

CS 4730 Algorithmic Game Theory

Homework #7

Student: Barak Barclay

Answers to homework problems:

1. For $0 \leq a \leq 1$, Nash flow = a
For $a > 1$, Nash flow = $1/a$
2. For $0 \leq a \leq 1$, total latency of a Nash flow = a
For $a > 1$, total latency of a Nash flow = 1
3. For $0 \leq a < 1$,
 $x_H^{\text{Opt}} = 1$
 $x_L^{\text{Opt}} = 0$

$$L = x_H * ax_H + x_L$$

$$L = ax_H^2 + 1 - x_H$$

$$d/dx(L) = 2ax_H - 1 = 0$$
For $a \geq 1$,
 $x_H^{\text{Opt}} = 1/2a$
 $x_L^{\text{Opt}} = 1 - 1/2a$
4. For $0 \leq a < 1$, $L = a$

$$L = 1/2a * a/2a + 1 - 1/2a$$
For $a \geq 1$, $L = 1 - 1/4a$
5. $\text{PoA}(G^{a \geq 0}) = 1 / (1 - 1/4a)$ where $a = 1$
 $\text{PoA}(G^{a \geq 0}) = 4/3$
6. $\text{PoA}(G^{a \geq 2}) = 1 / (1 - 1/4a)$ where $a = 2$
 $\text{PoA}(G^{a \geq 2}) = 8/7$