

1) Algorithm  $\text{alg}_1(L[0..n-1])$

$Q(1) \leftarrow \text{if } (n=1) \text{ return } L[0]$

$T(n-1) \leftarrow \text{temp} = \text{alg}_1(L[0..n-2])$

$Q(n) \leftarrow \text{if } (\text{temp} \leq L[n-1]) \text{ return temp}$

$Q(n) \leftarrow \text{else return } L[n-1]$

$$T(n) = T(n-1) + Q(1)$$

$$\hookrightarrow T(n-2) + Q(1)$$

$$\sum_{1}^{n-1} Q(1) = Q(n)$$

$$\boxed{T(n) = Q(n)}$$

b) Algorithm  $\text{alg}_2(X[l..r])$

$Q_1 \leftarrow \text{if } (l=r) \text{ return } X[l]$

$Q_1 \leftarrow \text{else } f\lfloor r = \text{floor}((l+r)/2)$

$T(n/2) \leftarrow \text{temp1} = \text{alg}_2(X[l..f\lfloor r])$

$T(n/2) \leftarrow \text{temp2} = \text{alg}_2(X[f\lfloor r+1..r])$

$Q_1 \leftarrow \text{if } (\text{temp1} \leq \text{temp2}) \text{ return temp1}$

$Q_1 \leftarrow \text{else return temp2}$

$$T(n) = 2T(n/2) + \underbrace{Q_1}_n$$

Master Theorem

$$a=2 \quad b=2 \quad k=0$$

$$a^{\frac{1}{k}} = 2^{\frac{1}{0}} > \frac{2}{b^{\frac{1}{k}}} = \Rightarrow T(n) = Q(n)$$

I would prefer the first algorithm first algorithm

uses less extra memory  
only +mp

2) def func(A[0,...n-1], x):

result = 0  
 $Q(n) \leftarrow \text{for } i \text{ in range}(0, n):$   
                result += (A[n-i-1] \* (x \*\* i))  
return result

$$T(n) = Q(n)$$

it is not possible to design an algorithm to have better complexity.

Because we have to see all elements.

3) def count\_str(str[0,...n-1], first, last)

Count = 0  
 $Q(n) \leftarrow \text{for } i \text{ in range}(0, \text{len}(\text{str})):$   
                if (str[i] == first):  
 $Q(n-i) \leftarrow \text{for } j \text{ in range}(i+1, \text{len}(\text{str})): \quad \begin{matrix} & \\ & \nearrow \end{matrix}$   
                    if (str[j] == last):  
                        Count += 1  
return Count.

$$T(n) = O(n^2)$$

4) def min\_distance(space[0,...n-1][0...k-1])

min = 999 \* maximum number

sum = 0

$Q(1) \leftarrow \text{for } i \text{ in range}(0, \text{len}(\text{space})): \quad \begin{matrix} & \\ & \nearrow \end{matrix}$

$Q(n-1) \leftarrow \text{for } j \text{ in range}(i+1, \text{len}(\text{space})): \quad \begin{matrix} & \\ & \nearrow \end{matrix}$

$Q(k) \leftarrow \text{for } k \text{ in range}(0, \text{len}(\text{space}[0])):$

$$\text{sum} += ((\text{space}[i][k] - \text{space}[j][k])^2)$$

$$\text{dist} = \sqrt{\text{sum}}$$

if dist < min : min = dist

$$\text{sum} = 0$$

return min

$$T(n) = O(kn^2)$$

5)

a) def most\_profitable (regions[0...n-1][0,1])

most = []

Max = 0

$\mathcal{O}(n)$  for i in range(0, len(regions)):

    sum = 0

$\mathcal{O}(n)$  ← for j in range(i, len(regions)):

            sum += regions[j][1]

            if sum > max:

                max = sum

                most = regions[i:j+1]  $\Rightarrow$

~~$\mathcal{O}(n^2)$~~

return most

$\mathcal{O}(n^2)$

$\mathcal{O}(n)$

b) def max\_profit(arr[l...r])

if len(arr) == 1:

    return arr[0][1]

index = int(len(arr)/2)

left = max\_profit(arr[0:index])  $\rightarrow T(n/2)$

right = max\_profit(arr[index:None])  $\rightarrow T(n/2)$

return left + right

$T(n) = 2T(n/2) + 1$

$\downarrow 2T(n/4) + 1 \quad \log_2 n$

$2^{\log_2 n} = n \Rightarrow T(n) = \underline{\underline{\mathcal{O}(n)}}$