

## CS 4730 Algorithmic Game Theory

## Homework #1

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**Answers to homework problems:**

1.

(a) A wins under Plurality Voting because it has the most first place votes.

(b) No candidate has more than 50% of the votes, so B is eliminated since it has the least first place votes. The next round looks like:

	4 voters	3 voters	2 voters
1 <sup>st</sup> Preference	A	C	C
2 <sup>nd</sup> Preference	C	A	A

So, C wins under Instant Runoff since it has more than 50% of the votes in the second round.

(c) B beats A 5 times. A beats B 4 times. C beats A 5 times. A beats C 4 times. B beats C 6 times. C beats B 3 time. So, B wins under Copeland's Method since A has 8 wins, B has 1 wins and C has 8 wins.

(d) The benefits of Instant Runoff over Plurality are: It's less likely that a candidate can syphon votes from another and help other candidates by doing so. It lessens strategic voting because of the former. It makes voters non-1<sup>st</sup> preference rankings relevant.

(e) B beats A (5 to 4) & B beats C (6 to 3).

(f) A good tie-breaking method would be to not allow head-to-head matchups that won by a greater margin to be overruled by matchups that won by a lesser margin. In the example, H beats S (7 to 5 with a margin of 2), E beats H (9 to 3 with a margin of 6), and S beats E (8 to 4 with a margin of 4). So, H beats S by a margin of 2 would be ignored and rather than tallying up the total wins, the social choice ranking would be determined by the remaining head-to-head matchups (E beats H and S beats E). This tie-breaking method would return a social choice ranking of:

1 <sup>st</sup> Preference	S
2 <sup>nd</sup> Preference	E
3 <sup>rd</sup> Preference	H

(g) Kemeny-Young Method:

	...over S	...over E	...over H
Prefer S...	-	8	5
Prefer E...	4	-	9
Prefer H..	7	3	-

S > E > H has a Kemeny Score of 22

	...over E	...over S	...over H
Prefer E...	-	4	9
Prefer S...	8	-	5
Prefer H..	3	7	-

E > S > H has a Kemeny Score of 18

The other 7 matrixes could be listed, but since it's obvious from the given preferences that H is last, they were omitted. This method gives a social choice ranking of:

1<sup>st</sup> Preference      S  
 2<sup>nd</sup> Preference      E  
 3<sup>rd</sup> Preference      H

2. The social choice function:

- Satisfies Axiom 1 because we gave it sets of ranked preferences (domain) and it gave us one set of ranked preferences (range).
- Is indeterminate for Axiom 2 because we don't have enough information to determine positive association with only one input and one output.
- Does not satisfy Axiom 3 because all voters prefer x to y, but the social choice ranks y over x.
- Is indeterminate for Axiom 4 because we don't have enough information to determine IIA with only one input and one output.
- Satisfies Axiom 5 because none of the input sets match the output set and therefore there isn't a dictator.

3. If Arrow's theorem holds, the social choice, preference profile and satisfied Axioms given doesn't make sense. Since Axiom's 1-4 are satisfied, according to Arrow's theorem Axiom 5 is not satisfied and there should be a dictator. Since none of the individual preferences match the social choice however, there is not a dictator. One way to fix this problem would be to change the social choice to individual 2's preferences. This would match Arrow's theorem and make the answer:

1<sup>st</sup> Preference    t  
 2<sup>nd</sup> Preference    y  
 3<sup>rd</sup> Preference    x

Another way to fix this problem would be to change which Axioms are satisfied. Axiom 1 probably isn't a good choice to change since even if other inputs don't give a legal output preference

ranking, it might be confusing and maybe impossible to answer. I believe Axiom 2 & Axiom 4 are necessary to predict most outputs, so they're probably not good choices. However, since neither preference profile has unanimity, Axiom 3 could easily be taken out. Then since the only change was individual 2 swapping t and y under Axiom 4 the answer would be:

1<sup>st</sup> Preference t  
 2<sup>nd</sup> Preference y  
 3<sup>rd</sup> Preference x

4. The decisive sets for the first preference profile are individuals 2 & 3 for (y, x) and individual 1 for (t, y) and individual 2 for (t, x). The minimal decisive set is all 3 individuals.

5.

- (a) No. x beats z. z beats t. t beats x. y beats z. t beats y. x beats y.
- (b) No, since Axiom's 1-4 are satisfied, there is a dictator, but since two individuals prefer y over z the dictator and the social decision cannot be determined.
- (c) Yes, individual 1 is the dictator.

1<sup>st</sup> Preference x  
 2<sup>nd</sup> Preference y  
 3<sup>rd</sup> Preference z  
 4<sup>th</sup> Preference t