

CS-105 (Probability and Statistics)
S.C. & S.S., J.N.U
End-semester Exam - 13/5/2019 - Paper A

Time: 2 hours

Total Marks: 50

1. (a) If X is any continuous random variable, show that $\text{Var}(X) = E(X^2) - E(X)^2$. (b) Give an example, if it exists, of a random variable X for which $E(X^2) = E(X)^2$. (c) Calculate the mean and variance of the Binomial distribution with parameters n and p by using the fact that it is a sum of n independent Bernoulli random variables. (10 marks)
2. (a) State and prove the precise relation between the general normal distribution $N(\mu, \sigma)$ and the standard normal distribution $N(0, 1)$. (b) Given $X \sim N(21.5, 3)$, calculate the probability that X lies in the interval $[17, 26]$ using the information given below. (10 marks)
3. (a) Define the terms random sample, statistic, population mean, sample mean and sample variance. (b) Give a precise statement of the law of large numbers and the central limit theorem. (10 marks)
4. Assume that the height of a certain (numerically large) group of students follows an unknown distribution with mean 60 inches and standard deviation 5 inches. (a) For a random sample of 100 students, what is the probability that the sample mean is between 60 and 61 inches? (b) If 80 different random samples of 100 students each are selected, approximately how many of them will have sample mean less than 59.5 inches? (10 marks)
5. A sample poll of 32 voters chosen at random from all voters in a given constituency showed that a proportion of $\frac{2}{3}$ were in favour of a given candidate. (a) What is the 95% confidence interval for the actual proportion of voters in favour of the candidate? (b) Can you conclude with 99.73% confidence that the candidate will surely win? (10 marks)

$P(z < z_c)$	99.73%	99%	95.45%	95%	68.27%	50%
z_c	3.0	2.58	2.0	1.96	1.0	0.67