CS 4730 Algorithmic Game Theory

Homework #8

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Answers to homework problems:

1.

(a) Society: 40 for X, 30 for Y, 35 for Z
The utilitarian choice is X

(b) The VCG mechanism can be used for this problem to incentivize truthful bidding. It makes it so that, given everyone's else bids, an individual can't be any better off by lying. The exception is if the individuals collude together. Payments are calculated by seeing what it would be like if each individual was not part of the bidding.

If x isn't there:

	X	Y	Z
y	10	40	0
z	0	-10	50
Society	10	30	50

The outcome would be Z.
x owes: 0 + 50 = 50
x discount: 10 + 0 = 10
x's payment: 50 - 10 = 40
x's utility: 30 - 40 = -10

If y isn't there:

	X	Y	Z
x	30	0	-15
z	0	-10	50
Society	30	-10	35

The outcome would be Z.
y owes: -15 + 50 = 35
y discount: 30 + 0 = 30
y's payment: 35 - 30 = 5
y's utility: 10 - 5 = 5

If z isn't there:

	X	Y	Z
X	30	0	-15
Y	10	40	0
Society	40	40	-15

The outcome could be either X or Y.
If X is chosen, z pays nothing and has a utility of 0.
If Y is chosen, z pays nothing and has a utility of -10.

(c)

For x:
If this was a non-dominant strategy, x would be able to change his numbers to get a better utility than -10.

1. Outcome X: Since his payment isn't based on his own bids, he can't change his utility that was calculated above. Utility = -10
2. Outcome Y: x would owe 50, be discounted 30 and have a utility of -20.
3. Outcome Z: Since he wouldn't change the outcome, he would pay nothing, but have a utility of -15.

For y:
If this was a non-dominant strategy, y would be able to change his numbers to get a better utility than 5.

1. Outcome X: Since his payment isn't based on his own bids, he can't change his utility that was calculated above. Utility = 5
2. Outcome Y: y would pay 50, be discounted -10 and have a utility of -20.
3. Outcome Z: Since he wouldn't change the outcome, he would pay nothing, but have a utility of 0.

For z:
If this was a non-dominant strategy, z would be able to change his numbers to get a better utility than 0.

1. Outcome X: Since his payment isn't based on his own bids, he can't change his utility that was calculated above. Utility = 0
2. Outcome Y: Since his payment isn't based on his own bids, he can't change his utility that was calculated above. Utility = -10
3. Outcome Z: z would pay 40, be discounted -15 and have a utility of -5.

Since the best utility for every player is outcome X, reporting truthfully is a dominant strategy for all players.

(d) Outcome X.
x would get the object, but y would get the most utility.

2.

(a) The utilitarian choice would be A.

(b) If y & z both bid 51 or more, the outcome is B and neither pay anything. This is because if you run the VCG mechanism without y or z when they both bid 51 or more, the outcome doesn't change so they wouldn't be charged anything.