a, b, c, d, e\}$ and the women $= \{A, B, C, D, E\}$

	A	B	C	D	E
a	5	4	5	4	5
b	4	5	4	5	3
c	1	1	3	3	1
d	2	3	1	1	2
e	3	2	2	2	4

Women's Preferences

	A	B	C	D	E
a	1	5	2	4	3
b	4	1	2	5	3
c	5	1	2	3	4
d	1	2	3	5	4
e	2	4	1	3	5

Men's Preferences

4. In class, we discussed how to deal with a situation where the number of men is not equal to the number of women. To handle this, simply fill out the short side of the table with "dummy" participants who are ranked last in the other side's preferences. If a person matches with a "dummy," this indicates that the person is does not have a mate. Consider the following preference structure

	A	B	C	D	E
a	1	1	2	3	3
b	2	3	1	1	2
c	3	2	3	2	1

Women's Preferences

	A	B	C	D	E
a	2	1	3	4	5
b	3	1	2	5	4
c	3	1	4	2	5

Men's Preferences

(a) Find a stable matching system using the Gale-Shapley algorithm when the women propose.

(b) Which women do not have a mate? Will these women remain without a mate in every other stable matching? Support your answer.