CS 4730/5730 Algorithmic Game Theory

Homework #1

Due: Thursday, February 14, 2019

If written: due in class

If typed and submitted on Canvas: 11:59 pm

Submission requirements:

5% extra credit if you submit *digital, typed* writeups in PDF format to the "Homework 1 Writeup" assignment on the Canvas site. However, you may also turn in handwritten assignments in class – but assignments submitted in person (or handwritten, scanned, and submitted on Canvas) will not receive the 5% extra credit.

Assignment:

We begin by reviewing 3 different voting rules and investigate their various benefits and drawbacks. First, we can revisit the example "Voter's Paradox" from class: There are 3 candidates: Health (H), Security (S), and Education (E) and 12 voters. The voters' preferences are follows:

	3 voters	4 voters	5 voters
1st Preference	H	E	S
2nd Preference	S	H	E
3rd Preference	E	S	H

3 voters think Health is better than Security and that Security is better than Education; 4 voters think Education is better than Health, and Health better than Security, and so forth.

1. **Plurality voting:** The candidate with the most 1st-place rankings wins; every voter's 2nd and 3rd choice is ignored. In the above example, Plurality voting would give Security the win, since Security has 5 first-place rankings, but Health and Education have only 3 and 4, respectively. (*Note: the plurality method is used in most elections in the United States.*)

2. **Instant Runoff:** First, add up each candidate's 1st-place rankings. If any candidate has more than 50% of the vote based on that count, that candidate wins. If no candidate has more than 50% of the based on 1st-place rankings, then we eliminate the lowest-ranked candidate and "virtually" re-run the election. In the above example, no candidate has over 50% of the votes, but Health had the fewest 1st-place rankings, so Health is eliminated. With Health gone, the rankings look like this:

	3 voters	4 voters	5 voters
1st Preference	S	E	S
2nd Preference	E	S	E

Now Security has 8 votes, which is more than 50%, so Security wins. (*Note: the Instant Runoff method is used in many elections in Australia and various other places in the world.*)

3. **Copeland's Method (simplified):** The winner is the candidate who wins the most hypothetical 2-person matchups with other candidates. (This method does not always produce a winner!) In the above example, we need to count how many "times" Security beats Health, how many times Health beats Education, how many times Education beats Security, and so on. First, just looking at Security and Health, we have

	3 voters	4 voters	5 voters
1st Preference	H		S
2nd Preference	S	H	
3rd Preference		S	H

Health beats Security 7 times, Security beats Health 5 times. Continuing with the other pairings (without re-drawing the table each time), we have that Health beats Education 3 times, Education beats Health 9 times; and Security beats Education 8 times, while Education beats Security 4 times. Health has a total of 10 wins, and Security and Education each have 13 wins, so in this case Copeland's method failed to choose a winner because Security and Education tied.

1. Consider the following fictional situation, inspired by 2016's U.S. Republican Presidential Primaries: There are 3 candidates: (A), (B), and (C) and 9 voters. The voters' preferences are follows:

	4 voters	3 voters	2 voters
1st Preference	A	C	B
2nd Preference	B	B	C
3rd Preference	C	A	A

4 voters think (A) is better than (B) and that (B) is better than (C); 3 voters think (C) is better than (B), and (B) is better than (A), and so forth.

(a) Which candidate wins under Plurality Voting?

(b) Which candidate wins under Instant Runoff?

(c) Does Copeland's Method choose a winner in this case? If so, which candidate?

(d) What are the benefits of Instant Runoff over Plurality?

(e) Copeland's Method is known as a *Condorcet Method*, which means that if it chooses a single winner, that winner would beat *every* other candidate in a head-to-head race. Verify that this is true with the (A)-(B)-(C) example.

(f) *(bonus):* In the Security-Education-Health example, Copeland's Method gave a tie. What would be a good tie-breaking method for that example?

(g) *(bonus):* Several other voting methods are listed at `https://en.wikipedia.org/wiki/Condorcet_method#Single-method_systems`. Choose one and perform it for the Security-Education-Health example.

2. A social choice function for a given preference profile is:

$$f\left(\begin{bmatrix} x & z & x \\ y & x & y \\ z & y & z \end{bmatrix}\right) = \begin{bmatrix} y \\ x \\ z \end{bmatrix}$$

   Go through each of the five axioms. Which axioms are not satisfied here?

3. Suppose a given social choice function satisfies Axioms #1-4. Further, suppose that the social choice function for a given preference profile is:

$$f\left(\begin{bmatrix} x & y & y \\ t & t & x \\ y & x & t \end{bmatrix}\right) = \begin{bmatrix} t \\ y \\ x \end{bmatrix}$$

   Can the social choice for the following preference profile be determined? If so, what is it?

$$f\left(\begin{bmatrix} x & t & y \\ t & y & x \\ y & x & t \end{bmatrix}\right) = ?$$

4. What are the decisive sets (with accompanying pairs) in the first preference profile in Question #3? What is the minimal decisive set? Assume the first column represents the preferences of individual 1, second column represents the preferences of individual 2, etc.

5. (a) Given the following preference profiles, can the social decision be determined when the guiding rule is to decide by a pairwise majority vote?

1	**2**	**3**
x	t	z
y	x	t
z	y	x
t	z	y

   (b) Now suppose that the social choice function satisfies Axioms #1-4. If it is also known that the social decision establishes a preference for y over z, is this information sufficient to predict the social decision?

   (c) Given the following preference profile and the information in (b), can the social decision be determined?

1	**2**	**3**
x	z	z
y	x	t
z	y	x
t	t	y